Definition of a wave:

A very general definition of a wave valid for mechanical and electromagnetic waves is the following:" a wave is a disturbance that transfers energy from one point to another without imparting net motion to the medium through which it travels. Thus, the particles of the medium are not permanently displaced, but just oscillate back and forth about their equilibrium position.

Types of waves:

There are three types of waves:

1. **Mechanical waves:** They require a medium for their propagation. Example: pulse waves on a rope or a string, sound waves, waves travelling across the surface of water, seismic waves, etc.

2. **Electromagnetic waves:** Those consist of transverse oscillations of $\vec{E}$ and $\vec{B}$ fields. In this case we have oscillations of fields and not material particles. They do not require a medium for their propagation (they can carry energy through the vacuum). Example: visible light, IR radiations, ultraviolet radiation, radio and TV waves, X – rays, gamma rays, ultrasound, etc.

3. **Matter waves:** Matter waves are waves which are associated to electrons and other micro- particles (protons, neutrons, alpha particles, atoms, molecules, etc.). Electrons, protons, neutrons, etc. are described, in quantum mechanics, by De Broglie waves – "probability waves" associated to these micro -particles.

Definition:

Wavelength: It is the distance travelled by the wave in the time in which the particle of the medium completes one vibration.

Frequency: It is the number of vibrations made by a particle in one second.

Amplitude: It is the maximum displacement of the particle from its mean position of rest.

Time period: It is the time taken by a particle to complete one vibration.

Sound:

Sound is one kind of mechanical wave that produces a sensation of hearing. It is produced when something makes vibratory motion.

Properties of sound:

The properties of sound are:

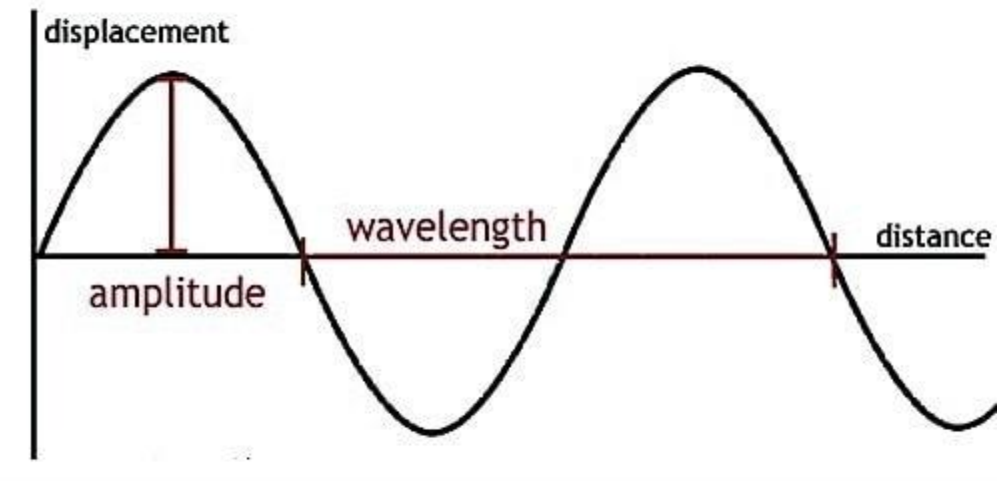
- It takes time to travel.
- It cannot pass through the vacuum.
- It may be reflected, refracted and deflected.
- It cannot be polarized.
- It exhibits interference.

Pitch:

Pitch is the quality that allows us to classify a sound as relatively high or low. Pitch is determined by the frequency of sound wave vibrations.

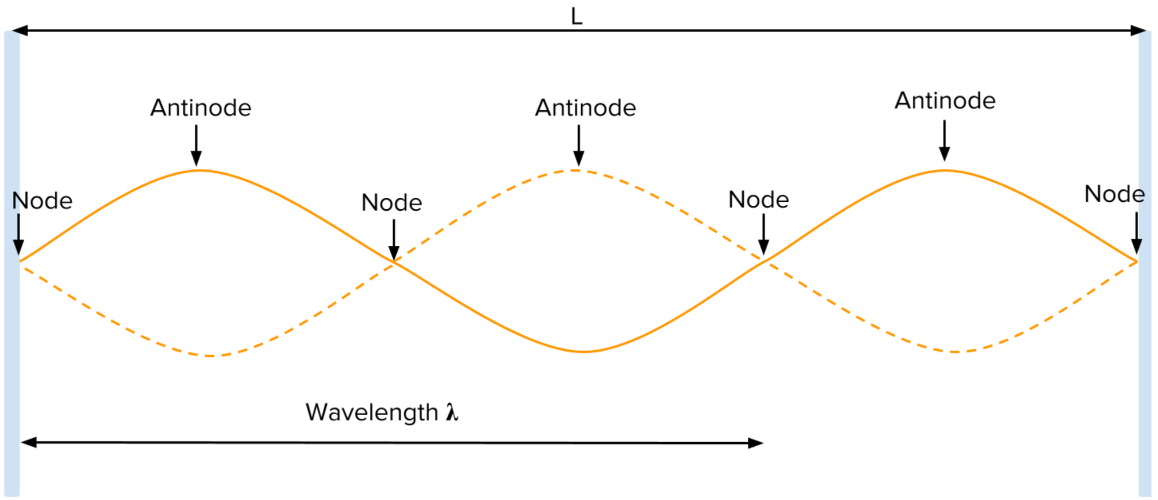
Progressive Wave:

A plane progressive wave is one which travels onwards through the medium in a given direction without attenuation, i.e., with its amplitude constant.



Standing wave:

Standing wave, also called stationary wave is the combination of two waves moving in opposite directions between two fixed points, each having the same amplitude and frequency. The phenomenon is the result of interference-that is, when waves are superimposed, their energies are either added together or cancelled out.



Distinction between progressive and stationary waves:

	Progressive waves	Stationary waves
1.	The vibration characteristic of each particle of the medium is the same and is handed on from particle to particle, so that there is an onward propagation of the wave through the medium.	The vibration characteristic of each particle of the medium is its own which it does not pass on to others, so that there is no onward propagation of the wave through the medium i.e., it remains confined to the limited space in which it is produced.
2.	All particles of the medium attain the same maximum displacement (positive or negative) but one after the other.	All particles attain their maximum displacement simultaneously but their maxima are different, decreasing progressively from an antinode to the adjoining node where it is reduced to zero.
3.	All particles pass through their mean positions with the same maximum velocity but one after the other.	All particles pass through their mean positions simultaneously but with different maximum velocities.
4.	No particles of the medium are permanently at rest.	Certain particles of the medium throughout remain at rest.
5.	All particles of the medium undergo the same changes of pressure and density but one after the other.	The changes of pressure and density are the maximum (positive or negative) at the nodal points and zero at the anti-nodal points but occur simultaneously at all points.
6.	There is a regular transference of energy across every section of the medium.	There is no transference of energy across any section of the medium.

Plane Progressive Harmonic Wave:

A plane progressive wave is one which travels onwards through the medium in a given direction without attenuation, i.e., with its amplitude constant.

Expression for a plane progressive harmonic wave:

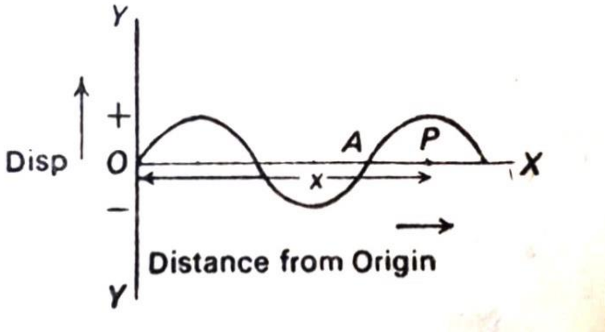


Figure-1: Plane progressive wave

Let us suppose a wave, originating at O, travels to the right along the axis of x. Then, if we start counting time when the particle at O just passes through its mean position in the positive direction, the equation of motion of this particle (at O) is obviously

$$y = a\ sin\ \omega t$$

where

y is the displacement at time t

a is the amplitude

ω is the angular velocity

Let us consider, for a particle at P, distant x from O, the phase lag is ϕ.

The equation of motion of the particle at P is thus

$$y = a \sin\ (\omega t - \phi) \dots \dots \dots \dots (i)$$

Now, we know, in a distance λ, the phase lag increases by 2π. In a distance x, therefore, the phase lag increases by $2\pi x/\lambda$, i.e.,

$$\phi = \frac{2\pi x}{\lambda}$$

Substituting this value of ϕ in the eq(i), we have

$$y = a \sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \dots \dots \dots \dots (ii)$$

where

$$\frac{2\pi}{\lambda} = \text{the propagation constant}$$

Again,

$$\omega = \frac{2\pi}{T}$$

where T is the time period of each particle of the medium.

And, since

$$v = n\lambda = \frac{\lambda}{T}$$

where,

v=the phase velocity or the wave velocity

n= the frequency of the oscillating particles or of the wave

we have,

$$\frac{1}{T} = \frac{v}{\lambda}$$

And, therefore,

$$\omega = \frac{2\pi v}{\lambda}$$

We thus have from the eq(ii) above,

$$y = a \sin \left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

Or,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

This equation is referred to as the equation of a plane progressive harmonic wave.

Particle Velocity

We have the equation of plane progressive wave motion,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \dots \dots \dots (i)$$

where

y is the displacement at distant x from the origin at time t

a is the amplitude

v is the phase velocity or the wave velocity

Differentiating this expression*(i)* for y with respect to t, we have

Particle velocity

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda}\, a \cos \frac{2\pi}{\lambda}(vt - x) \dots \dots \dots \dots (ii)$$

And, differentiating the same expression for y with respect to x, we have

Slope of the displacement curve

$$\frac{dy}{dx} = -a \cos \frac{2\pi}{\lambda}(vt - x) \dots \dots \dots \dots \dots (iii)$$

From *(ii)* and *(iii)* we thus obtain

$$U = \frac{dy}{dt} = -v \frac{dy}{dx}$$

i.e.,

particle velocity at a point = − (wave velocity) × (slope of displacement curve at that point)

Differential equation of a wave equation:

Differentiating eq(ii) for the particle velocity again with respect to t, we have

Acceleration of the particle,

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2}\ a \sin\frac{2\pi}{\lambda}(vt - x)$$

$$= -\frac{4\pi^2 v^2}{\lambda^2}\ y \ldots\ldots\ldots (iv)$$

Similarly, differentiating eq(iii) gain with respect to x, we have

Rate of change of compression with distance,

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2} a \sin\frac{2\pi}{\lambda}(vt - x)$$

$$= -\frac{4\pi^2}{\lambda^2} y \ldots\ldots\ldots (v)$$

From relations (iv) and (v), we thus obtain

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

i.e.

particle acceleration at a point = (wave velocity)^2 × (curvature of displacement curve at that point)

This is referred to as the differential equation of a plane or one dimensional progressive.

Wave velocity and Phase velocity:

We know that the equation of a plane progressive harmonic wave is,

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

where

y is the displacement at distant x from the origin at time t

a is the amplitude

v is the wave velocity, equal to λ/T

We have the from the above,

$$y = a\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

Since

$$\text{the frequency of the wave } n = \frac{v}{\lambda} = \frac{1}{T}$$

and

$$\text{the propagation constant } k = \frac{2\pi}{\lambda}$$

The wave equation takes the form

$$y = a\sin(2\pi nt - kx)$$

Or, since the angular frequency $\omega = 2\pi n$, we have

$$y = a\sin(\omega t - kx)$$

showing that the constant phase $(\omega t - kx)$ of the wave travels along the positive direction of the x-axis with velocity dx/dt , i.e., the phase velocity of the wave = dx/dt.

Now, since $(\omega t - kx) = \text{constant}$, we have

$$\omega - k\left(\frac{dx}{dt}\right) = 0$$

Or,

$$\omega = k\left(\frac{dx}{dt}\right)$$

whence, the phase velocity of the wave

$$\frac{dx}{dt} = \frac{\omega}{k}$$

But,

$$\frac{\omega}{k} = \frac{2\pi n}{\frac{2\pi}{\lambda}} = n\lambda = \frac{\lambda}{T} = wave\ velocity$$

Thus, for a single wave, in any given medium,

$$Wave\ velocity\ =\ Phase\ velocity\ =\ v = \frac{\lambda}{T} = \frac{\omega}{k}$$

Doppler Effect:

Doppler effect is commonly observed that the pitch apparently changes when either the source or the observer are in motion relative to each other. When the source approaches the observer or when the observer approaches the source or when both approach each other, the apparent pitch is higher than the actual pitch of the sound produced by the source. Similarly, When the source moves away from the observer or when the observer moves away from the source or when both move away from each other, the apparent pitch is lower than the actual pitch of the sound produced by the source.

[Doppler effect in sound is asymmetric. When the source moves towards the observer with a certain velocity, the apparent pitch is different to the case when the observer is moving towards the source with the same velocity. But it is not so in the case of light. Doppler effect in light is symmetric.]

Doppler Effect Formula:

Doppler effect is the apparent change in the pitch of waves due to the relative motion between the source of the sound and the observer. We can deduce the apparent pitch in the Doppler effect using the following equation:

$$n' = \frac{v \pm b}{v \pm a} n$$

Where,

n' = The apparent pitch

n = The actual pitch

v = The velocity of sound waves

b = The velocity of the observer

a = The velocity of the source

1. Observer at Rest and Source in Motion:

(a) When the source moves towards the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v.

Let the source move with a velocity a towards the observer. In one second, n waves will be contained in a length $(v - a)$ and the apparent wavelength,

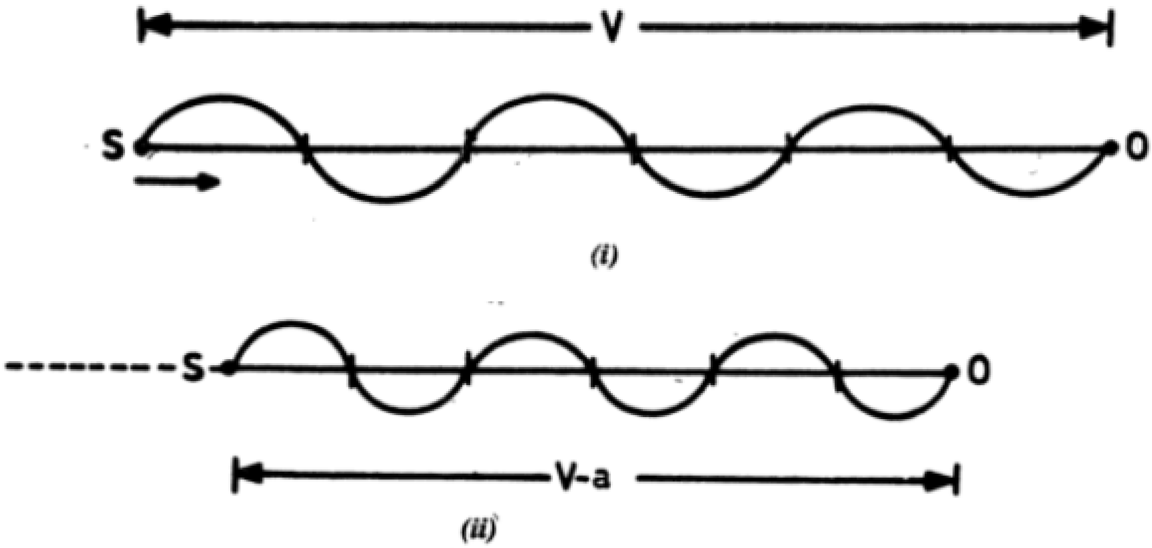
$$\lambda' = \frac{(v-a)}{n}$$

The apparent pitch,

$$n' = \frac{v}{\lambda'}$$

$$\therefore n' = \frac{v}{(v-a)} n$$

Thus, the apparent pitch increases when the source moves towards a stationary observer.



(b) When the source moves away from the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v.

Let the source move with a velocity a away from the observer. In one second, n waves will be contained in a length $(v + a)$ and the apparent wavelength,

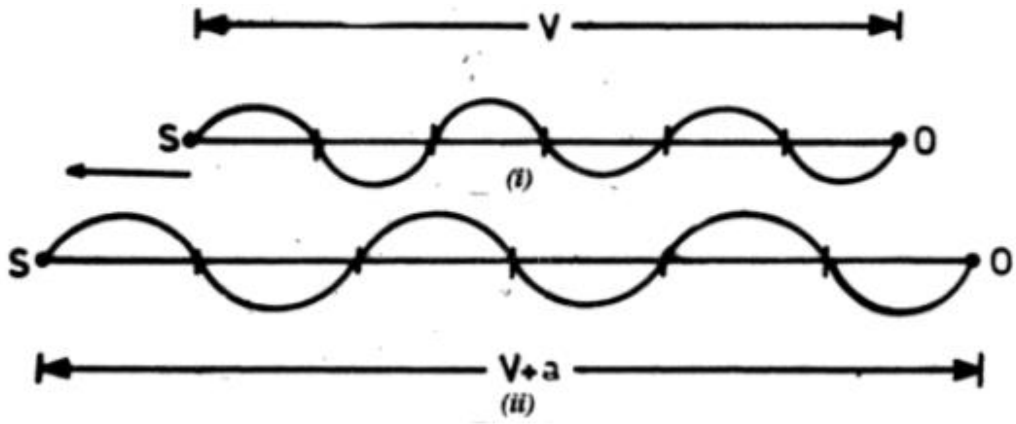
$$\lambda' = \frac{(v + a)}{n}$$

The apparent pitch,

$$n' = \frac{v}{\lambda'}$$

$$\therefore n' = \frac{v}{(v + a)} n$$

Thus, the apparent pitch decreases when the source moves away from a stationary observer.



2. Source at Rest and Observer in Motion

(a) When the observer moves towards a stationary source

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v. Let the observer move with a velocity b towards a stationary source. In this case the observer receives more number of waves in one second. The apparent wavelength remains the same.

The apparent frequency,

$$n' = n + \frac{b}{\lambda}$$

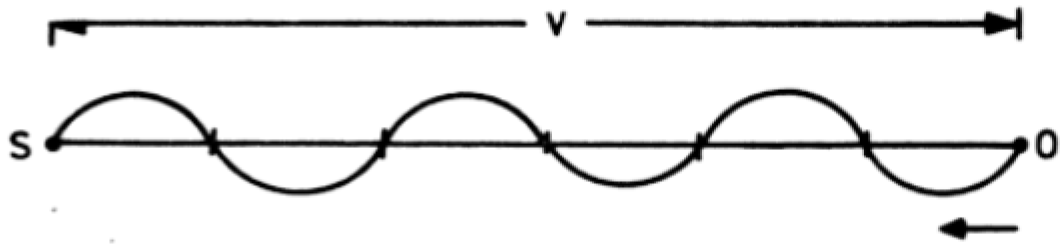
$$\Rightarrow n' = \frac{v}{\lambda} + \frac{b}{\lambda}$$

$$\Rightarrow n' = \frac{v+b}{\lambda}$$

But $\lambda = \frac{v}{n}$

$$\therefore\ n' = \frac{v+b}{v} n$$

Thus, the apparent pitch increases when the observer moves towards a stationary source.



(i)

(ii)

(b) When the observer moves away from a stationary source

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v. Let the observer move with a velocity b away from a stationary source. In this case the observer receives less number of waves in one second. The apparent wavelength remains the same.

The apparent frequency,

$$n' = n - \frac{b}{\lambda}$$

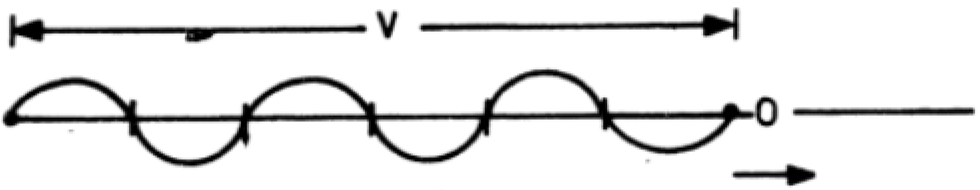
$$\Rightarrow n' = \frac{v}{\lambda} - \frac{b}{\lambda}$$

$$\Rightarrow n' = \frac{v-b}{\lambda}$$

But $\lambda = \frac{v}{n}$

$$\therefore \; n' = \frac{v-b}{v} n$$

Thus, the apparent pitch decreases when the observer moves away from a stationary source.



(i)

(ii)

3. When both the source and the observer are in motion

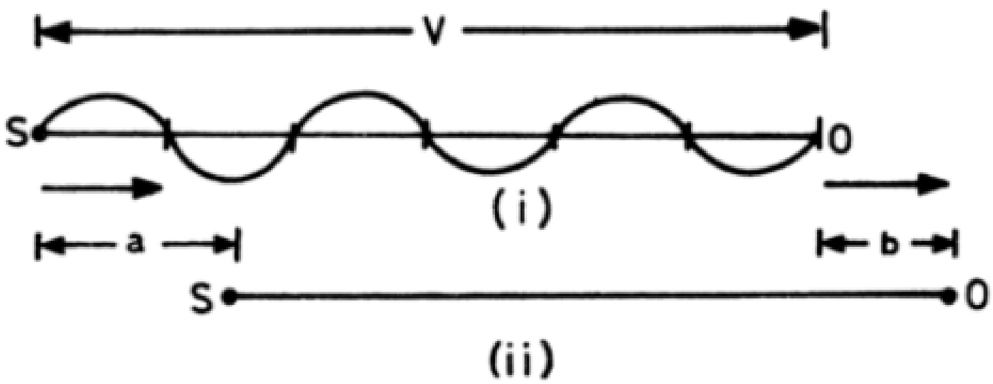
When the source moves towards the observer and the observer moves away from the source.

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is *a* and the velocity of sound is *b*.
Let the source move towards the observer with a velocity *a* and the observer move away from the source with a velocity *b*. The apparent wavelength,

$$\lambda' = \frac{(v-a)}{n}$$

and $$n' = \frac{v-b}{\lambda'}$$

$$\therefore\ n' = \left(\frac{v-b}{v-a}\right)n \cdots\cdots\cdots\cdots\cdots\cdots (1)$$



Special cases:

(a) When the source and the observer move towards each other

In equation (1), taking b to be negative,

$$n' = [\frac{v-(-b)}{v-a}]n$$

$$\therefore\ n' = \left(\frac{v+b}{v-a}\right)n$$

(b) When the source and the observer move away from each other

In equation (1), taking a to be negative

$$n' = [\frac{v-b}{v-(-a)}]n$$

$$\therefore\ n' = \left(\frac{v-b}{v+a}\right)n$$

When the source moves away from the observer and the observer moves towards the source.

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is *a* and the velocity of sound is *b*.

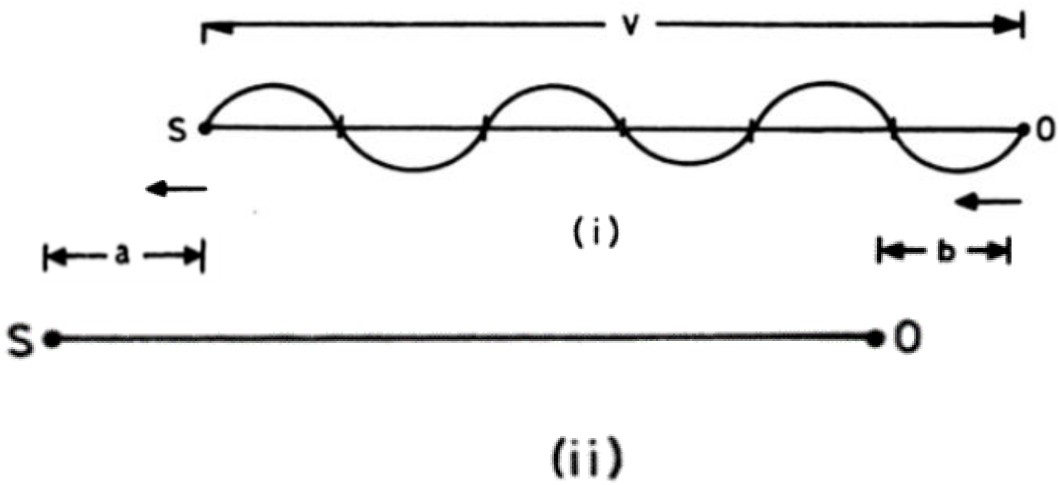
Let the source move away from the observer with a velocity *a* and the observer move towards the source with a velocity *b*.

The apparent wavelength,

$$\lambda' = \frac{(v+a)}{n}$$

and $$n' = \frac{v+b}{\lambda'}$$

$$\therefore\ n' = \left(\frac{v+b}{v+\ a}\right) n$$



Math-1

1. When a simple harmonic wave is propagated through a medium, the displacement of a particle at any instant of time is given by

$$y = 10\, sin \frac{2\pi}{100}(36000t - 20)$$

where y is expressed in centimeters and t in seconds.
Calculate the wave velocity, wavelength and frequency and time period.

Solution:

For any type of wave, we have the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Comparing it with the given equation, we get

Amplitude

$$a = 10\ cm$$

Velocity

$$v = 36000\ cms^{-1}$$

Wavelength

$$\lambda = 100\ cm$$

Frequency

$$v = n\lambda$$
$$n = \frac{v}{\lambda}$$
$$n = \frac{36000}{100}$$
$$n = 360\ Hz$$

Time period

$$T = \frac{1}{n} = \frac{1}{360}\ second$$

When a simple harmonic wave is propagated through a medium, the displacement of a particle at any instant of time is given by

$$y = 10 \sin \frac{2\pi}{200}(200t - 20)$$

where y is expressed in centimeters and t in seconds.

Calculate the wave velocity, wavelength and frequency and time period.

Solution:

For any type of wave, we have the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Comparing it with the given equation, we get

Amplitude

$$a = 10\ cm$$

Velocity

$$v = 200\ cms^{-1}$$

Wavelength

$$\lambda = 200\ cm$$

Frequency

$$v = n\lambda$$
$$n = \frac{v}{\lambda}$$

$$n = \frac{200}{200}$$

$$n = 1\ Hz$$

Time period

$$T = \frac{1}{n} = \frac{1}{1} = 1\ second$$

Math-2

2. The equation of a transverse wave travelling along a stretched string is given by

$$y = 10 \sin \pi \, (2t - 0.01x)$$

where y and x are expressed in centimeters and t in seconds.

Find the amplitude, velocity, wavelength and frequency of the wave.

Solution:

We can rewrite the given equation as,

$$y = 10 \sin \pi \, \left(2t - \frac{x}{100}\right)$$

$$or, \qquad y = 10 \sin 2\pi \, \left(t - \frac{x}{200}\right)$$

$$or, \qquad y = 10 \sin \frac{2\pi}{200} \, (200t - x) \ldots \ldots \ldots (i)$$

For any type of wave, we have the equation

$$y = a \sin \frac{2\pi}{\lambda} (vt - x)$$

Comparing it with the given equation, we get

Amplitude

$$a = 10 \ cm$$

Velocity

$$v = 200 \ cms^{-1}$$

Wavelength

$$\lambda = 200 \ cm$$

Frequency

$$n = \frac{v}{\lambda} = \frac{200}{200} = 1 \ Hz$$

Math-3

1. A person is standing on a platform. A railway engine moving away from the person with a speed of 72 km/hr blows a whistle of pitch 740 hertz. Calculate the apparent pitch of the whistle. The velocity of sound is 350 m/s.

Solution:

Here $V = 350$ m/s

$a = 72$ km/hr $= 20$ m/s

$n = 740$ hertz

The source is moving away from a stationary observer.

$$\therefore \quad n' = \left(\frac{V}{V+a}\right)n$$

$$n' = \left(\frac{350}{350+20}\right)\times 740$$

$$n' = \mathbf{700\ hertz.}$$

Fourier's Theorem

We may state Fourier's theorem as:

Any finite, single-valued periodic function, which is either continuous or which possesses only a finite number of discontinuities of slope or magnitude (all within the interval of one time-period), may be regarded as a combination of simple harmonic vibrations whose frequencies are integral multiples of that of the given function.

Example:

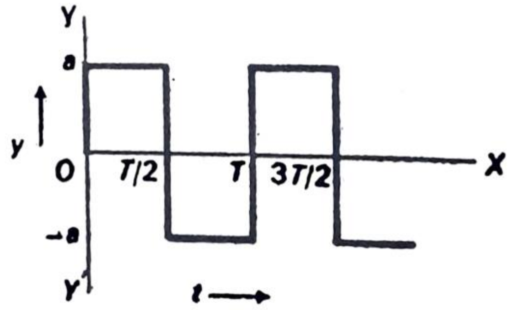


Fig. 9.14