The Basic Elements

The Basic Elements

DERIVATIVE

To understand the response of the basic *R, L,* and *C* elements to a sinusoidal signal, you need to understand the concept of the **derivative**

derivative *dx/dt* is defined as the rate of change of *x* with respect to time. If *x* fails to change at a particular instant, *dx* = 0, and the derivative is zero.

For the sinusoidal waveform, *dx/dt* is zero only at the positive and negative peaks (w*t* = π/2 and 3/2π in Fig. 14.1), since *x* fails to change at these instants of time. The derivative *dx/dt* is actually the slope of the graph at any instant of time.

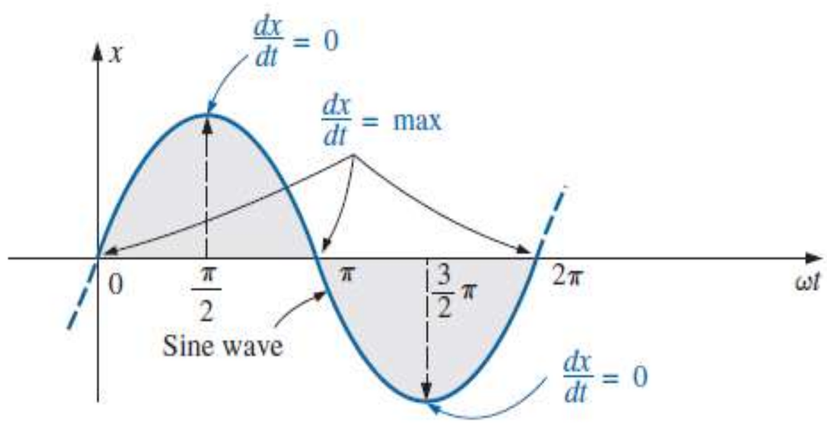

FIG. 14.1
Defining those points in a sinusoidal waveform that have maximum and minimum derivatives.

The Basic Elements

DERIVATIVE

Greatest change in *x* occurs at the instants v*t* = 0, π, and 2π

the derivative of a sine wave is a cosine wave.

both waveforms (*x*1 and *x*2) have the same peak value, the sinusoidal function with the higher frequency produces the larger peak value for the derivative

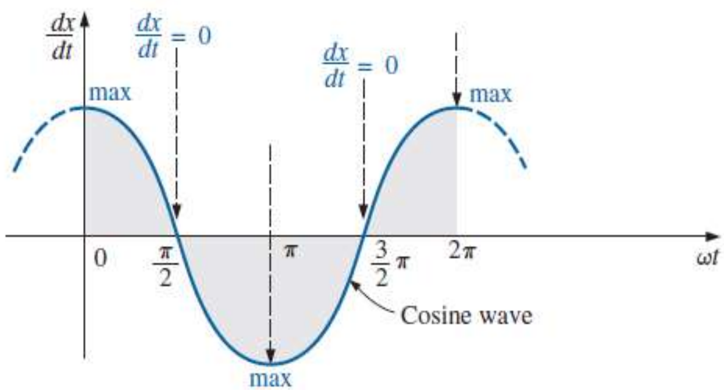

FIG. 14.2
Derivative of the sine wave of Fig. 14.1.

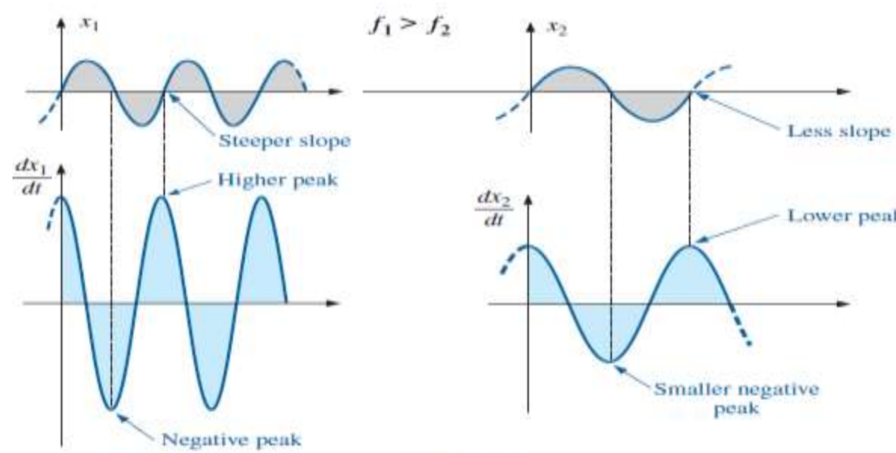

FIG. 14.3
Effect of frequency on the peak value of the derivative.

The Basic Elements

DERIVATIVE

the derivative of a sine wave has the same period and frequency as the original sinusoidal waveform.

For the sinusoidal voltage

$$e(t) = E_m \sin(\omega t \pm \theta)$$

the derivative can be found directly by differentiation (calculus) to produce the following:

$$\boxed{\begin{aligned}\frac{d}{dt}e(t) &= \omega E_m \cos(\omega t \pm \theta)\\ &= 2\pi f E_m \cos(\omega t \pm \theta)\end{aligned}} \qquad \textbf{(14.1)}$$

The Basic Elements

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Resistor

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

For v= *Vm* sin w*t,*

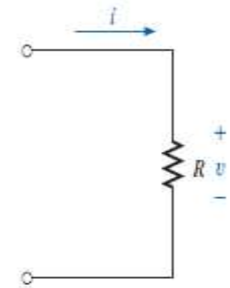

FIG. 14.4
Determining the sinusoidal response for a resistive element.

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where $$\boxed{I_m = \frac{V_m}{R}} \quad (14.2)$$

In addition, for a given i,

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

where $$\boxed{V_m = I_m R} \quad (14.3$$

A plot of v and i in Fig. 14.5 reveals that

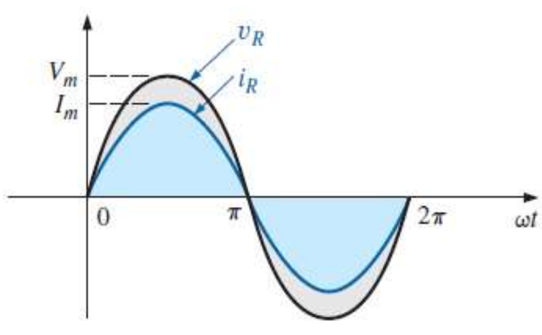

FIG. 14.5
The voltage and current of a resistive element are in phase.

The Basic Elements

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Inductor

For v= *Vm* sin w*t,*

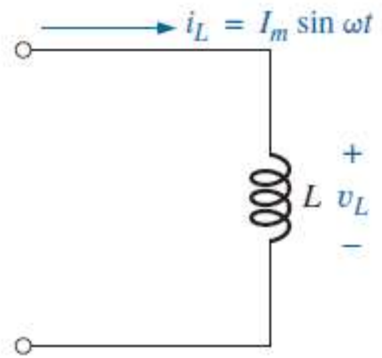

FIG. 14.8
Investigating the sinusoidal response of an inductive element.

$$v_L = L\frac{di_L}{dt}$$

and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

Therefore, $$v_L = L\frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$$

or $$v_L = V_m \sin(\omega t + 90°)$$

where $$V_m = \omega L I_m$$

Note that the peak value of v_L is directly related to ω (= $2\pi f$) and L a predicted in the discussion above.

A plot of v_L and i_L in Fig. 14.9 reveals that

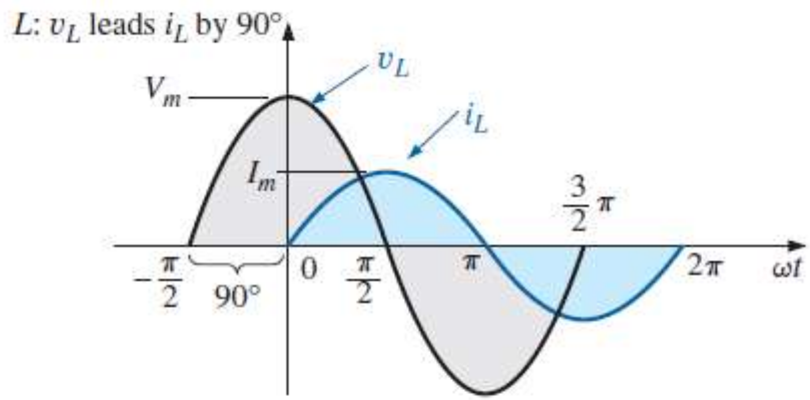

FIG. 14.9
For a pure inductor, the voltage across the coil leads the current through the coil by 90°.

The Basic Elements

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Capacitor

for a capacitor, i_C leads V_C by 90°, or

V_C lags i_C by 90°.

For v= *Vm* sin w*t,*

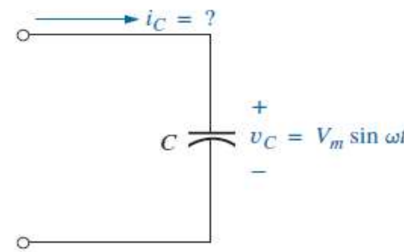

FIG. 14.11
Investigating the sinusoidal response of a capacitive element.

$$i_C = C\frac{dv_C}{dt}$$

and, applying differentiation,

dv_C d

Therefore,

$$i_C = C\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

or $$i_C = I_m \sin(\omega t + 90°)$$

where $$I_m = \omega C V_m$$

Note that the peak value of i_C is directly related to ω (= $2\pi f$) and as predicted in the discussion above.

A plot of v_C and i_C in Fig. 14.12 reveals that

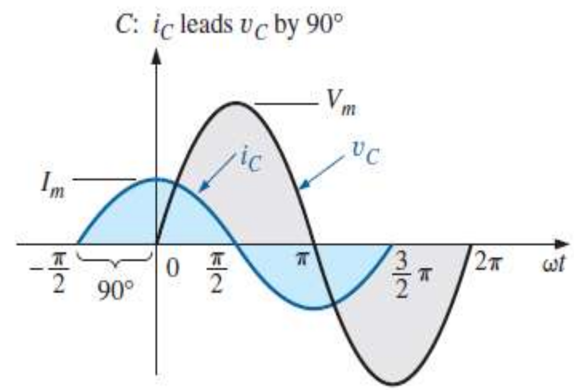

FIG. 14.12
The current of a purely capacitive element leads the voltage across the element by 90°.

The Basic Elements

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

EXAMPLE 14.1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and *i.*

a. y =100 sin 377*t*

b. y _=25 sin(377*t* + 60°)

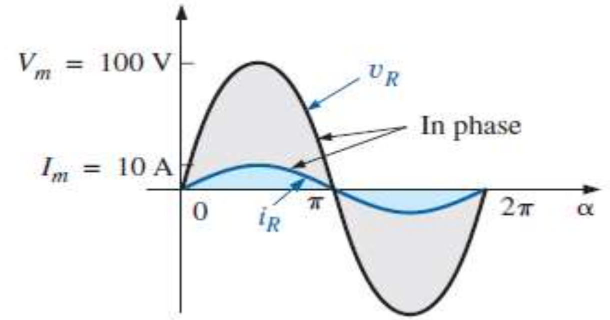

FIG. 14.13
Example 14.1(a).

Solutions:

a. Eq. (14.2): $I_m = \frac{V_m}{R} = \frac{100\text{ V}}{10\ \Omega} = 10\text{ A}$

(v and i are in phase), resulting in

$$\mathbf{i = 10 \sin 377t}$$

The curves are sketched in Fig. 14.13.

b. Eq. (14.2): $I_m = \frac{V_m}{R} = \frac{25\text{ V}}{10\ \Omega} = 2.5\text{ A}$

(v and i are in phase), resulting in

$$\mathbf{i = 2.5 \sin(377t + 60^\circ)}$$

The curves are sketched in Fig. 14.14.

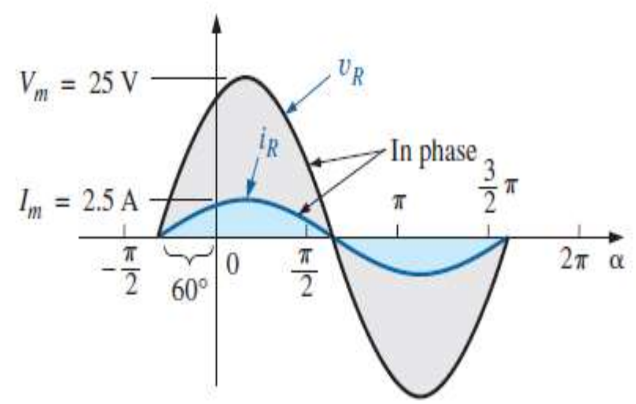

FIG. 14.14
Example 14.1(b).

The Basic Elements

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

EXAMPLE 14.3 The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$
b. $i = 7 \sin(377t - 70°)$

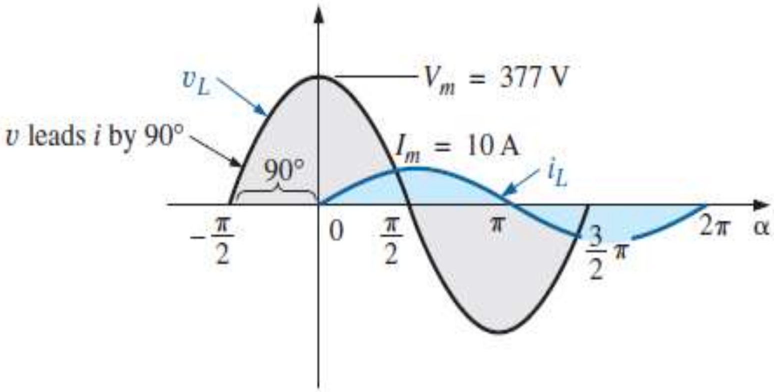

FIG. 14.15
Example 14.3(a).

Solutions:

a. Eq. (14.4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7\ \Omega$
Eq. (14.5): $V_m = I_m X_L = (10 \text{ A})(37.7\ \Omega) = 377$ V

and we know that for a coil v leads i by 90°. Therefore,

$$\boldsymbol{v = 377 \sin(377t + 90°)}$$

The curves are sketched in Fig. 14.15.

The Basic Elements

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

EXAMPLE 14.3 The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$
b. $i = 7 \sin(377t - 70°)$

b. X_L remains at 37.7 Ω.

$V_m = I_m X_L = (7\text{ A})(37.7\ \Omega) = 263.9\text{ V}$

and we know that for a coil v leads i by 90°. Therefore,

$$v = 263.9 \sin(377t - 70° + 90°)$$

and

$$\boldsymbol{v = 263.9 \sin(377t + 20°)}$$

The curves are sketched in Fig. 14.16.

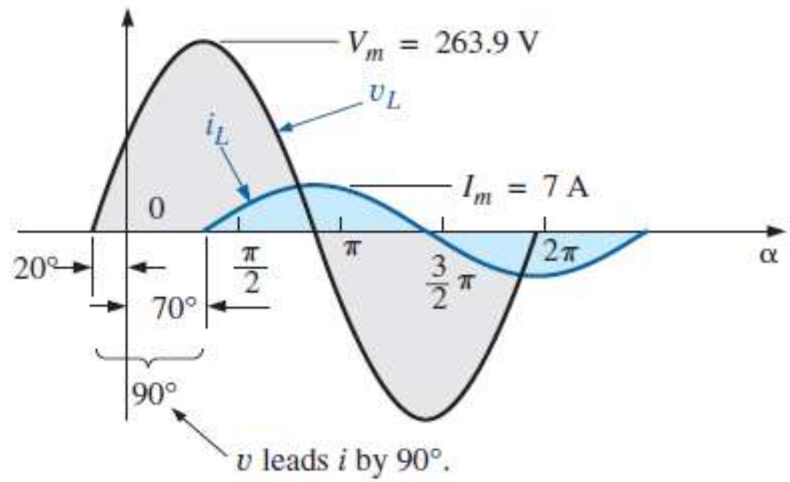

FIG. 14.16
Example 14.3(b).

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

Ideal Response

Resistor *R* For an ideal resistor, you can assume that *frequency will have absolutely no effect on the impedance level,*
as shown by the response in Fig. 14.19.
Note that at 5 kHz or 20 kHz, the resistance of the resistor remains at 22 Ω;

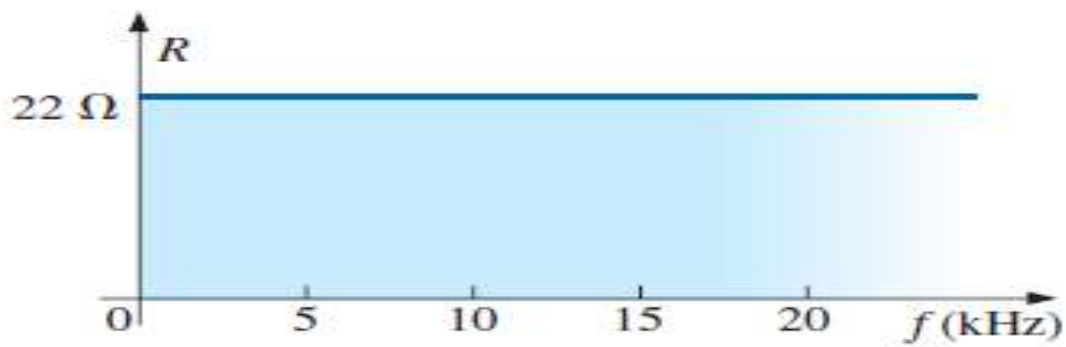

FIG. 14.19
R versus f for the range of interest.

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

Ideal Response

Inductor *L*: For the ideal inductor, the equation for the reactance can

$$X_L = \omega L = 2\pi f L = (2\pi L)f = kf \quad \text{with } k = 2\pi L$$

at a frequency of 0 Hz, an inductor takes on the characteristics of a short circuit, as shown in Fig. 14.21.

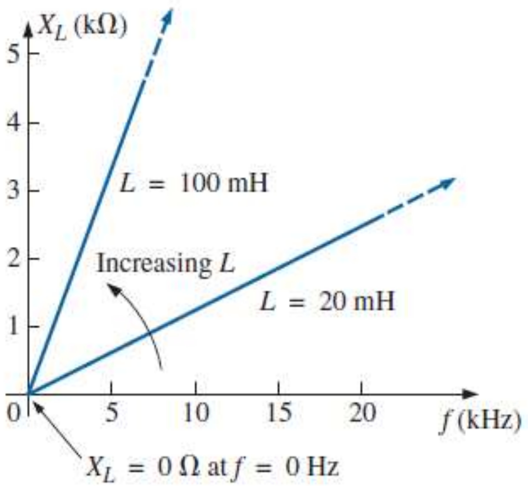

FIG. 14.20
X_L versus frequency.

at very high frequencies, the characteristics of an inductor approach those of an open circuit, as shown in Fig. 14.21.

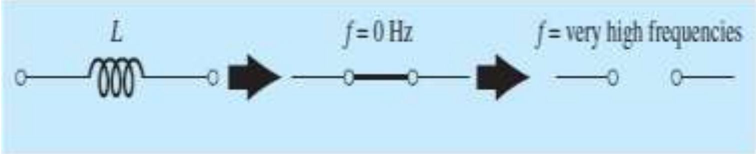

FIG. 14.21
Effect of low and high frequencies on the circuit model of an inductor.

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

Ideal Response

Capacitor *C* : For the capacitor, the equation for the reactance

$$X_C = \frac{1}{2\pi f C}$$

can be written as

$$X_C f = \frac{1}{2\pi C} = k \qquad \text{(a constant)}$$

which matches the basic format for a hyberbola:

$$yx = k$$

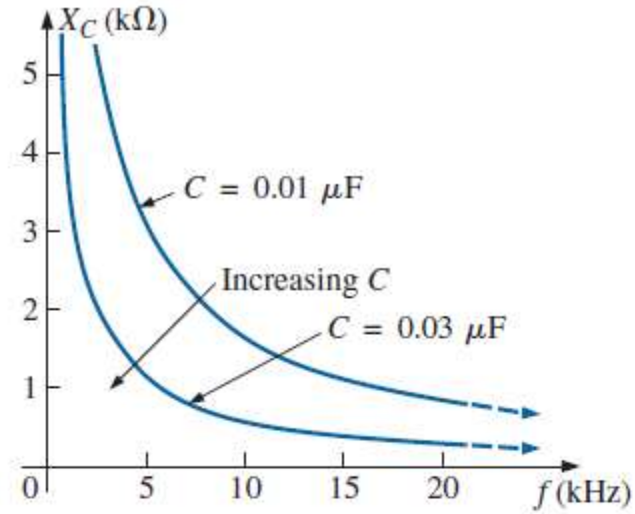

FIG. 14.22
X_C versus frequency.

where *XC* is the *v* variable, *f* the *x* variable, and *k* a constant equal to 1/(2π*C*).At or near 0 Hz, the reactance of any capacitor is extremely high, as determined by the basic equation for capacitance:

$$X_C = \frac{1}{2\pi f c} = \frac{1}{2\pi (0\ \text{Hz}) C} \Rightarrow \infty\ \Omega$$

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS
Ideal Response

at or near 0 Hz, the characteristics of a capacitor approach those of an open circuit, as shown in Fig. 14.23.

at very high frequencies, a capacitor takes on the characteristics of a short circuit, as shown in Fig. 14.23.

Finally, recognize the following:
As frequency increases, the reactance of an inductive element increases while that of a capacitor decreases, with one approaching an open-circuit equivalent as the other approaches a short-circuit equivalent.

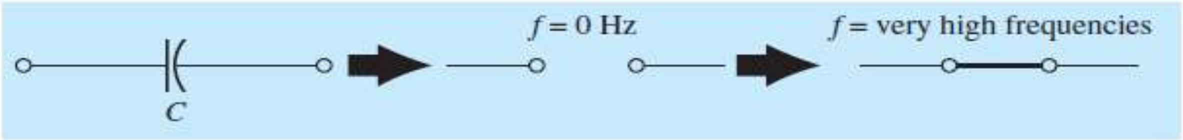

FIG. 14.23
Effect of low and high frequencies on the circuit model of a capacitor.

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

Practical Response

Resistor *R* In the manufacturing process, every resistive element inherits some stray capacitance levels and lead inductances

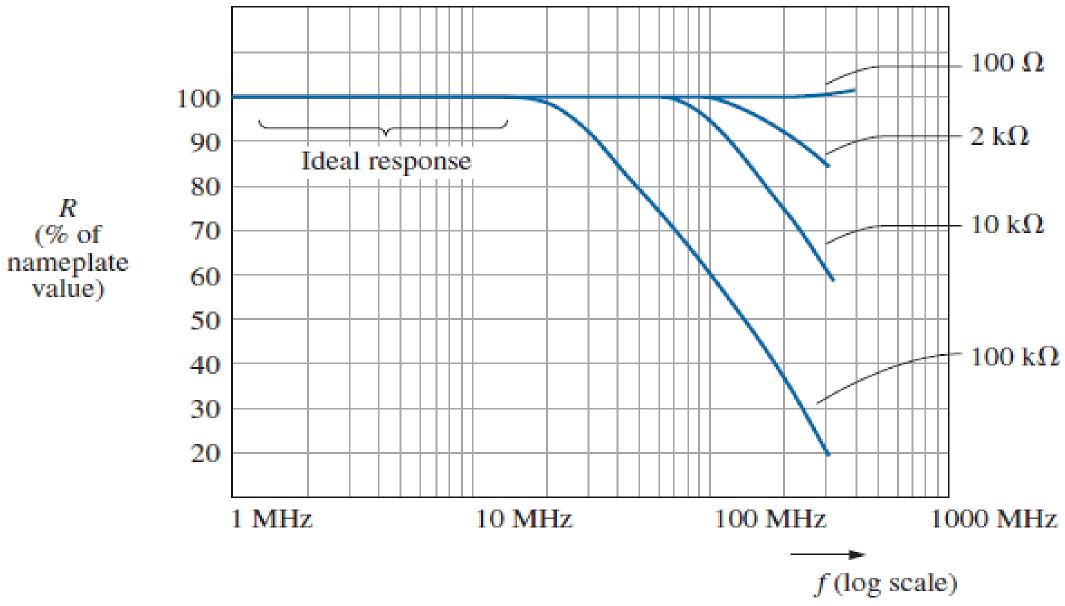

FIG. 14.24

Typical resistance-versus-frequency curves for carbon composition resistors.

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

Practical Response

Inductor *L* : In reality, inductance can be affected by frequency, temperature, and current. A true equivalent for an inductor appears in Fig. 14.25. The series resistance *Rs* represents the copper losses ,the eddy current losses ,and the hysteresis losses .The capacitance *Cp* is the stray capacitance that exists between the windings of the inductor

In general, therefore, the frequency of application for a coil becomes important at increasing frequencies. Inductors lose their ideal characteristics and, in fact, begin to act as capacitive elements with increasing losses at very high frequencies.

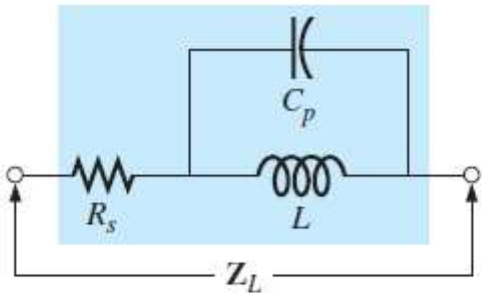

FIG. 14.25
Practical equivalent for an inductor.

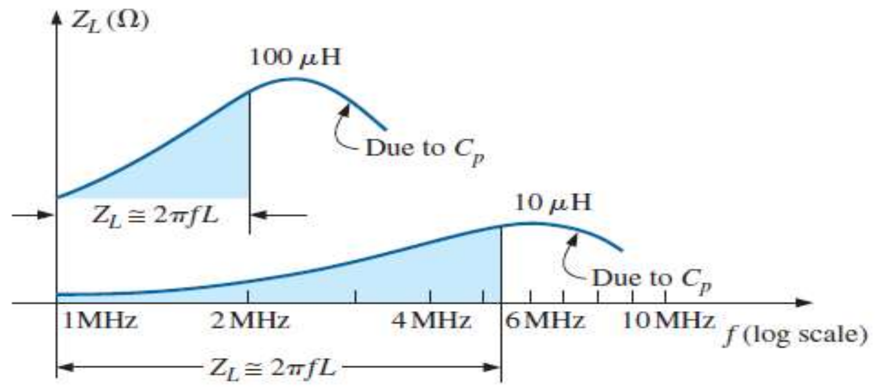

FIG. 14.26
Z_L versus frequency for the practical inductor equivalent of Fig. 14.25.

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

Practical Response

Capacitor *C* : The capacitor, like the inductor, is not ideal at higher frequencies. In fact, a transition point can be defined where the characteristics of the capacitor will actually be inductive. The complete equivalent model for a capacitor is provided in Fig. 14.27

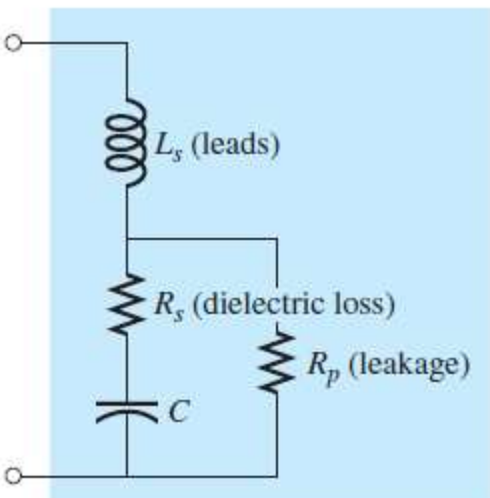

FIG. 14.27
Practical equivalent for a capacitor.

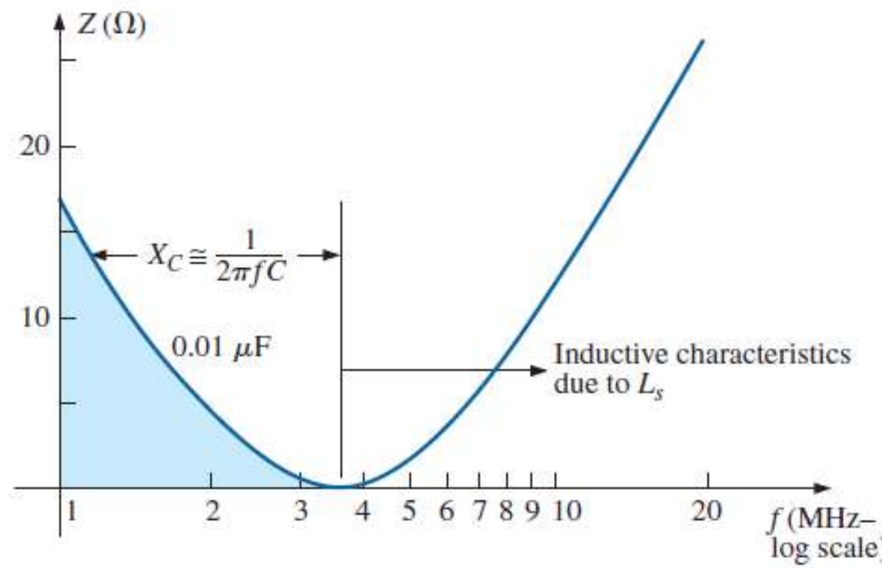

FIG. 14.28
Impedance characteristics of a 0.01 μF metalized film capacitor versus frequency.

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

EXAMPLE 14.8 At what frequency will the reactance of a 200 mH inductor match the resistance level of a 5 k_ resistor?

Solution: The resistance remains constant at 5 kΩ for the frequency range of the inductor. Therefore,

$$R = 5000\ \Omega = X_L = 2\pi f L = 2\pi L f$$
$$= 2\pi(200 \times 10^{-3}\ \text{H})f = 1.257f$$

and
$$f = \frac{5000\ \text{Hz}}{1.257} \cong \mathbf{3.98\ kHz}$$

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

EXAMPLE 14.9 At what frequency will an inductor of 5 mH have the same reactance as a capacitor of 0.1 μF?

Solution:

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \times 10^{-3}\ \text{H})(0.1 \times 10^{-6}\ \text{F})}}$$

$$= \frac{1}{2\pi\sqrt{5 \times 10^{-10}}} = \frac{1}{(2\pi)(2.236 \times 10^{-5})} = \frac{10^5\ \text{Hz}}{14.05} \cong \mathbf{7.12\ kHz}$$

The Basic Elements

AVERAGE POWER AND POWER FACTOR

A common question is, How can a sinusoidal voltage or current deliver power to a load if it seems to be delivering power during one part of its cycle and taking it back during the negative part of the sinusoidal cycle? The equal oscillations above and below the axis seem to suggest that over one full cycle there is no net transfer of power or energy. However, as mentioned in the last chapter, there is a net transfer of power over one full cycle because power is delivered to the load *at each instant* of the applied voltage or current (except when either is crossing the axis) no matter what the direction is of the current or polarity of the voltage.

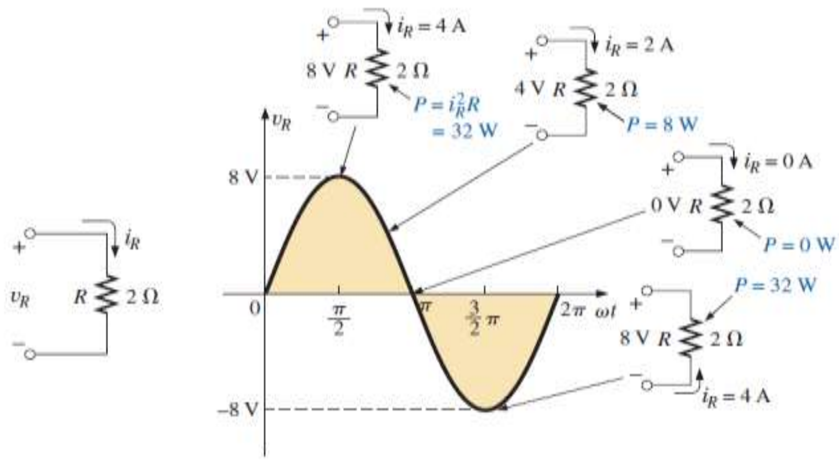

FIG. 14.29
Demonstrating that power is delivered at every instant of a sinusoidal voltage waveform (except $v_R = 0$ V).

The Basic Elements

AVERAGE POWER AND POWER FACTOR

In total, therefore,

even though the current through and the voltage across reverse direction and polarity, respectively, power is delivered to the resistive load at each instant of time.

If we plot the power delivered over a full cycle, the curve in Fig. 14.30 results.

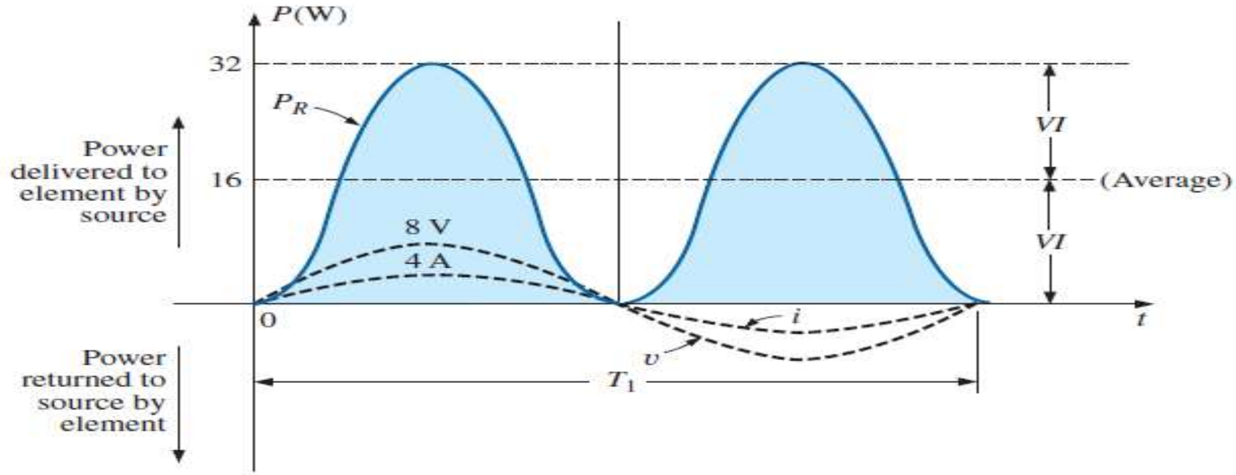

FIG. 14.30
Power versus time for a purely resistive load.

The Basic Elements

AVERAGE POWER AND POWER FACTOR

The fact that the power curve is always above the horizontal axis reveals that power is being delivered to the load at each instant of time of the applied sinusoidal voltage.

The average value of the power curve occurs at a level equal to *Vm Im*/2 as shown in Fig. 14.30.

This power level is called the **average** or **real power** level .If we substitute the equation for the peak value in terms of the rms value as follows:

$$P_{av} = \frac{V_m I_m}{2} = \frac{(\sqrt{2}\,V_{rms})(\sqrt{2}\,I_{rms})}{2} = \frac{2\,V_{rms} I_{rms}}{2} \qquad \boxed{P_{av} = V_{rms}\, I_{rms}} \qquad (14.10)$$

The magnitude of average power delivered is independent of whether v leads i or i leads v.

$$\boxed{P = \frac{V_m I_m}{2}\cos\theta} \quad \text{(watts, W)} \qquad (14.11) \qquad \boxed{P = V_{rms} I_{rms}\cos\theta} \qquad (14.12)$$

The Basic Elements

AVERAGE POWER AND POWER FACTOR

Resistor

In a purely resistive circuit, since v and i are in phase, $|\theta_v - \theta_i| = \theta = 0°$, and $\cos\theta = \cos 0° = 1$, so that

$$\boxed{P = \frac{V_m I_m}{2} = V_{\text{rms}} I_{\text{rms}}} \qquad \text{(W)} \qquad \textbf{(14.13)}$$

Or, since

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

then

$$\boxed{P = \frac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R} \qquad \text{(W)} \qquad \textbf{(14.14)}$$

The Basic Elements

AVERAGE POWER AND POWER FACTOR

Inductor

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

In a purely inductive circuit, since v leads i by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

Capacitor

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos(90°) = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The Basic Elements

AVERAGE POWER AND POWER FACTOR

EXAMPLE 14.10 Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5\sin(\omega t + 40°)$$
$$v = 10\sin(\omega t + 40°)$$

Solution: Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10\text{ V})(5\text{ A})}{2} = \mathbf{25\ W}$$

or
$$R = \frac{V_m}{I_m} = \frac{10\text{ V}}{5\text{ A}} = 2\ \Omega$$

and
$$P = \frac{V_{rms}^2}{R} = \frac{[(0.707)(10\text{ V})]^2}{2} = \mathbf{25\ W}$$

or
$$P = I_{rms}^2 R = [(0.707)(5\text{ A})]^2(2) = \mathbf{25\ W}$$

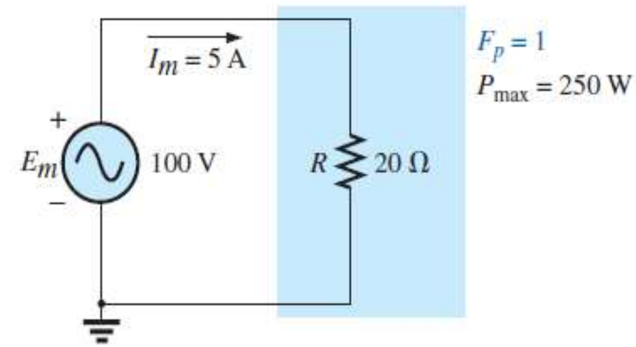

FIG. 14.33
Purely resistive load with $F_p = 1$.

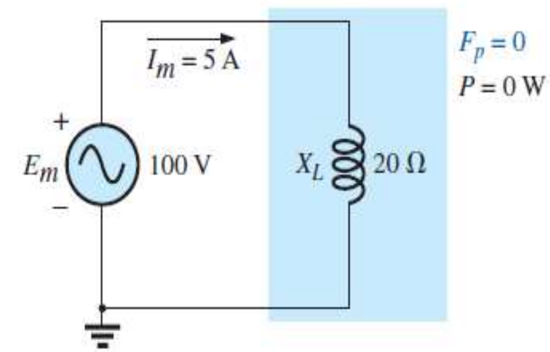

FIG. 14.34
Purely inductive load with $F_p = 0$.

Power Factor

$$\boxed{\text{Power factor} = F_p = \cos\theta} \qquad \mathbf{(14.15)}$$

capacitive networks have leading power factors, and inductive networks have lagging power factors.

HOME WORK

EXAMPLE 14.5 The voltage across a 1 μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

EXAMPLE 14.6 The current through a 100 μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor.

$$i = 40 \sin(500t + 60°)$$