Thapar Institute of Engineering & Technology, Patiala

School of Mathematics,

End-Semester Examination, December-2019

UMA061: Advanced Numerical Analysis

Time Limit: 03 Hours, Maximum Marks: 100

Instructor(s): Dr. Sapna Sharma

Instructions:

- This question paper contains two printed pages and eight questions. You are expected to answer all the questions.
- Organize your work in a reasonably neat, organized, and coherent way.

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1. Approximate the polynomial $x^3 + 5x^2 + 2x - 1 = 0$ to a quadratic polynomial with minimum error in the interval (3, 4). (10)

2. Explain Birge Vieta method. (10)

3. Using cubic spline find the value of the function $f(x) = \ln(x)$ for $x = 1.8$ from the following data

x	1.0	1.2	1.6	2.0
f(x)	0.0	0.18232	0.4700	0.69315

When values of second derivative *f''(x)* are given as end conditions
When values of first derivatives *f''(x)* are given as end conditions. (10)

4. Define the Initial and Boundary value problem and categorize them. (8)

5. The linear system $x_1 - x_3 = 0.2,\quad -0.5x_1 + x_2 - 0.25x_3 = -1.425,\ x_1 - 0.5x_2 + x_3 = 2$, has (0.9,-0.8,0.7)' the solution.
   (a) Is the coefficient matrix is diagonally dominant?
   (b) Perform two iterations of Gauss Seidel method using initial approximation (0, 0, 0)'. (12)

6. For any $x^0 \in R^n$, the sequence $\{x\}_{k=0}^{k=\infty}$ define by $x^k = T\,x^{k-1} + c$ for each $k \geq 1$, converges to the unique solution $x = Tx + c$ if only if $\rho(T) < 1$. (10)

7. Solve the differential equation, $xy'' + (x-1)y' - y = 0,\qquad 0 \leq x \leq .75$
   Subject to conditions, $y'(0) = 1, y(0.75) = 1.3125$.
   Replace the derivative boundary condition by second order formula, taking step size 0.25. (20)

P.T.O.

8. All electrical components, especially off-the-shelf components do not match their nominal value. Variations in materials and manufacturing as well as operating conditions can affect their value. Suppose a circuit is designed such that it requires a specific component value, how confident can we be that the variation in the component value will result in acceptable circuit behavior? To solve this problem a probability density function is needed to be integrated to determine the confidence interval. For an oscillator to have its frequency within 5% of the target of 1 kHz, the likelihood of this happening can then be determined by finding the total area under the normal distribution for the range in question: $(1-\alpha)=\int_{-2.15}^{2.9}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\,dx$

n	1	2	4	8
Trapezoidal Rule	0.11489	0.99637	0.96969	0.97901

(a) Use Richardson's extrapolation formula to find the frequency. Use the 2-segment(h/2) and 4-segment(h/4) Trapezoidal rule results given in Table 1.
(b) Complete the table by Romberg integration. (20)

		1^{st} Order	2^{nd} Order	3^{rd} Order
1-segme	0.11489			
		1.2902		
			0.93884	
2-segment	0.99637			0.98425
4-segment	0.96969			
		0.98212		
8-segment	0.97901			

***********The End***********