Complex Expand Command

Command Complex Expand

ComplexExpand[(x + I y) ^ 3]

$x^3 - 3 x y^2 + i (3 x^2 y - y^3)$

ComplexExpand[Sin[x + I y]]

Cosh[y] Sin[x] + i Cos[x] Sinh[y]

ComplexExpand[Sin[x + I y] ^ 3]

$\text{Cosh}[y]^3 \text{Sin}[x]^3 - 3 \text{Cos}[x]^2 \text{Cosh}[y] \text{Sin}[x] \text{Sinh}[y]^2 + i (3 \text{Cos}[x] \text{Cosh}[y]^2 \text{Sin}[x]^2 \text{Sinh}[y] - \text{Cos}[x]^3 \text{Sinh}[y]^3)$

$$\textbf{ComplexExpand}\left[\frac{12}{\sqrt{3} + I}\right]$$

$-3 i + 3 \sqrt{3}$

$$\textbf{ComplexExpand}\left[(1 + I)\left(\sqrt{3} + I\right)\right]$$

$-1 + \sqrt{3} + i (1 + \sqrt{3})$

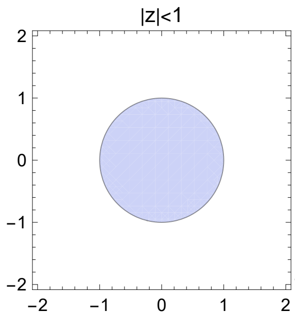
$$\textbf{ComplexExpand}\left[\left\{(2 + I)\frac{(2 - I)^2}{(1 + 2 I)^2}, \frac{\left(\sqrt{3} - I\right)^2 (1 + I)^2}{\left(\sqrt{3} + I\right)^2}\right\}\right]$$

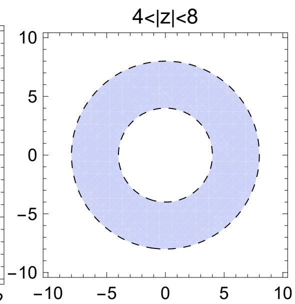
$\{-2 - i, -i + \sqrt{3}\}$

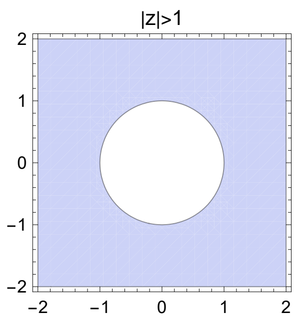
Plot the following regions
1. |z|<1
2. 4<|z|<8
3. Absolute(x+iy),x>1

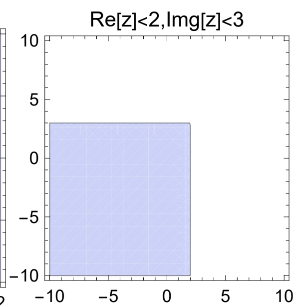
4. Re(z)<2, Img(z)<3
5. Absolute[(z-1)/(z+1)>=2
6. $z\bar{z}$=1
7. |z|=1
8. |z-(1+i)|=1

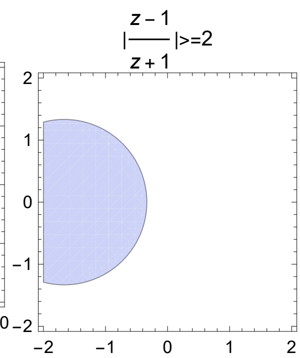
```
z = x + I y;
A1 = RegionPlot[Abs[z] < 1, {x, −2, 2}, {y, −2, 2}, ImageSize → 150, PlotLabel → "|z|<1"];
A2 = RegionPlot[4 < Abs[z] < 8, {x, −10, 10}, {y, −10, 10}, ImageSize → 150,
    BoundaryStyle → Dashed, PlotLabel → "4<|z|<8"];
A3 = RegionPlot[Abs[x + I * y] > 1, {x, −2, 2}, {y, −2, 2}, ImageSize → 150,
    PlotLabel → "|z|>1"];
A4 = RegionPlot[Re[z] < 2 && Im[z] < 3, {x, −10, 10}, {y, −10, 10}, ImageSize → 150,
    PlotLabel → "Re[z]<2,Img[z]<3"];
A5 = RegionPlot[Abs[(z − 1) / (z + 1)] ≥ 2, {x, −2, 2}, {y, −2, 2}, ImageSize → 150,
    PlotLabel → "|(z − 1)/(z + 1)|>=2"];
Print[A1, A2, A3, A4, A5]
A6 = ContourPlot[z * Conjugate[z] − 1 == 1, {x, −2, 2},
    {y, −2, 2}, Axes → True, ImageSize → 150, PlotLabel → "zz̄=1"];
A7 = ContourPlot[Abs[z] == 1, {x, −2, 2},
    {y, −2, 2}, Axes → True, ImageSize → 150, PlotLabel → "|z|=1"];
```



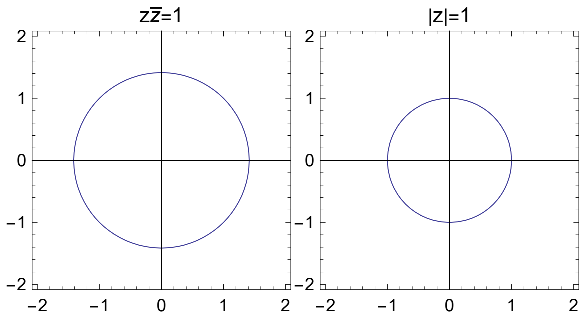






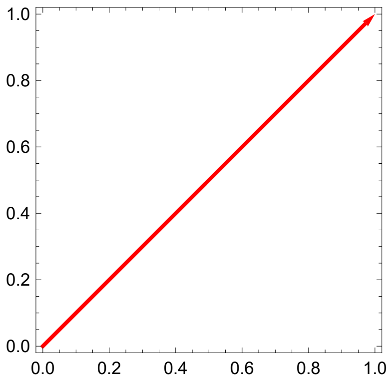


```
Print[A6, A7]
```



Line joining any two complex numbers

```
z1 = Input["Enter first complex number"];
z2 = Input["Enter second complex number"];
Graphics[{Thick, Red, Arrow[{{{Re[z1], Im[z1]}, {Re[z2], Im[z2]}}}]},
  Axes → True, ImageSize → 200, Frame → True]
```



PEACTICAL 1

Make a geometrical plot to show that nth roots of unity are equally spaced points that lie on the unit circle $C_1(0)$={z:|z|=1} and form the vertices of a regular polygon with n sides for n=4,5,6,7,8.

For n=4

```
ComplexExpand[z /. Solve[z^4 - 1 == 0]]
```

{−1, −ⅈ, ⅈ, 1}

```
A1 = Show[Graphics[{Thick, Red, Circle[{0, 0}, 1]}],
     ListLinePlot[{{−1, 0}, {0, −1}, {1, 0}, {0, 1}, {−1, 0}},
       PlotStyle → Blue, PlotMarkers → Automatic], Axes → True,
     ImageSize → 100, PlotLabel → "For n=4"];
```

```
ComplexExpand[z /. N[Solve[z^5 - 1 == 0]]]
```

```
{1., -0.809017 - 0.587785 ⅈ, 0.309017 + 0.951057 ⅈ,
  0.309017 - 0.951057 ⅈ, -0.809017 + 0.587785 ⅈ}
```

```
A2 = Show[Graphics[{Thick, Blue, Circle[{0, 0}, 1]}],
      ListLinePlot[{{1, 0}, {0.309017, 0.951057}, {-0.809017, 0.587785},
          {-0.809017, -0.587785}, {0.309017, -0.951057}, {1, 0}},
        PlotStyle → {Purple, Dashed, Thick}, PlotMarkers → Automatic], Axes → True,
      ImageSize → 100, PlotLabel → "For n=5"];
```

```
ComplexExpand[z /. N[Solve[z^6 - 1 == 0]]]
```

```
{-1., 1., -0.5 - 0.866025 ⅈ, 0.5 + 0.866025 ⅈ, 0.5 - 0.866025 ⅈ, -0.5 + 0.866025 ⅈ}
```

```
A3 = Show[Graphics[{Thick, Blue, Circle[{0, 0}, 1]}],
      ListLinePlot[{{1, 0}, {0.5, 0.866025},
          {-0.5, 0.8662025}, {-1, 0}, {-0.5, -0.866025}, {0.5, -0.866025}, {1, 0}},
        PlotStyle → Green, PlotMarkers → Automatic], Axes → True,
      ImageSize → 100, PlotLabel → "For n=6"];
```

```
ComplexExpand[z /. N[Solve[z^7 - 1 == 0]]]
```

```
{1., -0.900969 - 0.433884 ⅈ, 0.62349 + 0.781831 ⅈ, -0.222521 - 0.974928 ⅈ,
  -0.222521 + 0.974928 ⅈ, 0.62349 - 0.781831 ⅈ, -0.900969 + 0.433884 ⅈ}
```

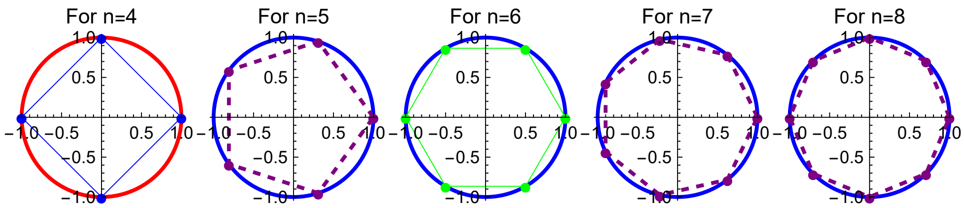
```
A4 = Show[Graphics[{Thick, Blue, Circle[{0, 0}, 1]}],
      ListLinePlot[{{1, 0}, {0.62349, 0.781831}, {-0.222521, 0.974928}, {-0.900969, 0.433884},
          {-0.900969, -0.433884}, {-0.222521, -0.974928}, {0.62349, -0.781831}, {1, 0}},
        PlotStyle → {Purple, Dashed, Thick}, PlotMarkers → Automatic], Axes → True,
      ImageSize → 100, PlotLabel → "For n=7"];
```

```
ComplexExpand[z /. N[Solve[z^8 - 1 == 0]]]
```

```
{-1., 0. - 1. ⅈ, 0. + 1. ⅈ, 1., -0.707107 - 0.707107 ⅈ,
  0.707107 + 0.707107 ⅈ, 0.707107 - 0.707107 ⅈ, -0.707107 + 0.707107 ⅈ}
```

```
A5 = Show[Graphics[{Thick, Blue, Circle[{0, 0}, 1]}],
      ListLinePlot[{{1, 0}, {0.707107, 0.707107}, {0, 1}, {-0.707107, 0.707107},
          {-1, 0}, {-0.707107, -0.707107}, {0, -1}, {0.707107, -0.707107}, {1, 0}},
        PlotStyle → {Purple, Dashed, Thick}, PlotMarkers → Automatic], Axes → True,
      ImageSize → 100, PlotLabel → "For n=8"];
```

```
Print[A1, A2, A3, A4, A5]
```



PRACTICAL 2

Find the solutions of the equation $z^3 = 8\,i$ and represent these geometrically.

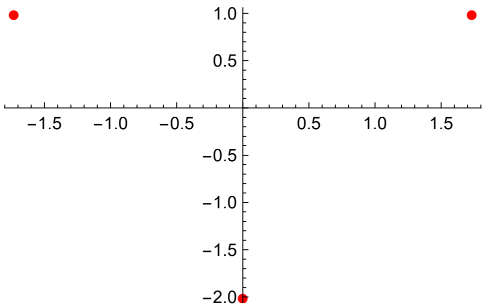
```
Quit[]
```

```
pts = ComplexExpand[z /. Solve[z^3 - 8 * I == 0]]
```

$\{-2\,i, i - \sqrt{3}, i + \sqrt{3}\}$

```
ComplexListPlot[{{-2 i, i - √3, i + √3}},
  PlotStyle → {RGBColor[1, 0, 0], Thickness[3.5]}, PlotMarkers → Automatic, ImageSize → 250]
```

```
ListPlot[{{0, -2}, {-√3, 1}, {√3, 1}},
  PlotStyle → {Red, Thickness[3.5]}, PlotMarkers → Automatic,
  ImageSize → 250]
```



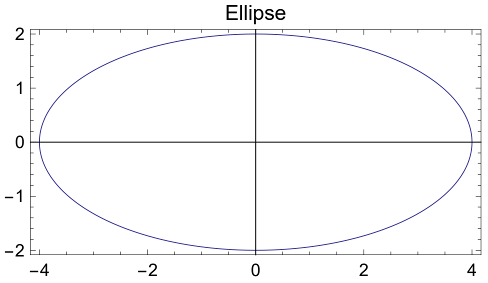
PRACTICAL 3

Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units. Show the effect of rotation of this ellipse by an angle of
$\frac{\pi}{6}$ radians and shifting of the centre from (0, 0) to (2, 1) by making *a* parametric plot.

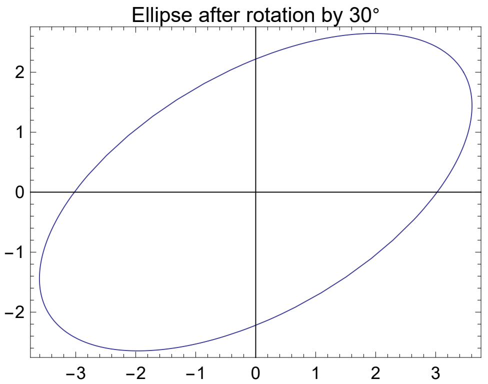
Ellipse : $\frac{x^2}{16} + \frac{y^2}{4} = 1$

The parametric equation is : $x = 4$ Cost, $y = 2$ Sint : t[0, 2 Pi]

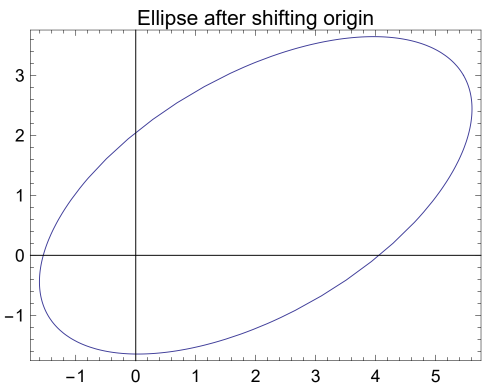
```
A1 = ParametricPlot[{4 Cos[t], 2 Sin[t]}, {t, 0, 2 Pi},
    Frame → True, ImageSize → 250, PlotLabel → "Ellipse"];
Print[A1]
```




```
A2 = ParametricPlot[
    {4 Cos[t] Cos[π/6] - 2 Sin[t] Sin[π/6], 4 Cos[t] Sin[π/6] + 2 Sin[t] Cos[π/6]}, {t, 0, 2 Pi},
    Frame → True, ImageSize → 250, PlotLabel → "Ellipse after rotation by 30°"];
Print[A2]
```




```
A3 = ParametricPlot[
    {4 Cos[t] Cos[π/6] - 2 Sin[t] Sin[π/6] + 2, 4 Cos[t] Sin[π/6] + 2 Sin[t] Cos[π/6] + 1}, {t, 0, 2 Pi},
    Frame → True, ImageSize → 250, PlotLabel → "Ellipse after shifting origin"];
Print[A3]
```



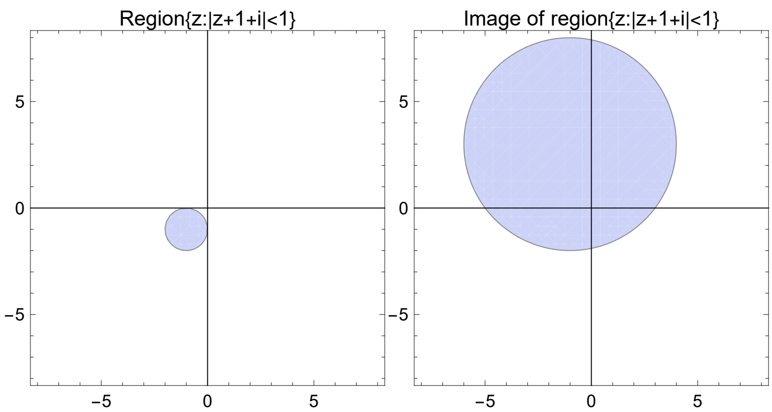
PRACTICAL-4

Find the image of D1={z:|z+1+i|<1} under the mapping f(z)=(3-4i)z+6+2i

```
Solve[w1 == (3 - 4 I) z1 + (6 + 2 I), z1]
```

$$\left\{\left\{z1 \to \frac{1}{25}\,(-10 + 3\,w1 + 2\,i\,(-15 + 2\,w1))\right\}\right\}$$

```
z = x + I y;
A1 = RegionPlot[Abs[z + (1 + I)] < 1, {x, -8, 8}, {y, -8, 8}, Axes → True, ImageSize → 200,
    PlotLabel → "Region{z:|z+1+i|<1}"];
A2 = RegionPlot[Abs[1/25 (-10 + 3 z + 2 I (-15 + 2 z)) + 1 + I] < 1,
    {x, -8, 8}, {y, -8, 8}, Axes → True, ImageSize → 200,
    PlotLabel → "Image of region{z:|z+1+i|<1}"];
Print[A1, A2]
```

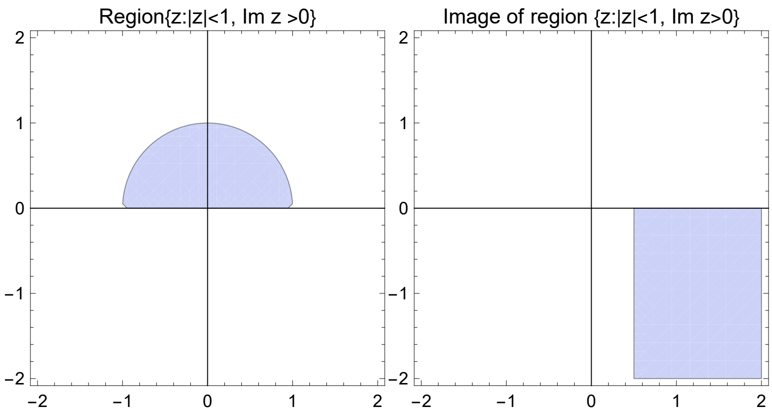


Find the image of the semi-disc S={z:|z|<1, Im z>0} under the mapping f(z)=1/(z+1)

$$\text{Solve}\left[w1 == \frac{1}{z_1 + 1},\, z_1\right]$$

$$\left\{\left\{x + i\,y_1 \to \frac{1 - w1}{w1}\right\}\right\}$$

```
A1 = RegionPlot[Abs[z] < 1 && Im[z] > 0, {x, -2, 2}, {y, -2, 2}, Axes → True, ImageSize → 200,
    PlotLabel → "Region{z:|z|<1, Im z >0}"];
A2 = RegionPlot[Abs[1/z - 1] < 1 && Im[1/z - 1] > 0,
    {x, -2, 2}, {y, -2, 2}, Axes → True, ImageSize → 200,
    PlotLabel → "Image of region {z:|z|<1, Im z>0}"];
Print[A1, A2]
```



PRACTICAL-5

Show that image of D_1 ={z:Re(z)>1} under the mapping f(z)=(-1+i)z-2+3i is the half plane v>u+7, where u=Re(w) etc. Plot the map.

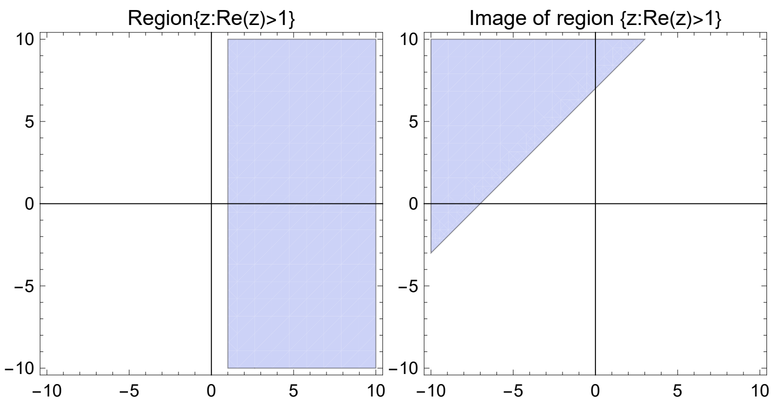
Solve[w1 == (−1 + I) z_1 − 2 + 3 I, z_1]

$$\left\{\left\{x + i\, y_1 \to \frac{1}{2}\left(-5 + i\,(1 - w1) - w1\right)\right\}\right\}$$

A1 = RegionPlot[Re[z] > 1, {x, −10, 10}, {y, −10, 10}, Axes → True, ImageSize → 200, PlotLabel → "Region{z:Re(z)>1}"];

A2 = RegionPlot$\left[\mathrm{Re}\left[\frac{1}{2}(-5 + I(1 - z) - z)\right] > 1,\right.$ {x, −10, 10}, {y, −10, 10}, Axes → True, ImageSize → 200, PlotLabel → "Image of region {z:Re(z)>1}"$\left.\right]$;

Print[A1, A2]



PRACTICAL-6

Show that the image of the right half plane A={z:Re $z \geq \frac{1}{2}$ under the mapping $w = f(z) = \frac{1}{z}$ is the closed disk $D_1(1) = \{w : |w - 1| \leq 1\}$ in the w – plane.

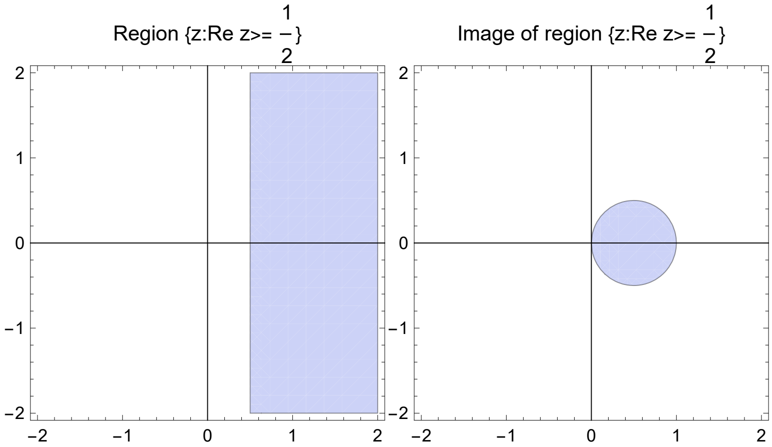
Quit[]

Solve[w1 == 1 / z1, z1]

$\left\{\left\{z1 \to \frac{1}{w1}\right\}\right\}$

z = x + I y;

```
A1 = RegionPlot[Re[z] > 0.5, {x, -2, 2}, {y, -2, 2},
    Axes → True, ImageSize → 200, PlotLabel → "Region {z:Re z>= 1/2}"];
A2 = RegionPlot[Re[1/z] > 1, {x, -2, 2}, {y, -2, 2}, Axes → True,
    ImageSize → 200, PlotLabel → "Image of region {z:Re z>= 1/2}"];
Print[A1, A2]
```



PRACTICAL-7

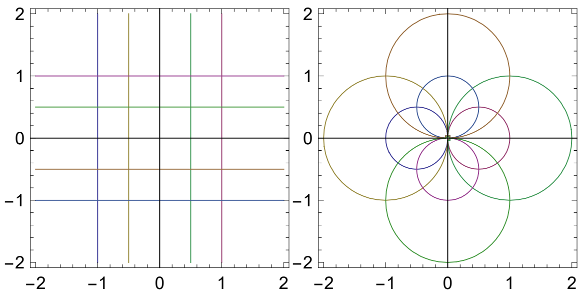
Make a plot of the vertical lines x=a, for $a = -1, \frac{-1}{2}, \frac{1}{2}, 1$ and the horizontal lines, y=b for $b = -1, \frac{-1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $w = f(z) = \frac{1}{z}$.

z = x + I y;

A1 = ContourPlot[{Re[z] == −1, Re[z] == 1, Re[z] == $\frac{-1}{2}$, Re[z] == $\frac{1}{2}$, Im[z] == −1, Im[z] == 1, Im[z] == $\frac{-1}{2}$, Im[z] == $\frac{1}{2}$}, {x, −2, 2}, {y, −2, 2}, Axes → True, ImageSize → 150];

A2 = ContourPlot[{Re[$\frac{1}{z}$] == −1, Re[$\frac{1}{z}$] == 1, Re[$\frac{1}{z}$] == $\frac{-1}{2}$, Re[$\frac{1}{z}$] == $\frac{1}{2}$, Im[$\frac{1}{z}$] == −1, Im[$\frac{1}{z}$] == 1, Im[$\frac{1}{z}$] == $\frac{-1}{2}$, Im[$\frac{1}{z}$] == $\frac{1}{2}$}, {x, −2, 2}, {y, −2, 2}, Axes → True, ImageSize → 150];

Print[A1, A2]



PRACTICAL-8

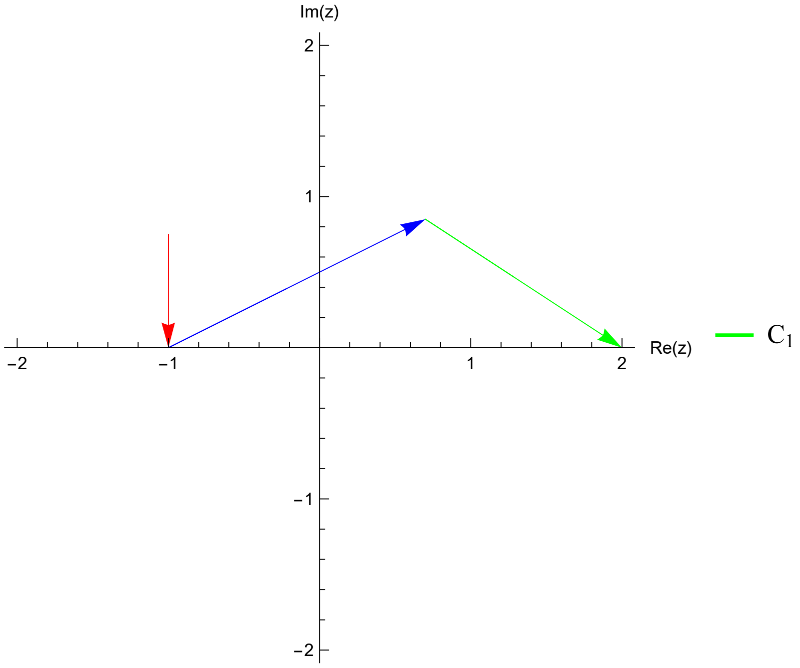
Find a parametrization of the polygon path C=C1+C2+C3 from -1+i to 3+i, where C_1 is the line from : $-1+i$ to -1, C_2 is the line from:-1 to 1+i and C_3 is the line from 1+i to 3-i. Make a plot of this path.

Parametric Plot of polygon path $c(t) = C_1(t) + C_2(t) + C_3(t)$

Quit[]

C_1[t_] = ComplexExpand[(−1 + I) ∗ (1 − t) + (−1) ∗ t];
C_2[t_] = ComplexExpand[(−1) ∗ (1 − t) + (1 + I) ∗ t];
C_3[t_] = ComplexExpand[(1 + I) ∗ (1 − t) + (3 − I) ∗ t];

```
Show[ParametricPlot[{ReIm[C1[t]], ReIm[C2[t]], ReIm[C3[t]]},
    {t, 0, 1}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
    PlotLegends → {"C1", "C2", "C3"}, AxesLabel → {"Re(z)", "Im(z)"}],
  Graphics[{Red, Arrow[{{-1, 0.75}, {-1, 0}}]}],
  Graphics[{Blue, Arrow[{{-1, 0}, {0.7, 0.85}}]}],
  Graphics[{Green, Arrow[{{0.7, 0.85}, {2, 0}}]}], PlotRange → 2]
```



PRACTICAL-9

Plot the line segment 'L' joining the point A=0 to $B = 2 + \frac{\pi}{4}$i and give an exact calculation of $\int_L e^z \, dz$

$$\text{Integrate}\left[e^z, \left\{z, 0, 2 + \frac{\pi}{4} \text{I}\right\}\right]$$

$$\frac{-1 + e^{2+\frac{i\pi}{4}}}{\text{Log}[e]}$$

Some other examples: Find $\int_{2+3i}^{5+2i} z^2 \, dz$, $\int_{2+3i}^{5+2i} e^z \, dz$, $\int_0^{2+i} \bar{z} \, dz$, $\int_{-2}^{-2+i} (z+2)^2 \, dz$.

Print["The value of intergral $\int_{2+3i}^{5+2i} z^2 dz$ is : ", Integrate[z^2, {z, 2 + 3 I, 5 + 2 I}]]

The value of intergral $\int_{2+3i}^{5+2i} z^2 dz$ is : $37 + \frac{133\, i}{3}$

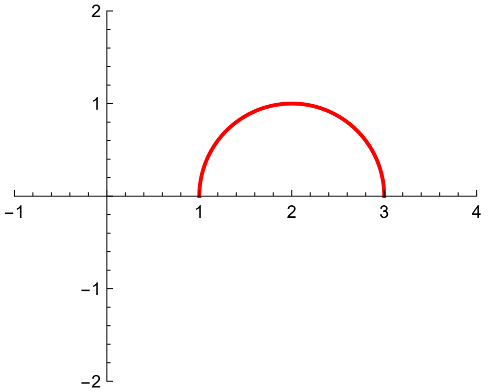
Print["The value of intergral $\int_{2+3i}^{5+2i} e^z dz$ is : ", Integrate[e^z, {z, 2 + 3 I, 5 + 2 I}]]

The value of intergral $\int_{2+3i}^{5+2i} e^z dz$ is : $\frac{-e^{2+3i} + e^{5+2i}}{\text{Log}[e]}$

PRACTICAL-10

Plot the semicircle ‘C’ with radius 1 centered at z=2 and evaluate the contour integral $\int_C \frac{1}{z-2}\, dz$

```
ParametricPlot[{2 + Cos[t], Sin[t]}, {t, 0, Pi},
  PlotRange → {{-1, 4}, {-2, 2}}, PlotStyle → {Red, Thick}, ImageSize → {250}]
```



Quit[]

f[z_] := $\frac{1}{z-2}$;
z[t_] := 2 + Cos[t] + I Sin[t];
$\int_0^{\pi}$ f[z[t]] × z'[t] dt

$i\,\pi$

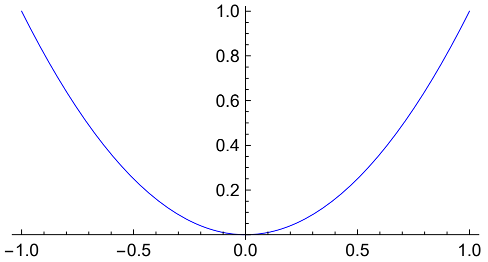
Some other examples: $\int_C \frac{1}{z}\, dz$ where c is a circle with centre at origin and radius r

```
f[z_] := 1/z;
z[t_] := r Cos[t] + I r Sin[t];
∫_0^{2π} f[z[t]] × z'[t] ⅆt
```

$2 i \pi$

Evaluate $\int_C (z-i)\, dz$ where C is a parabola $y = x^2$: -1<= x<= 1

```
ParametricPlot[{t, t^2}, {t, -1, 1}, PlotStyle → {Red, Blue}, ImageSize → {250}]
```




```
f[z_] := z - I;
z[t_] := t + I t^2;
∫_{-1}^{1} f[z[t]] × z'[t] ⅆt
```

0

PRACTICAL-11

Show that $\int_C z\, dz = \int_C z\, dz$ =4+2i where C_1 is the line segment from -1-i to 3+i and C_2 is the portion of the parabola $x = y^2$+2y joining from -1-i to 3+i. Make plot of two contours C_1, C_2 joining -1-i to 3+i

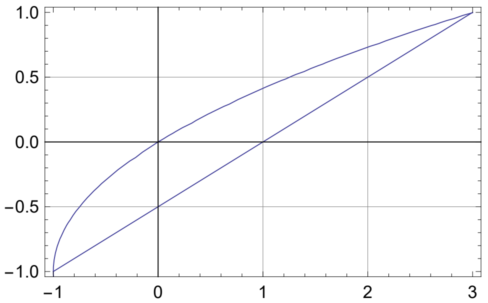
```
Integrate[z, {z, -1 - I, 3 + I}]
```

$4 + 2 i$

```
f[z_] := z;
z[t_] := (t^2 + 2 t) + I t;
∫_{-1}^{1} f[z[t]] × z'[t] ⅆt
```

$4 + 2 i$

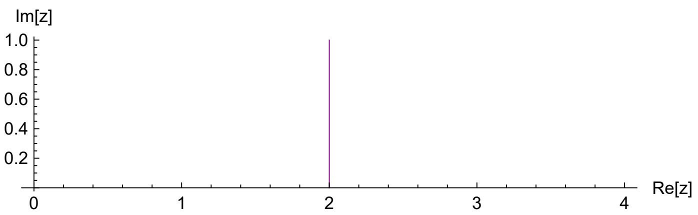
```
A1 = ListLinePlot[{{Re[-1 - I], Im[-1 - I]}, {Re[3 + I], Im[3 + I]}},
      ImageSize → 250, GridLines → Automatic, Frame → True];
A2 = ContourPlot[{x == y^2 + 2 y}, {x, -2, 3},
      {y, -3, 3}, Axes → {True, True}, GridLines → Automatic,
      ImageSize → 150, Frame → True, PlotLabel → "Graph of x==y^2+2y"];
Show[A1, A2]
```



PRACTICAL-12

Use ML inequality to show that $|\int_C \frac{1}{z^2+1}\, dz| <= \frac{1}{2\sqrt{5}}$ where C is the straight line segment from 2+2i while solving represent the distance from the point z to the point i and -i i.e, |z-i| & |z+i| in the complex plane.

```
f[z_] := 1 / (z^2 + 1)
z[t_] := 2 + I * t
ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, 0, 1},
  PlotStyle → Purple, Axes → True, AxesLabel → {"Re[z]", "Im[z]"}]
```

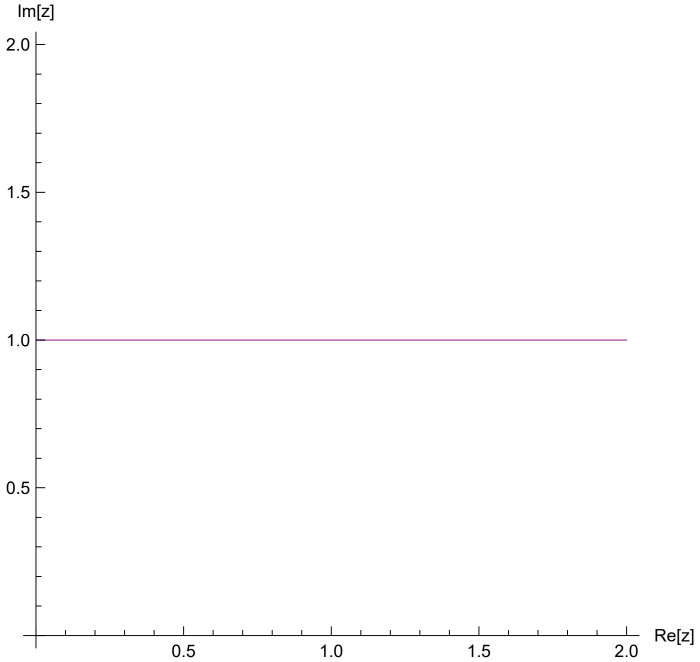


The length of the curve,L is given by

```
Integrate[Abs[z'[t]], {t, 0, 1}]
```

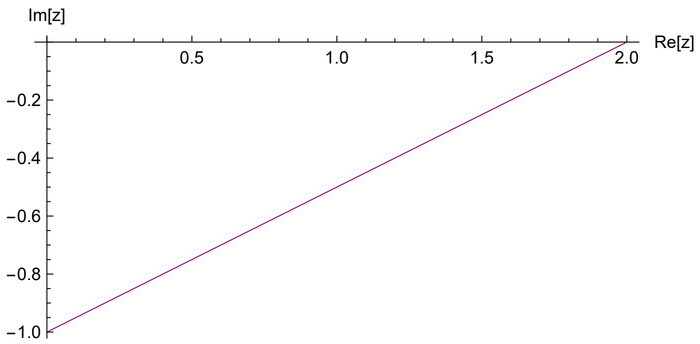
1

z[t_] := 2 * (1 – t) + I
ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, 0, 1},
PlotStyle → Purple, Axes → True, AxesLabel → {"Re[z]", "Im[z]"}]



Again the distance |z+i| i.e the distance from z to -i least when z=2

z[t_] := 2 (1 – t) – I * t
ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, 0, 1},
PlotStyle → Purple, Axes → True, AxesLabel → {"Re[z]", "Im[z]"}]



g[t_] = 4 + (4 + 6 I) t

4 + (4 + 6 ⅈ) t

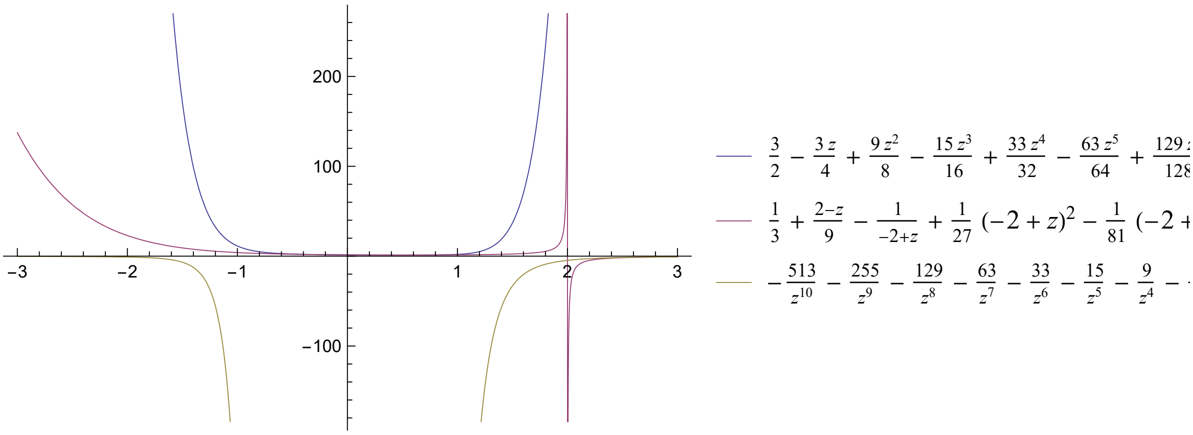
PRACTICAL-13

Find and plot three different Laurent series representation for the function $f(z) = \frac{3}{2+z-z^2}$ involving powers of z

Grid[Table[{n, Normal[Series[3 / (2 + z – z ^ 2), {z, n, 10}]]}, {n, {0, 2, Infinity}}], Frame → All]

0	$\frac{3}{2} - \frac{3z}{4} + \frac{9z^2}{8} - \frac{15z^3}{16} + \frac{33z^4}{32} - \frac{63z^5}{64} + \frac{129z^6}{128} - \frac{255z^7}{256} + \frac{513z^8}{512} - \frac{1023z^9}{1024} + \frac{2049z^{10}}{2048}$
2	$\frac{1}{3} + \frac{2-z}{9} - \frac{1}{-2+z} + \frac{1}{27}(-2+z)^2 - \frac{1}{81}(-2+z)^3 + \frac{1}{243}(-2+z)^4 - \frac{1}{729}(-2+z)^5 + \frac{(-2+z)^6}{2187} - \frac{(-2+z)^7}{6561} + \frac{(-2+z)^8}{19683} - \frac{(-2+z)^9}{59049} + \frac{(-2+z)^{10}}{177147}$
∞	$-\frac{513}{z^{10}} - \frac{255}{z^9} - \frac{129}{z^8} - \frac{63}{z^7} - \frac{33}{z^6} - \frac{15}{z^5} - \frac{9}{z^4} - \frac{3}{z^3} - \frac{3}{z^2}$

Plot[Evaluate[Table[{Normal[Series[3 / (2 + z – z ^ 2), {z, n, 10}]]}, {n, {0, 2, Infinity}}]], {z, –3, 3}, PlotLegends → "Expressions"]



PRACTICAL-14

Locate the poles of $f(z) = \frac{1}{5z^4+26z^2+5}$.

```
f[z_] := 1 / (5 z ^ 4 + 26 z ^ 2 + 5);
Print["f[z]=", f[z]];
Print[Denominator[f[z]] == 0];
solset = Sort[Solve[Denominator[f[z]] == 0, z]];
Print[solset];
Print["The singularities are"];
z1 = ReplaceAll[z, solset[[1]]];
Print["z1=", z1];
z2 = ReplaceAll[z, solset[[2]]];
Print["z2=", z2]
z3 = ReplaceAll[z, solset[[3]]];
Print["z3=", z3];
z4 = ReplaceAll[z, solset[[4]]];
Print["z4=", z4];
```

$$f[z]=\frac{1}{5+26 z^2+5 z^4}$$

$$5+26 z^2+5 z^4 == 0$$

$$\left\{\left\{z \to -\frac{i}{\sqrt{5}}\right\}, \left\{z \to \frac{i}{\sqrt{5}}\right\}, \{z \to -i\sqrt{5}\}, \{z \to i\sqrt{5}\}\right\}$$

The singularities are

$$z1=-\frac{i}{\sqrt{5}}$$

$$z2=\frac{i}{\sqrt{5}}$$

$$z3=-i\sqrt{5}$$

$$z4=i\sqrt{5}$$

PRACTICAL-15

Locate the zeroes and poles of $g(z) = \frac{\pi \text{Cot}[\pi z]}{z^2}$. Determine their order, also justify that $\text{Res}(g, 0) = \frac{-\pi^2}{3}$.

```
g[z_] := π * Cot[π * z] / z ^ 2;
Reduce[g[z] == 0, z]
```

$$C[1] \in \text{Integers} \,\&\&\, z \neq 0 \,\&\&\, z == \frac{\frac{\pi}{2} + \pi\, C[1]}{\pi}$$

Poles = Solve[Denominator[g[z]] == 0, z]

{{z → 0}, {z → 0}}

residue = Residue[g[z], {z, 0}]

$-\frac{\pi^2}{3}$

PRACTICAL-16

Evaluate $\int_{C^+{}_{1\,(0)}} e^{2/z}\, dz$ where $C_1{}^+(0)$ denotes the circle{z : |z|=1} with positive orientation. Similarly evaluate $\int_{C_1{}^+(0)} \frac{1}{z^4+z^3-2\,z^2}\, dx$

Quit[]

f[z_] := Exp[2 / z]
g[t_] := Cos[t] + I * Sin[t]
$\int_0^{2\pi}$ **f[g[t]] * g'[t] dt**

$4\,i\,\pi$

h[z_] := 1 / z ^ 4 + z ^ 3 − 2 z ^ 2
g[t_] := Cos[t] + I * Sin[t]
$\int_0^{2\pi}$ **h[g[t]] * g'[t] dt**

0