(17) If the position vectors of three points $A(2,4,-1)$ $B(1,2,-3)$ and $C(3,1,2)$ find a vector perpendicular to the plane ABC.

$\vec{AB} = (-1,-2,-2)$ $\vec{BC} = (2,-1,5)$

$= -\hat{i} - 2\hat{j} - 2\hat{k}$ $= 2\hat{i} - \hat{j} + 5\hat{k}$

$\therefore$ perpendicular vector $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -2 \\ 2 & -1 & 5 \end{vmatrix}$

$= \hat{i}(-10-2) - \hat{j}(-5+4) + \hat{k}(1+4)$

$= -12\hat{i} + \hat{j} + 5\hat{k}$

(18) Show that the points whose position vectors are given by $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$, $3\hat{i}+4\hat{j}+5\hat{k}$ are collinear.

If they are collinear $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

$\Rightarrow 1(15-16) - 2(10-12) + 3(8-9)$

$\Rightarrow -1+4-3 = 0$

$\therefore$ those position vectors are collinear.

(ii) $\underline{a}\times\underline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 1 & -1 \end{vmatrix} = \hat{i}(-2+3) - \hat{j}(-1+6) + \hat{k}(-3)$

$= \hat{i} - 5\hat{j} - 3\hat{k}$

$\underline{b}\times\underline{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix} = \hat{i}(-2+3) - \hat{j}(-4+1) + \hat{k}(5)$

$= \hat{i} + 3\hat{j} + 5\hat{k}$

$(\underline{a}\times\underline{b})\times(\underline{b}\times\underline{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -5 & -3 \\ 1 & 3 & 5 \end{vmatrix}$

$= \hat{i}(-25+9) - \hat{j}(5+3) + \hat{k}(15+5)$

$= -14\hat{i} - 8\hat{j} + 20\hat{k}$

$\therefore$ magnitude $= \sqrt{14^2+8^2+20^2}$

$= 2\sqrt{165}$

(16) Find the area of the parallelogram whose diagonals are $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$

Area $= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = \frac{1}{2}|\hat{i}(-2) - \hat{j}(14) + \hat{k}(-10)|$

$= \frac{1}{2}\sqrt{2^2+14^2+10^2} = 5\sqrt{3}$

(14) Determine a unit vector perpendicular to the ~~unit vector~~ plane of $A = 3\hat{i} - 5\hat{j} + \hat{k}$ and $B = 2\hat{i} - 4\hat{j} - 7\hat{k}$

perpendicular vector,

$$\underline{A} \times \underline{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & 1 \\ 2 & -4 & -7 \end{vmatrix} = \hat{i}(35+4) - \hat{j}(-21-2) + \hat{k}(12+10)$$

$$= 39\hat{i} + 23\hat{j} + 22\hat{k}$$

$$\therefore \text{unit vector} = \frac{39\hat{i} + 23\hat{j} + 22\hat{k}}{\sqrt{39^2 + 23^2 + 22^2}}$$

$$= \frac{1}{\sqrt{2534}}(39\hat{i} + 23\hat{j} + 22\hat{k})$$

(15) $a = \hat{i} + 2\hat{i} - 3\hat{k}$, $b = 2\hat{i} + \hat{j} - \hat{k}$, $c = \hat{i} + 3\hat{j} - 2\hat{k}$

find (i) $a \cdot (b \times c)$ (ii) $(a \times b) \times (b \times c)$ and also magnitude

(i) $$\underline{b} \times \underline{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 1 & -1 \end{vmatrix} = \hat{i}(-2+3) - \hat{j}(-1+6) + \hat{k}(-3)$$

$$= \hat{i} - 5\hat{j} - 3\hat{k}$$

$$\underline{a} \cdot (\underline{b} \times \underline{c}) = (\hat{i} + 2\hat{i} - 3\hat{k}) \cdot (\hat{i} - 5\hat{j} - 3\hat{k})$$

$$= 1 - 10 + 9 = 0$$

① $\underline{a} = 9\hat{i} + \hat{j} - 6\hat{k}$

$\underline{b} = 4\hat{i} - 6\hat{j} + 5\hat{k}$

$\therefore \underline{a}.\underline{b} = 36 - 6 - 30 = 0$

$\therefore$ $\underline{a}$ and $\underline{b}$ perpendicular to each other

(ii) Let $\underline{u} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\underline{v} = 8\hat{i} - 12\hat{j} - 5\hat{k}$

$\underline{u}.\underline{v} = 16 - 36 + 20 = 0$

$\therefore$ u and v perpendicular to each other

(iii) Let $\underline{u} = 2\hat{i} + a\hat{j} + 2\hat{k}$ and $\underline{v} = 4\hat{i} - 2\hat{j} - 2\hat{k}$

Hence u, v perpendicular,

$\underline{u}.\underline{v} = 0$

$8 - 2a - 4 = 0$

$2a = 4$

$\therefore a = 2$

(13) Find the unit vector parallel to xy plane and perpendicular to the unit vector $4\hat{i} - 3\hat{j} + \hat{k}$

unit vector $4\hat{i} - 3\hat{j} + \hat{k}$ parallel to xy plane and perpendicular to k that is

$k \times (4\hat{i} - 3\hat{j} + \hat{k}) = 4(\hat{i} \times \hat{k}) - 3(\hat{k} \times \hat{j}) + (\hat{k} \times \hat{k})$

$= 4\hat{j} + 3\hat{i}$ $\therefore$ unit vector $= \frac{1}{5}(4\hat{j} + 3\hat{i})$

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Let, $\underline{u} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\underline{v} = 2\hat{i} + 3\hat{j} - k$

$$\underline{u} \times \underline{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(-1-6) - \hat{j}(-3-4) + \hat{k}(9-2)$$

$$= -7\hat{i} + 7\hat{j} + 7\hat{k}$$

$\underline{u} \cdot \underline{v} = uv\cos\theta$

$6 + 3 - 2 = \sqrt{14} \cdot \sqrt{14} \cos\theta$

$\cos\theta = \dfrac{7}{\sqrt{14 \cdot 14}}$

$u = \sqrt{9+1+4}$
$= \sqrt{14}$
$v = \sqrt{4+9+1}$
$= \sqrt{14}$

$\cos\theta = \dfrac{1}{2}$

(12) (i) Show that the vectors $a = 9\hat{i} + \hat{j} - 6k$ and $b = 4\hat{i} - 6\hat{j} + 5k$ are at right angles to one another, where i, j, k are unit vectors along x, y, and z axes respectively.

(ii) Show that the two vectors $2\hat{i} + 3\hat{j} - 4k$ and $8\hat{i} - 12\hat{j} - 5k$ are perpendicular to each other.

(iii) Determine the value of a so that $2\hat{i} + a\hat{j} + 2k$ and $4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular.

(iii) Let $\underline{u} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\underline{v} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$|\underline{u} \times \underline{v}| = \left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 3 & -2 & 1 \end{vmatrix}\right| = |\hat{i}(1) - \hat{j}(5) + \hat{k}(-10)|$$

$$= |\hat{i} - 5\hat{j} - 10\hat{k}|$$

$$= \sqrt{1 + 5^2 + 10^2} = \sqrt{126}$$

$\therefore$ Unit vector $= \dfrac{\hat{i} - 5\hat{j} - 10\hat{k}}{\sqrt{126}}$

(11) Find the vector product of the following pairs of vectors and the angle between them.

(i) $4\hat{i} - 5\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$

(ii) $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{j} - \hat{k}$

(i) Let $\underline{u} = 4\hat{i} - 5\hat{j} + \hat{k}$ and $\underline{v} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\underline{u} \times \underline{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}(-17) - \hat{j}(11) + \hat{k}(13)$$

$$= -17\hat{i} - 11\hat{j} + 13\hat{k}$$

$$\cancel{u \times v = uv} \sin\theta = \frac{\sqrt{17^2 + 11^2 + 13^2}}{\sqrt{4^2 + 5^2 + 1^2} \cdot \sqrt{1^2 + 2^2 + 3^2}}$$

$\therefore$ ~~$\sin\theta =$~~

$$\sin\theta = \frac{\sqrt{579}}{\sqrt{42}\sqrt{14}}$$

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(10) Find the unit vectors perpendicular to each of the:
(i) $\hat{i}+\hat{j}+2\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ (ii) $4\hat{i}-\hat{j}+3\hat{k}$ (iii), $-2\hat{i}+\hat{j}-2\hat{k}$
(iii) $2\hat{i}+3\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$

(i) Let $\underline{u}=\hat{i}+\hat{j}+2\hat{k}$, $\underline{v}=2\hat{i}+3\hat{j}+\hat{k}$

$$|\underline{u}\times\underline{v}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = |\hat{i}(-5)-\hat{j}(-3)+\hat{k}(1)|$$
$$= |-5\hat{i}+3\hat{j}+\hat{k}|$$
$$= \sqrt{5^2+3^2+1^2} = \sqrt{35}$$

$\therefore$ unit vector $= \dfrac{-5\hat{i}+3\hat{j}+\hat{k}}{\sqrt{35}}$

(ii) Let $\underline{u}=4\hat{i}-\hat{j}+3\hat{k}$, $\underline{v}=-2\hat{i}+\hat{j}-2\hat{k}$

$$|\underline{u}\times\underline{v}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = |\hat{i}(-1)-\hat{j}(-2)+\hat{k}(2)|$$
$$= |-\hat{i}+2\hat{j}+2\hat{k}|$$
$$= \sqrt{1^2+2^2+2^2} = 3$$

$\therefore$ Unit vector $= \dfrac{-\hat{i}+2\hat{j}+2\hat{k}}{3}$

⑨ What is the unit vector perpendicular to each of the vectors $u = 2\hat{i} + \hat{j} - \hat{k}$ and $v = -6\hat{i} + 3\hat{j} + 5\hat{k}$ calculate the sin of the angle between these vectors.

perpendicular unit vector of u and v is

$$= \pm \frac{\underline{u} \times \underline{v}}{|\underline{u} \times \underline{v}|} \quad \therefore \underline{u} \times \underline{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ -6 & 3 & 5 \end{vmatrix}$$

$$= \hat{i}(8) - \hat{j}(4) + \hat{k}(12)$$

$$= 8\hat{i} - 4\hat{j} + 12\hat{k}$$

$$\therefore |\underline{u} \times \underline{v}| = \sqrt{8^2 + 4^2 + 12^2} = \sqrt{224}$$

The required unit vector $= \pm \dfrac{8\hat{i} - 4\hat{j} + 12\hat{k}}{\sqrt{224}}$

We know, $\sin\theta = \dfrac{|\underline{u} \times \underline{v}|}{|\underline{u}| \times |\underline{v}|}$

$$= \frac{\sqrt{224}}{\sqrt{6}\,\sqrt{70}}$$

$$u = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

$$v = \sqrt{6^2 + 3^2 + 5^2} = \sqrt{70}$$

$$\therefore \theta = \sin^{-1} \sqrt{\frac{56}{105}}$$

(7) Find the area of the triangle whose vert
are $A(1,3,2)$, $B(2,-1,1)$, $C(-1,2,3)$

$\vec{AB} = (1,-4,-1)$ $\vec{AC} = (-2,-1,1)$

$\therefore$ Area of the triangle is $\frac{1}{2}|AB \times AC|$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & -1 \\ -2 & -1 & 1 \end{vmatrix} = \frac{1}{2}\left|\hat{i}(-5) - \hat{j}(-1) + \hat{k}(-9\right.$$

$$= \frac{1}{2}\sqrt{5^2+1^2+9^2} = \frac{1}{2}\sqrt{107}$$

(8) Find the volume of the parallelopiped wh
edges are represented by $a = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $b = \hat{i} +$
and $c = 3\hat{i} - \hat{j} + 2\hat{k}$

Volume of abc is $\underline{a} \cdot (\underline{b} \times \underline{c})$

$$= 2\hat{i} - 3\hat{j} + 4\hat{k} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= 2\hat{i} - 3\hat{j} + 4\hat{k} \cdot \hat{i}(3) - \hat{j}(5) + \hat{k}(-7)$$

$$= 6 + 15 - 28 = -7$$

Volume can not be negative so
volume is 7 ~~unit~~ cubic unit.

⑤ Find the unit vector perpendicular to each of the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+2\hat{k}$

Let $\underline{a}=2\hat{i}+\hat{j}+\hat{k}$ and $\underline{b}=\hat{i}-\hat{j}+2\hat{k}$. Perpendicular of ~~a~~ a and b is $\underline{a}\times\underline{b}$

$$\therefore \underline{a}\times\underline{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 1\\ 1 & -1 & 2\end{vmatrix}=\hat{i}(3)-\hat{j}(3)+\hat{k}(-3)$$
$$=3\hat{i}-3\hat{j}-3\hat{k}$$

$$\therefore \text{Unit vector}=\frac{3\hat{i}-3\hat{j}-3\hat{k}}{\sqrt{9+9+9}}=\frac{\hat{i}}{\sqrt{3}}-\frac{\hat{j}}{\sqrt{3}}-\frac{\hat{k}}{\sqrt{3}}$$

⑥ Find the angles which the vector $3\hat{i}-6\hat{j}+2\hat{k}$ makes, with the co-ordinate axes.

Let $\underline{a}=3\hat{i}-6\hat{j}+2\hat{k}$ made angles α, β, γ with the positive direction of x, y and z respectively

$$|\underline{a}|=\sqrt{3^2+6^2+2^2}=7$$

$$\therefore \underline{a}\cdot\hat{i}=\underline{a}\cos\alpha=7\cos\alpha \Rightarrow (3\hat{i}-6\hat{j}+2\hat{k})\cdot\hat{i}=7\cos\alpha$$

$$\therefore 7\cos\alpha=3 \quad \therefore \cos\alpha=3/7$$

$$\underline{a}\cdot\hat{j}=\underline{a}\cos\beta=7\cos\beta \Rightarrow (3\hat{i}-6\hat{j}+2\hat{k})\cdot\hat{j}=7\cos\beta$$

$$7\cos\beta=-6 \quad \therefore \cos\beta=-6/7$$

$$\underline{a}\cdot\hat{k}=\underline{a}\cos\gamma=7\cos\gamma \Rightarrow (3\hat{i}-6\hat{j}+2\hat{k})\cdot\hat{k}=7\cos\gamma$$

$$7\cos\gamma=2 \quad \therefore \cos\gamma=2/7$$

③ Find the cross product of the two vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-4\hat{j}+2\hat{k}$

Let $\underline{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\underline{b}=3\hat{i}-4\hat{j}+2\hat{k}$

$$\underline{a}\times\underline{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 3 & -4 & 2\end{vmatrix}=\hat{i}(4+12)-\hat{j}(2-9)+\hat{k}(-4-6)$$

$$=16\hat{i}+7\hat{j}-10\hat{k}$$

④ Find the sine of the angle between the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-4\hat{j}+2\hat{k}$

From question ③ we found $\underline{a}\times\underline{b}=16\hat{i}+7\hat{j}-10\hat{k}$

$|\underline{a}\times\underline{b}|=\sqrt{16^2+7^2+10^2}=9\sqrt{5}$

we know that,

$\underline{a}\times\underline{b}=ab\sin\theta$

$\therefore$ ~~$\theta=$ Si~~ $9\sqrt{5}=\sqrt{14}\sqrt{29}\sin\theta$

$\therefore \sin\theta=\dfrac{9\sqrt{5}}{\sqrt{14}\sqrt{29}}$

$a=\sqrt{1^2+2^2+3^2}$
$=\sqrt{14}$

$b=\sqrt{3^2+4^2+2^2}$
$=\sqrt{29}$

① Find the angle between the vectors $a = 3\hat{i}+2\hat{j}+6\hat{k}$ and $b = 2\hat{i}+4\hat{j}-4\hat{k}$

Let θ be the angle between a and b.

We know that,

$\underline{a}\cdot\underline{b} = ab\cos\theta$

$\therefore \cos\theta = \dfrac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(2\hat{i}+4\hat{j}-4\hat{k})}{ab}$

$= \dfrac{6+8-24}{6\cdot 7} = \dfrac{-10}{42} = \dfrac{-5}{21}$

$\therefore \theta = \cos^{-1}(-5/21)$

$a = \sqrt{3^2+2^2+6^2}$
$= \sqrt{49} = 7$

$b = \sqrt{\cancel{4}+16+16}$
$= \sqrt{36} = 6$

② Find the scalar product of the vectors (2,3,1) and (3,1,-2) and also find the angle between them

Let $\underline{a} = 2\hat{i}+3\hat{j}+\hat{k}$ and $\underline{b} = 3\hat{i}+\hat{j}-2\hat{k}$

$\therefore \underline{a}\cdot\underline{b} = (2\hat{i}+3\hat{j}+\hat{k})\cdot(3\hat{i}+\hat{j}-2\hat{k}) = 6+3-2 = 7$

again,

$\underline{a}\cdot\underline{b} = ab\cos\theta$

$\therefore \theta = \cos^{-1}\dfrac{a\cdot b}{ab} = \dfrac{7}{\sqrt{14}\cdot\sqrt{14}}$

$\theta = \cos^{-1}\dfrac{7}{14} = \cos^{-1}\dfrac{1}{2}$

$\therefore \theta = 60^\circ$

$a = \sqrt{2^2+3^2+1^2}$
$= \sqrt{14}$

$b = \sqrt{3^2+1^2+2^2}$
$= \sqrt{14}$