Magnetic Field:

We describe the space around a permanent magnet or a current-carrying conductor as the location of a magnetic field.

Generally, a region of space in which a magnetic pole experiences an applied force is called a magnetic field.

Absolute and Relative Permeabilities of a Medium:

The phenomena of magnetism and electromagnetism are dependent upon a certain property of the medium called its permeability. Every medium is supposed to possess two permeabilities:

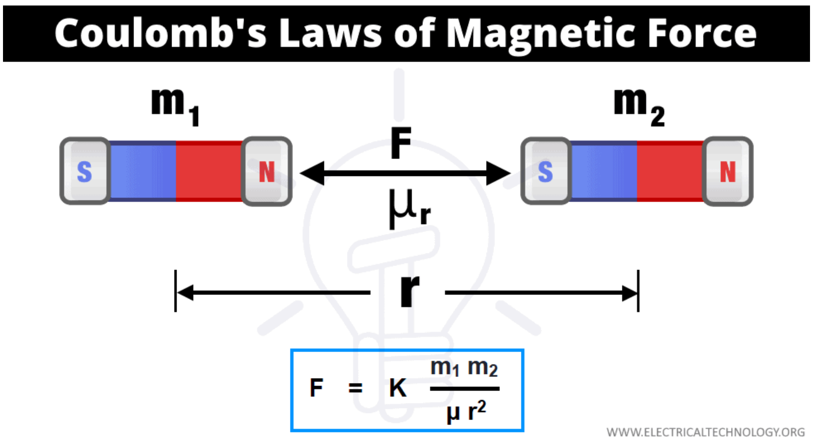
(i) absolute permeability (μ) and
(ii) relative permeability (μ_r).

$$Here, \mu = \mu_0 \, \mu_r \text{ H/m.}$$

Laws of Magnetic Force:

The force between two magnetic poles placed in a medium is:

(i) directly proportional to their pole strengths
(ii) inversely proportional to the square of the distance between them and
(iii) inversely proportional to the absolute permeability of the surrounding medium.



Magnetic Field Strength ($\vec{H}$):

Magnetic field strength at any point within a magnetic field is numerically equally to the force experienced by a N-pole of one weber placed at that point. Hence, unit of H is N/Wb.

Magnetic Potential:

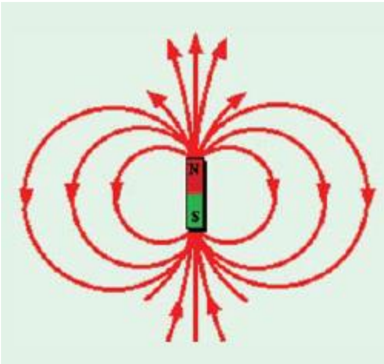
The magnetic potential at any point within a magnetic field is measured by the work done in shifting a N-pole of one weber from infinity to that point against the force of the magnetic field.

It is a scalar quantity.

Flux per Unit Pole:

A unit N-pole is supposed to radiate out a flux of one weber. Its symbol is Φ. Therefore, the flux coming out of a N-pole of m weber is given by

$$\Phi = m\,Wb$$



Flux Density (B):

It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter B and is measured in weber/meter2. It is a Vector Quantity.

If ΦWb is the total magnetic flux passing normally through an area of $A\ m^2$, then

$$B = \frac{\Phi}{A}\ Wb\,m^{-2}\ or\ tesla\ (T)$$

Intensity of Magnetization (I):

It may be defined as the induced pole strength developed per unit area of the bar. Also, it is the magnetic moment developed per unit volume of the bar.

Let m = pole strength induced in the bar in Wb

A = face or pole area of the bar in m^2

Then

$$I = \frac{m}{A}\ Wb\,m^{-2}$$

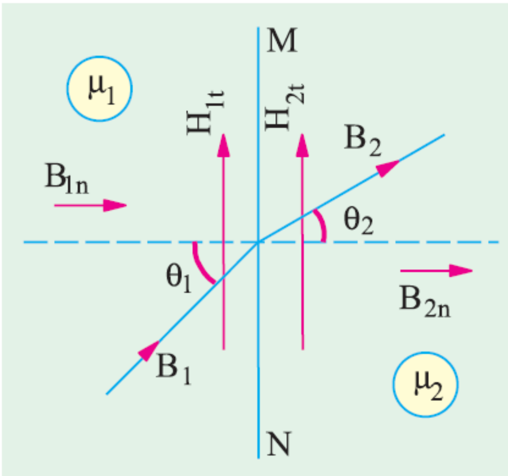
Susceptibility (K):

Susceptibility is defined as the ratio of intensity of magnetization I to the magnetizing force H.

$$\therefore K = I/H \text{ henry/metre.}$$

Boundary Conditions:

The two boundary conditions between two materials of different permeabilities are:



(i) the normal component of flux density is continuous across boundary.

$$B_{1n} = B_{2n} \qquad \ldots \quad (i)$$

(ii) the tangential component of H is continuous across boundary.

$$H_{1t} = H_{2t} \qquad \ldots \quad (ii)$$

Also, it can be shown that

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$

This is called the law of magnetic flux refraction.

The Magnetic Force on A Moving Charge:

Let us therefore consider a set of measurements that, least in principle, could be done to study the magnetic force that may act on an electric charge. (In these experiments, we consider only electric or magnetic forces; we assume that the experiments are carried out in an environment where other forces, such as gravity, may be neglected.)

1. We first test for the presence of an electric force by placing a small test charge at rest at various locations. Later we can subtract the electric force (if any) from the total force, which presumably leaves only the magnetic force. We assume this has been done, so that from now on we can ignore any electric force that acts on the charge.

2. Next, we project the test charge q through a particular point P with a velocity $\vec{v}$. We find that the magnetic force $\vec{F}$, if it is present, always acts sideways, that is, at right angles to the direction of $\vec{v}$. We can repeat the experiment by projecting the charge through p in different directions; we find that, no matter what the direction of $\vec{v}$, the magnetic force is always at right angles to that direction.

3. As we vary the direction of $\vec{v}$ through point P, we also find that the magnitude of $\vec{F}$ changes from zero when $\vec{v}$ has a certain direction to a maximum when it is at right angles to that direction. At intermediate angles, the magnitude of $\vec{F}$ varies as the sine of the angle ϕ that the velocity vector makes with that particular direction. (Note that there are actually two directions of $\vec{v}$ for which $\vec{F}$ is zero; these directions are opposite to each other, that is, $\phi = 0° \; or \; 180°$.)

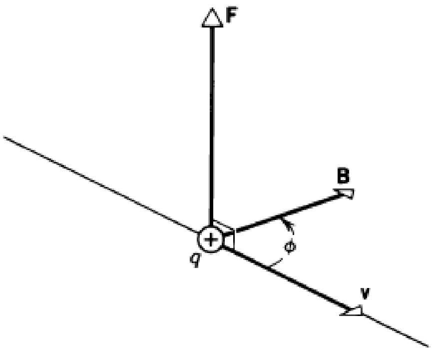


Figure: A particle with a positive charge q moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ experiences a magnetic deflecting force $\vec{F}$.

4. As we vary the magnitude of the velocity, we find that the magnitude of F varies in direct proportion.

5. We also find that F is proportional to the magnitude of the test charge q, and that F reverses direction when q changes sign.

We now define the magnetic field $\vec{B}$ in the following way, based on these observations: the direction of $\vec{B}$ at point P is the same as one of the directions of $\vec{v}$ in which the force is zero, and the magnitude of $\vec{B}$ is determined from the magnitude $F_{\perp}$. of the maximum force exerted when the test charge is projected perpendicular to the direction of $\vec{B}$; that is,

$$B = \frac{F_{\perp}}{qv} \dots \dots \dots (i)$$

At arbitrary angles, our observations are summarized by the formula

$$F = qvB \, sin\phi \dots \dots \dots (ii)$$

where ϕ is the smaller angle between $\vec{v}$ and $\vec{B}$. Because F, v, and B are vectors, Eq.(ii) can be written as a vector product:

$$\vec{F} = q \, \vec{v} \times \vec{B}$$

Lorenz Force:

If a positive test charge q_0 is fired with velocity $\vec{v}$ through a point P and if both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ act on the charge, the resultant force $\vec{F}$ acting on the charge is found by

$$\vec{F} = q_0\vec{E} + q_0\ (\vec{v} \times \vec{B})$$

This force is called the Lorentz force.

Ampere’s Law:

For any closed loop, sum of the length elements times the magnetic field in the direction of the length elements is equal to the permeability times the electric current enclosed in the loop.

Ampere’s law states that the line integral of the magnetic field around a closed loop is equal to the permeability μ_0 times the electric current passing through that loop.

Mathematically,

$$\oint \vec{B}.d\vec{l} = \mu_0 i$$

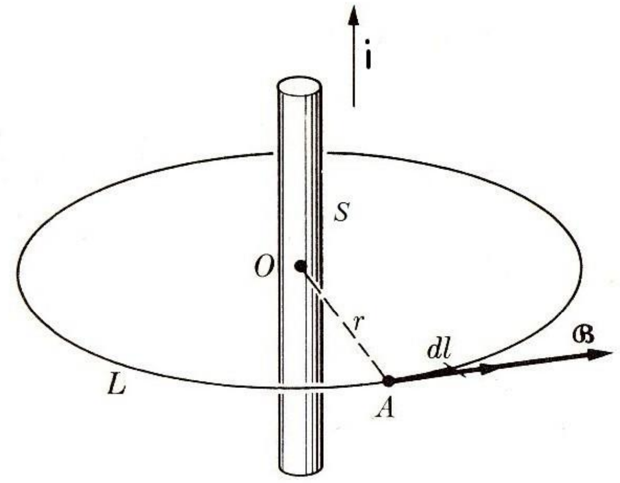


Figure-1: A circular path of integration surrounding a wire

Explanation:

Let us consider a conducting wire carrying a current i and $\vec{B}$ is the magnetic field at a distance r from the center of the wire as shown in fig-1.

The experimental result can be expressed by the proportionality.

$$B \propto \frac{i}{r}$$

$$\Rightarrow B = \frac{\mu_0}{2\pi}\frac{i}{r}$$

$$\Rightarrow B(2\pi r) = \mu_0 i \cdots\cdots\cdots\cdots\cdots (1)$$

For all points on this circle $\vec{B}$ has same magnitude. Thus,

$$\oint \vec{B}.d\vec{l} = \oint B\, dl cos\theta$$

$$\Rightarrow \oint \vec{B}.d\vec{l} = \oint B\, dl$$

$$\Rightarrow \oint \vec{B}\,.\,d\vec{l} = B\oint dl$$

$$\Rightarrow \oint \vec{B}\,.\,d\vec{l} = B(2\pi r) \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

From equation (1) and (2), we get,

$$\oint \vec{B}.d\vec{l} = \mu_0 i$$. This is Ampere's law.

Application of Ampere's Law:

<u>$\vec{B}$ Near a Long Wire:</u>

Let us consider a conducting wire carrying a current i and $\vec{B}$ is the magnetic field at a distance r from the center of the long wire.

Applying the Ampere's law, we get,

$$\oint \vec{B}.d\vec{l} = \mu_0 i$$

$$\Rightarrow \oint B dl cos\theta = \mu_0 i$$

$$\Rightarrow \oint B dl = \mu_0 i$$

$$\Rightarrow B\oint dl = \mu_0 i \qquad [\because \oint dl = 2\pi r]$$

$$\Rightarrow B(2\pi r) = \mu_0 i$$

$$\therefore B = \frac{\mu_0 i}{2\pi r}$$

Biot-Savart Law:

The magnetic effects of a current must be formulated in terms of line elements of current. Since each portion of a complete circuit in which a circuit flow makes its own contribution to the magnetic field.

The magnetic field (dB) at any point is proportional to the length of conductor of small part (dl), proportional to the conductor carrying current (i), and proportional to the sine angle between the small length of conductor and the displacement vector from the element to the point P and inversely proportional to the square of the displacement vector from the element to P.

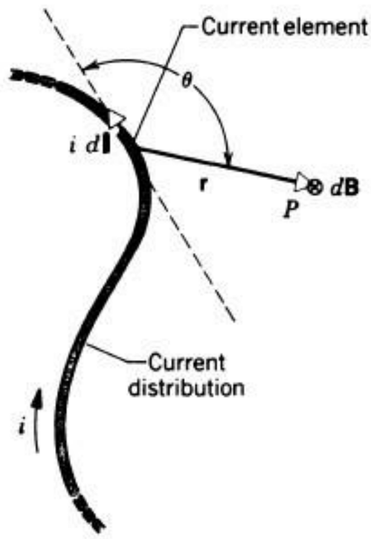


Figure-1: The current element d**l** establishes a magnetic field contribution d**B** at point P.

Explanation:

Let us consider dl be a line element of a conductor carrying current i.

According to Biot-savart law the magnetic field dB is given by,

$$dB = \frac{\mu_0 i dl sin\theta}{4\pi r^2} \text{...............} (1)$$

Where,

dB = Magnitude of the magnetic induction

i = Conductor carrying current

dl = Length of the conductor

r = Displacement vector from the element to P and θ = The angle between dl and r

From equation (1), we get,

$$dB = \frac{\mu_0 i}{4\pi} \frac{dlrsin\theta}{r^3}$$

$$\therefore d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{\vec{r} \times d\vec{l}}{r^3}$$

This is the vector form of Biot-savart law.

The resultant field of the point P is given by

$$\vec{B} = \int d\vec{B}$$

$$= \frac{\mu_0 i}{4\pi} \int \frac{\vec{r} \times d\vec{l}}{r^3}$$

Applications of Biot-savart Law:

A Long Straight Wire:

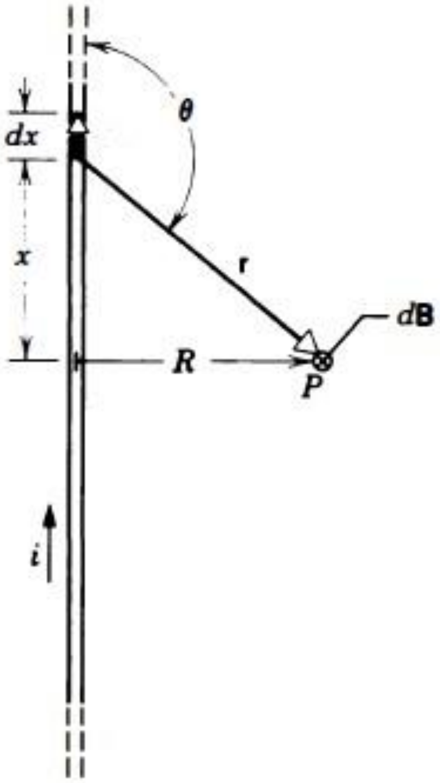


Figure-2: The magnetic field d**B** established by a current element in a long straight wire points into the page at P.

Let us consider a point P is taken at a distance r from the current element of a long straight wire dx which carrying a current i and R is the perpendicular distance from the wire to the point P.

Applying Biot-savart law, we get,

$$dB = \frac{\mu_0 i dx sin\theta}{4\pi r^2}$$ where θ is the angle between dx and r

$$\therefore B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{dx sin\theta}{r^2} \quad \cdots\cdots\cdots\cdots\cdots (1)$$

Now, $r = \sqrt{x^2 + R^2}$ ……………. (2)

and $sin\theta = \sin(\pi - \theta)$

$$= \frac{R}{r} = \frac{R}{\sqrt{x^2 + R^2}} \text{..............} (3)$$

Putting the value of equation (2) and (3) in equation (1), we get,

$$B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{+\infty} \frac{dx}{(\sqrt{x^2 + R^2})^2} \frac{R}{\sqrt{x^2 + R^2}}$$

$$= \frac{\mu_0 i}{4\pi} \int_{-\infty}^{+\infty} \frac{dxR}{(\sqrt{x^2 + R^2})^{\frac{3}{2}}}$$

$$= \frac{\mu_0 i}{4\pi R} \left[\frac{x}{\sqrt{x^2 + R^2}}\right]_{-\infty}^{+\infty}$$

$$= \frac{\mu_0 i}{4\pi R} * 2$$

$$\therefore B = \frac{\mu_0 i}{2\pi R}$$

A Circular Current Loop:

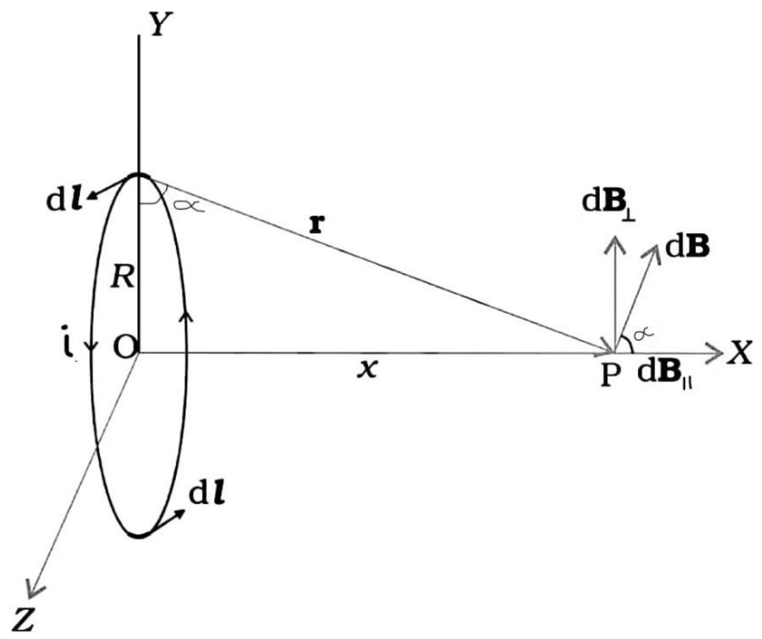


Figure-3: A circular loop of radius R carrying a current i.

Let us consider $d\vec{B}$ into two components, one, $d\vec{B}_{\parallel}$, along the axis of the loop and another, $d\vec{B}_{\perp}$, at right angle to the axis. Only $d\vec{B}_{\parallel}$ contributes to the total magnetic field $\vec{B}$ at point P. Because the components $d\vec{B}_{\perp}$ point in different directions perpendicular to the axis, and their resultant for the complete loop is zero. Thus,

$$B = \int dB_{\parallel} \cdots\cdots\cdots\cdots\cdots (1)$$

Applying Biot-savart law, we get,

$$dB = \frac{\mu_0 i dl sin\theta}{4\pi r^2}$$

$$= \frac{\mu_0 i dl}{4\pi r^2} \qquad [\because \theta = 90^0]$$

Now, $$\frac{dB_{\parallel}}{dB} = cos\alpha$$

or, $$dB_{\parallel} = dBcos\alpha$$

$$\therefore \quad dB_{\parallel} = \frac{\mu_0 i dl cos\alpha}{4\pi r^2}$$

From figure-1,

$$r = \sqrt{x^2 + R^2} \text{ and } cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{x^2 + R^2}}$$

$$\therefore \quad dB_{\parallel} = \frac{\mu_0 i dl R}{4\pi(\sqrt{x^2 + R^2})^2\sqrt{x^2 + R^2}}$$

$$= \frac{\mu_0 i dl R}{4\pi(x^2 + R^2)^{\frac{3}{2}}}$$

From equation (1),

$$B = \int dB_{\parallel}$$

$$= \int \frac{\mu_0 i dl R}{4\pi(x^2 + R^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 i R}{4\pi(x^2 + R^2)^{\frac{3}{2}}} \int dl$$

$$= \frac{\mu_0 i R}{4\pi(x^2 + R^2)^{\frac{3}{2}}} * 2\pi R$$

If x >> R, then,

$$\therefore B = \frac{\mu_0 i \pi R^2}{2\pi x^3}$$

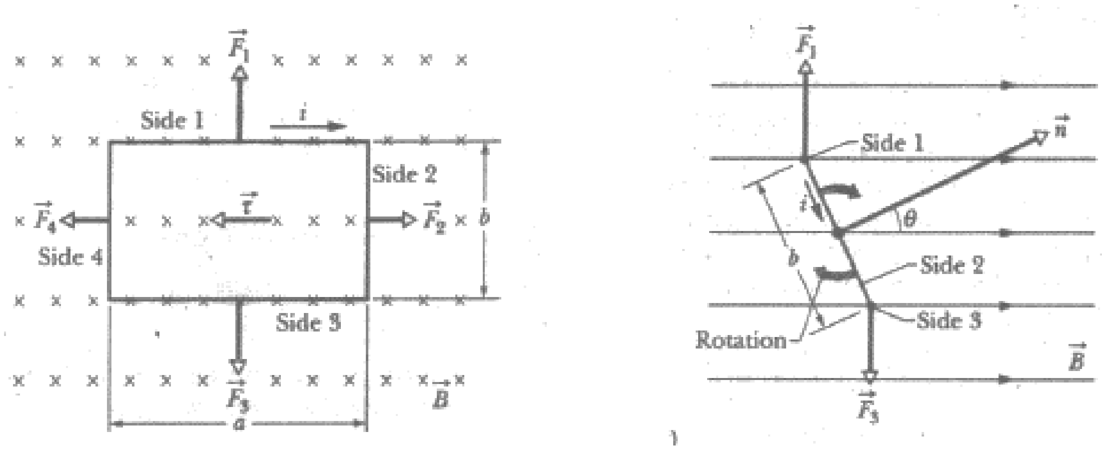
$$= \frac{\mu_0 i A}{2\pi x^3}$$ [∵ $A = \pi R^2$ = Area of the loop]

For N turns,

$$B = \frac{\mu_0 N i A}{2\pi x^3}$$

$$\therefore B = \frac{\mu_0 \mu}{2\pi x^3}$$ where, $\mu = NiA$ = Magnetic dipole moment

Torque on a Current Loop:



(a) **(b)**

Figure-1: A rectangular coil carrying a current i is placed in a uniform external magnetic field.

Let us consider a rectangular loop of wire of length a and width b placed in a uniform field of induction $\vec{B}$ as shown in Fig.-1.

The net force on the loop is the resultant of the forces on the four sides of the loop.

Force on side 2 is $\vec{F_2} = i\vec{b} \times \vec{B}$

The magnitude of the force on this side is $F_2 = ibBsin(90^0 - \theta)$

$= ibBcos\theta$

Force on side 4 is $\vec{F_4} = i\vec{b} \times \vec{B}$

The magnitude of the force on this side is $F_4 = ibBsin(270^0 - \theta)$

$= -\,ibBcos\theta$

The force $\vec{F_2}$ and $\vec{F_4}$ has the same magnitude and opposite direction. The net force is zero. So, they do not make torque.

Force on side 1 is $\vec{F_1} = i\vec{a} \times \vec{B}$

The magnitude of the force on this side is $F_1 = iaBsin90^0$

$= iaB$

Force on side 3 is $\vec{F_3} = i\vec{a} \times \vec{B}$

The magnitude of the force on this side is $F_3 = iaBsin90^0$

$$= iaB$$

The force $\overrightarrow{F_1}$ and $\overrightarrow{F_3}$ do not act on the same line. So, they make torque.

The net force on the loop, $F = F_1 + F_2 + F_3 + F_4$

$$= iaB + 0 + iaB + 0$$

$$= 2iaB$$

The magnitude of torque τ'on the loop is given by

$$\tau' = 2iaB(\frac{b}{2})sin\theta$$

$$= iabB\ sin\theta$$

$$= iABsin\theta \qquad [\because ab = A = \text{The area of the coil}]$$

For N turns,

$$\tau = N\tau'$$

$$\Rightarrow \tau = NiABsin\theta$$

$$\Rightarrow \tau = \mu Bsin\theta$$ where, $\mu = NiA$ = Magnetic dipole moment

$$\therefore\ \vec{\tau} = \overrightarrow{\mu} \times \vec{B}$$

Math

1. Calculate the magnetising force and flux density at a distance of 5 cm from a long straight circular conductor carrying a current of 250 A and placed in air.

Here,

Current, $I = 250\ A$

$radius, r = 5\ cm = \frac{5}{100}\ m = \ 0.05\ m$

The magnetising force H $= ?$

The flux density $B = ?$

We know,

$$H = \frac{I}{2\pi r}$$

$$H = \frac{250}{2 \times 3.1416 \times 0.05} = 795.77\frac{AT}{m}$$

Again, we know,

$$B = \mu_0 H$$

$$B = 4\pi \times 10^{-7} \times 795.77$$

$$B = 0.001\ Wb\ m^{-2}$$

Sample Problem 2 In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5.29×10^{-11} m at a frequency ν of 6.63×10^{15} Hz (or rev/s). (*a*) What value of B is set up at the center of the orbit? (*b*) What is the equivalent magnetic dipole moment?

Solution (*a*) The current is the rate at which charge passes any point on the orbit and is given by

$$i = e\nu = (1.60 \times 10^{-19}\ \mathrm{C})(6.63 \times 10^{15}\ \mathrm{Hz}) = 1.06 \times 10^{-3}\ \mathrm{A}.$$

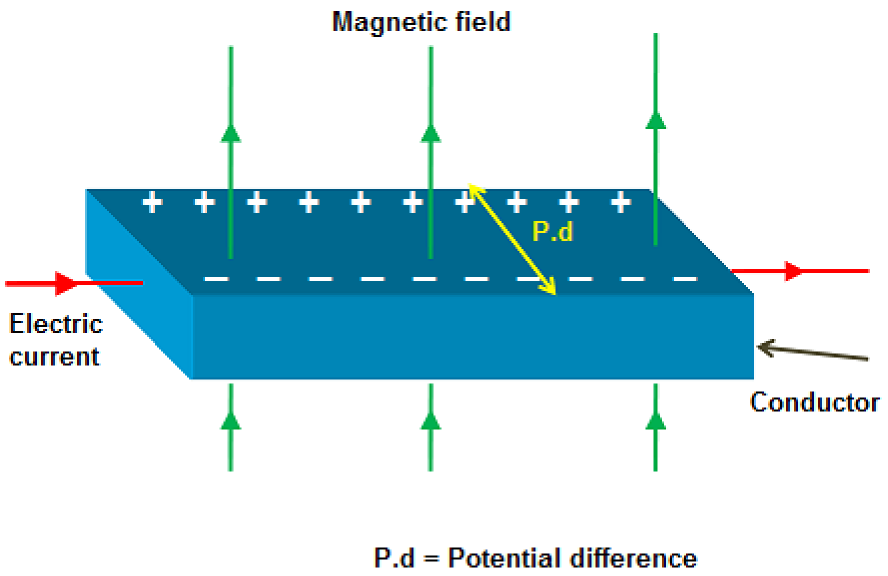
The magnetic field B at the center of the orbit is given by Eq. 16,

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(1.06 \times 10^{-3}\ \mathrm{A})}{2(5.29 \times 10^{-11}\ \mathrm{m})} = 12.6\ \mathrm{T}.$$

(*b*) From Eq. 36 of Chapter 34 with N (the number of loops) = 1, we have

$$\mu = iA = (1.06 \times 10^{-3}\ \mathrm{A})(\pi)(5.29 \times 10^{-11}\ \mathrm{m})^2$$
$$= 9.31 \times 10^{-24}\ \mathrm{A \cdot m^2}.$$

Hall Effect



Hall Effect

Mainly Lorentz force is responsible for Hall effect. All of we know that when we place a current carrying conductor inside a magnetic field, the conductor experiences a mechanical force to a direction depending upon the direction of magnetic field and the direction of current in the conductor. The electric current means a flow of charge. In metal it is entirely due to the flow of electrons, in semiconductor, it is due to flow of free electrons as well as holes. In semiconductor, holes move in the direction of conventional current and free electrons move in the opposite of the direction of conventional current. As the electrons have charge, they experience a force while flowing through a conductor placed inside a magnetic field. Due to this force, the electrons get diverted towards one side of the conducted during flowing. As the following charges get shifted to one side of the conductor, there may be a tiny potential difference appeared across the cross-section of the conductor. We call this entire phenomenon as hall effect.

Application of Hall Effect:

Hall effect is a very useful phenomenon and helps to

- Determine the Type of Semiconductor
- Calculate the Carrier Concentration
- Measure Magnetic Flux Density
- Determine if a substance is a semiconductor or an insulator