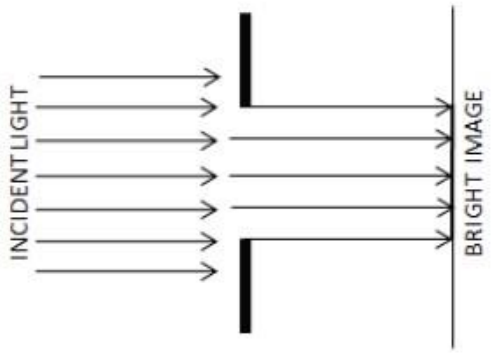
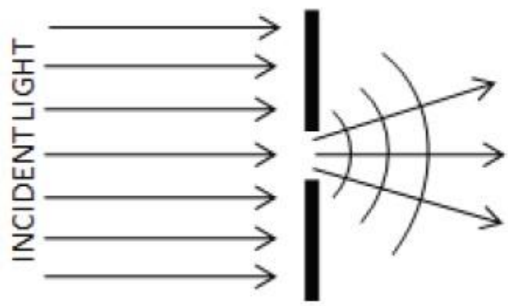
Diffraction of Light

The phenomenon of bending and spreading of light waves into the geometrical shadow around the corners of an obstacle or aperture in its path is known as diffraction of light. In fact, any departure from rectilinear path may be called diffraction. The effects can be observed only when the size of the obstacle is very small and comparable to the wavelength of light.



No Diffraction



Diffraction

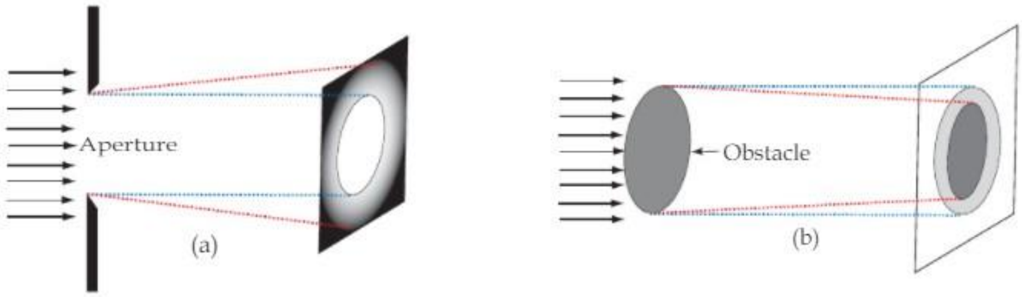


Figure-1 : Diffraction of light: (a) bending of light at the edge of a aperture, (b) bending of light around an obstacle.

Types of Diffraction

The phenomenon of diffraction is broadly divided into two categories:

1. Fraunhofer diffraction, and
2. Fresnel diffraction

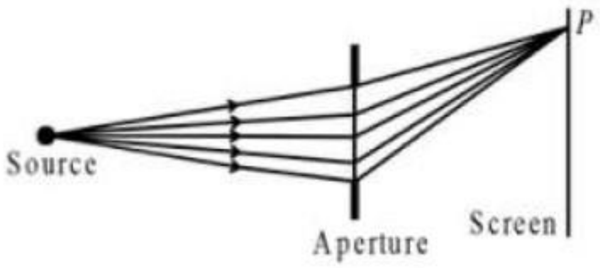
1. **Fraunhofer diffraction:**

In Fraunhofer diffraction, the source of light and the screen on which the diffraction pattern is observed are at infinite distance from the obstacle or aperture. This condition is achieved by using two convex lenses, one for rendering the incident rays parallel before it falls on the aperture and another, to focus the diffracted light on the screen.

Source and screen are moved to a large distance resulting in Fraunhofer diffraction

2. **Fresnel diffraction:**

In Fresnel diffraction, the source of light and the screen on which the diffraction pattern is observed are at finite distance from the obstacle or aperture. No lenses are used in this case. In this type of diffraction, interference takes place between light waves reaching a point (near or in the shadow of the obstacle) from different parts of the same wavefront at the aperture without modification by lenses.



Fresnel diffraction

Figure-2: Fraunhofer diffraction and Fresnel diffraction.

Fresnel's assumptions

1. According to Fresnel, a wavefront can be divided into a large number of strips or zones called Fresnel zones of small area. The resultant effect at any point will depend on the combined effect of all the secondary waves coming from various zones.

2. The effect at a point due to any particular zone depends on distance of the point from the zone.

3. The effect will also depend on the obliquity (inclination) of the point with reference to the zone under consideration.

Distinction between Fresnel diffraction and Fraunhofer diffraction:

	Fresnel diffraction	Fraunhofer diffraction
1.	The source of light and the screen on which the diffraction pattern is observed are at finite distance from the obstacle or aperture.	The source of light and the screen on which the diffraction pattern is observed are at infinite distance from the obstacle or aperture.
2.	The incident wavefront and the diffracted wavefronts are spherical or cylindrical.	The incident wavefront and the diffracted wave fronts are plane wave fronts.
3.	The incident beam is a divergent beam whereas the diffracted beam is a convergent beam.	The incident beam is a parallel beam and the diffracted beam is also parallel beam.
4.	No changes in the wavefront are made by using either lenses or mirrors.	The incident rays from a source are made parallel using a convex lens and the diffracted rays are brought to focus on a screen using another convex lens (converging lenses).
5.	The center of the diffraction pattern is either bright or dark. The pattern is the image of the obstacle or aperture.	The center of the diffraction pattern is always bright. The pattern is the image of the source itself.

Fraunhofer diffraction at a single slit

Consider a point source of light *S*, in figure-1, placed at the principle focus of lens L_1. The parallel rays strike the single slit *AB* of width a. *XY* is the incidental spherical wavefront. Each Point on the slit *AB* act as sources of secondary disturbances and sends out secondary waves in all directions. The diffracted rays after passing through lens L_2 are brought to focus on the screen *MN*. The diffraction pattern consists of central bright and alternate bright and dark bands of decreasing intensity.

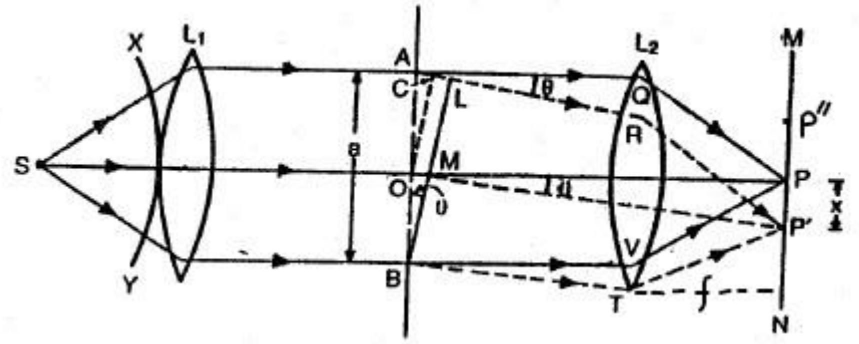


Figure-1: Fraunhofer Diffraction at a single slit.

The secondary waves travelling in the direction parallel to *OP* viz. *AQ* and *BV* corne to focus at *P* and a bright central image is observed. The secondary from points equidistant from *O* and situated in the upper and lower halves *OA* and *OB* of the wavefront travel the same distance in reaching *P* and hence the path difference is zero. The secondary waves reinforce one another and *P* will be a point of maximum intensity.

Secondary Minima or Dark Bands:

Now, consider the secondary waves travelling in the direction *AR*, inclined at an angle θ to the direction *OP*. All the secondary wave travelling in this direction reach the point P'on the screen. The point P' wil be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wavefront. Draw *OC* and *BL* perpendicular to *AR*.

Then, in the ΔABL

$$sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

or $AL = asin\theta$

where a is the width of the slit and AL is the path difference between the secondary waves originating from A and B. If this path difference is equal to λ the wavelength of light used, then P' will be a point of minimum intensity.

[**This is because if we consider the whole wavefront at the slit to be divided into equal halves, then the path difference between the waves originating in the first half and the corresponding point in the second half will be $\frac{\lambda}{2}$ (or a phase difference of π) when they meet P'. Thus, all these waves will interfere destructively and so the point P' will be a point of minimum intensity.**

The points for which the path difference is:

$$\boldsymbol{a sin\theta = \lambda}$$

form the first dark band or the first minimum. The points for which the path difference is:

$$\boldsymbol{a sin\theta = 2\lambda}$$

form the second dark band or the second minimum.]

In general, the points on the screen for which the path difference is:

$$a sin\theta_n = n\lambda$$

$$\therefore \qquad sin\theta_n = \frac{n\lambda}{a}$$

form the $n\ th$ dark band or the $n\ th$ minimum.

where, θ_n gives the direction of the $n\ th$ minimum. Here, n is an integer.

Secondary Maxima or Bright Bands:

If the path difference is odd multiples of $\frac{\lambda}{2}$, the directions of the secondary maxima can be obtained. In this case,

$$a sin\theta_n = (2n+1)\frac{\lambda}{2}$$

or $$sin\theta_n = \frac{(2n+1)\lambda}{2a}$$ where, n = 1, 2, 3 etc.

Thus, the diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides. The intensity distribution on the screen is given in Fig.- 2.

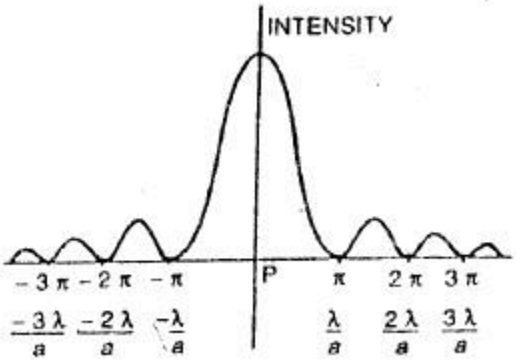


Figure-2: The intensity distribution on the screen.

P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points *A* and *B* is $\lambda, 2\lambda$ etc., correspond to the positions of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point *P* outwards.

Width of the Central Maximum:

If the lens L_2 is very near the slit or the screen is far away from the lens L_2, then

$$sin\theta = \frac{x}{f}$$

Where, f is the focal length of the lens L_2.

But, $$sin\theta = \frac{\lambda}{a}$$

$$\therefore \quad \frac{x}{f} = \frac{\lambda}{a}$$

or $$x = \frac{f\lambda}{a}$$

where, x is the distance of the secondary minimum from the point *P*.

Thus, the width of the central maximum = $2x$.

$$\therefore \quad 2x = \frac{2f\lambda}{a}$$

The width of the central maximum is proportional to λ, the wavelength of light and inversely proportional to the width a of the slit.

Math

1. A lens whose focal length is 50 cm forms a Fraunhofer diffraction pattern of slit 0.0025cm wide. Calculate the width of the central maximum on the screen. (Wavelength of the light used is 500nm)

   Solution:

   Here,

   Focal length, f = 50 cm
   Width of slit, a = 0.0025 cm
   Wavelength, λ = 500 nm = 500$\times 10^{-7}$cm
   Width of the central maximum, $2x$ =?

   We know,

   The width of the central maximum, $2x = \frac{2f\lambda}{a}$

   $= \frac{2\times50\times500\times10^{-7}}{0.0025}$

   $=$ 2 cm

2. In Fraunhofer diffraction due to a narrow slit a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of the central maximum, find the wavelength of light.

   Solution:

   Here,

   a = 0.2 mm = 0.02 cm

   x = 5 mm = 0.5 cm

D = 2m = 200 cm

n = 1

λ =?

We know,

$$a sin\theta = n\lambda \quad \cdots\cdots\cdots\cdots\cdots (1)$$

Again,

$$sin\theta = \frac{x}{D}$$

Putting the value of $sin\theta$ in equation (1)

$$a \times \frac{x}{D} = n\lambda$$

$$\therefore \lambda = \frac{ax}{nD}$$

$$= \frac{0.02 \times 0.5}{1 \times 200}$$

$$= 5 \times 10^{-5} \text{ cm}$$

$$= 5000 \text{ Å}$$

3. Find the angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.

Solution:

Here,

Width of the slit, a = 12×10^{-5} cm = 12×10^{-7} m

Wavelength, λ = 6000 Å = 6000×10^{-10} m

Angular width of the central bright maximum, 2θ =?

We know,

$$sin\theta = \frac{\lambda}{a}$$ [where, θ is the half angular width of the central maximum]

$$= \frac{6000 \times 10^{-10}}{12 \times 10^{-7}}$$

$$= 0.5$$

$\therefore$ $\theta = \sin^{-1}(0.5)$

$= 30^0$

Angular width of the central bright maximum,

$$2\theta = 2 \times 30^0$$

$$= 60^0$$

Diffraction Grating

A diffraction grating is an optical device having a large number of equidistant narrow rectangular slits of equal width, placed side by side and parallel to one another. The slits are separated by opaque spaces.

Diffraction in a Plane Transmission Grating

When a plane wavefront is incident on a grating surface, light is transmitted though the slits are obstructed by the opaque regions. Such a grating is called transmission grating.

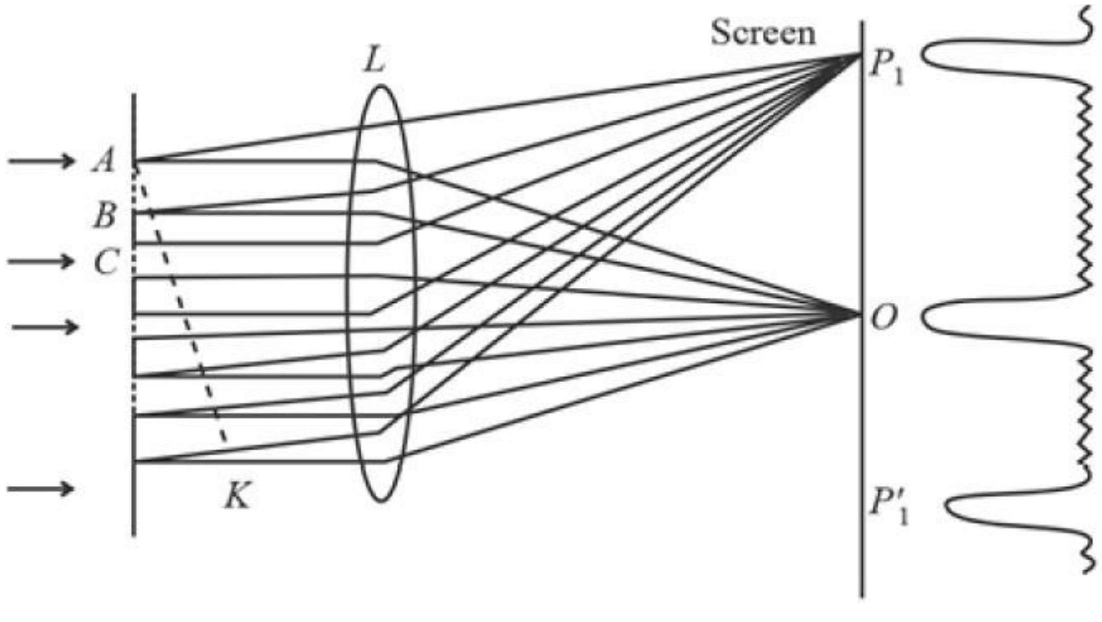


Figure-1: Principal maximum in diffraction grating.

In figure-1, AB is a slit and BC is an opaque portion. The width of each slit is a and the opaque spacing between any two consecutive slits is b.

Let a parallel beam of monochromatic light of wavelength λ is incident on a plane transmission grating having N numbers of slits separated by opaque regions. Then all the secondary waves travelling in the same direction as that of the incident light will come to focus at the point O on the screen. The point O where all the secondary waves reinforce one another corresponds to the position of the central bright maximum.

Now, let us consider the secondary waves at an angle θ with the direction of the incident light. The collecting lens is suitably rotated such that the axis of the lens is parallel to the direction of the secondary waves. These secondary waves come to focus at the point P_1 on the screen. The intensity at P_1 will depend on the path difference between the corresponding points in two adjacent slits.

If AK is the normal drawn to the direction of the diffracted rays, then CN is the path difference between the rays diffracted from the corresponding points A and C at an angle θ.

$$\therefore \text{Path difference, } CN = ACsin\theta$$
$$= (a+b)sin\theta$$

If this path difference is an even multiple of $\frac{\lambda}{2}$, the point P_1 will be bright. Thus,

$$(a+b)sin\theta_n = 2n \cdot \frac{\lambda}{2}$$
$$= n\lambda \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

Whereas, when the path difference is odd multiple of $\frac{\lambda}{2}$, the point P_1 will be dark. Thus,

$$(a+b)sin\theta_n = (2n+1) \cdot \frac{\lambda}{2} \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

Where n = 0, 1, 2, 3, ⋯⋯ Eq. (1) is often referred to as the grating rule and gives the direction of the nth order principal maximum which depends on λ and $(a+b)$. These conditions are valid for any pair of corresponding points in the adjoining slits in the grating.

Secondary Maxima and Minima

The angle of diffracting θ_n corresponding to the direction of the $n\,th$ principal maximum is given by the equation

$$(a+b)sin\theta_n = n\lambda$$

Where, $(a+b)$ is called the grating element. Here a is the width of the slit and b is the width of the opaque portion.

If there is small change in the angle of diffraction such that the path difference between the rays diffracted at the extreme ends of the grating λ, then the path difference between the rays diffracted at the corresponding points of the adjacent slits will be $\frac{\lambda}{N}$.

If we imagine the grating surface to be divided into two halves, the path difference between the secondary waves diffracted in the first half and the corresponding points in the second half will be $\frac{\lambda}{2}$ cancelling out each other. Thus, this gives the position of the first secondary minimum. Similarly, we can see if the angle of diffraction is such that the path difference between the extreme ends of the grating is $2\lambda, 3\lambda \cdots\cdots (N-1)\lambda$ so that

The path difference between the corresponding points between the adjoining slits is $\frac{2\lambda}{N}, \frac{3\lambda}{N} \cdots\cdots$ $\cdots\cdots \frac{(N-1)\lambda}{N}$, corresponding directions will be the direction of $2, 3, \cdots, (N-1)th$ secondary minimum respectively.

In between any two secondary minima, there is a secondary maximum. If the path difference between the extreme diffracted rays is $\frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \cdots\cdots$ corresponding directions give the directions of the secondary maxima. In all, there are $(N-1)$ secondary minima separated by $(N-2)$ secondary maxima between any two consecutive principal maxima.

Width of Principal Maxima

Let the angular separation between the first secondary minimum after the nth principal maximum be $\Delta\theta$. We have seen that the additional path difference between the corresponding points of successive slits should be $\frac{\lambda}{N}$. Thus, for the first minimum after nth principal maximum:

$$(a+b)\sin(\theta_n + \Delta\theta) = n\lambda + \frac{\lambda}{N}$$

Expanding, we have,

$$(a+b)[\sin\theta_n cos\Delta\theta + sin\Delta\theta cos\theta_n] = n\lambda + \frac{\lambda}{N}$$

As $\Delta\theta$ is very small, cos$\Delta\theta$ = 1 and sin$\Delta\theta$ = $\Delta\theta$. Thus,

$$(a+b)[\sin\theta_n + \Delta\theta cos\theta_n] = n\lambda + \frac{\lambda}{N}$$

$$\text{or, } (a+b)\sin\theta_n + (a+b)\Delta\theta cos\theta_n = n\lambda + \frac{\lambda}{N}$$

$$\text{or, } n\lambda + (a+b)\Delta\theta cos\theta_n = n\lambda + \frac{\lambda}{N} \qquad [\text{ As } (a+b)\sin\theta_n = n\lambda]$$

$$\text{or, } (a+b)\Delta\theta cos\theta_n = \frac{\lambda}{N}$$

$$\therefore\ \Delta\theta = \frac{\lambda}{N(a+b)cos\theta_n}$$

$\therefore$ Angular width of nth principal maximum = $2\Delta\theta$

$$= \frac{2\lambda}{N(a+b)cos\theta_n}$$

Math

1. A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having 1250 lines per cm and a second order spectral line is observed to be deviated through 30^0. Calculate the wavelength of the spectral line.

Solution:

Here,

Total number of lines, N = 1250 lines

Order, n = 2

Angle, θ = 30^0

Wavelength, λ =?

We know,

$$(a + b)\sin\theta = n\lambda \;\cdots\cdots\cdots\cdots\cdots\cdots\; (1)$$

Again,

Grating Element,

$$(a + b) = \frac{1}{N} = \frac{1}{1250} \text{ cm}$$

From equation (1)

$$\lambda = \frac{(a+b)\sin\theta}{n}$$

$$= \frac{\sin 30^0}{2 \times 1250}$$

$$= 2 \times 10^{-4} \text{ cm}$$

2. A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having 5000 lines per cm and a second order spectral line is observed to be deviated through 30^0. Calculate the wavelength of the spectral line.

Solution:

Here,

Total number of lines, N = 5000 lines

Order, n = 2

Angle, θ = 30^0

Wavelength, λ =?

We know,

$$(a+b)\sin\theta = n\lambda \text{ ·····················} (1)$$

Again,

Grating Element,

$$(a+b) = \frac{1}{N} = \frac{1}{5000} \text{ cm}$$

From equation (1)

$$\lambda = \frac{(a+b)\sin\theta}{n}$$

$$= \frac{\sin 30^0}{2 \times 5000}$$

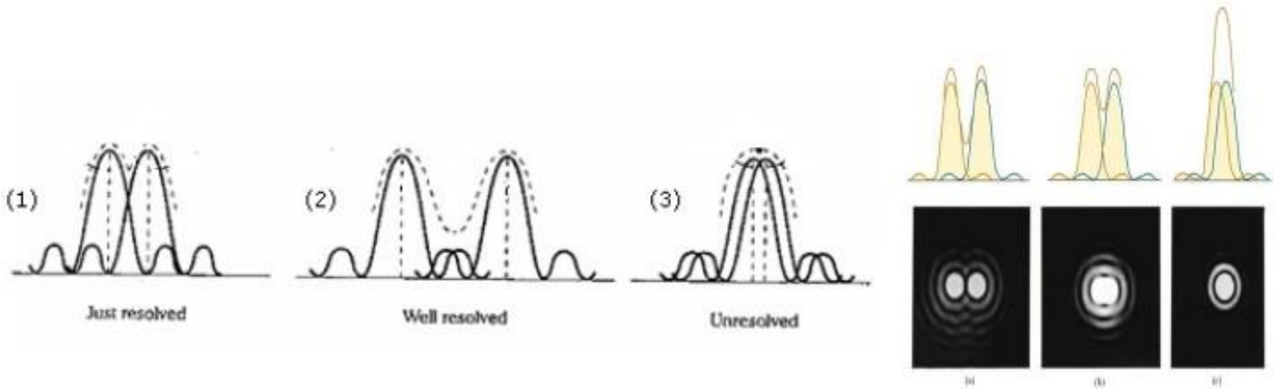
$$= 5 \times 10^{-5} \text{ cm}$$

$$= 5000 \text{ Å}$$

Resolving power

The ability of an optical instrument to show two close lying point objects as well separated point objects is called its resolving power. The resolution is limited by the diffraction patterns of the two close lying point objects which overlap as shown.

Rayleigh criterion for resolution



1. **Condition for just resolved** – Two close lying sources of light or point objects

   are said to be just resolved, if the central maximum of the diffraction pattern due to one source coincides with the first minimum of the diffraction pattern due to the second source. It also means that the distance between two central maxima due to two sources is equal to the distance between the central maximum and first minimum of any one of them.

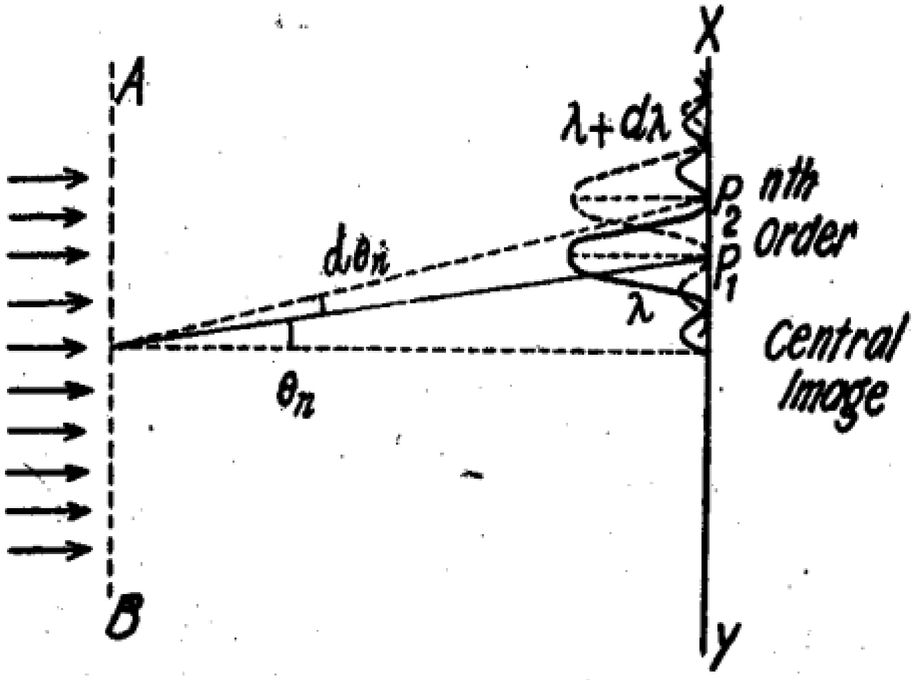
2. **Condition for well resolved** – Two close lying sources of light or point objects are said to be well resolved, if the distance between two central maxima of the diffraction pattern due to two sources is greater than the distance between the central maximum and first minimum of any one of them.

3. **Condition for unresolved** – Two close lying sources of light or point objects are said to be unresolved, if the distance between two central maxima of the diffraction pattern due to two sources is less than the distance between the central maximum and first minimum of any one of them.

Resolving power of grating

The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in wavelength between this line and a neighboring line such that the two lines appear to be just resolved. Thus, the resolving power of a grating appear to be just resolved. Thus, the resolving power of a grating

$$= \frac{\lambda}{d\lambda}$$



It is the capacity of the grating to form separate diffraction maxima of two wavelengths that are close to each other.

The direction of the $n\ th$ principal maximum for a wavelength λ is

$$(a + b)\ sin\theta_n = n\lambda \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

The equation for minima is $N(a + b)sin\theta_n = m\lambda$

where m has all integral values except $0, N, 2N \cdots\cdots nN$ because for these values of m, the condition for maxima is satisfied.

Thus, the first minimum adjacent to nth principal maximum in the direction $\theta_n + d\theta$ is obtained by substituting the value of m as $(nN + 1)$. Thus, the first minimum in the direction of $(\theta_n + d\theta)$ is

$$N(a + b)sin(\theta_n + d\theta) = (nN + 1)\ \lambda \cdots\cdots\cdots\cdots\cdots (2)$$

The direction of the nth principal maximum for a wavelength $\lambda + d\,\lambda$ is

$$(a+b)sin(\theta_n + d\theta) = n(\lambda + d\lambda) \cdots\cdots\cdots\cdots\cdots (3)$$

Multiplying (3) by N, we have

$$N(a\ +\ b)sin(\theta_n + d\theta) = nN(\lambda + d\lambda) \cdots\cdots\cdots\cdots\cdots (4)$$

The two lines appear just resolved if the angle diffraction $(\theta_n + d\theta)$ also correspond to the direction of first secondary minimum due to the first diffraction pattern.

Comparing (2) and (4)

$$nN(\lambda + d\lambda) = (nN\ +\ 1)\lambda$$

or $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$

or $n\,d\lambda = \frac{\lambda}{N}$

or $\frac{\lambda}{d\lambda} = nN \rightarrow$ Expression for resolving power of grating

Thus, the resolving power is (1) directly proportional to the order of the spectrum and (2) the total number of lines on the grating surface.

Math

1. What should be the minimum number of lines in a grating which will just resolve in the second order the lines whose wavelengths are 5890 Å and 5896 Å.

   Solution:

   Here,

   $n = 2$

   $\lambda = 5890$ Å

   $d\lambda = (5896 - 5890)$ Å $= 6$ Å

   $N = ?$

   We Know,

   Resolving Power,

   $$\frac{\lambda}{d\lambda} = nN$$

   $$\therefore N = \frac{\lambda}{nd\lambda}$$

   $$= \frac{5890}{2 \times 6}$$

   $$= 490.833$$

   $= 491$ approximately.

2. What should be the minimum number of lines in a grating which will just resolve in the first order the lines whose wavelengths are 5890 Å and 5896 Å.

Solution:

Here,

$n = 1$

$\lambda = 5890$ Å

$d\lambda = (5896 - 5890)$ Å $= 6$ Å

$N = ?$

We Know,

Resolving Power,

$$\frac{\lambda}{d\lambda} = nN$$

$$\therefore N = \frac{\lambda}{nd\lambda}$$

$$= \frac{5890}{1 \times 6}$$

$= 981.667$

$= 982$ approximately.

Distinction Between Interference and Diffraction

	Interference	Diffraction
1.	It is the modification in the intensity of light due to superposition of two or more light waves.	It is the bending of light around the corners of small obstacles and hence it's spreading into the region of geometrical shadow.
2.	It is due to the superposition of finite number of waves from different coherent sources.	It is due to the superposition of infinite number of secondary waves from different points of the same wavefront.
3.	Interference fringes are of equal width.	Diffraction fringes are of unequal width. The width of the central band is maximum and the widths of the less bright bands gradually decrease.
4.	Interference pattern consists of alternately bright and dark bands, all the bright bands being of the same brightness.	Diffraction pattern consists of a central bright band of maximum brightness, surrounded on either side by alternately dark and less bright bands called secondary maxima.
5.	In an interference pattern, a good contrast between dark and bright bands exists. The intensity of dark bands is nearly zero.	In a diffraction pattern the contrast between the secondary maxima and minima are comparatively lesser. The intensity of secondary maxima decreases with distance.