United International University (UIU)

Data Structures & Algorithms – 1 (DSA 1)

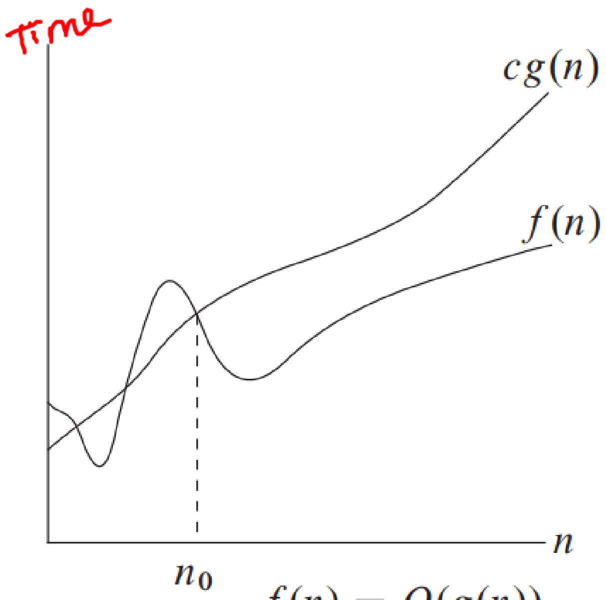
Topic: Asymptotic Notations

Previous Mid-Question Solve of UIU

Learn With Mahfuz

big-oh, big-omega, theta

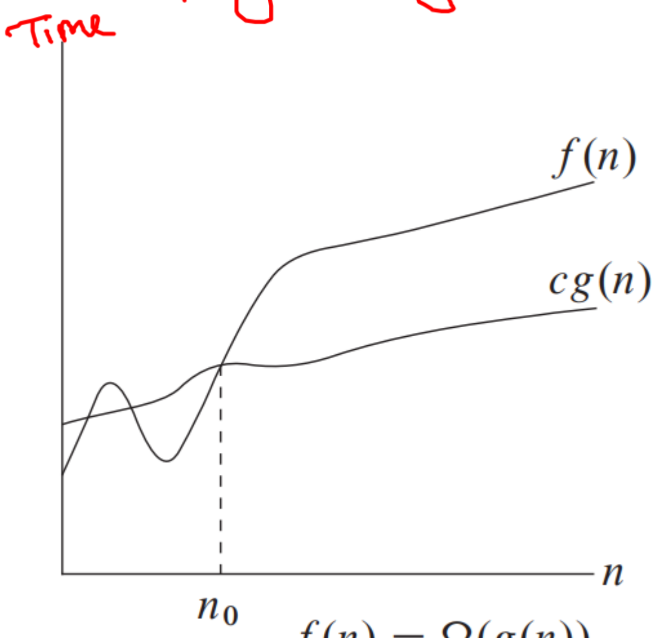
Big-oh



$f(n) = O(g(n))$

$f(n) \leq c \cdot g(n)$

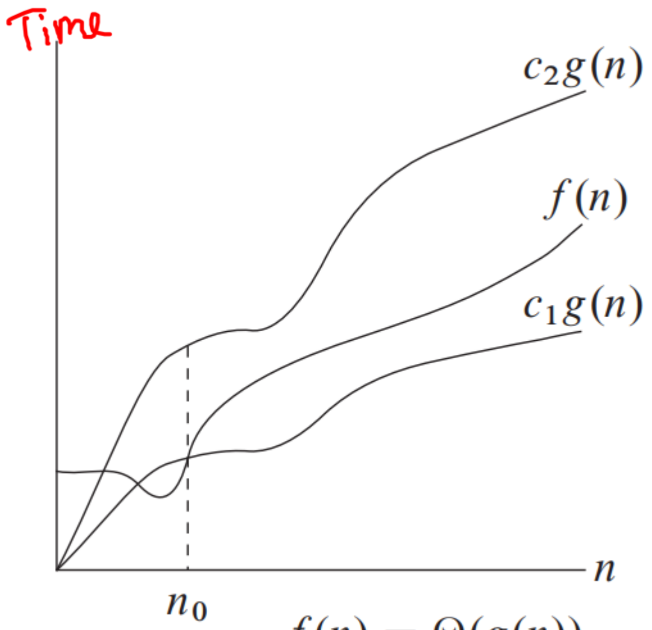
Big-Omega



$f(n) = \Omega(g(n))$

$f(n) \geq c \cdot g(n)$

Theta



$f(n) = \Theta(g(n))$

$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

* $\boxed{c, c_1, c_2, n_0 \rightarrow +ve \quad n \geq n_0}$

Spring 2022 – Spring 2025

All Questions Solve of Asymptotic Notations

c) If $f(n)=kn^2-3$, prove that $f(n)=O(n^2)$. Here, k=last digit of your student id+2. [2]

$k = 3 + 2 = 5$

UIU DSA-1 Mid Question: Spring-2022

For Big-Oh notation

$$f(n) \le c \cdot g(n)$$

$$5n^2 - 3 \le c \cdot n^2$$

$$5 - \frac{3}{n^2} \le c$$

$$\therefore c = 5, \; n_0 = 1$$

$$5n^2 - 3 \le 5 \cdot n^2 ; \quad n \ge 1$$

$$\therefore f(n) = O(n^2)$$

n	$5 - \frac{3}{n^2}$
1	2
2	4.25
3	4.66
4	4.81
5	4.88
⋮	
∞	5

$$\frac{3}{1^2} = 3$$

$$\frac{3}{2^2} = 0.75$$

$$\frac{3}{100^2} = 0.0003$$

$$\frac{3}{(10000000)} \approx 0$$

$$\frac{3}{\infty^2} = 0$$

c) If f(n)=kn-5, prove that f(n)=Θ(n). Here, k=last digit of your student id+4. [2]

$f(n) = 5n - 5$ $\quad$ $k = 1 + 4 = 5$

for Θ-notation:

$$c_1 \cdot n \le f(n) \le c_2 \cdot n \quad [\because g(n) = n]$$

$c_1 \cdot n \le 5n - 5$

$c_1 \le 5 - \frac{5}{n}$

$\therefore c_1 = 2.5$

$5n - 5 \le c_2 \cdot n$

$5 - \frac{5}{n} \le c_2$

$\therefore c_2 = 5$

n	$5 - \frac{5}{n}$
1	0
2	2.5 →
3	3.33
4	3.75
⋮	⋮
10	4.5
⋮	⋮
∞	5 →

$2.5n \le 5n - 5 \le 5 \cdot n$ $\quad$ $n \ge 2$

$n=1$ $\quad 2.5 \le 0 \le 5$ False $\quad$ $n_0 = 2$

$n=2$ $\quad 5 \le 5 \le 10$ T

$n=3$ $\quad 7.5 \le 10 \le 15$ T

⋮

$\therefore f(n) = \Theta(n)$

b) If $f(n)=kn^2-5$, prove that $f(n)=\Theta(n^2)$. Here, k=last digit of your student id+2. [3]

$f(n) = 3n^2 - 5$ ← $g(n)$ $1+2=3$

UIU DSA-1 Mid Question: Fall-2022

$$c_1 \cdot n^2 \le 3n^2 - 5 \le c_2 \cdot n^2$$

$c_1 \cdot n^2 \le 3n^2 - 5$

$c_1 \le 3 - \frac{5}{n^2}$

$\therefore c_1 = 1.75$

$3n^2 - 5 \le c_2 \cdot n^2$

$3 - \frac{5}{n^2} \le c_2$

$\therefore c_2 = 3$

n	$3 - \frac{5}{n^2}$
1	-2
2	$1.75 \rightarrow c_1$
3	2.44
⋮	⋮
∞	$3 \longrightarrow c_2$

$[1.75, 3]$

↖ 2

$$1.75n^2 \le 3n^2 - 5 \le 3n^2 \;;\; n \ge 2$$

$n=1$ $1.75 \le -2 \le 3$ ✗

$n=2$ $7 \le 7 \le 12$ ✓

$n=3$

⋮

$\therefore n_0 = 2$

$\therefore f(n) = \Theta(n^2)$

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b) If f(n)=kn-4, prove that f(n)=Θ(n). Here, k=last digit of your student id+5. [2]

UIU DSA-1 Mid Question: Spring-2023

$f(n) = 6n - 4$ $g(n)$ $= 1 + 5 = 6$

$c_1 \cdot n \leq 6n - 4 \leq c_2 \cdot n$

$c_1 \cdot n \leq 6n - 4$

$c_1 \leq 6 - \frac{4}{n}$

$2 \leq 6 - \frac{4}{n}$

$\therefore c_1 = 2$

$6n - 4 \leq c_2 \cdot n$

$6 - \frac{4}{n} \leq c_2$

$\therefore c_2 = 6$

n	$6 - \frac{4}{n}$
1	$2 \rightarrow c_1$
2	4
3	4.67
⋮	⋮
∞	$6 \rightarrow c_2$

$2n \leq 6n - 4 \leq 6n$

$n = 1$ $2 \leq 2 \leq 6$

$n = 2$

⋮

$n \geq 1$

$\therefore n_0 = 1$

$\therefore f(n) = \Theta(n)$

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b) Prove that running time of $f(n) = 3n^3 + 2n^2 + 5n + 1$ is $O(n^3)$ [2]

UIU DSA-1 Mid Question: Summer-2023

$f(n) = 3n^3 + 2n^2 + 5n + 1$, $g(n) = n^3$

$3n^3 + 2n^2 + 5n + 1 \leq c \cdot n^3$

$3 + \frac{2}{n} + \frac{5}{n^2} + \frac{1}{n^3} \leq c$

n	$3 + \frac{2}{n} + \frac{5}{n^2} + \frac{1}{n^3}$
1	11
2	5.38
3	4.25
4	3.83
⋮	⋮
∞	3

$\therefore c = 11$

$3n^3 + 2n^2 + 5n + 1 \leq 11 \cdot n^3$ $n \geq 1$, $n_0 = 1$

Let, $n=1$: $11 \leq 11$

$n=2$: $31 \leq 81$

⋮

$\therefore f(n) = O(n^3)$

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b) If $f(n)=kn^2-3n+5$, prove that $f(n)=\Theta(n^2)$. Here, k=last digit of your student id+4. [3]

$g(n)$

$1+4=5$

UIU DSA-1 Mid Question: Fall-2023

$f(n)=5n^2-3n+5$

$c_1 \cdot n^2 \le 5n^2-3n+5 \le c_2 \cdot n^2$

$c_1 \cdot n^2 \le 5n^2-3n+5$

$c_1 \le 5-\frac{3}{n}+\frac{5}{n^2}$

$c_1 = 4.5$

$5-\frac{3}{n}+\frac{5}{n^2} \le c_2$

$c_2 = 7$

n	$5-\frac{3}{n}+\frac{5}{n^2}$
1	7
2	4.75
3	4.5556
4	4.5625
5	4.6
6	4.69
7	4.67
$\vdots$	$\vdots$
∞	5

$4.5n^2 \le 5n^2-3n+5 \le 7n^2$

$n \ge 1$

$n_0 = 1$

$n=1$ $\quad 4.5 \le 7 \le 7$ ✓

$n=2$ $\quad 18 \le 19 \le 28$ ✓

$n=3$ $\quad 40.5 \le 41 \le 63$ ✓

$\vdots$

$\therefore f(n)=\Theta(n^2)$

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b) Prove the running time of $f(n) = \frac{1}{3}n^3 - 2n^2$ is $\theta(n^3)$. [2]

$g(n)$

UIU DSA-1 Mid Question: Spring-2025

$c_1 \cdot n^3 \le \frac{1}{3}n^3 - 2n^2 \le c_2 \cdot n^3$

$c_1 \le \frac{1}{3} - \frac{2}{n}$ | $\frac{1}{3} - \frac{2}{n} \le c_2$

$c_1 = 0.04$ $c_2 = 0.34$

$0.04n^3 \le \frac{1}{3}n^3 - 2n^2 \le 0.34n^3$

$n=1$ $0.04 \le -1.67$ False

$n=2$ $0.32 \le -5.34$ False

$\vdots$

$n=6$ $8.64 \le 0$ False

$n=7$ $13.72 \le 16.33 \le 116.62$ True

$n \ge 7$

$n_0 = 7$

$\therefore f(n) = \theta(n^3)$

n	$\frac{1}{3} - \frac{2}{n}$
1	-1.6667
2	-0.6667
3	-0.3334
4	-0.17
5	-0.07
6	0
7	0.05
8	0.08
$\vdots$	$\vdots$
∞	0.33

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Click here to go to the **GitHub repository**

Click [here]() to see this video!

THANK YOU!

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