Interference

Interference of Light

When two or more light waves propagate in a medium simultaneously, they superimpose each other. As a result, redistribution of energy takes place. This phenomenon is called interference. Thus, interference of light is the phenomenon which takes place two or more waves superimpose each other resulting in the redistribution of light energy.

Constructive Interference

The points at which the waves meet in phase, they reinforce each other thereby increasing the amplitude and intensity of the resultant wave. This is called constructive interference. Thus, at the points of constructive interference, the waves meet in the same phase or at a phase difference of $2n\pi$ $(n = 0, 1, 2, \cdots\cdots)$.

In terms of path difference, path difference for constructive interference

$$= 2n \cdot \frac{\lambda}{2} \qquad n = 0, 1, 2, \cdots\cdots$$

That is, for the constructive interference, the path difference should be even multiple of $\frac{\lambda}{2}$. The points of constructive interference are the points of maximum brightness or maxima.

Destructive interference

The points at which the waves meet in opposite phase, they cancel each other. This is called destructive interference. Thus, at the points of destructive interference, the waves meet in opposite phase or at a phase difference of $(2n + 1)\pi$ $(n = 0, 1, 2, \cdots\cdots)$.

In terms of path difference, path difference for destructive interference

$$= (2n + 1) \cdot \frac{\lambda}{2} \qquad n = 0, 1, 2, \cdots\cdots$$

That is, for the destructive interference, the path difference should be an odd multiple of $\frac{\lambda}{2}$. These points are the points of minimum brightness or darkness or minima.

Monochromatic Light

The visible light is a continuous spectrum which consist a large number of wavelengths (approximately 3500Å to 7800 Å). Every single wavelength (or frequency) of this continuous spectrum is called monochromatic light.

Ordinary light or white light, coming from sun, electric bulb, LED etc. consists a large number of wavelengths and hence non-monochromatic. But some specific sources like sodium lamp and helium neon laser emit monochromatic lights with wavelengths 589.3 nm and 632.8 nm respectively.

Plane Wave

A plane wave is a wave whose wavefront remains in a plane during the propagation of wave. In light wave, the maximum amplitude of electric vector remains constant and confined in a plane perpendicular to direction of propagation. Such type of wave called plane wave.

Unpolarized Light

Light coming from many sources like sun, flame, incandescent lamp produce unpolarized light in which electric vector are oriented in all possible directions perpendicular to direction of propagation.

Polarized Light

In polarized light, electric vector are confined to only a single direction.

Phase Difference

If there are two waves have some frequency then the phase difference is the angle (or time) after which the one wave achieves the same position and phase as of first wave. In the figure-1, two waves with phase difference θ are shown.

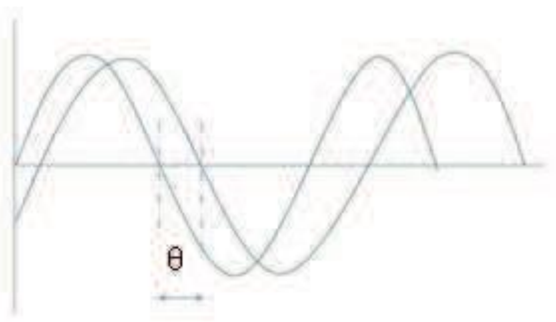


Figure-1

Coherence

If two or more waves of same frequencies are in same phase or have constant phase difference, those waves are called coherent wave. Figure-1 shows coherent wave with same phase (zero phase difference) and with constant phase difference.

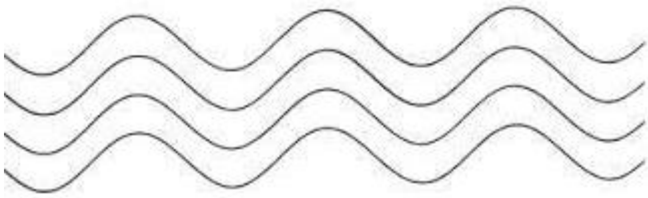


Figure-1

Wavefront

The locus of all the particles in the medium which are disturbed at the same instant of time and are in the same phase or same state of vibration is called a wavefront.

If the distance of the source is small [Fig.-1], the wavefront is spherical. When the source is at a large distance, then any small portion of the wavefront can be considered plane [Fig.-2]. Thus, rays of light diverging from or converging to a point give rise to a spherical wavefront and a parallel beam of light gives rise to a plane wavefront.

Huygens' Principle

According to Huygens, a source of light sends out waves in all directions, through a hypothetical medium called ether.

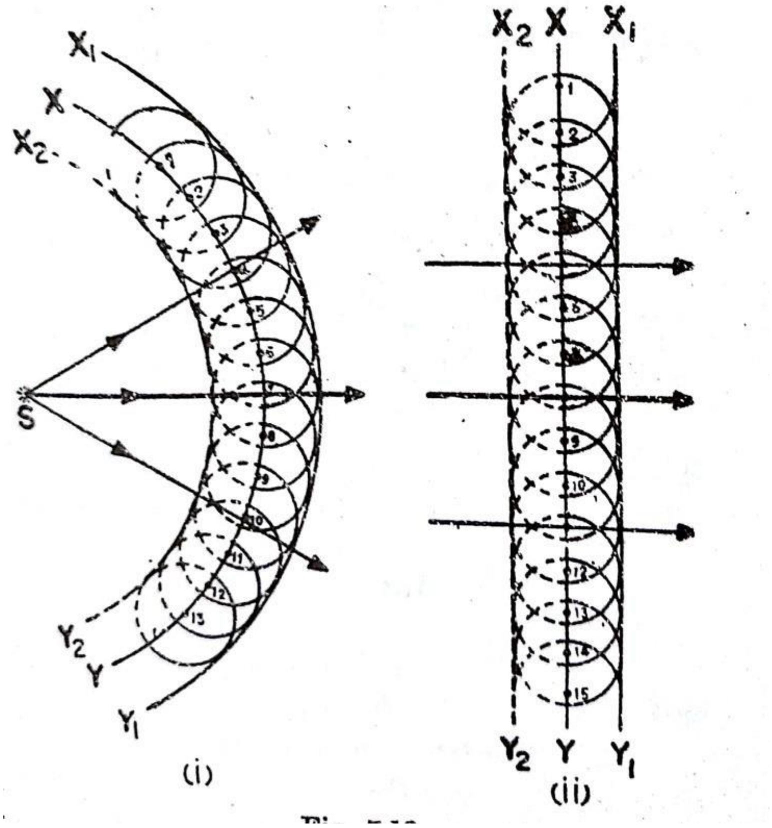


Figure-1

In Figure-1 (i), S is a source of light sending light energy in the form of waves in all directions. After any given interval of time (t), all the particles of the medium on the surface XY will be vibrating in phase. Thus, XY is a portion of the sphere of radius vt and centre S. v is the velocity of propagation of the wave. XY is called the primary wavefront.

According to Huygens' principle, all points on the primary wavefront (1, 2, 3,…etc.) are sources of secondary disturbance. These secondary waves travel through space with the same velocity as the original wave and the envelope of all the secondary wavelets after any given interval of time gives rise to the secondary wavefront. In Fig.-1(i), XY is the primary spherical wavefront and in In Fig.-1(ii), XY is the primary plane wavefront. After an interval of time t' the secondary waves travel a distance vt'. With the points 1,2,3…etc. as centres, spheres of radii vt' are drawn. The surfaces X_1Y_1 and X_2Y_2 refer to the secondary wavefront. X_1Y_1 is the forward wavefront and X_2Y_2 is the backward wavefront. But according to Huygens' principle, the secondary wavefront is confined only to the forward wavefront X_1Y_1 and not the backward wavefront X_2Y_2.

Interference phenomena

The phenomenon of interference of light is due to the superposition of two trains within the region of cross over. Let us consider the waves produced on the surface of water.

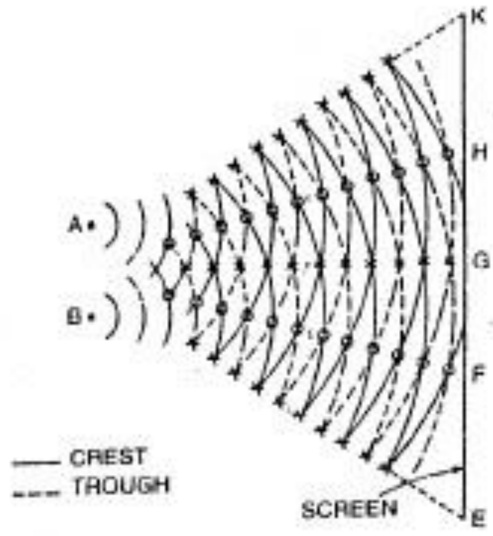


Figure-1

In Fig.-1 points A and B are the two sources which produce waves of equal amplitude and constant phase difference. Waves spread out on the surface of water which are circular in shape. At any instant, the particle will be under the action of the displacement due to both the waves. The points shown by circles in the diagram will have minimum displacement because the crest of one wave falls on the trough of the other and the resultant displacement is zero. The points shown by crosses in the diagram will have maximum displacement because, either the crest of one will combine with the crest of the other or the trough of one will combine with the trough of the other. In such a case, the amplitude of the displacement is twice the amplitude of either of the waves. Therefore, at these points the waves reinforce with each other. As the intensity (energy) is directly proportional to the square of the amplitude ($I \propto A^2$) the intensity ar these points is four times the intensity due to one wave. It should be remembered that there is no loss of energy due to interference. The energy is only transferred from the points of minimum displacement to the points of maximum displacement.

Young's Experiment

In 1802, Thomas Young performed double slit experiment in which a light first entered through a pinhole, then again divided into two pinholes and finally brought to superimpose on each other and obtained interferences. He allowed sunlight to fall on a pinhole *S* and the at some distance away on two pinholes *A* and *B* (Fig.-1).

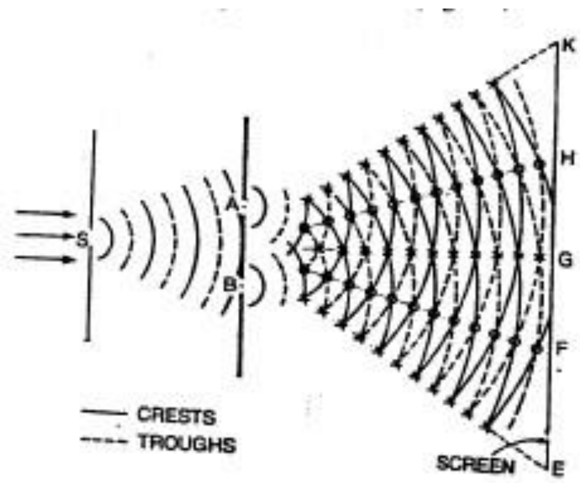


Figure-1

A and *B* are equidistant from *S* and are close to each other. Spherical waves spread out from *S*. Spherical waves also spread out from *A* and *B*. These waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright. The points such as *E* are bright because the crest due to one wave coincides with the crest due to the other and therefore, they reinforce with each other. The points such as *F* are dark because the crest of one falls on the trough of the other and they neutralize the effect of each other. Points, similar to *E*, where the trough of one falls on the trough of the other, are also bright because the two waves reinforce.

It is not possible to show interference due to two independent sources of light, because a large number of difficulties are involved. The two sources may emit light waves of largely different amplitude and wavelength and the phase difference between the two may change with time.

Principle of Superposition

When two or more waves travelling through a medium superimpose each other, they lose their individual identity and a new wave is formed, whose amplitude is determined using the superposition principle.

According to the principle of superposition,

"*The resultant amplitude at a point at any instant of time is the algebraic sum of the amplitudes due to the individual waves.*"

That is, if A is the resultant amplitude and A_1, A_2, A_3,······ are the amplitudes due to the individual waves, then

$$A = A_1 \pm A_2 \pm A_3 \pm \cdots \cdots (1)$$

The +ve sign is taken when amplitudes of the waves are in the same direction and -ve sign when they are in opposite direction. The superposition principle is illustrated in Fig.-1. The resultant intensity is the square of the resultant amplitude.

$$I = A^2 \cdots \cdots (2)$$

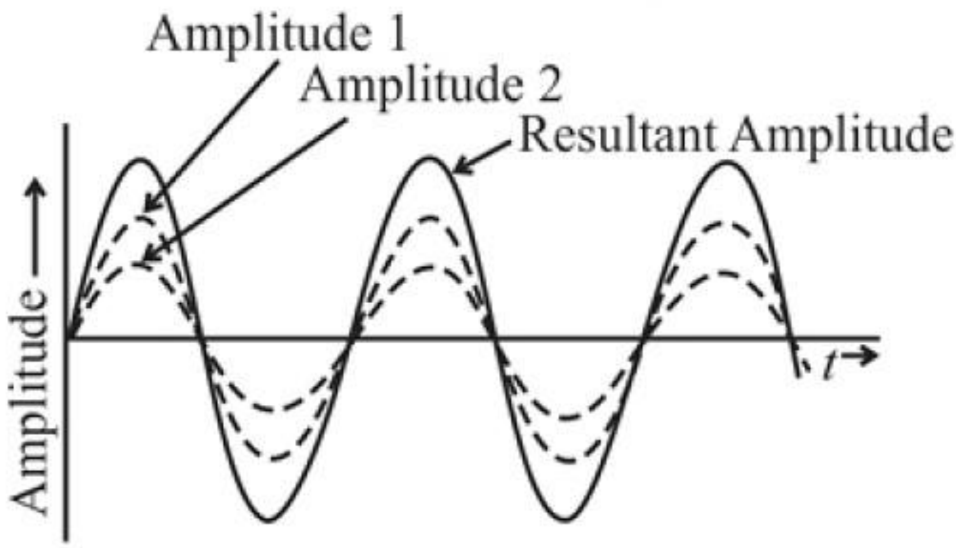


Figure-1: Superposition of wave of the same phase and frequency.

Conditions for Permanent Interference of Light

To obtain a sustained interference pattern, the following conditions must be satisfied.

1. The two sources of light should be coherent.
2. The two sources should emit light waves continuously of same wavelength and time period or frequency. That is, the two sources should be monochromatic.
3. The separation between the two sources should be small.
4. The separation between the sources and the screen should be large.
5. The amplitudes of the interfering waves should be equal or nearly equal.
6. The two sources should be extremely narrow.
7. The two interfering waves should meet at a small angle.

Relationship Between Phase Difference and Path Difference

If the path difference between the two waves is λ, the phase difference $= 2\pi$.

Suppose for a path difference x, the phase difference is δ.

For a path difference λ, the phase difference $= 2\pi$

$\therefore$ For a path difference x, the phase difference $= \frac{2\pi x}{\lambda}$

$\therefore$ Phase difference, $\delta = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \text{path difference}$

Production of Coherent Sources

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour (wavelength). In actual practice, it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources. There are two methods of obtaining these sources.

1. **Division of Wavefront:** In this method, a wavefront is divided into two or more parts by reflection, refraction or diffraction. These portions then meet at a small angle to undergo interference and produce an interference pattern. This method is employed in Young's doub1e slit experiment, Fresnel's biprism, Llyod's mirror, Prism etc.

2. **Division of Amplitude:** In this method, the amplitude of a beam is divided into two or more parts by a combination of reflection and refraction. These divided parts then reunite to undergo interference, after traversing different paths. In such cases, extended broad source of light is used so that bright fringes are obtained. This method is used in thin film devices, Newton’s rings, Michelson interferometer, etc.

Conditions for Constructive and Destructive Interference

Let the two coherent light waves undergoing interference be represented by

$$y_1 = asin\omega t$$

and $$y_2 = asin(\omega t + \delta)$$

where, a and b are their respective amplitudes and δ is the phase difference between the two waves.

Applying the principle of superposition, the resultant displacement y is given by

$$y = y_1 + y_2$$

$$= asin\omega t + asin(\omega t + \delta)$$

$$= asin\omega t + asin\omega t\, cos\delta + acos\omega t\, sin\delta$$

$$= asin\omega t(1 + cos\delta) + cos\omega t \cdot asin\delta$$

Putting $a(1 + cos\delta) = Rcos\theta \cdots\cdots\cdots (1)$

and $asin\delta = Rsin\theta \cdots\cdots\cdots\cdots (2)$

$$y = Rsin\omega tcos\theta + Rcos\omega tsin\theta$$

$$\therefore\ y = Rsin(\omega t + \theta)\ \cdots\cdots\cdots\cdots (3)$$

which represents the equation of simple harmonic vibration of amplitude R.

Squaring (1) and (2) and adding

$$R^2sin^2\theta + R^2cos^2\theta = a^2sin^2\delta + a^2(1 + cos\delta)^2$$

or, $$R^2 = a^2sin^2\delta + a^2(1 + cos^2\delta + 2\ cos\delta)$$

$$= a^2sin^2\delta + a^2 + a^2cos^2\delta + 2a^2\ cos\delta$$

$$= 2a^2 + 2a^2cos\delta$$

$$= 2a^2(1 + cos\delta)$$

$$= 2a^2 \cdot 2cos^2\frac{\delta}{2}$$

$$\therefore R^2 = 4a^2cos^2\frac{\delta}{2} \cdots\cdots\cdots\cdots (4)$$

The intensity at a point is given by the square of the amplitude

$$I = R^2$$

or, $$I = 4a^2cos^2\frac{\delta}{2}$$

Special Cases:

1. When the phase difference $\delta = 0, 2\pi, 2(2\pi), \cdots\cdots n(2\pi)$, or the path difference $x = 0$, $\lambda, 2\lambda, \cdots\cdots\cdots n\lambda$.

$$I = 4a^2$$

Intensity is maximum when the phase difference is a whole number multiple of 2π or the path difference is a whole number multiple of wavelength.

2. When the phase difference $\delta = 0, 3\pi, \cdots\cdots (2n+1)\pi$, or the path difference $x = \frac{\lambda}{2}$, $\frac{3\lambda}{2}, \frac{5\lambda}{2}, \cdots\cdots\cdots (2n+1)\frac{\lambda}{2}$.

$$I = 0$$

Intensity is minimum when the path difference is an odd number multiple of half wavelength.

3. **Energy distribution:** From equation (4), it is found that the intensity at bright points is $4a^2$ and at dark points it is zero.

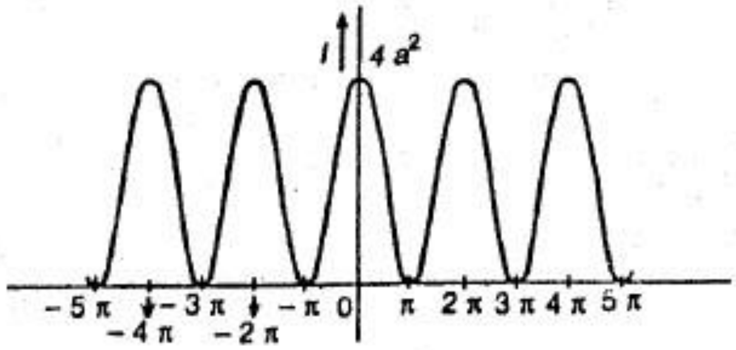


Figure-1: Energy Distribution

According to the law of conservation of energy, the energy cannot be destroyed. Here also the energy is not destroyed but only transferred from the points of minimum intensity to the points of maximum intensity. It varies from zero to $4a^2$ and the average value is $2a^2$ as shown. If net intensity is taken due to the two waves without interference, then it turns out be proportional to $2a^2$ $(a^2 + a^2 = 2a^2)$. Thus, interference phenomenon is in accordance with law of conservation of energy.

Young's Double Slit Experiment

Consider a narrow monochromatic source *S* and two pinholes A and B, equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen be placed at a distance D from the coherent sources. The point C on the screen is equidistant from A and B. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity.

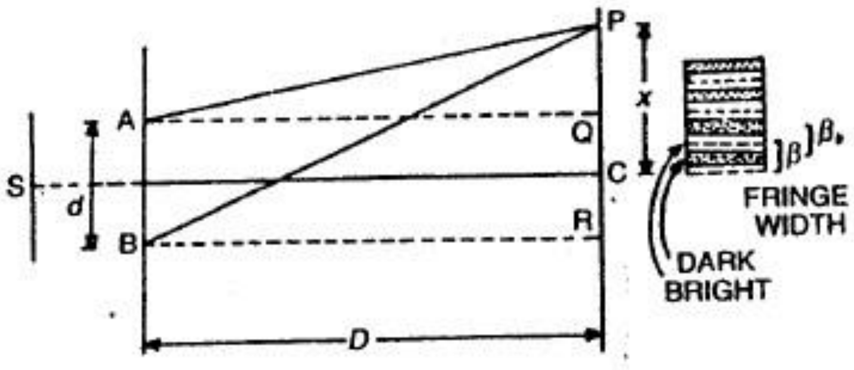


Figure-1: Young's Double Slit Experiment

Consider a point P at a distance x from C. The waves reach at the point P from A and B.

Here, $PQ = x - \frac{d}{2}$, $PR = x + \frac{d}{2}$

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$(BP)^2 - (AP)^2 = 2xd$$

$$BP - AP = \frac{2xd}{BP+AP}$$

But, BP = AP = D (approximately)

∴ Path difference = $BP - AP$

$$= \frac{2xd}{2D}$$

$$= \frac{xd}{D} \quad \cdots\cdots\cdots (1)$$

Phase difference = $\frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) \cdots\cdots\cdots (2)$

1. **Bright Fringes:** If the path difference is a whole number multiple of wavelength λ, the point P is bright.

$$\therefore \quad \frac{xd}{D} = n\lambda \qquad \text{where, } n = 0, 1, 2, \cdots$$

$$\text{or} \quad x = \frac{n\lambda D}{d} \cdots\cdots\cdots (3)$$

This equation gives the distances of the bright fringes from the point C. At C, the path difference is zero and a bright fringe is formed.

When n = 1, $x_1 = \frac{\lambda D}{d}$

n = 2 $x_2 = \frac{2\lambda D}{d}$

n = 3, $x_3 = \frac{3\lambda D}{d}$

$x_n = \frac{n\lambda D}{d}$

Therefore, the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$= \frac{\lambda D}{d} \cdots\cdots\cdots (4)$$

2. **Dark Fringes:** If the path difference is an odd number multiple of half wavelength, the point P is dark.

$$\therefore \quad \frac{xd}{D} = (2n + 1)\frac{\lambda}{2} \qquad \text{where, } n = 0, 1, 2, \cdots$$

$$\text{or} \quad x = \frac{(2n+1)\lambda D}{2d} \cdots\cdots\cdots (5)$$

This equation gives the distances of the dark fringes from the point C.

When n = 0, $x_0 = \frac{\lambda D}{2d}$

n = 1 $x_1 = \frac{3\lambda D}{2d}$

n = 2, $x_2 = \frac{5\lambda D}{2d}$

$x_n = \frac{(2n+1)\lambda D}{2d}$

Therefore, the distance between any two consecutive dark fringes

$$x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d}$$

$$= \frac{\lambda D}{d} \cdots\cdots\cdots (6)$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Therefore, alternatively bright and dark parallel fringes are formed. The fringes are formed on both sides of C. Moreover, from equations (5) and (6), it is clear that the width of the bright fringe is equal to the width of the dark fringe.

The fringe width, $\beta = \frac{\lambda D}{d}$.

Therefore,

i. The width of the fringe is directly proportional to the wavelength of light, $\beta \propto \lambda$.
ii. The width of the fringe is directly proportional to the distance of the screen from the two sources, $\beta \propto D$.
iii. The width of the fringe is inversely proportional to the distance between the two sources, $\beta \propto \frac{1}{d}$.

Math

1. Two coherent sources are 0.18 mm apart and the fringes are observed on a screen 80 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of l0.8 mm from the central fringe. Calculate the wavelength of light.

Solution:

Here,

D = 80 cm

d = 0.18 mm

= 0.018 cm

n = 4

x = 10.8 mm

= 1.08 cm

λ =?

We know,

$$x = \frac{n\lambda D}{d}$$

or, $$\lambda = \frac{xd}{nD}$$

$$= \frac{1.08 \times 0.018}{4 \times 80}$$

$$= 6075 \times 10^{-8} \text{ cm}$$

$$= 6075 \text{ Å}$$

2. Two straight and narrow parallel slits 1 mm apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of 100 cm from the slits are 0.5 mm apart. What is the wavelength of light?

Solution:

Here,

β = 0.5 mm

= 0.05 cm

d = 1 mm

= 0.1 cm

D = 100 cm

λ =?

We know,

$$\beta = \frac{\lambda D}{d}$$

$$\therefore \quad \lambda = \frac{\beta d}{D}$$

$$= \frac{0.05 \times 0.1}{100}$$

$$= 5 \times 10^{-5} \text{ cm}$$

= 5000 Å

3. Two coherent sources of monochromatic light of wavelength 6000 Å produce an interference pattern on a screen kept at a distance of 1 m from them. The distance between two consecutive bright fringes on the screen is 0.5 mm. Find the distance between the two coherent sources.

Solution:

Here,

$\lambda = 6000$ Å $= 6 \times 10^{-7}$

D = 1 m

$\beta = 0.5$ mm $= 5 \times 10^{-4}$ m

$d = ?$

We know,

$$\beta = \frac{\lambda D}{d}$$

$$\therefore \quad d = \frac{\lambda D}{\beta}$$

$$= \frac{6 \times 10^{-7} \times 1}{5 \times 10^{-4}}$$

$$= 1.2 \times 10^{-3} \text{ m}$$

$$= 1.2 \text{ mm}$$

4. Light of wavelength 5500 Å from a narrow slit is incident on a double slit. The overall separation of 5 fringes on a screen 200 cm away is 1 cm, calculate (a) the slit separation and (b) the fringe width.

Solution:

Here,

$n = 5$

$D = 200$ cm

$= 2$ m

$\lambda = 5500$ Å

$= 5.5 \times 10^{-7}$ m

$x = 1$ cm

$= 10^{-2}$ m

$d = ?$

a) We know,

$$x = \frac{n\lambda D}{d}$$

$$d = \frac{n\lambda D}{x}$$

$$= \frac{5 \times 5.5 \times 10^{-7} \times 2}{10^{-2}}$$

$= 5.5 \times 10^{-4}$ m

$= 0.055$ cm

b) We know,

$$\beta = \frac{x}{n}$$

$$= \frac{1}{5}$$

$= 0.2$ cm

Interferometry

Interferometry is a branch of science in which optical waves or any other electromagnetic waves are superimposed on each other and interference phenomenon occurs. Interferometry plays important role to study in the field of optics, astronomy, fiber optics, spectroscopy, remote sensing, velocity measurement, etc. Interferometers are devices use for different measurement of path difference, fringe widths, refractive index, and many other parameters with the help of interference phenomenon.

Michelson interferometer

Michelson interferometer is a device used for the formation and study of interference fringes by a monochromatic light. In this apparatus, a beam of light coming from an extended source of light is divided into two parts, one is reflected part and another is refracted part after passing through a partially polished glass plate. These two beams are brought together after reflected from plane mirrors, and finally interference fringes are produced in the field of view.

Construction

The apparatus is shown in Figure-1. The main part of the apparatus is a half-silvered glass plate A so that light coming from the source S is equally reflected and transmitted by it. The plate A inclined at an angle 45^0 with incident light as shown in figure-1, the incident light then divided into two parts, one is reflected part and another is transmitted part. The transmitted light is then passes through another glass plate C which is of equal thickness as of A, and parallel to plate A, this plate C is called compensating plate. The transmitted and reflected parts of light are normally incident on two highly polished mirrors M_2 and M_1 respectively. The mirrors M_1 and M_2 are perpendicular to each other as shown in figure. The mirror M_1 is fixed in a carriage and can be moved with help of a screw and micro scale. Therefore, mirror M_1 is movable and the mirror M_2 is fixed. A telescope E is also fixed as shown in figure. The light reflected from mirror M_1 and M_2 are superimposed and interference fringes are formed in the field of view.

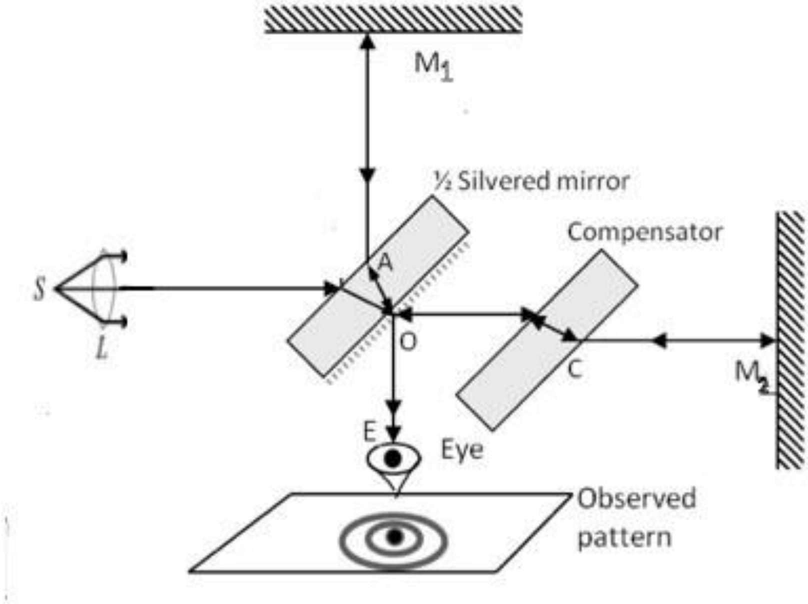


Figure-1: Michelson Interferometer

Working

S is a source of monochromatic light. Light from a monochromatic source S after passing through the lens L, falls on the plate A. The lens L makes the beam parallel. One half of the energy of the incident beam is reflected by the plate A towards the mirror M_1 and the other half is transmitted towards the mirror M_2. These two beams return to the plate A. The beam reflected back by M_1 is transmitted through the glass plate A and the beam reflected back by M_2 is reflected by the glass plate A towards the telescope E. The beam going towards the mirror M_1 and reflected back, has to pass twice through the glass plate A. Therefore, to compensate for the path, the plate C is used between the mirror M_2 and A. The light beam going towards the mirror M_2 and reflected back towards A also passes twice through the compensation plate C. Now in the direction of telescope E, we have two coherent beams of light reflected from mirror M_1 and M_2, and interference takes place and we observed interference pattern in the field of view.

Types of Fringes

1. **Circular fringes**
   Circular fringes are produced with monochromatic light in a Michelson interferometer. Here, the mirror M_1 and the virtual mirror M_2' which is the image of M_2 must be parallel.

2. **Localized fringes**
   When the mirror M_1 and the virtual mirror M_2' (image of M_2) are inclined, the air film enclosed is wedge-shaped and straight-line fringes are observed.

3. **White light fringes**
   With white light, the fringes are observed only when the path difference is small. The different colors overlap on one another and only the first colored fringes are visible. The central fringe is dark and the other fringes are colored. White light fringes are useful for the determination of zero path difference, especially in the standardization of the metre.

Applications of Michelson Interferometer

Michelson interferometer can be used to determine

1. the wavelength of a given monochromatic source of light.
2. the difference between the two neighboring wavelengths or resolution of the spectral lines.
3. refractive index and thickness of various thin transparent materials.
4. for the measurement of the standard metre in terms of the wavelength of light.

Determination of the wavelength of monochromatic light

The mirrors M_1 and M_2 are adjusted so that circular fringes are visible in the field of view. If M_1 and M_2 are equidistant from the glass plate, the field of view will be perfectly dark. The mirror M_2 is kept fixed and the mirror M_1 is moved with the help of the handle of the micrometer screw and the number of fringes that cross the field of view is counted.

Suppose for the monochromatic light of wavelength λ, the distance through which the mirror is moved = *d* and the number of fringes that cross the centre of the field of view = *n*. Then,

$$2d = n\,\lambda$$

$$\therefore \qquad d = \frac{n\lambda}{2},$$

because for one fringe shift, the mirror moves through a distance equal to half the wavelength. Hence λ can be determined.

Math

1. In moving one mirror in a Michelson interferometer through a distance of 0.1474 mm, 500 fringes cross the centre of the field of view. What is the wavelength of light?

Solution:

Here,

$n = 500$

$d = 0.1474$ mm

$= 0.01474$ cm

We know,

$$d = \frac{n\lambda}{2}$$

$$\therefore \quad \lambda = \frac{2d}{n}$$

$$= \frac{2 \times 0.01474}{500}$$

$$= 5896 \times 10^{-8} \text{ cm}$$

$$= 5896 \text{ Å}$$

2. A shift of 100 circular fringes is observed when the movable mirror of the Michelson interferometer is shifted by 0.0295 mm. Calculate the wavelength of light.

Solution:

Here,

Here,

$n = 100$

$d = 0.0295$ mm

$= 2.95 \times 10^{-3}$ cm

We know,

$$d = \frac{n\lambda}{2}$$

$$\therefore \qquad \lambda = \frac{2d}{n}$$

$$= \frac{2 \times 2.95 \times 10^{-3}}{100}$$

$$= 5.90 \times 10^{-5} \text{ cm}$$

$$= 5900 \text{ Å}$$

Interference due to reflected light (thin films)

Consider a transparent film of thickness r and refractive index μ. A ray SA incident on the upper surface of the film is partly reflected along AT and partly refracted along AB. At B part of it is reflected along BC and finally emerges out along CQ. The difference in path between the two rays AT and CQ can be calculated. Draw CN normal to AT and AM normal to BC. The angle of incidence is i and the angle of refraction is r. Also produce CB to meet AE produced at P. Here, $\angle APC = r$.

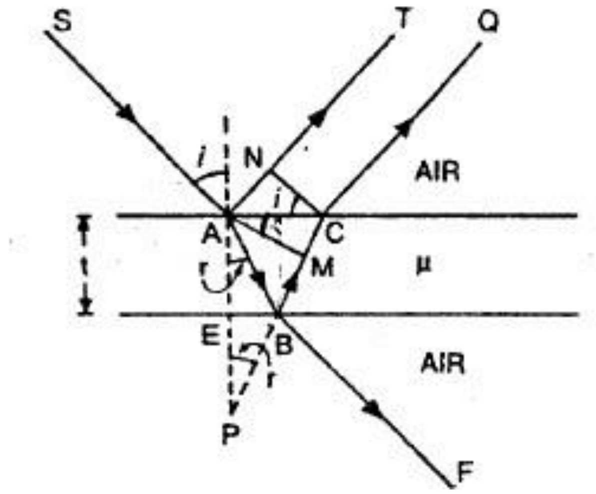


Figure-1: Reflected Light (Thin Films)

The optical path difference,

$$x = \mu(AB + BC) - AN$$

Here, $\mu = \frac{sini}{sinr} = \frac{AN}{CM}$

$\therefore \quad AN = \mu \cdot CM$

$$x = \mu(AB + BC) - \mu \cdot CM$$

$$= \mu(AB + BC - CM)$$

$$= \mu(PC - CM)$$

$$= \mu \cdot PM$$

In the ΔAPM,

$$\cos r = \frac{PM}{AP}$$

$$PM = AP \cos r$$

$$= (\text{AE} + \text{EP}) \cos r$$

$$= 2t\cos r \qquad (\because AE = EP = t)$$

$$\therefore \quad x = \mu \cdot PM$$

$$= 2\mu t\cos r \quad \cdots\cdots\cdots (1)$$

This equation (1), in the case of reflected light does not represent the correct path difference but only the apparent.

The correct path difference in this case,

$$x = 2\mu t\cos r - \frac{\lambda}{2} \qquad \cdots\cdots\cdots (2)$$

1. If the path difference $x = n\lambda$ where n = 0, 1, 2, etc., constructive interference takes place and the film appears bright.

$$2\mu t\cos r - \frac{\lambda}{2} = n\lambda$$

$$\therefore \quad 2\mu t\cos r = (2n+1)\,\frac{\lambda}{2} \cdots\cdots\cdots (3)$$

2. If the path difference $x = (2n+1)\,\frac{\lambda}{2}$ where n = 0, 1, 2, etc., destructive interference takes place and the film appears dark.

$$2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

` $$\therefore \quad 2\mu t\cos r = n\lambda \cdots\cdots\cdots (4)$$

It should be remembered that the interference pattern will not be perfect because the intensities of the rays AT and CQ will not be same and their amplitudes are different.

Interference due to multiple reflection

Consider reflected rays 1, 2, 3, etc. as shown in figure-1. The amplitude of the incident ray is a. Let r be the reflection coefficient, t the transmission coefficient from rarer to denser medium and t' the transmission coefficient from denser to rarer medium.

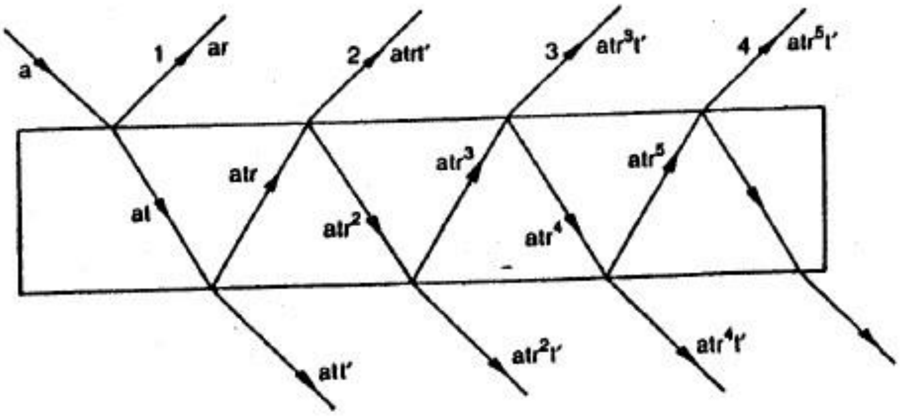


Figure-1: Multiple Reflection

The amplitudes of the reflected rays are: ar, $atrt'$, atr^3t', atr^5t' ···· $etc.$ The ray 1 is reflected at the surface of a denser medium. It undergoes a phase change π. The rays 2, 3, 4, etc. are all in phase but out of phase with ray 1 by π.

The resultant amplitude of 2, 3, 4, etc. is given by

$$A = atrt' + atr^3t' + atr^5t' + \cdots$$

$$= att'r\ [\ 1 + r^2 + r^4 + \cdots]$$

As r is less than 1, the terms inside the brackets form a geometric series.

$$A = att'r \left[\frac{1}{1-r^2}\right]$$

$$= \left[\frac{att'r}{1-r^2}\right]$$

According to the principle of reversibility,

$$tt = 1 - r^2$$

$$\therefore \qquad A = \frac{a(1-r^2)r}{1-r^2} = ar$$

Thus, the resultant amplitude of 2, 3, 4, etc. is equal in magnitude of the amplitude of rays 1 but out of phase with it. Therefore, the minima of the reflected system will be of zero intensity.

Newton’s Rings

When a plano-convex lens of large focal length with its convex surface is placed on a plane glass plate, a thin film of air of increasing thickness is formed. The interference fringes formed are circular bright and dark rings. This formation is due to division of amplitude wherein the superposition of reflected or transmitted waves from the air film occurs. When viewed with white light, the fringes are colored.

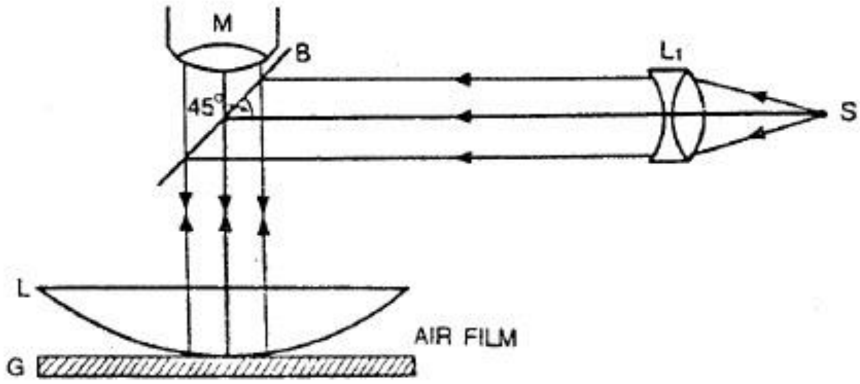


Figure-1: Newton’s Rings

S is a Source of monochromatic light at the focus of the lens L_1, (Figure-1). A horizontal beam of light falls on the glass plate B at 45^0. The glass plate B reflects a part of the incident light towards the air film enclosed by the lens *L* and the plane glass plate *G*. The reflected beam from the air film is viewed with a microscope. Interference takes place and dark and bright circular fringes are produced. This is due to the interference between the light reflected from the lower surface of the lens and the upper surface of the glass plate *G*.

Theory

1. Newton's Rings by Reflected Light

Suppose the radius of curvature of the lens is R and the air film is of thickness t at a distance of $OQ = r$, from the point of contact O.

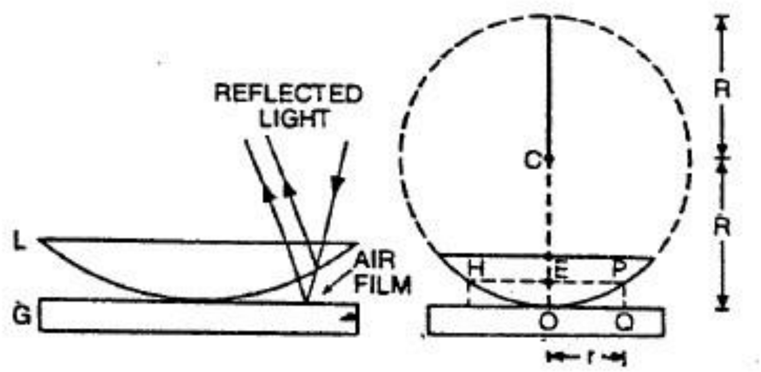


Figure-2: Newton's Rings by Reflected Light

Here, interference is due to reflected light. Therefore, for the bright rings

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2} \cdots\cdots\cdots (1)$$

where, n = 1, 2, 3, etc.

Here, θ is small, therefore $\cos\theta = 1$

For air, $\mu = 1$

$$\therefore \quad 2t = (2n-1)\frac{\lambda}{2} \cdots\cdots\cdots (2)$$

For the dark rings,

$$2\mu t \cos\theta = n\lambda \qquad \text{where, n = 0, 1, 2, etc.}$$

$$\therefore \quad 2t = n\lambda \cdots\cdots\cdots (3)$$

In figure-2,

$$EP \times HE = OE \times (2R - OE)$$

But, $EP = HE = r, \quad OE = PQ = t$

and $2R - t = 2R$ (approximately)

$$r^2 = 2R \cdot t$$

$$\therefore \quad t = \frac{r^2}{2R}$$

Substituting the value of t in equation (2) and (3),

For bright rings

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

$$\therefore \qquad r = \sqrt{\frac{(2n-1)\lambda R}{2}} \cdots\cdots\cdots (4)$$

Figure-3

For dark rings,

$$r^2 = n\lambda R$$

$$\therefore \qquad r = \sqrt{n\lambda R} \cdots\cdots\cdots (5)$$

When n = 0, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore, the center is dark. Alternatively dark and bright rings are produced as shown in figure-3.

Result

The radius of the dark ring is proportional to

i. $\sqrt{n}$
ii. $\sqrt{\lambda}$ and
iii. $\sqrt{R}$

Similarly, the radius of the bright ring is proportional to

1. $\sqrt{\frac{(2n-1)}{2}}$
2. $\sqrt{\lambda}$
3. $\sqrt{R}$.

If D is the diameter of the dark rings,

$$D = 2r = 2\sqrt{n\lambda R}$$

For the central dark ring

$$n = 0$$

$$D = 2\sqrt{n\lambda R} = 0$$

This corresponds to the center of the Newton's rings.

For the *n*th dark ring,

$$D_n = 2\sqrt{n\lambda R}$$

For the nth bright ring,

$$D_n = 2\sqrt{\frac{(2n-1)\lambda R}{2}}$$

2. **Newton's Rings by transmitted Light**

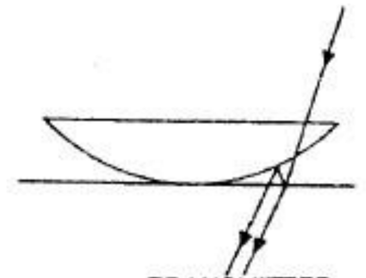


Figure-4

In the case of transmitted light in figure-4, the interference fringes are produced such that for bright rings,

$$2\mu t \cos\theta = n\lambda$$

and for dark rings

$$2\mu t \cos\theta = (2n-1)\,\frac{\lambda}{2} \cdots\cdots\cdots (6)$$

Here,

θ is small, therefore $\cos\theta = 1$

For air, $\mu = 1$

For the bright rings,

$$2\mu t \cos\theta = n\lambda \quad \cdots\cdots\cdots (7)$$ where, n = 0, 1, 2, etc.

$$\therefore \quad 2t = n\lambda$$

and for dark rings

$$2t = (2n-1)\,\frac{\lambda}{2}$$

Taking the value of $t = \frac{r^2}{2R}$, where r is the radius of the ring and R is the radius of curvature of the lower surface of the lens, the radius for the bright and dark rings can be calculated.

For bright rings,

$$r^2 = n\lambda R$$

$$\therefore \quad r = \sqrt{n\lambda R}$$

For dark rings

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

$$\therefore \quad r = \sqrt{\frac{(2n-1)\lambda R}{2}}$$

Where, n = 0, 1, 2, etc.

Figure-5

When n = 0,
For bright rings

$$r = 0$$

Therefore, in the case of Newton's rings due to transmitted light, the central ring is bright as shown in figure-5.

Determination of the Wavelength of Sodium Light Using Newton's Rings

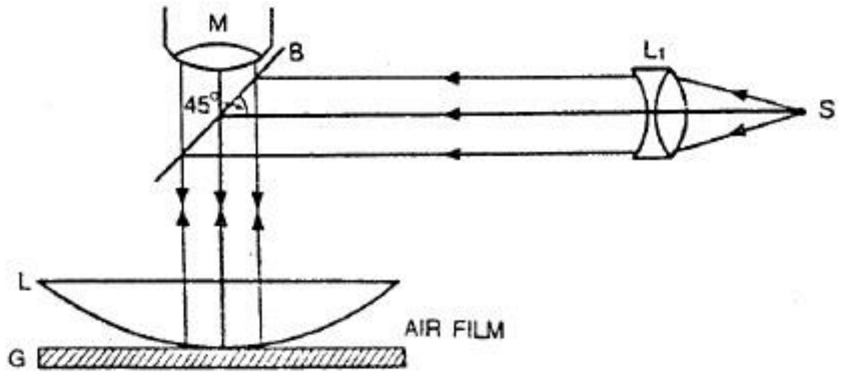


Figure-1

The arrangement used is shown in figure-1. S is a source of sodium light. A parallel beam of light from the lens L_1 is reflected by the glass plate B inclined at an angle of 45^0 to the horizontal. L is a plano-convex lens of large focal length. Newton's rings are viewed through B by the travelling microscope M focused on the air film. Circular bright and dark rings are seen with the centre dark. With the help of a travelling microscope, measure the diameter of the *n*th dark ring.

Suppose, the diameter of the *n* th ring = D_n

$$r_n{}^2 = n\lambda R$$

But $$r_n = \frac{D_n}{2}$$

$$\therefore \quad \frac{D_n{}^2}{4} = n\lambda R$$

or, $$D_n{}^2 = 4n\lambda R \cdots\cdots\cdots\cdots (1)$$

Measure the diameter of the $(n+m)\ th$ dark ring.

Let it be D_{n+m}

$$\therefore \quad \frac{(D_{n+m})^2}{4} = n\lambda R$$

or, $$(D_{n+m})^2 = 4(n+m)\lambda R \cdots\cdots\cdots\cdots (2)$$

Subtracting equation (1) and (2)

$$(D_{n+m})^2 - D_n{}^2 = 4m\lambda R$$

$$\therefore \quad \lambda = \frac{(D_{n+m})^2 - D_n{}^2}{4mR} \cdots\cdots\cdots\cdots (3)$$

Hence, λ can be calculated.

Math

1. A plano-convex lens of radius 300 cm is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of the 8 th dark ring in the transmitted system is 0.72 cm. Calculate the wavelength of light used.

   Solution:

   Here,

   $n = 8$

   $D = 0.72$ cm

   $r = 0.36$ cm

   $R = 300$ cm

   $\lambda = ?$

   We know,

   For the transmitted system,

   $$r^2 = \frac{(2n-1)\lambda R}{2}$$

   or, $$\lambda = \frac{2r^2}{(2n-1)R}$$

   $$= \frac{2 \times (0.36)^2}{(2\times 8 - 1)300}$$

   $= 5760 \times 10^{-8}$ cm

   $= 57600$ Å

2. In a newton's rings experiment the diameter of the 15 th ring was found to be 0.590 cm and that of the 5 th ring was 0.336 cm. If the radius of the plano-convex lens is 100 cm, calculate the wavelength of light used.

   Solution:

   Here,

   m = 10

   D_5 = 0.336 cm

   D_{15} = 0.590 cm

   R = 100 cm

   λ =?

   We know,

$$\lambda = \frac{(D_{n+m})^2 - {D_n}^2}{4mR}$$

$$= \frac{(D_{15})^2 - {D_5}^2}{4mR}$$

$$= \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$= 5880 \times 10^{-8}$ cm

= 5880 Å

3. In a newton's rings experiment the diameter of the 5 th ring was 0.336 cm and the diameter of the 15 th ring was 0.590 cm. Find the radius of curvature of the plano-convex lens, if wavelength of light used is 5890 Å.

Solution:

Here,

$m = 10$

$D_5 = 0.336$ cm

$D_{15} = 0.590$ cm

$\lambda = 5880 \times 10^{-8}$ cm

$R = ?$

We know,

$$\lambda = \frac{(D_{n+m})^2 - D_n{}^2}{4mR}$$

or, $$R = \frac{(D_{15})^2 - D_5{}^2}{4m\lambda}$$

$$= \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 5880 \times 10^{-8}}$$

$$= 99.82 \text{ cm}$$

4. In a newton's rings experiment, find the radius of curvature of the lens surface in contact with the glass plate when with a light of wavelength 5890×10^{-8} cm, the diameter of the third dark ring is 3.2 mm. The light is falling at such an angle that it passes through the air film at an angle of zero degree to the normal.

   Solution:

   Here,

   $n = 3$

   $D = 3.2$ mm

   $r = 1.6$ mm $= 0.16$ cm

   $\lambda = 5890 \times 10^{-8}$ cm

   $R = ?$

   We know,

   $r^2 = n\lambda R$

   or, $R = \dfrac{r^2}{n\lambda}$

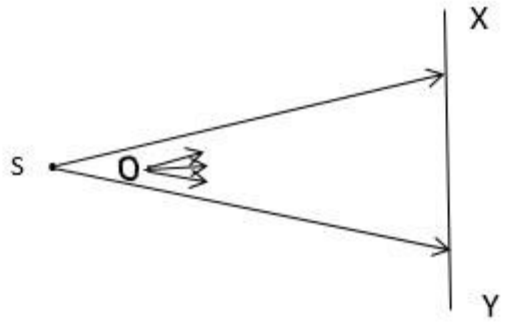
   $= \dfrac{(0.16)^2}{3 \times 5890 \times 10^{-8}}$

   $= 144.9$ cm

Holography

Holography is a technique that enables a light field (which is generally the result of a light source scattered off objects) to be recorded and later reconstructed when the original light field is no longer present, due to the absence of the original object.

S is a point source of light and O is a small object. On XY, the secondary wavelets from O superimpose on the strong primary waves from S. As the primary wave is uniform and more intense than the secondary wave, the variation in intensity across XY is dependent on the variation in phase across it. It is not determined by the variation in intensity across the secondary wave. In other words, the presence of strong coherent background helps to record information about the phase of the diffracted light. The pattern obtained on XY is called a hologram and its photograph is taken, keeping the time of exposure extremely small.



For reconstructing the field of view, the photographic plate is developed by reversal and if this developed plate is inserted at the place XY and only a source S is used, on looking through this plate towards S, the object will appear at the point O. In this way the original field of view is observed.

Applications of Holography

1. Holography has been used in holographic interferometry.
2. Holography is also useful in the microscopic examination of certain kinds of specimens.
3. Holography is also useful to provide a high-capacity system for image storage reexamination.