CSE 121: ELECTRICAL CIRCUITS

MAXIMUM POWER TRANSFER THEOREM

Network Theorems

1. Superposition Theorem
2. Thevenin’s Theorem
3. Norton’s Theorem
4. Maximum Power Transfer Theorem

Maximum Power Transfer Theorem

When designing a circuit, it is often important to be able to answer one of the following questions

What load should be applied to a system to ensure that the load is receiving maximum power from the system?

Maximum Power Transfer Theorem

When designing a circuit, it is often important to be able to answer one of the following questions

conversely:

For a particular load, what conditions should be imposed on the source to ensure that it will deliver the maximum power available?

Maximum Power Transfer Theorem

Maximum Power Transfer Theorem, which states the following:

A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance of the network applied to the load.

That is,

$$\boxed{R_L = R_{Th}}$$

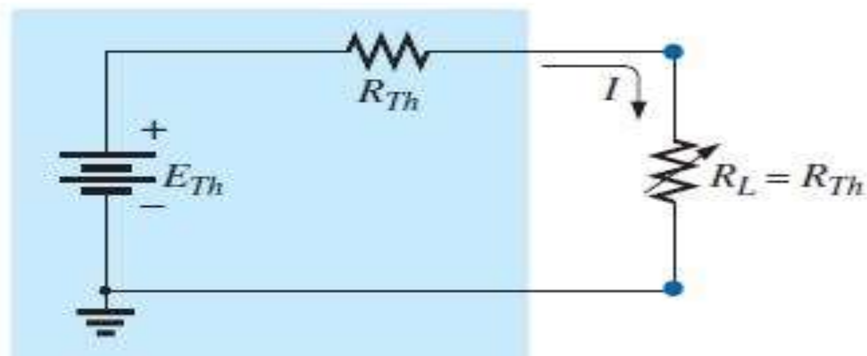


FIG. 9.78
Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

Maximum Power Transfer Theorem

Guess what value of *RL* would result in maximum power transfer to *RL*

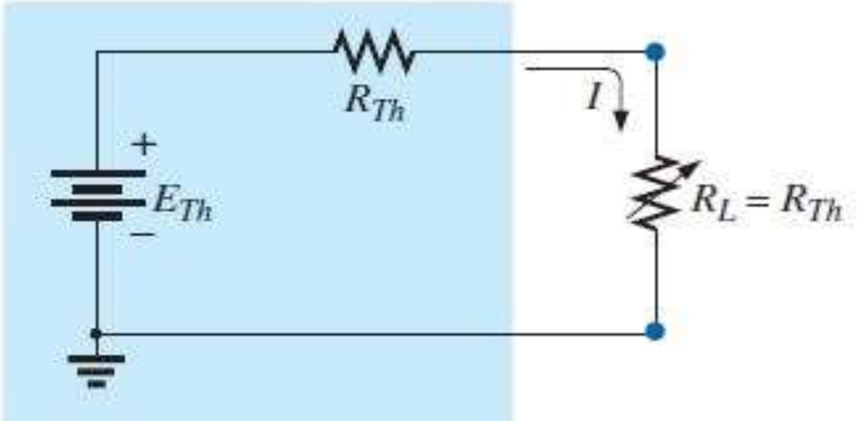


FIG. 9.78

Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

the smaller the value of *RL* ,

But, $P_L = I_L^2 R_L,$

When R_L decreases, I_L will increases (*I= V/R*)and power would be increase, however R_L is the multiplier.???

Maximum Power Transfer Theorem

Guess what value of *RL* would result in maximum power transfer to *RL*

larger values of *RL*

$$P_L = V_L^2/R_L.$$

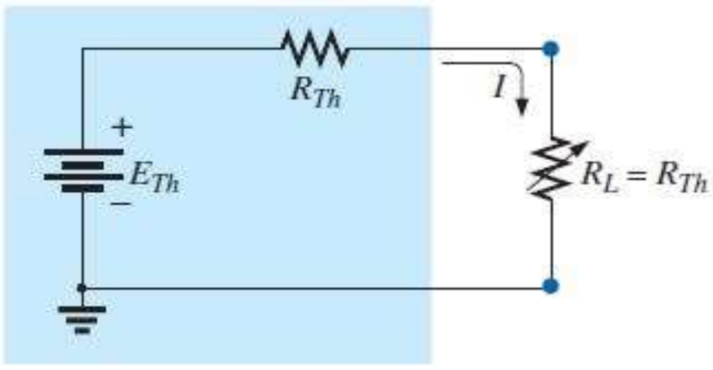


FIG. 9.78
Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

If we increase R_L, it will increases V_L ($V = IR$) and power would be increase, however R_L is the denominator.???

Maximum Power Transfer Theorem

Maximum Power transfer theorem Proof

A variable resistance R_L is connected to a DC source network as shown in the circuit diagram in figure A

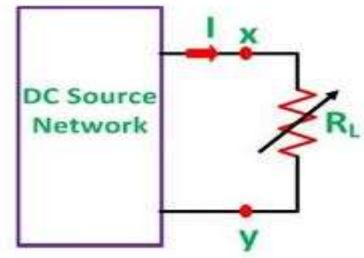


Figure A

figure B represents the Thevenin's voltage V_{TH} and Thevenin's resistance R_{TH} of the source network.

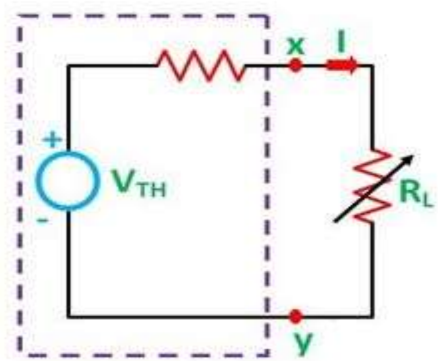


Figure B

In figure B , current will be calculated by the equation

$$I = \frac{V_{TH}}{R_{TH} + R_L} \dots\dots\dots..(1)$$

Maximum Power Transfer Theorem

Maximum Power transfer theorem Proof

power delivered to the resistive load is given by the equation

$$P_L = I^2 R_L \ldots\ldots\ldots\ldots (2)$$

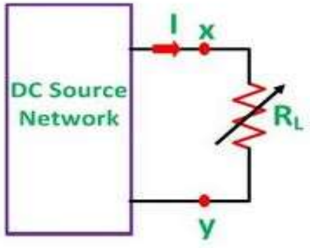


Figure A

Putting the value of I from the equation (1) in the equation (2) we will get

$$P_L = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 x\, R_L$$

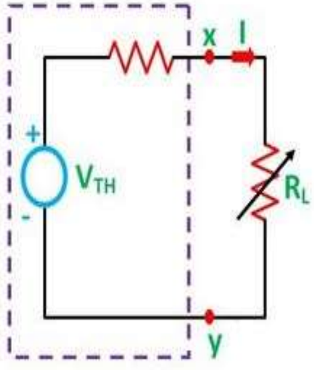


Figure B

P_L can be maximized by varying R_L and hence, maximum power can be delivered when $(dP_L/dR_L) = 0$,

Maximum Power Transfer Theorem

Maximum Power transfer theorem Proof

However,

$$\frac{dP_L}{dR_L} = \frac{1}{[(R_{TH} + R_L)^2]^2}\left[(R_{TH} + R_L)^2 \frac{d}{dR_L}(V_{TH}^2 R_L) - V_{TH}^2 R_L \frac{d}{dR_L}(R_{TH} + R_L)^2\right]$$

$$\frac{dP_L}{dR_L} = \frac{1}{(R_{TH} + R_L)^4}[(R_{TH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L \, x \, 2(R_{TH} + R_L)]$$

$$\frac{dP_L}{dR_L} = \frac{V_{TH}^2 (R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} = \frac{V_{TH}^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^2}$$

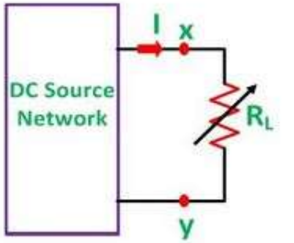


Figure A

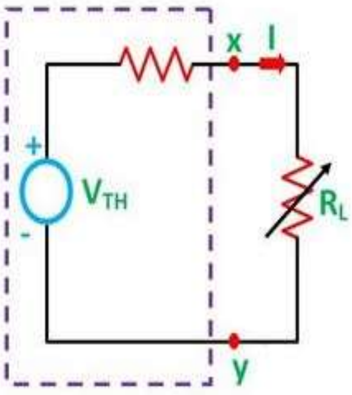


Figure B

Maximum Power Transfer Theorem

Maximum Power transfer theorem Proof

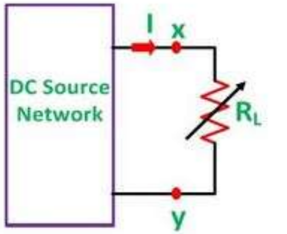


Figure A

But as we know, Condition for Maximum Power is
$(dP_L/dR_L) = 0$,Therefore,

$$\frac{V_{TH}^2\,(R_{TH} - R_L)}{(R_{TH} + R_L)^2} = 0$$

Which gives

$$(R_{TH} - R_L) = 0 \quad \text{or} \quad R_{TH} = R_L$$

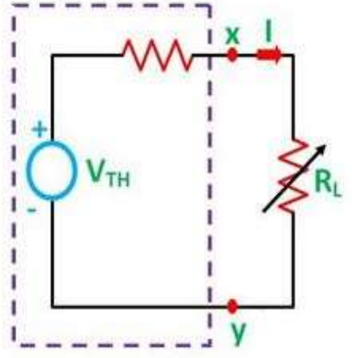


Figure B

Maximum Power will be transferred when

Rth = RL

PROOF

Maximum Power Transfer Theorem

Maximum Power Delivered to the LOAD:

$$P_{max} = \frac{V_{TH}^2 R_{TH}}{(R_{TH} + R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

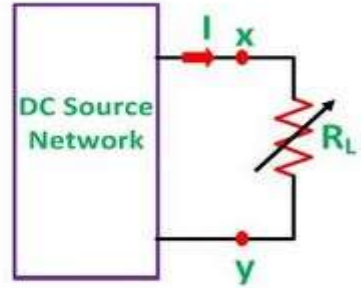


Figure A

Total Power delivered by the SOURCE

the total power supplied is given by the equation

$$P = 2\,\frac{V_{TH}^2}{4R_{TH}} = \frac{V_{TH}^2}{2R_{TH}}$$

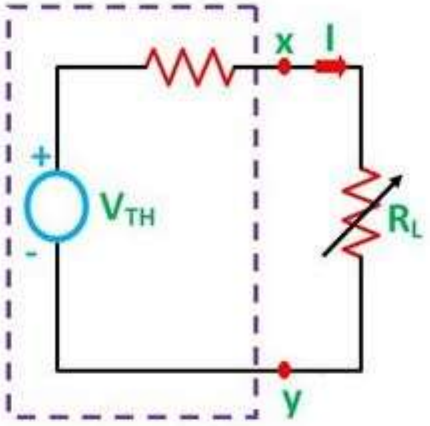


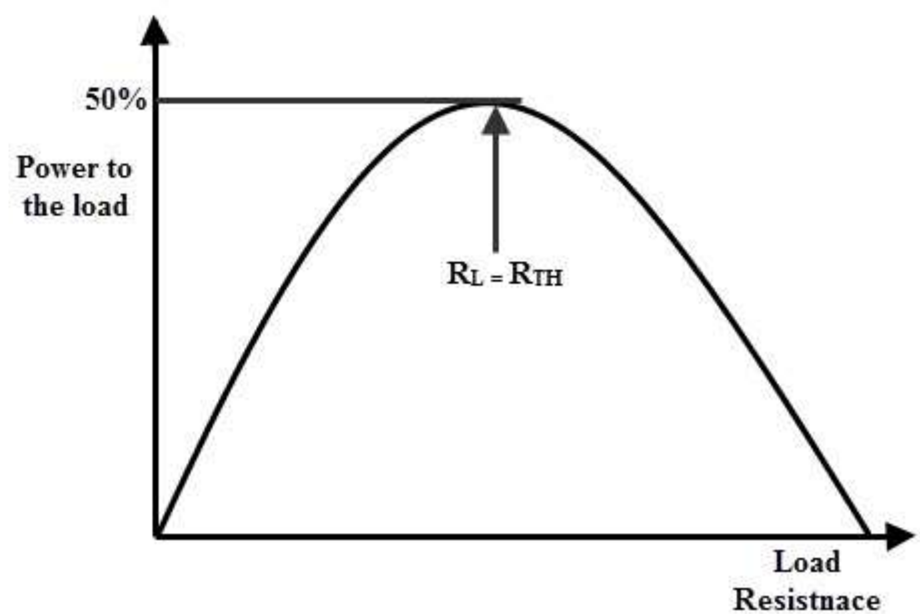
Figure B

Maximum Power Transfer Theorem

Efficiency:

During Maximum Power Transfer the efficiency η becomes:

$$P = 2\,\frac{V_{TH}^2}{4R_{TH}} = \frac{V_{TH}^2}{2R_{TH}}$$



Maximum Power Transfer Theorem

Steps for Solving Network Using Maximum Power Transfer Theorem

Step 1 – Remove the load resistance of the circuit.

Step 2 – Find the Thevenin's resistance (R_{TH}).

Step 3 -this R_{TH} is the load resistance of the network, i.e., $R_L = R_{TH}$ that allows maximum power transfer.

Step 4 – Maximum Power Transfer is calculated by the equation shown below

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

Maximum Power Transfer Theorem

Demonstration of Maximum Power tanrasfer theorem:

Considerthe Thévenin equivalent circuit in Fig. 9.79

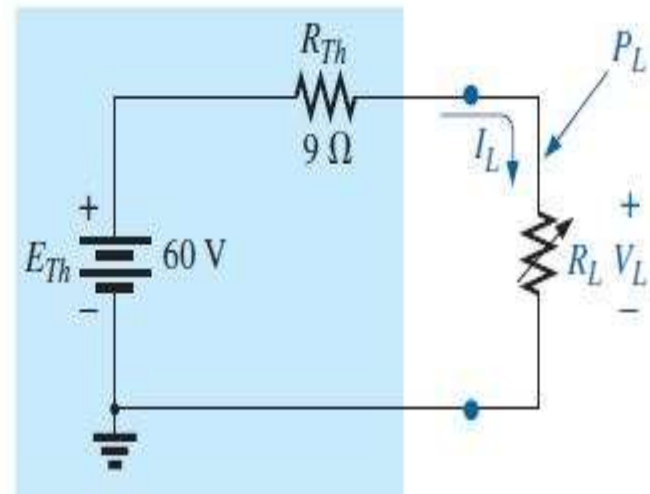


FIG. 9.79

For the circuit in Fig. 9.79, the current through the load is determined By

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\text{ V}}{9\ \Omega + R_L}$$

The voltage is determined by

$$V_L = \frac{R_L E_{Th}}{R_L + R_{Th}} = \frac{R_L(60\text{ V})}{R_L + R_{Th}}$$

and the power by

$$P_L = I_L^2 R_L = \left(\frac{60\text{ V}}{9\ \Omega + R_L}\right)^2 (R_L) = \frac{3600 R_L}{(9\ \Omega + R_L)^2}$$

Maximum Power Transfer Theorem

Demonstration of Maximum Power tanrasfer theorem:

If we tabulate the three quantities versus a range of values for *RL* from
1._ to 30 _, we obtain the results appearing in Table 9.1

TABLE 9.1

R_L (Ω)	P_L (W)	I_L (A)	V_L (V)
0.1	4.35	6.60	0.66
0.2	8.51	6.52	1.30
0.5	19.94	6.32	3.16
1	36.00	6.00	6.00
2	59.50	5.46	10.91
3	75.00	5.00	15.00
4	85.21	4.62	18.46
5	91.84	4.29	21.43
6	96.00	4.00	24.00
7	98.44 Increase	3.75 Decrease	26.25 Increase
8	99.65	3.53	28.23
9 (R_{Th})	100.00 (Maximum)	3.33 ($I_{max}/2$)	30.00 ($E_{Th}/2$)
10	99.72	3.16	31.58
11	99.00	3.00	33.00
12	97.96	2.86	34.29
13	96.69	2.73	35.46
14	95.27	2.61	36.52
15	93.75	2.50	37.50
16	92.16	2.40	38.40
17	90.53	2.31	39.23
18	88.89	2.22	40.00
19	87.24	2.14	40.71
20	85.61	2.07	41.38
25	77.86	1.77	44.12
30	71.00	1.54	46.15
40	59.98	1.22	48.98
100	30.30	0.55	55.05
500	6.95 Decrease	0.12 Decrease	58.94 Increase
1000	3.54	0.06	59.47

when*RL* is equal to the Thévenin resistance of 9 _, the power has a maximum value of 100 W,

Maximum Power Transfer Theorem

Demonstration of Maximum Power tanrasfer theorem:

The power to the load versus the range of resistor values is provided in Fig. 9.80.

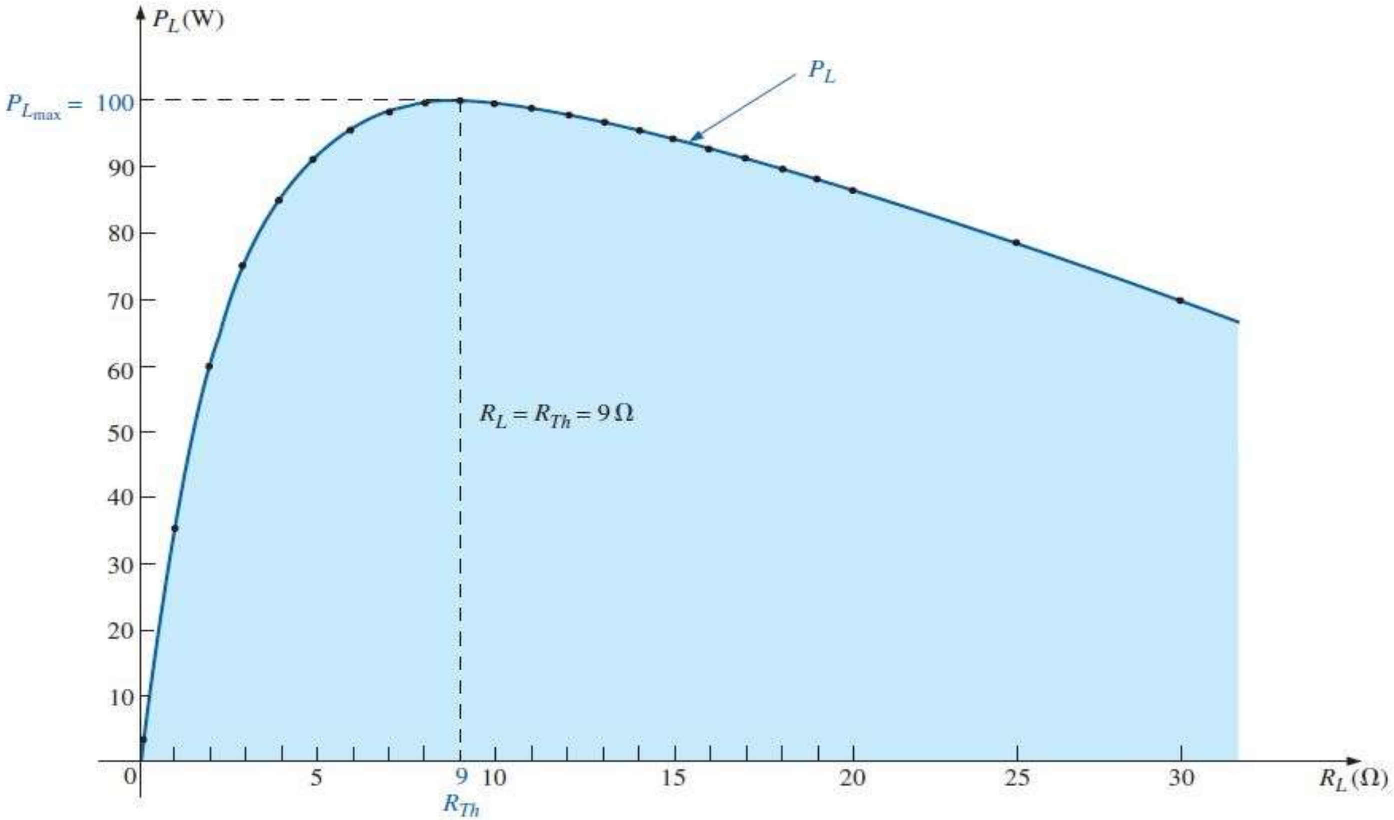


FIG. 9.80

P_L versus R_L for the network in Fig. 9.79.

Maximum Power Transfer Theorem

Demonstration of Maximum Power tanrasfer theorem:

This is important because it tells you the following:

If the load applied is less than the Théven in resistance, the power to the load will drop off rapidly as it gets smaller. However, if the applied load is greater than the Théven in resistance, the power to the load will not drop off as rapidly as it increases

important to remember the following:

The total power delivered by a supply such as Eth is absorbed by both the Théven in equivalent resistance and the load resistance. Any power delivered by the source that does not get to the load is lost to the Théven in resistance.

Maximum Power Transfer Theorem

EXAMPLE 9.14 A dc generator, battery, and laboratory supply are connected to resistive load *RL* in Fig. 9.85.

a. For each, determine the value of *RL* for maximum power transfer to *RL.*

b. Under maximum power conditions, what are the current level and the power to the load for each configuration?

c. What is the efficiency of operation for each supply in part (b)?

d. If a load of 1 k_ were applied to the laboratory supply, what would the power delivered to the load be? Compare your answer to the level of part (b). What is the level of efficiency?

e. For each supply, determine the value of *RL* for 75% efficiency.

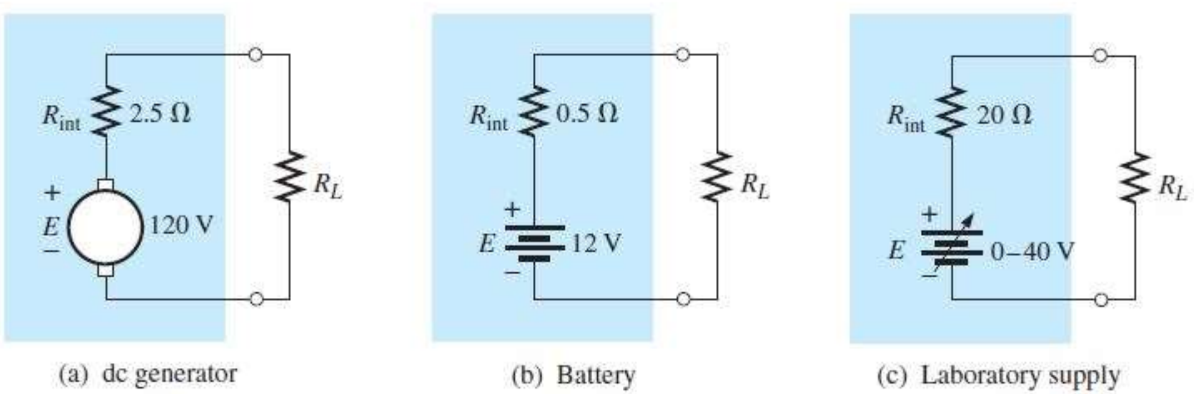


FIG. 9.85
Example 9.14.

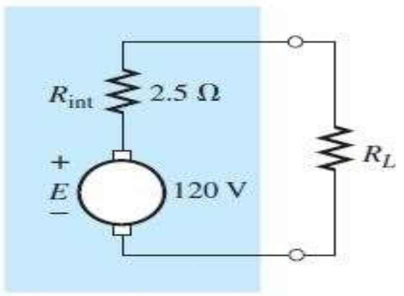
Maximum Power Transfer Theorem

Solutions:

a. For the dc generator

.

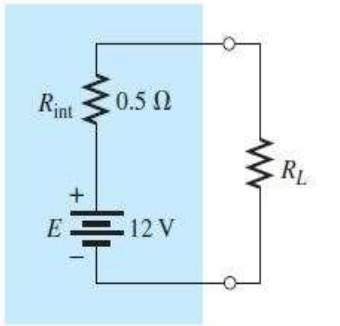
$$R_L = R_{Th} = R_{int} = 2.5\ \Omega$$



(a) dc generator

For the 12 V car battery,

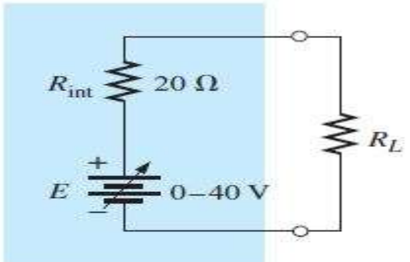
$$R_L = R_{Th} = R_{int} = 0.05\ \Omega$$



(b) Battery

For the dc laboratory supply,

$$R_L = R_{Th} = R_{int} = 20\ \Omega$$



(c) Laboratory supply

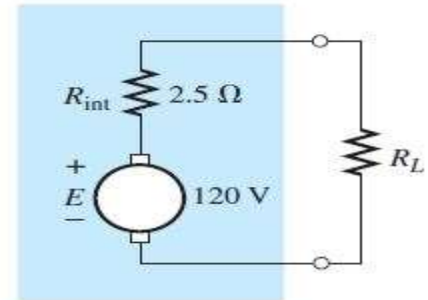
Maximum Power Transfer Theorem

Solutions:

b. For the dc generator

.

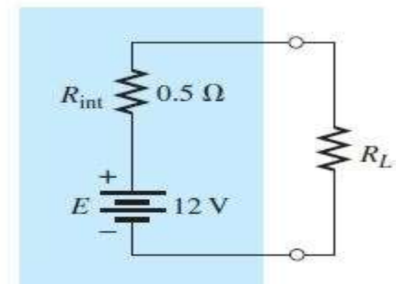
$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(120\text{ V})^2}{4(2.5\ \Omega)} = \mathbf{1.44\ kW}$$



(a) dc generator

For the 12 V car battery

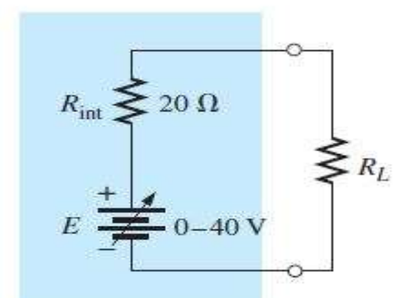
$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(12\text{ V})^2}{4(0.05\ \Omega)} = \mathbf{720\ W}$$



(b) Battery

For the dc laboratory supply,

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(40\text{ V})^2}{4(20\ \Omega)} = \mathbf{20\ W}$$



(c) Laboratory supply

Maximum Power Transfer Theorem

c. They are all operating under a 50% efficiency level because *RL_RTh.* b. For the dc generator

.

d. The power to the load is determined as follows:

$$I_L = \frac{E}{R_{\text{int}} + R_L} = \frac{40\text{ V}}{20\ \Omega + 1000\ \Omega} = \frac{40\text{ V}}{1020\ \Omega} = 39.22\text{ mA}$$

and $\quad P_L = I_L^2 R_L = (39.22\text{ mA})^2(1000\ \Omega) = \mathbf{1.54\ W}$

The power level is significantly less than the 20 W achieved in part (b). The efficiency level is

$$\eta\% = \frac{P_L}{P_s} \times 100\% = \frac{1.54\text{ W}}{EI_s} \times 100\% = \frac{1.54\text{ W}}{(40\text{ V})(39.22\text{ mA})} \times 100\%$$

$$= \frac{1.54\text{ W}}{1.57\text{ W}} \times 100\% = \mathbf{98.09\%}$$

which is markedly higher than achieved under maximum power conditions—albeit at the expense of the power level.

Maximum Power Transfer Theorem

e. For the dc generator,

$$\eta = \frac{P_o}{P_s} = \frac{R_L}{R_{Th} + R_L} \quad (\eta \text{ in decimal form})$$

and

$$\eta = \frac{R_L}{R_{Th} + R_L}$$

$$\eta(R_{Th} + R_L) = R_L$$

$$\eta R_{Th} + \eta R_L = R_L$$

$$R_L(1 - \eta) = \eta R_{Th}$$

and

$$\boxed{R_L = \frac{\eta R_{Th}}{1 - \eta}} \tag{9.7}$$

$$R_L = \frac{0.75(2.5\ \Omega)}{1 - 0.75} = \mathbf{7.5\ \Omega}$$

For the battery,

$$R_L = \frac{0.75(0.05\ \Omega)}{1 - 0.75} = \mathbf{0.15\ \Omega}$$

Maximum Power Transfer Theorem

For the laboratory supply,

$$R_L = \frac{0.75(20\ \Omega)}{1 - 0.75} = \mathbf{60\ \Omega}$$

QUIZ

1. Maximum Power will be transferred---------to -------.

A. source, source
B. source, load
C. load, source
D. load, load

ANS: B. source, load

QUIZ

2. Maximum Power will be transferred from source to load when------is made equal to ----

A. R_L, R_{th}
B. R_{th}, R_L
C. Both
D. None of the above

ANS: A. RL, Rth

QUIZ

3. Maximum ----- percent Power will be transferred from source to load.

A. 25
B. 50
C. 75
D. 100

ANS: B. 50

QUIZ

4. Maximum ----- percent Power will be transferred from source to load.

A. 25
B. 50
C. 75
D. 100

ANS: B. 50

QUIZ

5. By which theorem, we can find the internal resistance of a source.

A. Superposition theorem
B. Thevenin's theorem
C. Norton's theorem
D. Maximum Power transfer theorem

ANS: D. Maximum Power transfer theorem

Maximum Power Transfer Theorem

Home Work: Find out Load Resistance and Maximum Power