## Thapar Institute of Engineering and Technology, Patiala

School of Mathematics End Semester Examination

B.E.(sem IV &VI) Course Code: UMA035/UMA031 Course Name: Optimization Techniques Time: 2 Hours, M.M. 35 Name of Faculty: MKS,AK,MKR,SJK,NK,PN,MG, JPR, BHU,SPNP, TV

**Note:** Attempt all five questions. Calculator without graphing mode and alphanumeric memory is permitted.

**1. (a)** Consider a transportation problem with 3 plants and 4 warehouses. The plant capacities are 10, 25 and 20. The warehouse demands are 25, 10, 15 and 5. The unit cost of shipping is given by the following table:

Plants	Warehouse						
	I	II	III	IV	Supply		
A	5	10	4	5	10		
В	6	8	7	2	25		
C	4	2	5	7	20		
Demand	25	10	15	5			

The solution of this transportation problem is  $X_{13}=10$ ,  $X_{21}=20$ ,  $X_{24}=5$ ,  $X_{31}=5$ ,  $X_{32}=10$ ,  $X_{33}=5$  and the remaining variables are zero. Answer the following questions:

(i) Is this solution degenerate basic feasible solution?

(ii) Is this solution optimal basic feasible solution?

(iii) Does this problem have more than one optimal solution? If yes find alternate solution.

(4.5 M

**(b)** Amazon supply chain wishes to send items from a particular warehouse (P<sub>i</sub>) to another warehouse (Q<sub>i</sub>) to maximize its profit on certain routes. Following table gives the per unit profit on each route. Find the intial basic feasible solution using least cost method and the corresponding profit.

Wareh ouse	$Q_1$	$Q_2$	$Q_3$	Demand
P <sub>1</sub>	9	10	7	15
P <sub>2</sub>	6	1	5	20
P <sub>3</sub>	4	3	8	10
Supply	20	10	15	

(2.5 M)

**2.** (a) Determine the stationary points for the function and classify them for the function  $f(x,y)=x^2+y^2-xy$ . Also check convexity of the function. (3 M)

(b) Consider the following Assignment problem (AP), representing the cost of performing the jobs  $J_i$  by person  $P_i$ :

	$J_1$	$J_2$	$J_3$
$P_1$	20	27	30
P <sub>2</sub>	10	18	16
P <sub>3</sub>	14	16	12

(i) Express the above AP as an linear programming problem.

(ii) Solve the given AP for optimal assignment at minimum cost by using Hungarian method.

(4 M)

3. (a) Draw the network of the following project.

Activity	A	В	C	D	E	F	G	Н	I	J	K
Predecessor(s)				A	A,B	С	C	E.F	E.F	D.H	IG

(2 M)

**(b)** The following table gives the normal duration, normal cost, crash duration and crash cost for different activities of a project.

Activity	Normal duration(days)	Normal cost (In Rs.)	Crash duration(days)	Crash Cost (In Rs.)
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	820
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1000	9	1200

- (i) Determine the critical path, normal duration, and normal cost of the project.
- (ii) Determine the optimal cost for completing the project if the normal duration is to be reduced by one day.

(5 M)

**4. (a)** Use Lagrange multipliers method to solve the following non-linear programming problem (NLPP).

Maximize 
$$Z = -(x_1 - x_2)^2 + 5x_1 + x_2$$
, Subject to  $x_1 + x_2 = 4$ ;  $x_1, x_2 \ge 0$ .

(3 M)

(b) Solve the following NLPP using KKT conditions.

Minimize 
$$Z = -10 x_1 - 4 x_2 + 2 x_1^2 + x_2^2$$
, Subject to  $2 x_1 + x_2 \le 5$ ;  $x_1, x_2 \ge 0$ 

(4 M)

5. **(a)** Consider the following multiobjective linear programming problem (MLPP).

Minimize 
$$x_1 + 2x_2$$
; Maximize  $-2x_1 + x_2$ ; Minimize  $2x_1 - x_2$ ,

Subject to 
$$2x_1 + 3x_2 \le 6$$
;  $x_1, x_2 \ge 0$ 

(3 M)

**(b)** Use Fibonacci Search Method to find the minimum of  $f(x) = x^2 - 3x$ ,  $0 \le x \le 2$  within the interval of uncertainty  $0.25 L_0$  where  $L_0$  is the length of an initial interval of uncertainty.

(4 M)

## **End of Question Paper**