# ELECTROSTATICS

Electrostatics is the study of stationary electric charges or fields as opposed to electric currents.

We begin our study of electrostatics by investigating the two fundamental laws governing electrostatic fields:

(1) Coulomb's law and

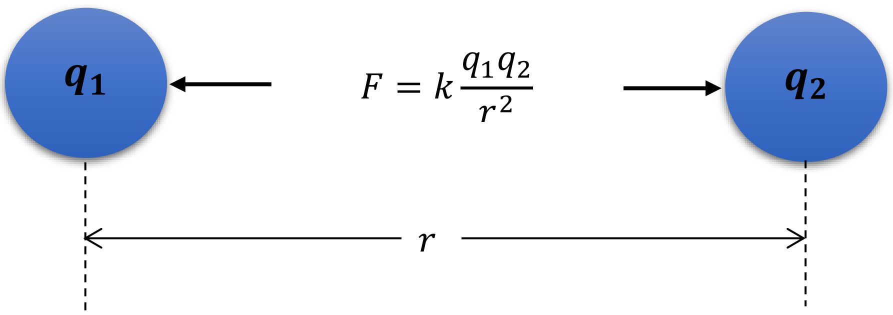
(2) Gauss's law.

# COULOMB'S LAW

**Basics:**

- The polarity of charges may be positive or negative.
- Like charges repel
- Unlike charges attract

**<u>Coulomb's law:</u>**




**Coulomb's law** states that the force $F$ between two-point charges $q_1$ and $q_2$ is:

i. Along the line joining them
ii. Directly proportional to the product $q_1 q_2$ of the charges
iii. Inversely proportional to the square of the distance $r$ between them.

Expressed mathematically,

$$F = k\frac{q_1 q_2}{r^2}$$

where, $k$ is the proportionality constant whose value depends on the choice of system of units.

In SI units, charges $q_1$ and $q_2$are in coulombs (C), the distance $r$ is in meters (m), and the force $F$ is in newtons (N) so that

$$k = \frac{1}{4\pi\varepsilon_0}$$

The constant $\varepsilon_0$ is known as the permittivity of free space (in farads per meter) and has the value

$$\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$$

Hence,

$$k = \frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9\ m/F$$

Coulomb's Law:

1. A point charge of $\mathbf{+3.00 \times 10^{-6} C}$ is $\mathbf{12.0\ cm}$ distant from a second point charge of $\mathbf{-1.50 \times 10^{-6} C}$. Calculate the magnitude of the force on each charge. $[Here, k = \mathbf{8.99 \times 10^{9}\ Nm^{2}C^{-2}}]$

Here,

$$12cm = 12 \times 10^{-2}\ m$$

Being of opposite signs, the two charges *attract* one another, and the magnitude of this force is given by **Coulomb's law**

$$F = k\frac{|q_1 q_2|}{r^2}$$

$$F = \mathbf{8.99 \times 10^{9}\ Nm^{2}C^{-2}} \frac{(\mathbf{3.00 \times 10^{-6} C})(\mathbf{1.50 \times 10^{-6} C})}{(12 \times 10^{-2}\ m)^2}$$

$$F = 2.81\ N$$

Each charge experiences a force of attraction of magnitude 2.81N.

2. What must be the distance between point charge $\mathbf{q_1 = 26.0 \times 10^{-6}\ C}$ and point charge $\mathbf{q_2 = -47.0 \times 10^{-6}\ C}$ for the electrostatic force between them to have a magnitude of $\mathbf{5.70\ N}$ ? $[Here, k = \mathbf{8.99 \times 10^{9}\ Nm^{2}C^{-2}}]$

We are given the charges and the magnitude of the (attractive) force between them.

Here,

$$\mathbf{q_1 = 26.0 \times 10^{-6}\ C}$$
$$\mathbf{q_2 = -47.0 \times 10^{-6}\ C}$$
$$\mathbf{F = 5.70\ N}$$
$$\mathbf{r} = ?$$

We can use **Coulomb's law** to solve for **r**, the distance between the charges:

$$F = k\frac{|q_1 q_2|}{r^2}$$
$$=> \ r^2 = k\frac{|q_1 q_2|}{F}$$

Putting given values in the equation:

$$r^2 = \mathbf{8.99 \times 10^9 \ Nm^2C^{-2}} \frac{(\mathbf{26.0} \ \times \mathbf{10^{-6}} \ \boldsymbol{C})(\mathbf{47.0} \ \times \mathbf{10^{-6}} \ \boldsymbol{C})}{\mathbf{5.70 \ N}}$$
$$r^2 = 1.93 \ m^2$$

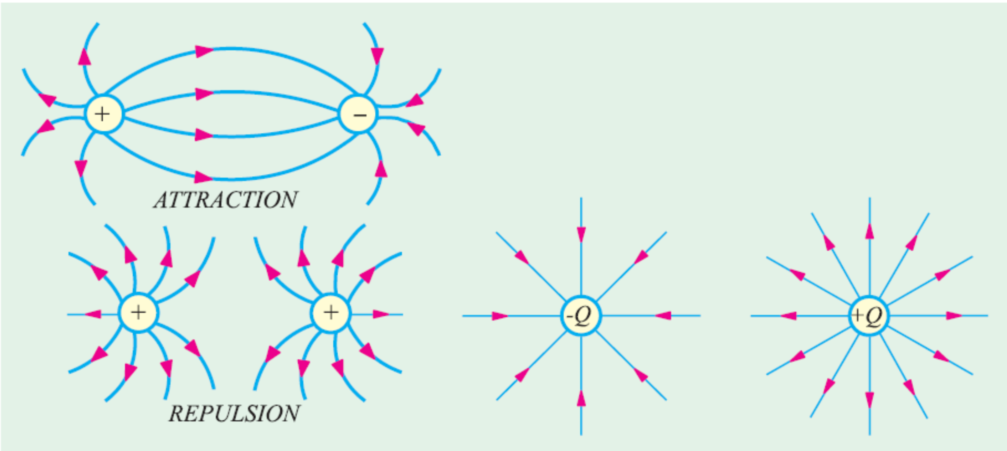
$$Hence, \qquad r = \sqrt{1.93 \ m^2} = 1.39 \ m$$

3. Two equal and opposite charges of magnitude $5 \times 10^5 C$ are attracted by a force of 2.25N. Find the distance r between the charges?

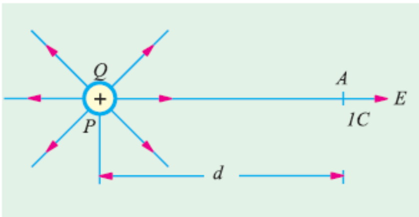
4. Two equal and opposite charges of magnitude $5 \times 10^5 C$ are at a distance of d apart such that their force of attraction is 4.5N. Calculate the value of d.

## Electric Filed:

It is found that in the medium around a charge, a force acts on a positive or negative charge when placed in that medium. The region in which the electric forces act, is called an electric field or electrostatic field.




## Electric Field Intensity:




The electric field intensity (or electric field strength) $\vec{E}$ is the force that a unit positive charge experiences when placed in an electric field.

$$\vec{E} = \frac{\vec{F}}{Q}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

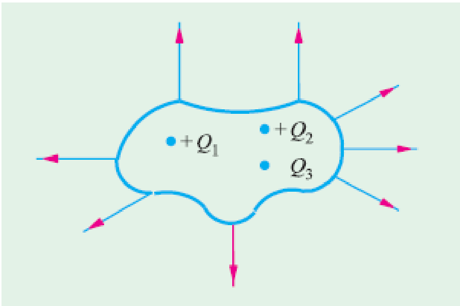
## Flux:

The meaning of flux is just ***the number of field lines*** passing through the surface.

$$\Phi_E = \oint \vec{E}.d\vec{s}$$

Irrespective of where the charge Q is placed within a closed surface completely surrounding it, the total normal flux is Q and the total number of lines of force passing out normally is $Q/\varepsilon_0$.

In fact, if there are placed charges of value $Q_1$, $Q_2$, $-Q_3$ inside a closed surface, the total i.e. net charge enclosed by the surface is $(Q_1 + Q_2 - Q_3)/\varepsilon_0$ through the closed surface.




## Electric Flux Density or Electric Displacement:

It is given by the normal flux per unit area.
If a flux of $\Psi$ coulombs passes normally through an area of $A\ m^2$ , then flux density is

$$D = \frac{\Psi}{A}\ C/m^2$$

It is related to electric field intensity by the relation

$$D \;=\; \varepsilon_0 \varepsilon r\ E \text{ ...in a medium}$$
$$= \varepsilon_0\ E \text{ ...in free space}$$

In other words, the product of electric intensity $E$ at any point within a dielectric medium and the absolute permittivity $\varepsilon\ (= \ \varepsilon_0\varepsilon_r)$ at the same point is called the ***displacement*** at that point.

## Statement of Gauss's law:

Gauss's law states that the total electric flux $\Phi_E$ through any closed surface is equal to the total charge enclosed by that surface.

Mathematically,

$$\Phi_E = \frac{q}{\varepsilon_0}$$

or, $\varepsilon_0 \Phi_E = q$

or, $\varepsilon_0 \oint \vec{E}.d\,\vec{s} = q$ $[\because\ \Phi_E = \oint \vec{E}.d\vec{s}\ ]$

Where,

$\varepsilon_0$ = Permittivity constant

$\Phi_E$ = Electric flux for the surface

q = Net charge of the surface

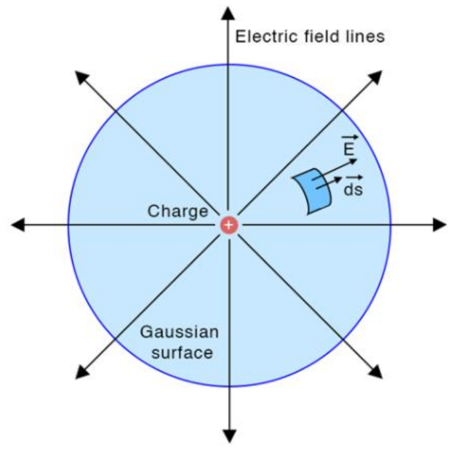


Figure-1: A Spherical Gaussian Surface

## Explanation:

Let us consider a +ve charge q is enclosed in a spherical Gaussian surface of radius r, the net flux through the Gaussian surface is given by

$$\Phi_E = \oint \vec{E}.d\vec{s}$$

$$= \oint E ds \cos\theta \quad [\because \theta \text{ is the angle between } \vec{E} \text{ and } d\vec{s}\ ]$$

$$= \oint \frac{q}{4\pi\varepsilon_0 r^2}\ ds \cos\theta \quad [\because E = \frac{q}{4\pi\varepsilon_0 r^2}\ ]$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \oint ds \cos\theta$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{4\pi r^2}{r^2} \qquad [\because\ \oint ds\ cos\theta = 4\pi r^2\ ]$$

$$= \frac{q}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \oint \vec{E}.d\,\vec{s} = \frac{q}{\varepsilon_0}$$

$$\therefore\ \varepsilon_0 \oint \vec{E}.d\,\vec{s} = q$$

# Applications of Gauss’s Law

## Line of Charge:

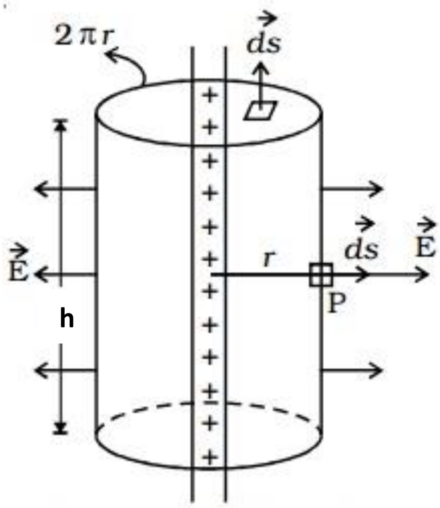



*Fig 1.17 Infinitely long straight charged wire*

Let us consider a section of an infinite rod of charge, the linear charge density λ being constant for all points on the line.

Let us consider a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis. There is no flux through the circular caps because $\vec{E}$ here lies in the surface at every point.

The charge enclosed by the Gaussian surface of Fig.-1.17 is λh.

Applying Gauss’s law, we have

$$\varepsilon_0 \oint \vec{E}.d\,\vec{s} = q$$

$$\Rightarrow \quad \varepsilon_0 \oint \vec{E}.d\,\vec{s} = \lambda\text{h} \qquad [\because\; q = \lambda\text{h}\,]$$

$$\Rightarrow \quad \varepsilon_0 \oint Eds\cos\theta = \lambda\text{h}$$

$$\Rightarrow \quad \varepsilon_0 \oint_a Eds\cos\theta + \varepsilon_0 \oint_b Eds\cos\theta + \varepsilon_0 \oint_c Eds\cos\theta = \lambda\text{h}$$

$$\Rightarrow \quad \varepsilon_0 \oint_a Eds\cos 90^0 + \varepsilon_0 \oint_b Eds\cos 0^0 + \varepsilon_0 \oint_c Eds\cos 90^0 = \lambda\text{h}$$

$$\Rightarrow \quad 0 + \varepsilon_0 \oint_b Eds + 0 = \lambda\text{h}$$

$$\Rightarrow \quad \varepsilon_0 E \oint_b ds = \lambda\text{h}$$

$$\Rightarrow \quad \varepsilon_0 ES = \lambda\text{h}$$ where, $S$ = The area of the surface

$$\Rightarrow \quad \varepsilon_0 E(2\pi r h) = \lambda h \quad [\because\ S = 2\pi r h\ ]$$

$$\therefore\ E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

## Spherically Symmetric Charge Distribution:

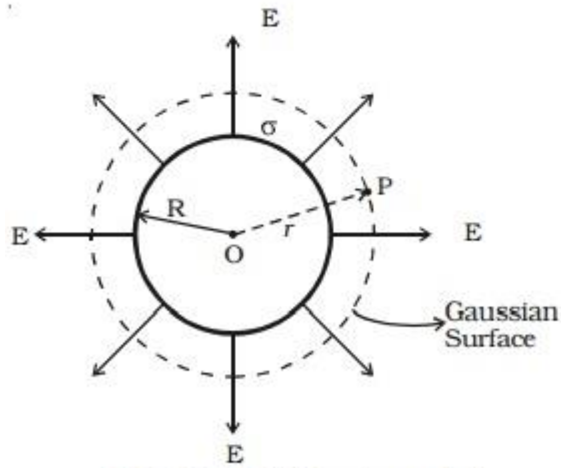



*Fig1.20a. Field at a point outside the shell*

Electric field intensity $\vec{E}$ for outside the sphere (r > R).

Let us consider a spherical Gaussian surface of radius r, outside the charge distribution i.e., r > R.

Applying Gauss's law, we get,

$$\varepsilon_0 \oint \vec{E}.d\,\vec{s} = q \qquad \text{where q is the total charge}$$

$$\Rightarrow \quad \varepsilon_0 \oint E ds \cos 0^0 = q$$

$$\Rightarrow \quad \varepsilon_0 E S = q \qquad \text{where, } S = \text{The area of the surface}$$

$$\Rightarrow \quad \varepsilon_0 E(4\pi r^2) = q \qquad [\because\ S = 4\pi r^2\ ]$$

$$\therefore E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Electric field intensity $\vec{E}$ for inside the sphere (r < R).

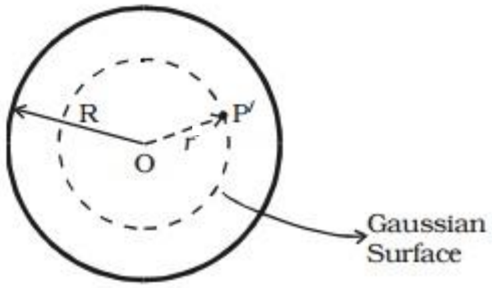



*Fig 1.20b Field at a point inside the shell*

Let us consider a spherical Gaussian surface of radius r, outside the charge distribution i.e., r < R.

Applying Gauss's law, we get,

$$\Rightarrow \quad \varepsilon_0 \oint \vec{E}.d\,\vec{s} = q'$$

where, $q'$ is the part of q contained within the sphere of radius r.

$$\Rightarrow \quad \varepsilon_0 \oint E ds \cos 0^0 = q'$$

$$\Rightarrow \varepsilon_0 ES = q'$$

$$\Rightarrow \quad \varepsilon_0 E(4\pi r^2) = q'$$

$$\Rightarrow \quad E = \frac{q'}{4\pi\varepsilon_0 r^2} \text{......................} (1)$$

Now $q' = q\,\dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$

where, $\frac{4}{3}\pi R^3$ is the volume of the spherical charge distribution

$$\Rightarrow \; q' = q\,\frac{r^3}{R^3}$$

Putting the value of $q'$ in equation (1), we get,

$$E = \frac{1}{4\pi\varepsilon_0 r^2} * q\,\frac{r^3}{R^3}$$

$$\therefore \; E = \frac{qr}{4\pi\varepsilon_0 R^3} \text{..................} (2)$$

We know, $\sigma = \dfrac{q}{\frac{4}{3}\pi R^3}$

$$\therefore \frac{q}{R^3} = \frac{4\pi\sigma}{3} \text{..................} (3)$$

Putting the value of equation (3) in equation (2), we get

$$E = \frac{r}{4\pi\varepsilon_0} * \frac{4\pi\sigma}{3}$$

$$\therefore E = \frac{r\sigma}{3\varepsilon_0}$$

## Gauss's law Math:

1. A plastic rod, whose length h is **220 cm** and whose radius r is **3.6 mm**, carries a negative charge q of magnitude $\mathbf{3.8 \times 10^{-7}\ C}$, spread uniformly over its surface. What is the electric field near the midpoint of the rod, at a point on its surface?

$[\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}]$

Here,

$$Lenth\ h = 220\ cm = 2.2\ m$$
$$Charge\ q = -3.8 \times 10^{-7}\ C$$
$$radius\ r = 3.6\ mm = \frac{3.6}{1000}\ m = 0.0036\ m$$
$$Electric\ Field\ E = ?$$

The linear charge density for the rod is

$$\lambda = \frac{q}{h} = \frac{-3.8 \times 10^{-7}\ C}{2.2\ m}$$

$$\Longrightarrow \lambda = -1.73 \times 10^{-7}\ C/m$$

We know,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$
$$= \frac{-1.73 \times 10^{-7}\ Cm^{-1}}{(2\pi)(8.85 \times 10^{-12}\ C^2N^{-1}m^{-2})(0.0036\ m)}$$
$$= -8.6 \times 10^5\ NC^{-1}$$

The minus sign tells us that, because the rod is negatively charged, the direction of the electric field is radially inward, toward the axis of the rod.