Thapar Institute of Engineering & Technology, Patiala

School of Mathematics,

End-Semester Examination, December-2019

UMA061: Advanced Numerical Analysis

Time Limit: 03 Hours, Maximum Marks: 100

Instructor(s): Dr. Sapna Sharma

Instructions:

- This question paper contains two printed pages and eight questions. You are expected to answer all the questions.
- Organize your work in a reasonably neat, organized, and coherent way.
- 1. Approximate the polynomial $x^3 + 5x^2 + 2x 1 = 0$ to a quadratic polynomial with minimum error in the interval (3, 4). (10)
- 2. Explain Birge Vieta method. (10)
- 3. Using cubic spline find the value of the function $f(x) = \ln(x)$ for x = 1.8 from the following data

X	1.0	1.2	1.6	2.0
f(x)	0.0	0.18232	0.4700	0.69315

When values of second derivative f''(x) are given as end conditions When values of first derivatives f''(x) are given as end conditions. (10)

- 4. Define the Initial and Boundary value problem and categorize them. (8)
- 5. The linear system $x_1 x_3 = 0.2$, $-0.5x_1 + x_2 0.25x_3 = -1.425$, $x_1 0.5x_2 + x_3 = 2$, has (0.9, -0.8, 0.7)' the solution.
 - (a) Is the coefficient matrix is diagonally dominant?
 - (b) Perform two iterations of Gauss Seidel method using initial approximation (0, 0, 0)'. (12)
- 6. For any $x^0 \in \mathbb{R}^n$, the sequence $\{x\}_{k=0}^{k=\infty}$ define by $x^k = T \ x^{k-1} + c$ for each $k \ge 1$, converges to the unique solution x = Tx + c if only if $\rho(T) < 1$.
- 7. Solve the differential equation, xy'' + (x-1)y' y = 0, $0 \le x \le .75$ Subject to conditions, y'(0) = 1, y(0.75) = 1.3125. Replace the derivative boundary condition by second order formula, taking step size 0.25.

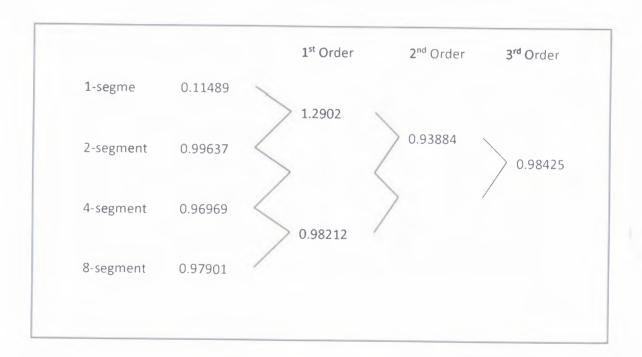
8. All electrical components, especially off-the-shelf components do not match their nominal value. Variations in materials and manufacturing as well as operating conditions can affect their value. Suppose a circuit is designed such that it requires a specific component value, how confident can we be that the variation in the component value will result in acceptable circuit behavior? To solve this problem a probability density function is needed to be integrated to determine the confidence interval. For an oscillator to have its frequency within 5% of the target of 1 kHz, the likelihood of this happening can then be determined by finding the total area under the normal

distribution for the range in question: $(1-\alpha) = \int_{-2.15}^{2.9} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

n	1	2	4	8
Trapezoidal Rule	0.11489	0.99637	0.96969	0.97901

(a) Use Richardson's extrapolation formula to find the frequency. Use the 2-segment(h/2) and 4-segment(h/4) Trapezoidal rule results given in Table 1.

(b) Complete the table by Romberg integration. (20)



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