Roll Number:____________________

Thapar Institute of Engineering & Technology, Patiala

School of Mathematics

End Semester Examination

office copy

B. E. (Second Year)

Course Code: UMA035

Course Name: Optimization Techniques

Feb 5, 20221

Time: 2 Hours, M. Marks: 50

Name of Faculty: AK, MKS

Note: *Attempt any five questions.*

Q1 (a). Find all the basic and non-basic solutions for the following system of inequalities
$x_1 + x_2 \geq 2, 2x_1 + x_2 \geq 3, x_1 \geq 0, x_2 \geq 0.$ (6)

Q1 (b). Check graphically whether the set $S = \{(x_1, x_2): 0 < x_1^2 + x_2^2 \leq 1\}$ is a convex set or not. (2)

Q1 (c). Check graphically whether the set $S = \{(x_1, x_2): x_1^2 + x_2^2 \geq 1\}$ is a convex set or not. (2)

Q2(a). Solve the following linear programming problem by Simplex method.

Maximize $z = 6x_1 + 5x_2$ subject to $x_1 + x_2 \leq 5, 3x_1 + 2x_2 \leq 12, x_1 \geq 0, x_2 \geq 0.$ (5)

Q2(b). Solve the following system of linear equations by Two-phase method
$4x_1 + 3x_2 = 25, 2x_1 + x_2 = 11, x_1 \geq 0, x_2 \geq 0.$ (5)

Q3(a). On solving the linear programming problem, Maximize $Z = 4x_1 + 3x_2 + 5x_3$ subject to
$x_1 + 2x_2 + 3x_3 + S_1 = 9,\ 2x_1 + 3x_2 + x_3 + S_2 = 12,\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, S_1 \geq 0, S_2 \geq 0$, the following optimal table is obtained (S_1 and S_2 are slack variables).

	x_1	x_2	x_3	S_1	S_2	Solution
$z_j - c_j$	0	$\frac{18}{5}$	0	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{138}{5}$
x_3	0	$\frac{1}{5}$	1	$\frac{2}{5}$	$-\frac{1}{5}$	$\frac{6}{5}$
x_1	1	$\frac{7}{5}$	0	$-\frac{1}{5}$	$\frac{3}{5}$	$\frac{27}{5}$

If the right hand side value of the first constraint 9 is replaced with 40. Then, using the post optimality analysis, find the optimal solution of the modified problem. (5)

Q3 (b) A company needs to assign five jobs to five workers. The following table summarizes the cost of the assignments. Determine the optimal assignment using the Hungerian method. (5)

Worker \ Job	1	2	3	4	5
1	10	12	15	12	8
2	7	16	14	14	11
3	13	14	7	9	9
4	12	10	11	13	10
5	8	13	15	11	15

P.T.O

Q4 (a) Use Vogel's approximation method to find a basic feasible solution of the following minimization transportation problem. Hence, find an optimal solution by $(u - v)$ method.

Destinations

Sources		D_1	D_2	D_3	D_4	Availability
	S_1	19	14	23	11	11
	S_2	15	16	12	21	13
	S_3	30	25	16	39	19
	Demand	6	10	12	15	

(7)

Q4(b). Solve the following integer linear programming problem graphically using Branch and Bound method.

Minimize $z = 4x_1 + 9x_2$ subject to $x_1 + x_2 \geq 7, 2x_1 + 3x_2 \geq 17, x_1 \leq 8, x_1 \geq 0, x_2 \geq 0$. (3)

Q5 Use the critical path method to find the minimum normal time and the corresponding normal cost for completing the following project. Also, find the minimum cost for completing the project in 24 weeks (Draw separate network for each iteration and find the critical path(s) for network of each iteration using the earliest starting time and latest completion time).

Activity	Normal time (weeks)	Normal cost (Rs.)	Crash time (weeks)	Crash cost (Rs.)
1-2	13	700	9	900
1-3	5	400	4	460
1-4	7	600	4	810
2-5	12	800	11	865
3-2	6	900	4	1130
3-4	5	1000	3	1180
4-5	9	1500	6	1800

(10)

Q6 (a) Solve the following non-linear programming problem by using KKT conditions.

Maximize $z = 2x_1 - x_1^2 + 3x_2 - 2x_2^2$ subject to $x_1 + x_2 \leq 3, x_1 \geq 0, x_2 \geq 0$. (7)

Q6 (b)Find the extreme point for the function $x_1 + x_1x_2 + 2x_2 + 3x_3 - x_1^2 - 2x_2^2 - x_3^2$. Also, justify that the obtained extreme point is a maximum point or a minimum point. (3)

Q7(a) Perform two iterations of Steepest descent method for the non-linear programming problem Minimize $z = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ with the initial approximation (0,0). (7)

Q7(b) Use the graphical method to find all efficient solutions of the following multi-objective linear programming problem

Maximize $(2x_1 + 3x_2)$

Maximize $(3x_1 + 2x_2)$

subject to

$x_1 + x_2 \leq 3, x_1 \geq 0, x_2 \geq 0$. (3)