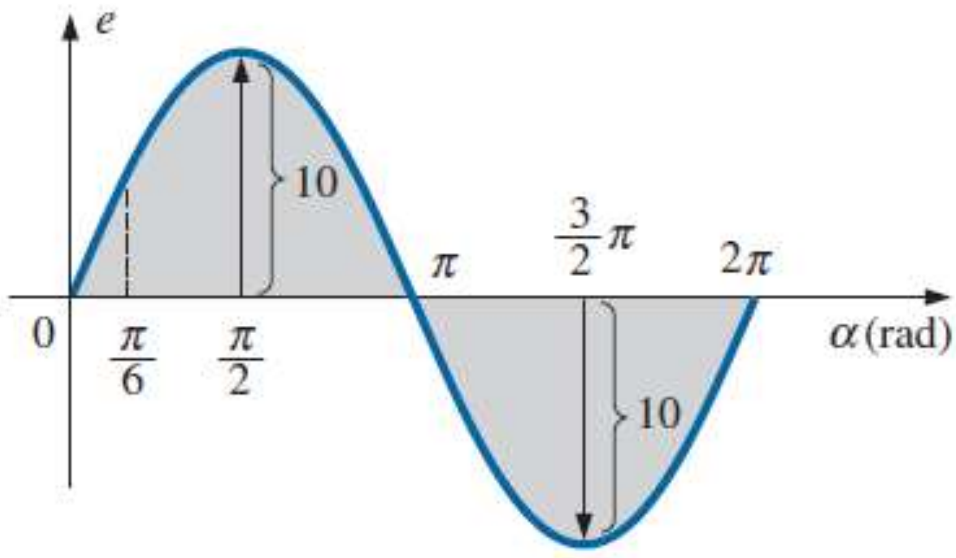
Alternating Voltage and Current

Phase Relations

- We have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0, π, and 2π, as shown in Fig.

$$\boxed{A_m \sin \omega t}$$

$$= 10 \sin \omega t$$



Phase Relations

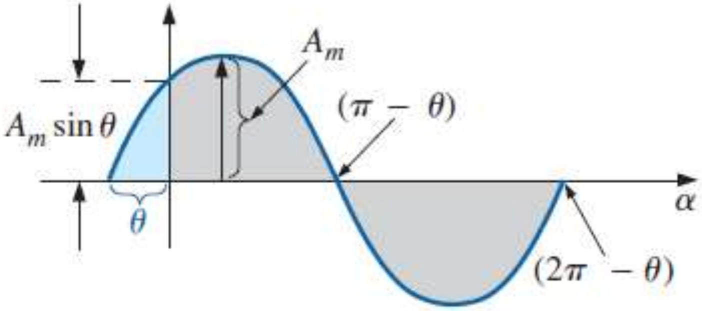
- **If the waveform is shifted to the right or left of *0°*, the expression becomes**

$$\boxed{A_m \sin(\omega t \pm \theta)}$$

Shifted angle in degrees or radians

For *positive-going* (increasing with time) slope *before* 0°, *as shown in Fig., the* expression is

$$\boxed{A_m \sin(\omega t + \theta)}$$

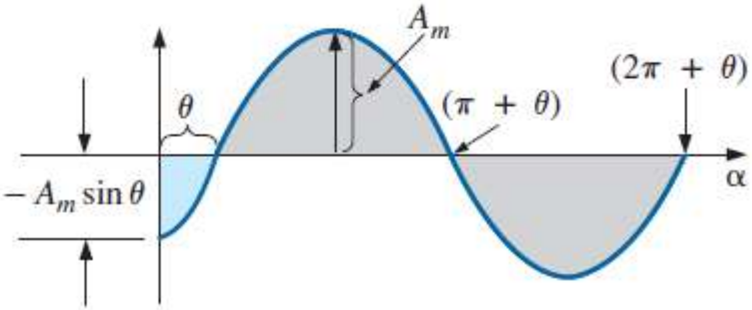


At $\omega t = \alpha = 0°$, the magnitude is determined by $A_m \sin\theta$.

Phase Relations

For positive-going (increasing with time) slope *after 0°, as shown in Fig., the* expression is

$$\boxed{A_m \sin(\omega t - \theta)}$$



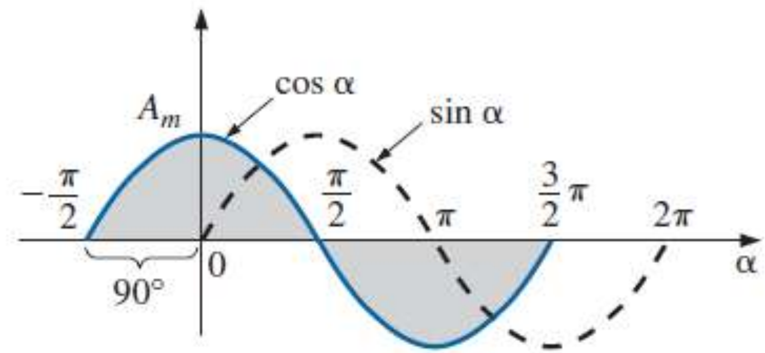
at $\omega t = \alpha = 0°$, the magnitude is $A_m \sin(-\theta)$ $-A_m \sin\theta$.

Phase Relations

If the waveform crosses the horizontal axis with a positive-going slope 90° (π/2) sooner, as shown in Fig., it is called a *cosine wave; that is,*

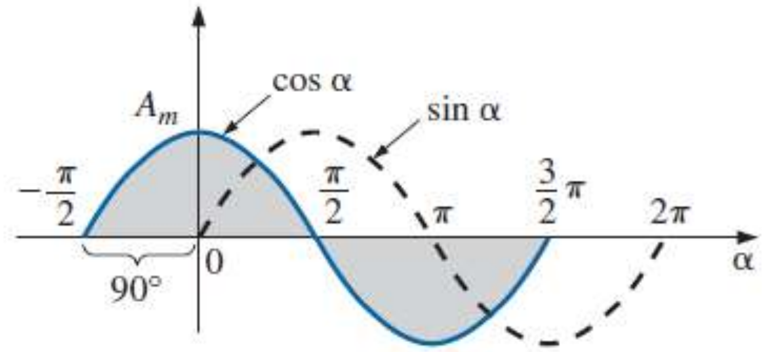
$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\sin \omega t = \cos(\omega t - 90^\circ) = \cos\left(\omega t - \frac{\pi}{2}\right)$$



Phase Relations: Leading and Lagging

The terms **leading and lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted *on the* same set of axes.



In Fig., the cosine curve is said to *lead the sine curve by 90°*, and the sine curve is said to *lag the cosine curve by 90°.*

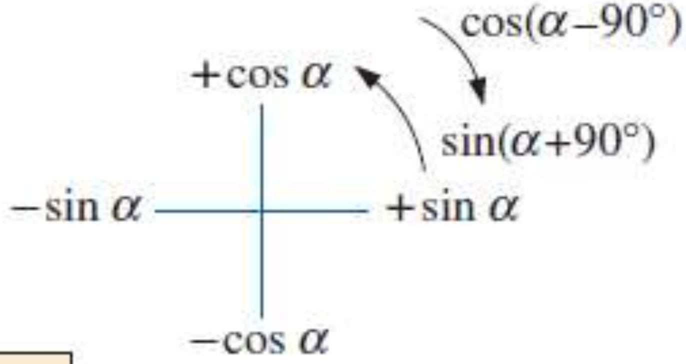
The 90° is referred to as the phase angle between the two waveforms.

The waveforms are *out of phase by 90°.*

If both waveforms cross the axis at the same point with the same slope, they are *in phase.*

Phase Relations: Leading and Lagging

The geometric relationship between various forms of the sine and cosine functions can be derived from Fig.



$$\begin{aligned}
\cos\alpha &= \sin(\alpha + 90^\circ)\\
\sin\alpha &= \cos(\alpha - 90^\circ)\\
-\sin\alpha &= \sin(\alpha \pm 180^\circ)\\
-\cos\alpha &= \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)
\end{aligned}$$

etc.

$$\begin{aligned}
\sin(-\alpha) &= -\sin\alpha\\
\cos(-\alpha) &= \cos\alpha
\end{aligned}$$

Phase Relations: Leading and Lagging

$$e = -E_m \sin \omega t$$

The negative sign is associated in the above Eq. with the sine portion of the expression, not the peak value E_m. *In other words,*

$$e = E_m(-\sin \omega t)$$

Since $$-\sin \omega t = \sin(\omega t \pm 180°)$$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180°)$$

$$e = -E_m \sin \omega t = E_m \sin(\omega t + 180°) = E_m \sin(\omega t - 180°)$$

Phase Relations: Problem

Problem: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30^\circ)$
$i = 5 \sin(\omega t + 70^\circ)$

b. $i = 15 \sin(\omega t + 60^\circ)$
$v = 10 \sin(\omega t - 20^\circ)$

c. $i = 2 \cos(\omega t + 10^\circ)$
$v = 3 \sin(\omega t - 10^\circ)$

d. $i = -\sin(\omega t + 30^\circ)$
$v = 2 \sin(\omega t + 10^\circ)$

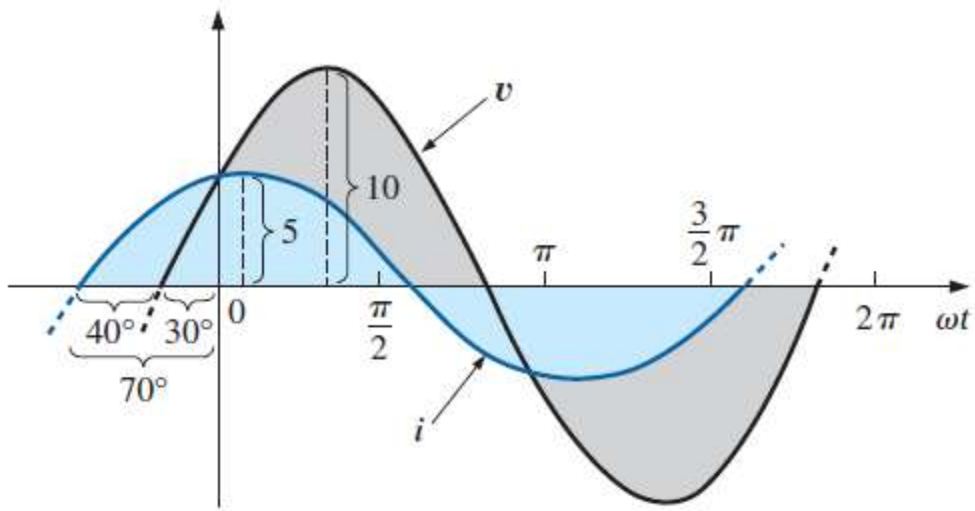
e. $i = -2 \cos(\omega t - 60^\circ)$
$v = 3 \sin(\omega t - 150^\circ)$

Phase Relations: Problem

Problem: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
$\quad i = 5 \sin(\omega t + 70°)$

Solution:



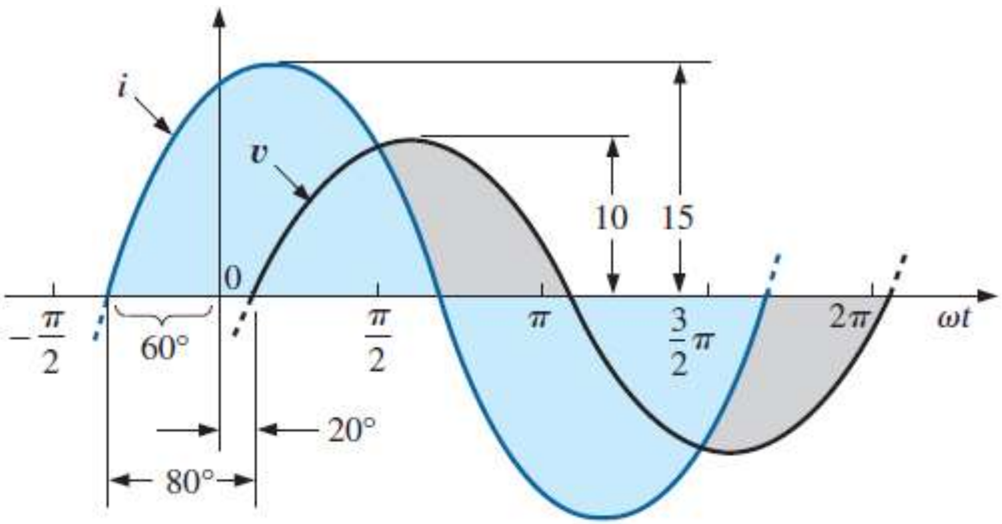
i leads v by 40°, or v lags i by 40°.

Phase Relations: Problem

Problem: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

b. $i = 15 \sin(\omega t + 60^\circ)$
$v = 10 \sin(\omega t - 20^\circ)$

Solution:



i leads v by 80º, or v lags i by 80º.

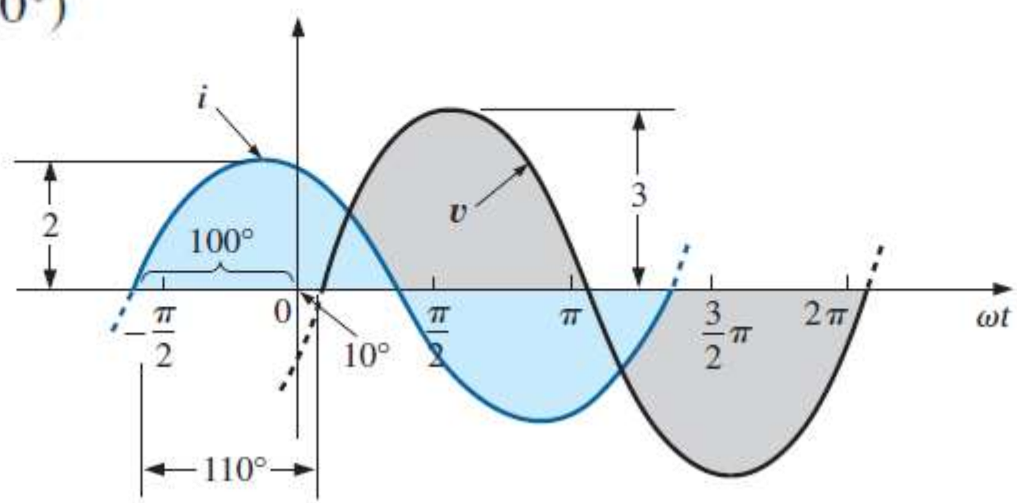
Phase Relations: Problem

Problem: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

c. $i = 2\cos(\omega t + 10°)$
$v = 3\sin(\omega t - 10°)$

Solution:

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$



***i* leads *v* by 110°, or *v* lags *i* by 110°.**

Phase Relations: Problem

Problem: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

$$\text{d. } i = -\sin(\omega t + 30^\circ)$$
$$v = 2\sin(\omega t + 10^\circ)$$

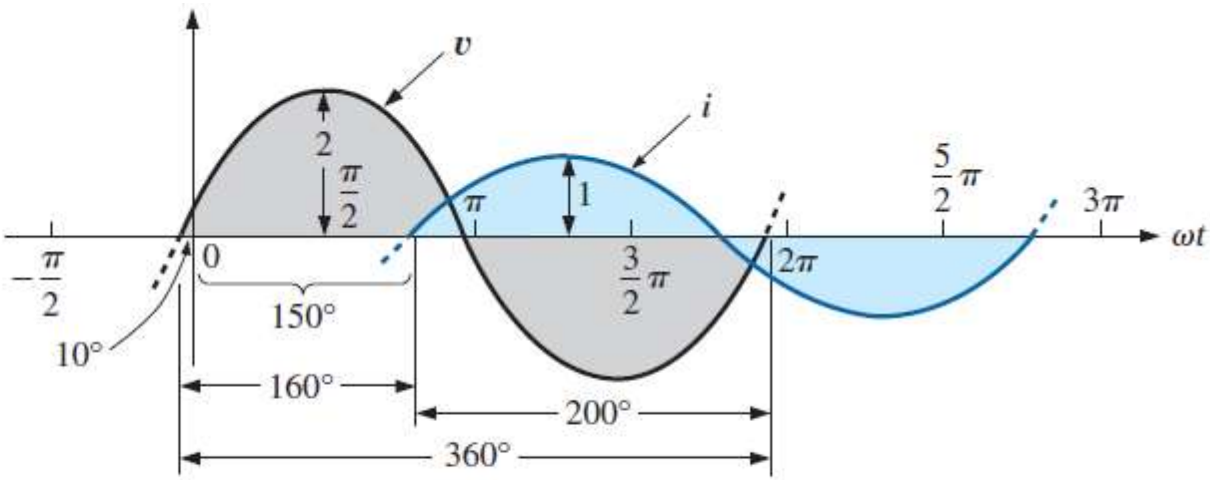
Solution:

$$-\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ - 180^\circ) = \sin(\omega t - 150^\circ)$$

Or using

Note

$$-\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ + 180^\circ) = \sin(\omega t + 210^\circ)$$



***v* leads *i* by 160°, or *i* lags *v* by 160°.**

***i* leads *v* by 200°, or *v* lags *i* by 200°.**

Phase Relations: Problem

Problem: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

$$\text{e.}\quad i = -2\cos(\omega t - 60^\circ)$$
$$v = 3\sin(\omega t - 150^\circ)$$

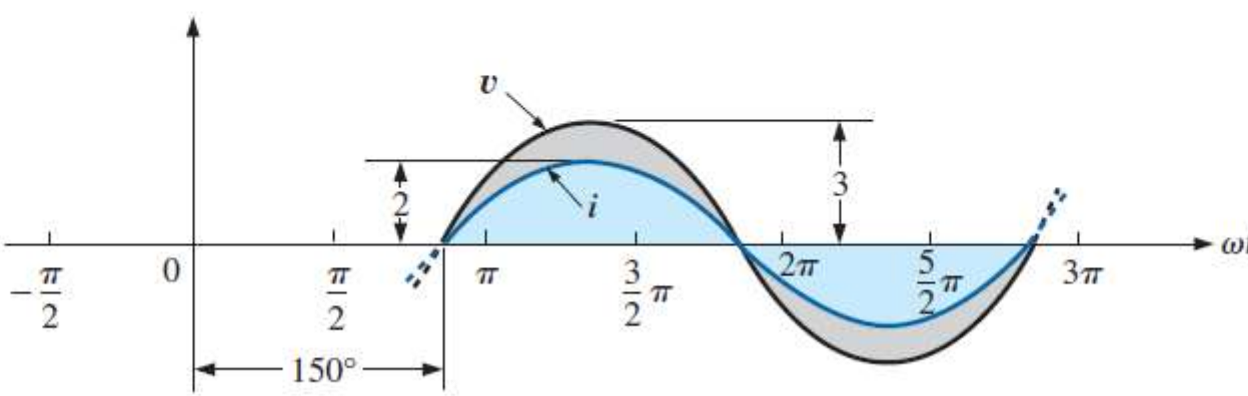
Solution:

$$i = -2\cos(\omega t - 60^\circ) = 2\cos(\omega t - 60^\circ \overset{\text{By choice}}{-} 180^\circ)$$
$$= 2\cos(\omega t - 240^\circ)$$

However, $\cos\alpha = \sin(\alpha + 90^\circ)$

so that $2\cos(\omega t - 240^\circ) = 2\sin(\omega t - 240^\circ + 90^\circ)$
$$= 2\sin(\omega t - 150^\circ)$$

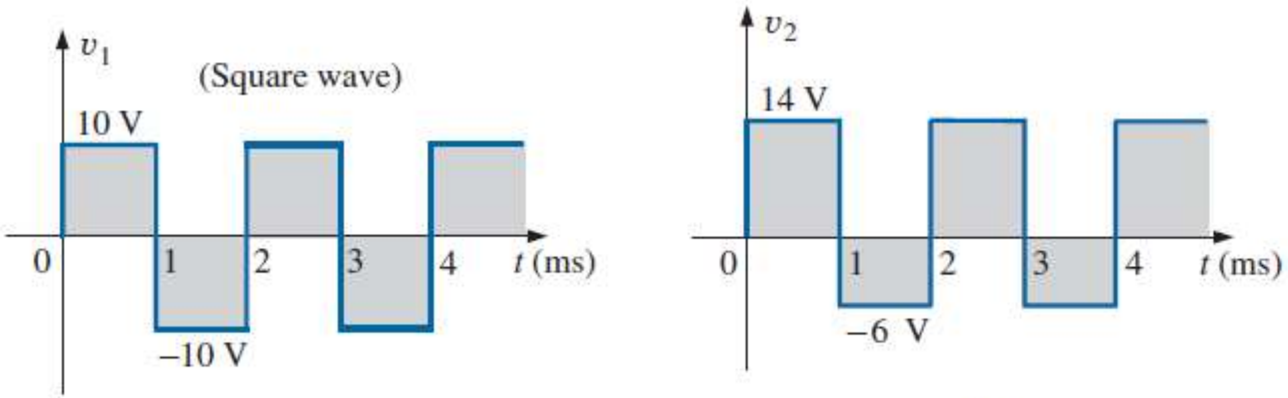
v and i are in phase.



Average Value

$$Average\,Value\,Under\,the\,Curve = \frac{Area\,Under\,the\,Curve}{Lenght\,of\,the\,Curve}$$

Determine the average value of the waveforms in Fig.



(a) (b)

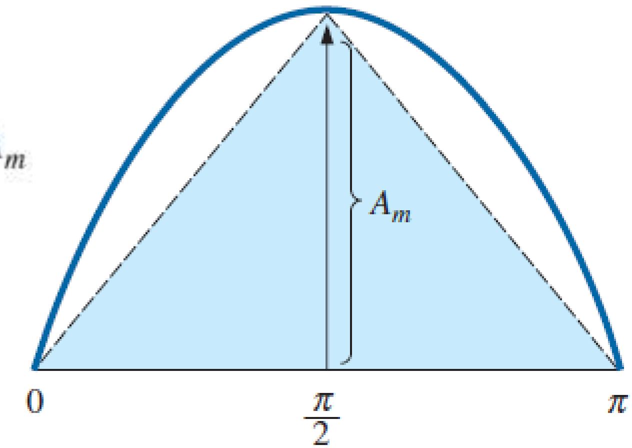
$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}}$$

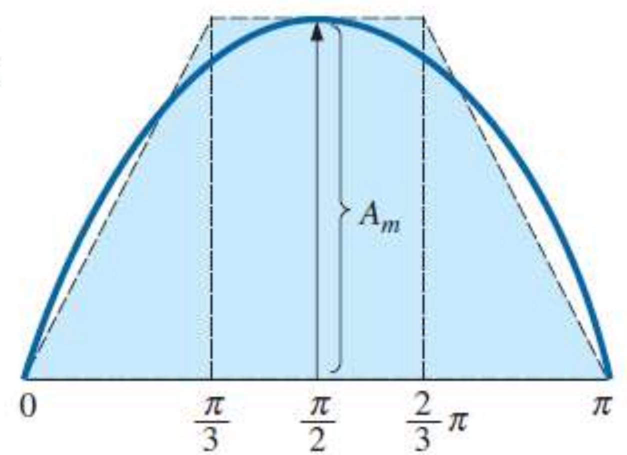
$$= \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

Average Value

$$\text{Area shaded} = 2\left(\frac{1}{2}bh\right) = 2\left[\left(\frac{1}{2}\right)\overbrace{\left(\frac{\pi}{2}\right)}^{b}\overbrace{(A_m)}^{h}\right] = \frac{\pi}{2}A_m \cong 1.58A_m$$



$$\text{Area} = A_m\frac{\pi}{3} + 2\left(\frac{1}{2}bh\right) = A_m\frac{\pi}{3} + \frac{\pi}{3}A_m = \frac{2}{3}\pi A_m = 2.094A_m$$



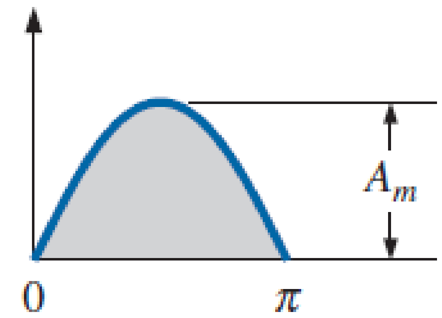
Average Value

$$\text{Area} = \int_0^{\pi} A_m \sin\alpha \, d\alpha$$

$$\text{Area} = A_m[-\cos\alpha]_0^{\pi}$$

$$= -A_m(\cos\pi - \cos 0^\circ)$$

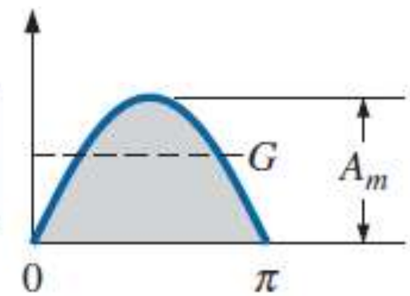
$$= -A_m[-1 - (+1)] = -A_m(-2)$$



$$\boxed{\text{Area} = 2A_m}$$

Average, $G = \dfrac{2A_m}{\pi}$

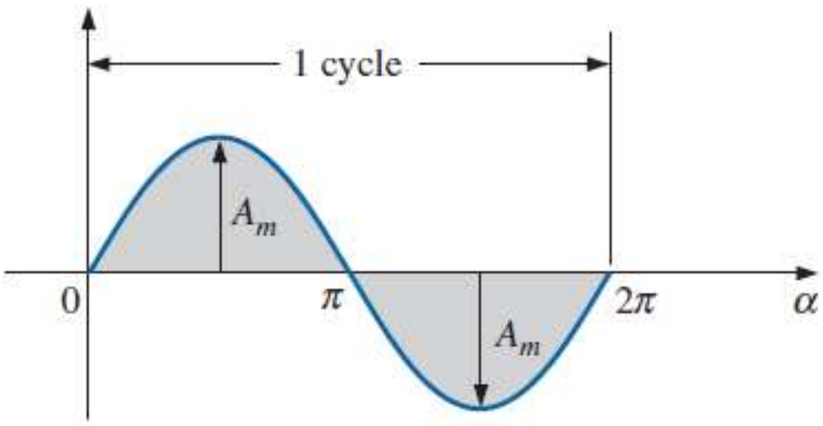
$$\boxed{G = \frac{2A_m}{\pi} = 0.637A_m}$$



Average Value

Determine the average value of the sinusoidal waveform in Fig.

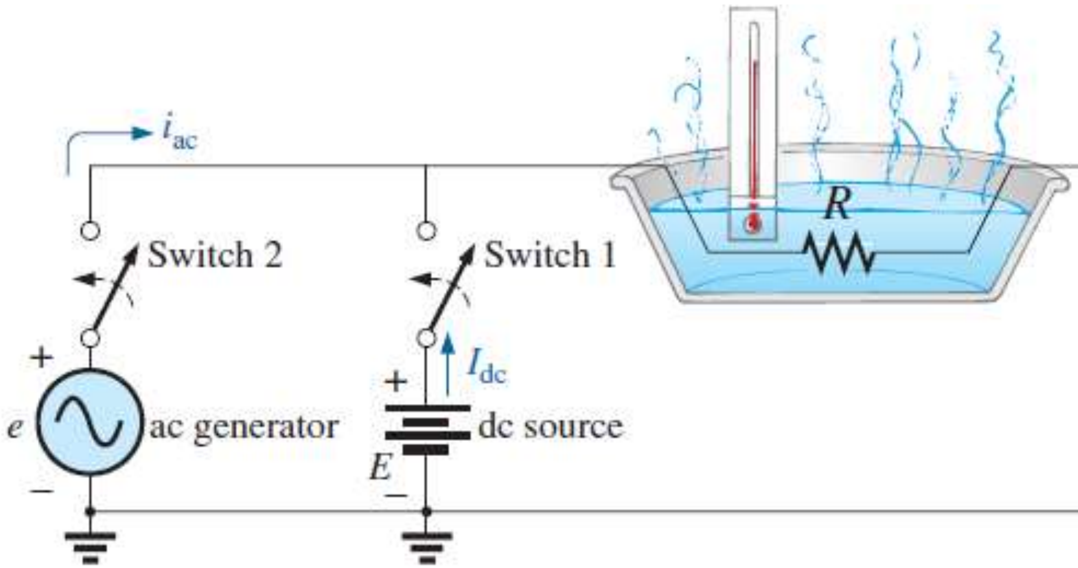
$$G = \frac{+2A_m - 2A_m}{2\pi} = 0 \text{ V}$$



The average value of a pure sinusoidal waveform over one full cycle is zero.

Effective (rms) Values

The rms value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when following through the same circuit for the same time.

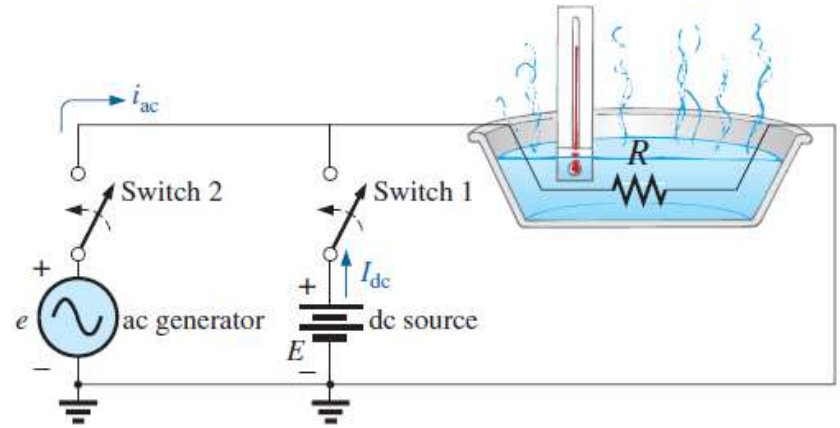


Effective (rms) Values

Relationship of *dc* and *ac* quantities with respect to the power delivered to a load.

The power delivered by the ac supply at any instant of time is

$$P_{ac} = (i_{ac})^2 R = (I_m \sin \omega t)^2 R = (I_m^2 \sin^2 \omega t) R$$



However, $$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$P_{ac} = I_m^2 \left[\frac{1}{2}(1 - \cos 2\omega t)\right] R$$

Effective (rms) Values

$$P_{ac} = I_m^2\left[\frac{1}{2}(1 - \cos 2\omega t)\right]R$$

$$\boxed{P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2}\cos 2\omega t}$$

The *average power delivered by the ac source is just the first term,* since the average value of a cosine wave is zero.

$$P_{av(ac)} = P_{dc} \Rightarrow \frac{I_m^2 R}{2} = I_{dc}^2 R \Rightarrow I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

The equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its peak value.

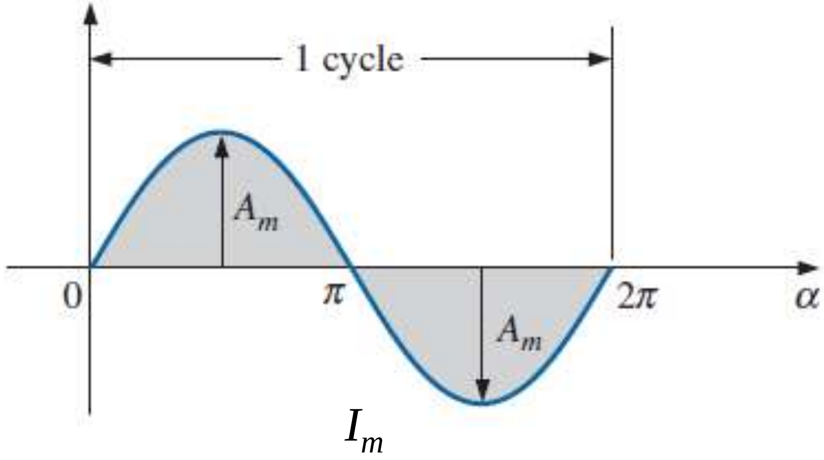
The equivalent dc value is called the ***rms or effective value*** **of the sinusoidal** quantity.

Analytical Method for Determining rms Value

The mean of square of the instantaneous values of current over one complete cycle is:

$$mean\,of\; i^2 = \frac{area\,of\; i^2}{2\pi}$$

$$= \frac{\int_0^{2\pi} i^2 d\alpha}{2\pi}$$



$$rms\,value\,of\; i = \sqrt{\frac{\int_0^{2\pi} i^2 d\alpha}{2\pi}}$$

$$= \sqrt{\frac{\int_0^{2\pi} (I_m \sin \alpha)^2 \, d\alpha}{2\pi}} = \sqrt{\frac{I_m^2}{2\pi}\left(\int_0^{2\pi} \sin^2 \alpha \, d\alpha\right)}$$

Analytical Method for Determining rms Value

$$I_{rms} = \sqrt{\frac{I_m^2}{2\pi}\left(\int_0^{2\pi} \frac{(1-\cos 2\alpha)}{2}\, d\alpha\right)} = \sqrt{\frac{I_m^2}{4\pi}\left(\int_0^{2\pi} (1-\cos 2\alpha)\, d\alpha\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left(\left[\alpha - \frac{\sin 2\alpha}{2}\right]_0^{2\pi}\right)} = \sqrt{\frac{I_m^2}{4\pi}\left((2\pi - \frac{\sin 4\pi}{2}) - (0 - \frac{\sin 0}{2})\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \times 2\pi} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

HOME WORK

PROBLEM : Find the Average value and RMS value of the following Signal and also plot the following signals on the same plot.

A. $v_1 (t) = 10 \sin (200t + 0)$
B. $v_2(t) = 20 \sin (200t + 180)$
C. $v_3(t) = 30 \sin (200t - 90)$

Reference: *PPTX Collected ,* ***Prof. Dr. Dipankar Das***