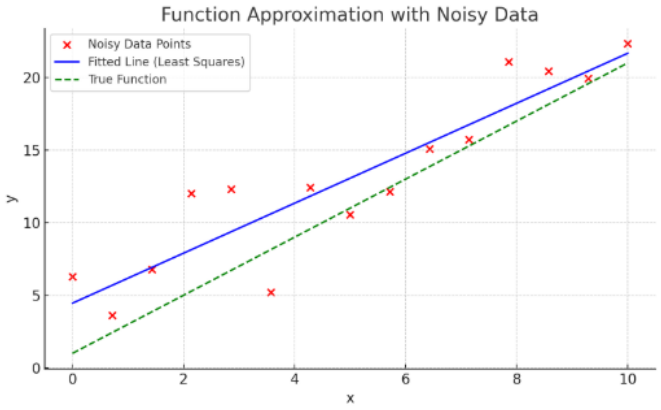
Function approximation:

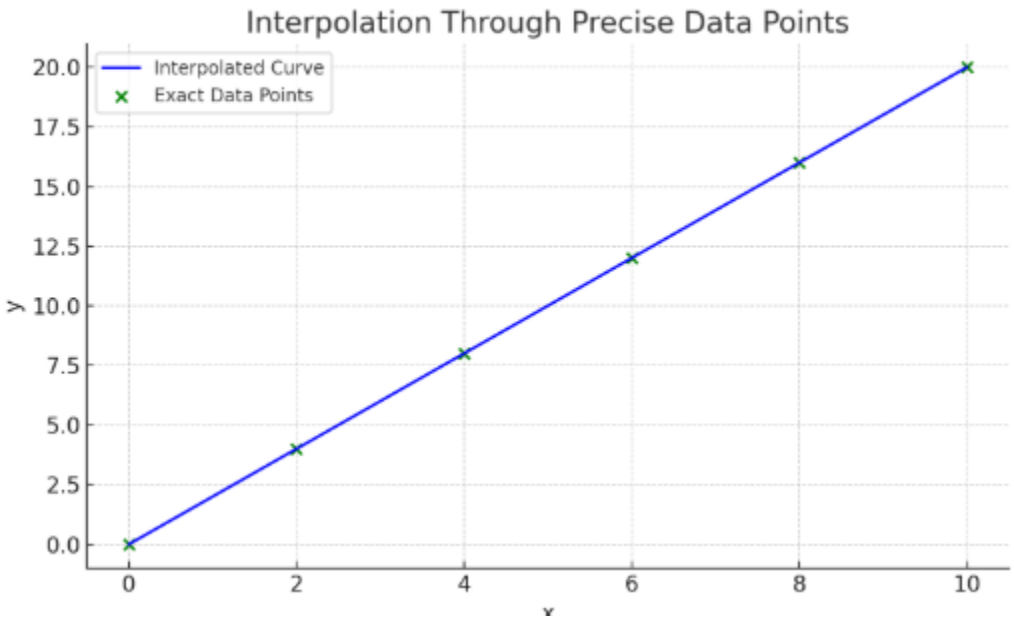
Function approximation means finding a simple function that is close enough to a more complicated or unknown function.

a) where these data exhibit a significant degree of error or “noise,” the strategy is to derive a single curve that represents the general trend of these data. Because any individual data point may be incorrect, we make no effort to intersect every point. Rather, the curve is designed to follow the pattern of the points taken as a group. One approach of this nature is called least squares regression.



- 🔴 **Red dots**: These are the data points, but they include some **noise** (errors or randomness).
- 🔵 **Blue line**: This is the **least squares regression line** — it doesn't try to pass through every dot but shows the general trend.
- 🟢 **Green dashed line**: The original function (`y = 2x + 1`) before noise was added — which we usually don't know in real problems.

b) Second, where these data are known to be very precise, the basic approach is to fit a curve or a series of curves that pass directly through each of the points. Such data usually originate from tables. Examples are values for the density of water or for the heat capacity of gases as a function of temperature. The estimation of values between well-known discrete points is called interpolation.



- 🟢 **Green dots**: Exact data points (like from a scientific table — e.g., water density at certain temperatures).
- 🔵 **Blue line**: The **interpolated curve** that passes exactly through **each data point**.

• Least Square Curve fitting:

Curve fitting is a way to draw the best possible curve or line through a set of data points. It helps us find a mathematical relationship (equation) that describes how the data behaves.

Least squares curve fitting methods are mathematical techniques used to find the curve that best fits a set of data points by minimizing the sum of the squared differences (errors) between the observed values and the values predicted by the curve.

Here are the main types of least squares curve fitting methods:

1. Linear Least Squares

- Fits a straight line (or linear function of parameters) to the data.
- Equation: $y = ax + b$
- Extension: Multiple linear regression
- **Use:** When data follows an approximately linear relationship.

2. Polynomial Least Squares

- Fits a polynomial curve of degree n.
- Equation: $y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$
- Higher degrees can capture nonlinear patterns but may cause overfitting.
- **Use:** For nonlinear trends in data.

3. Nonlinear Least Squares

- Used when the model equation is nonlinear in parameters.
- Example: $y = ae^{bx}$, $y = \frac{a}{b+x}$
- Requires iterative methods (Gauss–Newton, Levenberg–Marquardt).
- **Use:** When relationships are nonlinear and cannot be transformed easily

4. Weighted Least Squares (WLS)

- Assigns different weights to data points, reducing the influence of points with higher variance or lower reliability.
- Error term minimized: $\sum w_i (y_i - f(x_i))^2$
- **Use:** When data has unequal variances.

N.B.: For details, see *Introduction to Numerical Analysis* by S.S. Sastry, Chapter 4