In this project, critical review of Bayesian Online Changepoint Detection algorithm (BOCD) developed by Adams and MacKay[1] is done and further the algorithm is extended by integrating with Recursive Least Squares(RLS) algorithm.
BOCD algorithm uses recursive Bayesian estimation to detect changes in streaming data in an online fashion.

Problem 1

1. Perform a critical review of the paper by Adams and MacKay clearly highlighting its usefulness, limitations and shortcomings.

2. Implement the algorithm described in the paper for the well-drilling nuclear magnetic response data. The data (transformed for ease of implementation) is available in mat file names as `NMRMatlog.m`
A helpful resource to understand the paper can be found in github repository (blog resource)
This process can be assumed to be piece-wise Gaussian and has multiple changepoints in mean. You can assume the standard deviation to be constant across changepoints but unknown. Historic evidence suggests that the changepoint occurrence in this process is memoryless and can thus be modelled as a geometric distribution with the timescale, $\lambda_{CP} = 250$
Since both the mean(μ) and variance(σ^2) are unknown, use a normal-gamma prior μ and $\frac{1}{\sigma^2}$ (also called precision or λ). This prior can be constructed as

$$
\begin{aligned}
p(\mu, \lambda, \mu_0, k, , \beta) &= p(\mu|\lambda; \mu_0, k) \times p(\lambda; \alpha, \beta) \\
\text{where}, p(\mu|\lambda; \mu_0, k) &= \mathcal{N}(\mu_0, \frac{1}{k\lambda}) \\
p(\lambda; \alpha, \beta) &= \text{Gamme}(\alpha, \beta)
\end{aligned}
$$

where μ_0, k, α and β, are the hyperparameters of the prior. Since, the normal-gamma prior is conjugate for a gaussian likelihood, the posterior distribution for μ and λ have the same family as the prior. Thus, you can use the above equations to derive recursive Bayesian updates for the hyperparameters[2]. You can start with . $\mu_0 = 1.15, k = 0.01, \alpha = 20$ and $\beta = 2$
Report the run length posterior in the format given in the paper. Note that your run length posterior will not match with the plot in the paper as the priors and the data have been modified.

Note: We define the Gaussian and Gamma pdfs as follows (use only this definition in the implementation to ensure closed form expressions):

$$
\begin{aligned}
\mathcal{N}(\mu, \sigma^2) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
\text{Gamma}(\alpha, \beta) &= \frac{\beta^\alpha}{\Gamma(\alpha)} x^{(\alpha-1)} e^{(\beta x)}
\end{aligned}
$$

(Hint: If you are facing over ow or under ow errors try to transform your data meaningfully to fix it.)

Problem 2

1. In this question, you are going to extend the algorithm by integrating it with the Recursive Least Squares (RLS) algorithm.

   A small historic sample of a mystery time series signal that can help you predict the future accurately is provided in `historic.mat`. Use this for identifying an AR model of the appropriate order for the signal (using the LS method) and to build an initial model.

   At a later time, you start to observe the signal one data point at a time (streaming) as given in `new.mat`. Keep refining your parameter estimates as something interesting is going to happen! At some instant after 200 time steps, the series is known to undergo a change because the mystery channel that transmitted the signal corroded increasing the variance of the signal's innovations (endogenous white-noise driving force). Use your knowledge of RLS and your newfound knowledge of the Bayesian Online Changepoint Detection (BOCD) algorithm to detect the time of change and report it.

   Use the following prior parameters for the mean and precision (inverse of variance) of the innovations: Normal-Gamma prior with $\mu_0 = 0, k = 1, \alpha = 10$ and $\beta = 1$. Use a memoryless changepoint prior with $\lambda_{CP} = 50$.
   (Hint: Start the BOCD algorithm after 200 time steps have passed and stop it as soon as you detect the changepoint for computational ease.)

2. Upon the successful detection of the changepoint, update the model (including pa- rameters and variance of driving white-noise) and report the same.

References

1. R. P. Adams and D. J. C. MacKay, "Bayesian Online Changepoint Detection," arXiv:0710.3742 [stat], Oct. 2007. arXiv: 0710.3742.

2. K. P. Murphy, "Conjugate bayesian analysis of the gaussian distribution," tech. rep., 2007.