Lecture-2
10.4.25

$$\Gamma n = \int_0^{\infty} e^{-x} \cdot x^{n-1} \, dx \quad \text{—(i)}$$

$$\lfloor n = \int_0^{\infty} e^{-x} \cdot x^{n} \, dx \quad \text{—(ii)}$$

$$\Gamma(n+1) = \int_0^{\infty} e^{-x} \cdot x^{n} \, dx \quad \text{—(iii)}$$

for gamma

$$\Gamma 6 = 5 \Gamma 5$$

$\Gamma(n+1) = \lfloor n$

$\Gamma 7 = \lfloor 6$

$\lfloor n-1 = \dfrac{\lfloor n}{n}$

$0! = 1$

$\lfloor n = n \cdot \lfloor n-1$

$\Gamma(n+1) = n \cdot \Gamma n$

Reduction formula

বিটা এবং গামা ফাংশনের অনেকগুলো রূপ আছে।

Let $x = y^2$

$dx = 2y \, dy$

$$\Gamma n = \int_0^{\infty} e^{-y^2} (y^2)^{n-1} \cdot 2y \, dy$$

$$= 2 \int_0^{\infty} e^{-y^2} \cdot y^{2n-1} \, dy$$

Let

$$x = my \quad or = \delta y$$

$$\Gamma n = \int_0^{\infty} e^{-mx} (my)^{n-1} m dy$$

$$\frac{\Gamma n}{m^n} = \int_0^{\infty} e^{-mx} y^{n-1} dy$$

$$\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$$

Let $x = \frac{y}{1+y}$ [x-কে y-এর মাধ্যমে]

$$dx = \frac{(1+y).1 - y.1}{(1+y)^2} dy$$

$$dx = \frac{1}{(1+y)^2} dy$$

Modify

$$y = \frac{x}{1-x}$$ [y-কে x এর মাধ্যমে]

$= \infty$

$x = 0 \quad y = 0$

$x = 1, \ y = \infty$

$$\int_1^1 x\,dx = \left[\frac{x^2}{2}\right]_1^1 = \frac{1^2}{2} - \frac{1^2}{2} = 0$$

$$\beta(m,n) = \int_0^\infty \left(\frac{y}{1+y}\right)^{m-1} \left(1 - \frac{y}{1+y}\right)^{n-1} \cdot \frac{1}{(1+y)^2}\,dy$$

$$= \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy \longrightarrow$$ পাওয়ার গুলোর (যোগ ($m-1$ $\cancel{+}$, $n-1$, 2 এগুলো))

$\longrightarrow$ formula

$$\rightarrow \int_0^\infty \frac{y^5}{(1+y)^{13}}\, dy$$ [Integration previous page formula

$$= \int_0^\infty \frac{y^{6-1}}{(1+y)^{6+7}}\, dy$$

$$= \beta(6,7)$$

Ans

Let $x = \sin^2\theta$

$dx = 2\sin\theta . \cos\theta \, d\theta$

$$\beta(m,n) = \int_0^{\pi/2} (\sin^2\theta)^{m-1} . (1-\sin^2\theta)^{n-1} . 2\sin\theta . \cos\theta \, d\theta$$

$$= 2\int_0^{\pi/2} (\sin\theta)^{2m-1} (\cos\theta)^{2n-1} \, d\theta$$

$$\int_0^{\pi/2} \sin^8\theta$$ $\boxed{2.9/2}$

Let $2m-1 = p$ | $2n-1 = q$

$m = \frac{p+1}{2}$ | $n = \frac{q+1}{2}$

$$\boxed{\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right) = 2\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta}$$

$$\frac{1}{2} \cdot 2\int_0^{\pi/2} \sin^8\theta \, d\theta$$

$$= \frac{1}{2}\, \beta\left(\frac{8+1}{2}, \frac{0+1}{2}\right)$$

$\boxed{H.W}$ β – এবং গামা ফাংশনের Type – গুলো

কত Type এবং কি কি

Lecture-3
16.4.25

Definition of Gamma Function

$$\Gamma n = \int_0^\infty e^{-t} \cdot t^{n-1}\, dt \longleftarrow (1)$$

Let

$t = x^2 \Rightarrow dt = 2x\,dx$ Same limit

Express $t = x^2$ $\boxed{t = x^2}$

$$\Gamma n = \int_0^\infty e^{-x^2} \cdot x^{2n-2}\, 2x\,dx$$

এতএব আসবে হবে।

$$\Gamma n = 2\int_0^\infty e^{-x^2} x^{2n-1}\, dx \longleftarrow (ii)$$

[2 হবে।

অন্যবর্তী function - এখানে $\boxed{y \text{ use}}$

Similary,
$$\Gamma m = 2\int_0^\infty e^{-y^2} y^{2m-1}\, dy \longleftarrow (iii)$$

Multimple (ii) and (iii)

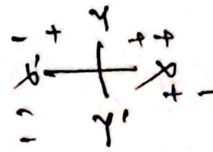
$$\Gamma n\, \Gamma m = 4\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, x^{2n-1}\, y^{2m-1}\, dx\,dy \longleftarrow (iv)$$

↑ দুইটার Integration নেওয়া হয়েছে

3N একসাথে power থেকে

Replacing cartesian into polar convert

$x = r\cos\theta$ $y = r\sin\theta$

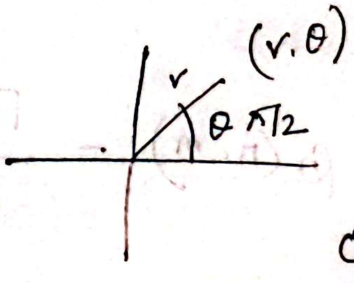


calculating

$dxdy = r\,dr\,d\theta$ এইটাই মনে রাখব

$\boxed{\text{show that } dxdy = r\,dr\,d\theta}$

$\boxed{x \text{ ও } y \text{ positive হবে 1st coordinate}}$



$\theta = \pi/2$

$r = \infty$

x এবং y এর মান বসিয়ে

$$\Gamma n \Gamma m = 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2}(r\cos\theta)^{2n-1}(r\sin\theta)^{2m-1}\, r\,dr\,d\theta$$

$$= \underbrace{2\int_{r=0}^{\infty} e^{-r^2} r^{2(n+m)+1}\,dr}_{\Gamma n+m} \times \overbrace{2\int_{\theta=0}^{\pi/2} \cos^{2n-1}\theta \cdot \sin^{2m-1}\theta\, d\theta}^{\beta(n,m)}$$

$2n-1+2m-1+1 = 2(n+m)-1$

$$= \Gamma n+m \cdot \beta(n,m)$$ using eqn (11)

$\Gamma n = 2\int_0^\infty e^{-x^2} x^{2n-1}\,dx$

$$\therefore \beta(n,m) = \frac{\Gamma n \cdot \Gamma m}{\Gamma n+m}$$

Bitta function is denoted by $B(m,n)$ and defined by $\int_0^1 x^{m-1} \cdot (1-x)^{n-1}\, dx$

which is ~~converscnte~~ convergent for all positive values of m, n.

$$B(m,n) = \int_0^1 x^{m-1} \cdot (1-x)^{n-1}\, dx$$

$(1-x) = y$ ধরতে হবে

also Symmetric function

হয়ে আছে

$$B(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$$

Proman কর যে,

$\gamma \frac{1}{2} = \sqrt{\pi}$

Evaluate $\gamma \frac{1}{2} = ?$

P- 96.97

Problem

$$\int_0^\infty e^{-t^2}\, dt$$

$$= \frac{1}{2}\, 2\int_0^\infty e^{-t^2} \cdot t^{2\cdot\frac{1}{2}-1}\, dt$$ [2 লাগাতে তাই নিয়ে নিছি

$$= \frac{1}{2}\, \Gamma(1/2) \quad \Gamma\tfrac{1}{2}$$

$$= \frac{1}{2} \cdot \sqrt{\pi}$$

problem - 2 : find the value

$$\int_0^1 \frac{1}{\sqrt{1-x^3}}\, dx$$

$$= \int_0^1 (1-x^3)^{-\frac{1}{2}}\, dx$$ কাটার জন্য $\frac{1}{2}$

Let $x^3 = z \Rightarrow x = z^{\frac{1}{3}} \Rightarrow x^2 = z^{2/3}$

$$= dz = 3x^2\, dx$$

$$\frac{dz}{3 z^{2/3}} = dx$$ যত বার লাগবে নিবো, $x^2 = z^{2/3}$

$$= \int_0^1 (1-z)^{\frac{-1}{2}} \frac{dz}{3\, z^{2/3}}$$

Simplify

$$= -\frac{1}{3}\int_0^1 z^{\frac{1}{3}-1} (1-z)^{\frac{1}{2}-1}\, dz$$

1st β Function

$$= \frac{1}{3}\, \beta\left(\frac{1}{3}, \frac{1}{2}\right)$$

2nd

$$\frac{1}{3\sqrt{1-x^3}}\, dx$$

$$(1-x^3)^{-\frac{1}{3}}$$

$$\frac{1}{3}\int z^{\frac{1}{3}-1} (1-z)^{\frac{2}{3}-1}\, dz$$

$$= \frac{1}{3}\, \beta\left(\frac{1}{3}, \frac{2}{3}\right)$$

$$= \frac{1}{3} \frac{\Gamma\frac{1}{3}\, \Gamma\frac{2}{3}}{\Gamma\frac{1}{3}+\frac{2}{3}}$$

Formula

$$\Gamma n\, \Gamma 1-n = \frac{\pi}{\sin n\pi}$$

$$\frac{1}{3} \cdot \Gamma\frac{1}{3}\, \Gamma 1-\frac{1}{3}$$

Using Formula

$$= \frac{1}{3} \frac{\pi}{\sin \frac{1}{3}\pi} \qquad \left[\Gamma n \, \Gamma 1-n = \frac{\pi}{\sin n\pi} \right.$$

$$= \frac{1}{3} \cdot \frac{\pi}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2\pi}{3\sqrt{3}} \qquad \sin 60^\circ = \sqrt{3}/2$$

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3 এর (i)

P - 123

7 এর 11

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10 - এর (i) (ii)

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15 এর ii

17 এর ii

Assignment

Lecture-4

17.04.2025

Page-86 2.9

$$\beta(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy = \int_0^\infty \frac{y^{n-1}}{(1+y)^{n+m}} =$$

$$\int_0^\infty \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}}\,dy$$

We know,

$$\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1}\,dx$$

Replacing ~~m and~~ m/n by n and n by m $n/3$

$$B(n,m) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{n+m}}\,dy \quad \text{— (11)}$$

Since

Since β Function is a symmetric function that mean $\beta(m,n) = \beta(n,m)$

From eq^n (II) $\beta(n,m) = \int_0^1 \frac{y^{n-1}}{(1+y)^{n+m}}\,dy + \int_0^\infty \frac{y^{n-1}}{(1+y)^{n+m}}$

—— (III)

Now,

let $y = \frac{1}{z}$ $dy \Rightarrow dy = -\frac{1}{z^2}\,dz$ for 2nd Integral in (III)

y	1	∞
z	1	0

Only for (II) Integram

$$\int_{-1}^{\infty} \frac{y^{n-1}}{(1+y)^{n+m}}\,dy = \int_1^0 \frac{\left(\frac{1}{z}\right)^{n-1}}{\left(1+\frac{1}{z}\right)^{n+m}}\left(-\frac{1}{z^2}\right)dz$$

$$= + \int_0^1 \frac{1}{z^{n+1}} \left(\frac{1}{\frac{z+1}{z}} \right)^{n+m} dz$$

- লিমিট (-) হয় না
যার (-) কে
Int আগে (-) হল
তার আগের (-)কে
কে নিয়ে
গুণ হয়ে +1

$$= \int_0^1 \frac{z^{n+m-n+1-2}}{(1+z)^{n+m}} dz$$

$$= \int_0^1 \frac{z^{m-1}}{(1+z)^{n+m}} dz$$

Variable fact
না। শুধু
replace
করতে হবে।

$$= \int_0^1 \frac{y^{m-1}}{(1+y)^{n+m}} dy \qquad \boxed{z = y}$$

From (4) $\beta(n,m) = \int_0^1 \frac{y^{n-1}}{(1+y)^{n+m}} dy + \int_0^1 \frac{y^{m-1}}{(1+y)^{n+m}} dy$

$$\therefore \beta(n,m) = \int_0^1 \frac{y^{n-1} + y^{m-1}}{(1+y)^{n+m}} dy \quad \text{——— (IV)}$$

দুইটোকে Compare

$$\int_0^1 \frac{x^7 + x^9}{(1+x)^{18}}\,dx$$

$$= \int_0^1 \frac{x^{8-1} + x^{10-1}}{(1+x)^{8+10}}\,dx$$

$$= \beta(8,10)$$