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Arden's Theorem

It states that "If P & Q are 2 Regular expressions over ϵ, and if P does not contain ϵ, then the following eq^n $\boxed{R = Q + RP}$ has a unique sol^n, that is, $\boxed{R = QP^*}$"

Proof: Given $R = Q + RP \longrightarrow$ (i)

substitute $R = QP^*$ in (i)

$$R = Q + (QP^*)P = Q(\epsilon + P^* P)$$

$$= QP^* \qquad [\because \text{ As } \epsilon + R^* R = R^*]$$

We have proved $R = QP^*$ is a sol^n (but not yet unique proved)

Now to prove that $R = QP^*$ is the unique sol^n to $R = Q + RP$

Given $R = Q + RP$

$$R = Q + (Q + RP)P$$

$$= Q + QP + RP^2 \longrightarrow \text{(ii)}$$

$$= Q + QP + (Q + RP)P^2$$

$$= Q + QP + QP^2 + RP^3 \longrightarrow \text{(iii)}$$

$$= Q + QP + QP^2 + Q(Q + RP)P^3$$

$$= Q + QP + QP^2 + QP^3 + RP^4 \longrightarrow \text{(iv)}$$

$$\vdots$$

$$= Q + QP + QP^2 + \cdots + QP^n + RP^{n+1}$$

$$= Q(\epsilon + P + P^2 + \cdots + P^n) + RP^{n+1}$$

$$= Q(\epsilon + P + P^2 + \cdots + P^n) + QP^* P^{n+1}$$

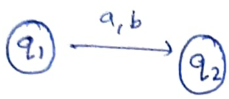
$$= Q\{(\epsilon + P + P^2 + \cdots + P^n) + P^* P^{n+1}\}$$

$$= QP^*$$

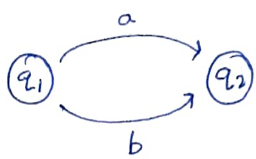
Conversion of Regular Expression to Finite Automata

Rules :

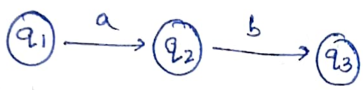
① $R = a + b$ { union operation }



OR



② $R = ab$

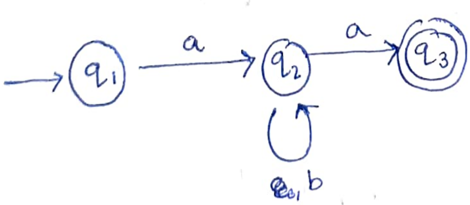


③ $R = a^*$

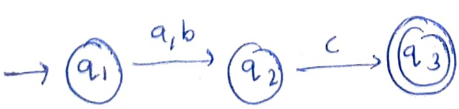
Eg ① Convert the following Regular expression into equivalent FA

i) $R = a b^* a$

Sol^n $L = aa, aba, abba, abbba, \ldots$

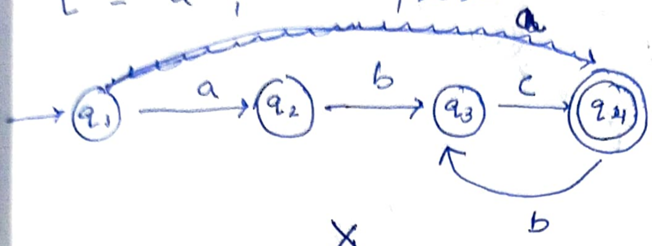


ii) $R = (a+b)c$

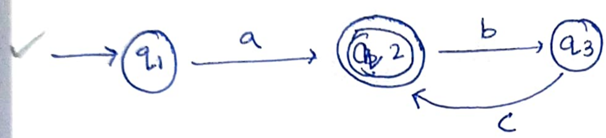


iii) $R = a(bc)^*$

$L = a, abc, abcbc$



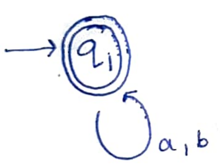
Or



(More efficent) :)

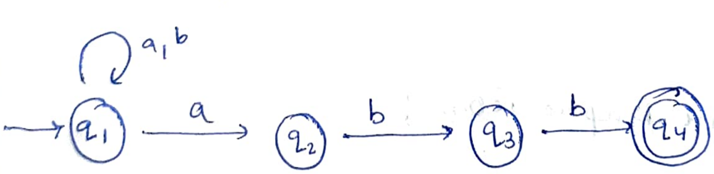
iv) $R = (a+b)^*$

$L = \varepsilon, a, b, aa, ab$



v) $R = (a+b)^* abb$

$L = abb, aabb, babb, \ldots$



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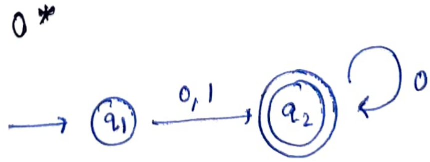
Q ① Convert the following Regular expression to FA :

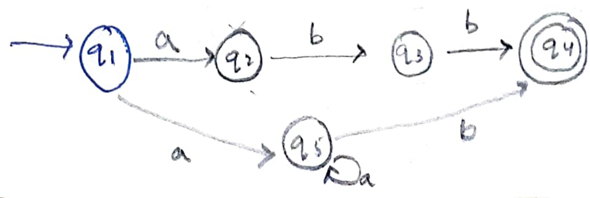
$R = (0+1)\,0^*$

Ans FA :



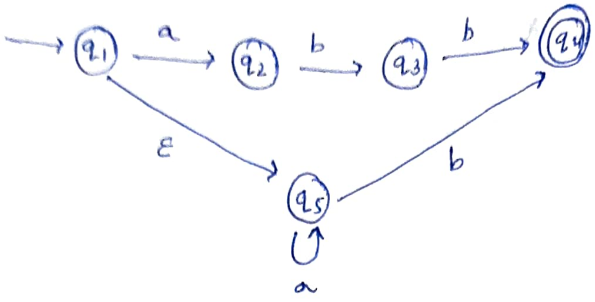
② $R = abb + a^+ b$

Ans

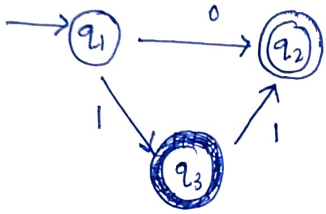


(3) $R = abb + a^* b$

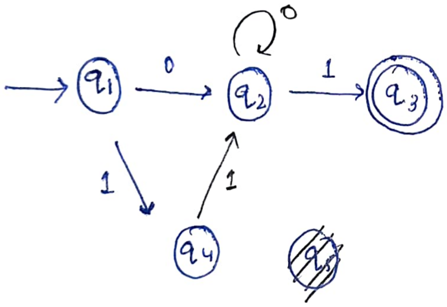
Ans



(4) $R = 0 + 11$; $\Sigma = \{0, 1\}$

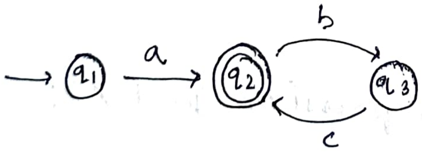


(5) $R = (0 + 11)\, 0^* 1$



$L = \{01, 111, 001, 1101, \ldots$

Q(6) Convert FA To Regular Expression :



Ans

$R = a + a(bc)^*$

$= a(\varepsilon + (bc)^*)$

$= a(bc)^*$

Ans

$q_1 \Rightarrow \varepsilon \longrightarrow (i)$

$q_2 = q_1 a + q_3 c \longrightarrow (ii)$

$q_3 = q_2 b \longrightarrow (iii)$

solving (ii) $q_2 = q_1 a + q_3 c$

Put the values of q_1 & q_3 from eqⁿ (i) & (iii)

$q_2 = \varepsilon a + q_2 bc$

$\Rightarrow \underbrace{q_2}_{R} = \underbrace{a}_{Q} + \underbrace{q_2}_{R} \underbrace{bc}_{P}$

using Arden's theorem,

$q_2 = a(bc)^*$

So the regular expression will be, ~~$a(bc)^*$~~

$$\boxed{R = a(bc)^*}$$

For more than 1 final states (let R_1 & R_2)

solve for, $R = R_1 + R_2$

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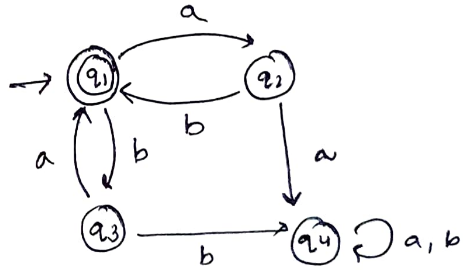
Conversion from FA To Regular Expression

steps

1. Find out the transition or edge which comes from other state to that particular state.
2. Then find out solⁿ for the state which is in final state & leave other states as it is.

Example Convert the following FA to equivalent Regular expression :



solⁿ

$q_1 = \varepsilon + q_2 b + q_3 a \longrightarrow$ (i)

$q_2 = q_1 a \longrightarrow$ (ii)

$q_3 = q_1 b \longrightarrow$ (iii)

$q_4 = q_2 a + q_3 b + q_4 \cancel{ab}$
$+ q_4 b \longrightarrow$ (iv)

solving the final stage q_1

$q_1 = \varepsilon + (q_1 a) b + (q_1 b) a$ {substituting q_2 & q_3}

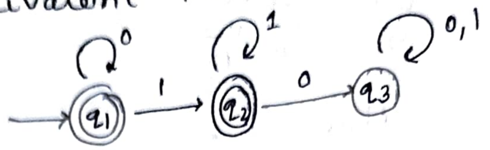
$= \varepsilon + q_1 ab + q_1 ba$

$\underbrace{q_1}_{R} = \underbrace{\varepsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{(ab + ba)}_{P}$

$q_1 = \varepsilon\,(ab+ba)^*$

$q_1 = (ab+ba)^*$ ← Required regular expression

Eg ② Convert FA To equivalent RE



Solⁿ

$q_1 = \varepsilon + q_1 0$

$q_2 = q_1 1 + q_2 1$

$q_3 = q_2 0 + q_3 0 + q_3 1$

Solving q_1 & q_2 ;

$q_1 \cancel{+ q_2} = \varepsilon + q_1 0$

$q_2 = q_1 + q_2 1$

$= (\varepsilon + q_1 0) + q_2 1$

$= \varepsilon + q_1 0 + q_2 1$

$q_1 + q_2 = \varepsilon + q_1 0 + q_1 1 + q_2 1$

$= \varepsilon + q_1 (0 + 1) +$

$q_1 = \varepsilon 0^* = 0^*$

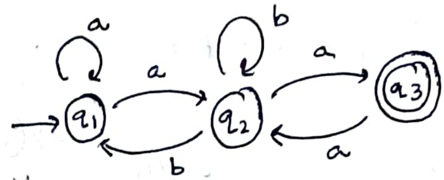
$q_2 = q_1 1 + q_2 1$

$= 0^* 1 + q_2 1$

$q_2 = 0^* 1 (1^*) = 0^* 1^+$

$q_1 + q_2 = 0^* + 0^* (1^+)$

(4) Convert FA $\rightarrow$ RE



Sol^n

$$q_1 = \varepsilon + q_1 a + q_2 b \rightarrow (i)$$

$$q_2 = q_1 a + q_2 b + q_3 a \rightarrow (ii)$$

$$q_3 = q_2 a \rightarrow (iii)$$

Solving q_3 :

$$q_3 = q_2 a = (q_1 a + q_2 b + q_3 a) a$$

$$= \{(\varepsilon + q_1 a + q_2 b) a + q_2 b + q_3 a\} a$$

$$q_2 = q_1 a + q_2 b + q_3 a$$

$$\cancel{= q_1 a + q_3 a + q_2 b} \quad q_1 a + q_2 b + q_2 aa$$

$$q_2 = q_1 a + q_2 (b + aa)$$

Applying Arden's theorem :

$$q_2 = q_1 a (b + aa)^* \longrightarrow (iv)$$

Now, $q_1 = \varepsilon + q_1 a + q_2 b$

$$= \varepsilon + q_1 a + q_1 a (b + aa)^* b$$

$$q_1 = \varepsilon + q_1 \{a + a(b + aa)^* b\}$$

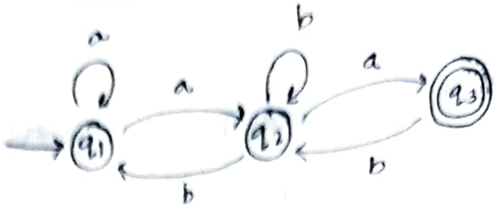
So, $q_1 = \{a + a(b+aa)^* b\}^* \longrightarrow (v)$

Again, $q_3 = q_2 a$

$$= q_1 a (b + aa)^* a$$

$$= \{a + a(b + aa)^* b\}^* a (b + aa)^* a \rightarrow (vi)$$

27/10/23 (5) FA → RE



Sol^n

$q_1 = \varepsilon + q_1 a + q_2 b \longrightarrow (i)$

$q_2 = q_1 a + q_2 b + q_3 b \longrightarrow (ii)$

$q_3 = q_2 a \longrightarrow (iii)$

Now (ii) becomes

$q_2 = q_1 a + q_2 b + q_3 b$

$= q_1 a + q_2 b + q_2 a b$

$q_2 = q_1 a + q_2 (b + ab)$

$q_2 = q_1 a (b + ab)^* \longrightarrow (iv)$

Now $q_1 = \varepsilon + q_1 a + q_2 b$

$= \varepsilon + q_1 a + q_1 a (b + ab)^* b$

$q_1 = \varepsilon + q_1 \{a + a (b + ab)^* b\}$

$q_1 = \varepsilon \{a + a (b + ab)^* b\}^* \longrightarrow (v)$

Next $q_3 = q_2 a$

$= q_1 a (b + ab)^* a$

$q_3 = \{a + a (b + ab)^* b\}^* a (b + ab)^* a$

Ans

Equivalence of 2 finite Automata

Rules

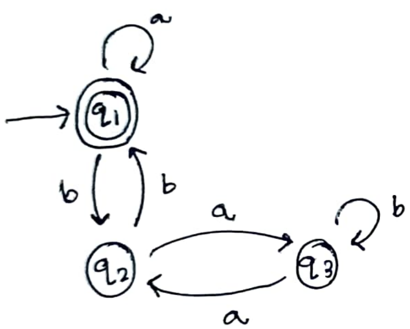
1. For any pair of states $\{q_i, q_j\}$ the Transition for input $a \in \Sigma$ is defined by $\{q_a, q_b\}$ where $\delta(q_i, a) = q_a$ and

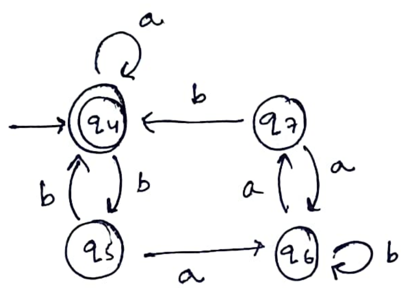
$\delta(q_j, a) = q_b$.

The 2 automaton are not equivalent if for a pair $\{q_a, q_b\}$ one is intermediate state & the other is final state.

(2) If initial state is the final state of 1 automaton then in 2nd automaton also initial state must be final state for them to be equivalent.

Example (1) Find whether the given 2 automata are equivalent or not.





M_1 M_2

Sol^n

States	a	b
$\{q_1, q_4\}$	$\{\underbrace{q_1}_{FS}, \underbrace{q_4}_{FS}\}$	$\{\underbrace{q_2}_{IS}, \underbrace{q_5}_{IS}\}$
$\{q_2, q_5\}$	$\underbrace{\{q_3, q_6\}}_{IS}$	$\underbrace{\{q_1, q_4\}}_{FS}$
$\{q_3, q_6\}$	$\underbrace{\{q_2, q_7\}}_{IS}$	$\underbrace{\{q_3, q_6\}}_{IS}$
$\{q_2, q_7\}$	$\{q_3, q_6\}$	$\{q_1, q_4\}$

FS → final state
IS → Intermediate state

(2) M_1 M_2