Roll Number: _____

Thapar Institute of Engineering and Technology, Patiala
School of Mathematics

B.E. (Generic Elective)	Course Code: UMA066/UMAQ69
	Course Name: Graph Theory and Applications
Date: 10-12-2022	Day/Duration: Saturday/3 Hours,
M. Marks: 45 marks	Name of Faculty: Dr. Mamta Gulati

**Note: Attempt all 5 questions. Calculator without graphing mode and alphanumeric memory is permitted.**

1. (a) Are the simple graphs with following adjacency matrix isomorphic?

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

   (b) Out degree's of vertices of a directed graph are $1, 5, 5, 2, 0$. Find the number of edges in the graph. If the ordered sequence for indegree of the same graph is $6, 1, 2, 2, 0$, identify if the graph has a pendant vertex or an isolated vertex?

   (c) Can you construct a simple graph which does not have a Hamiltonian circuit such that degree of every vertex is atleast $n/2$, where $n \geq 3$ is the number of vertices?

   (d) Show that a graph is disconnected iff its vertex set V can be partioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and the other in subset V_2.

   [3 + 2 + 2 + 2 marks]

2. (a) Using Homeomorphic graphs, identify if the following graph is planar or not:
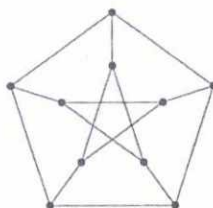
   (b) Using Boolean algrebra, find the chromatic partition and hence the chromatic number of the following graph:

   [4 + 5 marks]

3. (a) Show that the length of the shortest path between vertices v and u in a connected simple graph equals the level number of u in the breadth-first spanning tree of the graph with the root as v.
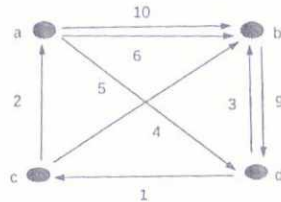
(b) Show that every tree has either one or two centers.

(c) For a graph G and its geometric dual G_*, what is the relation between rank of G and nullity of G_*. Justify your answer.

[3 + 4 + 2 marks]

4. (a) Using Floyd-Warshall's algorithm, find the shortest distance between all the pairs of vertices in the following directed graph,

(b) Give proof or a counter-example for each of the statement below:

   i. Every k-connected graph has edge connectivity of k.

   ii. Graph with edge connectivity 1 is always separable.

[4 + 5 marks]

5. (a) Show that for a connected bipartite (not necessarily complete) planar simple graph having e-edges and v-vertices, $e \leq 2v - 4$ if $v \geq 3$.

(b) Find the number of internal vertices in a complete rooted binary tree with n-vertices.

(c) What is the radius and diameter of a complete bipartite graph $K_{m,n}$.

[3 + 3 + 3 marks]

—————————End of Question Paper—————————
—————————Note: Answer sheets will be shown on 16-12-2022—————————
—————————at 3:00 PM in room number G253A—————————