ELECTROSTATICS

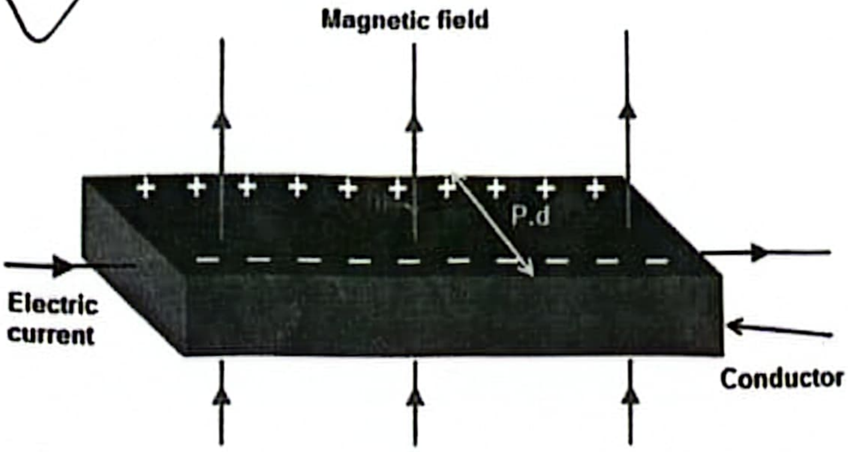
Electrostatics is the study of stationary electric charges or fields as opposed to electric currents.

We begin our study of electrostatics by investigating the two fundamental laws governing electrostatic fields:

(1) Coulomb's law and

(2) Gauss's law.

Hall Effect



P.d = Potential difference

Hall Effect

Mainly Lorentz force is responsible for Hall effect. All of we know that when we place a current carrying conductor inside a magnetic field, the conductor experiences a mechanical force to a direction depending upon the direction of magnetic field and the direction of current in the conductor. The electric current means a flow of charge. In metal it is entirely due to the flow of electrons, in semiconductor, it is due to flow of free electrons as well as holes. In semiconductor, holes move in the direction of conventional current and free electrons move in the opposite of the direction of conventional current. As the electrons have charge, they experience a force while flowing through a conductor placed inside a magnetic field. Due to this force, the electrons get diverted towards one side of the conducted during flowing. As the following charges get shifted to one side of the conductor, there may be a tiny potential difference appeared across the cross-section of the conductor. We call this entire phenomenon as hall effect.

Application of Hall Effect:

Hall effect is a very useful phenomenon and helps to

- Determine the Type of Semiconductor
- Calculate the Carrier Concentration
- Measure Magnetic Flux Density
- Determine if a substance is a semiconductor or an insulator

Math

1. Calculate the magnetising force and flux density at a distance of 5 cm from a long straight circular conductor carrying a current of 250 A and placed in air.

Here,

Current, $I = 250\ A$

$radius, r = 5\ cm = \frac{5}{100}\ m = \ 0.05\ m$

The magnetising force $\mathrm{H} = ?$

The flux density $B = ?$

We know,

$$H = \frac{I}{2\pi r}$$

$$H = \frac{250}{2 \times 3.1416 \times 0.05} = 795.77 \frac{AT}{m}$$

Again, we know,

$$B = \mu_0 H$$

$$B = 4\pi \times 10^{-7} \times 795.77$$

$$B = 0.001\ Wb\ m^{-2}$$

Biot-Savart Law:

The magnetic effects of a current must be formulated in terms of line elements of current. Since each portion of a complete circuit in which a circuit flow makes its own contribution to the magnetic field.

The magnetic field (dB) at any point is proportional to the length of conductor of small part (dl), proportional to the conductor carrying current (i), and proportional to the sine angle between the small length of conductor and the displacement vector from the element to the point P and inversely proportional to the square of the displacement vector from the element to P.

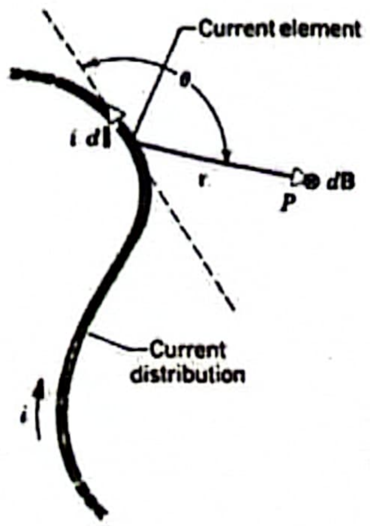


Figure-1: The current element dl establishes a magnetic field contribution dB at point P.

Explanation:

Let us consider dl be a line element of a conductor carrying current i.

According to Biot-savart law the magnetic field dB is given by,

$$dB = \frac{\mu_0 i dl \sin\theta}{4\pi r^2} \quad \text{................} \quad (1)$$

Where,

dB = Magnitude of the magnetic induction

i = Conductor carrying current

dl = Length of the conductor

r = Displacement vector from the element to P

$$\Rightarrow \oint \vec{B}\,.\,d\vec{l} = B(2\pi r) \text{ .... } (2)$$

From equation (1) and (2), we get,

$\oint \vec{B}.d\vec{l} = \mu_0 i$. This is Ampere's law.

Application of Ampere's Law:

$\vec{B}$ Near a Long Wire:

Let us consider a conducting wire carrying a current i and $\vec{B}$ is the magnetic field at a distance r from the center of the long wire.

Applying the Ampere's law, we get,

$$\oint \vec{B}.d\vec{l} = \mu_0 i$$

$$\Rightarrow \oint Bdl\cos\theta = \mu_0 i$$

$$\Rightarrow \oint Bdl = \mu_0 i$$

$$\Rightarrow B\oint dl = \mu_0 i \qquad [\because \oint dl = 2\pi r]$$

$$\Rightarrow B(2\pi r) = \mu_0 i$$

$$\therefore\ B = \frac{\mu_0 i}{2\pi r}$$

Applications of Biot-savart Law:

A Long Straight Wire:

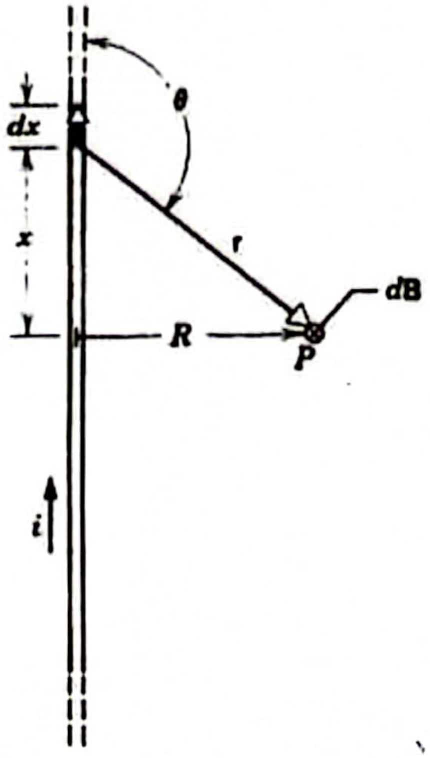


Figure-2: The magnetic field dB established by a current element in a long straight wire points into the page at P.

Let us consider a point P is taken at a distance r from the current element of a long straight wire dx which carrying a current i and R is the perpendicular distance from the wire to the point P.

Applying Biot-savart law, we get,

$$dB = \frac{\mu_0 i dx sin\theta}{4\pi r^2}$$ where θ is the angle between dx and r

$$\therefore B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{dx sin\theta}{r^2} \quad \text{..............} (1)$$

Now, $r = \sqrt{x^2 + R^2}$ .... (2)

$$\Rightarrow \oint \vec{B}.d\vec{l} = B(2\pi r) \text{ .... } (2)$$

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$$\Rightarrow B\oint dl = \mu_0 i \qquad [\because \oint dl = 2\pi r]$$

$$\Rightarrow B(2\pi r) = \mu_0 i$$

$$\therefore\ B = \frac{\mu_0 i}{2\pi r}$$

Lorenz Force:

If a positive test charge q_0 is fired with velocity $\vec{v}$ through a point P and if both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ act on the charge, the resultant force $\vec{F}$ acting on the charge is found by

$$\vec{F} = q_0\vec{E} + q_0\ (\vec{v} \times \vec{B})$$

This force is called the Lorentz force.

Ampere's Law:

For any closed loop, sum of the length elements times the magnetic field in the direction of the length elements is equal to the permeability times the electric current enclosed in the loop.

Ampere's law states that the line integral of the magnetic field around a closed loop is equal to the permeability μ_0 times the electric current passing through that loop.

Mathematically,

$$\oint \vec{B}.d\vec{l} = \mu_0 i$$

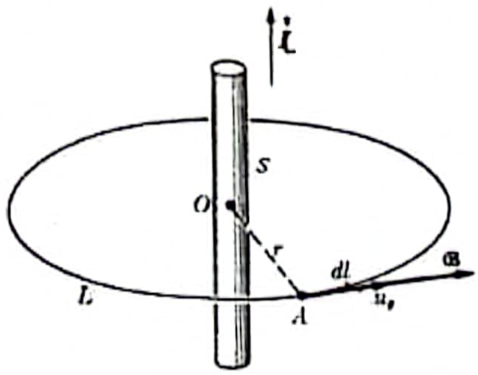


Figure-1: A circular path of integration surrounding a wire

Explanation:

Let us consider a conducting wire carrying a current i and $\vec{B}$ is the magnetic field at a distance r from the center of the wire as shown in fig-1.

The experimental result can be expressed by the proportionality.

$$B \propto \frac{i}{r}$$

$$\Rightarrow B = \frac{\mu_0}{2\pi}\frac{i}{r}$$

$$\Rightarrow B(2\pi r) = \mu_0 i \cdots\cdots\cdots\cdots\cdots (1)$$

For all points on this circle $\vec{B}$ has same magnitude. Thus,

$$\oint \vec{B}.d\vec{l} = \oint B\, dl cos\theta$$

$$\Rightarrow \oint \vec{B}.d\vec{l} = \oint B\, dl$$

$$\Rightarrow \oint \vec{B}.d\vec{l} = B \oint dl$$

The Magnetic Force on A Moving Charge:

Let us therefore consider a set of measurements that, least in principle, could be done to study the magnetic force that may act on an electric charge. (In these experiments, we consider only electric or magnetic forces; we assume that the experiments are carried out in an environment where other forces, such as gravity, may be neglected.)

1. We first test for the presence of an electric force by placing a small test charge at rest at various locations. Later we can subtract the electric force (if any) from the total force, which presumably leaves only the magnetic force. We assume this has been done, so that from now on we can ignore any electric force that acts on the charge.

2. Next, we project the test charge q through a particular point P with a velocity $\vec{v}$. We find that the magnetic force $\vec{F}$, if it is present, always acts sideways, that is, at right angles to the direction of $\vec{v}$. We can repeat the experiment by projecting the charge through P in different directions; we find that, no matter what the direction of $\vec{v}$, the magnetic force is always at right angles to that direction.

3. As we vary the direction of $\vec{v}$ through point P, we also find that the magnitude of $\vec{F}$ changes from zero when $\vec{v}$ has a certain direction to a maximum when it is at right angles to that direction. At intermediate angles, the magnitude of $\vec{F}$ varies as the sine of the angle ϕ that the velocity vector makes with that particular direction. (Note that there are actually two directions of $\vec{v}$ for which $\vec{F}$ is zero; these directions are opposite to each other, that is, $\phi = 0° \; or \; 180°$.)

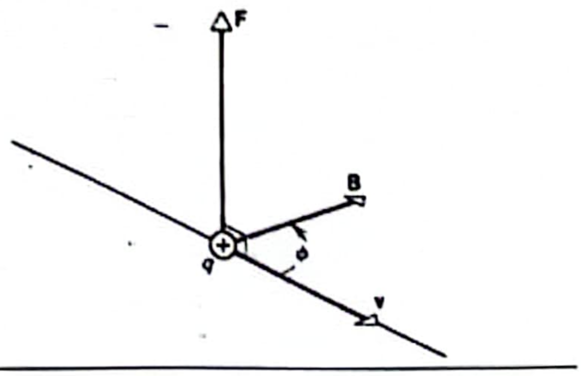


Figure: A particle with a positive charge q moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ experiences a magnetic deflecting force $\vec{F}$.

4. As we vary the magnitude of the velocity, we find that the magnitude of F varies in direct proportion.

5. We also find that F is proportional to the magnitude of the test charge q, and that F reverses direction when q changes sign.

Magnetic Field:

We describe the space around a permanent magnet or a current-carrying conductor as the location of a magnetic field.

Generally, a region of space in which a magnetic pole experiences an applied force is called a magnetic field.

Absolute and Relative Permeabilities of a Medium:

The phenomena of magnetism and electromagnetism are dependent upon a certain property of the medium called its permeability. Every medium is supposed to possess two permeabilities:

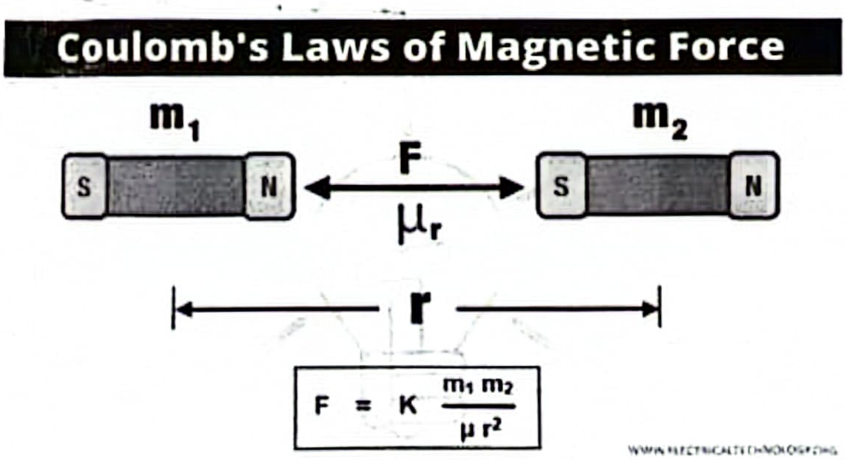
(i) absolute permeability (μ) and
(ii) (ii) relative permeability (μ_r).

$$Here, \mu = \mu_0 \mu_r \text{ H/m.}$$

Laws of Magnetic Force:

The force between two magnetic poles placed in a medium is:

(i) directly proportional to their pole strengths
(ii) inversely proportional to the square of the distance between them and
(iii) inversely proportional to the absolute permeability of the surrounding medium.



Magnetic Field Strength ($\vec{H}$):

Magnetic field strength at any point within a magnetic field is numerically equally to the force experienced by a N-pole of one weber placed at that point. Hence, unit of H is N/Wb.

$$\Rightarrow \quad \varepsilon_0 E(2\pi r h) = \lambda h \quad [\because S = 2\pi r h]$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Spherically Symmetric Charge Distribution:

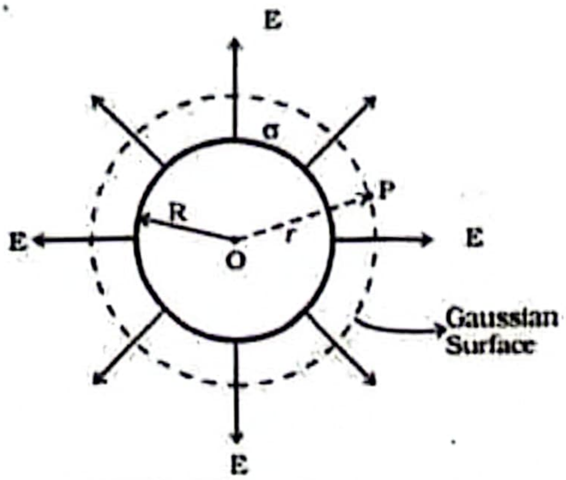


Fig1.20a. Field at a point outside the shell

Electric field intensity $\vec{E}$ for outside the sphere (r > R).

Let us consider a spherical Gaussian surface of radius r, outside the charge distribution i.e., r > R.

Applying Gauss's law, we get,

$$\varepsilon_0 \oint \vec{E}.d\vec{s} = q \quad \text{where q is the total charge}$$

$$\Rightarrow \varepsilon_0 \oint E ds \cos 0^0 = q$$

$$\Rightarrow \varepsilon_0 E S = q \quad \text{where, } S = \text{The area of the surface}$$

$$\Rightarrow \varepsilon_0 E(4\pi r^2) = q \quad [\because S = 4\pi r^2]$$

$$\therefore E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Electric field intensity $\vec{E}$ for inside the sphere (r < R).

1

Gauss's law Math:

1. A plastic rod, whose length h is 220 cm and whose radius r is 3.6 mm, carries a negative charge q of magnitude 3.8×10^{-7} C, spread uniformly over its surface. What is the electric field near the midpoint of the rod, at a point on its surface?

$[\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}]$

Here,

$$Lenth\ h = 220\ cm = 2.2\ m$$
$$Charge\ q = -3.8 \times 10^{-7}\ C$$
$$radius\ r = 3.6\ mm = \frac{3.6}{1000}\ m = 0.0036\ m$$
$$Electric\ Field\ E = ?$$

The linear charge density for the rod is

$$\lambda = \frac{q}{h} = \frac{-3.8 \times 10^{-7}\ C}{2.2\ m}$$
$$\Rightarrow \lambda = -1.73 \times 10^{-7}\ C/m$$

We know,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$
$$= \frac{-1.73 \times 10^{-7} \cdot Cm^{-1}}{(2\pi)(8.85 \times 10^{-12}\ C^2N^{-1}m^{-2})(0.0036\ m)}$$
$$= -8.6 \times 10^5\ NC^{-1}$$

The minus sign tells us that, because the rod is negatively charged, the direction of the electric field is radially inward, toward the axis of the rod.

Statement of Gauss's law:

Gauss's law states that the total electric flux Φ_E through any closed surface is equal to the total charge enclosed by that surface.

Mathematically,

$$\Phi_E = \frac{q}{\varepsilon_0}$$

or, $\varepsilon_0 \Phi_E = q$

or, $\varepsilon_0 \oint \vec{E}.d\vec{s} = q \quad [\because \Phi_E = \oint \vec{E}.d\vec{s}\ ]$

Where,

ε_0 = Permittivity constant

Φ_E = Electric flux for the surface

q = Net charge of the surface

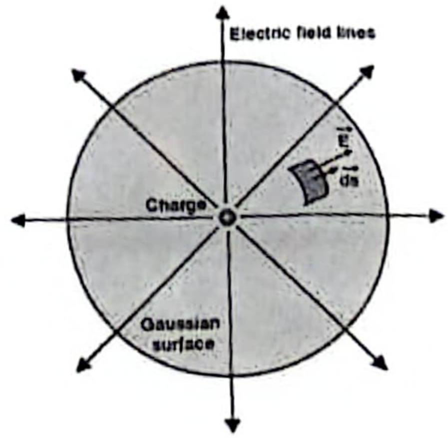


Figure-1: A Spherical Gaussian Surface

Explanation:

Let us consider a +ve charge q is enclosed in a spherical Gaussian surface of radius r, the net flux through the Gaussian surface is given by

$$\Phi_E = \oint \vec{E}.d\vec{s}$$

$$= \oint Eds\cos\theta \quad [\because \theta \text{ is the angle between } \vec{E} \text{ and } d\vec{s}\,]$$

$$= \oint \frac{q}{4\pi\varepsilon_0 r^2} ds\cos\theta \quad [\because E = \frac{q}{4\pi\varepsilon_0 r^2}]$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\oint ds\cos\theta$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4\pi r^2}{r^2} \quad [\because \oint ds\cos\theta = 4\pi r^2\,]$$

$$= \frac{q}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

$$\therefore \varepsilon_0 \oint \vec{E}.d\vec{s} = q$$

$\oint ds$

total

Applications of Gauss's Law

Line of Charge:

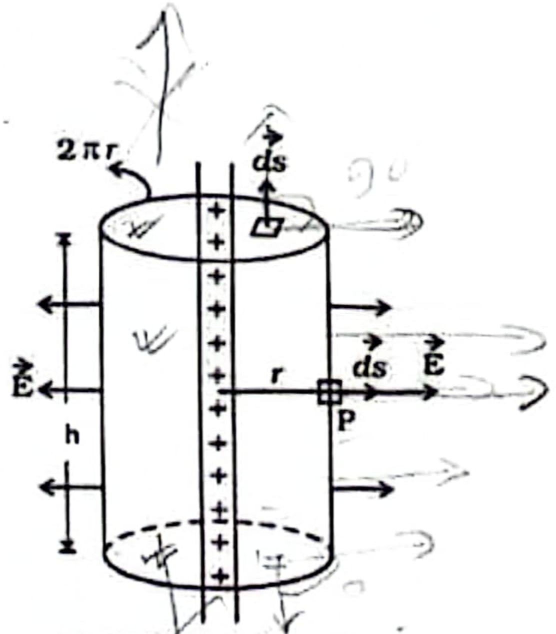


Fig 1.17 Infinitely long straight charged wire

Let us consider a section of an infinite rod of charge, the linear charge density λ being constant for all points on the line.

Let us consider a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis. There is no flux through the circular caps because $\vec{E}$ here lies in the surface at every point.

The charge enclosed by the Gaussian surface of Fig.-1.17 is λh.

Applying Gauss's law, we have

$$\varepsilon_0 \oint \vec{E}.d\vec{s} = q$$

$$\Rightarrow \varepsilon_0 \oint \vec{E}.d\vec{s} = \lambda h \qquad [\because q = \lambda h]$$

$$\Rightarrow \varepsilon_0 \oint E ds \cos\theta = \lambda h$$

$$\Rightarrow \varepsilon_0 \oint_a E ds \cos\theta + \varepsilon_0 \oint_b E ds \cos\theta + \varepsilon_0 \oint_c E ds \cos\theta = \lambda h$$

$$\Rightarrow \varepsilon_0 \oint_a E ds \cos 90^0 + \varepsilon_0 \oint_b E ds \cos 0^0 + \varepsilon_0 \oint_c E ds \cos 90^0 = \lambda h$$

$$\Rightarrow 0 + \varepsilon_0 \oint_b E ds + 0 = \lambda h$$

$$\Rightarrow \varepsilon_0 E \oint_b ds = \lambda h$$

$$\Rightarrow \varepsilon_0 E S = \lambda h \qquad \text{where, } S = \text{The area of the surface}$$

Coulomb's Law:

1. A point charge of $+3.00 \times 10^{-6} C$ is $12.0\ cm$ distant from a second point charge of $-1.50 \times 10^{-6} C$. Calculate the magnitude of the force on each charge. $[Here, k = 8.99 \times 10^{9}\ Nm^{2}C^{-2}]$

Here,

$$12cm = 12 \times 10^{-2}\ m$$

Being of opposite signs, the two charges *attract* one another, and the magnitude of this force is given by Coulomb's law

$$F = k\frac{|q_1 q_2|}{r^2}$$

$$F = 8.99 \times 10^{9}\ Nm^{2}C^{-2}\frac{(3.00 \times 10^{-6} C)(1.50 \times 10^{-6} C)}{(12 \times 10^{-2}\ m)^2}$$

$$F = 2.81\ N$$

Each charge experiences a force of attraction of magnitude 2.81N.

2. What must be the distance between point charge $q_1 = 26.0 \times 10^{-6}\ C$ and point charge $q_2 = -47.0 \times 10^{-6}\ C$ for the electrostatic force between them to have a magnitude of $5.70\ N$? $[Here, k = 8.99 \times 10^{9}\ Nm^{2}C^{-2}]$

We are given the charges and the magnitude of the (attractive) force between them.

Here,

$$q_1 = 26.0 \times 10^{-6}\ C$$
$$q_2 = -47.0 \times 10^{-6}\ C$$
$$F = 5.70\ N$$
$$r = ?$$

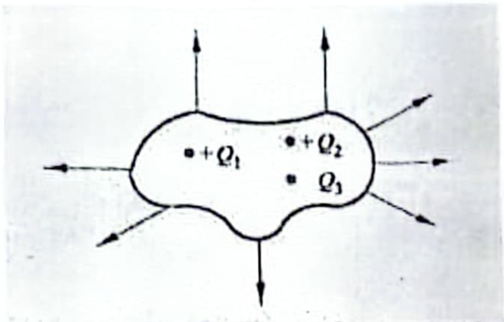
Flux:

The meaning of flux is just ***the number of field lines*** passing through the surface.

$$\Phi_E = \oint \vec{E}.d\vec{s}$$

Irrespective of where the charge Q is placed within a closed surface completely surrounding it, the total normal flux is Q and the total number of lines of force passing out normally is Q/ε_0.

In fact, if there are placed charges of value Q_1, Q_2, $-Q_3$ inside a closed surface, the total i.e. net charge enclosed by the surface is $(Q_1 + Q_2 - Q_3)/\varepsilon_0$ through the closed surface.



Electric Flux Density or Electric Displacement:

It is given by the normal flux per unit area.
If a flux of Ψ coulombs passes normally through an area of $A\ m^2$, then flux density is

$$D = \frac{\Psi}{A}\ C/m^2$$

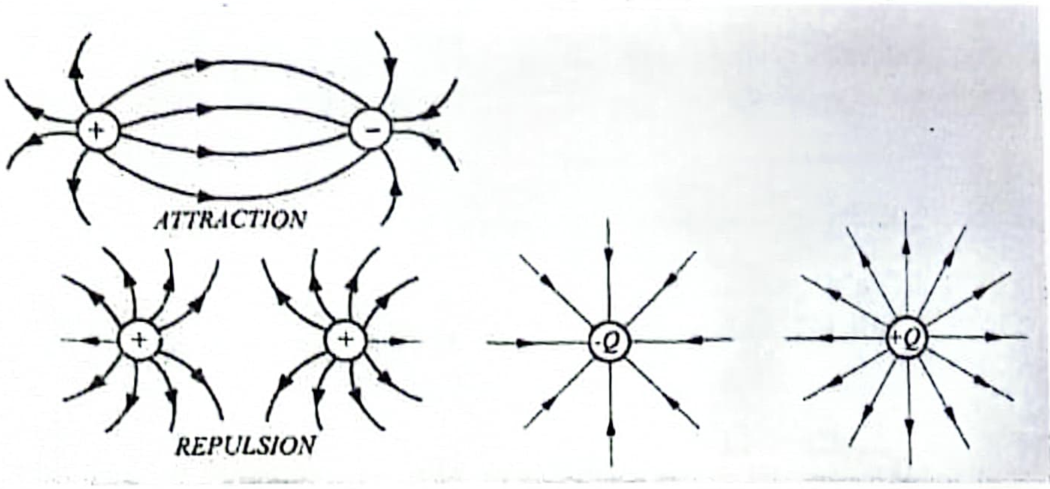
It is related to electric field intensity by the relation

$$D = \varepsilon_0\varepsilon_r E \text{ ...in a medium}$$
$$= \varepsilon_0 E \text{ ...in free space}$$

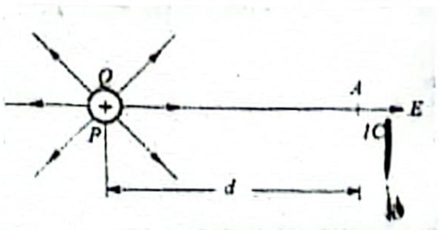
In other words, the product of electric intensity E at any point within a dielectric medium a the absolute permittivity ε (= $\varepsilon_0\varepsilon_r$) at the same point is called the ***displacement*** at that poi

Electric Filed:

It is found that in the medium around a charge, a force acts on a positive or negative charge when placed in that medium. The region in which the electric forces act, is called an electric field or electrostatic field.



Electric Field Intensity:



The electric field intensity (or electric field strength) $\vec{E}$ is the force that a unit positive c experiences when placed in an electric field.

$$\vec{E} = \frac{\vec{F}}{Q}$$

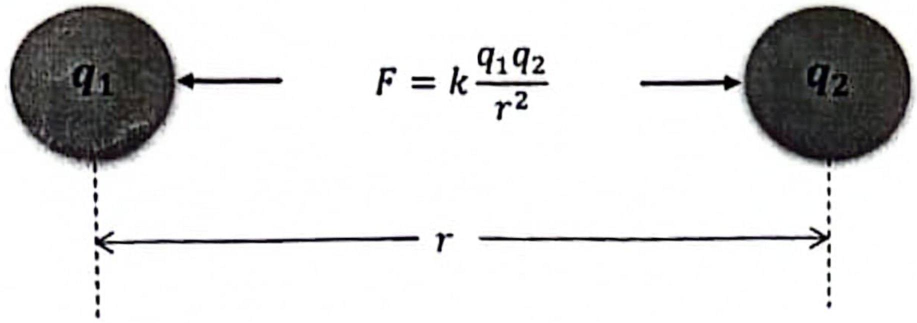
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

COULOMB'S LAW

Basics:

- The polarity of charges may be positive or negative.
- Like charges repel
- Unlike charges attract

Coulomb's law:



Coulomb's law states that the force F between two-point charges q_1 and q_2 is:

i. Along the line joining them
ii. Directly proportional to the product $q_1 q_2$ of the charges
iii. Inversely proportional to the square of the distance r between them.

Expressed mathematically,

$$F = k\frac{q_1 q_2}{r^2}$$

where, k is the proportionality constant whose value depends on the choice of system of units.

In SI units, charges q_1 and q_2 are in coulombs (C), the distance r is in meters (m), and the force F is in newtons (N) so that

$$k = \frac{1}{4\pi\varepsilon_0}$$

The constant ε_0 is known as the permittivity of free space (in farads per meter) and has the value

$$\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$$

Hence,

$$k = \frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9\ m/F$$

Definition of a wave:

A very general definition of a wave valid for mechanical and electromagnetic waves is the following:" a wave is a disturbance that transfers energy from one point to another without imparting net motion to the medium through which it travels. Thus, the particles of the medium are not permanently displaced, but just oscillate back and forth about their equilibrium position.

Types of waves:

There are three types of waves:

1. **Mechanical waves:** They require a medium for their propagation. Example: pulse waves on a rope or a string, sound waves, waves travelling across the surface of water, seismic waves, etc.

2. **Electromagnetic waves:** Those consist of transverse oscillations of $\vec{E}$ and $\vec{B}$ fields. In this case we have oscillations of fields and not material particles. They do not require a medium for their propagation (they can carry energy through the vacuum). Example: visible light, IR radiations, ultraviolet radiation, radio and TV waves, X – rays, gamma rays, ultrasound, etc.

3. **Matter waves:** Matter waves are waves which are associated to electrons and other micro- particles (protons, neutrons, alpha particles, atoms, molecules, etc.). Electrons, protons, neutrons, etc. are described, in quantum mechanics, by De Broglie waves – "probability waves" associated to these micro -particles.

Definition:

Wavelength: It is the distance travelled by the wave in the time in which the particle of the medium completes one vibration.

Frequency: It is the number of vibrations made by a particle in one second.

Amplitude: It is the maximum displacement of the particle from its mean position of rest.

Time period: It is the time taken by a particle to complete one vibration.

Sound:

Sound is one kind of mechanical wave that produces a sensation of hearing. It is produced when something makes vibratory motion.

Properties of sound:

The properties of sound are:

- It takes time to travel.
- It cannot pass through the vacuum.
- It may be reflected, refracted and deflected.
- It cannot be polarized.
- It exhibits interference.

Pitch:

Pitch is the quality that allows us to classify a sound as relatively high or low. Pitch is determined by the frequency of sound wave vibrations.

Plane Progressive Harmonic Wave:

A plane progressive wave is one which travels onwards through the medium in a given direction without attenuation, i.e., with its amplitude constant.

Expression for a plane progressive harmonic wave:

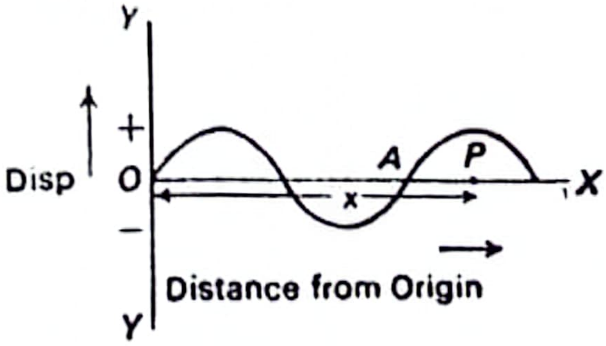


Figure-1: Plane progressive wave

Let us suppose a wave, originating at O, travels to the right along the axis of x. Then, if we start counting time when the particle at O just passes through its mean position in the positive direction, the equation of motion of this particle (at O) is obviously

$$y = a \sin \omega t$$

where

y is the displacement at time t

a is the amplitude

ω is the angular velocity

Let us consider, for a particle at P, distant x from O, the phase lag is ϕ.

The equation of motion of the particle at P is thus

$$y = a \sin\ (\omega t - \phi) \dots\dots\dots\dots (i)$$

Now, we know, in a distance λ, the phase lag increases by 2π. In a distance x, therefore, the phase lag increases by $2\pi x/\lambda$, i.e.,

$$\phi = \frac{2\pi x}{\lambda}$$

Substituting this value of ϕ in the eq(i), we have

$$y = a \sin \left(\omega t - \frac{2\pi x}{\lambda}\right) \dots\dots\dots\dots (ii)$$

where

$$\frac{2\pi}{\lambda} = \text{the propagation constant}$$

Again,

$$\omega = \frac{2\pi}{T}$$

where T is the time period of each particle of the medium.

And, since

$$v = n\lambda = \frac{\lambda}{T}$$

where,

v=the phase velocity or the wave velocity

n= the frequency of the oscillating particles or of the wave

we have,

$$\frac{1}{T} = \frac{v}{\lambda}$$

And, therefore,

$$\omega = \frac{2\pi v}{\lambda}$$

We thus have from the eq(ii) above,

$$y = a\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

Or,

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

This equation is referred to as the equation of a plane progressive harmonic wave.

Doppler Effect:

Doppler effect is commonly observed that the pitch apparently changes when either the source or the observer are in motion relative to each other. When the source approaches the observer or when the observer approaches the source or when both approach each other, the apparent pitch is higher than the actual pitch of the sound produced by the source. Similarly, When the source moves away from the observer or when the observer moves away from the source or when both move away from each other, the apparent pitch is lower than the actual pitch of the sound produced by the source.

[Doppler effect in sound is asymmetric. When the source moves towards the observer with a certain velocity, the apparent pitch is different to the case when the observer is moving towards the source with the same velocity. But it is not so in the case of light. Doppler effect in light is symmetric.]

Doppler Effect Formula:

Doppler effect is the apparent change in the pitch of waves due to the relative motion between the source of the sound and the observer. We can deduce the apparent pitch in the Doppler effect using the following equation:

$$n' = \frac{v \pm b}{v \pm a} n$$

Where,

n' = The apparent pitch

n = The actual pitch

v = The velocity of sound waves

b = The velocity of the observer

a = The velocity of the source

2~~1.~~ Observer at Rest and Source in Motion:

~~(a)~~When the source moves towards the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v.

Let the source move with a velocity a towards the observer. In one second, n waves will be contained in a length $(v - a)$ and the apparent wavelength,

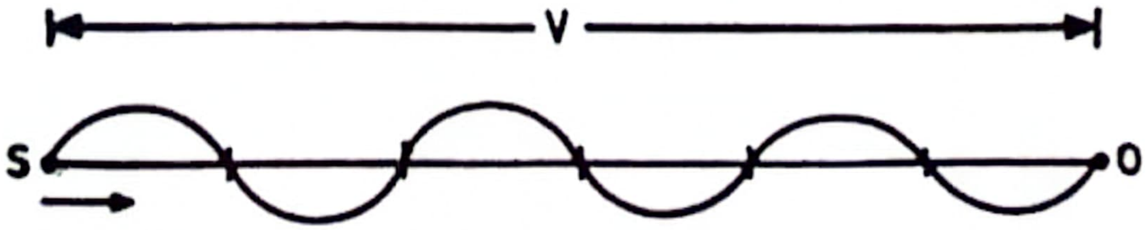
$$\lambda' = \frac{(v-a)}{n}$$

The apparent pitch,

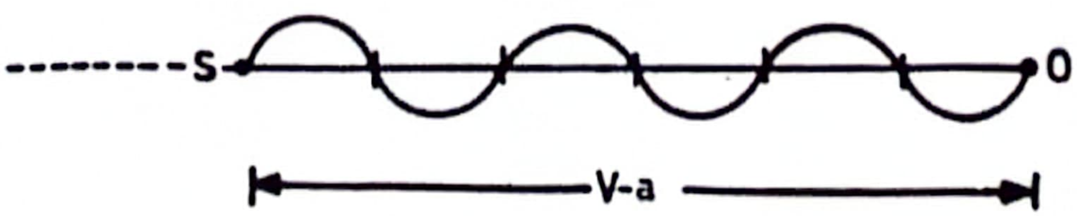
$$n' = \frac{v}{\lambda'}$$

$$\therefore n' = \frac{v}{(v-a)} n$$

Thus, the apparent pitch increases when the source moves towards a stationary observer.



(i)



(ii)

(b) When the source moves away from the stationary observer

Suppose a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v.

Let the source move with a velocity a away from the observer. In one second, n waves will be contained in a length $(v + a)$ and the apparent wavelength,

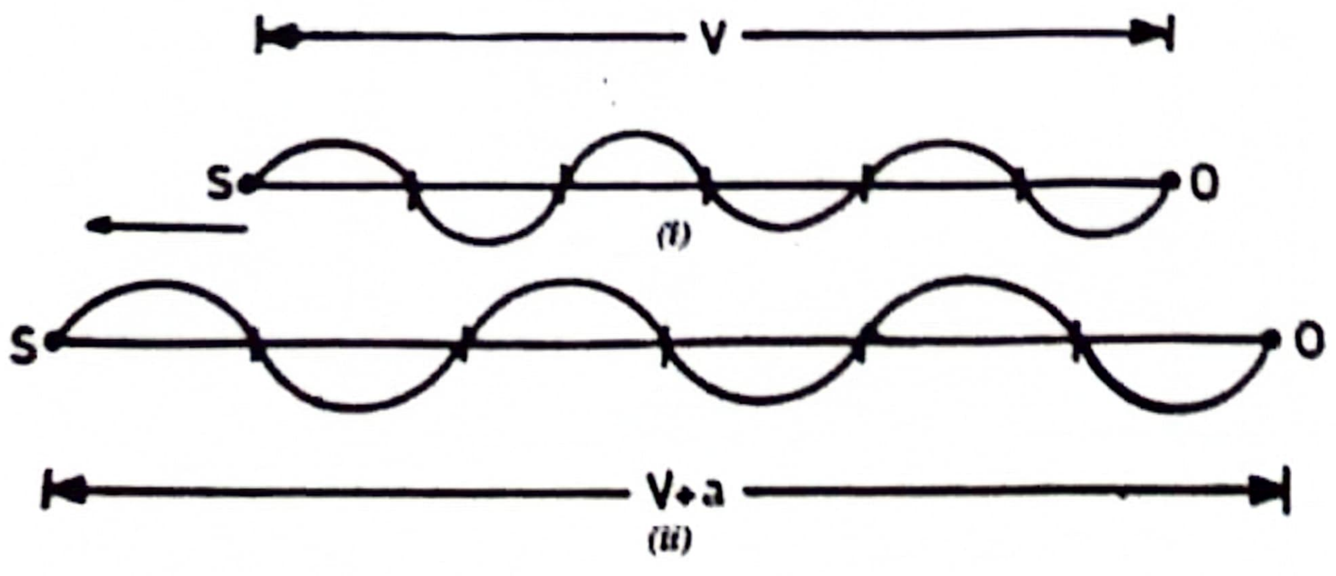
$$\lambda' = \frac{(v + a)}{n}$$

The apparent pitch,

$$n' = \frac{v}{\lambda'}$$

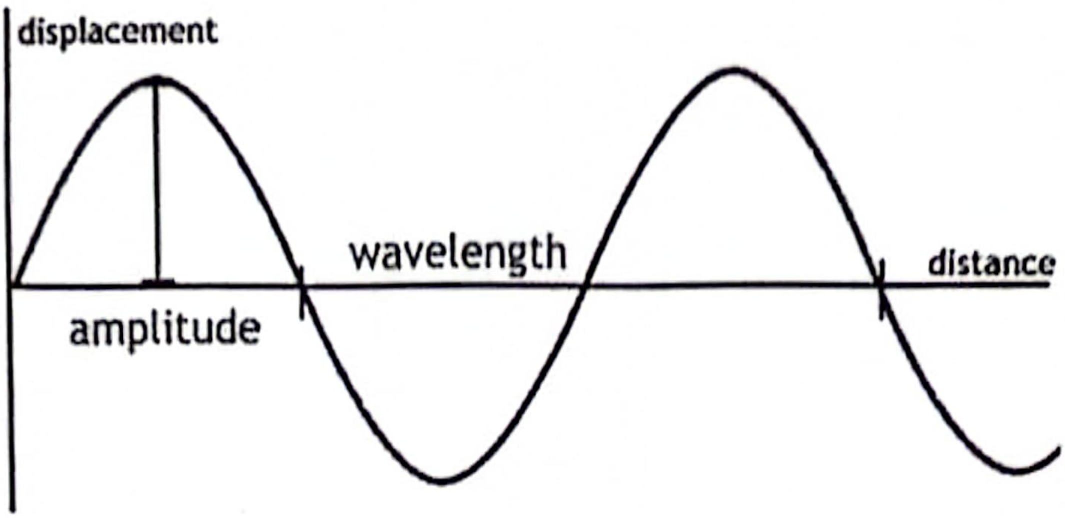
$$\therefore n' = \frac{v}{(v + a)} n$$

Thus, the apparent pitch decreases when the source moves away from a stationary observer.



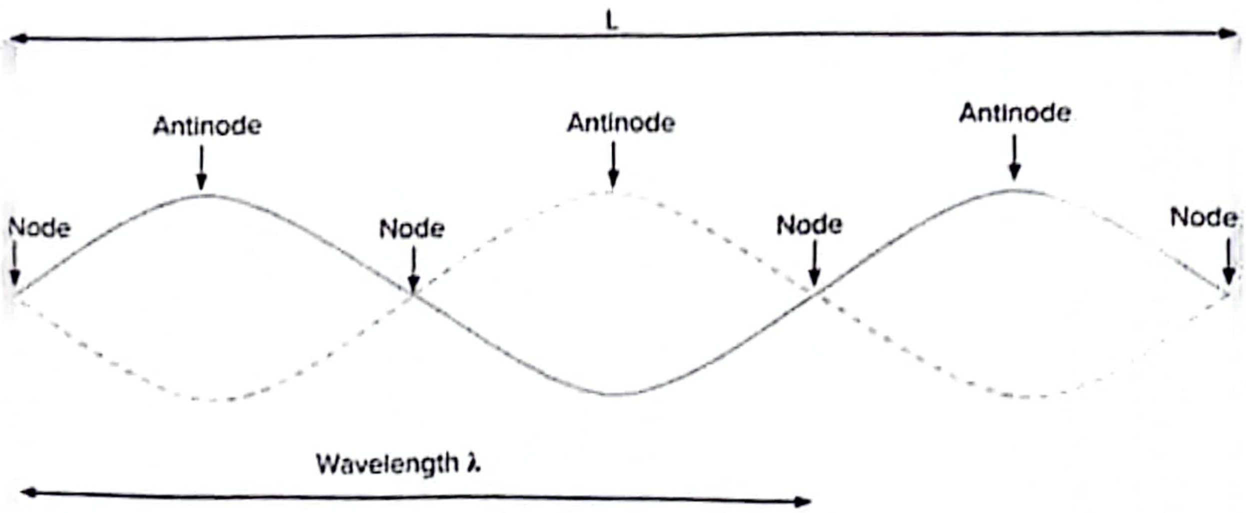
Progressive Wave:

A plane progressive wave is one which travels onwards through the medium in a given direction without attenuation, i.e., with its amplitude constant.



Standing wave:

Standing wave, also called stationary wave is the combination of two waves moving in opposite directions between two fixed points, each having the same amplitude and frequency. The phenomenon is the result of interference-that is, when waves are superimposed, their energies are either added together or cancelled out.



Math-2

2. The equation of a transverse wave travelling along a stretched string is given

$$y = 10 \sin \pi\,(2t - 0.01x)$$

where y and x are expressed in centimeters and t in seconds.

Find the amplitude, velocity, wavelength and frequency of the wave.

Solution:

We can rewrite the given equation as,

$$y = 10 \sin \pi \left(2t - \frac{x}{100}\right)$$

$$or, \quad y = 10 \sin 2\pi \left(t - \frac{x}{200}\right)$$

$$or, \quad y = 10 \sin \frac{2\pi}{200}\,(200t - x) \ldots\ldots\ldots (i)$$

For any type of wave, we have the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Comparing it with the given equation, we get

Amplitude

$$a = 10\ cm$$

Velocity

$$v = 200\ cms^{-1}$$

Wavelength

$$\lambda = 200\ cm$$

Frequency

$$n = \frac{v}{\lambda} = \frac{200}{200} = 1\ Hz$$

When a simple harmonic wave is propagated through a medium, the displacement of a particle a
any instant of time is given by

$$y = 10 \sin \frac{2\pi}{200}(200t - 20)$$

where y is expressed in centimeters and t in seconds.

Calculate the wave velocity, wavelength and frequency and time period.

Solution:

For any type of wave, we have the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Comparing it with the given equation, we get

Amplitude

$$a = 10\ cm$$

Velocity

$$v = 200\ cms^{-1}$$

Wavelength

$$\lambda = 200\ cm$$

Frequency

$$v = n\lambda$$

$$n = \frac{v}{\lambda}$$

$$n = \frac{200}{200}$$

$$n = 1\ Hz$$

Time period

$$T = \frac{1}{n} = \frac{1}{1} = 1\ second$$

Math-3

1. A person is standing on a platform. A railway engine moving away from the person with a speed of 72 km/hr blows a whistle of pitch 740 hertz. Calculate the apparent pitch of the whistle. The velocity of sound is 350 m/s.

Solution:

Here $V = 350$ m/s

$a = 72$ km/hr $= 20$ m/s

$n = 740$ hertz

The source is moving away from a stationary observer.

$$\therefore \quad n' = \left(\frac{V}{V+a}\right)n$$

$$n' = \left(\frac{350}{350+20}\right) \times 740$$

$$n' = 700 \text{ hertz.}$$

Math-1

1. When a simple harmonic wave is propagated through a medium, the displacement of a particle at any instant of time is given by

$$y = 10\, sin \frac{2\pi}{100}(36000t - 20)$$

where y is expressed in centimeters and t in seconds.
Calculate the wave velocity, wavelength and frequency and time period.

Solution:

For any type of wave, we have the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Comparing it with the given equation, we get

Amplitude

$$a = 10\ cm$$

Velocity

$$v = 36000\ cms^{-1}$$

Wavelength

$$\lambda = 100\ cm$$

Frequency

$$v = n\lambda$$

$$n = \frac{v}{\lambda}$$

$$n = \frac{36000}{100}$$

$$n = 360\ Hz$$

Time period

$$T = \frac{1}{n} = \frac{1}{360}\ second$$