Experiment No.: 09
Experiment Name: Implementation of Trapezoidal Method in Numerical Integration.
Theory:
The Trapezoidal Method is a numerical technique used to approximate the value of a definite integral when the exact analytical integration is difficult or when the function is given only as a set of values. It works by dividing the area under a curve into small trapezoids instead of rectangles, giving a more accurate result than simple Riemann sums.
Suppose we want to approximate the integral:

$$\int_{a}^{b} f(x)dx$$

We divide the interval [a,b][a, b][a,b] into nnn equal subintervals, each of width:

$$\mathrm{h} = \frac{\text{b-a}}{\mathrm{n}}$$

The Trapezoidal Rule approximates the area under the curve by treating each pair of points as the left and right sides of a trapezoid. The area of one trapezoid is:

$$\text{Area} = \frac{\mathrm{h}}{2}[f(xi) + f(xi + 1)]$$

Adding all trapezoidal areas, the formula becomes:

$$\int_{a}^{b} f(x)dx \approx \frac{\mathrm{h}}{2}[f(x0) + 2(f(x1) + f(x2) + \cdots + f(xn - 1)) + f(xn)]$$

Here,

- f(x0) and f(xn) are the first and last function values,
- all middle values f(x1),f(x2),...,f(xn−1) are multiplied by 2 because each of them is shared by two trapezoids.

This method assumes the curve between two points is approximately a straight line forming a trapezoid. While simpler than Simpson's Method, it still provides a good level of accuracy, especially when the number of subintervals n is large.

Program 1: Programming Code

```
import math
def func(x):
    return (1/(1+x*x))

def Trapezoidal_Rule(x0, xn, n):
    h = (xn-x0)/n
    total = func(x0) + func(xn)
    xi = 0.0
    for i in range (1,n):
        xi = x0 + h
        total += 2*func(xi)
        x0 = xi

    return (h/2) * total

x0  = float(input("Enter the lower limit: "))
```

```python
xn  = float(input("Enter the upper limit: "))
n  = int(input("Enter the number of interval: "))

approx_area = Trapezoidal_Rule(x0, xn, n)
print("=> By using trapezoidal method: ")
print(f"The obtained approximate area is: {approx_area:.4f}")

exact_sol = math.atan(xn) - math.atan(x0)
print(f"The actual area is: {exact_sol:.4f}")
error = abs(exact_sol - approx_area)
print(f"The error is: {error:.4f}")
```

Output:

```
=> By using trapezoidal method:
The obtained approximate area is: 1.2478
The actual area is: 1.2490
The error is: 0.0012
```

Discussion & Conclusion

The Trapezoidal Method provides a straightforward and effective way to approximate definite integrals, especially when the exact form of the function is difficult to integrate analytically or when only discrete data points are available. By replacing small segments of the curve with straight-line trapezoids, the method offers an improved estimate over basic rectangular approximations. Although its accuracy depends on the number of subintervals used, increasing the value of n significantly reduces the error and makes the approximation closer to the true integral value.

Experiment No.: 10
Experiment Name: Implementation of Simpson’s Method in Numerical Integration.
Theory:
Simpson’s Method is a numerical technique used to approximate the value of a definite integral when the exact analytical solution is difficult or not possible. Unlike the Trapezoidal Method, which approximates the curve using straight lines, Simpson’s Method uses **parabolic curves (quadratics)** to approximate the area under the function. Because parabolas fit smooth curves more accurately, Simpson’s Method generally provides much better accuracy.
To apply Simpson’s Method, the interval [a,b][a, b][a,b] is divided into an **even number** of equal subintervals. The width of each subinterval is:

$$h = \frac{\text{b-a}}{\text{n}}$$

Simpson’s 1/3 Rule replaces every pair of subintervals with a parabola passing through three points. The formula for approximating the integral is:

$$\int_{a}^{b} f(x)dx \approx \frac{h}{3}[f(x0) + 4f(x1) + 2f(x2) + 4f(x3) + \cdots + 4f(xn-1) + f(xn)]$$

Here,

- The first and last values f(x0) and f(xn) appear once.
- The odd-indexed values f(x1),f(x3),…are multiplied by **4**.
- The even-indexed interior values f(x2),f(x4),…are multiplied by **2**.

These weights come from integrating the quadratic polynomial that fits each segment of the curve.
Because Simpson’s Method uses quadratic approximation, it captures the shape of the curve more accurately than methods using straight-line approximations. As a result, it requires fewer subintervals to achieve high precision.
Simpson’s Method is widely used for scientific and engineering calculations due to its balance between simplicity and accuracy.

Program 1: Programming Code

```python
import math
def func(x):
    return (1/(1+x*x))

def Simpsons_Rule(x0, xn, n):
    h = (xn-x0)/n
    total = func(x0) + func(xn)
    xi = 0.0
    for i in range (1,n):
        xi = x0 + h
        if (i%2 == 0):
            total += 2*func(xi)
        else:
            total += 4*func(xi)
```

```python
        x0 = xi

    return (h/3) * total

x0  = float(input("Enter the lower limit: "))
xn  = float(input("Enter the upper limit: "))
n  = int(input("Enter the number of interval: "))

approx_area = Simpsons_Rule(x0, xn, n)
print("=> By using Simpsons method: ")
print(f"The obtained approximate area is: {approx_area:.4f}")

exact_sol = math.atan(xn) - math.atan(x0)
print(f"The actual area is: {exact_sol:.4f}")
error = abs(exact_sol - approx_area)
print(f"The error is: {error:.4f}")
```

Output:

```
=> By using Simpsons method:
The obtained approximate area is: 1.2471
The actual area is: 1.2490
The error is: 0.0020
```

Discussion & Conclusion

Simpson's Method is a highly accurate and efficient numerical technique for approximating definite integrals, especially when the underlying function is smooth and continuous. By replacing small intervals of the curve with parabolic segments instead of straight lines, Simpson's Method greatly reduces the approximation error compared to the Trapezoidal Rule. Its use of weighted function values, particularly the emphasis on midpoints, helps capture the curvature of the function more effectively.

Although the method requires an even number of subintervals, this small restriction is balanced by its high level of precision and reliability. With increasing subdivisions, the approximation quickly converges to the exact value of the integral. Overall, Simpson's Method is widely used in scientific, engineering, and mathematical applications due to its simplicity, stability, and superior accuracy in numerical integration tasks.