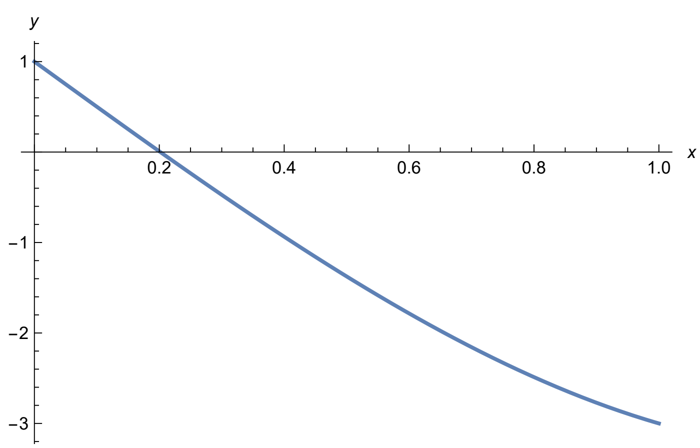
Practical 1 - Bisection Method

In[∘]:=
```
Bisection[a0_, b0_, k_] :=
 Module[{},
  a = N[a0];
  b = N[b0];
  m = (a + b) / 2;
  i = 0;
  output = {{i, a, b, m, f[a], f[b], f[m], Sign[f[a] * f[m]]}};
  While[i < k, If[Sign[f[b]] * Sign[f[m]] < 0, a = m, b = m;];
   m = (a + b) / 2;
   i = i + 1;
   output = Append[output, {i, a, b, m, f[a], f[b], f[m], Sign[f[a] * f[m]]}];];
  Print[
   NumberForm[
    TableForm[output,
     TableHeadings → {None,
        {"i", "a0", "a1", "a2", "f[a0]", "f[a1]", "f[a2]", "Sign[f[a0]*f[a2]]"}}], 16]];
  Print["m= ", NumberForm[m, 16]];]
```

In[∘]:=
```
f[x_] := x^3 - 5 * x + 1
Plot[f[x], {x, 0, 1}, PlotStyle → {Thick}, AxesLabel → {x, y}]
Print["y=f[x]= ", f[x]];
Bisection[0, 1, 5]
```

Out[∘]=



y=f[x]= $1 - 5x + x^3$

i	a0	a1	a2	f[a0]	f[a1]	f[a2]
0	0.	1.	0.5	1.	−3.	−1.375
1	0.	0.5	0.25	1.	−1.375	−0.234375
2	0.	0.25	0.125	1.	−0.234375	0.376953125
3	0.125	0.25	0.1875	0.376953125	−0.234375	0.069091796875
4	0.1875	0.25	0.21875	0.069091796875	−0.234375	−0.083282470703125
5	0.1875	0.21875	0.203125	0.069091796875	−0.083282470703125	−0.007244110107421875

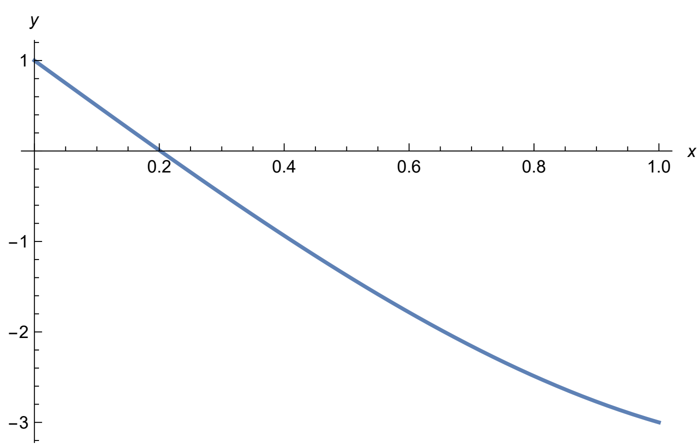
m= 0.203125

Practical 2 - Newton Raphson

In[◦]:=
```
NewtonRaphson[x0_, n_, f_] := Module[{xk1, xk = N[x0]}, k = 0;
   Output = {{k, x0, f[x0]}};
   While[k < n, fPrimexk = f'[xk];
    If[fPrimexk == 0, Print["The Derivative of function at ", k,
      "th iteration is zero, we cannot proceed further with the iterative scheme"];
     Break[]];
    xk1 = xk - f[xk] / fPrimexk;
    xk = xk1;
    k++;
    Output = Append[Output, {k, xk, f[xk]}];];
   Print[
    NumberForm[TableForm[Output, TableHeadings → {None, {"k", "xk", "f[xk]"}}], 10]];
   Print["Root After", n, "iterations xk=", NumberForm[xk, 10]];
   Print["Function value at approximated root, f[xk]=", NumberForm[f[xk], 6]];];
```

In[◦]:=
```
f[x_] := x^3 - 5 * x + 1
Plot[f[x], {x, 0, 1}, PlotStyle → {Thick}, AxesLabel → {x, y}]
Print["y=f[x]=", f[x]];
NewtonRaphson[0.5, 5, f]
```

Out[◦]=



y=f[x]=$1 - 5x + x^3$

k	xk	f[xk]
0	0.5	-1.375
1	0.1764705882	0.1231426827
2	0.2015680743	0.0003492763989
3	0.2016396751	$3.100484314 \times 10^{-9}$
4	0.2016396757	$1.110223025 \times 10^{-16}$
5	0.2016396757	$1.110223025 \times 10^{-16}$

Root After5iterations xk=0.2016396757

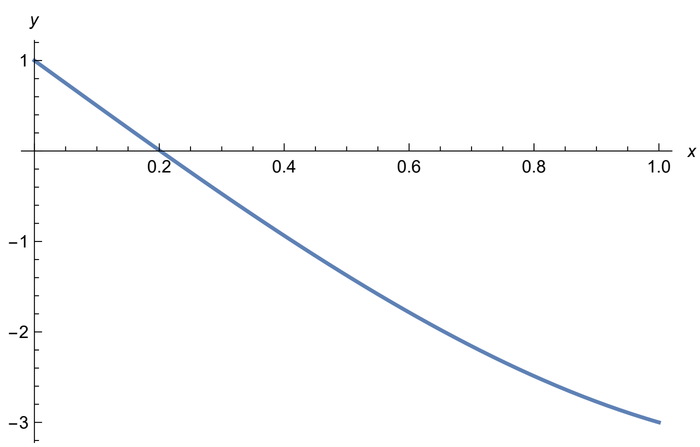
Function value at approximated root, f[xk]=1.11022×10^{-16}

Practical 3 - Secant Method

In[∘]:=
```
Secant[a0_, b0_, n_] :=
 Module[{},
  a = N[a0];
  b = N[b0];
  c = (a * f[b] - b * f[a]) / (f[b] - f[a]);
  i = 0;
  Output = {{i, a, b, c, f[c]}};
  While[i < n, a = b; b = c;
   c = (a * f[b] - b * f[a]) / (f[b] - f[a]);
   i = i + 1;
   Output = Append[Output, {i, a, b, c, f[c]}];];
  Print[
   NumberForm[
    TableForm[Output, TableHeadings → {None, {"i", "a", "b", "f[c]"}}], 16]];
  Print["c= ", NumberForm[c, 16]];]
```

In[∘]:=
```
f[x_] := x^3 - 5 * x + 1
Plot[f[x], {x, 0, 1}, PlotStyle → {Thick}, AxesLabel → {x, y}]
Print["y=f[x]= ", f[x]]
Secant[0, 1, 6]
```

Out[∘]=



y=f[x]= $1 - 5x + x^3$

i	a	b	f[c]	
0	0.	1.	0.25	−0.234375
1	1.	0.25	0.1864406779661017	0.07427731170178065
2	0.25	0.1864406779661017	0.2017362561791272	−0.0004711161687362964
3	0.1864406779661017	0.2017362561791272	0.2016398528913041	$-8.64229303898512 \times 10^{-7}$
4	0.2017362561791272	0.2016398528913041	0.2016396757212823	$1.035271868232712 \times 10^{-11}$
5	0.2016398528913041	0.2016396757212823	0.2016396757234047	$-2.220446049250313 \times 10^{-16}$
6	0.2016396757212823	0.2016396757234047	0.2016396757234046	$1.110223024625157 \times 10^{-16}$

c= 0.2016396757234046

Practical 4 - LU Decomposition

In[ ]:=
```
LUDecom[A0_, n_] :=
  Module[{A = A0, i, p}, U = A0; L = IdentityMatrix[n];
   Print[MatrixForm[L], MatrixForm[U], "=", MatrixForm[A0]];
   For[p = 1, p ≤ n - 1, p++,
    For[i = p + 1, i ≤ n, i++,
      m = A[[i,p]] / A[[p,p]];
      L[[i,p]] = m;
      A[[i]] = A[[i]] - m A[[p]];
      U = A;
      Print[MatrixForm[L], MatrixForm[U], "=", MatrixForm[A0]];];];];
Print["L", "=", MatrixForm[L]];
Print["U", "=", MatrixForm[U]];
A = (4  2 3
     -3 1 4
     2  4 5);
LUDecom[A, 3]
```

L=L

U=U

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 3 \\ 0 & \frac{5}{2} & \frac{25}{4} \\ 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 3 \\ 0 & \frac{5}{2} & \frac{25}{4} \\ 0 & 3 & \frac{7}{2} \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ \frac{1}{2} & \frac{6}{5} & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 3 \\ 0 & \frac{5}{2} & \frac{25}{4} \\ 0 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

Practical 5 - Gauss Jacobi

```
In[ ]:= Jacobi[A0_, B0_, X0_, max_] :=
  Module[{A = N[A0], B = N[B0], i, j, k = 0, n = Length[X0], X = X0, Xold = X0},
   Print["X", 0, "=", X];
   While[k < max, For[i = 1, i ≤ n, i = i + 1,
     X[[i]] = (B[[i]] + A[[i, i]] * Xold[[i]] - Sum[A[[i, j]] * Xold[[j]], {j, 1, n}]) / A[[i, i]]];
    Print["X", k + 1, "=", X];
    Xold = X;
    k = k + 1;];];
```

$$X[[i]] = \left(B[[i]] + A[[i, i]] * Xold[[i]] - \sum_{j=1}^{n} A[[i, j]] * Xold[[j]]\right) \Big/ A[[i, i]]$$

```
In[ ]:= A0 = {{4, 1, 0, 1}, {1, 4, 1, 0}, {0, 1, 4, 1}, {1, 0, 1, 4}};
B0 = {2, -2, 2, -2};
X0 = {0, 0, 0, 0};
Jacobi[A0, B0, X0, 10];
```

```
X0={0, 0, 0, 0}

X1={0.5, -0.5, 0.5, -0.5}

X2={0.75, -0.75, 0.75, -0.75}

X3={0.875, -0.875, 0.875, -0.875}

X4={0.9375, -0.9375, 0.9375, -0.9375}

X5={0.96875, -0.96875, 0.96875, -0.96875}

X6={0.984375, -0.984375, 0.984375, -0.984375}

X7={0.992188, -0.992188, 0.992188, -0.992188}

X8={0.996094, -0.996094, 0.996094, -0.996094}

X9={0.998047, -0.998047, 0.998047, -0.998047}

X10={0.999023, -0.999023, 0.999023, -0.999023}
```

Practical 6 - Gauss Seidel

In[ ]:=
```
Seidel[A0_, B0_, X0_, max_] :=
 Module[{A = N[A0], B = N[B0], i, j, k = 0, n = Length[X0], X = X0, Xold = X0},
  Print["X", 0, "=", X];
  While[k < max, For[i = 1, i ≤ n, i = i + 1,
     X[[i]] = (B[[i]] - Sum[A[[i, j]] * X[[j]], {j, 1, i-1}] - Sum[A[[i, j]] * Xold[[j]], {j, i+1, n}]) / A[[i, i]]];
   Print["x", k + 1, "=", NumberForm[X, 10]];
   Xold = X;
   k = k + 1;];];
```

$$X[\![i]\!] = \left(B[\![i]\!] - \sum_{j=1}^{i-1} A[\![i,j]\!] * X[\![j]\!] - \sum_{j=i+1}^{n} A[\![i,j]\!] * Xold[\![j]\!]\right) \Big/ A[\![i,i]\!]$$

In[ ]:=
```
A0 = {{2, -1, 0, 0}, {-1, 2, -1, 0}, {0, -1, 2, -1}, {0, 0, -1, 2}};
B0 = {1, 0, 0, 1};
X0 = {0.5, 0.5, 0.5, 0.5};
Seidel[A0, B0, X0, 10]
```

```
X0={0.5, 0.5, 0.5, 0.5}
x1={0.75, 0.625, 0.5625, 0.78125}
x2={0.8125, 0.6875, 0.734375, 0.8671875}
x3={0.84375, 0.7890625, 0.828125, 0.9140625}
x4={0.89453125, 0.861328125, 0.8876953125, 0.9438476563}
x5={0.9306640625, 0.9091796875, 0.9265136719, 0.9632568359}
x6={0.9545898438, 0.9405517578, 0.9519042969, 0.9759521484}
x7={0.9702758789, 0.9610900879, 0.9685211182, 0.9842605591}
x8={0.9805450439, 0.9745330811, 0.9793968201, 0.98969841}
x9={0.9872665405, 0.9833316803, 0.9865150452, 0.9932575226}
x10={0.9916658401, 0.9890904427, 0.9911739826, 0.9955869913}
```

Practical 7 - Lagranges Interpolation

```
In[◦]:= lagrange[x0_, f0_] :=
  Module[{xi = x0, fi = f0, n, m, polynomial},
   n = Length[xi];
   m = Length[fi];
   For[i = 1, i ≤ n, i = i + 1,
    L[i, x_] = (∏_{j=1}^{i-1} (x - xi[[j]]) / (xi[[i]] - xi[[j]])) ×
       (∏_{j=i+1}^{n} (x - xi[[j]]) / (xi[[i]] - xi[[j]]));];
   polynomial[x] = ∑_{k=1}^{n} L[k, x] * fi[[k]];
   Return[polynomial[x]];]
```

```
In[1]:= nodes = {0, 1, 3};
values = {1, 3, 55};
lagrangepolynomial[x_] = lagrange[nodes, values]
lagrangepolynomial[x_] = Simplify[lagrangepolynomial[x]]
Print["Lagrange Polynomial= ", lagrangepolynomial[x]]
lagrangepolynomial[2]

Out[3]= lagrange[{0, 1, 3}, {1, 3, 55}]

Out[4]= lagrange[{0, 1, 3}, {1, 3, 55}]

Lagrange Polynomial= lagrange[{0, 1, 3}, {1, 3, 55}]

Out[6]= lagrange[{0, 1, 3}, {1, 3, 55}]
```

Practical 8 - Newton interpolation

Question.Use the table given below to estimate the
value of f (1.5) through Newton ' s Interpolation Polynomial
i 0 × 1 × 2 × 3
x 1 × 2 × 3 × 4
log (xi) 0 × 0.3010 × 0.4771 × 0.6021

In[7]:=
```
sum = 0;
points = {{1, 0}, {2, 0.3010}, {3, 0.4771}, {4, 0.6021}};
n = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
dd[k_] := Sum[(f[[i]] / Product[If[Equal[j, i], 1, (y[[i]] - y[[j]])], {j, 1, k}]), {i, 1, k}]
p[x_] := Sum[(dd[i] * Product[If[i ≤ j, 1, x - y[[j]]], {j, 1, i - 1}]), {i, 1, n}]
Simplify[p[x]]
Evaluate[p[1.5]]
```

Out[9]= 4

Out[10]=
{1, 2, 3, 4}

Out[11]=
{0, 0.301, 0.4771, 0.6021}

Out[14]=
$-0.4997 + 0.62365\, x - 0.13625\, x^2 + 0.0123\, x^3$

Out[15]=
0.170725

Practical 9 - Trapezoidal Rule

In[ ]:=
```
Quit[]
```

In[ ]:=
```
Trap[a0_, b0_, n0_] := Module[{a = N[a0], b = N[b0], k, n = n0, X}, h = (b - a) / n;
  X_k_ = a + k * h;
  Return[h/2 (f[a] + f[b]) + h Sum_{k=1}^{n-1} f[X_k]];]
f[x_] := 1/(1 + x);
Print["The value of ∫_0^1 1/(1 + x) dx, using TR is: ", N[Trap[0, 1, 10]]]
```

The value of $\int_0^1 \frac{1}{1+x}\,dx$, using TR is: 0.693771

Practical 10 - Simpsons Rule

In[ ]:=
```
Simp[a0_, b0_, n0_] := Module[{a = N[a0], b = N[b0], k, n = n0, X}, h = (b - a) / (2 * n);
  X_k_ = a + k * h;
  Return[h/3 (f[a] + f[b]) + (2 h)/3 Sum[f[X_2k], {k, 1, n-1}] + (4 h)/3 * Sum[f[X_2k-1], {k, 1, n}];]
f[x_] := 1/(1 + x);
Print["The value of ∫_0^1 1/(1 + x) dx, using Simpsons Rule is: ", N[Simp[0, 1, 2]]]
```

$$\text{Simp}[a0\_, b0\_, n0\_] := \text{Module}\Big[\{a = N[a0], b = N[b0], k, n = n0, X\}, h = (b - a) / (2 * n); X_{k\_} = a + k * h; \text{Return}\Big[\frac{h}{3}(f[a] + f[b]) + \frac{2\,h}{3}\sum_{k=1}^{n-1} f[X_{2\,k}] + \frac{4\,h}{3} * \sum_{k=1}^{n} f[X_{2\,k-1}]\Big];\Big]$$

$$f[x\_] := \frac{1}{1 + x};$$

$$\text{Print}\Big[\text{"The value of } \int_0^1 \frac{1}{1+x}\,dx\text{, using Simpsons Rule is: "}, N[\text{Simp}[0, 1, 2]]\Big]$$

The value of $\int_0^1 \frac{1}{1+x}\,dx$, using Simpsons Rule is: 0.693254

Practical 11 - Eulers Method

In[ ]:=
```
Euler[{t_, y_}] := {t + h, y + h * f[t, y]}
f[t_, y_] := 1 + (y / t)
h = 0.5
points = NestList[Euler, {1, 1}, 10]
TableForm[points, TableHeadings → {Automatic, {"t_i", "y_i"}}]
```

Out[ ]=

0.5

Out[ ]=

{{1, 1}, {1.5, 2.}, {2., 3.16667}, {2.5, 4.45833}, {3., 5.85}, {3.5, 7.325}, {4., 8.87143}, {4.5, 10.4804}, {5., 12.1448}, {5.5, 13.8593}, {6., 15.6193}}

Out[ ]//TableForm=

	t_i	y_i
1	1	1
2	1.5	2.
3	2.	3.16667
4	2.5	4.45833
5	3.	5.85
6	3.5	7.325
7	4.	8.87143
8	4.5	10.4804
9	5.	12.1448
10	5.5	13.8593
11	6.	15.6193

Practical 12 - RK Method

```
Quit[]
RK4[a0_, b0_, α_, m0_] :=
 Module[ {a = a0, b = b0, j, m = m0}, h = (b - a) / m;
Y = T = Table[0, {m + 1}];
T[[1]] = a;
Y[[1]] = α;
For[j = 1, j ≤ m, j++,
K1 = h f[T[[j]], Y[[j]]];
K2 = h f[T[[j]] + h/2, Y[[j]] + K1/2 ];
K3 = h f[T[[j]] + h/2, Y[[j]] + K2/2 ];
K4 = h * f[T[[j]] + h, Y[[j]] + K3];
Y[[j + 1]] = Y[[j]] + 1/6 (K1 + 2 * K2 + 2 * K3 + K4);
T[[j + 1]] = a + h j; ];
Return[Transpose[{T, Y}]] ]
f[t_, y_] := 1 + y/t;
points = RK4[1, 5, 1, 8];
TableForm[points, TableHeadings → {Automatic, {"ti", "yi3"}}]
```

Syntax: "RK4[a0_, b0_, α_, m0_] := Module[{a = a0, b = b0, j, m = m0}, h = (b – a) / m;" is incomplete; more input is needed.