EEE 1131 : Basic Electrical Circuits

Series Parallel Circuit, Branch Current Analysis

SOURCE CONVERSIONS

All sources—whether they are voltage sources or current sources—have some internal resistance in the relative positions shown in Fig. 8.5.

For the voltage source, if *Rs*=0, or if it is so small compared to any series resistors that it can be ignored, then we have an “ideal” voltage source for all practical purposes. For the current source, since the resistor *Rp* is in parallel, if *Rl* _ ∞ _, or if it is large enough compared to any parallel resistive elements that it can be ignored, then we have an “ideal” current source.

ideal sources *cannot be converted* from one type to another. That is, a voltage source cannot be converted to a current source, and vice versa—*the internal resistance must be present*

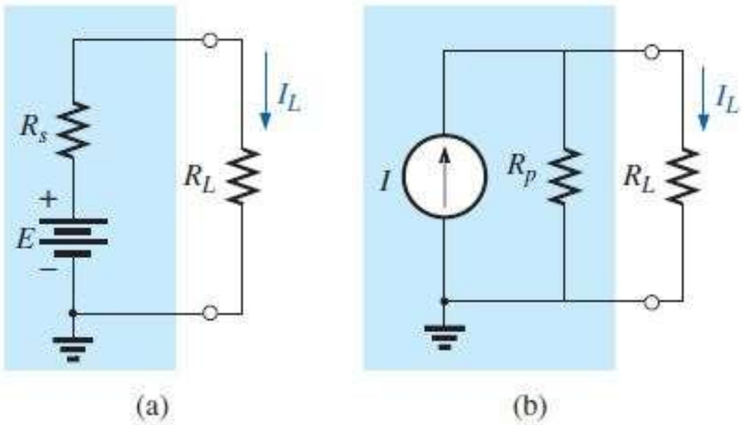


FIG. 8.5
Practical sources: (a) voltage; (b) current.

SOURCE CONVERSIONS

First note that the resistance is the same in each configuration—a nice advantage. For the voltage source equivalent, the voltage is determined by a simple application of Ohm's law to the current source: *$E = IR_p$.* For the current source equivalent, the current is again determined by applying Ohm's law to the voltage source: *$I = E/R_s$.*

It is important to realize, however, that
the equivalence between a current source and a voltage source exists only at their external terminals.

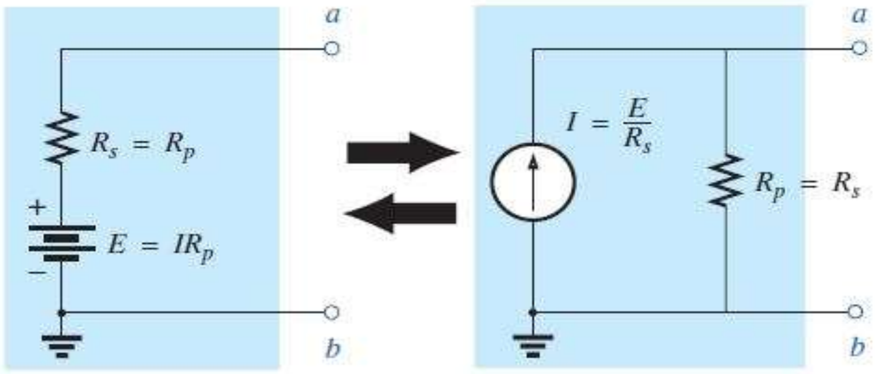


FIG. 8.6
Source conversion.

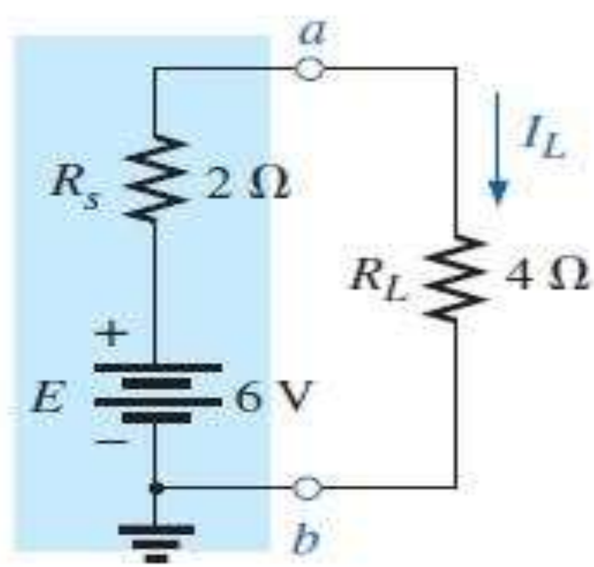
SOURCE CONVERSIONS

EXAMPLE 8.4 For the circuit in Fig. 8.7:

a. Determine the current I_L.

b. Convert the voltage source to a current source.

c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).



SOURCE CONVERSIONS

Solutions:

a. Applying Ohm's law:

$$I_L = \frac{E}{R_s + R_L} = \frac{6\ \text{V}}{2\ \Omega + 4\ \Omega} = \frac{6\ \text{V}}{6\ \Omega} = \mathbf{1\ A}$$

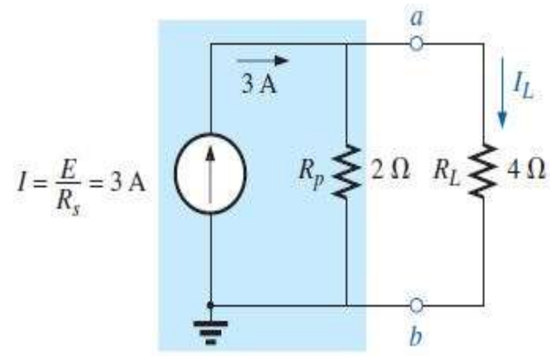


FIG. 8.8
Equivalent current source and load for the voltage source in Fig. 8.7.

b. Using Ohm's law again:

$$I = \frac{E}{R_s} = \frac{6\ \text{V}}{2\ \Omega} = \mathbf{3\ A}$$

and the equivalent source appears in Fig. 8.8 with the load reapplied.

c. Using the current divider rule:

$$I_L = \frac{R_p I}{R_p + R_L} = \frac{(2\ \Omega)(3\ \text{A})}{2\ \Omega + 4\ \Omega} = \frac{1}{3}(3\ \text{A}) = \mathbf{1\ A}$$

We find that the current I_L is the same for the voltage source as it was for the equivalent current source—the sources are therefore equivalent.

CURRENT SOURCES IN PARALLEL

current sources of different values cannot be placed in series due to a violation of Kirchhoff's current law.

two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.

CURRENT SOURCES IN PARALLEL

EXAMPLE 8.6 Reduce the parallel current sources in Fig. 8.11 to a single current source.

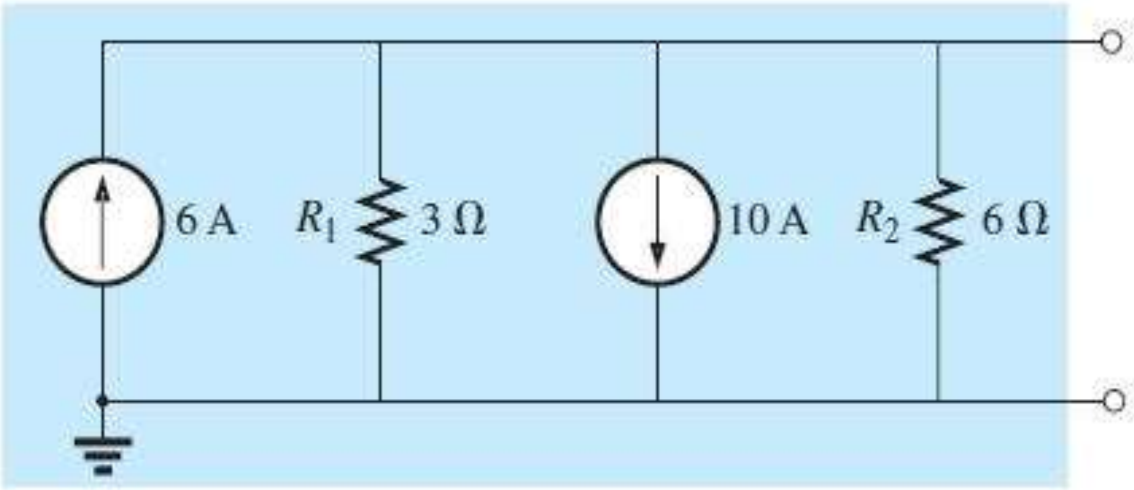


FIG. 8.11

Parallel current sources for Example 8.6.

CURRENT SOURCES IN PARALLEL

Solution: The net source current is

$$I = 10\ \text{A} - 6\ \text{A} = \mathbf{4\ A}$$

with the direction of the larger.

The net internal resistance is the parallel combination of resistors, R_1 and R_2:

$$R_p = 3\ \Omega \parallel 6\ \Omega = \mathbf{2\ \Omega}$$

The reduced equivalent appears in Fig. 8.12.

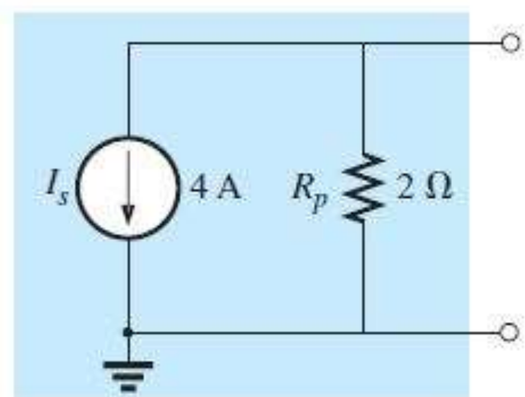


FIG. 8.12
Reduced equivalent for the configuration of Fig. 8.11.

CURRENT SOURCES IN SERIES

current sources of different current ratings are not connected in series,
just as voltage sources of different voltage ratings are not connected in parallel.

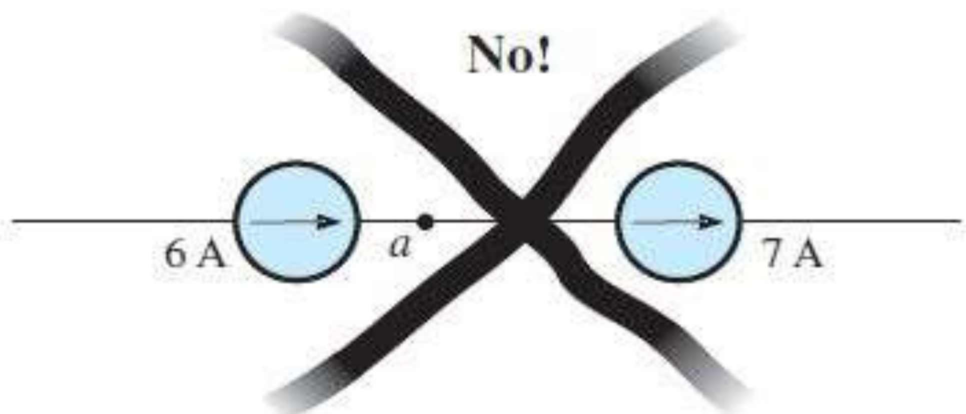


FIG. 8.18
Invalid situation.

BRANCH-CURRENT ANALYSIS

1. Assign a distinct current of arbitrary direction to each branch of the network.
2. Indicate the polarities for each resistor as determined by the assumed current direction.
3. Apply Kirchhoff's voltage law around each closed, independent loop of the network.
4. Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.
5. Solve the resulting simultaneous linear equations for assumed branch currents.

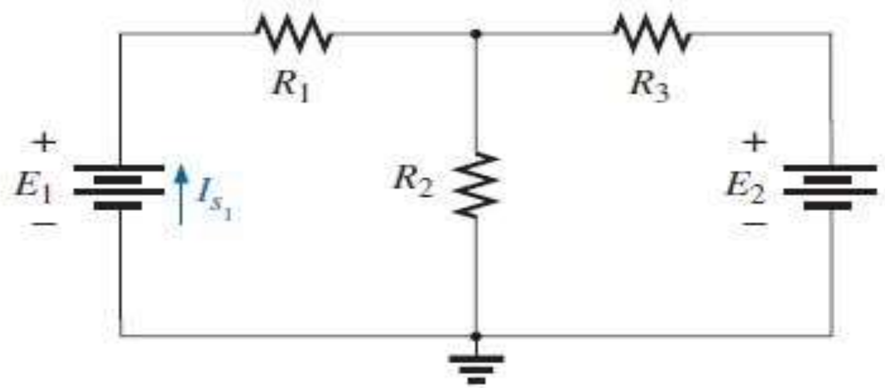


FIG. 8.19
Demonstrating the need for an approach such as branch-current analysis.

BRANCH-CURRENT ANALYSIS

EXAMPLE 8.9 Apply the branch-current method to the network in Fig. 8.22.

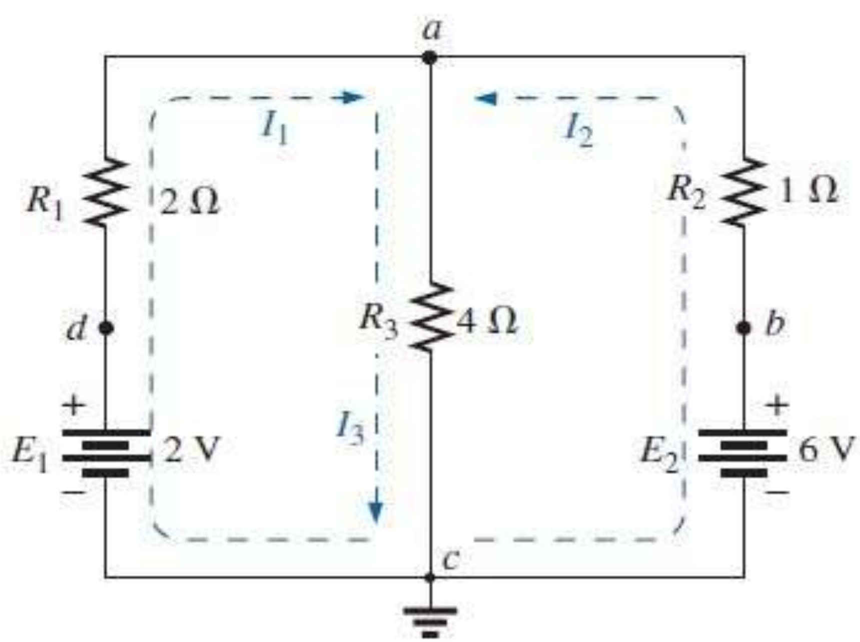


FIG. 8.22
Example 8.9.

BRANCH-CURRENT ANALYSIS

Solution 1:

*Step 1: S*ince there are three distinct branches (*cda, cba, ca*), three currents of arbitrary directions (*I*1, *I*2, *I*3) are chosen, as indicated in Fig. 8.22. The current directions for *I*1 and *I*2 were chosen to match the “pressure” applied by sources *E*1 and *E*2, respectively. Since both *I*1 and *I*2 enter node *a, I*3 is leaving.

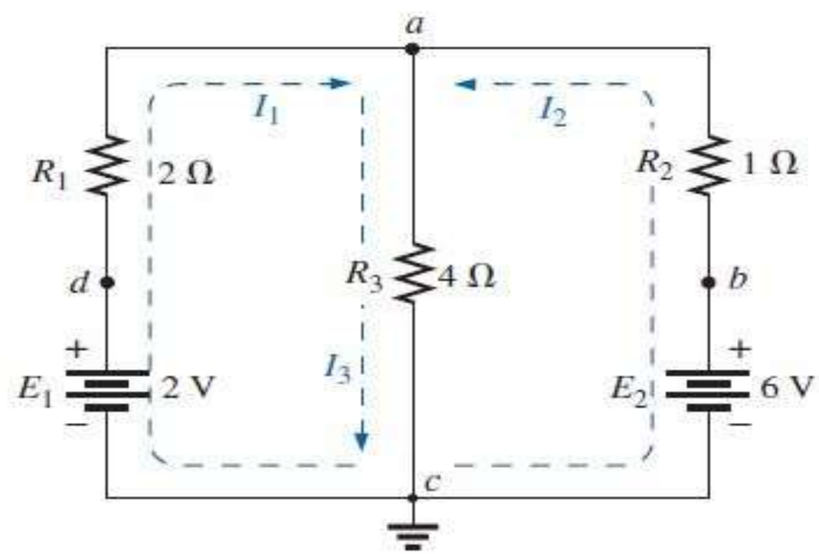


FIG. 8.22
Example 8.9.

BRANCH-CURRENT ANALYSIS

Solution :

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as indicated in Fig. 8.23.

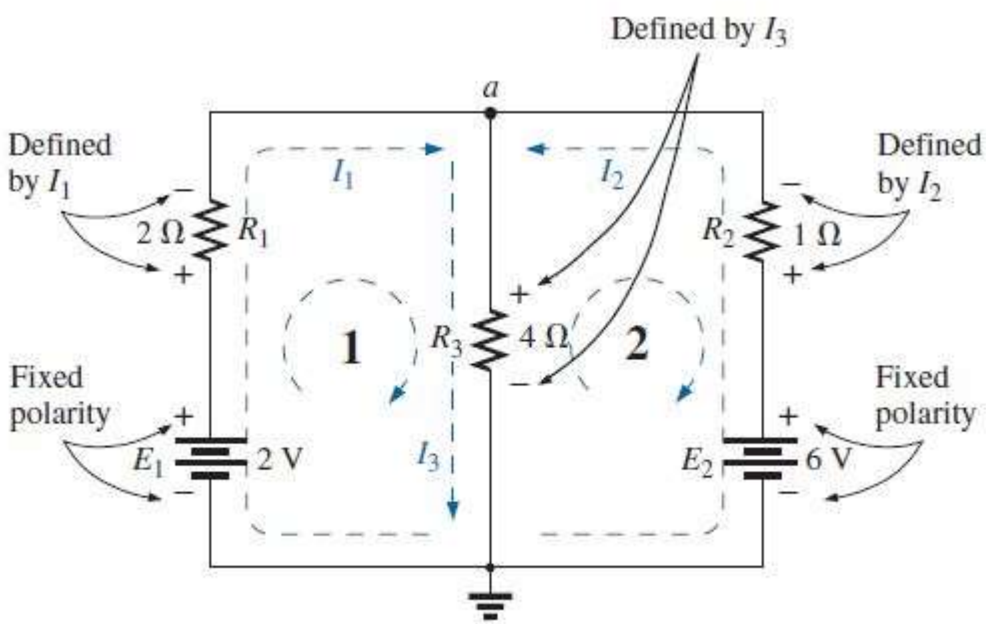


FIG. 8.23

Inserting the polarities across the resistive elements as defined by the chosen branch currents.

BRANCH-CURRENT ANALYSIS

Solution :

Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

Rise in potential

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Drop in potential

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = \underbrace{+2\text{ V}}_{\text{Battery potential}} - \underbrace{(2\ \Omega)I_1}_{\text{Voltage drop across }2\ \Omega\text{ resistor}} - \underbrace{(4\ \Omega)I_3}_{\text{Voltage drop across }4\ \Omega\text{ resistor}} = 0$$

Rise in potential

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

Drop in potential

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$

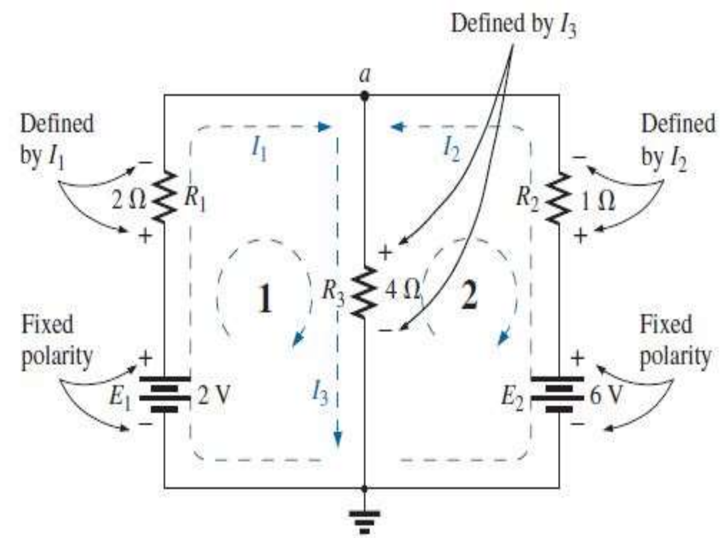


FIG. 8.23
Inserting the polarities across the resistive elements as defined by the chosen branch currents.

BRANCH-CURRENT ANALYSIS

Solution :
Step 4: Applying Kirchhoff's current law at node *a* (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$\begin{aligned} 2 - 2I_1 - 4I_3 &= 0 \\ 4I_3 + 1I_2 - 6 &= 0 \\ I_1 + I_2 &= I_3 \end{aligned} \qquad \text{Rewritten:} \quad \begin{aligned} 2I_1 + 0 + 4I_3 &= 2 \\ 0 + I_2 + 4I_3 &= 6 \\ I_1 + I_2 - I_3 &= 0 \end{aligned}$$

BRANCH-CURRENT ANALYSIS

Using third-order determinants (Appendix D), we have

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = -1\ \mathbf{A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = 2\ \mathbf{A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = 1\ \mathbf{A}$$

BRANCH-CURRENT ANALYSIS

Solution 2: Instead of using third-order determinants as in Solution 1, we can reduce the three equations to two by substituting the third equation in the first and second equations:

$$\left.\begin{aligned} 2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} &= 0 \\ 4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 &= 0 \end{aligned}\right\} \begin{aligned} 2 - 2I_1 - 4I_1 - 4I_2 &= 0 \\ 4I_1 + 4I_2 + I_2 - 6 &= 0 \end{aligned}$$

or

$$\begin{aligned} -6I_1 - 4I_2 &= -2 \\ +4I_1 + 5I_2 &= +6 \end{aligned}$$

Multiplying through by -1 in the top equation yields

$$\begin{aligned} 6I_1 - 4I_2 &= +2 \\ 4I_1 + 5I_2 &= +6 \end{aligned}$$

and using determinants,

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = \mathbf{-1\ A}$$

BRANCH-CURRENT ANALYSIS

EXAMPLE 8.10 Apply branch-current analysis to the network in Fig.8.28

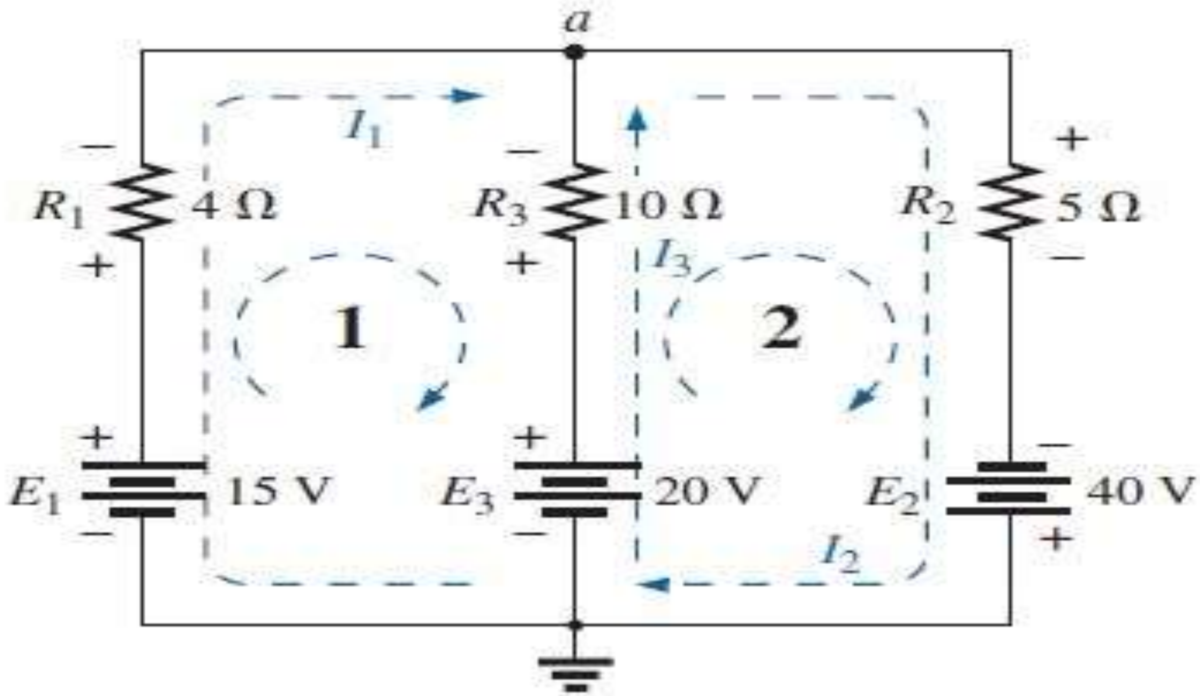
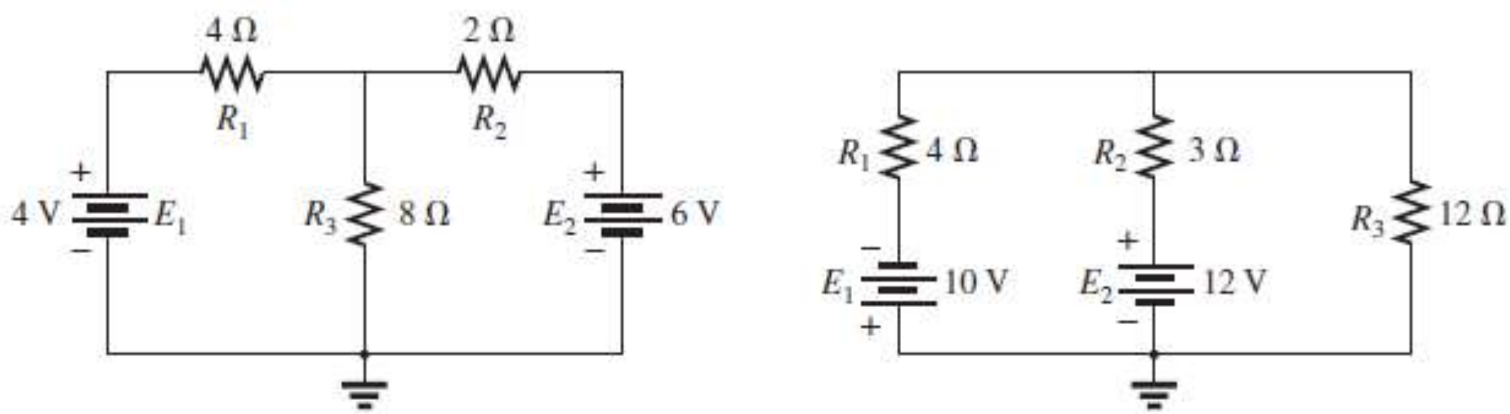


FIG. 8.28
Example 8.10.

HOME WORK

Using branch-current analysis, find the magnitude and direction of the current through each resistor for the networks in the following Figure.



(a)

(b)