# Design of a PI controller for an RRC Circuit

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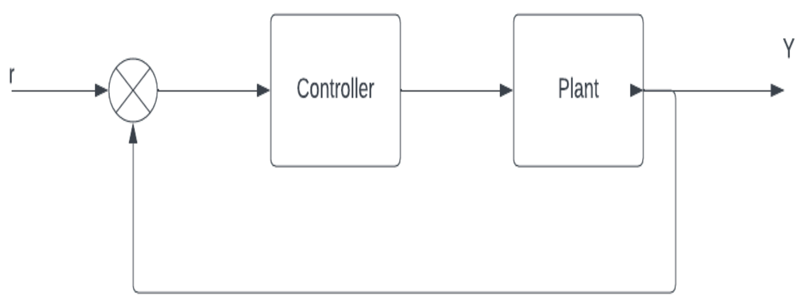
**Abstract**

This report describes the process of making an analog PI controller for an RRC circuit using op amps and also comparing the output of this analog version with that of a simulated model in Matlab.

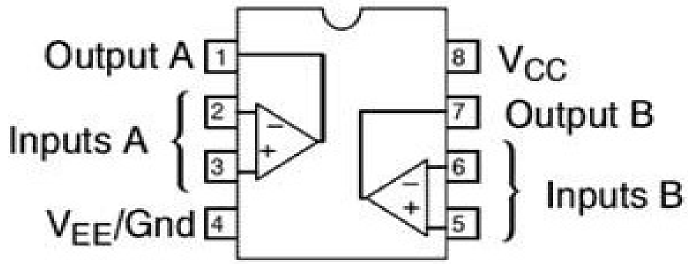
## 1 Introduction

Our goal in this project is to develop a controller for the RRC circuit, which is our plant. A controller can modify the characteristics of the system such as the rise time, the overshoot, and also the steady state error. I begin by developing and testing the proportional controller and then move on to build the Proportional Integral controller for the plant.




This circuit was built using three LM358 op amps two 3.3µF capacitors, two 5.1KΩ resistors for the RRC circuit, nine 10KΩ resistors, one 22KΩ resistor, one 47KΩ resistor, one 100KΩ resistor. The 22k, 47k, 100kΩ resistors were used to change the values of the gain for the proportional controller.

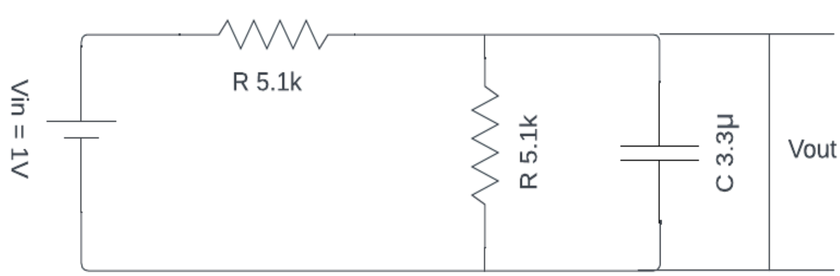
The LM358 op-amps were powered using +15V/-15V, the pin-out diagram for these op amps is given below:




Each LM358 op-amp consists of two op-amps with a supply range of 3V to 32V

The output we are trying to control in the plant is the voltage across the capacitor for a given reference input. The reference input that we would give would be a step input in the form of a square wave oscillating between 1V and 0V at a frequency of 10Hz, this wave can be generated using a waveform generator. We can measure the input and output against each other using an Oscilloscope, by changing the values of the gain and implementing the PI controller, we can see how the characteristics of the system changes.

# 2 The Plant (RRC Circuit)



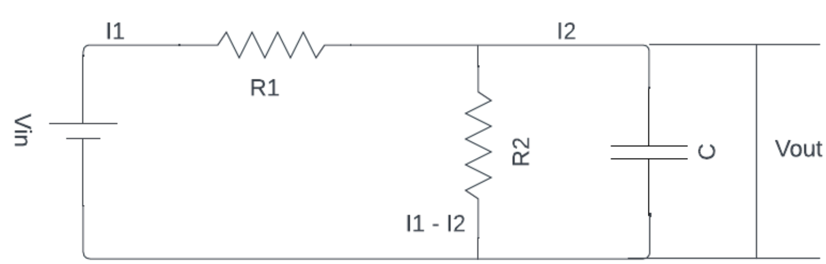

Let us first discuss how the RRC circuit works, when a voltage is applied across this circuit, the capacitor which is connected across the resistor in parallel charges until the voltage across the capacitor becomes equal to the voltage across the resistor in parallel. When the capacitor is fully charged, ideally, no current should flow through the capacitor due to the negative voltage that is developed across the capacitor, as if the capacitor does not exist in the circuit. Let us calculate the voltage across the capacitor at time infinity when a DC voltage of 1V is applied to the circuit. At time infinity, we can ignore the capacitor as the no current would flow through it, hence, the potential difference across the resistor which the capacitor is connected in parallel to using Kirchhoff's law is:

$$V = IR + IR$$
$$I = V/2R$$

The potential difference across the resistor is given by, $IR$, so, the voltage across it would be:

$$\frac{V}{2R} * \frac{R}{1}$$
$$\frac{V}{2}$$

This would be the voltage across the resistor and the capacitor as well, hence, when we input a voltage of 1 volt to the RRC circuit, we would get an output of 0.5 volts across the capacitor. This means that the steady state error would be 50 percent. Let us try to find the transfer function for the system using Kirchhoff's law so that we can simulate the system, to view its response to a




step function.

$$Loop1 : Vin = I1R1 + R2(I1 - I2)$$
$$Loop2 : 0 = \frac{1}{C}\int_0^t I2\,dt - (I1 - I2)R2$$
$$Loop3 : Vin = I1R1 + \frac{1}{C}\int_0^t I2\,dt$$

Applying the Laplace transform on all the equations:

$$Vin(s) = I1(s)[R1 + R2] - R2I2(s)$$
$$0 = -I1(s)R2 + I2(s)[\frac{1}{CS} + R2]$$
$$Vin = I1(s)R1 + \frac{1}{CS}I2(s)$$

We also know that voltage across the capacitior is equal to:

$$Vout = \frac{1}{C}\int_0^t I2\,dt$$

Applying the laplace transform:

$$Vout = \frac{1}{CS}I2(s)$$

also,

$$Vout = VR2$$

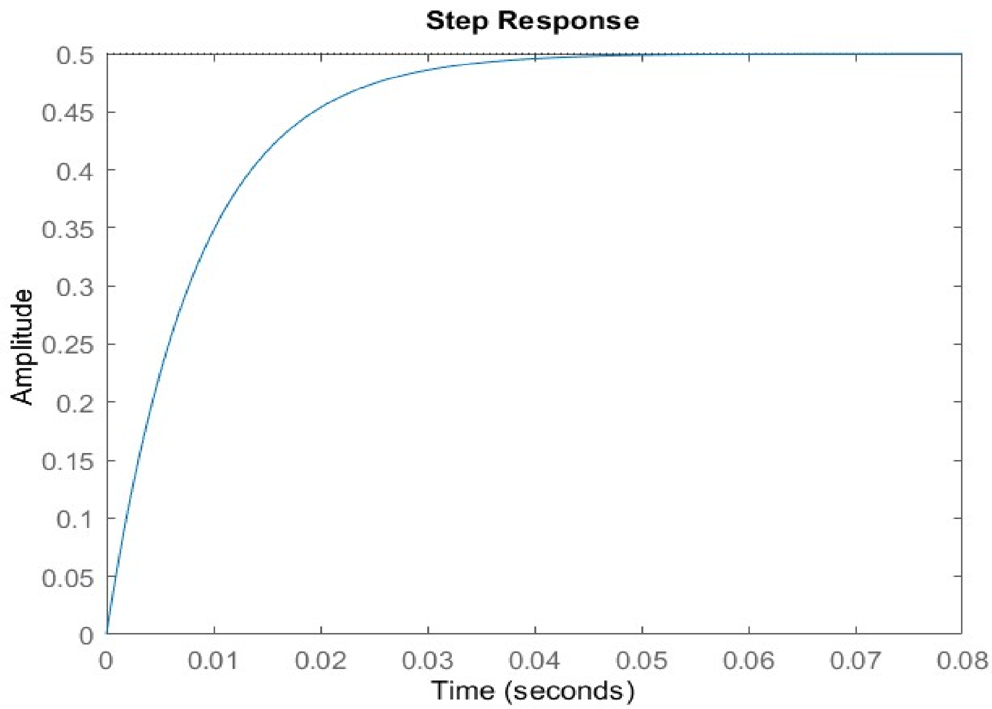
Solving the above equations, we get:

$$\frac{Vout}{Vin} = \frac{R2}{CSR1R2 + R1 + R2}$$

which is the transfer function of the plant, but as the resistances are equal, the transfer function becomes:

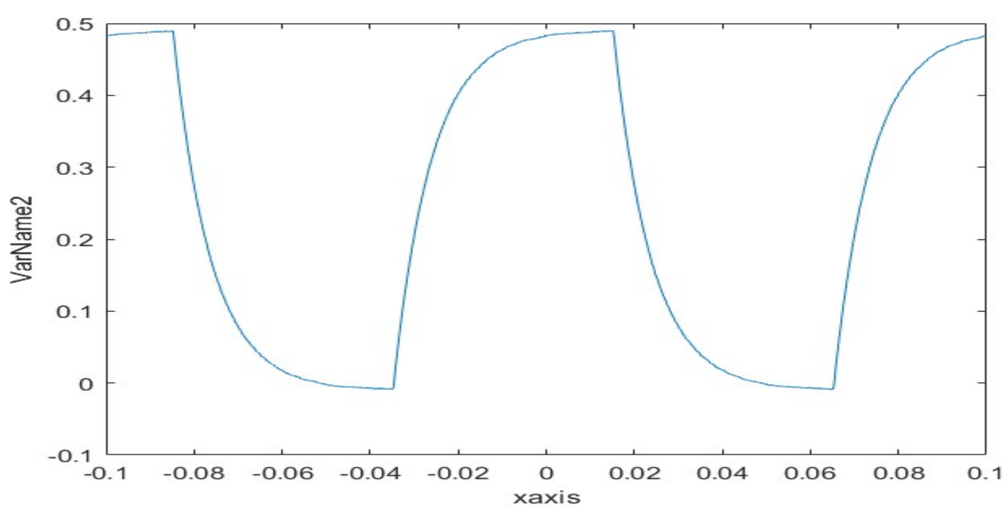
$$G(s) = \frac{R}{CSR + 2}$$

Now that we have the transfer function of the plant, we can simulate the open loop step response of the plant and see what happens:

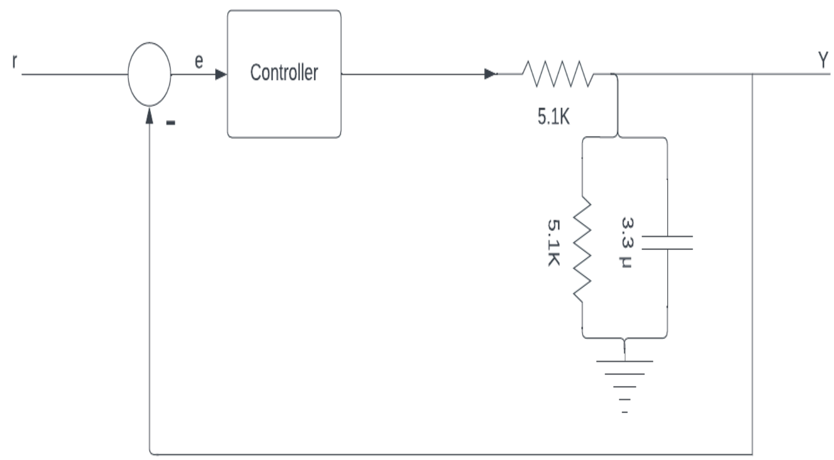



Just as we predicted, the response or the output is 0.5 V.

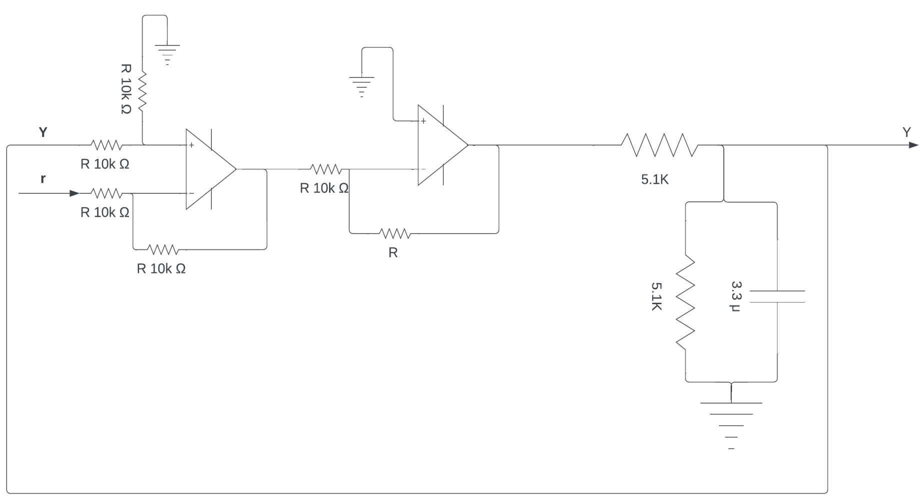
The experimental graph for the open loop step response of the plant that we obtained using the oscilloscope is shown below:



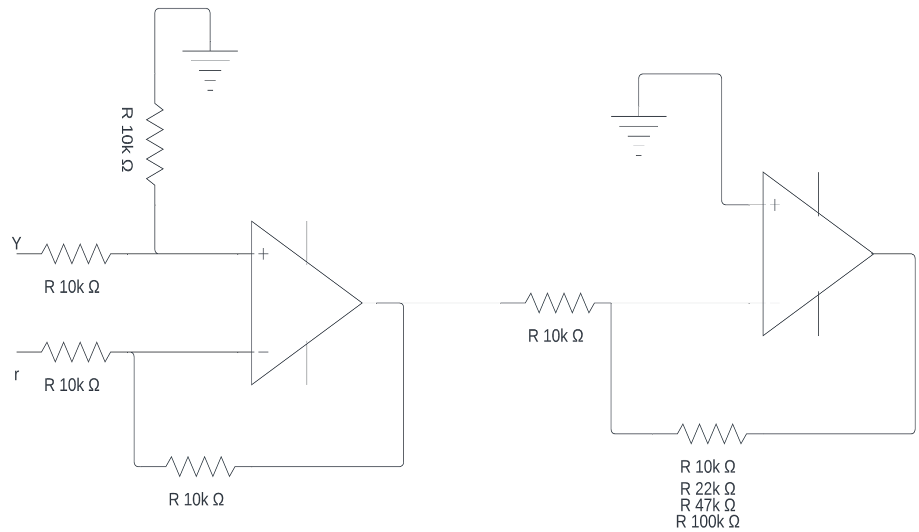

The experimental graph is almost similar to the one that we have simulated using Matlab. From the above observations, it is clear that we want to improve the steady state error of the system, to do this, we can use a proportional controller. The controller is connected to the plant as shown below:




# 3 Building the Proportional Controller



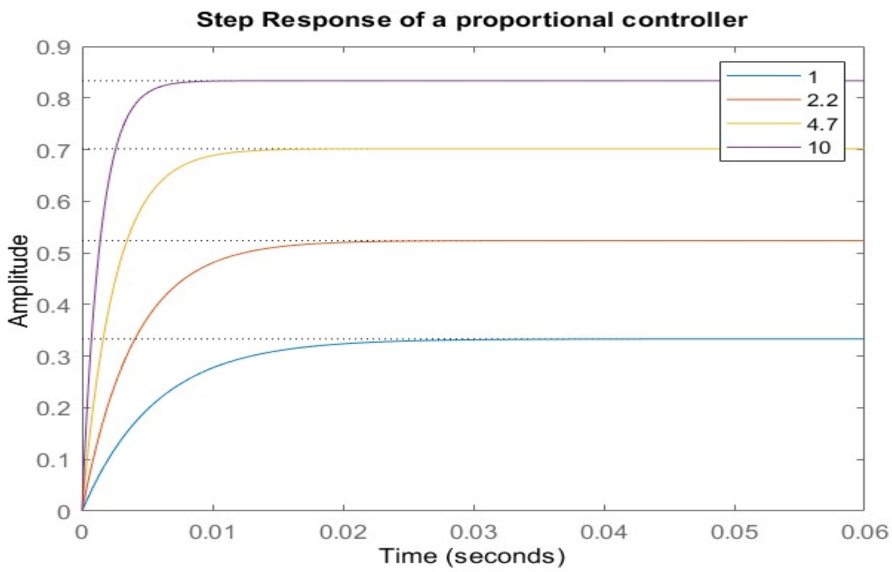

As we can see from the circuit diagram, the proportional controller consists of two parts, one is the differential amplifier circuit and the other one is the inverting amplifier circuit. The reference and feedback is applied as shown. The output from the differential amplifier would be $-(r-Y)$ and as it passes through the inverting amplifier the output becomes positive as it is inverted again, $G(r-Y)$, where 'G' can be considered the gain.



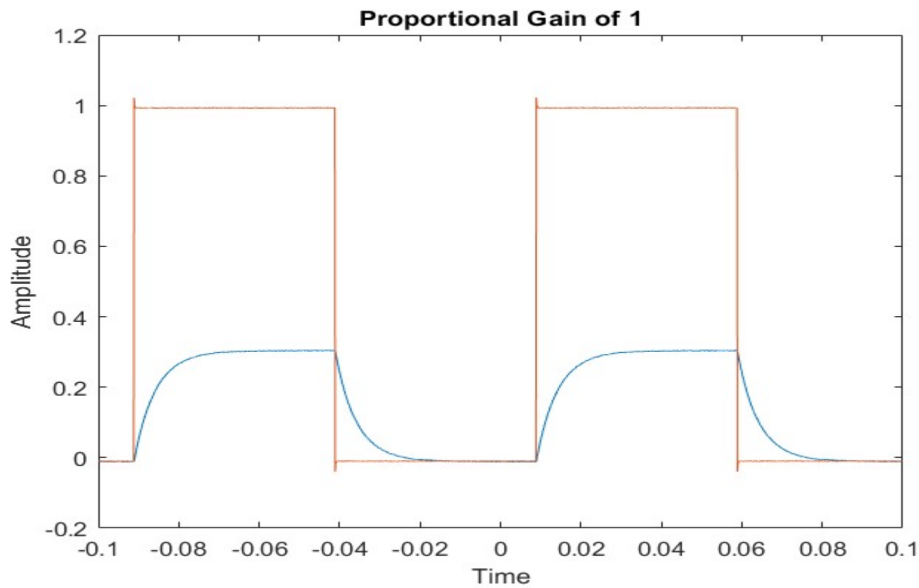

First, I started to build the differential amplifier circuit. To construct this, I used all 10k$\Omega$ resistors, so that the output of this circuit would be $-(r-Y)$ without any gain. The testing of this circuit was done by supplying a constant voltage of 5 volts as the reference input and then I supplied 3 volts to the other terminal using the DC voltage supplier in the ITLL lab, when the output came out to be -2V, it was confirmed that the differential amplifier circuit was working. Now, coming to the inverting amplifier circuit, it was very straight-forward. This segment was constructed independently from the differential amplifier circuit. A resistor of 10k$\Omega$ is used as a reference value to calculate the gain as we change the resistance for the feedback resistor which includes the values, 10k, 22k, 47k, 100k$\Omega$. These resistances are used to create varying gains for the proportional controller, by varying gain we can observe the effect on the system characteristics. The gain of the inverting amplifier is calculated as follows:

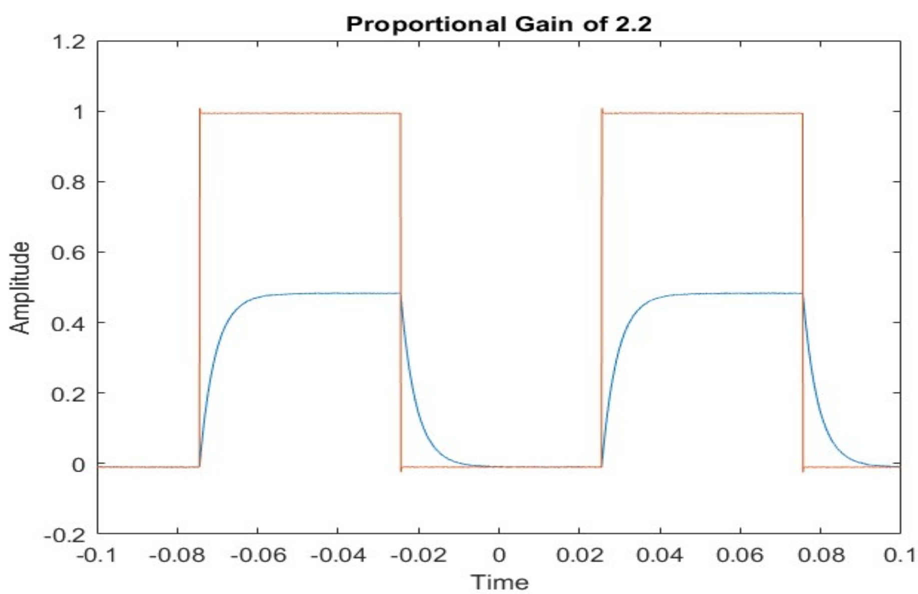
$$Vout = \frac{-Rfeedback}{R}Vin$$

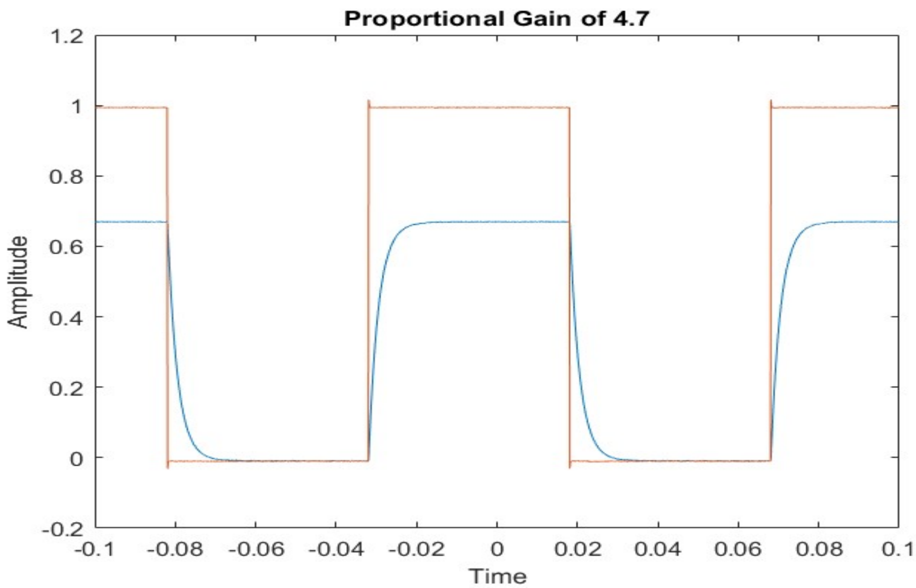
Once the inverting amplifier was tested for various gain values and confirmed to be working as intended, both the differential amplifier circuit and the inverting amplifier circuit can be connected together. Now, the proportional controller can be connected to the plant to be tested for a step input reference, but before testing it experimentally, I simulated the model on Matlab to get some reference on how the graphs would look. Below are the simulated graphs for a step input for different values of gain:

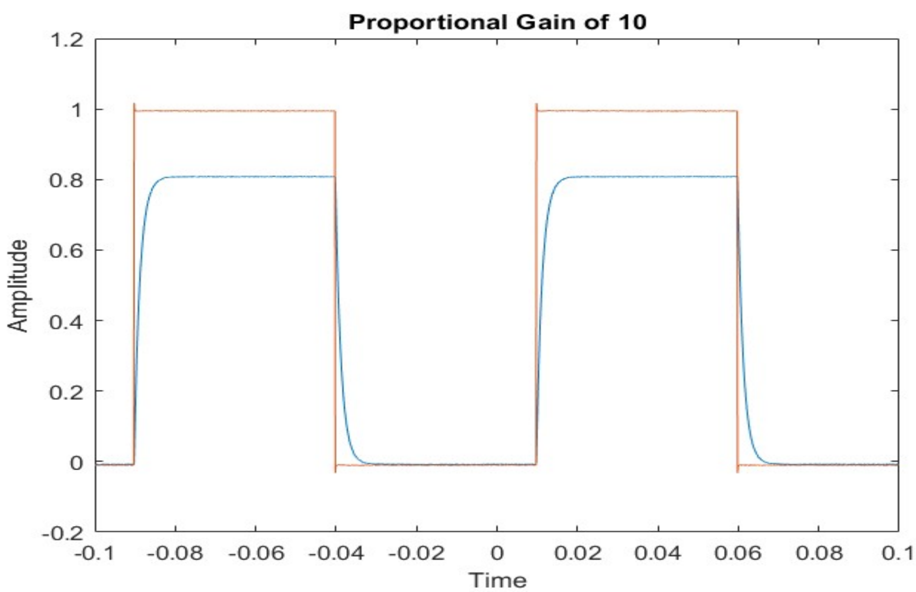



The different values of gain are represented in the legend. As it can be seen from the plots, as the value of the gain is increased, the steady state error decreases and also the rise time decreases which is a good thing. Now, let us see the experimental plots:

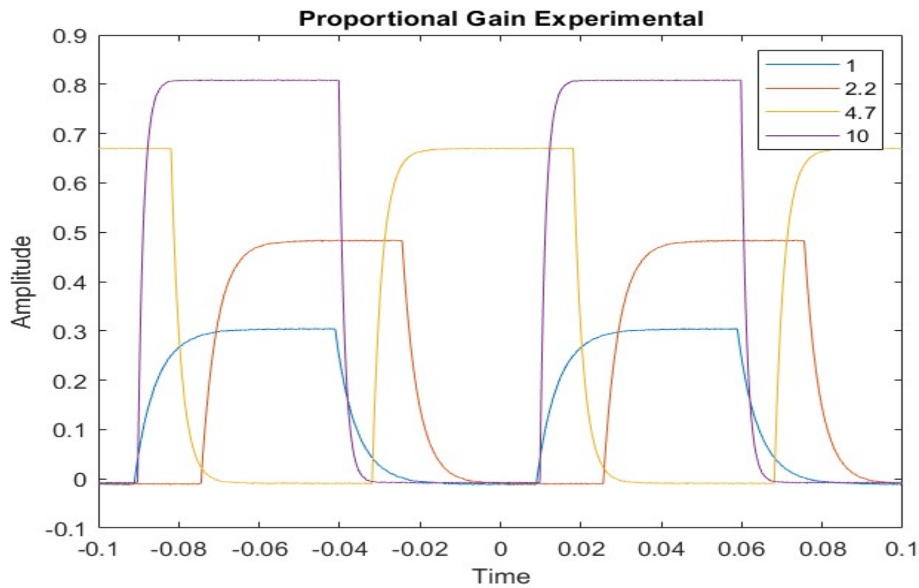

Proportional Gain of 1
Amplitude
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1


Proportional Gain of 2.2
Amplitude
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1

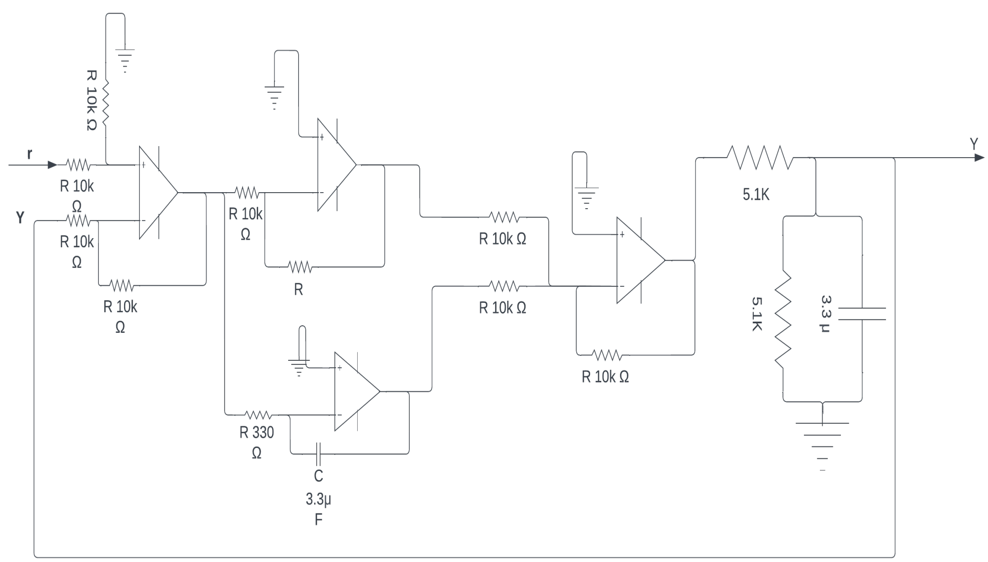



The experimental plots also follow the similar trend of decreasing steady state errors and rise times, but they don't match the exact values as represented by the simulated plots but are close enough and this is understandable because in real equipment there exists some loses which is the simulation does not account for.




# 4 Building the Proportional Integral Controller




Although the steady state error was reduced by a good margin by increasing the proportional gain, there still exists a significant amount. To reduce this steady state error even further or more conveniently, if we can make it zero, then that would be something very favourable. To achieve this, we can use a proportional integral controller which can in theory reduce the steady state error to zero.

In the above diagram is the layout for my PI controller, as shown, I have added an Integrator circuit in parallel to the proportional circuit and both of these are then connected to the summer circuit. The reference and feedback are given as shown in the diagram, here, the output from the difference circuit is positive, as it then goes into both the Integrator and the amplifier, the output then becomes negative, but once the this signal passes through the summer circuit, the output is inverted and it becomes positive once again.

The values of the resistance and the capacitance for the Integrator were chosen as 330Ω and 3.3µF respectively. The reason a low resistance value is chosen for the Integrator is because as the resistance value increases the rise time also increases, so, a low resistance ensures that we have an optimal rise time. Below is the equation for the output voltage of an Integrator circuit:

$$Vout = \frac{-1}{RC} \int_0^t Vin\, dt$$

Taking the Laplace transform gives us:

$$\frac{Vout(s)}{Vin(s)} = \frac{-1}{RCS}$$

Substituting the values of our resistor and capacitor gives us the transfer function of the integrator:

$$\frac{Vout(s)}{Vin(s)} = \frac{-918.27}{S}$$

The equation for our summer looks like:

$$Vout = -Rfeedback(\frac{V1}{R1} + \frac{V2}{R2})$$

As I chose all of the resistances to be equal to each other in the summer circuit, the output voltage of the summer becomes:

$$Vout = -(V1 + V2)$$

The transfer function of the PI controller is given as:

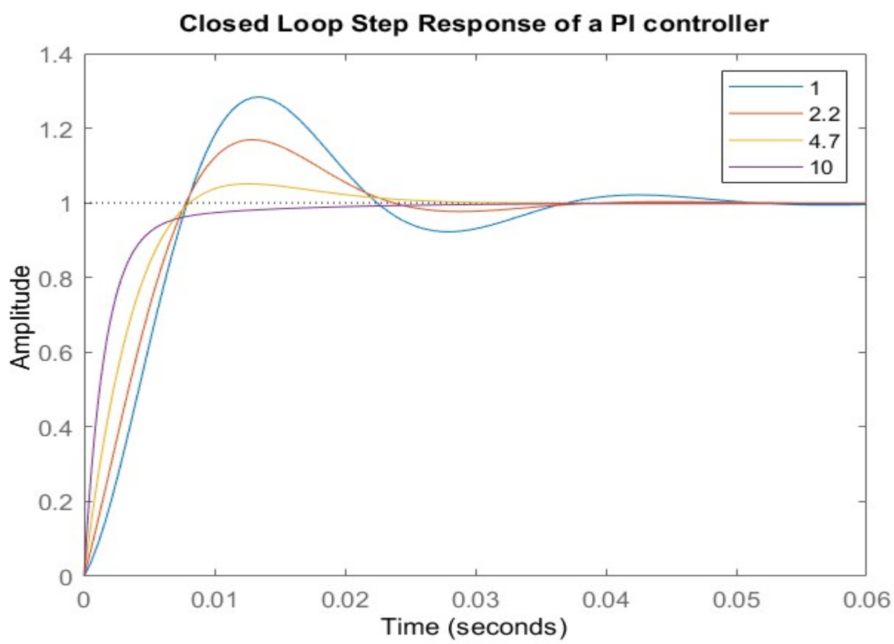
$$C(S) = Kp + \frac{Ki}{S}$$

Where, $Kp$ is the gain of the amplifier and $Ki$ is 918.27 as given above. The open loop transfer function of the system is given by:

$$L(S) = (Kp + \frac{Ki}{S}) * G(S)$$

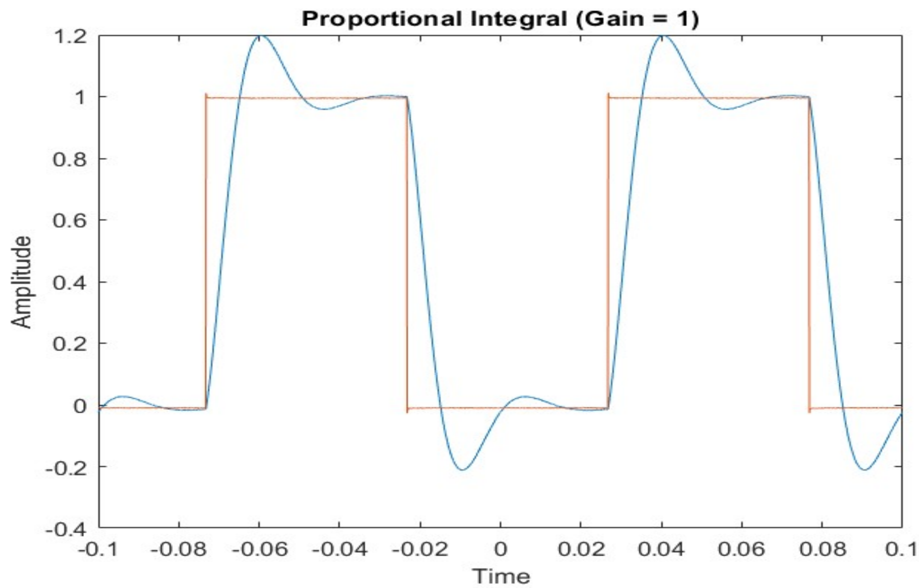
where, $G(S)$ is the transfer function of the plant.

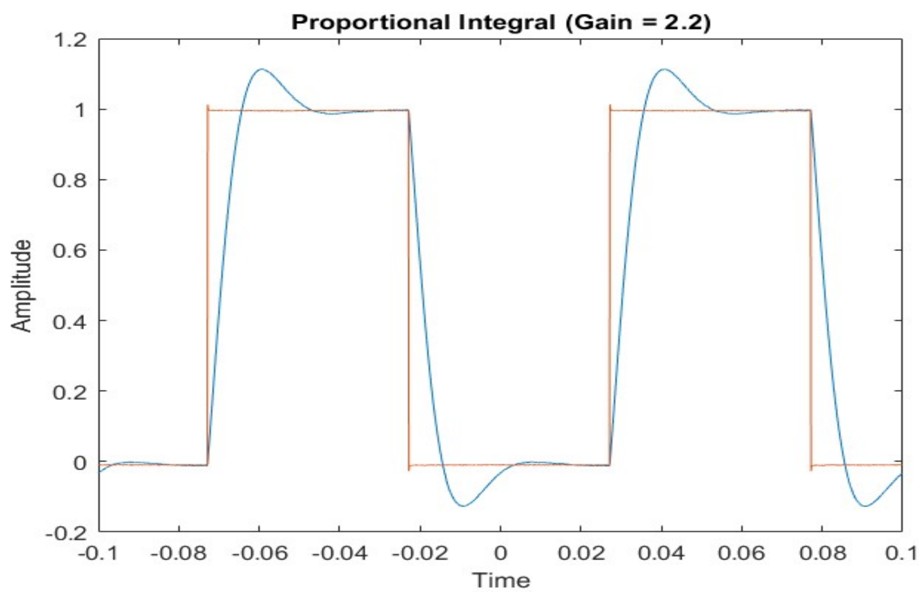
Using this information, we can simulate the step response of the system for various values of gain:

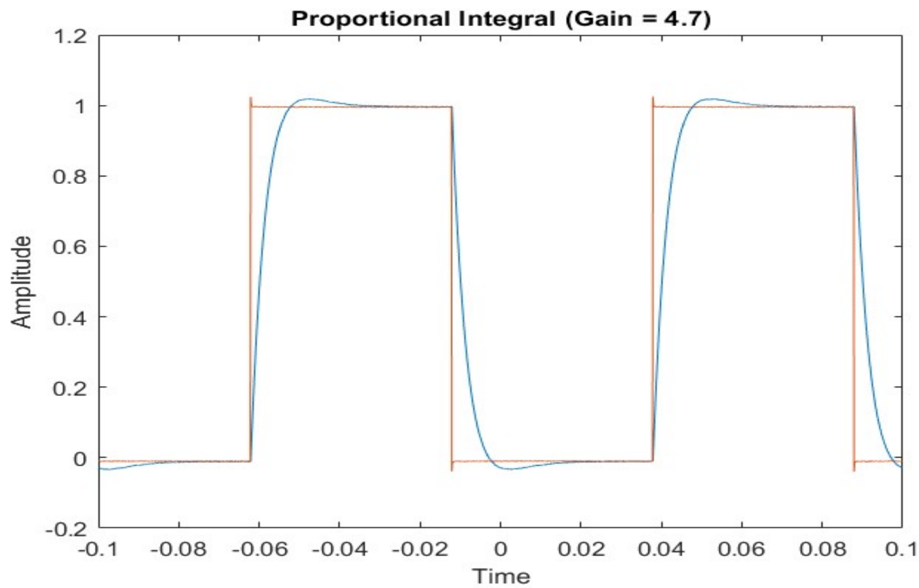


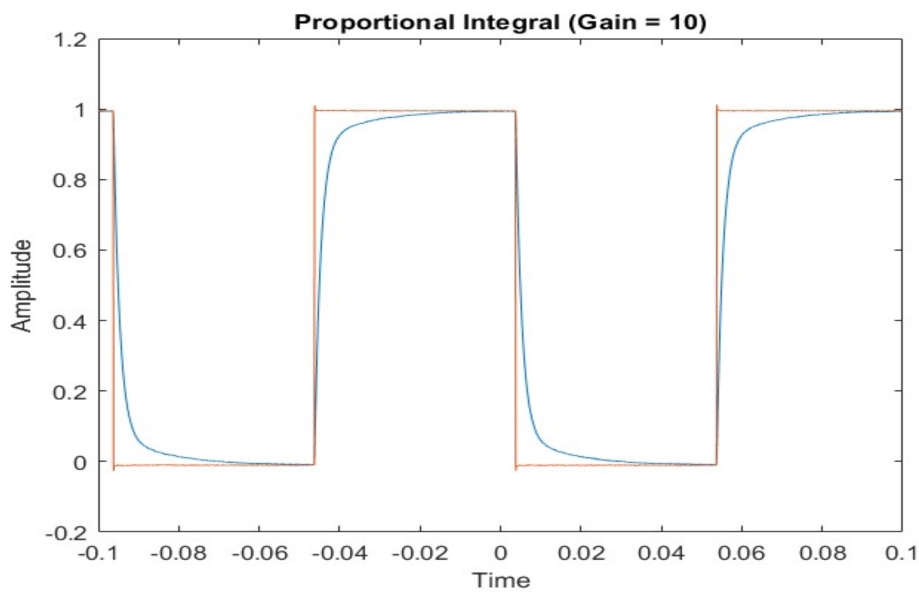

As we can see in the diagram, with the addition of the Integrator, we were able to reduce the steady state error to zero. It is observed that at low gain there is a large overshoot and as the gain is increased, the overshoot is consequently reduced and also as the gain is increased the rise time of the system decreases as also observed previously in the proportional controller.

Now, let us look at the experimental counterparts:


Proportional Integral (Gain = 1)
Amplitude
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1

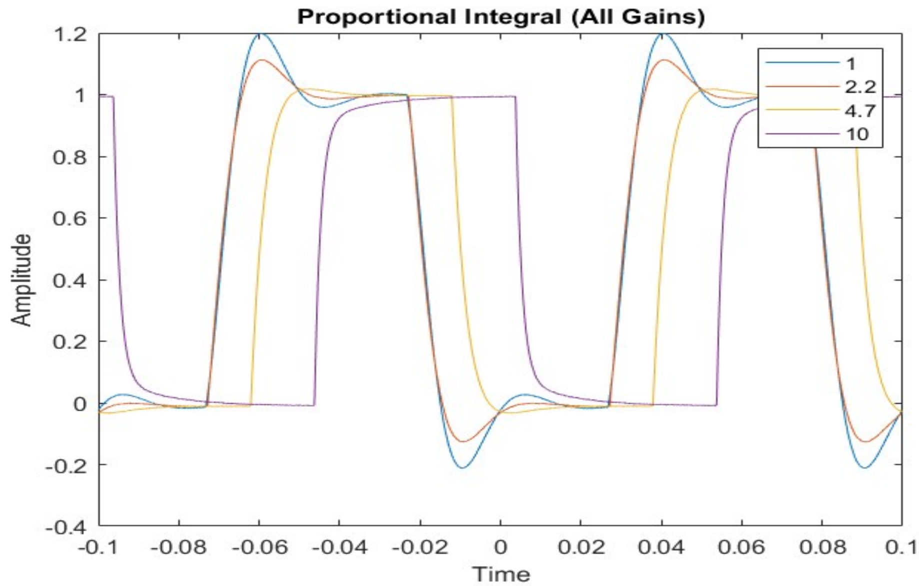

Proportional Integral (Gain = 2.2)
Amplitude
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1



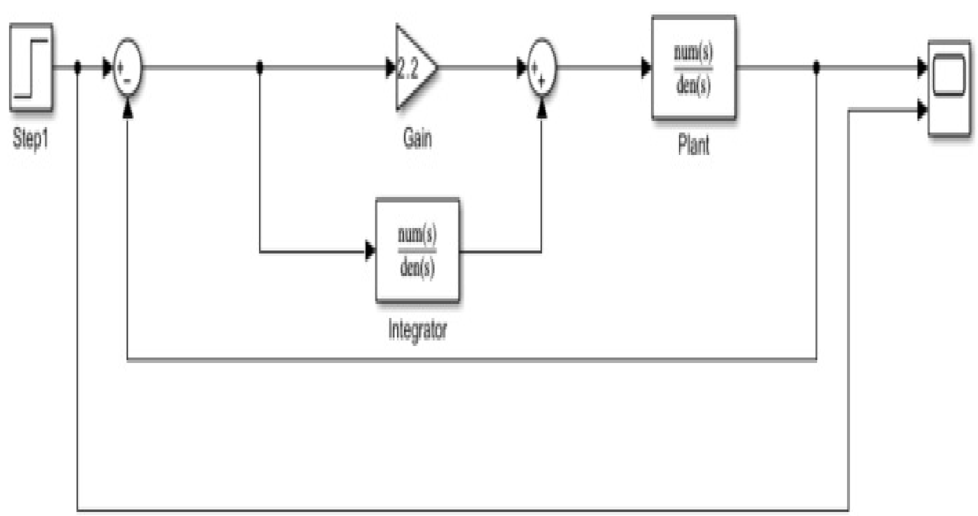




From the above experimental plots it can be seen that a similar trend of decreasing overshoots with increasing gain is followed as seen with the simulated plots. Because of the Integral part we achieve a zero steady state error and at gain = 10 we achieve a nice smooth curve for the response of our system to a step input.

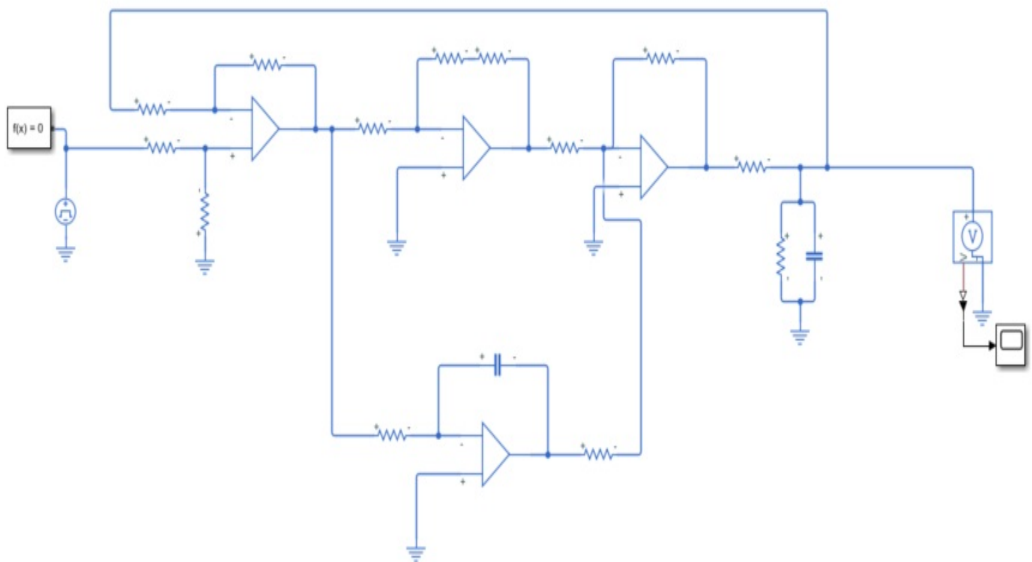
The experimental plots of all the step responses for various gains is given below:



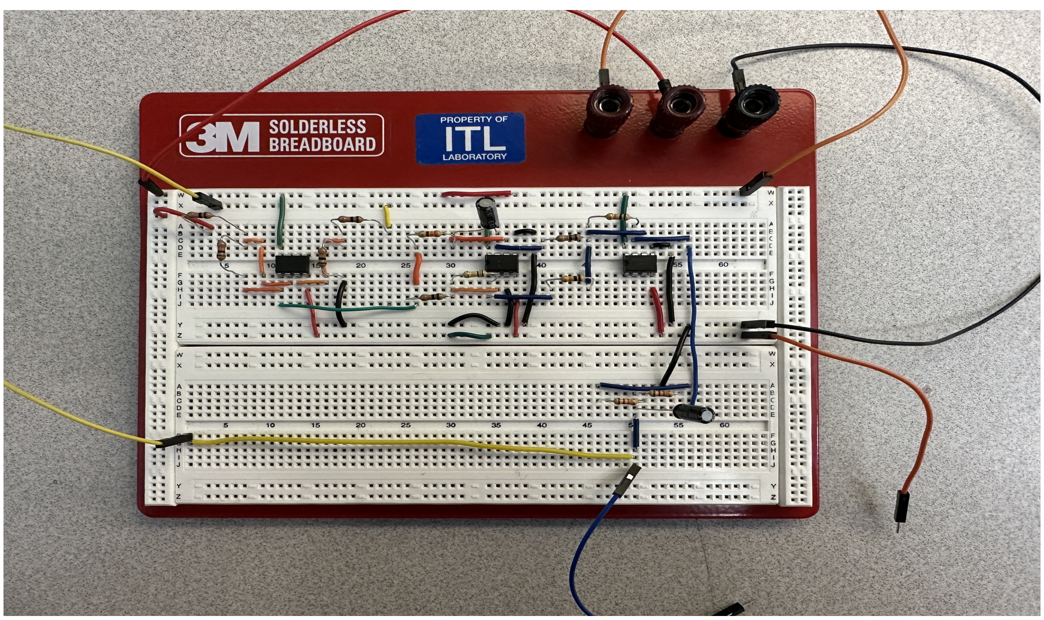

The Simulink model using transfer functions:



Simulink Model:




## 5 Breadboard Setup of my PI Controller



# 6 Conclusion

In this project, we explored the the influence of a controller in determining characteristics such as the rise time, overshoot, and also steady state error of a system, also, the influence of changing gain was understood. With the help of op-amps, controllers such as the proportional, proportional integral were implemented. Also, the difficulty in implementing an analogous circuit was understood, with various errors being spotted time to time.

Through the journey of this project there were many hassles and at times there seemed to be no answer to the bugs that were occurring, one such instance was when the differential circuit kept giving me a constant voltage regardless of the voltages that I input. The problem seemed to fix itself after I gave the circuit on my breadboard a makeover and made it as clean as possible, also making it much easier to debug and avoiding the possibility of a short circuit. The project allowed me to explore how controllers could influence system characteristics visually, and also the implementation of a controller in the ground level using op-amps.