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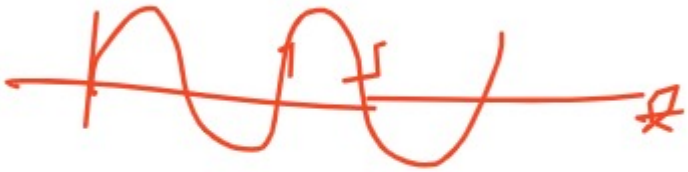
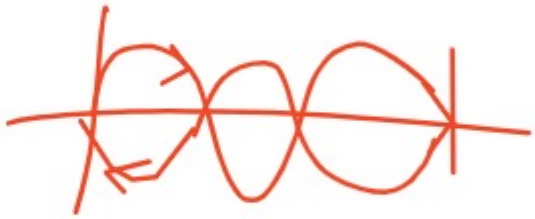
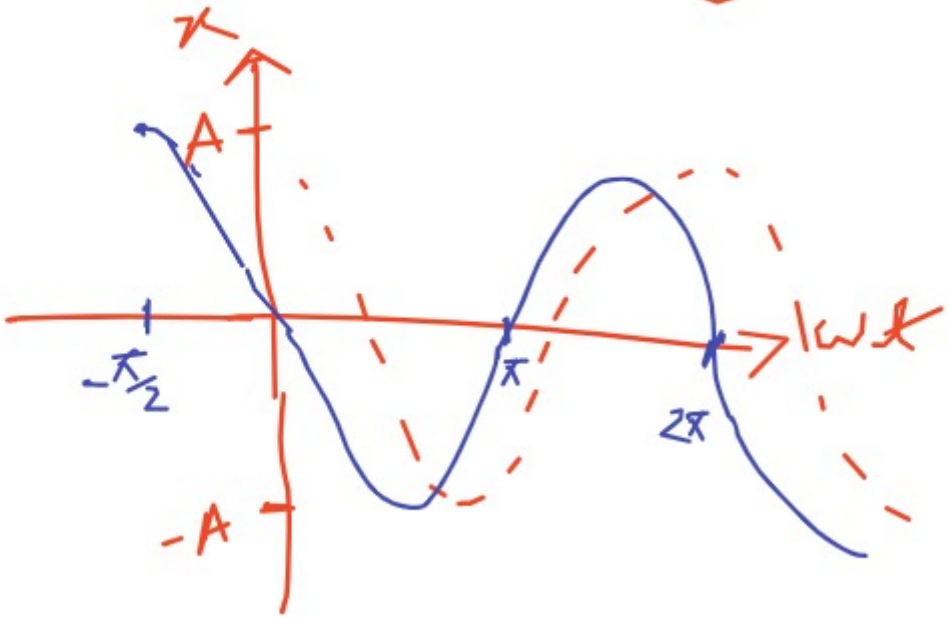
SHM



2-wave



$x = A\cos\left(\omega t \pm \frac{\pi}{2}\right)$



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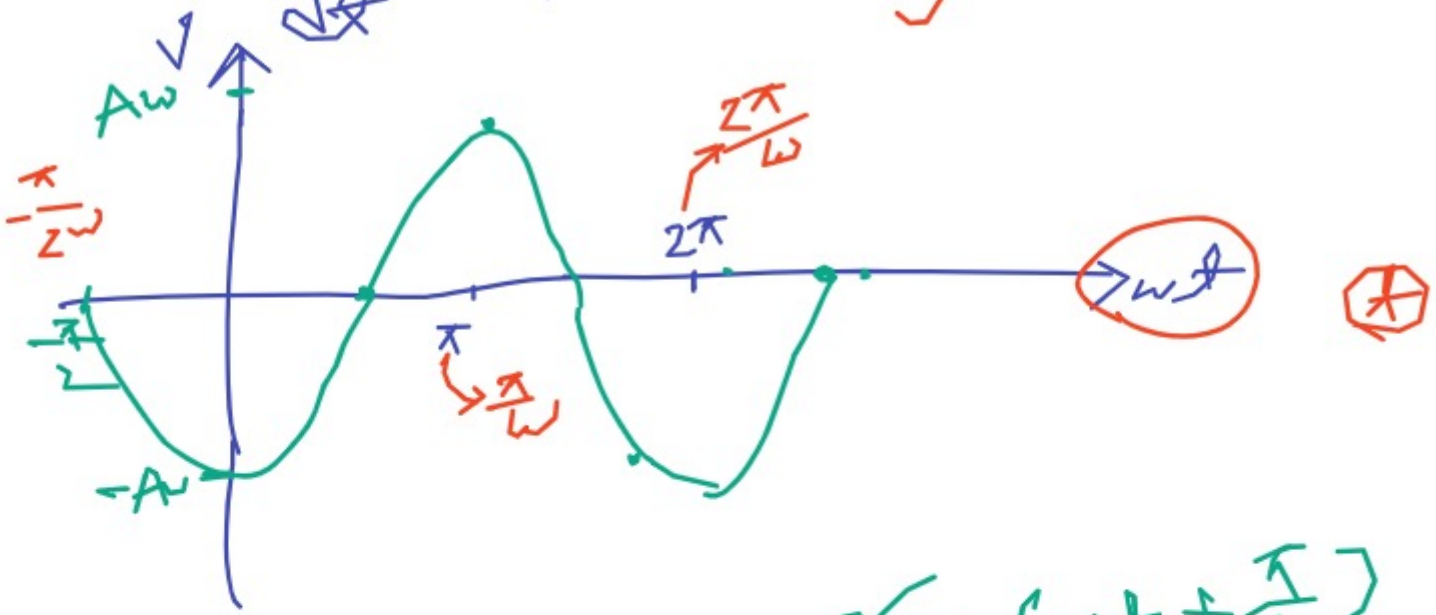
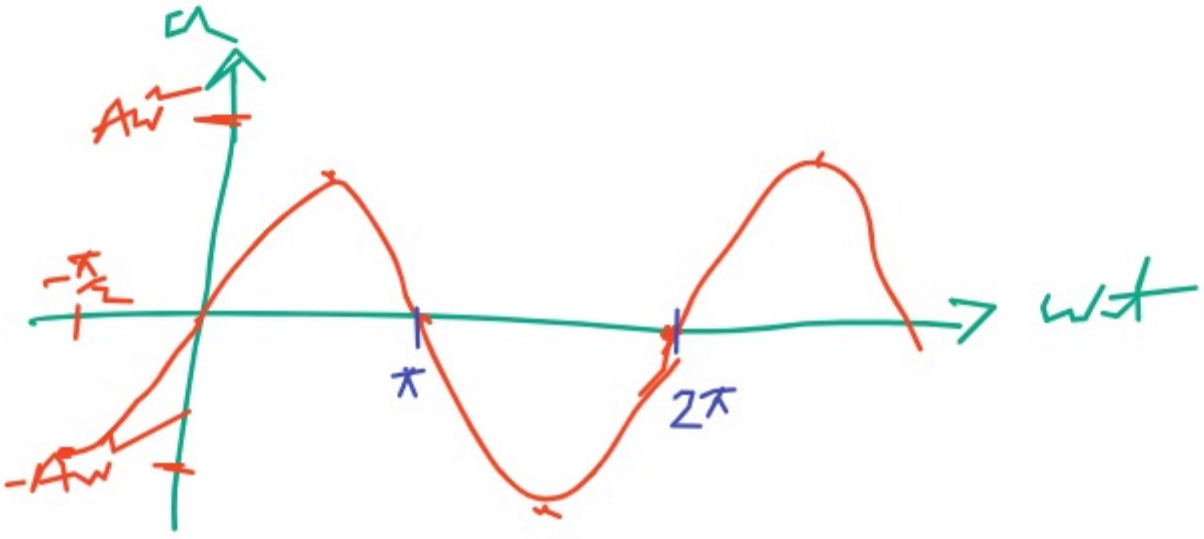
$$v = \frac{dx}{dt} = -A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$



$$a = \frac{dv}{dt} = -A\omega^2 \cos\left(\omega t + \frac{\pi}{2}\right)$$



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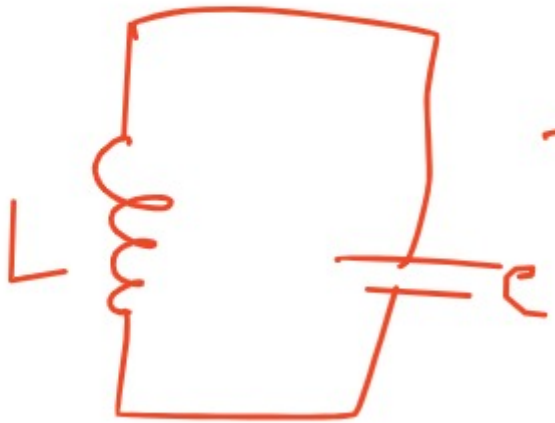
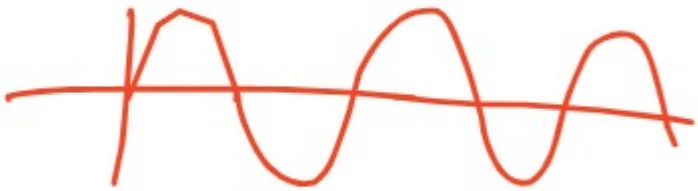
1(c)

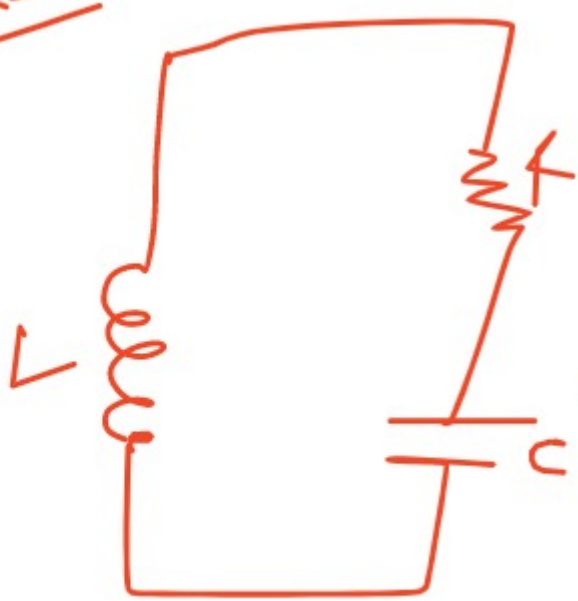
LC



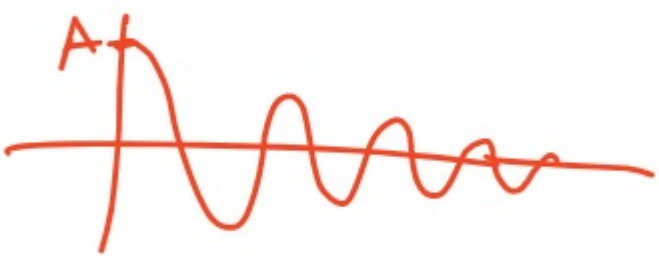


SHM

RLC



Energy



DHM

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(2)

$m = 0.12\ kg$

$A = 8.5\ cm = 0.085\ m$

$T = 0.20\ s$

(i)

$f_{max} = ma_{max}$

$= m\omega^2 A$

$= 0.12 \times (31.92)^2 \times 0.085$

$= 10.07\ N$

$\omega = \frac{2\pi}{T}$

$= \frac{2\pi}{0.20}$

$= 31.92\ rads^{-1}$

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(ii) $F \propto x$

$F = -kx$ ← spring const.

$\omega^2 = \frac{k}{m}$

$\therefore k = m\omega^2$

$= 0.12 \times (31.92)^2$

$= 118.47 \ Nm^{-1}$

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2(b)

$m = 300\ gm = 0.3\ kg$

$K = 5000\ dynes/cm$

$= 5000 \times 10^{-3}\ Nm^{-1} = 5\ Nm^{-1}$

$A = 7\ cm = 7 \times 10^{-2}\ m$

$= 0.07\ m$

(i) $f = \frac{1}{T}$

$= \frac{1}{1.59}$

$= 0.65\ Hz$

$T = 2\pi\sqrt{\frac{m}{K}}$

$= 2\pi\sqrt{\frac{0.3}{5}}$

$= 1.54\ s$

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(ii) $\omega = 2\pi f$

$= 2\pi \times 0.65$

$= 4.08 \text{ rad/s}$

(iii) $E = \frac{1}{2} k A^2$

$= \frac{1}{2} \times 5 \times (0.07)^2$

$= 0.01 \text{ J}$

2(c)

$$x = 9 \sin\left(16t - \frac{\pi}{6}\right)$$

$$x = A \sin(\omega t - \varphi)$$

(i) $x_{max} = A = 9\ m$

(ii) $V_{max} = \omega A$

$= 16 \times 9$

$= 144\ ms^{-1}$

(iii) $a_{max} = \omega^2 A$

$= (16)^2 \times 9$

$= 2304\ ms^{-2}$

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3(a)

$m = 990\ gm = 990 \times 10^{-3}\ kg$

$= 0.99\ kg$

$k = 190\ N/m$

$b = 75\ gm/s = 0.075\ kg/s$

$A = 8\ cm = 0.08\ m$

(i) $T = \frac{2\pi}{\omega_d}$

$= \frac{2\pi}{19.69}$

$= 0.32\ s$

$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$= \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ $\quad \gamma = \frac{b}{m}$

$= \sqrt{\frac{191}{0.79} - \frac{0.075^2}{4 \times (0.79)^2}}$

$= 19.69\ rad\,s^{-1}$

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(ii) $A = \frac{A_0}{4}$

$\Rightarrow A e^{-\frac{\gamma}{2}t} = \frac{A_0}{4}$

$\Rightarrow e^{-\frac{\gamma}{2}t} = \frac{1}{4}$

$\Rightarrow \ln\left(e^{-\frac{\gamma}{2}t}\right) = \ln\left(\frac{1}{4}\right)$

$\Rightarrow -\frac{\gamma}{2}t \cdot \ln e = \ln\frac{1}{4}$

$\Rightarrow -\frac{\gamma}{2}t = -1.39$

$\Rightarrow -\frac{0.15}{2} \times t = -1.39$

$\Rightarrow t = 18.53\ s$

$x = Ae^{-\frac{\gamma}{2}t}\cos(\omega t + \varphi)$

Here, $\gamma = \frac{b_0}{m}$

$= \frac{0.075}{0.99}$

$= 0.15\ s^{-1}$

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(iii) $\text{Lifetime} = \frac{1}{\gamma}$

$= \frac{1}{0.15}$

$= 6.67\ s$

3(b)

$C = 0.0005\ mF = 0.0005 \times 10^{-3}\ F$

$= 5 \times 10^{-7}\ F$

$L = 0.1\ h$

$R = 250\ \Omega$

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(*) $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \rightarrow$ osci.

$$\frac{1}{LC} = \frac{1}{0.1 \times 5 \times 10^{-7}} = 20 \times 10^6$$

$$\left(\frac{R}{2L}\right)^2 = \frac{R^2}{4L^2} = \frac{(250)^2}{4 \times (0.1)^2} = 1.56 \times 10^6$$

$$\frac{1}{LC} > \frac{R^2}{4L^2} \quad \therefore \text{osci...}$$

20×10^6 $\quad$ 1.56×10^6

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$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\Rightarrow \omega_0 = \sqrt{20 \times 10^6 - 1.56 \times 10^6}$$

$$\Rightarrow \omega_0 = 4299.18$$

$$\Rightarrow 2\pi f = 4299.18$$

$$\Rightarrow f = \frac{4299.18}{2\pi}$$

$$= 683.99 \text{ Hz}$$

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3(e)

$y = A\sin\frac{2\pi}{\lambda}(vt - x)$

$y = 4\sin\pi(250t - 0.25x)$

$y = 4\sin\pi \times 0.25\ (1000t - x)$

$y = 4\sin\frac{2\times\pi\times 0.25}{2}(1000t - x)$

$y = 4\sin\frac{2\pi}{8}(1000t - x)$

(i) $v = 1000\ ms^{-1}$

(ii) $\lambda = 8\ m$

(iii) $f = \frac{V}{\lambda} = \frac{1000}{8} = 125\ Hz$

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7(a)

$x = A\cos\omega t$

PE

$$PE = \frac{1}{2}kx^2$$

$$= \frac{1}{2}k(A\cos\omega t)^2$$

$$= \frac{1}{2}kA^2\cos^2(\omega t)$$

↑ PE is depend on time (proven)

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KE

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2}m\left(-A\omega\sin(\omega t)\right)^2$$

$$= \frac{1}{2}mA^2\omega^2\sin^2(\omega t)$$

KE is depend on time (power)

Total Energy

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$$E = KE + PE$$

$$= \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t) + \frac{1}{2} k A^2 \cos^2(\omega t)$$

$$= \frac{1}{2} k A^2 \sin^2(\omega t) + \frac{1}{2} k A^2 \cos^2(\omega t)$$

$$= \frac{1}{2} k A^2 \left(\sin^2(\omega t) + \cos^2(\omega t) \right)$$

$$= \frac{1}{2} k A^2 \times 1$$

$$= \frac{1}{2} k A^2$$

$\therefore E$ is not dependent on time

(proved)

$\omega^2 = \frac{k}{m}$

$\therefore k = \omega^2 m$

$\sin^2\theta + \cos^2\theta = 1$

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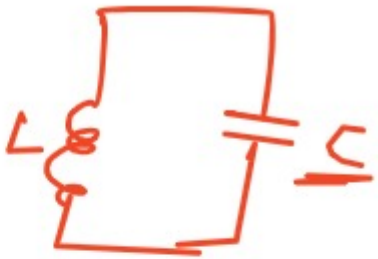
9(b)



$V_c = \frac{Q}{C}$

$V_L = L\frac{di}{dt}$

$Q = CV_c$

$V_c = \frac{Q}{C}$

$V_L \propto \frac{di}{dt}$

$V_L = L\frac{di}{dt}$

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KVL

$$V_C + V_L = 0$$

$$\Rightarrow \frac{q}{C} + L\frac{di}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{q}{C} + L\frac{di}{dt}\right) = \frac{d}{dt}(0)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{q}{C}\right) + \frac{d}{dt}\left(L\frac{di}{dt}\right) = 0$$

$$\Rightarrow \frac{1}{C}\frac{d}{dt}(q) + L\frac{d}{dt}\left(\frac{di}{dt}\right) = 0$$

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$$\Rightarrow \frac{1}{LC}\left(\frac{d}{dt}(q)\right) + \frac{d}{dt}\left(\frac{di}{dt}\right) = 0$$

$\because i = \frac{dq}{dt}$

$$\Rightarrow \frac{1}{LC} i + \frac{d^2 i}{dt^2} = 0$$

$$\Rightarrow \boxed{\frac{d^2 i}{dt^2} + \frac{1}{LC} i = 0}$$

Sol^n of this eq.

SHM: $\frac{d^2 x}{dt^2} + \omega^2 x = 0$

$\omega^2 = \frac{1}{LC}$

$$Q(t) = Q_0 \cos(\omega t + \phi)$$

$$\boxed{x = A\cos(\omega t + \phi)}$$

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Σ(a) DHM

$F_d \propto -v$

$f_d = -bv$

$F' \propto -x$

$F = -kx$

DHM

$F = F_d + F'$

$\Rightarrow ma = -bv + (-kx)$

$\Rightarrow ma = -bv - kx$

$\Rightarrow m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx$

$\Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

Let, $\gamma = \frac{b}{m}$

$\therefore \omega_0^2 = \frac{k}{m}$

$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0}$

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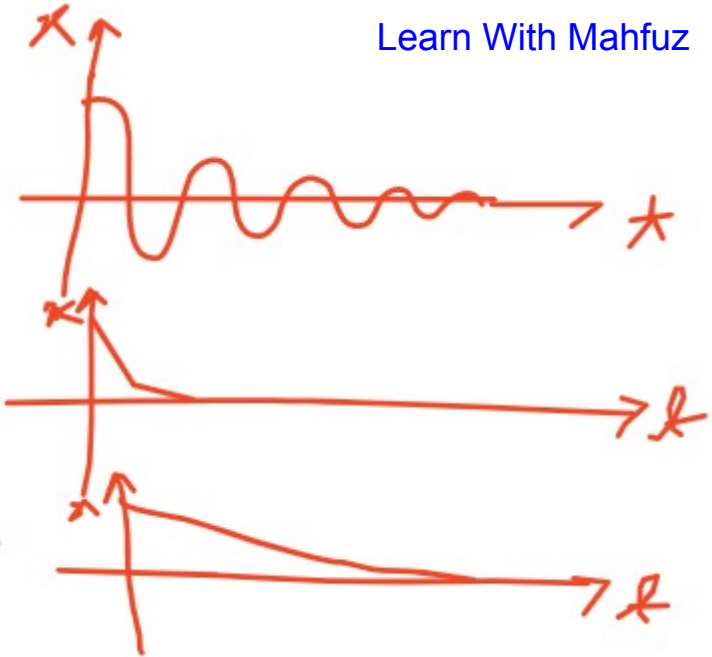
$\omega_0^2 > \frac{\gamma^2}{4}$ ⇒ oscillatory

$\omega_0^2 = \frac{\gamma^2}{4}$ ⇒ Critical

$\omega_0^2 < \frac{\gamma^2}{4}$ ⇒ Over damping



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Q[6]

$$y = A\sin\frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(A\sin\frac{2\pi}{\lambda}(vt - x)\right)$$

$$= A\cos\left(\frac{2\pi}{\lambda}(vt - x)\right) \cdot \frac{d}{dt}\left(\frac{2\pi}{\lambda}(vt - x)\right)$$

$$= A\cos\left(\frac{2\pi}{\lambda}(vt - x)\right) \cdot \left(\frac{2\pi}{\lambda} \cdot v + (vt - x) \cdot 0\right)$$

$$= A \cdot \frac{2\pi}{\lambda} \cdot v \cdot \cos\frac{2\pi}{\lambda}(vt - x)$$

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$$\frac{d^2y}{dt^2} = -A\left(\frac{2\pi}{\lambda}\cdot v\right)^2 \cdot \sin\frac{2\pi}{\lambda}(vt-x) \quad \text{---(1)}$$

Now,

$$\frac{dy}{dx} = \frac{d}{dx}\left(A\sin\frac{2\pi}{\lambda}(vt-x)\right)$$

$$= -A\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x)$$

$$\frac{d^2y}{dx^2} = A\frac{2\pi}{\lambda}\sin\frac{2\pi}{\lambda}(vt-x)\cdot -\frac{2\pi}{\lambda}$$

$$\frac{d^2y}{dx^2} = -A\left(\frac{2\pi}{\lambda}\right)^2\sin\frac{2\pi}{\lambda}(vt-x) \quad \text{---(2)}$$

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After comparing eqn (1) & (2),

$$\frac{d^2y}{dt^2} = -A\left(\frac{2\pi}{\lambda}\cdot v\right)^2 \cdot \sin\frac{2\pi}{\lambda}(vt-x)$$

$$\frac{d^2y}{dt^2} = v^2\cdot\left(-A\left(\frac{2\pi}{\lambda}\right)^2 \sin\frac{2\pi}{\lambda}(vt-x)\right)$$

$$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$$