NODE ANALYSIS

Circuit terminologies: (Branch, Node)

- **Branch:** It represents single circuit element such as voltage source, resistor, etc.

- **Node:** A point in a network where two or more elements are connected is called node.

NODAL ANALYSIS

Nodal analysis—a method that provides the nodal voltages of a network, that is, the voltage from the various **nodes**(junction points) of the network to ground.

The number of nodes for which the voltage must be determined using nodal analysis is 1 less than the total number of nodes.

The number of equations required to solve for all the nodal voltages of
a network is 1 less than the total number of independent nodes.

NODAL ANALYSIS

Nodal Analysis Procedure

1. ***Determine the number of nodes within the network.***

2. ***Pick a reference node, and label each remaining node with a subscripted value of voltage: V1, V2, and so on.***

3. ***Apply Kirchhoff's current law at each node except the reference.***

4. ***Solve the resulting equations for the nodal voltages.***

NODAL ANALYSIS

EXAMPLE 8.19 Apply nodal analysis to the network in Fig. 8.46

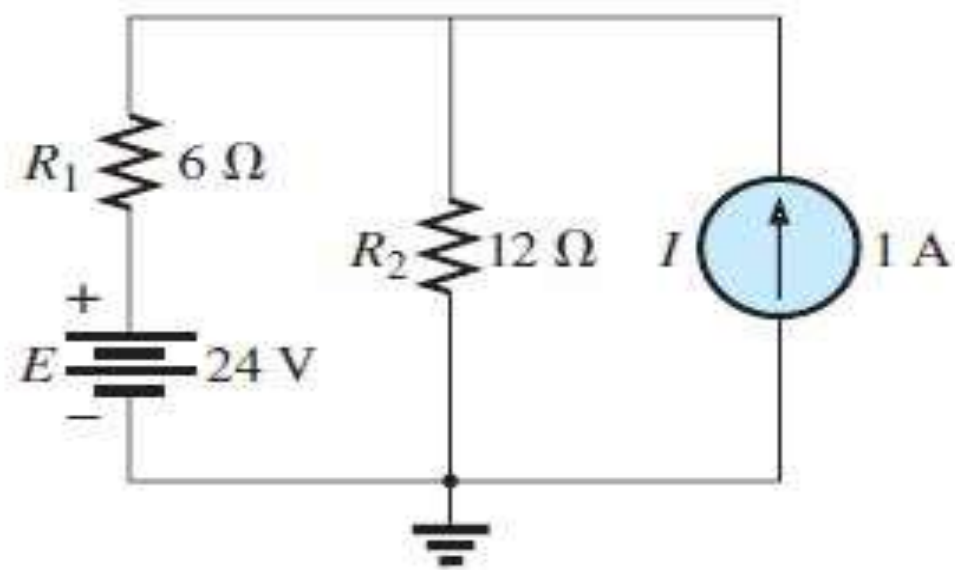

FIG. 8.46
Example 8.19.

NODAL ANALYSIS

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 8.47. The lower node is defined as the reference node at ground potential (zero volts), and the other node as *V*1, the voltage from node 1 to ground.

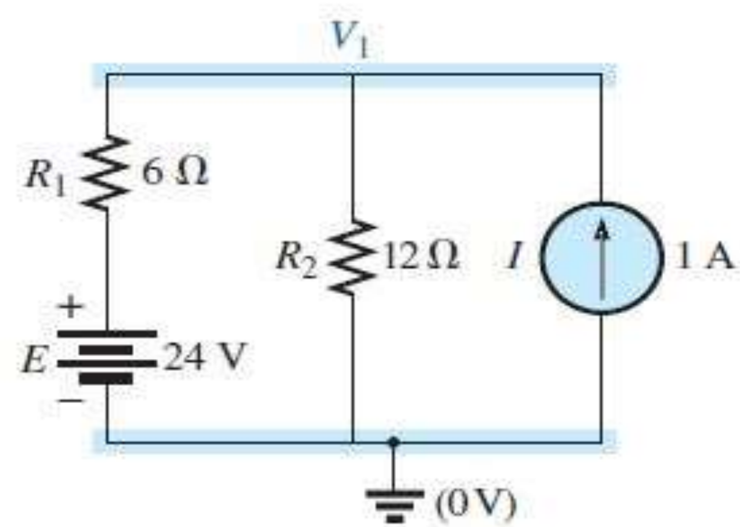

FIG. 8.47
Network in Fig. 8.46 with assigned nodes.

NODAL ANALYSIS

Solution:

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 8.48, and Kirchhoff's current law is applied as follows:

.

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

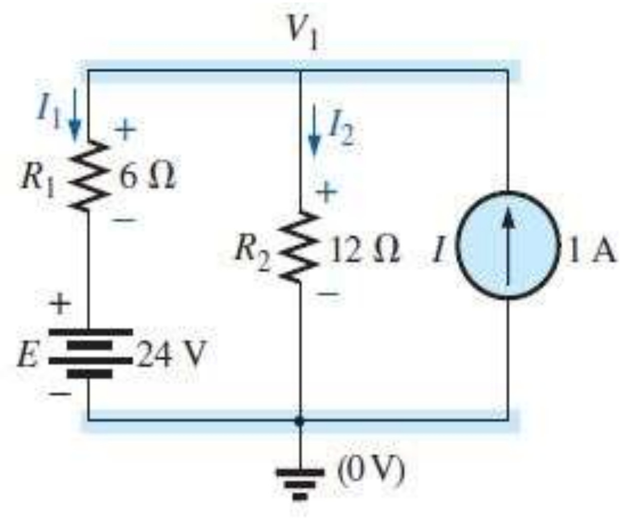

FIG. 8.48

Applying Kirchhoff's current law to the node V_1.

NODAL ANALYSIS

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with $$V_{R_1} = V_1 - E$$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or $$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + 1$$

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega}\right) = \frac{24\ \text{V}}{6\ \Omega} + 1\ \text{A} = 4\ \text{A} + 1\ \text{A}$$

$$V_1\left(\frac{1}{4\ \Omega}\right) = 5\ \text{A}$$

$$V_1 = \mathbf{20\ V}$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\ \text{V} - 24\ \text{V}}{6\ \Omega} = \frac{-4\ \text{V}}{6\ \Omega}$$
$$= \mathbf{-0.67\ A}$$

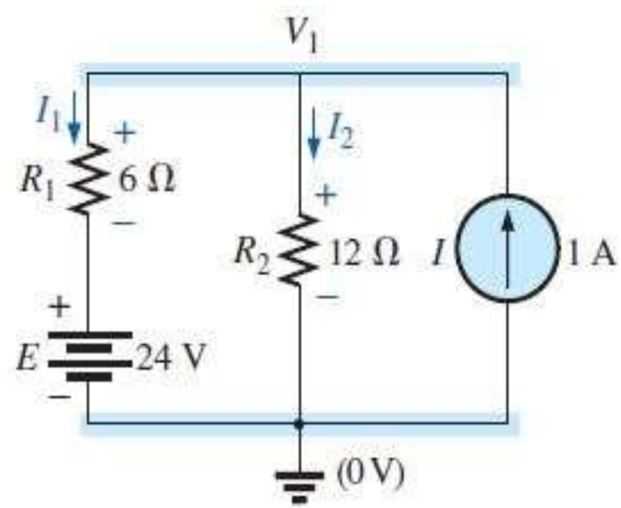

FIG. 8.48

Applying Kirchhoff's current law to the node V_1.

NODAL ANALYSIS

The minus sign indicates that the current I_1 has a direction opposite to that appearing in Fig. 8.48.

$$I_2 = \frac{V_1}{R_2} = \frac{20\ \text{V}}{12\ \Omega} = \mathbf{1.67\ A}$$

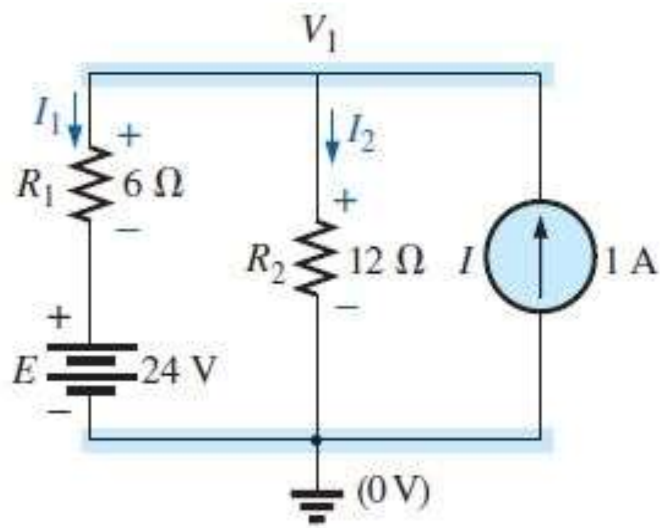

FIG. 8.48

Applying Kirchhoff's current law to the node V_1.

NODAL ANALYSIS

EXAMPLE 8.20 Apply nodal analysis to the network in Fig. 8.49

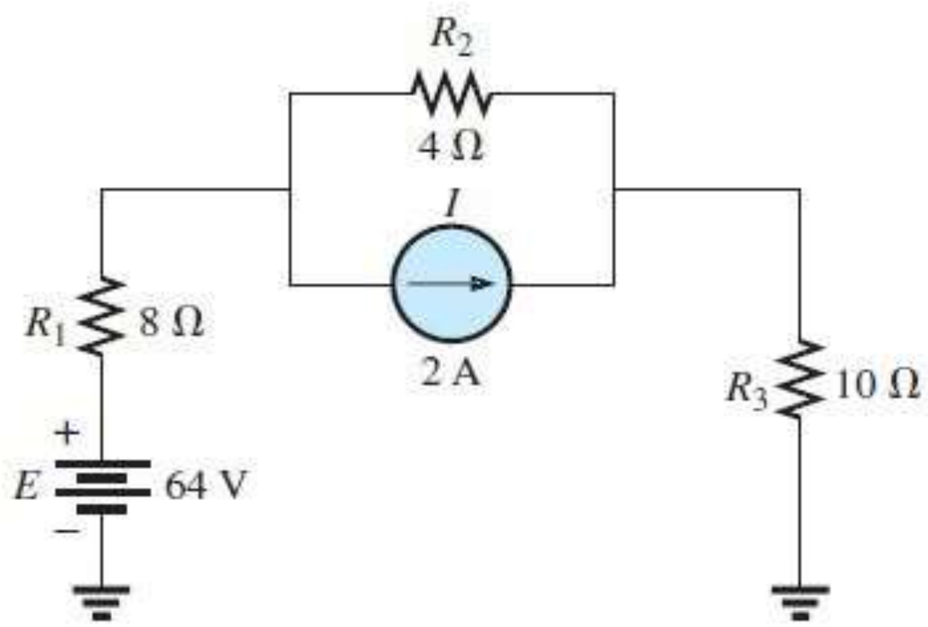

FIG. 8.49
Example 8.20.

NODAL ANALYSIS

Solution:

Steps 1 and 2: The network has three nodes, as defined in Fig. 8.50, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as *V*1 and *V*2

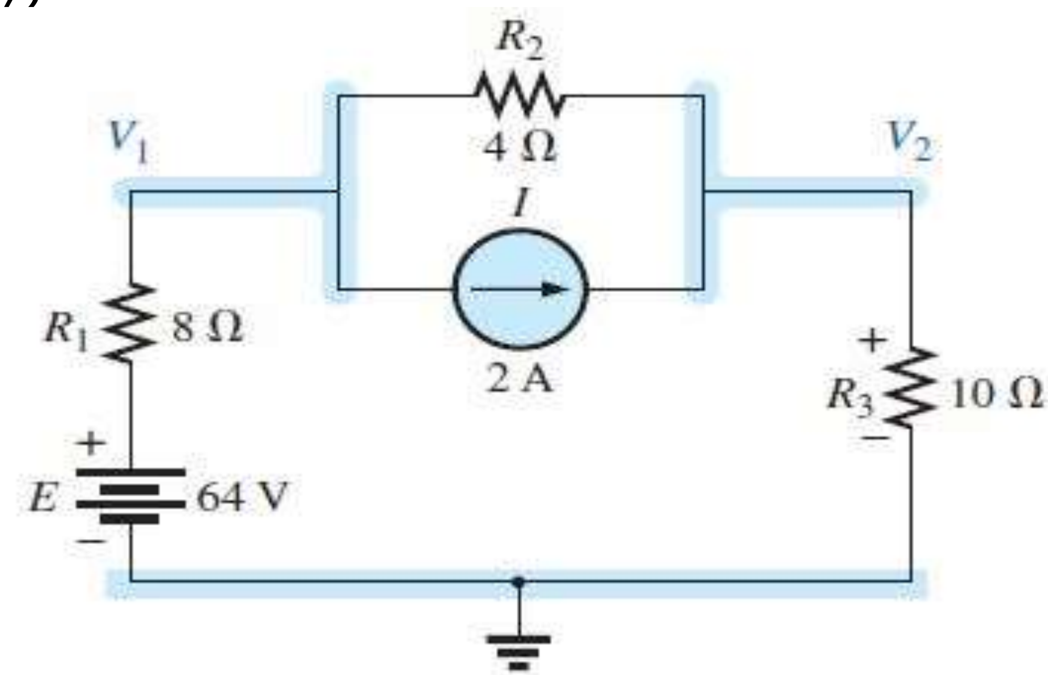

FIG. 8.50

Defining the nodes for the network in Fig. 8.49.

NODAL ANALYSIS

Solution:

Step 3: For node *V*1, the currents are defined as shown in Fig. 8.51 and Kirchhoff's current law is applied:

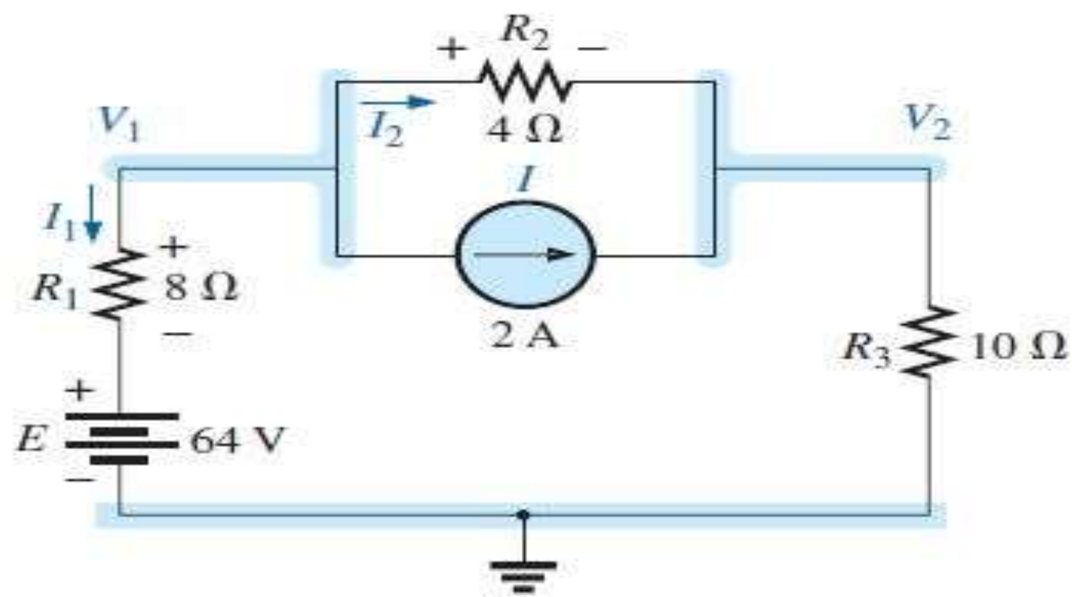

FIG. 8.51

Applying Kirchhoff's current law to node V_1.

NODAL ANALYSIS

$$0 = I_1 + I_2 + I$$

with $$I_1 = \frac{V_1 - E}{R_1}$$

and $$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that $$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or $$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and $$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

Substituting values:

$$V_1\left(\frac{1}{8\,\Omega} + \frac{1}{4\,\Omega}\right) - V_2\left(\frac{1}{4\,\Omega}\right) = -2\text{ A} + \frac{64\text{ V}}{8\,\Omega} = 6\text{ A}$$

For node V_2 the currents are defined as shown in Fig. 8.52, an Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with $$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

or $$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

and $$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

Substituting values:

$$V_2\left(\frac{1}{4\,\Omega} + \frac{1}{10\,\Omega}\right) - V_1\left(\frac{1}{4\,\Omega}\right) = 2\text{ A}$$

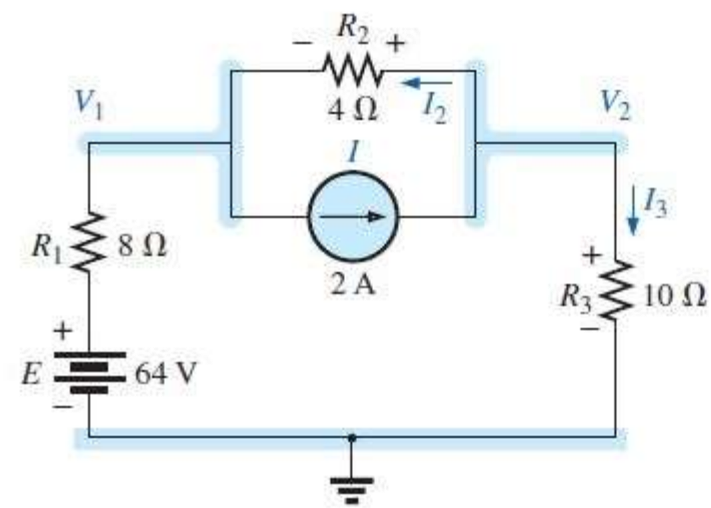

FIG. 8.52
Applying Kirchhoff's current law to node V_2.

NODAL ANALYSIS

Step 4: The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\ \Omega}+\frac{1}{4\ \Omega}\right)-V_2\left(\frac{1}{4\ \Omega}\right)=6\ \text{A}$$

$$-V_1\left(\frac{1}{4\ \Omega}\right)+V_2\left(\frac{1}{4\ \Omega}+\frac{1}{10\ \Omega}\right)=2\ \text{A}$$

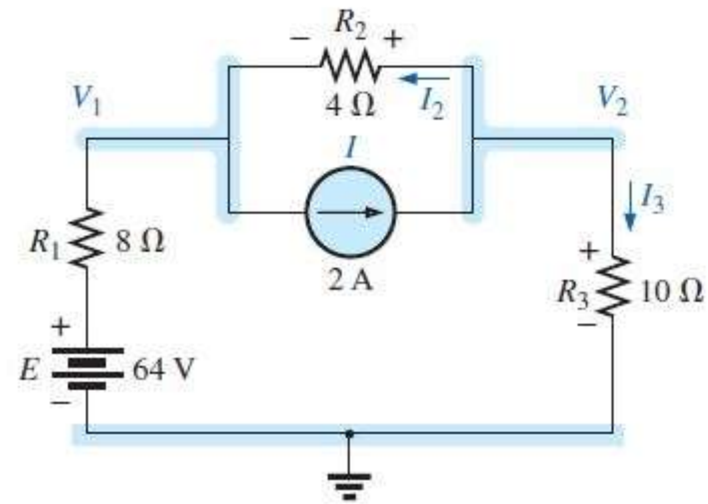

FIG. 8.52
Applying Kirchhoff's current law to node V_2.

NODAL ANALYSIS

which become

$$\begin{aligned} 0.375V_1 - 0.25V_2 &= 6 \\ -0.25V_1 + 0.35V_2 &= 2 \end{aligned}$$

Using determinants,

$$V_1 = \mathbf{37.82\ V}$$
$$V_2 = \mathbf{32.73\ V}$$

Since E is greater than V_1, the current I_1 flows from ground to V_1 and is equal to

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\ \text{V} - 37.82\ \text{V}}{8\ \Omega} = \mathbf{3.27\ A}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.73\ \text{V}}{10\ \Omega} = \mathbf{3.27\ A}$$

Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.82\ \text{V} - 32.73\ \text{V}}{4\ \Omega} = \mathbf{1.27\ A}$$

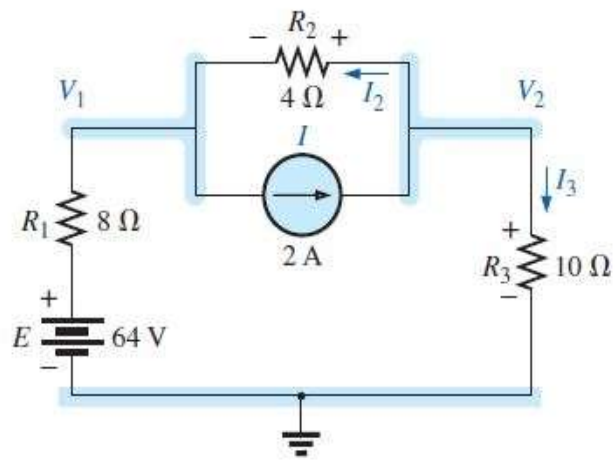

FIG. 8.52
Applying Kirchhoff's current law to node V_2.