Practical 1

Make a geometric plot to show that the nth roots of unity are equally spaced points that lie on the unit circle C1(0)={z:|z|<1} and form the vertices off a regular polygon with n sides, for n = 4, 5, 6 ,7 ,8.

In[1]:=
```
For z = 5;
```

Set: Tag Times in For z is Protected.

```
Solve[z^5 == 1, z]
```

Out[#]=

$$\left\{\{z \to 1\},\ \left\{z \to -(-1)^{1/5}\right\},\ \left\{z \to (-1)^{2/5}\right\},\ \left\{z \to -(-1)^{3/5}\right\},\ \left\{z \to (-1)^{4/5}\right\}\right\}$$

```
roots = ComplexExpand[Solve[z^5 == 1, z]]
```

Out[#]=

$$\left\{\{z \to 1\},\ \left\{z \to -\frac{1}{4}-\frac{\sqrt{5}}{4}-i\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\right\},\ \left\{z \to -\frac{1}{4}+\frac{\sqrt{5}}{4}+i\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\right\},\ \left\{z \to -\frac{1}{4}+\frac{\sqrt{5}}{4}-i\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\right\},\ \left\{z \to -\frac{1}{4}-\frac{\sqrt{5}}{4}+i\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\right\}\right\}$$

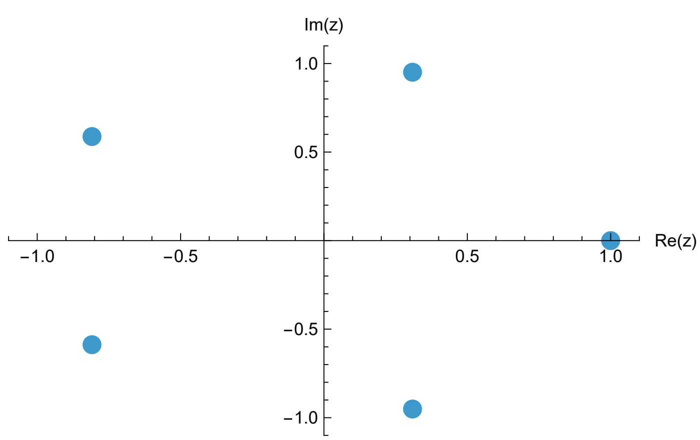
```
z /. roots
```

Out[#]=

$$\left\{1,\ -\frac{1}{4}-\frac{\sqrt{5}}{4}-i\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}},\ -\frac{1}{4}+\frac{\sqrt{5}}{4}+i\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},\ -\frac{1}{4}+\frac{\sqrt{5}}{4}-i\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},\ -\frac{1}{4}-\frac{\sqrt{5}}{4}+i\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\right\}$$

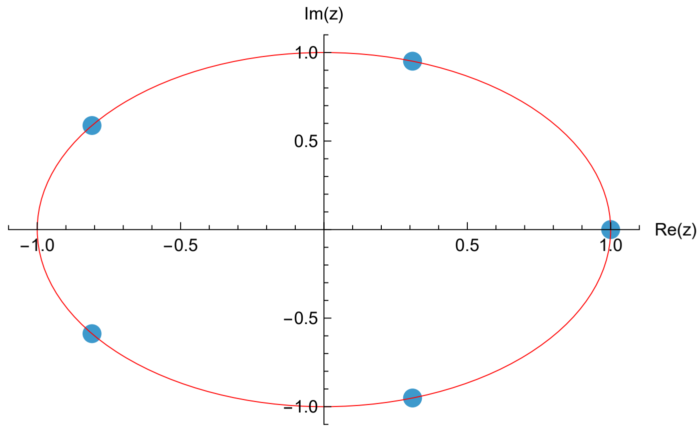
```
rootPlot = ListPlot[{Re[z], Im[z]} /. roots, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}},
AxesLabel → {"Re(z)", "Im(z)"}, PlotStyle → PointSize[0.03]]
```

Out[#]=



```
Show[rootPlot, Graphics[{Red, Circle[{0, 0}, 1]}]]
```

Out[ ]=



In[7]:=
```
For z = 4;
```

Set: Tag Times in For z is Protected.

```
Solve[z^4 == 1, z]
```

Out[ ]=

```
{{z → -1}, {z → -i}, {z → i}, {z → 1}}
```

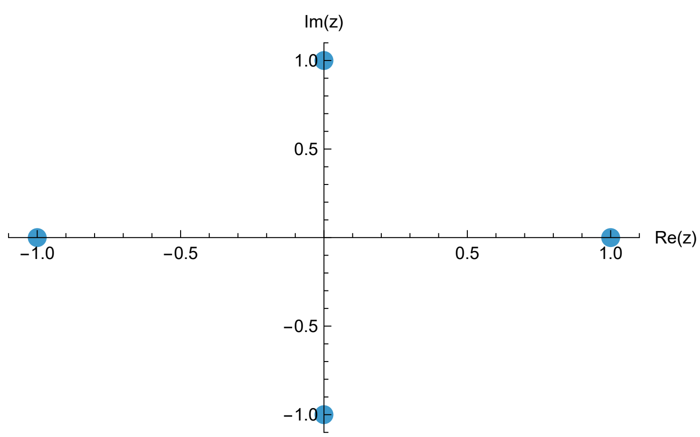
```
roots1 = ComplexExpand[Solve[z^4 == 1, z]]
```

Out[ ]=

```
{{z → -1}, {z → -i}, {z → i}, {z → 1}}
```

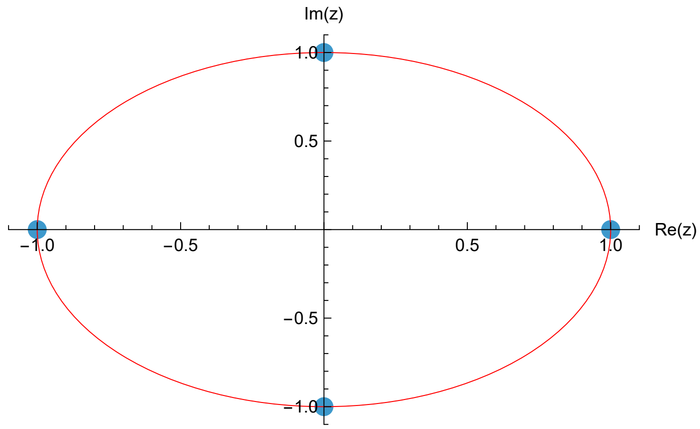
```
rootPlot1 = ListPlot[{Re[z], Im[z]} /. roots1, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}},
AxesLabel → {"Re(z)", "Im(z)"}, PlotStyle → PointSize[0.03]]
```

Out[ ]=



```
Show[rootPlot1, Graphics[{Red, Circle[{0, 0}, 1]}]]
```

Out[#]=



In[12]:=
```
For z = 6;
```

Set: Tag Times in For z is Protected.

```
Solve[z^6 == 1, z]
```

Out[#]=

$$\{\{z \to -1\}, \{z \to 1\}, \{z \to -(-1)^{1/3}\}, \{z \to (-1)^{1/3}\}, \{z \to -(-1)^{2/3}\}, \{z \to (-1)^{2/3}\}\}$$

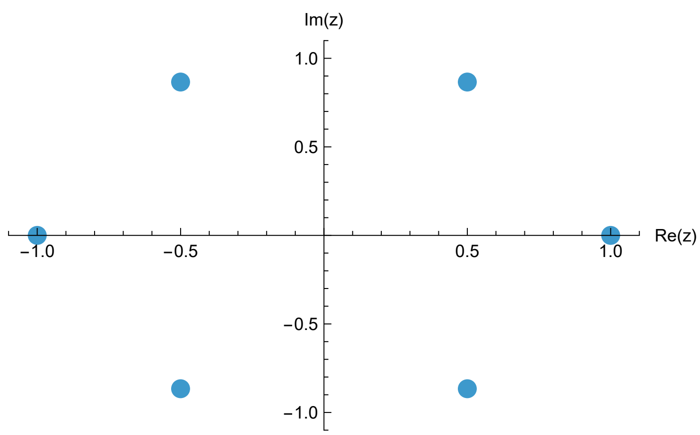
```
roots2 = ComplexExpand[Solve[z^6 == 1, z]]
```

Out[#]=

$$\left\{\{z \to -1\}, \{z \to 1\}, \left\{z \to -\frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}, \left\{z \to \frac{1}{2} + \frac{i\sqrt{3}}{2}\right\}, \left\{z \to \frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}, \left\{z \to -\frac{1}{2} + \frac{i\sqrt{3}}{2}\right\}\right\}$$

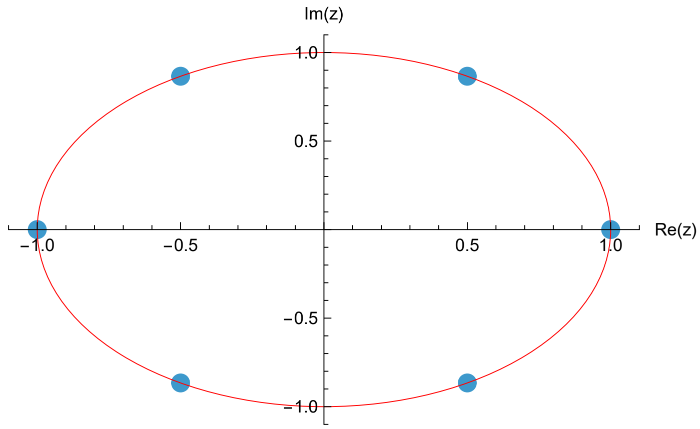
```
rootPlot2 = ListPlot[{Re[z], Im[z]} /. roots2, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}},
AxesLabel → {"Re(z)", "Im(z)"}, PlotStyle → PointSize[0.03]]
```

Out[#]=



```
Show[rootPlot2, Graphics[{Red, Circle[{0, 0}, 1]}]]
```

Out[ ]=



In[17]:=
```
For z = 7;
```

Set: Tag Times in For z is Protected.

```
Solve[z^7 == 1, z]
```

Out[ ]=

$\{\{z \to 1\}, \{z \to -(-1)^{1/7}\}, \{z \to (-1)^{2/7}\}, \{z \to -(-1)^{3/7}\}, \{z \to (-1)^{4/7}\}, \{z \to -(-1)^{5/7}\}, \{z \to (-1)^{6/7}\}\}$

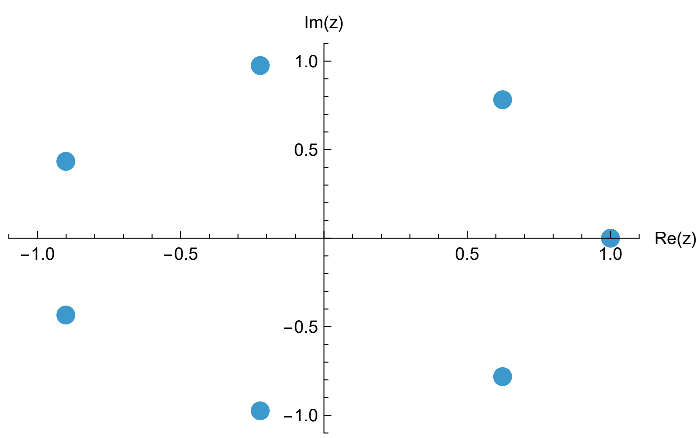
```
roots3 = ComplexExpand[Solve[z^7 == 1, z]]
```

Out[ ]=

$\left\{\{z \to 1\}, \left\{z \to -\mathrm{Cos}\left[\frac{\pi}{7}\right] - i\,\mathrm{Sin}\left[\frac{\pi}{7}\right]\right\}, \left\{z \to i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right] + \mathrm{Sin}\left[\frac{3\pi}{14}\right]\right\}, \left\{z \to -i\,\mathrm{Cos}\left[\frac{\pi}{14}\right] - \mathrm{Sin}\left[\frac{\pi}{14}\right]\right\}, \left\{z \to i\,\mathrm{Cos}\left[\frac{\pi}{14}\right] - \mathrm{Sin}\left[\frac{\pi}{14}\right]\right\}, \left\{z \to -i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right] + \mathrm{Sin}\left[\frac{3\pi}{14}\right]\right\}, \left\{z \to -\mathrm{Cos}\left[\frac{\pi}{7}\right] + i\,\mathrm{Sin}\left[\frac{\pi}{7}\right]\right\}\right\}$

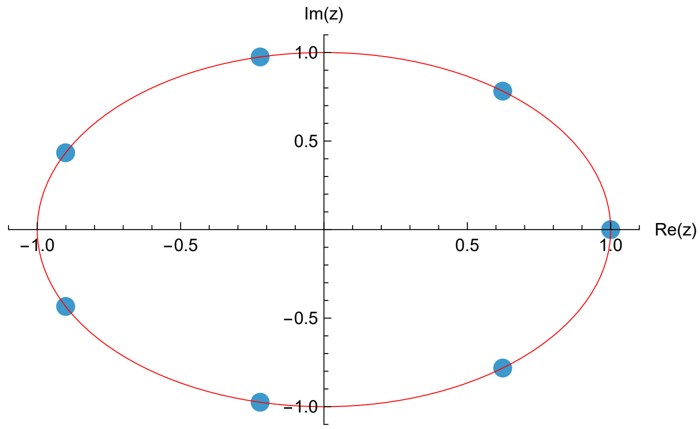
```
rootPlot3 = ListPlot[{Re[z], Im[z]} /. roots3, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}},
AxesLabel → {"Re(z)", "Im(z)"}, PlotStyle → PointSize[0.03]]
```

Out[ ]=



```
Show[rootPlot3, Graphics[{Red, Circle[{0, 0}, 1]}]]
```

Out[ ]=



In[22]:=
```
For z = 8;
```

Set: Tag Times in For z is Protected.

```
Solve[z^8 == 1, z]
```

Out[ ]=

$$\{\{z \to -1\}, \{z \to -i\}, \{z \to i\}, \{z \to 1\}, \{z \to -(-1)^{1/4}\}, \{z \to (-1)^{1/4}\}, \{z \to -(-1)^{3/4}\}, \{z \to (-1)^{3/4}\}\}$$

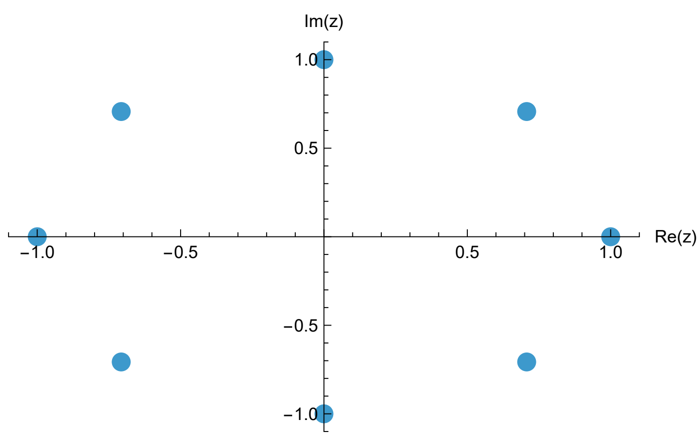
```
roots4 = ComplexExpand[Solve[z^8 == 1, z]]
```

Out[ ]=

$$\left\{\{z \to -1\}, \{z \to -i\}, \{z \to i\}, \{z \to 1\}, \left\{z \to -\frac{1+i}{\sqrt{2}}\right\}, \left\{z \to \frac{1+i}{\sqrt{2}}\right\}, \left\{z \to \frac{1-i}{\sqrt{2}}\right\}, \left\{z \to -\frac{1-i}{\sqrt{2}}\right\}\right\}$$

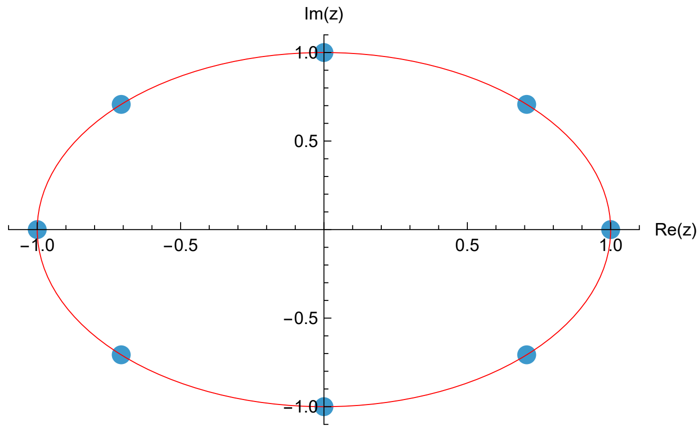
```
rootPlot4 = ListPlot[{Re[z], Im[z]} /. roots4, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}},
AxesLabel → {"Re(z)", "Im(z)"}, PlotStyle → PointSize[0.03]]
```

Out[ ]=



```
Show[rootPlot4, Graphics[{Red, Circle[{0, 0}, 1]}]]
```

Out[#]=




```
ClearAll
```

Out[#]=

```
ClearAll
```

Practical 2

Find all the solutions of the equation z^3=8i and represent these geometrically.

```
Solve[z^3 == 8 * I, z]
```

Out[#]=

$\{\{z \to -2\,i\}, \{z \to 2\,(-1)^{1/6}\}, \{z \to 2\,(-1)^{5/6}\}\}$

```
roots5 = ComplexExpand[Solve[z^3 == 8 * I, z]]
```

Out[#]=

$\{\{z \to -2\,i\}, \{z \to i + \sqrt{3}\}, \{z \to i - \sqrt{3}\}\}$

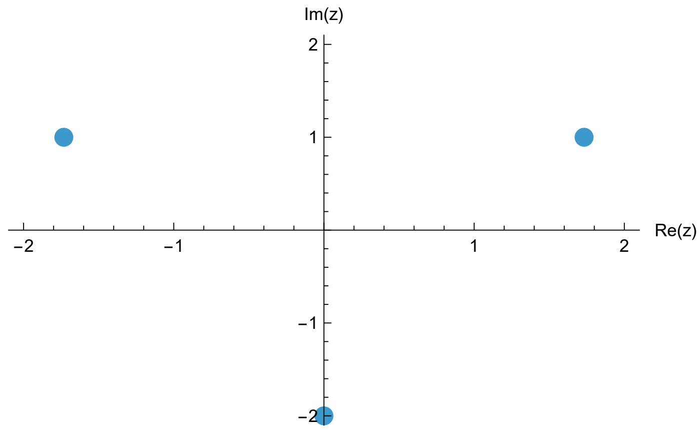
```
z /. roots5
```

Out[#]=

$\{-2\,i, i + \sqrt{3}, i - \sqrt{3}\}$

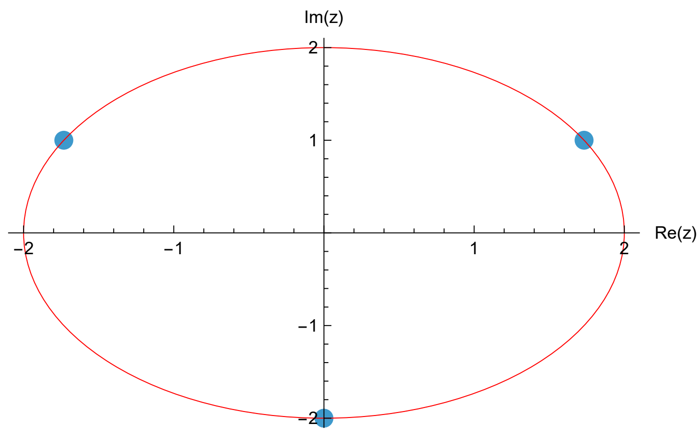
```
rootPlot5 = ListPlot[{Re[z], Im[z]} /. roots5, PlotRange → {{-2.1, 2.1}, {-2.1, 2.1}},
AxesLabel → {"Re(z)", "Im(z)"}, PlotStyle → PointSize[0.03]]
```

Out[∘]=




```
Show[rootPlot5, Graphics[{Red, Circle[{0, 0}, 2]}]]
```

Out[∘]=



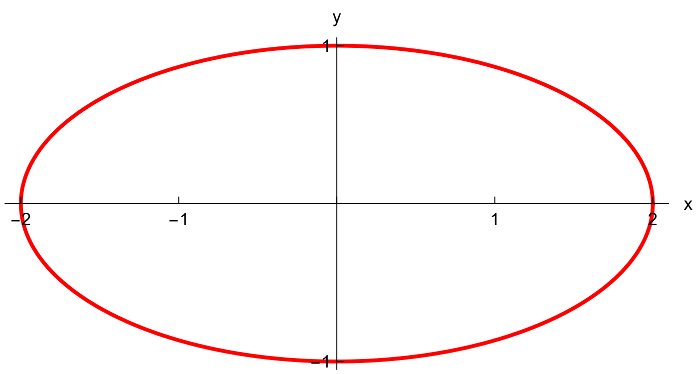
Practical 3

Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major-axis of 4 units and vertical minor-axis of 2 units.
Show the effect of rotation of this ellipse by an angle of Pi/6 radians and shifting of the center from (0,0) to (2,i) by making a parametric plot.

In[33]:=
```
Clear[r, s, t];
```

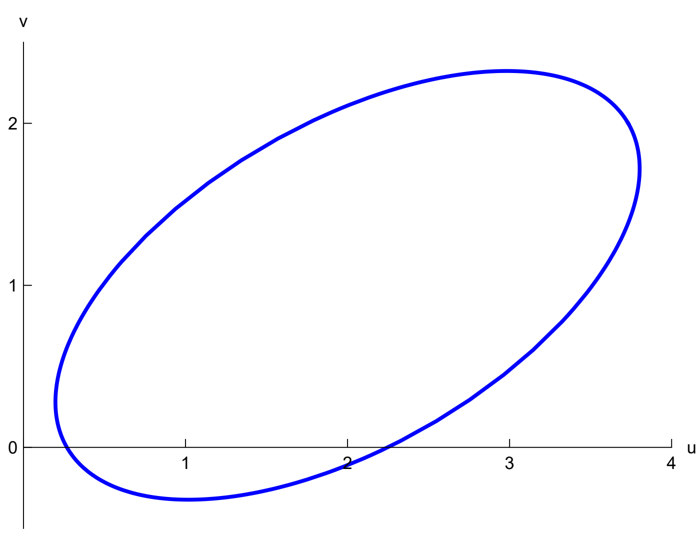
```
s[t_] = 2 Cos[t] + I * Sin[t];
r[t_] = s[t] * Exp[I * Pi / 6] + (2 + I);
ParametricPlot[{Re[s[t]], Im[s[t]]}, {t, 0, 2 * Pi}, PlotStyle → Red,
PlotRange → {{-2.1, 2.1}, {-1.05, 1.05}}, AspectRatio → 1 / 2,
Ticks → {Range[-2, 2, 1], Range[-2, 2, 1]}, AxesLabel → {"x", "y"}]
Print["s[t]=", s[t], "for 0<=t<=2*Pi"];
ParametricPlot[{Re[r[t]], Im[r[t]]}, {t, 0, 2 * Pi}, PlotStyle → Blue,
PlotRange → {{0, 4}, {-0.5, 2.5}}, AspectRatio → 3 / 4,
Ticks → {Range[0, 4, 1], Range[0, 3, 1]}, AxesLabel → {"u", "v"}]
Print["r[t]=", r[t], " for 0<=t<=2*Pi"]
```

Out[#]=



s[t]=2 Cos[t] + i Sin[t]for 0<=t<=2*Pi

Out[#]=



r[t] = (2 + i) + $e^{\frac{i\pi}{6}}$ (2 Cos[t] + i Sin[t]) for 0<=t<=2*Pi

Practical 4

Show that the image of the open disk D I ={z:|z+1+i<1|} under the linear transformation w= f(z)=(3-4i)z+6+2i is the open disk D2={w:|w+1-3i|<5}. Procedure: If w=f(z) then w=(3-4i)z+6+2i this impliesz=(w-6-2i)/(3-4i)

Now |z+1+i|<1 can be written as |(w-6-2i)/(3-4i)+1+i|<1
Plotting of f(D I) is equivalent to plotting of all those f(z) for which |z+1+i|<1 or in terms of w we can say that it is equivalent to plotting of w for which |(w-6-2i)/(3-4i)+1+i|<1.

```
z = x + i * y
Solve[w1 == (3 - 4 i) z1 + 6 + 2 i, z1]
```

Out[•]=

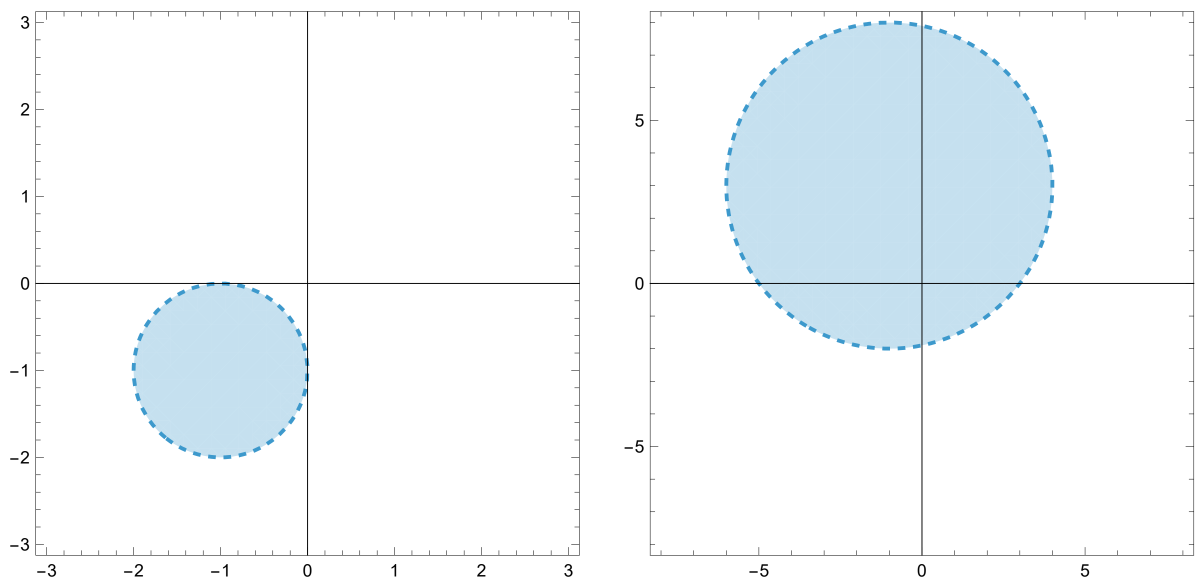
x + i y

Out[•]=

$$\left\{\left\{z1 \to \frac{1}{25} \left(-10 + 3\, w1 + 2\, i \left(-15 + 2\, w1\right)\right)\right\}\right\}$$

```
A1 = RegionPlot[Abs[z + 1 + i] < 1,
{x, -3, 3}, {y, -3, 3}, BoundaryStyle → Dashed, Axes → True];
A2 = RegionPlot[Abs[((z - 6 - 2 i) / (3 - 4 i)) + 1 + i] < 1, {x, -8, 8},
{y, -8, 8}, BoundaryStyle → Dashed, Axes → True];
GraphicsRow[{A1, A2}]
```

Out[•]=



Practical 5

Show that the image of the right half plane Re(z)=x>1 under the linear transformation w=(-1+i)z-2+3i is the half plane v>u+7, where u=Re(w), etc. Plot the map. Procedure= If w=f(z)=(-1+i)z-2+3i, then z=(w+2-3i)/(-1+i)={(u+2)+i(v-3)}/(-1+i=[(-u+v-5)+i(-u-v+1)]/2, so that Re(z)=x>1 => v>u+7.

```
z = x + i * y
Solve[w1 == (-1 + i) z1 - 2 + 3 i, z1]
```

Out[•]=

x + i y

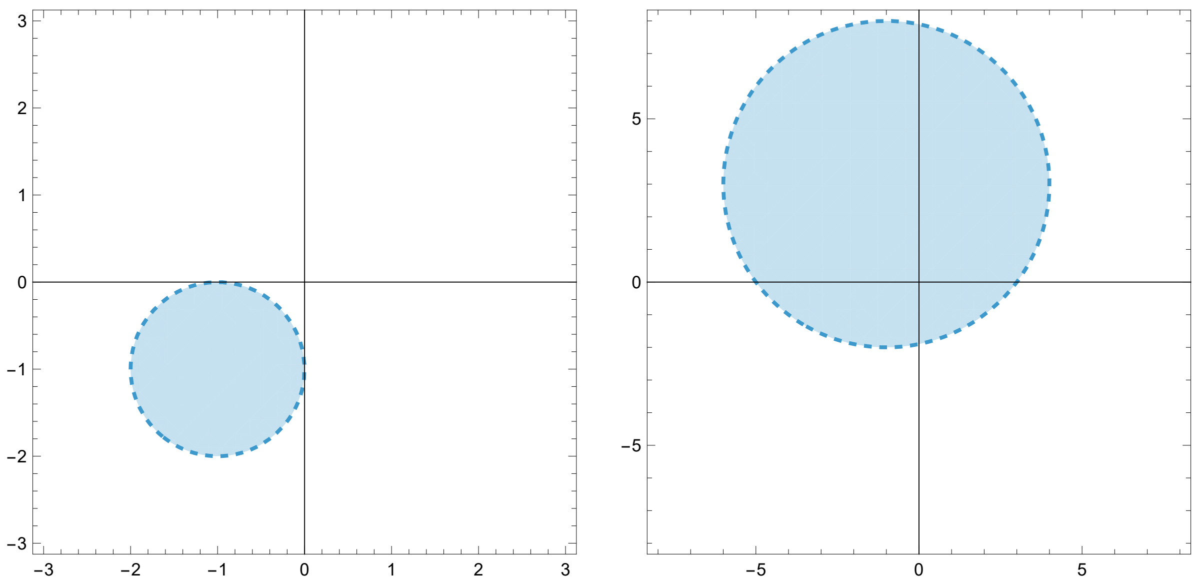
Out[•]=

$\left\{\left\{z1 \to \frac{1}{2}\,(-5 + i\,(1 - w1) - w1)\right\}\right\}$

In[8]:=

```
A3 = RegionPlot[Re[z] > 1, {x, -5, 5}, {y, -5, 5}, Axes → True];
A4 = RegionPlot[Re[(z + 2 - 3 i) / (-1 + i)] > 1, {x, -7, 7}, {y, -7, 7} Axes → True];
GraphicsRow[{A1, A2}]
```

Out[•]=



Practical 6

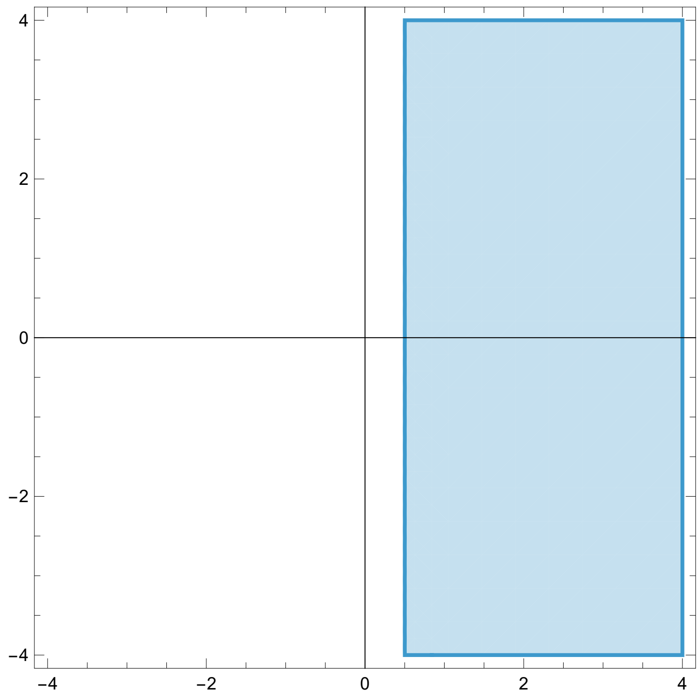
Show that the image of the right half-plane A = {z: Re z ≥ 1/ 2} under the mapping $w = f(z) = 1/z$ is the closed disk D_1(1)bar = {w: |w − 1| ≤ 1} in the w- plane.

```
z = x + ⅈ * y
A1 = RegionPlot[Re[z] ≥ 1 / 2, {x, -4, 4}, {y, -4, 4}, Axes → True]
```

Out[∘]=

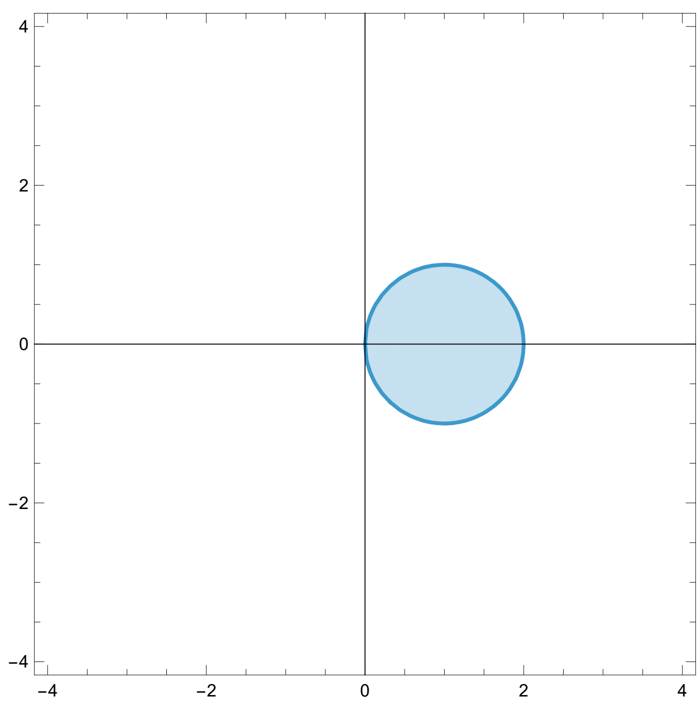
```
x + ⅈ y
```

Out[∘]=



```
A11 = RegionPlot[Re[1 / z] ≥ 1 / 2, {x, -4, 4}, {y, -4, 4}, Axes → True]
```

Out[∘]=



Practical 7

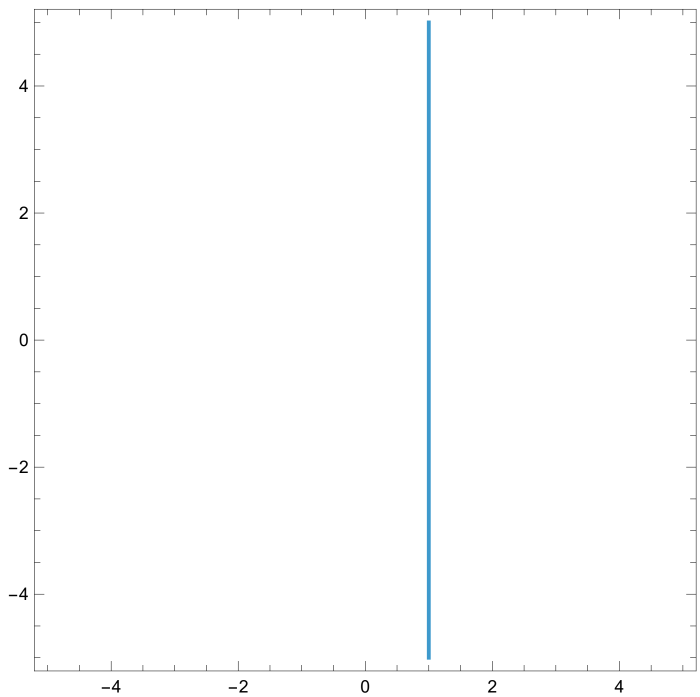
Make a plot of the vertical lines x = a, for $a = -1, -1/2, 1/2, 1$ and the horizontal lines y = b, for $b = -1, -1/2, 1/2, 1$. Find the plot of this grid under the mapping $f(z) = 1/z$.

```
z = x + i * y
A2 = ContourPlot[Re[z] == 1, {x, -5, 5}, {y, -5, 5}]
```

Out[ ]=

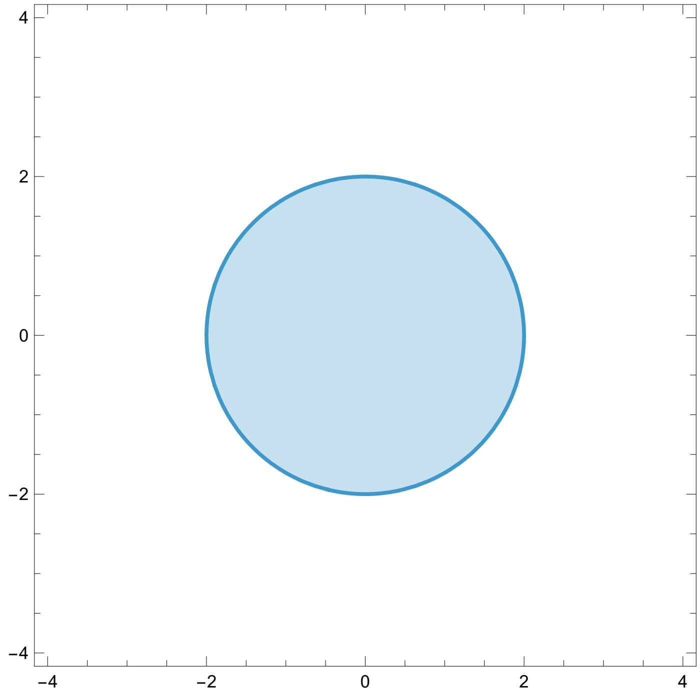
x + i y

Out[ ]=



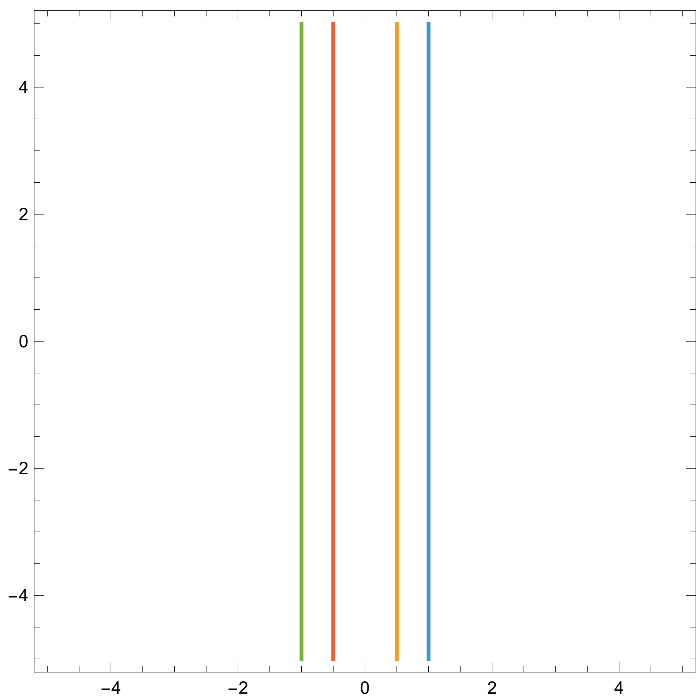
```
RegionPlot[Abs[z] ≤ 2, {x, -4, 4}, {y, -4, 4}]
```

Out[*]=



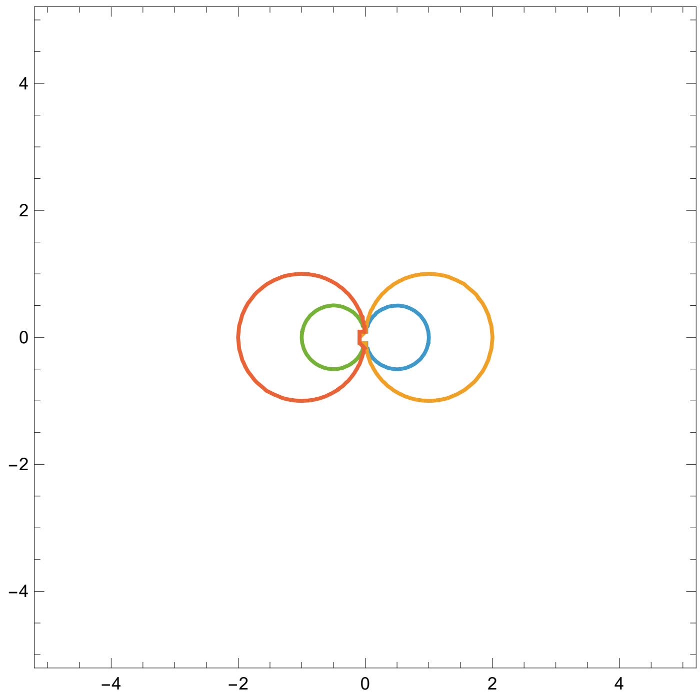

```
S1 = ContourPlot[{Re[z] == 1, Re[z] == 1 / 2, Re[z] == -1, Re[z] == -1 / 2}, {x, -5, 5}, {y, -5, 5}]
```

Out[*]=



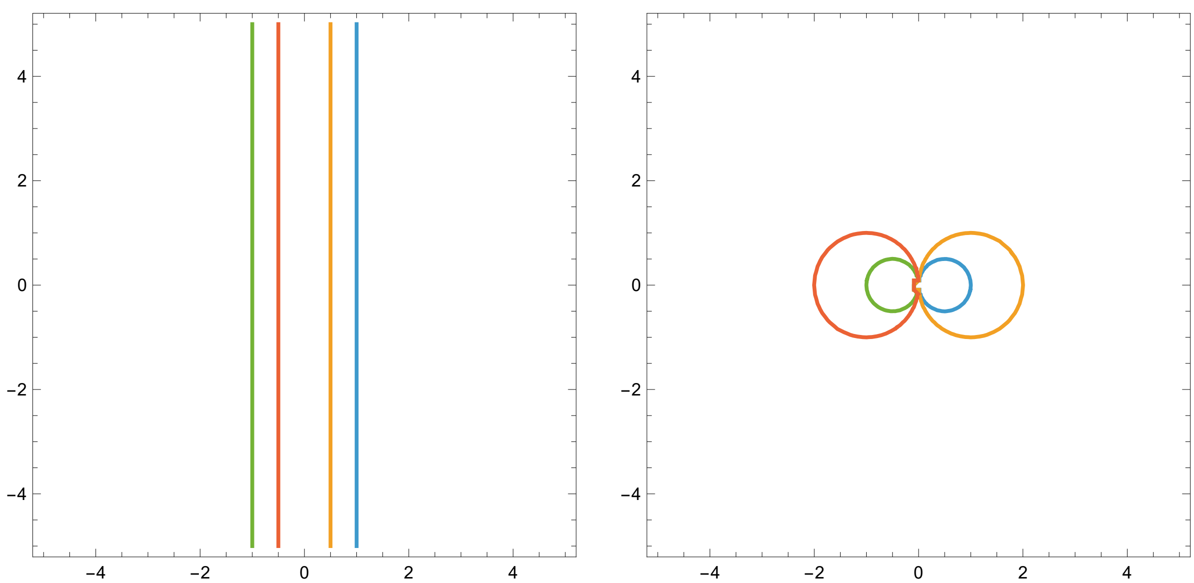
```
S2 = ContourPlot[
  {Re[1 / z] == 1, Re[1 / z] == 1 / 2, Re[1 / z] == -1, Re[1 / z] == -1 / 2}, {x, -5, 5}, {y, -5, 5}]
```

Out[#]=



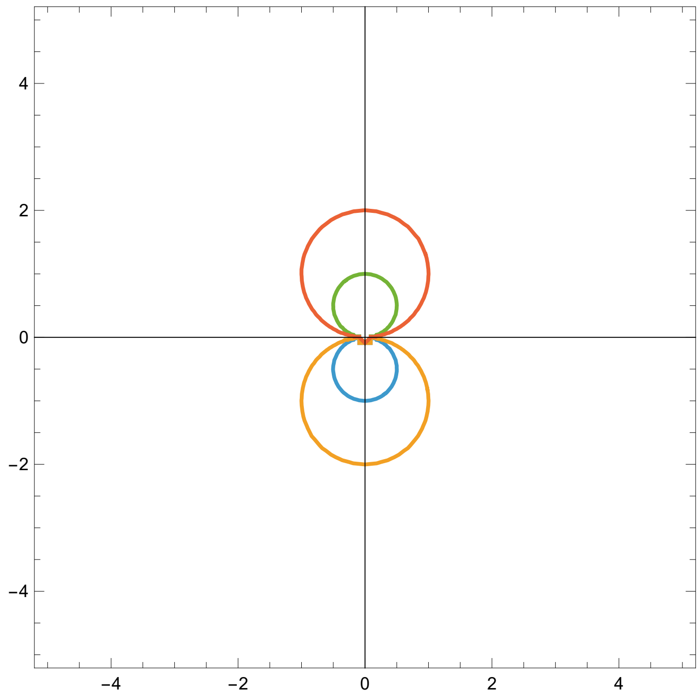
```
GraphicsRow[{S1, S2}]
```

Out[#]=



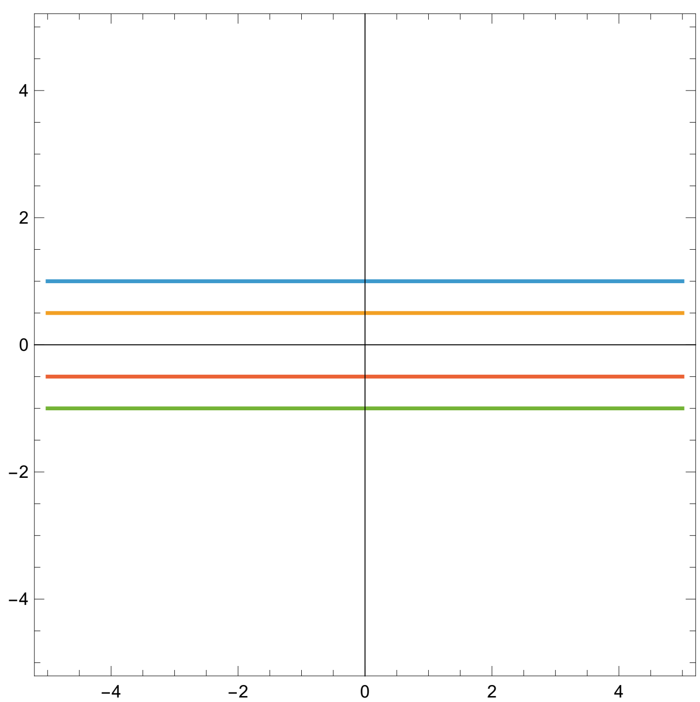
```
S3 = ContourPlot[{Im[1 / z] == 1, Im[1 / z] == 1 / 2, Im[1 / z] == -1, Im[1 / z] == -1 / 2},
  {x, -5, 5}, {y, -5, 5}, Axes → True]
```

Out[∘]=



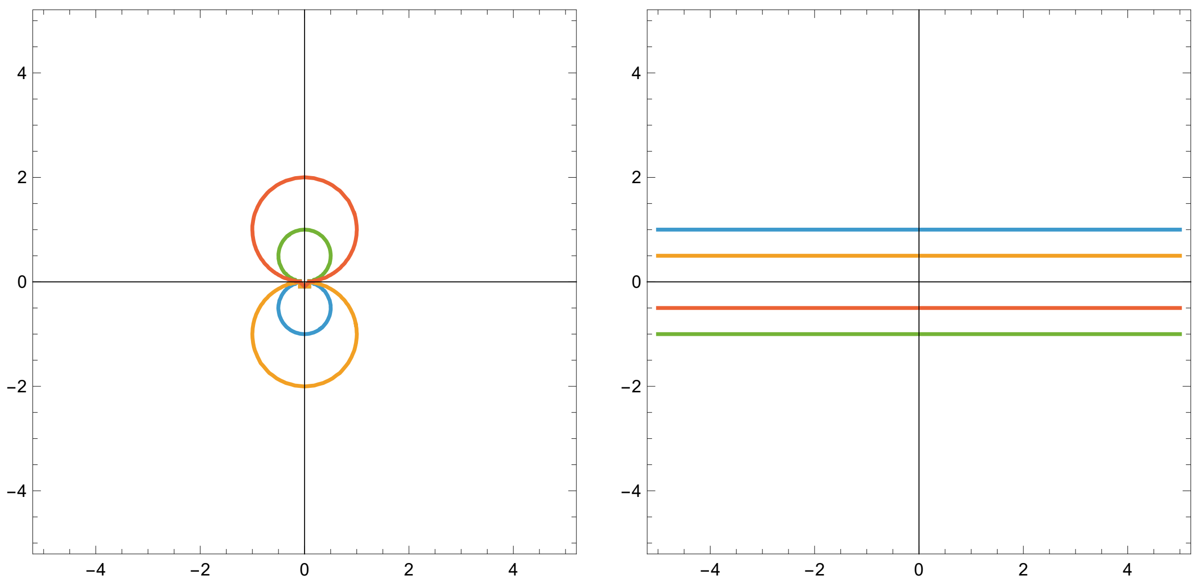

```
S4 = ContourPlot[{Im[z] == 1, Im[z] == 1 / 2, Im[z] == -1, Im[z] == -1 / 2},
  {x, -5, 5}, {y, -5, 5}, Axes → True]
```

Out[∘]=



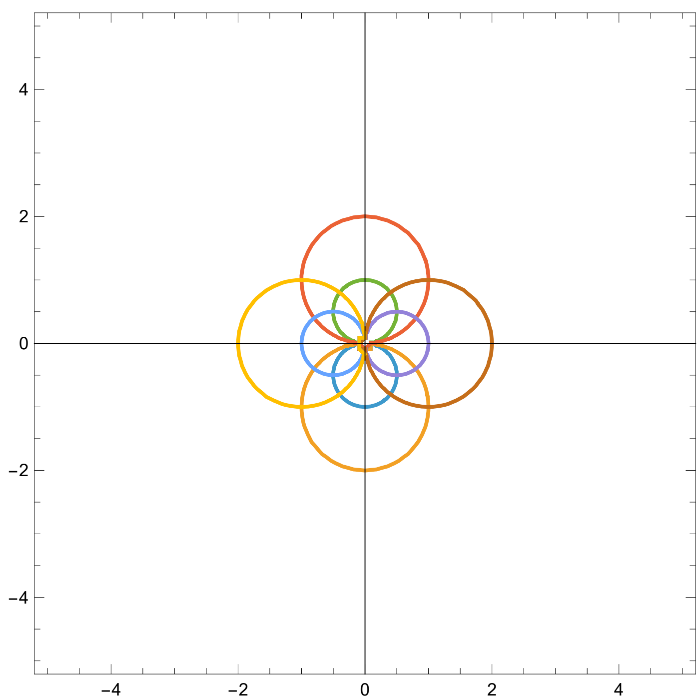
```
GraphicsRow[{S3, S4}]
```

Out[∘]=




```
S5 =
 ContourPlot[{Im[1 / z] == 1, Im[1 / z] == 1 / 2, Im[1 / z] == -1, Im[1 / z] == -1 / 2, Re[1 / z] == 1,
   Re[1 / z] == 1 / 2, Re[1 / z] == -1, Re[1 / z] == -1 / 2}, {x, -5, 5}, {y, -5, 5}, Axes → True]
```

Out[∘]=

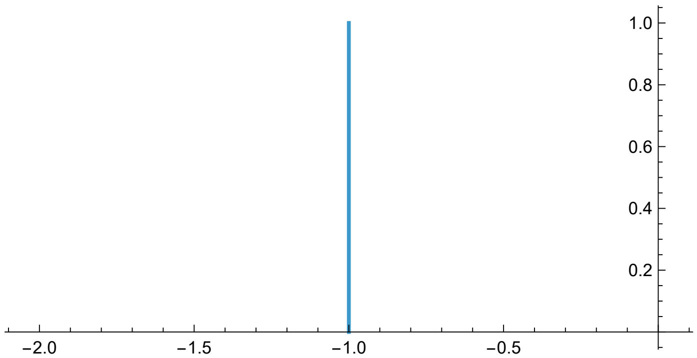


Practical 8

Find a parametrization of the polygonal path C = C1 + C2 + C3 from −1 + i to 3 − i, where C1 is the line from: $-1 + i$ to -1, C2 is the line from: -1 to $1 + i$ and C3 is the line from $1 + i$ to $3 - i$. Make a plot of this path.

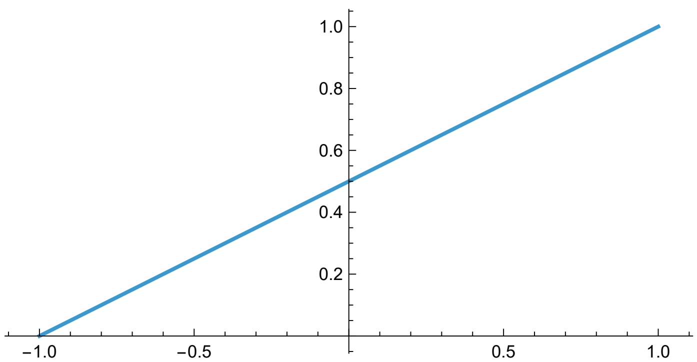
```
C1 = ParametricPlot[{-1, 1 - t}, {t, 0, 1}, Axes → True]
```

Out[∘]=



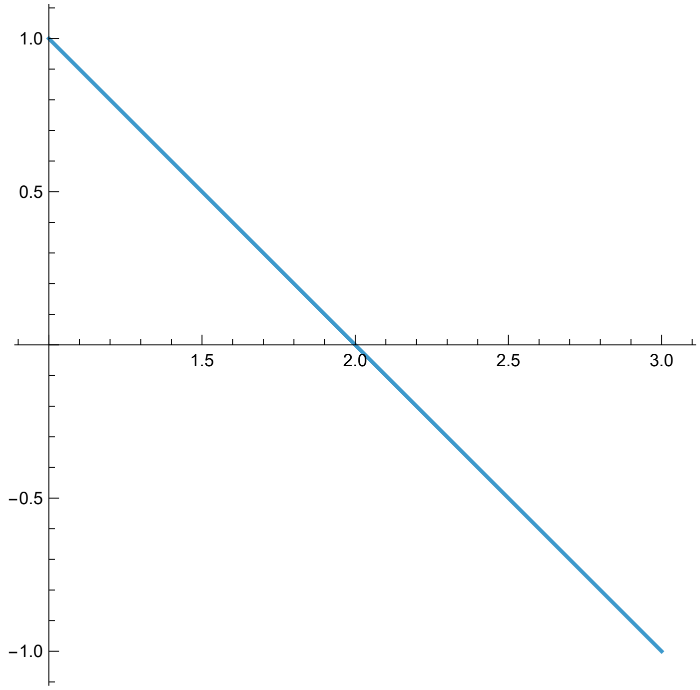

```
C2 = ParametricPlot[{-1 + 2 t, t}, {t, 0, 1}, Axes → True]
```

Out[∘]=



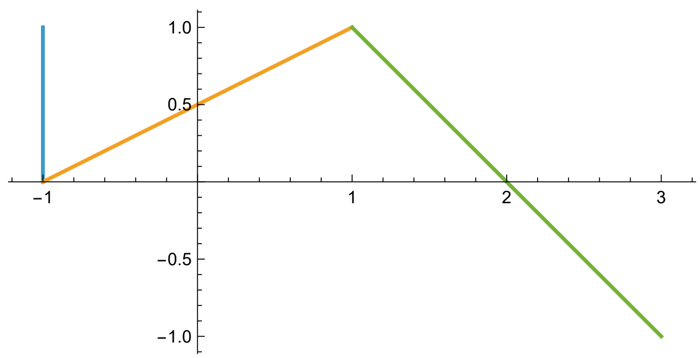
```
C3 = ParametricPlot[{1 + 2 t, 1 - 2 t}, {t, 0, 1}, Axes → True]
```

Out[ ]=




```
ParametricPlot[{{-1, 1 - t}, {-1 + 2 t, t}, {1 + 2 t, 1 - 2 t}}, {t, 0, 1}, Axes → True]
```

Out[ ]=

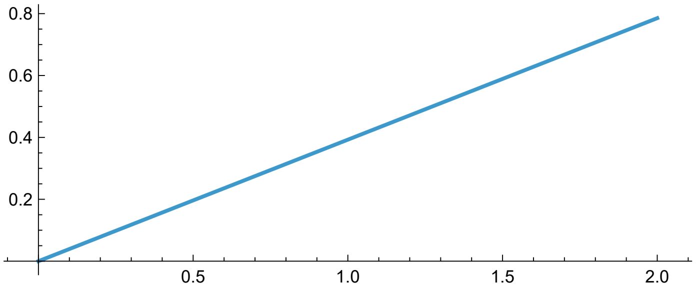


Practical 9

Plot the line segment ‘L’ joining the point A = 0 to B = 2 + $\pi/4$ of $\int e\hat{}zdz$.

```
ParametricPlot[{2*t, Pi*t/4}, {t, 0, 1}]
```

Out[]=




```
f[z_] := Exp[z]
g[t_] := 2*t + i*Pi*t/4
a = Integrate[f[g[t]] * g'[t], {t, 0, 1}]
```

Out[]=

$-1 + (-1)^{1/4} e^2$

```
N[a]
```

Out[]=

4.22485 + 5.22485 i

Practical 10

Evaluate $\int 1/\, z{-}2\; dz$, where C is the upper semicircle with radius 1 centered at z = 2 oriented in a positive direction.

```
f[z_] := 1/(z - 2)
g[t_] := 2 + Exp[i*t]
b = Integrate[f[g[t]] * g'[t], {t, 0, Pi}]
```

Out[]=

$i\,\pi$

```
N[b]
```

Out[]=

0. + 3.14159 i

Practical 11 (a)

Show that $\int zdz = \int zzddzz = 4{+}2i$, where C1 is the line segment from –1 – i to 3 + i and C2 is the portion of the parabola x = y2 + 2y joining –1 – i to 3 + i. Make plots of two contours C1 and C2 joining –1 – i to 3 + i .

```
f[z_] := z
g[t_] := -1 + 4 t + (-1 + 2*t) i
c = Integrate[f[g[t]] * g'[t], {t, 0, 1}]
```

Out[]=

4 + 2 i

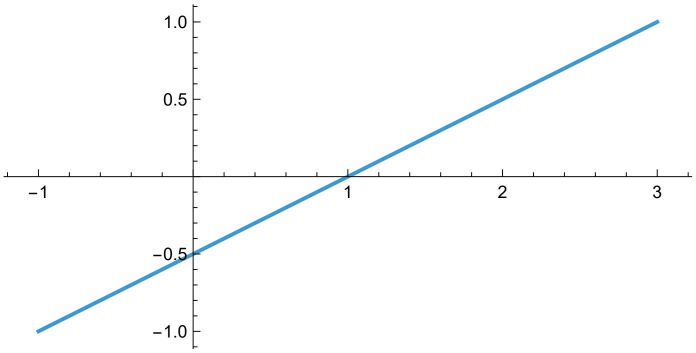
```
N[c]
```

Out[#]=

4. + 2. i

```
ParametricPlot[{-1 + 4 t, -1 + 2 t}, {t, 0, 1}]
```

Out[#]=



Practical 11 (b)

```
f[z_] := z
h[t_] := t^2 + 2 t + i * t
d = Integrate[f[h[t]] * h'[t], {t, -1, 1}]
```

Out[#]=

4 + 2 i

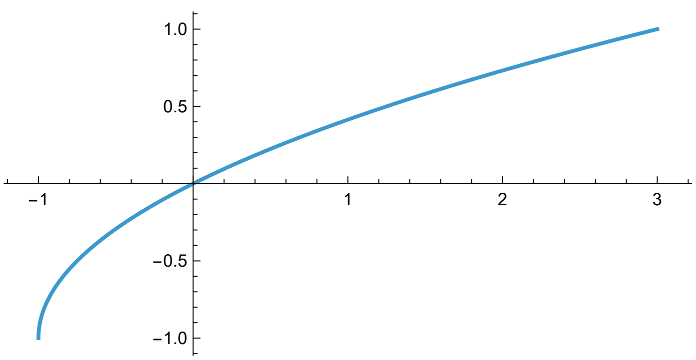
```
N[d]
```

Out[#]=

4. + 2. i

```
ParametricPlot[{t^2 + 2 t, t}, {t, -1, 1}]
```

Out[#]=



Practical 12

Use the ML inequality to show that $|\int 1/ z$^2+1 dz|<= 1/ 2 $\sqrt{5}$

, where C is the straight-line segment from 2 to 2 + i. While solving, represent the distance from the point z to the points i and – i, respectively, i.e., $|z - i|$ and $|z + i|$ on the complex plane $\mathbb{C}$.

```
f[z_] := 1 / (z^2 + 1)
g[t_] := 2 + i * t
va1 = Integrate[f[g[t]] * g'[t], {t, 0, 1}]
```

Out[ ]=

$$-\text{ArcTan}[2] + \text{ArcTan}[2 + i]$$

```
t1 = ComplexExpand[va1]
```

Out[ ]=

$$\frac{3\pi}{8} - \text{ArcTan}[2] + i\left(-\frac{\text{Log}[2]}{2} + \frac{\text{Log}[8]}{4}\right)$$

```
N[Abs[t1]]
```

Out[ ]=

0.187249

```
N[1 / 2 * Sqrt[5]]
```

Out[ ]=

1.11803

Practical 13 - Laurent Series

Find and plot three different Laurent series representations for the function:
$f(z) = 3/\ 2{+}z{-}z\text{^}2$, involving powers of z.

```
f[z_] := 3 / (2 + z - z^2)
Apart[f[z]] (*used to give partial fractions*)
```

Out[ ]=

$$-\frac{1}{-2 + z} + \frac{1}{1 + z}$$

```
? Series
```

Out[ ]=

Symbol

Series[f, {x, x_0, n}] generates a power series expansion for f about the point $x = x_0$ to order $(x - x_0)^n$, where n is an explicit integer.

Series[f, $x \to x_0$] generates the leading term of a power series expansion for f about the point $x = x_0$.

Series[f, {x, x_0, n_x}, {y, y_0, n_y}, …] successively finds series expansions with respect to x, then y, etc.

```
Series[f[z], {z, 0, 7}]
```

Out[ ]=

$$\frac{3}{2} - \frac{3z}{4} + \frac{9z^2}{8} - \frac{15z^3}{16} + \frac{33z^4}{32} - \frac{63z^5}{64} + \frac{129z^6}{128} - \frac{255z^7}{256} + O[z]^8$$

```
Series[f[z], {z, Infinity, 7}]
```

Out[#]=

$$-\frac{3}{z^2}-\frac{3}{z^3}-\frac{9}{z^4}-\frac{15}{z^5}-\frac{33}{z^6}-\frac{63}{z^7}+O\left[\frac{1}{z}\right]^8$$

```
In[9]:= f1[z_] := 1 / (1 + z)
```

```
In[11]:= f2[z_] := -1 / (z - 2)
```

```
t2 = Series[f1[z], {z, Infinity, 7}]
```

Out[#]=

$$\frac{1}{z}-\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3-\left(\frac{1}{z}\right)^4+\left(\frac{1}{z}\right)^5-\left(\frac{1}{z}\right)^6+\left(\frac{1}{z}\right)^7+O\left[\frac{1}{z}\right]^8$$

```
s1 = Normal[t2]
```

Out[#]=

$$\frac{1}{z^7}-\frac{1}{z^6}+\frac{1}{z^5}-\frac{1}{z^4}+\frac{1}{z^3}-\frac{1}{z^2}+\frac{1}{z}$$

```
t3 = Series[f2[z], {z, 0, 7}]
```

Out[#]=

$$\frac{1}{2}+\frac{z}{4}+\frac{z^2}{8}+\frac{z^3}{16}+\frac{z^4}{32}+\frac{z^5}{64}+\frac{z^6}{128}+\frac{z^7}{256}+O[z]^8$$

```
s2 = Normal[t3]
```

Out[#]=

$$\frac{1}{2}+\frac{z}{4}+\frac{z^2}{8}+\frac{z^3}{16}+\frac{z^4}{32}+\frac{z^5}{64}+\frac{z^6}{128}+\frac{z^7}{256}$$

```
Print["The Laurent series expansion of the given function in the domain 1<|z|<2 is",
  "...+", s1, "+", s2, "+..."]
```

The Laurent series expansion of the given function in the domain 1<|z|<2 is

$$...+\frac{1}{z^7}-\frac{1}{z^6}+\frac{1}{z^5}-\frac{1}{z^4}+\frac{1}{z^3}-\frac{1}{z^2}+\frac{1}{z}+\frac{1}{2}+\frac{z}{4}+\frac{z^2}{8}+\frac{z^3}{16}+\frac{z^4}{32}+\frac{z^5}{64}+\frac{z^6}{128}+\frac{z^7}{256}+...$$

Practical 14

Locate the poles of $f(z) = 1/\, 5z$^4+26z^2+5
and specify their order.

```
In[5]:= f[z_] := 1 / (5 * z^4 + 26 * z^2 + 5)
```

```
Apart[f[z]]
```

Out[#]=

$$-\frac{1}{24\left(5+z^2\right)}+\frac{5}{24\left(1+5 z^2\right)}$$

```
Solve[5 * z^4 + 26 * z^2 + 5 == 0, z]
```

Out[#]=

$$\left\{\left\{z \to -\frac{i}{\sqrt{5}}\right\}, \left\{z \to \frac{i}{\sqrt{5}}\right\}, \left\{z \to -i\sqrt{5}\right\}, \left\{z \to i\sqrt{5}\right\}\right\}$$

```
Series[f[z], {z, i / Sqrt[5], 7}]
```

Out[#]=

$$-\frac{i\sqrt{5}}{48\left(z-\frac{i}{\sqrt{5}}\right)}+\frac{25}{576}+\frac{185\, i\sqrt{5}\left(z-\frac{i}{\sqrt{5}}\right)}{6912}-\frac{5225\left(z-\frac{i}{\sqrt{5}}\right)^2}{82\,944}-$$
$$\frac{32\,725\, i\sqrt{5}\left(z-\frac{i}{\sqrt{5}}\right)^3}{995\,328}+\frac{965\,125\left(z-\frac{i}{\sqrt{5}}\right)^4}{11\,943\,936}+\frac{5\,848\,625\, i\sqrt{5}\left(z-\frac{i}{\sqrt{5}}\right)^5}{143\,327\,232}-$$
$$\frac{174\,670\,625\left(z-\frac{i}{\sqrt{5}}\right)^6}{1\,719\,926\,784}-\frac{1\,050\,548\,125\, i\sqrt{5}\left(z-\frac{i}{\sqrt{5}}\right)^7}{20\,639\,121\,408}+O\left[z-\frac{i}{\sqrt{5}}\right]^8$$

```
Residue[f[z], {z, i / Sqrt[5]}]
```

Out[#]=

$$-\frac{i\sqrt{5}}{48}$$

Observation - Pole of order 1 and residue = -i/Sqrt[5], complete practical 14

Practical 15

Locate the zeros and poles of $g(z) = \pi\cot(\pi z)/z$^2 and determine their order.Also justify that Res $(g, 0) = -\pi^2/3$.

In[13]:=
```
g[z] := Pi * Cos[Pi * z] / (z^2 * Sin [Pi * z])
```

```
Series[g[z], {z, 0, 5}]
```

Out[#]=

$$\frac{1}{z^3}-\frac{\pi^2}{3z}-\frac{\pi^4 z}{45}-\frac{2\pi^6 z^3}{945}-\frac{\pi^8 z^5}{4725}+O[z]^6$$

```
Residue[g[z], {z, 0}]
```

Out[#]=

$$-\frac{\pi^2}{3}$$

```
Series[g[z], {z, 2, 7}]
```

Out[#]=

$$\frac{1}{4\,(z-2)}-\frac{1}{4}+\frac{1}{48}\left(9-4\,\pi^2\right)\,(z-2)+\left(-\frac{1}{8\,\pi}+\frac{\pi}{12}\right)\pi\,(z-2)^2+\pi\left(\frac{5}{64\,\pi}-\frac{\pi}{16}-\frac{\pi^3}{180}\right)(z-2)^3+$$
$$\pi\left(-\frac{3}{64\,\pi}+\frac{\pi}{24}+\frac{\pi^3}{180}\right)(z-2)^4+\pi\left(\frac{7}{256\,\pi}-\frac{5\,\pi}{192}-\frac{\pi^3}{240}-\frac{\pi^5}{1890}\right)(z-2)^5+$$
$$\pi\left(-\frac{1}{64\,\pi}+\frac{\pi}{64}+\frac{\pi^3}{360}+\frac{\pi^5}{1890}\right)(z-2)^6+\pi\left(\frac{9}{1024\,\pi}-\frac{7\,\pi}{768}-\frac{\pi^3}{576}-\frac{\pi^5}{2520}-\frac{\pi^7}{18\,900}\right)(z-2)^7+O[z-2]^8$$

```
Residue[g[z], {z, 2}]
```

Out[#]=

$$\frac{1}{4}$$

Pole 2 is of order 1