

Resolution Exercise Solutions

2. Consider the following axioms:

- Every child loves Santa.
 $\forall x (CHILD(x) \rightarrow LOVES(x, Santa))$
- Everyone who loves Santa loves any reindeer.
 $\forall x (LOVES(x, Santa) \rightarrow \forall y (REINDEER(y) \rightarrow LOVES(x, y)))$
- Rudolph is a reindeer, and Rudolph has a red nose.
 $REINDEER(Rudolph) \wedge REDNOSE(Rudolph)$
- Anything which has a red nose is weird or is a clown.
 $\forall x (REDNOSE(x) \rightarrow WEIRD(x) \vee CLOWN(x))$
- No reindeer is a clown.
 $\neg \exists x (REINDEER(x) \wedge CLOWN(x))$
- Scrooge does not love anything which is weird.
 $\forall x (WEIRD(x) \rightarrow \neg LOVES(Scrooge, x))$
- (Conclusion) Scrooge is not a child.
 $\neg CHILD(Scrooge)$

3. Consider the following axioms:

- Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 $\forall x (BUY(x) \rightarrow \exists y (OWNS(x, y) \wedge (RABBIT(y) \vee GROCERY(y))))$
- Every dog chases some rabbit.
 $\forall x (DOG(x) \rightarrow \exists y (RABBIT(y) \wedge CHASE(x, y)))$
- Mary buys carrots by the bushel.
 $BUY(Mary)$
- Anyone who owns a rabbit hates anything that chases any rabbit.
 $\forall x \forall y (OWNS(x, y) \wedge RABBIT(y) \rightarrow \forall z \forall w (RABBIT(w) \wedge CHASE(z, w) \rightarrow HATES(x, z)))$
- John owns a dog.
 $\exists x (DOG(x) \wedge OWNS(John, x))$
- Someone who hates something owned by another person will not date that person.
 $\forall x \forall y \forall z (OWNS(y, z) \wedge HATES(x, z) \rightarrow \neg DATE(x, y))$
- (Conclusion) If Mary does not own a grocery store, she will not date John.
 $((\neg \exists x (GROCERY(x) \wedge OWN(Mary, x))) \rightarrow \neg DATE(Mary, John))$

4. Consider the following axioms:

- Every Austinite who is not conservative loves some armadillo.
 $\forall x (AUSTINITE(x) \wedge \neg CONSERVATIVE(x) \rightarrow \exists y (ARMADILLO(y) \wedge LOVES(x, y)))$
- Anyone who wears maroon-and-white shirts is an Aggie.
 $\forall x (WEARS(x) \rightarrow AGGIE(x))$
- Every Aggie loves every dog.
 $\forall x (AGGIE(x) \rightarrow \forall y (DOG(y) \rightarrow LOVES(x, y)))$
- Nobody who loves every dog loves any armadillo.
 $\neg \exists x ((\forall y (DOG(y) \rightarrow LOVES(x, y))) \wedge \exists z (ARMADILLO(z) \wedge LOVES(x, z)))$
- Clem is an Austinite, and Clem wears maroon-and-white shirts.
 $AUSTINITE(Clem) \wedge WEARS(Clem)$
- (Conclusion) Is there a conservative Austinite?
 $\exists x (AUSTINITE(x) \wedge CONSERVATIVE(x))$

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( ( (not (Austinite x)) (Conservative x) (Armadillo (f x)) )
  ( (not (Austinite x)) (Conservative x) (Loves x (f x)) )
  ( (not (Wears x)) (Aggie x) )
  ( (not (Aggie x)) (not (Dog y)) (Loves x y) )
  ( (Dog (g x)) (not (Armadillo z)) (not (Loves x z)) )
  ( (not (Loves x (g x))) (not (Armadillo z)) (not (Loves x z)) )
  ( (Austinite (Clem)) )
  ( (Wears (Clem)) )
  ( (not (Conservative x)) (not (Austinite x)) ) )
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5. Consider the following axioms:

- Anyone whom Mary loves is a football star.
 $\forall x (LOVES(Mary, x) \rightarrow STAR(x))$
- Any student who does not pass does not play.
 $\forall x (STUDENT(x) \wedge \neg PASS(x) \rightarrow \neg PLAY(x))$
- John is a student.
 $STUDENT(John)$
- Any student who does not study does not pass.
 $\forall x (STUDENT(x) \wedge \neg STUDY(x) \rightarrow \neg PASS(x))$

5. Anyone who does not play is not a football star.

$$\forall x (\neg PLAY(x) \rightarrow \neg STAR(x))$$

6. (Conclusion) If John does not study, then Mary does not love John.

$$\neg STUDY(John) \rightarrow \neg LOVES(Mary, John)$$

6. Consider the following axioms:

1. Every coyote chases some roadrunner.

$$\forall x (COYOTE(x) \rightarrow \exists y (RR(y) \wedge CHASE(x, y)))$$

2. Every roadrunner who says "beep-beep" is smart.

$$\forall x (RR(x) \wedge BEEP(x) \rightarrow SMART(x))$$

3. No coyote catches any smart roadrunner.

$$\neg \exists x \exists y (COYOTE(x) \wedge RR(y) \wedge SMART(y) \wedge CATCH(x, y))$$

4. Any coyote who chases some roadrunner but does not catch it is frustrated.

$$\forall x (COYOTE(x) \wedge \exists y (RR(y) \wedge CHASE(x, y) \wedge \neg CATCH(x, y)) \rightarrow FRUSTRATED(x))$$

5. (Conclusion) If all roadrunners say "beep-beep", then all coyotes are frustrated.

$$(\forall x (RR(x) \rightarrow BEEP(x)) \rightarrow (\forall y (COYOTE(y) \rightarrow FRUSTRATED(y))))$$

$$\begin{aligned} & ((\neg (COYOTE\ x)) \ (RR\ (f\ x))) \\ & (\neg (COYOTE\ x)) \ (CHASE\ x\ (f\ x))) \\ & (\neg (RR\ x)) \ (\neg (BEEP\ x)) \ (SMART\ x)) \\ & (\neg (COYOTE\ x)) \ (\neg (RR\ y)) \ (\neg (SMART\ y)) \ (\neg (CATCH\ x\ y))) \\ & (\neg (COYOTE\ x)) \ (\neg (RR\ y)) \ (\neg (CHASE\ x\ y)) \ (CATCH\ x\ y) \\ & (FRUSTRATED\ x)) \\ & (\neg (RR\ x)) \ (BEEP\ x)) \\ & (COYOTE\ (a))) \\ & (\neg (FRUSTRATED\ (a)))) \end{aligned}$$

7. Consider the following axioms:

1. Anyone who rides any Harley is a rough character.

$$\forall x ((\exists y (HARLEY(y) \wedge RIDES(x, y))) \rightarrow ROUGH(x))$$

2. Every biker rides [something that is] either a Harley or a BMW.

$$\forall x (BIKER(x) \rightarrow \exists y ((HARLEY(y) \vee BMW(y)) \wedge RIDES(x, y)))$$

3. Anyone who rides any BMW is a yuppie.

$$\forall x \forall y (RIDES(x, y) \wedge BMW(y) \rightarrow YUPPIE(x))$$

4. Every yuppie is a lawyer.

$$\forall x (YUPPIE(x) \rightarrow LAWYER(x))$$

5. Any nice girl does not date anyone who is a rough character.

$$\forall x \forall y (NICE(x) \wedge ROUGH(y) \rightarrow \neg DATE(x, y))$$

6. Mary is a nice girl, and John is a biker.

$$NICE(Mary) \wedge BIKER(John)$$

7. (Conclusion) If John is not a lawyer, then Mary does not date John.

$$\neg LAWYER(John) \rightarrow \neg DATE(Mary, John)$$

8. Consider the following axioms:

1. Every child loves anyone who gives the child any present.

$$\forall x \forall y \forall z (CHILD(x) \wedge PRESENT(y) \wedge GIVE(z, y, x) \rightarrow LOVES(x, z))$$

2. Every child will be given some present by Santa if Santa can travel on Christmas eve.

$$TRAVEL(Santa, Christmas) \rightarrow \forall x (CHILD(x) \rightarrow \exists y (PRESENT(y) \wedge GIVE(Santa, y, x)))$$

3. It is foggy on Christmas eve.

$$FOGGY(Christmas)$$

4. Anytime it is foggy, anyone can travel if he has some source of light.

$$\forall x \forall t (FOGGY(t) \rightarrow (\exists y (LIGHT(y) \wedge HAS(x, y)) \rightarrow TRAVEL(x, t)))$$

5. Any reindeer with a red nose is a source of light.

$$\forall x (RNR(x) \rightarrow LIGHT(x))$$

6. (Conclusion) If Santa has some reindeer with a red nose, then every child loves Santa.

$$(\exists x (RNR(x) \wedge HAS(Santa, x))) \rightarrow \forall y (CHILD(y) \rightarrow LOVES(y, Santa))$$

9. Consider the following axioms:

1. Every investor bought [something that is] stocks or bonds.

$$\forall x (INVESTOR(x) \rightarrow \exists y ((STOCK(y) \vee BOND(y)) \wedge BUY(x, y)))$$

2. If the Dow-Jones Average crashes, then all stocks that are not gold stocks fall.

$$DJCRASH \rightarrow \forall x ((STOCK(x) \wedge \neg GOLD(x)) \rightarrow FALL(x))$$

3. If the T-Bill interest rate rises, then all bonds fall.

$$TBRISE \rightarrow \forall x (BOND(x) \rightarrow FALL(x))$$

4. Every investor who bought something that falls is not happy.

$$\forall x \forall y (INVESTOR(x) \wedge BUY(x, y) \wedge FALL(y) \rightarrow \neg HAPPY(x))$$

5. (Conclusion) If the Dow-Jones Average crashes and the T-Bill interest rate rises, then any investor who is happy bought some gold stock.

$$(DJCRASH \wedge TBRISE) \rightarrow \forall x (INVESTOR(x) \wedge HAPPY(x) \rightarrow \exists y (GOLD(y) \wedge BUY(x, y)))$$

10. Consider the following axioms:

1. Every child loves every candy.
 $\forall x \forall y (CHILD(x) \wedge CANDY(y) \rightarrow LOVES(x,y))$
2. Anyone who loves some candy is not a nutrition fanatic.
 $\forall x ((\exists y (CANDY(y) \wedge LOVES(x,y))) \rightarrow \neg FANATIC(x))$
3. Anyone who eats any pumpkin is a nutrition fanatic.
 $\forall x ((\exists y (PUMPKIN(y) \wedge EAT(x,y))) \rightarrow FANATIC(x))$
4. Anyone who buys any pumpkin either carves it or eats it.
 $\forall x \forall y (PUMPKIN(y) \wedge BUY(x,y) \rightarrow CARVE(x,y) \vee EAT(x,y))$
5. John buys a pumpkin.
 $\exists x (PUMPKIN(x) \wedge BUY(John,x))$
6. Lifesavers is a candy.
 $CANDY(Lifesavers)$
7. (Conclusion) If John is a child, then John carves some pumpkin.
 $CHILD(John) \rightarrow \exists x (PUMPKIN(x) \wedge CARVE(John,x))$

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