Types of roots	**Complementary function**
Roots are real & distinct say $m_1, m_2, \ldots\ldots m_n$	$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \ldots\ldots c_n e^{m_n x}$
Roots are real & repeated say, $m_1 = m_2, \ \& \ m_3, m_4, \ldots\ldots m_n$ **are distinct.**	$y_c = (c_1 x + c_2) e^{m_1 x} + c_3 e^{m_3 x} + \ldots\ldots + c_n e^{m_1 x}$
Roots are complex and distinct , $m_1 = \alpha + i\beta$, $m_2 = \alpha - i\beta$	$y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$
Roots are complex and repeated , $m_1 = \alpha + i\beta,\ m_2 = \alpha - i\beta$, $m_3 = \alpha + i\beta,$ $m_4 = \alpha - i\beta$	$y_c = e^{\alpha x}((c_1 x + c_2)\cos \beta x + (c_3 x + c_4)\sin \beta x)$

Type I	$f(x) = e^{ax}$	$y_p = \dfrac{1}{\phi(D)} e^{ax} = \dfrac{1}{\phi(a)} e^{ax}$ provided $\phi(a) \neq 0$
		If $\phi(a) = 0$ then $y_p = \dfrac{1}{\phi(D)} e^{ax} = \dfrac{x}{\phi'(a)} e^{ax}$ provided $\phi'(a) \neq 0$
		If $\phi'(a) = 0$ then $y_p = \dfrac{1}{\phi(D)} e^{ax} = \dfrac{x^2}{\phi''(a)} e^{ax}$ provided $\phi''(a) \neq 0$
	$f(x) = a^x$	$y_p = \dfrac{1}{\phi(D)} a^x = \dfrac{1}{\phi(\log a)} a^x$ provided $\phi(\log a) \neq 0$
	$f(x) = k$, where k is constant	$y_p = \dfrac{1}{\phi(D)} k = \dfrac{1}{\phi(0)} k$ provided $\phi(0) \neq 0$
Type II	f(x)= sin(ax+b) or cos(ax+b)	$y_p = \dfrac{1}{\phi(D^2)} \sin(ax+b) = \dfrac{1}{\phi(-a^2)} \sin(ax+b)$ provided $\phi(-a^2) \neq 0$

		If $\phi(-a^2)=0$ then $\frac{1}{\phi(D^2)}\sin(ax+b)=\frac{x}{\phi'(-a^2)}\sin(ax+b)$ provided $\phi'(-a^2)\neq 0$
		If $\phi'(-a^2)=0$ then $\frac{1}{\phi(D^2)}\sin(ax+b)=\frac{x^2}{\phi''(-a^2)}\sin(ax+b)$ provided $\phi''(a)\neq 0$
Type III	$f(x)=x^m$	$y_p=\frac{1}{\phi(D)}x^m=[\phi(D)]^{-1}x^m$ where $\phi(D)$ is any one of the form (1+z) or (1-z) $(1+z)^{-1}=1-z+z^2-z^3+-....$ $(1-z)^{-1}=1+z+z^2+z^3+-....$
Type IV	$f(x)=e^{ax}V$ where V is a function of x	$y_p=\frac{1}{\phi(D)}e^{ax}V=e^{ax}\frac{1}{\phi(D+a)}V$
Type V	$f(x)=xV$ where V is a function of x	$y_p=\frac{1}{\phi(D)}xV=\left[x-\frac{\phi'(D)}{\phi(D)}\right]\frac{1}{\phi(D)}V$ Note: 1. Power of x is one 2. $\frac{1}{\phi(D)}V$ is not a case of failure

Method of variation of parameters

Step 1:Find Complimentary function of the given LDE form

$$C.F=c_1y_1+c_2y_2$$

Step 2:Find Wronkian,W= $\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$

Step 3: Find u&v, $u = \int \frac{-y_2 f(x)}{w} dx \qquad v = \int \frac{y_1 f(x)}{w} dx$

Step 4:Find $P.I = uy_1 + vy_2$

Step 5:Write complete solution y=C.F+P.I

Method to find Particular Integral using General Method

Given linear differential equation $\phi(D)y = f(x)$

$$y_p = \frac{1}{\phi(D)} f(x)$$

Note:

1. $\frac{1}{D-a} f(x) = e^{ax} \int e^{-ax} f(x) dx$
2. $\frac{1}{D+a} f(x) = e^{-ax} \int e^{ax} f(x) dx$
3. $\frac{1}{D} f(x) = \int f(x) dx$
4. $\frac{1}{(D+m_1)(D+m_2)} f(x) = \frac{1}{(D+m_1)}\left(\frac{1}{(D+m_2)} f(x)\right) = \frac{1}{(D+m_1)}\left(e^{-m_2 x} \int e^{m_2 x} f(x) dx\right)$

$$= e^{-m_1 x} \int e^{m_1 x} \left[e^{-m_2 x} \int e^{m_2 x} f(x) dx\right] dx$$

5. Partial fraction method

$$\frac{1}{(D+m_1)(D+m_2)} f(x) = \frac{1}{(m_1 - m_2)}\left[\frac{1}{(D+m_1)}\left(\frac{1}{(D+m_2)} f(x)\right)\right] = \frac{1}{(D+m_1)}\left(e^{-m_2 x} \int e^{m_2 x} f(x) dx\right)$$

Cauchy’s linear differential equation

Step 1: Convert Cauchy's LDE into LDE with constant coefficients by

substituting
$$x = e^z$$
$$x^2 \frac{d^2y}{dx^2} = D(D-1)y, D = \frac{d}{dz}$$
$$x\frac{dy}{dx} = Dy, D = \frac{d}{dz}$$

Step 2:Find C.F & P.I in terms of 'z'

Step 3:Find C.F &P.I in terms of 'x' by resubstituting z=logx

Legendre's linear differential equation

Step: I Reduce the given differential equation into linear differential equation by

substituting (ax+b) = e^z, D ≡ $\frac{d}{dz}$, $(ax+b)^2 \frac{d^2y}{dx^2} = a^2D(D-1)y$, $(ax+b)\frac{dy}{dx} = aDy$.

Step II: Find C.F and P.I in terms of z.

Step III: Find C.F and P.I in terms of x and y by substituting back z = log (ax+b).

Fourier Transform

FORMULAE

1. The Fourier integral representation of f(x) defined in the interval $-\infty < x < \infty$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(u) e^{-i\lambda(u-x)} du d\lambda$$

2. Fourier transform $F(\lambda)$ of function $f(x)$ defined in the interval $-\infty < x < \infty$

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du$$

3. The inverse Fourier transform $f(x)$ defined in $-\infty < x < \infty$ of $F(\lambda)$ is

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\lambda) e^{i\lambda x} d\lambda$$

4. The Fourier cosine integral representation of an even function f(x) defined in the interval $-\infty < x < \infty$ is

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(u) cos\lambda u cos\lambda x du d\lambda$$

5. The Fourier cosine transform F_c (λ) of an even function f(x) defined in the interval $-\infty < x < \infty$ is

$$F_c(\lambda) = \int_0^{\infty} f(u) \cos \lambda u du$$

6. The inverse Fourier cosine transform f(x) of F_c (λ) is

$$f(x) = \frac{2}{\pi} \int_0^{\infty} F_c(\lambda) \cos \lambda x d\lambda$$

The Fourier sine integral representation of an odd function f(x) defined in the interval $-\infty < x < \infty$ is

$$f(x) = \frac{2}{\pi}\int_0^\infty\int_0^\infty f(u) sin\lambda u sin\lambda x du d\lambda$$

7. The Fourier sine transform $F_s(\lambda)$ of an odd function f(x) defined in the interval $-\infty < x < \infty$ is

$$F_s(\lambda) = \int_0^\infty f(u)\sin\lambda u du$$

8. The inverse Fourier sine transform f(x) of $F_s(\lambda)$ is $f(x) = \frac{2}{\pi}\int_0^\infty F_s(\lambda) sin\lambda x d\lambda$

Useful Formulae

1. $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

2. $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

3. $\int_0^\infty \frac{\sin ax}{x}dx = \frac{\pi}{2}$, a is positive

$= -\frac{\pi}{2}$, if a is negative

$$Z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k) z^{-k} = \sum_{k=0}^{\infty} 2^k z^{-k} = \sum_{k=0}^{\infty} \left(\frac{2}{z}\right)^k = 1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + + \ldots\ldots$$

$$= \frac{1}{1-\frac{2}{z}} = \frac{z}{z-2} \qquad |z| > 2$$

Z-TRANSFORM OF STANDARD SEQUENCES

- If U(k)= $\{0,\ k < 0\ 1,\ k \geq 0$ then Z transform of U(k) I given by $\frac{z}{z-1}, |z| > 1$
- If$\delta(k) = \{1,\ k = 0\ 0,\ k \neq 0$,then Z transform of $\delta(k)$ is given by 1
- If f(k)= $a^k, k \geq 0,$ then Z transform of$\{a^k\}$ is given by $\frac{z}{z-a}, |z| > |a|$
- If f(k)= $a^k, k < 0,$ then Z transform of$\{a^k\}$ is given by $\frac{z}{a-z}, |z| < |a|$
- If f(k)= $\cos\alpha k, k \geq 0,$ then Z transform of $\{cos\propto k\}$ is given by

$$\frac{z(z-\cos\alpha)}{z^2 - 2z\cos\alpha + 1}, |z| < 1$$

- If f(k)= $\sin\alpha k, k \geq 0,$ then Z transform of $\{sin\propto k\}$ is given by

$$\frac{z\sin\alpha}{z^2 - 2z\cos\alpha + 1}, |z| > 1$$

- If f(k)= $\cosh\alpha k, k \geq 0,$ then Z transform of $\{cosh\propto k\}$ is given by

$$\frac{z(z-\cosh\alpha)}{z^2 - 2z\cosh\alpha + 1}, |z| > \max(|e^{\alpha}| or |e^{-\alpha}|)$$

- If f(k)= $\sinh \alpha k, k \geq 0,$ then Z transform of $\{sinh \propto k\}$ is given by

$$\frac{z \sinh \alpha}{z^2 - 2z \cosh \alpha + 1}, |z| < \max(|e^{\alpha}| or |e^{-\alpha}|)$$

PROPERTIES OF Z-TRANSFORM

1.Linearty: If $Z\{f(k)\} = F(Z)$, $Z\{af(k) + bg(k)\} = aF(Z) + bG(Z)$

2.Change of scale : If $Z\{f(k)\} = F(Z)$, $Z\{a^k f(k)\} = F(\frac{Z}{a})$

3. If $Z\{f(k)\} = F(Z)$, $Z\{e^{-ak} f(k)\} = F(e^{a} z)$

4.Shifting property: If $Z\{f(k)\} = F(Z)$,then

$$Z\{f(k+n)\} = z^n F(Z) \quad \& \quad Z\{f(k-n)\} = z^{-n} F(Z)$$

5. If $Z\{f(k)\} = F(Z)$, $Z\{k^n f(k)\} = \left(-z \frac{d}{dz}\right)^n F(z)$,

6. If $Z\{f(k)\} = F(Z)$, $Z\left\{\frac{f(k)}{k}\right\} = \int_z^{\infty} -z^{-1} F(Z)$

7.Initial value theorem: . If $Z\{f(k)\} = F(Z)$,then $F(z)$ =f(0)

8.Final value theorem: . If $Z\{f(k)\} = F(Z)$,$f(k)$ =$(z \ -) F(z)$

9.Convolution Theorem: If $Z\{f(k)\} = F(Z)$,and $Z\{g(k)\} = G(Z)$ then

$Z\{f(k) * g(k)\} = F(Z)G(z)$

Statistics

Type1: Measures of Central Tendency and Dispersion:

Measure of Central Tendency gives central value of data and dispersion gives variation of Data. Combining Central Tendency & Dispersion gives reliability of data.

This gives an idea about acceptance or rejection of sample for a particular survey.

Measure Of Central Tendency	**Formula**
Arithmetic Mean	**For individual Data** **A.M=** $\bar{x} = \frac{\sum X_i}{n}$ **For Frequency Distribution** **A.M=** $\bar{x} = \frac{\sum f_i X_i}{n}$
Variance	**For individual Data** $\sigma^2 = \frac{\sum X_i^2}{n} - (\frac{\sum X_i}{n})^2$ **For Frequency Distribution** $\sigma^2 = \frac{\sum f_i X_i^2}{n} - (\frac{\sum f_i X_i}{n})^2$
Standard Deviation	$\sigma = \sqrt{\sigma^2}$

Coefficient of Variation	**C.V.=** $\frac{\sigma}{\bar{x}} * 100$
Covariance between two variables x and y	$\mathrm{cov}(x,y) = \frac{1}{n}\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)$
Correlation coefficient r between two variables x and y	$r(x,y) = \frac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y}$
Line of regression y on x is	$y-\bar{y} = r\frac{\sigma_y}{\sigma_x}\left(x-\bar{x}\right)$
Line of regression x on y is	$x-\bar{x} = r\frac{\sigma_x}{\sigma_y}\left(y-\bar{y}\right)$
Moments about any number A μ_r'	$\mu_r' = \frac{1}{N}\sum f\left(x-A\right)^r$, $N = \sum f$
Moments about any number mean μ_r	$\mu_r = \frac{1}{N}\sum f\left(x-\bar{x}\right)^r$ $N = \sum f$
Relation between raw moments and central moments	$\mu_0 = 1$ $\mu_1 = 0$ $\mu_2 = \mu_2' - \left(\mu_1'\right)^2$ $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\left(\mu_1'\right)^3$ $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\left(\mu_1'\right)^2 - 3\left(\mu_1'\right)^4$
Coefficient of skewness	$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

Coefficient of Kurtosis	$\beta_2 = \frac{\mu_4}{\mu_2^2}$

Unit IV Probability & Probability Distribution

Probability

- The Probability of an Event is defined as

$$P(\text{event}) = \frac{\text{number of ways event can occur}}{\text{total number of outcomes}}$$

Laws of Probability

- $0 \leq P(A) \leq 1$ for any event A
- $P(\varnothing) = 0$, $P(S) = 1$ where S is sample space
- $P(A') = 1 - P(A)$
- If A and B are disjoint events, then
  - $P(A \cup B) = P(A) + P(B)$
- If A and B are independent events, then
  - $P(A \cap B) = P(A) \times P(B)$
- For <u>any</u> two events A and B,
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Probability Distributions:

1. Binomial Probability Distribution P(n,k,p)

probability of *k* successes in *n* trials where the probability of success <u>on any one trial</u> is *p*,*k* specified outcomes, *n* trials ,*p* probability of the specified outcome in 1 trial

P(X = *x*)=

mean) = n*p **Standard deviation = npq** **(note: q=1-p)**

- **Poisson Probability Distribution**

The Poisson probability distribution provides a good model for the probability distribution of the number of “rare events” that occur randomly in time, distance, or space.

The random variable X is said to follow the Poisson probability distribution if it has the probability function:

$$P(x=r)=\frac{e^{-z}z^{r}}{r},r=0,1,2,3$$

Where P(x) = the probability of x successes , given λ

z = the expected number of successes ; z > 0

The mean and variance of the Poisson probability distribution are:

$$\mu_x = E(X) = z \quad and \quad \sigma_x^2 = E[(X-\mu)^2] = z$$

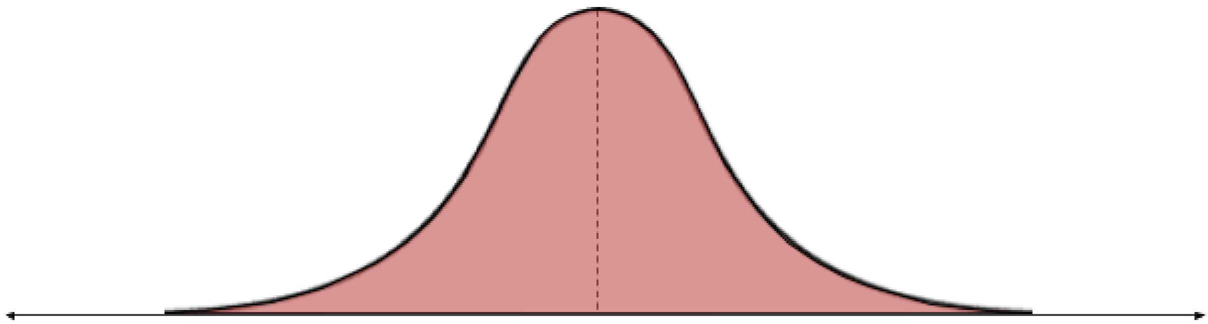
Normal Distribution

Symmetrical, bell-shaped curve Also known as Gaussian distribution

Mathematical formula

$$f(X)=\frac{1}{\sigma\sqrt{2\pi}}(e)^{-\frac{(X-\mu)^2}{2\sigma^2}}$$

The most important probability distribution in statistics is the **normal distribution**.



A normal distribution is a continuous probability distribution for a random variable, x.
The graph of a normal distribution is called the **normal curve**.

Properties of a Normal Distribution

1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and symmetric about the mean.
3. The total area under the curve is equal to one.
4. The normal curve approaches, but never touches the *x*-axis as it extends farther and farther away from the mean.
5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the *inflection points*.

Finding Areas Under the Standard Normal Curve

1. Sketch the standard normal curve and shade the appropriate area under the curve.
2. Find the area by following the directions for each case shown.
    a. To find the area to the *left* of *z*, find the area that corresponds to *z* in the Standard Normal Table.