Isentropic Property relations

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Nomenclature

$\dot{m}$	=	Mass flow rate
h	=	Static enthalpy
h_0	=	Stagnation enthalpy
T	=	Static temperature
T_0	=	Stagnation temperature
P	=	Static Pressure
P_0	=	Stagnation Pressure
D	=	Static density
D_0	=	Stagnation density
c_p	=	Specific heat at constant pressure
γ	=	Specific heat ratio
M	=	Mach Number
T^*	=	Critical temperature
P^*	=	Critical pressure
D^*	=	Critical density
A	=	Area at any location of the CD nozzle
A^*	=	Area at any location of the CD nozzle

Isentropic Relations

THIS document gives vivid information about the necessary equations on various static to stagnation property ratios and static to critical properties of isotropic Flow for a range of Mach numbers. I am directly referring to the section 2.5 from [1] Equation Numbers 2.47 2.48,2.49,2.50.

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

The above equation is the relation between the state properties of two points in flow. It is derived from the Equation of State. And the following relations are obtained from it using the Stagnation Ethalpy.

$$h_0 = h + \frac{V^2}{2} \quad (h = c_p T)$$

A. Total property relations

$$\frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \tag{2}$$

$$\frac{P}{P_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-\gamma}{\gamma-1}} \tag{3}$$

$$\frac{D}{D_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-1}{\gamma-1}} \tag{4}$$

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B. Critical property relations

Put M = 1 in the total property relations.

$$\frac{T}{T^*} = \left(\frac{1+\gamma}{2}\right)^{-1} \tag{5}$$

$$\frac{P}{P^*} = \left(\frac{1+\gamma}{2}\right)^{\frac{-\gamma}{\gamma-1}} \tag{6}$$

$$\frac{D}{D^*} = \left(\frac{1+\gamma}{2}\right)^{\frac{-1}{\gamma-1}} \tag{7}$$

References

[1] Rathakrishnan, E., *Gas Dynamics*, 4th ed., PHI, New Delhi, 2012, Chaps. 1,2,3,4.