### **NETWORK THEOREMS**

- 1. SUPERPOSITION THEOREM
- 2. THEVENIN'S THEOREM
- 3. NORTON'S THEOREM
- 4. MAXIMUM POWER TRANSFER THEOREM

#### **APPLICATION OF THEVENIN'S THEOREM**

- Analyze networks with sources that are not in series or parallel.
- Reduce the number of components required to establish the same characteristics at the output terminals.
- Investigate the effect of changing a particular component on the behaviour of a network without having to analyze the entire network after each change.

#### STATEMENT OF THEVENIN'S THEOREM

Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shown in Fig. 9.23

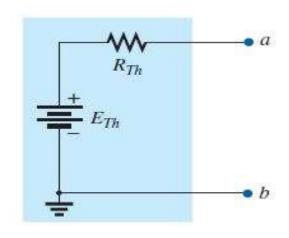


FIG. 9.23
Thévenin equivalent circuit.

#### STEPS OF THEVENIN'S THEOREM

To **demonstrate** the **power** of the theorem, consider the fairly complex network of Fig. 9.25(a)

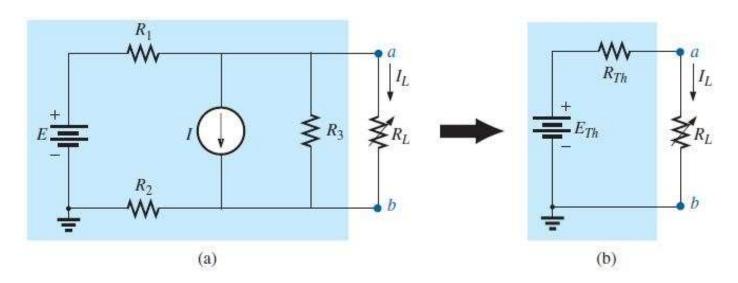


FIG. 9.25
Substituting the Thévenin equivalent circuit for a complex network.

#### STEPS OF THEVENIN'S THEOREM

- 1. Remove that portion of the network where the Thévenin equivalent circuit is found. In Fig. 9.25(a), this requires that the load resistor RL be temporarily removed from the network.
- 2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

#### **Determination of RTh:**

3. Calculate R<sub>Th</sub> by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

#### STEPS OF THEVENIN'S THEOREM

#### **Determination of Eth:**

4. Calculate  $E_{Th}$  by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the opencircuit potential between the two terminals marked in step 2.)

#### **Conclusion:**

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor RL between the terminals of the Thévenin equivalent circuit as shown in Fig. 9.25(b).

**EXAMPLE 9.6** Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.26. Then find the current through RL for values of 2  $\Omega$ , 10  $\Omega$ , and 100  $\Omega$ .

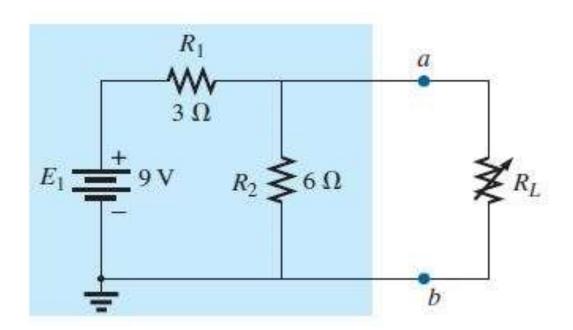


FIG. 9.26 Example 9.6.

**Solution:** Steps 1 and 2: These produce the network in Fig. 9.27. Note that the load resistor RL has been removed and the two "holding" terminals have been defined as a and b.

Steps 3: Replacing the voltage source E1 with a short-circuit equivalent yields the network in Fig. 9.28(a), where

$$R_{Th} = R_1 \| R_2 = \frac{(3 \Omega)(6 \Omega)}{3 \Omega + 6 \Omega} = 2 \Omega$$

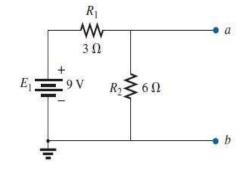
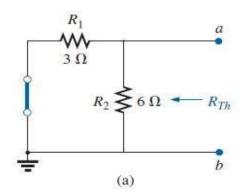


FIG. 9.27

Identifying the terminals of particular importance when applying Thévenin's theorem.



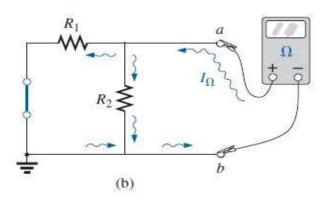


FIG. 9.28 Determining  $R_{Th}$  for the network in Fig. 9.27.

**Solution:** Step 4: Replace the voltage source (Fi 9.29). For this case, the open circuit Voltage *Eth* is the same as the voltage drop across the 6  $\Omega$  resistor. Applying the voltage divider rule,

$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6 \Omega)(9 \text{ V})}{6 \Omega + 3 \Omega} = \frac{54 \text{ V}}{9} = 6 \text{ V}$$

The use of a voltmeter to measure Eth appears in Fig. 9.30. Note that it is placed directly across the resistor R2 since Eth and  $V_{R2}$  are in parallel.

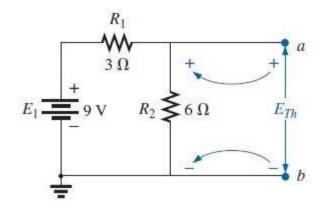


FIG. 9.29 Determining  $E_{Th}$  for the network in Fig. 9.27.

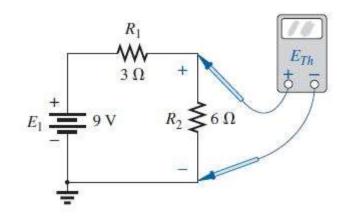


FIG. 9.30 Measuring  $E_{Th}$  for the network in Fig. 9.27.

**Solution:** Step 5 (Fig. 9.31):

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$
 $R_L = 2 \Omega$ :  $I_L = \frac{6 \text{ V}}{2 \Omega + 2 \Omega} = 1.5 \text{ A}$ 
 $R_L = 10 \Omega$ :  $I_L = \frac{6 \text{ V}}{2 \Omega + 10 \Omega} = 0.5 \text{ A}$ 
 $R_L = 100 \Omega$ :  $I_L = \frac{6 \text{ V}}{2 \Omega + 10 \Omega} = 0.06 \text{ A}$ 

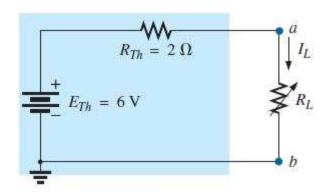


FIG. 9.31
Substituting the Thévenin equivalent circuit for the network external to R<sub>L</sub> in Fig. 9.26.

**EXAMPLE 9.7** Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.32.

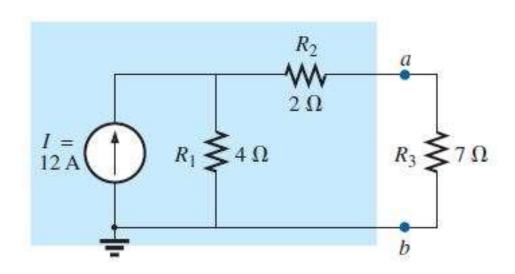


FIG. 9.32 Example 9.7.

**EXAMPLE 9.9** Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network in Fig. 9.43.

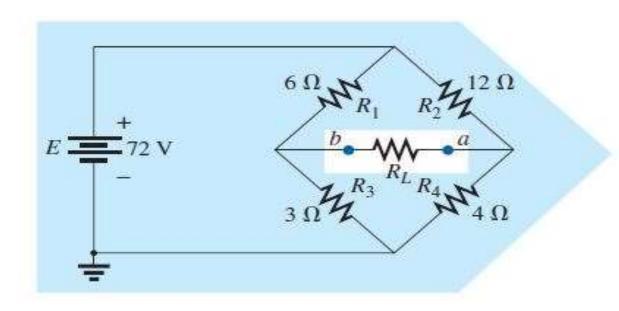


FIG. 9.43 Example 9.9.

FIG. 9.45

#### **Solution:**

Steps 1 and 2: See Fig. 9.44

Step 3: See Fig. 9.45. In this case, the short-circuit replacement of the voltage source E provides a direct connection between c and c\_ in Fig. 9.45(a), permitting a "folding" of the network around the horizontal line of a-b to produce the configuration in Fig. 9.45(b).

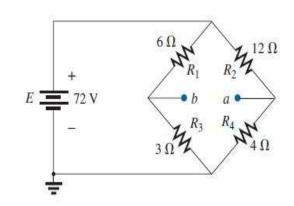
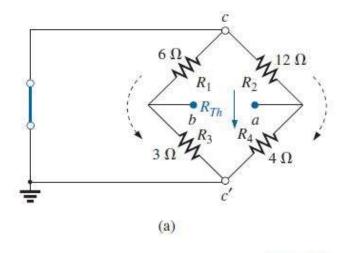
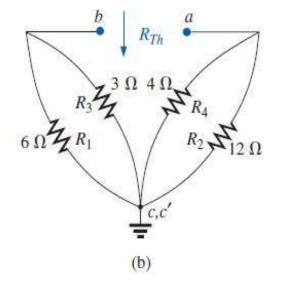


FIG. 9.44
Identifying the terminals of particular interest for the network in Fig. 9.43.





Step 4: The circuit is redrawn in Fig. 9.46. The absence of a direct connection between a and b results in a network with three parallel branches. The voltages V1 and V2 can therefore be determined using the voltage divider rule:

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6 \Omega)(72 \text{ V})}{6 \Omega + 3 \Omega} = \frac{432 \text{ V}}{9} = 48 \text{ V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12 \Omega)(72 \text{ V})}{12 \Omega + 4 \Omega} = \frac{864 \text{ V}}{16} = 54 \text{ V}$$

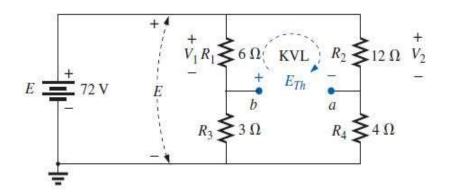


FIG. 9.46
Determining  $E_{Th}$  for the network in Fig. 9.44.

Assuming the polarity shown for  $E_{Th}$  and applying Kirchhoff's voltage law to the top loop in the clockwise direction results in

$$\Sigma_{C} V = +E_{Th} + V_1 - V_2 = 0$$

$$E_{Th} = V_2 - V_1 = 54 \text{ V} - 48 \text{ V} = 6 \text{ V}$$

and

Step 5: See Fig. 9.47

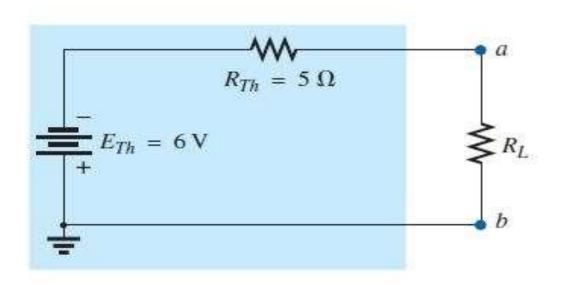


FIG. 9.47

Substituting the Thévenin equivalent circuit for the network external to the resistor  $R_L$  in Fig. 9.43.

**EXAMPLE 9.8:** Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.37. Note in this example that there is no need for the section of the network to be preserved to be at the "end" of the configuration.

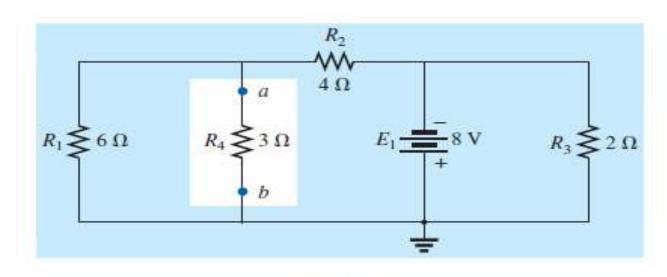


FIG. 9.37 Example 9.8.

Steps 1 and 2: See Fig. 9.38

Step 3: See Fig. 9.39. Steps 1 and 2 are relatively easy to apply, but now we must be careful to "hold" onto the terminals a and b as the Thévenin resistance and voltage are determined. In Fig. 9.39, all the remaining elements turn out to be in parallel, and the network can be redrawn as shown.

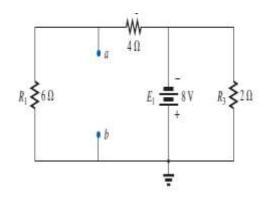


FIG. 9.38

Identifying the terminals of particular interest for the network in Fig. 9.37.

$$R_{Th} = R_1 \| R_2 = \frac{(6 \Omega)(4 \Omega)}{6 \Omega + 4 \Omega} = \frac{24 \Omega}{10} = 2.4 \Omega$$

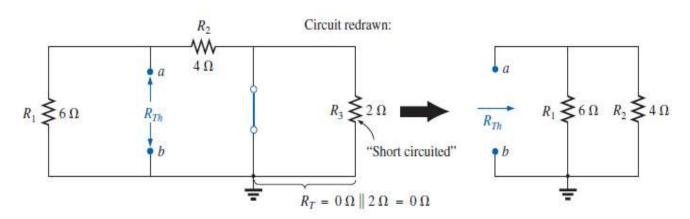


FIG. 9.39

Determining  $R_{Th}$  for the network in Fig. 9.38.

Step 4: See Fig. 9.40. In this case, the network can be redrawn as shown in Fig. 9.41. Since the voltage is the same across parallel elements, the voltage across the series resistors R1 and R2 is E1, or 8 V. Applying the voltage divider rule

$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6 \Omega)(8 \text{ V})}{6 \Omega + 4 \Omega} = \frac{48 \text{ V}}{10} = 4.8 \text{ V}$$

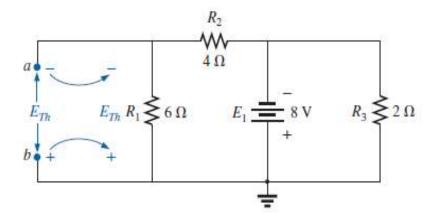


FIG. 9.40
Determining  $E_{Th}$  for the network in Fig. 9.38.

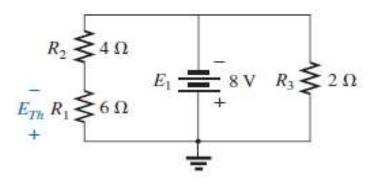


FIG. 9.41 Network of Fig. 9.40 redrawn.

Step 5: See Fig. 9.42.

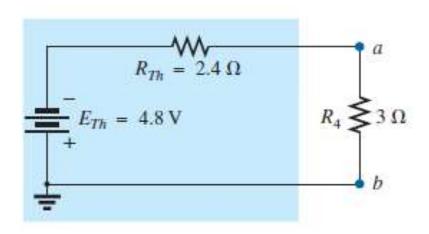


FIG. 9.42

Substituting the Thévenin equivalent circuit for the network external to the resistor R<sub>4</sub> in Fig. 9.37.

1. The Thevenin voltage is the \_\_\_\_\_

- a) Open circuit voltage
- b) Short circuit voltage
- c) Open circuit and short circuit voltage
- d) Neither open circuit nor short circuit voltage

ANS: A. Open circuit voltage

2. Thevenin's theorem is true for \_\_\_\_\_

- A. Linear networks
- B. Non-Linear networks
- C. Both linear networks and nonlinear networks
- D. Neither linear networks nor non-linear networks

ANS: A. Linear networks

3. Vth is found across the \_\_\_\_\_ terminals of the network.

- a) Input
- b) Output
- c) Neither input nor output
- d) Either input or output

ANS: B. Output

4. Thevenin's equivalent circuit consists of ---.

- a) Voltage source and resistor connected in series
- b) Voltage source and resistor connected in Parallel
- c) Current source and resistor connected in series
- d) Current source and resistor connected in Parallel

ANS: A. Voltage source and resistor connected in series

5. Current in Thevenin's equivalent circuit can determine using---.

- a) Ohm's law
- b) KCL
- c) KVL
- d) Current divider Law

ANS: A. Ohm's law

#### **HOME WORK**

**PRACTICE PROBLEM**: Find the Thévenin equivalent circuit for the network external to the resistor *R* for the network in the following circuit.

