SERIES RC, RL AND RLC CIRCUIT

Series A.C. Circuit

- **Introduction:**

In this chapter, phasor algebra is used to develop a quick, direct method for solving both series and parallel ac circuits.

The close relationship that exists between this method for solving for unknown quantities and the approach used for dc circuits will become apparent after a few simple

examples are considered. Once this association is established, many of the rules (current divider rule, voltage divider rule, and so on) for dc circuits can be readily applied to ac circuits

A.C. Circuit Containing Resistance Only

When an alternating voltage is applied across pure resistance, then free electrons flow (*i.e.* current) in one direction for the first half-cycle of the supply and then flow in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit.

Consider a circuit containing a pure resistance of $R\ \Omega$ connected across an alternating voltage source [See Fig. 11.60]. Let the alternating voltage be given by the equation :

$$v = V_m \sin \omega t \qquad ...(i)$$

As a result of this voltage, an alternating current i will flow in the circuit. The applied voltage has to overcome the drop in the resistance only *i.e.*

$$v = i R$$

or $$i = \frac{v}{R}$$

Substituting the value of v, we get,

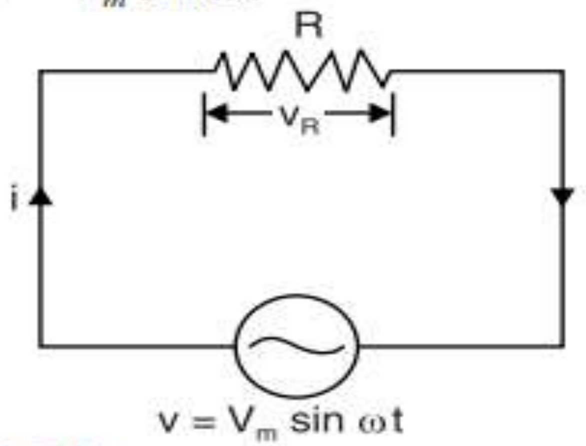


Fig. 11.60

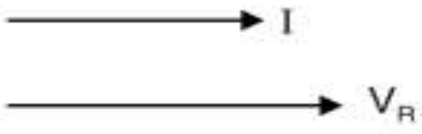


Fig. 11.61

$$i = \frac{V_m}{R} \sin \omega t \qquad ...(ii)$$

The value of i will be maximum (*i.e.* I_m) when $\sin \omega t = 1$.

$\therefore$ $$I_m = V_m/R$$

$\therefore$ Eq. (*ii*) becomes : $$i = I_m \sin \omega t \qquad ...(iii)$$

In terms of r.m.s. values, $$\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} \times R$$

or $$V = V_R = I R$$

A.C. Circuit Containing Pure Inductance Only

When an alternating current flows through a pure *inductive coil, a back e.m.f. (= $L\,di/dt$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. Since there is no ohmic drop, the applied voltage has to overcome the back e.m.f. only.

∴ Applied alternating voltage = Back e.m.f.

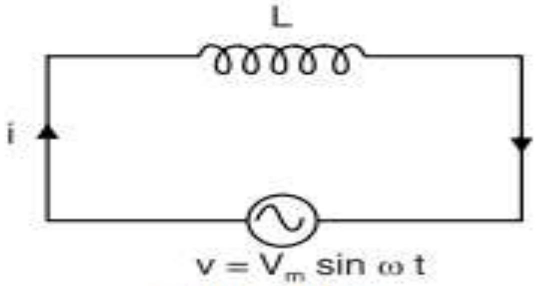


Fig. 11.65

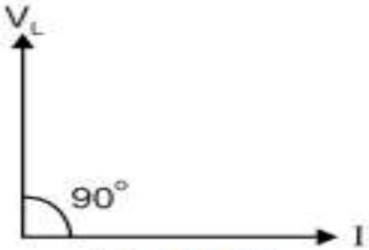


Fig. 11.66

Consider an alternating voltage applied to a pure inductance of L henry as shown in Fig. 11.65. Let the equation of the applied alternating voltage be :

$$v = V_m \sin \omega t \quad \text{...}(i)$$

Clearly, $$V_m \sin \omega t = L\frac{di}{dt}$$

or $$di = \frac{V_m}{L}\sin \omega t\, dt$$

Integrating both sides, we get, $i = \frac{V_m}{L}\int \sin \omega t\, dt = \frac{V_m}{\omega L}(-\cos \omega t)$

∴ $$i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2) \quad \text{....}(ii)$$

The value of i will be maximum (*i.e.* I_m) when sin ($\omega t - \pi/2$) is unity.

∴ $$I_m = V_m/\omega L$$

Substituting the value of $V_m/\omega L = I_m$ in eq. (ii), we get,

$$i = I_m \sin(\omega t - \pi/2) \quad \text{...}(iii)$$

Note that Ohm's law for an inductor states that peak current (I_m) through the inductor equals the peak voltage (V_m) across the inductor divided by the inductive reactance ($X_L = \omega L$).

(*i*) **Phase angle.** It is clear from eqs. (*i*) and (*iii*) that current lags behind the voltage by $\pi/2$ radians or 90°. *Hence in a pure inductance, current lags the voltage by 90°.* This is also indicated by the phasor diagram shown in Fig. 11.66. Note that r.m.s. values have been used in drawing the phasor diagram. The wave diagram shown in Fig. 11.67 also depicts that current lags the voltage by 90°. There is also physical explanation for the lagging of current behind voltage in an inductive coil. Inductance opposes the change in current and serves to delay the increase or decrease of current in the circuit. This causes the current to lag behind the applied voltage.

(*ii*) **Inductive reactance.** Inductance not only causes the current to lag behind the voltage but it also limits the magnitude of current in the circuit. We have seen above that :

$$I_m = V_m/\omega L$$

or $$\frac{V_m}{I_m} = \omega L$$

Clearly, the opposition offered by inductance to current flow is ωL. This quantity ωL is called the *inductive reactance* X_L of the coil. It has the same *dimensions as resistance and is, therefore, measured in Ω.

∴ $$I_m = V_m/X_L$$

or $$\frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{X_L}$$

or $$I = \frac{V_L}{X_L} \qquad (V = V_L)$$

where inductive reactance $X_L = \omega L = 2\pi f L$

Note that X_L will be in Ω if L is in henry and f in Hz.

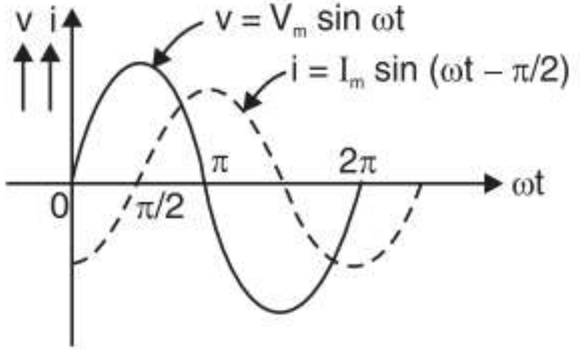


Fig. 11.67

A.C. Circuit Containing Capacitance Only

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electrons move to and fro around the circuit, connecting the plates, thus constituting alternating current.

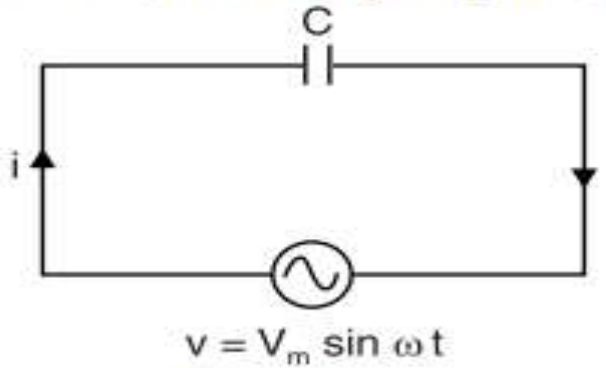


Fig. 11.69

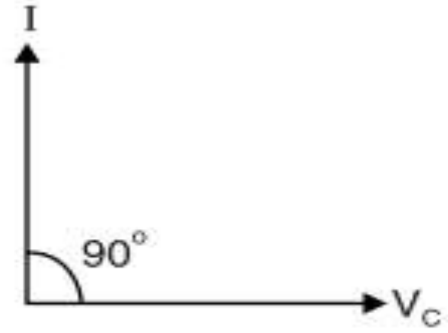


Fig. 11.70

Consider an alternating voltage applied to a capacitor of capacitance C farad as shown in Fig. 11.69. Let the equation of the applied alternating voltage be :

$$v = V_m \sin \omega t \qquad \text{...}(i)$$

As a result of this alternating voltage, alternating current will flow through the circuit. Let at any instant i be the current and q be the charge on the plates.

Charge on capacitor, $q = C\,v = C\,V_m \sin \omega t$

$\therefore$ Circuit current, $i = \dfrac{d}{dt}(q) = \dfrac{d}{dt}(C\,V_m \sin \omega t) = \omega\, C\, V_m \cos \omega\, t$

$$\therefore \quad i = \omega\, C\, V_m \sin(\omega\, t + \pi/2) \qquad \text{...}(ii)$$

The value of i will be maximum (*i..e.* I_m) when $\sin(\omega\, t + \pi/2)$ is unity.

$$\therefore \quad I_m = \omega\, C\, V_m$$

Substituting the value $\omega C\, V_m = I_m$ in eq. (*ii*), we get,

$$i = I_m \sin(\omega\, t + \pi/2) \qquad \text{...}(iii)$$

(*i*) **Phase angle.** It is clear from eqs. (*i*) and (*iii*) that current leads the voltage by $\pi/2$ radians or 90°. *Hence in a pure capacitance, current leads the voltage by 90°.* This is also indicated in the phasor diagram shown in Fig. 11.70. The wave diagram shown in Fig. 11.71 also reveals the same fact. There is also physical explanation for the lagging of voltage behind the current in a capacitor. Capacitance *opposes the change in voltage and serves to delay the increase or decrease of voltage across the capacitor. This causes the voltage to lag behind the current.

(*ii*) **Capacitive reactance.** Capacitance not only causes the voltage to lag behind current but it also limits the magnitude of current in the circuit. We have seen above that :

$$I_m = \omega C V_m$$

or
$$\frac{V_m}{I_m} = \frac{1}{\omega C}$$

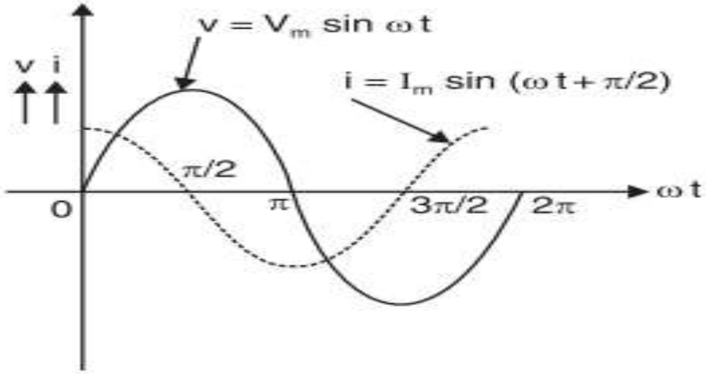


Fig. 11.71

If V_C and I are the r.m.s. values, then,

$$\frac{V_m}{I_m} = \frac{V_C}{I} = \frac{1}{\omega C} \qquad (V = V_C)$$

Clearly, the opposition offered by capacitance to current flow is $1/\omega C$. This quantity $1/\omega C$ is called the *capacitive reactance* X_C of the capacitor. It has the same dimensions as resistance and is, therefore, measured in Ω.

$$\therefore \qquad I = V_C/X_C$$

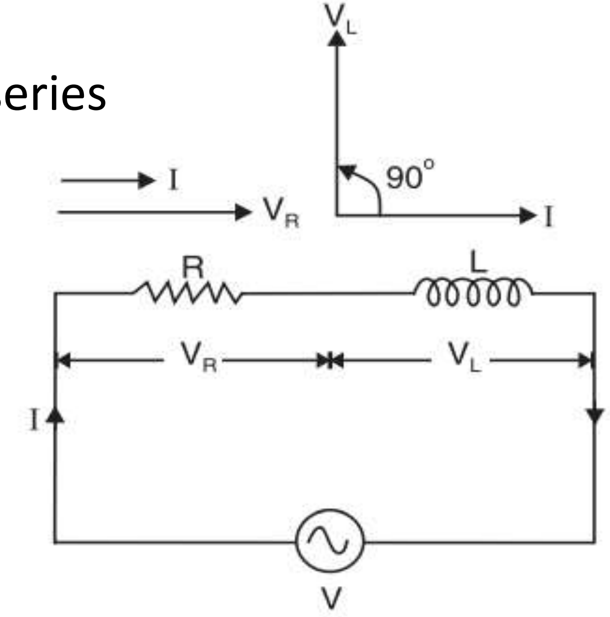
where capacitive reactance is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

Note that X_C will be in Ω if C is in farad and f in Hz.

Series A.C. Circuit

- **R-L Series A.C. Circuit:**

Fig. 12.1 (i) shows a pure resistance of R ohms connected in series with a coil of pure inductance L henry.



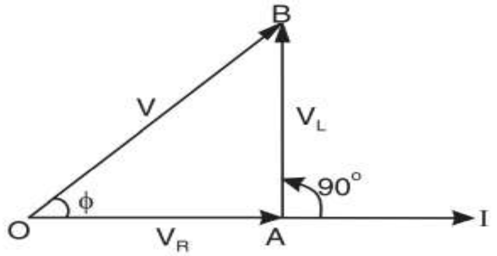
(i)

Let V = r.m.s. value of the applied voltage

I = r.m.s. value of the circuit current

$\therefore$ $V_R = IR$ .... where V_R is in phase with I

$V_L = IX_L$ .... where V_L leads I by 90°



(ii)

Series A.C. Circuit

Taking *current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 12.1 (*ii*). The voltage drop V_R (= $I R$) is in phase with current and is represented in magnitude and direction by the phasor OA. The voltage drop V_L (= $I X_L$) leads the current by 90° and is represented in magnitude and direction by the phasor AB. The applied voltage V is the phasor sum of these two drops *i.e.*

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$\therefore \qquad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called **impedance** of the circuit. It is represented by Z and is measured in ohms (Ω).

$$\therefore \qquad I = \frac{V}{Z} \quad \text{where } Z = \sqrt{R^2 + X_L^2}$$

Series A.C. Circuit

i) *Phase angle.* The value of phase angle Φ can be determined from the phasor diagram.

$$\tan\phi = \frac{V_L}{V_R} = \frac{I X_L}{I R} = \frac{X_L}{R}$$

(ii) *Impedance.* The total opposition offered to the flow of alternating current by a circuit is called impedance Z of the circuit. In R-L series circuit,

$$\text{Impedance, } Z = \sqrt{R^2 + X_L^2} \quad \text{where } X_L = 2\pi f L$$

Series A.C. Circuit

- **R-C Series A.C. Circuit**

Fig. 12.18 shows a resistance of R ohms connected in series with a capacitor of C farad.

Let

$$V = \text{r.m.s. value of applied voltage}$$

$$I = \text{r.m.s. value of the circuit current}$$

$$V_R = IR \text{ .... where } V_R \text{ is in phase with } I$$

$$V_C = IX_C \text{ .... where } V_C \text{ lags } I \text{ by } 90°$$

Series A.C. Circuit

Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 12.19. The voltage drop V_R (= IR) is in phase with current and is represented in magnitude and direction by the phasor OA. The voltage drop V_C (= IX_C) lags behind the current by 90° and is represented in magnitude and direction by the phasor AB. The applied voltage V is the phasor sum of these two drops *i.e.*

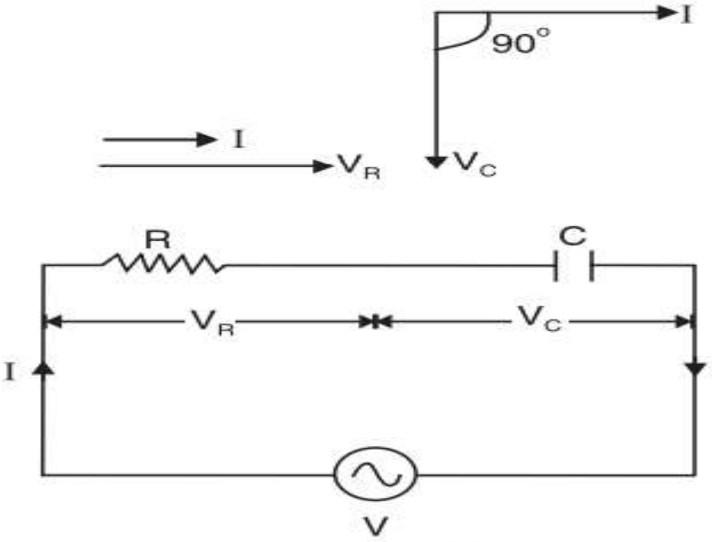


Fig. 12.18

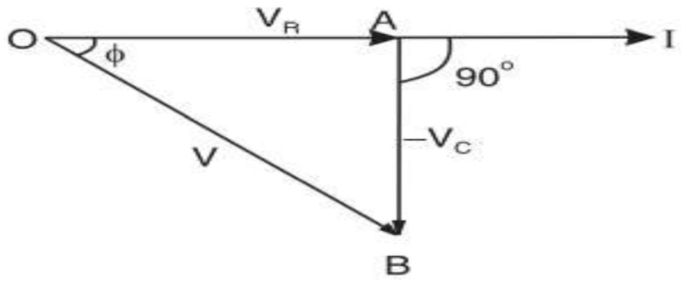


Fig. 12.19

$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

Series A.C. Circuit

The quantity $\sqrt{R^2 + X_C^2}$ offers opposition to current flow and is called **impedance** of the circuit.

$$\therefore \qquad I = V/Z \text{ where } Z = \sqrt{R^2 + X_C^2}$$

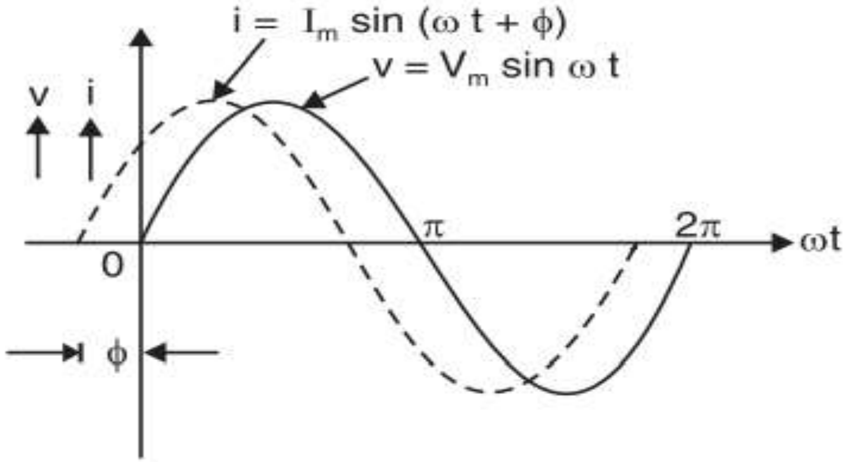
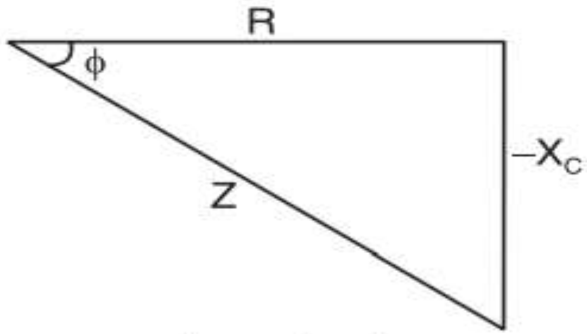


Fig. 12.20



Impedance triangle

Fig. 12.21

(i) Phase angle. The value of the phase angle can be determined as under :

$$\tan \phi = -\frac{V_C}{V_R} = -\frac{IX_C}{IR} = -\frac{X_C}{R}$$

Series A.C. Circuit

- **R-L-C Series A.C. Circuit**

This is a general series a.c. circuit. Fig. 12.32 shows R, L and C connected in series across a supply voltage V (r.m.s.). The resulting circuit current is I (r.m.s.).

$\therefore$ Voltage across R, $V_R = IR$ ... V_R is in phase with I

Voltage across L, $V_L = IX_L$ ... where V_L leads I by 90°

Voltage across C, $V_C = IX_C$ ... where V_C lags I by 90°

As before, the phasor diagram is drawn taking current as the reference phasor. In the phasor diagram (See Fig. 12.33), OA represents V_R, AB represents V_L and AC represents V_C. It may be seen that V_L is in phase opposition to V_C. It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. For the case considered,

Series A.C. Circuit

$V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ and is represented by AD. Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and is represented by OD.

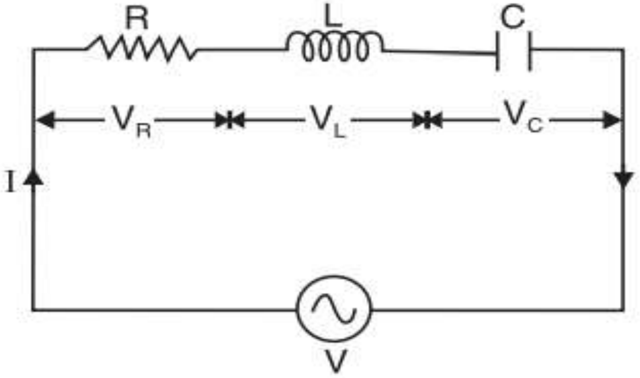


Fig. 12.32

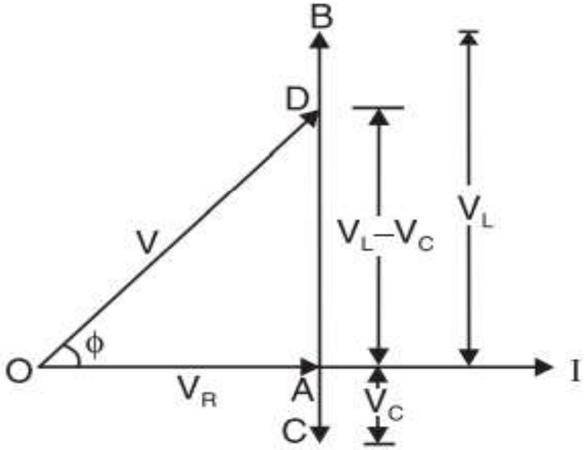


Fig. 12.33

$$\therefore \quad V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Series A.C. Circuit

The quantity $\sqrt{R^2+(X_L-X_C)^2}$ offers opposition to current flow and is called **impedance** of the circuit.

$$\text{Circuit power factor, } \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}} \quad ...(i)$$

$$\text{Also, } \quad \tan\phi = \frac{V_L-V_C}{V_R} = \frac{X_L-X_C}{R} \quad ...(ii)$$

Since X_L, X_C and R are known, phase angle ϕ of the circuit can be determined.

$$\text{Power consumed, } P = VI\cos\phi = {}^*I^2R$$

Three cases of *R-L-C* series circuit. We have seen that the impedance of a *R-L-C* series circuit is given by ;

$$Z = \sqrt{R^2+(X_L-X_C)^2}$$

Series A.C. Circuit

(*i*) **When $X_L - X_C$ is positive** (*i.e.* $X_L > X_C$), phase angle ϕ is positive and the circuit will be inductive.

(*ii*) **When $X_L - X_C$ is negative** (*i.e.* $X_C > X_L$), phase angle ϕ is negative and the circuit is capacitive.

(*iii*) **When $X_L - X_C$ is zero** (*i.e.* $X_L = X_C$), the circuit is purely resistive.

Series A.C. Circuit

Example 11.50. *A pure inductive coil allows a current of 10 A to flow from a 230 V, 50 Hz supply. Find (i) inductive reactance (ii) inductance of the coil (iii) power absorbed. Write down the equations for voltage and current.*

Solution. (*i*) Circuit current, $I = V/X_L$ $\quad (V_L = V)$

$\therefore$ Inductive reactance, $X_L = V/I = 230/10 = \mathbf{23\ \Omega}$

(*ii*) Now, $X_L = 2\pi f L \quad \therefore \quad L = \dfrac{X_L}{2\pi f} = \dfrac{23}{2\pi \times 50} = \mathbf{0.073\ H}$

(*iii*) Power absorbed = **Zero**

$$V_m = 230 \times \sqrt{2} = 325.27\ \text{V}\ ;\ I_m = 10 \times \sqrt{2} = 14.14\ \text{A}\ ; \quad \omega = 2\pi \times 50 = 314\ \text{rad/s}$$

Since in a pure inductive circuit, current lags behind the voltage by $\pi/2$ radians, the equations are :

$$\mathbf{v = 325.27 \sin 314\, t \quad ; \quad i = 14.14 \sin (314\, t - \pi/2)}$$

The Basic Elements

RESPONSE OF BASIC *R, L,* AND *C* ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

EXAMPLE 14.3 The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the υ and i curves.

a. $i = 10 \sin 377t$
b. $i = 7 \sin(377t - 70°)$

Solutions:

a. Eq. (14.4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7\ \Omega$
Eq. (14.5): $V_m = I_m X_L = (10 \text{ A})(37.7\ \Omega) = 377 \text{ V}$

and we know that for a coil υ leads i by 90°. Therefore,

$$\upsilon = \mathbf{377 \sin(377}t \mathbf{+ 90°)}$$

The curves are sketched in Fig. 14.15.

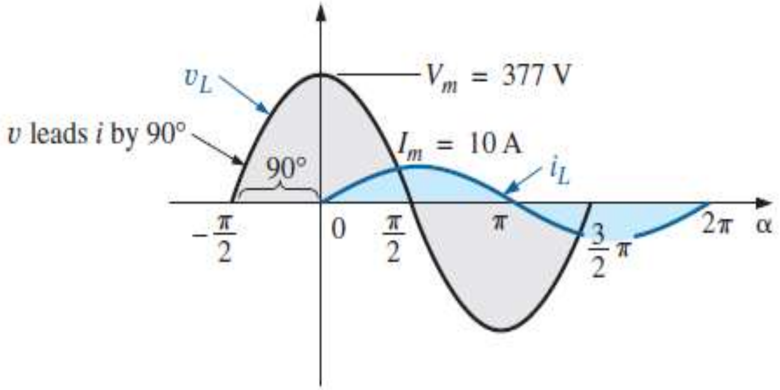


FIG. 14.15
Example 14.3(a).

The Basic Elements

FREQUENCY RESPONSE OF THE BASIC ELEMENTS

EXAMPLE 14.9 At what frequency will an inductor of 5 mH have the same reactance as a capacitor of 0.1 μF?

Solution:

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \times 10^{-3}\,\text{H})(0.1 \times 10^{-6}\,\text{F})}}$$

$$= \frac{1}{2\pi\sqrt{5 \times 10^{-10}}} = \frac{1}{(2\pi)(2.236 \times 10^{-5})} = \frac{10^5\,\text{Hz}}{14.05} \cong \mathbf{7.12\,kHz}$$

Thank you