Electromagnetic Induction

When electrons are in motion, they produce a magnetic field. The converse of this is also true i.e. when a magnetic field embracing a conductor moves relative to the conductor, it produces a flow of electrons in the conductor.

This phenomenon whereby an e.m.f. and hence current (i.e. flow of electrons) is induced in any conductor which is cut across or is cut by a magnetic flux is known as electromagnetic induction.

Faraday's Laws of Electromagnetic Induction

First Law:

It states that whenever a conductor cuts magnetic flux, an e.m.f. is induced in that conductor.

Second Law:

It states that the magnitude of the induced e.m.f. is equal to the rate of change of flux-linkages.

Explanation:

Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 webers to the final value of Φ_2 webers in time t seconds.

Then we have,

Initial flux linkages = $N\Phi_1$

Final flux linkages = $N\Phi_2$

∴ induced e.m.f.

$$e = \frac{N\Phi_2 - N\Phi_1}{t} \ \frac{Wb}{s} \ or\ volt$$

$$e = N\frac{\Phi_2 - \Phi_1}{t} \ volt$$

Putting the above expression in its differential form, we get

$$e = \frac{d}{dt}(N\Phi)\ volt$$

$$e = N\frac{d\Phi}{dt}\ volt$$

Usually, a minus sign is given to signify the fact that the induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it

$$e = -N\frac{d\Phi}{dt}\ volt$$

Math

1. The field coils of a 6-pole d.c. generator each having 500 turns, are connected in series. When the field is excited, there is a magnetic flux of 0.02 Wb/pole. If the field circuit is opened in 0.02 second and residual magnetism is 0.002 Wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current.

   Solution:

   Here,

   Total number of turns, $N = 6 \times 500 = 3000$

   Total initial flux = $6 \times 0.02 = 0.12$ Wb

   Total residual flux = $6 \times 0.002 = 0.012$ Wb

   Change in flux, $d\Phi = 0.12 - 0.012 = 0.108$ Wb

   Time of opening the circuit, $dt = 0.02$ second

   We know,

   Induced e.m.f., $e = N\frac{d\Phi}{dt}$

   $= 3000 \times \frac{0.108}{0.02}$

   $= 16200$ V

   The direction of this induced e.m.f. is the same as the initial direction of the exciting current.

2. A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it. Find the average e.m.f. and the current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.

Solution:

Here,

$d\Phi$ = 1 − 0.2 = 0.8 mWb = 0.8×10^{-3} Wb

dt = 1/10 = 0.1 second

N = 100

We know,

$$\text{Induced e.m.f., } e = N\frac{d\Phi}{dt}$$

$$= 100 \times \frac{0.8\times10^{-3}}{0.1}$$

$$= 0.8 \text{ V}$$

Again,

Total circuit resistance, R = 100 + 400 = 500 Ω

$$\therefore \quad \text{Current induced, I} = \frac{0.8}{500}$$

$$= 1.6 \times 10^{-3} \text{ A}$$

$$= 1.6 \text{ mA}$$

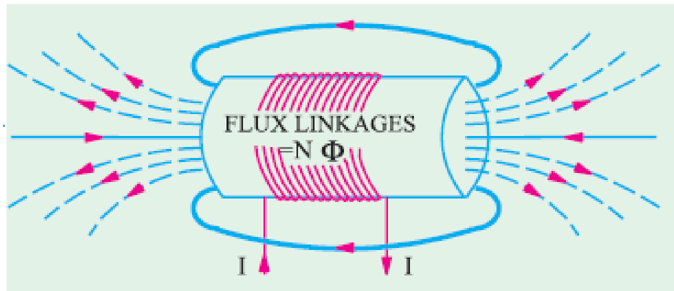
Lenz's Law:

Electromagnetically induced current always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

Self-inductance:

It is found that whenever an effort is made to increase current (and hence flux) through it, it is always opposed by the instantaneous production of counter e.m.f. of self-induction. Energy required to overcome this opposition is supplied by the battery.

If, now an effort is made to decrease the current (and hence the flux), then again it is delayed due to the production of self-induced e.m.f., this time in the opposite direction.

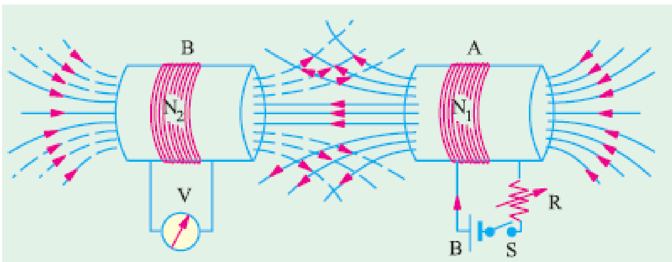


The property of the coil due to which it opposes any increase or decrease or current of flux through it, is known as self-inductance.

It is quantitatively measured in terms of coefficient of self-induction L.

Mutual Inductance:

Mutual inductance may, therefore, be defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil by induction when the current in the first coil changes.



This action being reciprocal, the second coil can also induce an e.m.f. in the first when current in the second coil changes.

This ability of reciprocal induction is measured in terms of the coefficient of mutual induction M.

Transformer:

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit.

Working Principle of a Transformer:

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is mutual induction between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance as shown in Fig.1. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction $e = MdI/dt$). If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil. The first coil, in which electric energy is fed from the a.c. supply mains, is called primary winding and the other from which energy is drawn out, is called secondary winding.

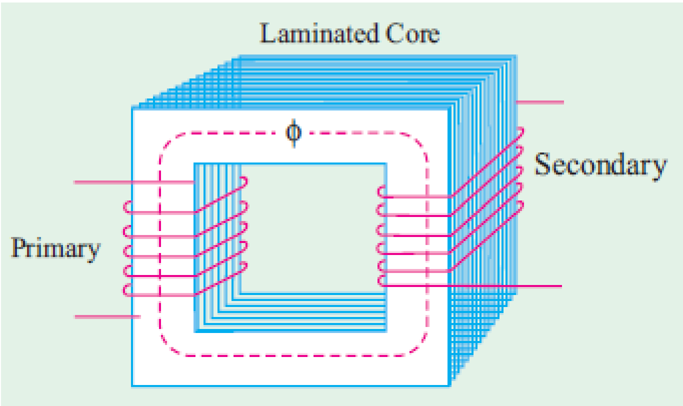


Figure-1: A Transformer

Ideal Transformer:

An ideal transformer is one which has no losses i.e. its windings have no ohmic resistance, there is no magnetic leakage and hence which has no I^2R and core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core.

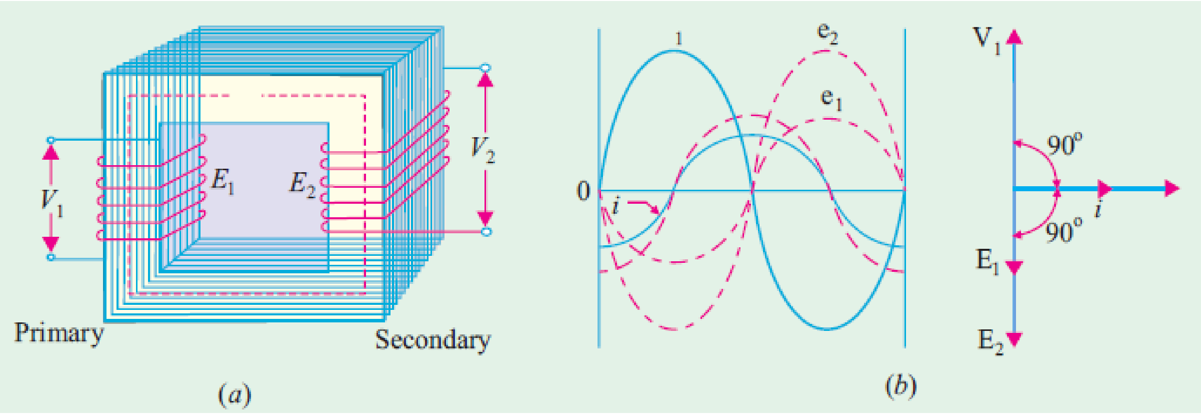


Figure-1

Consider an ideal transformer [Fig.1 (a)] whose secondary is open and whose primary is connected to sinusoidal alternating voltage V_1. This potential difference causes an alternating current to flow in the primary. Since the primary coil is purely inductive and there is no output (secondary being open) the primary draws the magnetising current $I\mu$ only. The function of this current is merely to magnetise the core, it is small in magnitude and lags V_1 by 90°. This alternating current $I\mu$ produces an alternating flux ϕ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, is in phase with it. This changing flux is linked both with the primary and the secondary windings. Therefore, it produces self-induced e.m.f. in the primary. This self-induced e.m.f. E_1 is, at every instant, equal to and in opposition to V_1. It is also known as counter e.m.f. or back e.m.f. of the primary.

Similarly, there is produced in the secondary an induced e.m.f. E_2 which is known as mutually induced e.m.f. This e.m.f. is antiphase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

E.M.F. Equation of a Transformer:

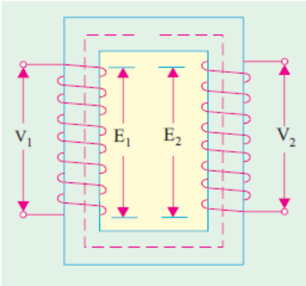


Figure-1: Transformer

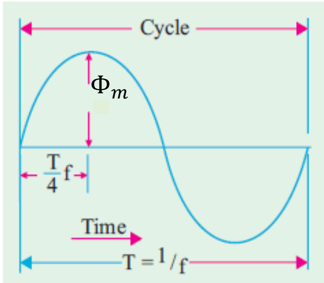


Figure-2: Flux Vs Time

Let us consider

N_1 = No. of turns in primary

N_2= No. of turns in secondary

Φ_m = Maximum flux in core in webers

f = Frequency of a.c. input in Hz

As shown in Fig-2, flux increases from its zero value to maximum value Φ_m in one quarter of the cycle i.e. in $1/4\ f$ second.

$$\therefore \text{ Average rate of change of flux } = \frac{\Phi_m}{\frac{1}{4} f}$$

$$= 4\ f\ \Phi_m \text{Wb/s or volt}$$

Now, rate of change of flux per turn means induced e.m.f. in volts,

$\therefore$ Average e.m.f./turn $= 4\ f\ \Phi_m$volt

If flux Φ varies sinusoidally, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = 1.11$$

$$\therefore \text{ r.m.s. value of e.m.f./turn} = 1.11 \times 4\ f\ \Phi_m \text{ volt}$$

$$= 4.44\ f\ \Phi_m \text{ volt}$$

Now, r.m.s. value of the induced e.m.f. in the whole of primary winding

$$= (\text{induced e.m.f/turn}) \times \text{No. of primary turns}$$

$$E_1 = 4.44\, f\, \Phi_m N_1 \quad \dots\dots\dots\dots\dots\dots\dots \text{(i)}$$

Similarly, r.m.s. value of the e.m.f. induced in secondary is,

$$E_2 = 4.44\, f\, \Phi_m N_2 \quad \dots\dots\dots\dots\dots\dots\dots \text{(ii)}$$

From equations (i) and (ii), we get

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \dots\dots\dots\dots\dots (iii)$$

In an ideal transformer on no-load, $V_1 = E_1$ and $V_2 = E_2$ where V_2 is the terminal voltage. So we can get,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} \quad \dots\dots\dots\dots\dots (iv)$$

Again, for an ideal transformer, input VA = output VA. i.e.

$$V_1 I_1 = V_2 I_2$$

$$or, \qquad \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Hence,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Math

1. The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is 1.2 Wb/m^2. If the e.m.f. per turn is 8 volt, determine (i) primary and secondary turns (ii) area of the core.

Solution:

Here,

E_1 = 250 volts

E_2 = 3000 volts

f = 50 Hz

B_m = 1.2 Wb/m^2

N_1 =?

N_2 =?

A =?

We know,

(i) $E_1 = N_1 \times$ e.m.f. induced/turn

$\therefore$ $N_1 = \frac{250}{8}$

$= 32$

$N_2 = \frac{3000}{8}$

$= 375$

(ii) $E_2 = 4.44\, f\, N_2\, B_m A$

or, $3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$

or, $A = 0.03\ m^2$

2. A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding be connected to a 50-Hz supply at 520 V, calculate (i) the peak value of flux density in the core (ii) the voltage induced in the secondary winding.

Solution:

Here,

N_1 = 400

N_2 = 1000

E_1 = 520 volts

f = 50 Hz

A = 60 cm^2

= $60 \times 10^{-4}\ m^2$

B_m =?

E_2 =?

(i) We know,

$$E_1 = 4.44\, f\, N_1\, B_m A$$

$$\therefore \quad B_m = \frac{E_1}{4.44\, f\, N_1 A}$$

$$= \frac{520}{4.44 \times 50 \times 400 \times 60 \times 10^{-4}}$$

$$= 0.976\ Wb/m^2$$

(ii) We know,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\therefore \quad E_2 = \frac{E_1 N_2}{N_1}$$

$$= \frac{520 \times 1000}{400}$$

= 1300 V

3. A single-phase transformer has 500 turns in the primary and 1200 turns in the secondary. The cross-sectional area of the core is 80 sq. cm. If the primary winding is connected to a 50 Hz supply at 500 V, calculate (i) Peak flux-density, and (ii) Voltage induced in the secondary.

Solution:

Here,

N_1 = 500

N_2 = 1200

E_1 = 500 volts

f = 50 Hz

A = 80 cm^2

= $80 \times 10^{-4}\ m^2$

B_m =?

E_2 =?

(i) We know,

$E_1 = 4.44\, f\, N_1\, B_m A$

$$\therefore \quad B_m = \frac{E_1}{4.44\, f\, N_1 A}$$

$$= \frac{500}{4.44 \times 50 \times 500 \times 80 \times 10^{-4}}$$

= 0.563 Wb/m^2

(ii) We know,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\therefore \quad E_2 = \frac{E_1 N_2}{N_1}$$

$$= \frac{500 \times 1200}{500}$$

= 1200 V

Energy Stored in a Magnetic Field:

When current through an inductive coil is gradually changed from zero to maximum value I, then every change of it is opposed by the self-induced e.m.f. produced due to this change. Energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil and is, later on, recovered when that field collapses. The value of this stored energy may be found in the following two ways:

(i) First Method.

Let, at any instant,

i = instantaneous value of current

e = induced e.m.f. at that instant = $L\frac{di}{dt}$

Then, work done in time dt in overcoming this opposition is

$$\begin{aligned} dW &= ei\,dt \\ &= L\frac{di}{dt}\times i\times dt \\ &= Li\,di \end{aligned}$$

Total work done in establishing the maximum steady current of I is

$$\begin{aligned} \int_0^W dW &= \int_0^I Li\,di \\ &= L\int_0^I i\,di \\ &= L\left[\frac{i^2}{2}\right]_0^I \\ &= L\left[\frac{I^2}{2}-\frac{0}{2}\right] \\ &= L\left[\frac{I^2}{2}-0\right] \\ &= \frac{1}{2}LI^2 \end{aligned}$$

$$\text{Or,}\qquad W = \frac{1}{2}LI^2$$

This work is stored as the energy of the magnetic field.

$$\therefore\ E = \frac{1}{2}LI^2\ joules$$

(ii) Second Method

If current grows uniformly from zero value to its maximum steady value I, then average current is

$$\frac{I}{2}$$

If L is the inductance of the circuit, then self-idcued e.m.f. is

$$e = L\frac{I}{t}$$

where 't' is the time for the current change from zero to I.

$$\therefore \text{ Average power absorbed } = \text{ induced e.m.f. } \times \text{ average current}$$

$$= L\frac{I}{t} \times \frac{I}{2}$$

$$= \frac{1}{2}\frac{LI^2}{t}$$

$$\text{Total energy absorbed } = \text{ power} \times \text{time}$$

$$= \frac{1}{2}\frac{LI^2}{t} \times t$$

$$= \frac{1}{2}\,LI^2$$

∴ Energy Stored

$$E = \frac{1}{2}\,LI^2$$

1. The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω when the exciting voltages is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.

Solution:

Here,

V = 230 volt

R = 50 Ω

N = 960

$\Phi = 0.005\ Wb$

L =?

E =?

We know,

Current through coil = $\frac{230}{50}$

$= 4.6$ A

Self-inductance, $L = \frac{N\Phi}{I}$

$= \frac{960 \times 0.005}{4.6}$

$= 1.0435\ H$

Energy stored, E = $\frac{1}{2}LI^2$

$= \frac{1}{2} \times 1.0435 \times (4.6)^2$

$= 11.04$ J