1| $f(x) = \sqrt{x-1} + \sqrt{5-x}$

$f(x)$ is real number such that

i.e., $x-1 \geqslant 0$ and $5-x \geqslant 0$

$\therefore x \geqslant 1$ i.e., $5 \geqslant x$

$\therefore x \leq 5$

so, domain of definition of (x) is $1 \leq x \leq 5$ or $[1,5]$

2| $f(x) = \sqrt{(3x-1)(7-x)}$

$f(x)$ is real number such that,

$3x-1 \geqslant 0$ and $7-x \geqslant 0$

i.e., $3x \geqslant 1$ i.e., $7 \geqslant x$

$\therefore x \geqslant 1/3$ $\therefore x \leq 7$

so, domain of definition of $f(x)$ is $1/3 \leq x \leq 7$ or $[1/3, 7]$

3| $f(x) = \sqrt{8+2x-3x^2}$

$f(x)$ is real number such that,

$8+2x-3x^2 \geqslant 0$

i.e., $8+6x-4x-3x^2 \geqslant 0$

i.e., $-(3x^2-6x+4x-8) \geqslant 0$

i.e., $3x^2-6x+4x-8 \leq 0$

i.e., $3x(x-2)+4(x-2) \leq 0$

$\therefore (3x+4)(x-2) \leq 0$

Now,

and $(3x+4) \leq 0$

$(x-2) \leq 0$

i.e, $3x \leq -4$

$\therefore x \leq 2$

$\therefore x \leq -4/3$

This inequality doesn't hold simutancely.

$\therefore f(x)$ is $-4/3 \leq x \leq 2$.

8) $f(x) = \log(x^2 - 5x + 6)$

we are given that,

$f(x) = \log(x^2 - 5x + 6)$

$f(x)$ is real such that,

$x^2 - 5x + 6 > 0$

i.e., $x^2 - 3x - 2x + 6 > 0$

i.e., $x(x-3) - 2(x-3) > 0$

$\therefore (x-3)(x-2) > 0$

$\therefore x - 3 > 0$ and $x - 2 > 0$

$\therefore x > 2$

$\therefore x > 3$

This doesn't hold simutancously.

$\therefore$ The domen of the function all the real numbers except $2 \leq x \leq 3$

$\therefore$ Domen $= R - \{2 \leq x \leq 3\}$

উঃ $f(x) = \sqrt{\log \frac{4x-x^2}{3}}$

$f(x)$ is neal such that

$$\log \frac{4x-x^2}{3} \geqslant 0$$

i.e, $\log \frac{4x-x^2}{3} \geqslant \log 1$

i.e, $\frac{4x-x^2}{3} \geqslant 1$

i.e, $4x-x^2 \geqslant 3$

i.e, $-x^2+4x-3 \geqslant 0$

i.e, $-(x^2-4x+3) \geqslant 0$

i.e, $x^2-4x+3 \leq 0$

i.e, $x^2-3x-x+3 \leq 0$

i.e, $x(x-3)-1(x-3) \leq 0$

$\therefore (x-3)(x-1) \leq 0$

Now,

$x-3 \leq 0$ and $x-1 \leq 0$

$\therefore x \leq 3$ $\quad x \leq 1$

This does not hold simutaneely.

The Domen of the function is $\{1 \leq x \leq 3\}$, $[1,3]$

6. If $f(x) = \cos(\log x)$ then show that

$$f(x).f(y) - \frac{1}{2}\{f(\frac{x}{y}) + f(xy)\} = 0$$

Solve:

$$f(x) = \cos(\log x)$$

$$f(x)(f y) = \cos(\log x) \cdot \cos(\log y) \text{ and}$$

$$f(\frac{x}{y}) + f(xy) \cdot$$

$$= \cos\{\log(\frac{x}{y})\} + \cos\{\log(xy)\}$$

$$= 2\cos\frac{\log\frac{x}{y} + \log xy}{2} \cos\frac{\log\frac{x}{y} - \log xy}{2}$$

$$= 2\cos\frac{(\log\frac{x}{y} \cdot xy)}{2} \cos\frac{(\log\frac{x}{y} \cdot \frac{1}{xy})}{2}$$

$$= 2\cos\left(\frac{1}{2} \cdot \log x^2\right) \cdot \cos\left(\frac{1}{2} \cdot \log\frac{1}{y^2}\right)$$

$$= 2\cos\left(\frac{1}{2} \cdot 2\log x\right) \cdot \cos\left(\frac{1}{2} \cdot \log y^{-2}\right)$$

$$= 2\cos(\log x) \cdot \cos\left(\frac{1}{2}(-2)\log y\right)$$

$$= 2\cos(\log x) \cdot \cos(\log y)$$

$$\therefore f(x).f(y) - \frac{1}{2}\{f(\frac{x}{y}) + f(xy)\}$$

$$= \cos(\log x) \cdot \cos(\log y) - \frac{1}{2} \cdot 2\cos(\log x) \cdot \cos(\log y)$$

$$= 0$$

7| Find the inverse of the function $f(x)$

$f(x) = \log_e (x+\sqrt{x^2+1})$

Let, $f(x) = y$

$\therefore x = f^{-1}(y)$

$\therefore y = \log_e (x+\sqrt{x^2+1})$

i.e, $e^y = x+\sqrt{x^2+1}$

i.e, $(e^y - x)^2 = x^2+1$

i.e, $(e^y)^2 - 2.e^y.x + x^2 = x^2+1$

i.e, $e^{2y} - 2e^y.x = 1$

i.e, $e^{2y} - 1 = 2e^y.x$

i.e $x = \dfrac{e^{2y}-1}{2e^y}$

i.e $x = \dfrac{e^{2x}-1}{e^x} \cdot \dfrac{1}{2}$

i.e $x = \frac{1}{2} \; e^{2x-x} - e^{-x}$

$\therefore x = \frac{1}{2}(e^x - e^{-x})$

8| Given $f(x) = x^2 - 10x + 3$, find $f(0)$ and $f(-2)$.

solve:

$f(x) = x^2 - 10x + 3$

$f(0) = 0^2 - 10.10 + 3$

$= 3$

$f(-2) = (-2)^2 - 10.(-2) + 3$

$= 9 + 20 + 3$

$= 27$

$\therefore f(a+b) = b\,\frac{a+b-a}{} + a\,\frac{a+b-b}{}$

9| If $f(x) = \sec x + \cos x$, then show that $f(x) = f(-x)$

Solve :

$$f(x) = \sec x + \cos x$$

$$\therefore f(-x) = (-\sec x) + (-\cos x)$$

$$= \sec x + \cos x$$

$[-\sec\theta = \sec\theta]$

$[(-\cos\theta) = \cos\theta]$

$$\therefore f(x) = f(-x)$$

10. If $f(x) = b\,\frac{x-a}{b-a} + a\,\frac{x-b}{a-b}$, then show that

$f(a) + f(b) = f(a+b)$

Let,

$$f(x) = b\,\frac{x-a}{b-a} + a\,\frac{x-b}{a-b}$$

$$f(a) = b\,\frac{a-a}{b-a} + a\,\frac{a-b}{a-b}$$

$$= 0 + a.1$$

$$= a$$

$$f(b) = b\,\frac{b-a}{b-a} + a\,\frac{b-b}{a-b}$$

$$= b.1 + 0$$

$$= b$$

$$\therefore f(a+b) = b\,\frac{a+b-a}{b-a} + a\,\frac{a+b-b}{a-b}$$

$$= b\,\frac{b}{b-a} + a\,\frac{a}{a-b}$$

$$= \frac{b^2}{b-a} + \frac{a^2}{a-b}$$

$$= \frac{b^2}{b-a} - \frac{a^2}{b-a}$$

$$= \frac{b^2-a^2}{b-a}$$

$$= \frac{(b+a)(b-a)}{(b-a)}$$

$$= b+a$$

$$= a+b$$

$\therefore f(a) + f(b) = a+b$

~~and~~ $\therefore f(a) + f(b) = f(a+b)$

11 If $f(x) = x^2 - 3x + 7$, then show that

$\{f(x+h) - f(x)\}/h = 2x - 3 + h$.

Here,

$$f(x+h) = (x+h)^2 - 3(x+h) + 7$$
$$= x^2 + 2hx + h^2 - 3x - 3h + 7$$

$$\therefore \{f(x+h) - f(x)\}/h = \frac{x^2 + 2hx + h^2 - 3x - 3h + 7 - x^2 + 3x - 7}{h}$$
$$= \frac{2hx + h^2 - 3h}{h}$$
$$= 2x + h - 3$$

$\therefore$ Hence, the result.

12. $\lim\limits_{x \to \pi} \frac{\sin x}{\pi - x} = 1$

$$\lim_{x \to \pi} \frac{\sin x}{\pi - x}$$
$$= \lim_{x \to 0} \frac{\sin(\pi - \theta)}{\theta}$$

where, $\pi - x = \theta$ $\therefore x = \pi - \theta$, and $\theta \to 0$ as $x \to \pi$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$
$$= 1$$

$f(x) = $ ~~1, 0 or~~ $1, 0$ or -1 according as

$x >, =$ or < 0

Show that it is discontinuous at $x = 0$

Here,

$L.H.L \lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0-0} (-1)$

$= -1$

$R.H.L = \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0+0} 1$

$= 1$

$\therefore L.H.L \neq R.H.L$

$f(x)$ is discontinuous at $x = 0$

13. $f(x) = ~~1~~$ $1, 0$ or -1 according as $x > , =$ or < 0

Show that it is discontinuous at $x = 0$

Here,

$$L.H.L \lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0-0} (-1)$$

$$= -1$$

$$R.H.L = \lim_{x \to 0^+} f(x)$$

$$= \lim_{x \to 0+0} 1$$

$$= 1$$

$\therefore$ $L.H.L \neq R.H.L$

$f(x)$ is discontinuous at $x = 0$

19. $\lim_{x \to 0} \frac{\sin x - \tan x}{x^3} = -\frac{1}{2}$

solve:

$$\lim_{x \to 0} \frac{\sin x - \tan x}{x^3}$$

$$\lim_{x \to 0} \frac{1}{x^3} (\sin x - \tan x)$$

$$\lim_{x \to 0} \frac{1}{x^3} \left(\sin x - \frac{\sin x}{\cos x}\right)$$

$$= \lim_{x \to 0} \frac{1}{x^3} \left(\frac{\cos x \cdot \sin x - \sin x}{\cos x}\right)$$

$$= \lim_{x \to 0} \frac{1}{x^3} \frac{\cancel{\cos}\ \sin x\,(\cos x - 1)}{\cos x}$$

$1 - \cos x = 2\sin^2 \frac{x}{2}$

$\cos x - 1 = -2\sin^2 \frac{x}{2}$

$$= \lim_{x \to 0} \frac{1}{x^2} \cdot \frac{\sin x}{x} \cdot \left(\frac{-2\sin^2 x/2}{\cos x}\right)$$

$$= \lim_{x \to 0} \frac{1}{x^2} \frac{\sin x}{x} \frac{1}{\cos x} \left(-2\sin^2 x/2\right)$$

$$= -2 \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} \cdot \left(\frac{\sin x/2}{x/2}\right)^2 \times \frac{1}{4}$$

$$= -2 \times \frac{1}{4} \cdot \lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{1}{\cos x} \lim_{x \to 0} \left(\frac{\sin x/2}{x/2}\right)^2$$

$$= -\frac{1}{2} \cdot 1 \cdot \frac{1}{\cos 0} \cdot 1^2$$

$$= -\frac{1}{2} \cdot 1 \cdot \frac{1}{1} \cdot 1$$

$$= -1/2$$

15. $$\lim_{h\to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$$

$$= \lim_{h\to 0} \frac{\{(a^2+2ah+h^2)\sin(a+h)\} - a^2\sin a}{h}$$

$$= \lim_{h\to 0} \frac{a^2\sin(a+h) + 2ah\sin(a+h) + h^2\sin(a+h) - a^2\sin a}{h}$$

$$= \lim_{h\to 0} \frac{a^2\{\sin(a+h) - \sin a\} + 2ah\sin(a+h) + h^2\sin(a+h)}{h}$$

$$= \lim_{h\to 0} \frac{a^2\left\{2\cos\left(\frac{a+h+a}{2}\right).\sin\left(\frac{a+h-a}{2}\right)\right\}}{h} + \lim_{h\to 0} 2a\sin(a+h) + \lim_{h\to 0} h\sin(a+h)$$

$$= \lim_{h\to 0} \frac{a^2\, 2\cos\left(a+\frac{h}{2}\right)\sin\frac{h}{2}}{h} + \lim_{h\to 0} 2a\sin(a+h) + \lim_{h\to 0} h\sin(a+h)$$

$$= a^2 \lim_{h\to 0} \cos\left(a+\frac{h}{2}\right). \lim \frac{\sin h/2}{h/2} + 2a\sin(a+h) + 0$$

$$= a^2\cos a . 1 + 2a\sin a$$

$$= a^2\cos a + 2a\sin a$$

16. $$\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$$

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x}\left\{\sqrt{1+\frac{1}{\sqrt{x}}\cdot\sqrt{x+\sqrt{x}}}\right.}$$

$$\lim_{x\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{x}{x^2}+\sqrt{\frac{x}{(x^2)^2}}}}}$$

$$\lim_{x\to\infty} \frac{1}{\sqrt{1+\sqrt{1/x+\sqrt{1/x^3}}}}$$

$$\cancel{\lim_{x\to}} = \frac{1}{\sqrt{1+\sqrt{1/\infty+\sqrt{1/\infty}}}}$$

$$= \frac{1}{\sqrt{1+\sqrt{0}+\sqrt{0}}}$$

$$= 1/1$$

$$= 1$$

17. $\underset{x \to 0}{Lt} \dfrac{x^2 \sin(1/x)}{\sin x}$

$\Rightarrow \underset{x \to 0}{Lt} \dfrac{x}{\sin x} \times \underset{x \to 0}{Lt} x \sin \dfrac{1}{x}$

$\sin \frac{1}{x} \leq 1$

$\therefore x \sin x \leq x \cdot 1$

$\therefore x \to 0$

$\Rightarrow 1 \times 0$

$\Rightarrow 0$

18| $\underset{x \to 0}{Lt} \dfrac{\operatorname{cosec} x - \cot x}{x}$

$\Rightarrow \underset{x \to 0}{Lt} \dfrac{1 - \cos x}{x \sin x}$

$\Rightarrow \underset{x \to 0}{Lt} \dfrac{2 \sin^2 1/2 x}{x \; 2 \sin 1/2 \cdot x \cos 1/2 x}$

$\Rightarrow \underset{x \to 0}{Lt} \dfrac{\sin \frac{1}{2} x}{1/2 x} \cdot \underset{x \to 0}{Lt} \dfrac{1}{2 \cos \frac{1}{2} x}$

$\Rightarrow 1 \times \frac{1}{2}$

$= \frac{1}{2}$

$$\underset{x \to \alpha}{Lt} \frac{x+1}{x^2+1}$$

$$\Rightarrow \underset{x \to \alpha}{Lt} \frac{\frac{1}{x} + \frac{1}{x^2}}{1 + \frac{1}{x^2}}$$

$$\Rightarrow \frac{0+0}{1+0}$$

$$\Rightarrow 0$$

$$\underset{x \to 0}{Lt} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)$$

$$\underset{x \to 0}{Lt} \frac{1 - \cos x}{\sin x}$$

$$\underset{x \to 0}{Lt} \frac{2\sin^2 \frac{1}{2}x}{2\sin \frac{1}{2}x \cdot \cos \frac{1}{2}x}$$

$$\underset{x \to 0}{Lt} \tan \frac{1}{2}x$$

$$= 0$$

If $\phi(x) = \{(x+2)^2 - 4\} / x$. show that

$\underset{x \to 0}{Lt} \phi(x) = 4$, although $\phi(0)$ does not exist.

$$\Rightarrow \underset{x \to 0}{Lt} \phi(x) = \underset{x \to 0}{Lt} \frac{x(x+4)}{x}$$

$$= \underset{h \to 0}{Lt} (x+4)$$

$$= 0 + 4$$

$$= 4$$

$$\phi(0) = \frac{2^2 - 4}{0}$$

$= \frac{0}{0}$ which is meaningless.

$\therefore \phi(0)$ does not exist.

22. Show that the function f defined by

$$F(x) = \frac{x-1}{1+e^{\frac{1}{x-1}}} \text{, for } x \neq 1$$

$$= 0 \text{, for } x = 1$$

is continuous at $x = 1$.

$\Rightarrow$ we are given that,

$$f(x) = \frac{x-1}{1 - e^{\frac{1}{x-1}}}$$

$$e^{\frac{1}{x-1}} \longrightarrow \alpha$$

$$\Rightarrow \underset{x \to 1^-}{Lt} f(x) = 0$$

$$e^{\frac{1}{x-1}} \longrightarrow \alpha$$

$$\underset{x \to 1^+}{Lt} f(x) = 0$$

$$\therefore \underset{x \to 1^-}{Lt} f(x) = f(1) = \underset{x \to 1^+}{Lt} f(x) = 0$$

Hence, $f(x)$ is continuous at $x = 1$

23. (i) $f(x) = x$ when $0 < x < 1$

$= 2 - x$ when $1 \leq x \leq 2$ $\Big\}$ at $x = 2$

$= x - \frac{1}{2}x^2$ when $x > 2$

$\Rightarrow$ we are given that,

$f(x) = x$ when $0 < x < 1$

$= 2 - x$ when $1 \leq x \leq 2$

$= x - \frac{1}{2}x^2$ when $x > 2$

$f(x)$ exist at $x = 2$

$\therefore$ L.H.L $= \lim\limits_{x \to 2^-} f(x)$

$= \lim\limits_{x \to 2-0} (2 - x)$

$= (2 - 2 + 0)$

$= 0$

R.H.L $= \lim\limits_{x \to 2^+} f(x)$

$= \lim\limits_{x \to 2+0} (x - \frac{1}{2}x^2)$

$= \{2 - \frac{1}{2} \cdot (2)^2\}$

$= 2 - \frac{1}{2} \times 4$

$= 0$

$\therefore$ L.H.L = R.H.L

Hence $f(x)$ is continuous at $x=2$

$$f(x) = \left.\begin{array}{ll} \dfrac{\tan^2 x}{3x}, & x \neq 0 \\ = \dfrac{2}{3}, & x = 0 \end{array}\right\} \text{ at } x = 0$$

$\Rightarrow$ we are given that,

$$f(x) = \frac{\tan^2 x}{3x}, \quad x \neq 0$$

$$= \frac{2}{3}, \quad x = 0$$

$f(x)$ is continuous at $x=0$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\tan^2 x}{3x}$$

$$= \lim_{x \to 0} \left\{ \frac{\sin^2 x}{x^2} \cdot \frac{1}{\cos^2 x} \cdot \frac{x}{3} \right\}$$

$$= \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^2 \cdot \lim_{x \to 0} \left(\frac{1}{\cos^2 x} \right) \cdot \lim_{x \to 0} \left(\frac{1}{3} x \right)$$

$$= 1^2 \times 1^2 \times 0$$

$$= 0$$

But $f(0) = \frac{2}{3}$

$\therefore \lim_{x \to 0} f(x) \neq f(0)$

Hence, $f(x)$ is discontinuous at $x=0$

25. $$f(x) = \left.\begin{aligned} &x^2+x, && 0 \le x \\ &= 2, && x=1 \\ &= 2x^3-x+1, && 1 < x \le 2 \end{aligned}\right\} \text{ at } x=1$$

we are given that,

$$f(x) = x^2+x, \quad 0 \le x$$
$$= 2, \quad x=1$$
$$= 2x^3-x+1, \quad 1 < x \le 2$$

$\therefore f(x)$ is continuous at $x=1$

$$L.H.L = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1-0} x^2+x$$

$$= 1^2+1$$

$$= 2$$

$$R.H.L = \lim_{x \to 1^+} f(x)$$

$$= \lim_{x \to 1+0} (2x^3 - x + 1)$$

$$= 2 \times 1^3 - 1 + 1$$

$$= 2 - 1 + 1$$

$$= 2$$

$\therefore L.H.L = R.H.L$

Hence, $f(x)$ is continuous at $x = 1$

26. $$f(x) = \begin{cases} \dfrac{x^4 + 4x^3 + 2x}{\sin x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{at } x = 0$$

we are given that

$$f(x) = \frac{x^4 + 4x^3 + 2x}{\sin x}$$

$$\Rightarrow \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{x^3 + 4x^2 + 2}{\frac{\sin x}{x}}$$

$$= \frac{\lim_{x \to 0} (x^3 + 4x^2 + 2)}{\lim_{x \to 0} \left(\frac{\sin x}{x}\right)}$$

$= \frac{2}{1}$

$= 2$

$f(0) = 0$

~~thus~~ $f(x) \neq f(0)$

Hence $f(x)$ is discontinuous at $x = 0$

$$f(x) = 3 + 2x \quad \text{for } -\frac{3}{2} \leq x < 0$$
$$= 3 - 2x \quad \text{for } 0 \leq x < 3/2$$
$$= -3 - 2x \quad \text{for } x \geq 3/2$$

show that $f(x)$ is continuous at $x = 0$ and discontinuous at $x = \frac{3}{2}$

we are given that,

$$f(x) = 3 + 2x \quad , -\frac{3}{2} \leq x < 0$$
$$= 3 - 2x \quad , 0 \leq x \leq 3/2$$
$$= -3 - 2x \quad , x \geq 3/2$$

$f(x)$ is continuous at $x = 0$

$\therefore$ LH.L $= \lim\limits_{x \to 0^-} f(x)$

$= \lim\limits_{x \to 0-0} 3 + 2x$

$= 3 + 2(0 - 0)$

$= 3$

$$\text{R.H.L} = \lim_{x \to 0^+} f(x)$$

$$= \lim_{x \to 0+0} (3-2x)$$

$$= 3-2(0+0)$$

$$= 3$$

$\therefore$ $f(x)$ is continuous at $x=0$

$\therefore$ $f(x)$ is discontinuous at $x = 3/2$

$$\therefore \text{LH.L} = \lim_{x \to \frac{3}{2}^-} f(x)$$

$$= \lim_{x \to \frac{3}{2}-0} (3-2x)$$

$$= 3 - 2 \times \frac{3}{2} - 0$$

$$= 0$$

$$\text{R.H.L} = \lim_{x \to 3/2^+} f(x)$$

$$= \lim_{x \to \frac{3}{2}+0} (-3-2x)$$

$$= -3 - 2 \times \frac{3}{2} + 0$$

$$= -3-3+0$$

$$= -6$$

$\therefore$ LH.L $\neq$ R.H.L

$f(x)$ is discontinuous at $\frac{3}{2}$ (Proved)

28. $f(x) = \dfrac{e^{-1/2}}{1+e^{1/x}}$, when $x \neq 0$

$= 1$, when $x = 0$

$\Rightarrow$ we are given that,

$$F(x) = \frac{e^{-1/x}}{1+e^{1/x}}, \text{ when } x \neq 0$$

$$= 1 \quad , \text{ when } x = 0$$

$\lim\limits_{x \to 0} \dfrac{e^{-1/x}}{1+e^{1/x}}$ does not exist.

$\therefore$ $f(x)$ is not continuous.

29. $f(x) = \dfrac{\log(1+ax) - \log(1-bx)}{x}$ is not defined. $x = 0$ find the value of $f(0)$ so that $f(x)$ is continuous at $x = 0$

$\Rightarrow$ Here, $\lim\limits_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{\log(1+ax) - \log(1-bx)}{x}$$

$$= a \lim_{x \to 0} \frac{\log(1+ax)}{ax} - (-b) \cdot \lim_{x \to 0} \frac{\log(1-bx)}{(-bx)}$$

$\therefore f(x)$ is continuous at $x=0$

$\therefore f(x)$ as $x \longrightarrow 0$

Hence, $f(0) = a+b$

30. $f(x) = -2\sin x$, $-\pi \leq x \leq -\frac{\pi}{2}$

$= a\sin x + b$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$= \cos x$, $\frac{\pi}{2} \leq x < \pi$

$f(x)$ is continuous in the interval $-\pi \leq x \leq \pi$

find the value of a and b.

$\Rightarrow f(x)$ is continuous at $x = -\pi/2$

$$= \lim_{x \to -\frac{\pi}{2}-} f(x) = \lim_{x \to -\frac{\pi}{2}+} f(x) = f\left(-\frac{\pi}{2}\right)$$

$$= \lim_{x \to -\pi/2-} (-2\sin x) = \lim_{x \to -\frac{\pi}{2}+0} (a\sin x + b) = -2\sin\left(\frac{\pi}{2}\right.$$

$= -a+b = 2$ (1)

and $f(x)$ is continuous at $x = \pi/2$

$$\lim_{x \to \frac{\pi}{2}-} f(x) = \lim_{x \to \pi/2+} f(x) = f\left(\frac{\pi}{2}\right)$$

$$= \lim_{x \to \pi/2-} (a\sin x + b) = \lim_{x \to \pi/2+} (\cos x) = \cos \pi/2$$

$= a + b = 0$ ——— (ii)

solving 1 and 2 we get $a = -1$ $b = 1$