Principle of Digital Communication

---Lecture 4

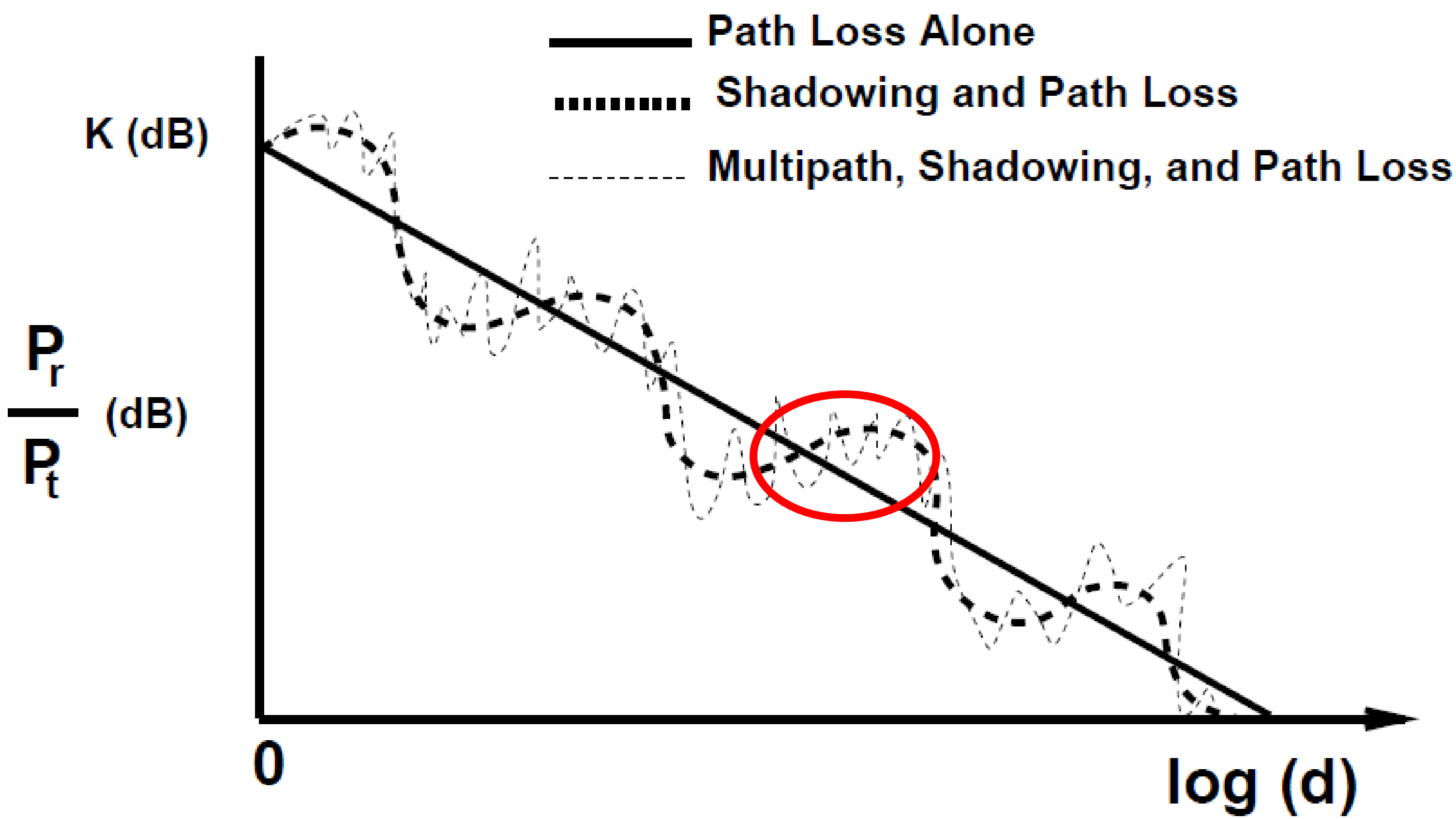
Part I Fading and Channel models

Spectrum for wireless communications

Frequency band	Frequency range
Extremely low frequency (ELF)	<3 kHz
Very low frequency (VLF)	3–30 kHz
Low frequency (LF)	30–300 kHz
Medium frequency (MF)	300 kHz–3 MHz
High frequency (HF)	3–30 MHz
Very high frequency (VHF)	30–300 MHz
Ultra high frequency (UHF)	300 MHz–3 GHz
Super high frequency (SHF)	3–30 GHz
Extra high frequency (EHF)	30–300 GHz

UHF and SHF have good propagation characteristics， and smaller antenna size， therefore, they are very suitable for wireless communications.

大尺度与小尺度衰落



Path Loss Alone
Shadowing and Path Loss
Multipath, Shadowing, and Path Loss
K (dB)
$\frac{P_r}{P_t}$ (dB)
0
log (d)

Path loss

- Definition of path loss

$$P_L = \frac{P_t}{P_r}.$$

$$P_L \text{ dB} = 10 \log_{10} \frac{P_t}{P_r} \text{ dB}.$$ in dB

- Path loss in free space

$$\frac{P_r}{P_t} = \left[\frac{\sqrt{G_l}\lambda}{4\pi d} \right]^2$$

$$P_L \text{ dB} = 10 \log_{10} \frac{P_t}{P_r} = -10 \log_{10} \frac{G_l \lambda^2}{(4\pi d)^2}.$$

Path loss

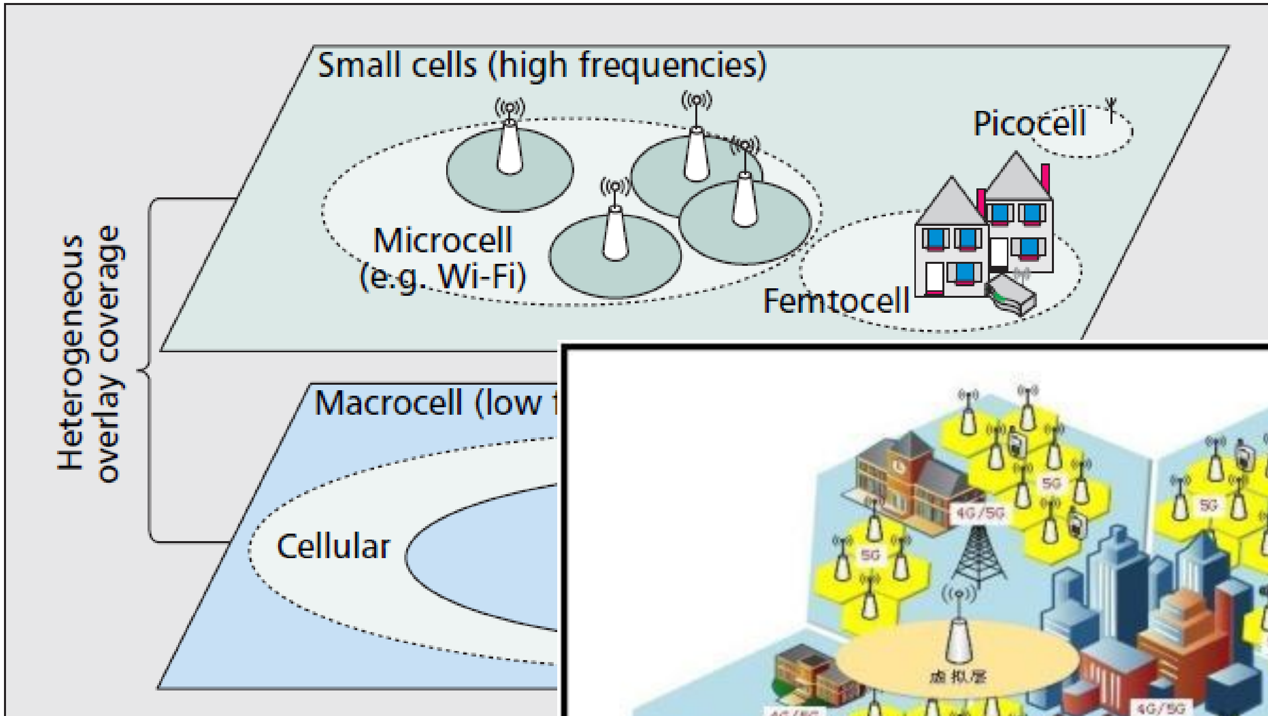
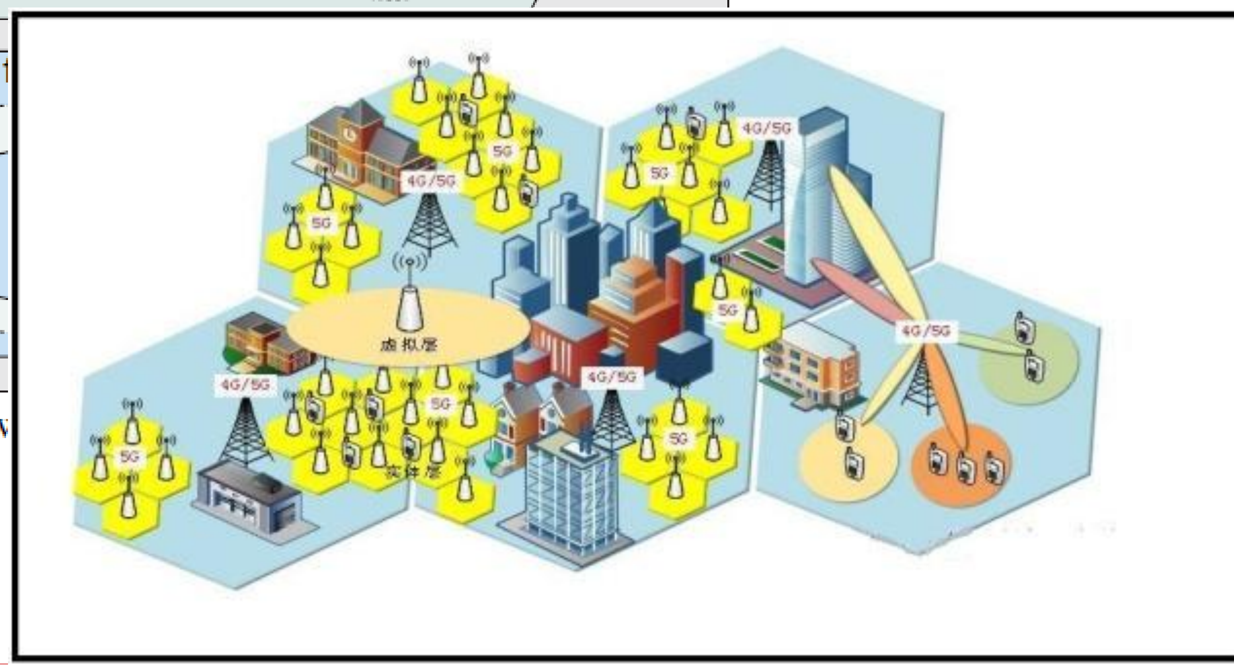


Figure 1. 5G heterogeneous netw
and overlay coverage.



Path loss

- Simplified path loss model

$$P_r = P_t K \left[\frac{d_0}{d}\right]^{\gamma}$$

in dB：

Pathloss parameter

$$P_r \text{ dBm} = P_t \text{ dBm} + K \text{ dB} - 10\gamma \log_{10}\left[\frac{d}{d_0}\right]$$

A constant that depends on the antenna characteristic and average path loss

$$K \text{ dB} = 20\log_{10}\frac{\lambda}{4\pi d_0}$$

Indoor 1~10m
Outdoor 10~100m
d>d0

路径损耗模型

$$P_r \text{ dBm} = P_t \text{ dBm} + K \text{ dB} - 10\gamma \log_{10}\left[\frac{d}{d_0}\right]$$

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

Small scale fading

- 时延扩展（delay spread）

The *delay spread* $\mathcal{L}$ is defined as the difference between the path delay on the longest significant path and that on the shortest significant path. That is,

$$\mathcal{L} = \max_j [\tau_j(t)] - \min_j [\tau_j(t)].$$

The difference between path lengths is rarely greater than a few kilometers, so $\mathcal{L}$ is rarely more than several microseconds. Since the path delays $\tau_j(t)$ are changing with time, $\mathcal{L}$ can also change with time, so we focus on $\mathcal{L}$ at some given t. Over the intervals of interest in modulation, however, $\mathcal{L}$ can usually be regarded as a constant.

- 信道相干带宽（channel coherence frequency）

$$\mathcal{F}_{\text{coh}} = \frac{1}{2\mathcal{L}}.$$

if the channel is badly faded at one frequency f,
how much does the frequency have to be changed to find an unfaded frequency?

to a very crude approximation, f must be changed by $\mathcal{F}_{\text{coh}}$.

Flat fading and frequency-selective fading

- When signal bandwidth is much smaller than the channel coherence time, the channel is flat fading, here T(symbol duration)>>T_d (time delay). Therefore, the channel is modeled as a single-tap filter.

- When signal bandwidth is much larger than the channel coherence time, the channel is frequency-selective fading. The channel is modeled as a multiple-tap filter.

- Whether the channel is flat or frequency-selective fading, it not only depends on the environments, but also depends on the specific implementation.

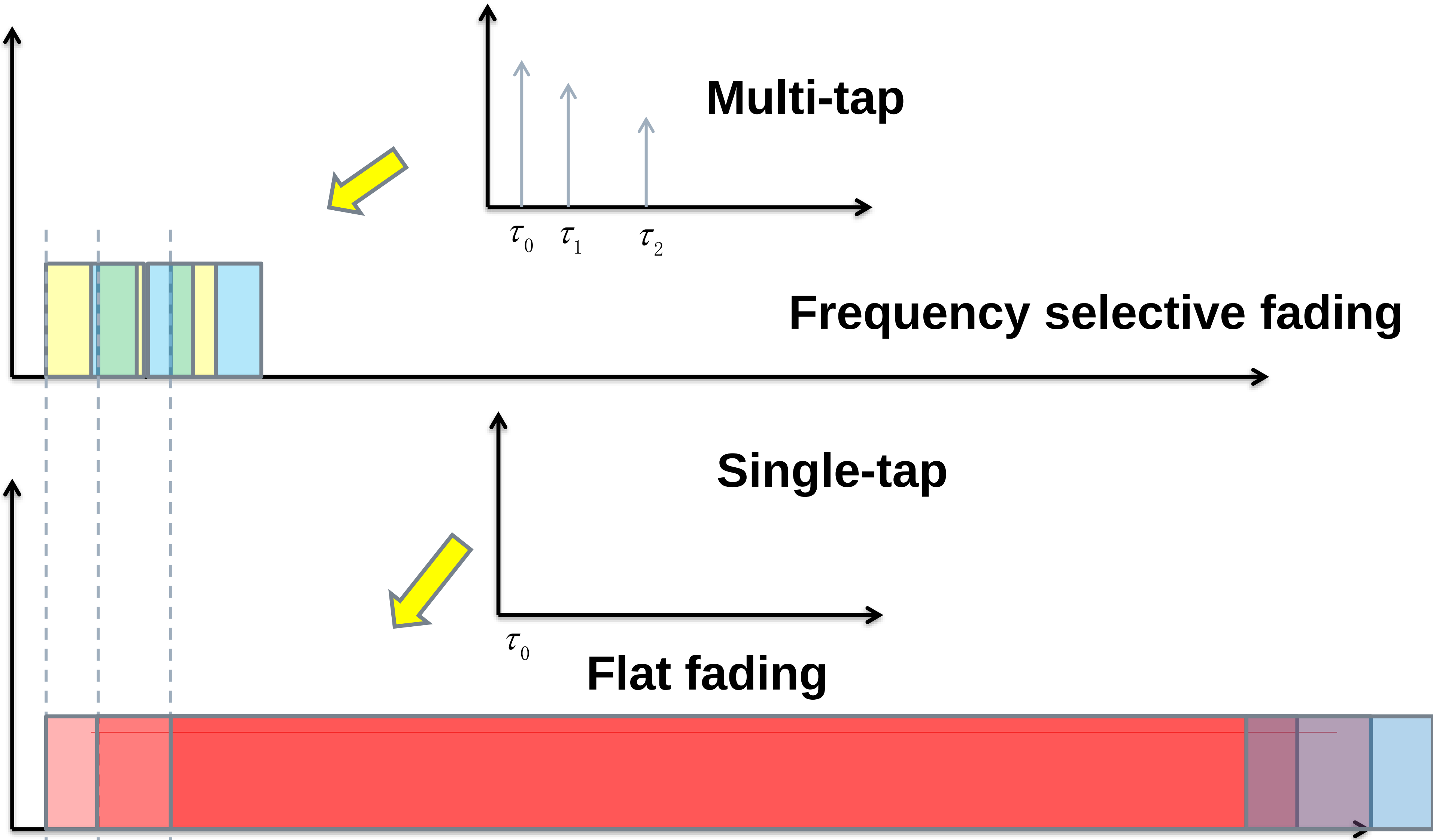
平坦衰落与频率选择性衰落

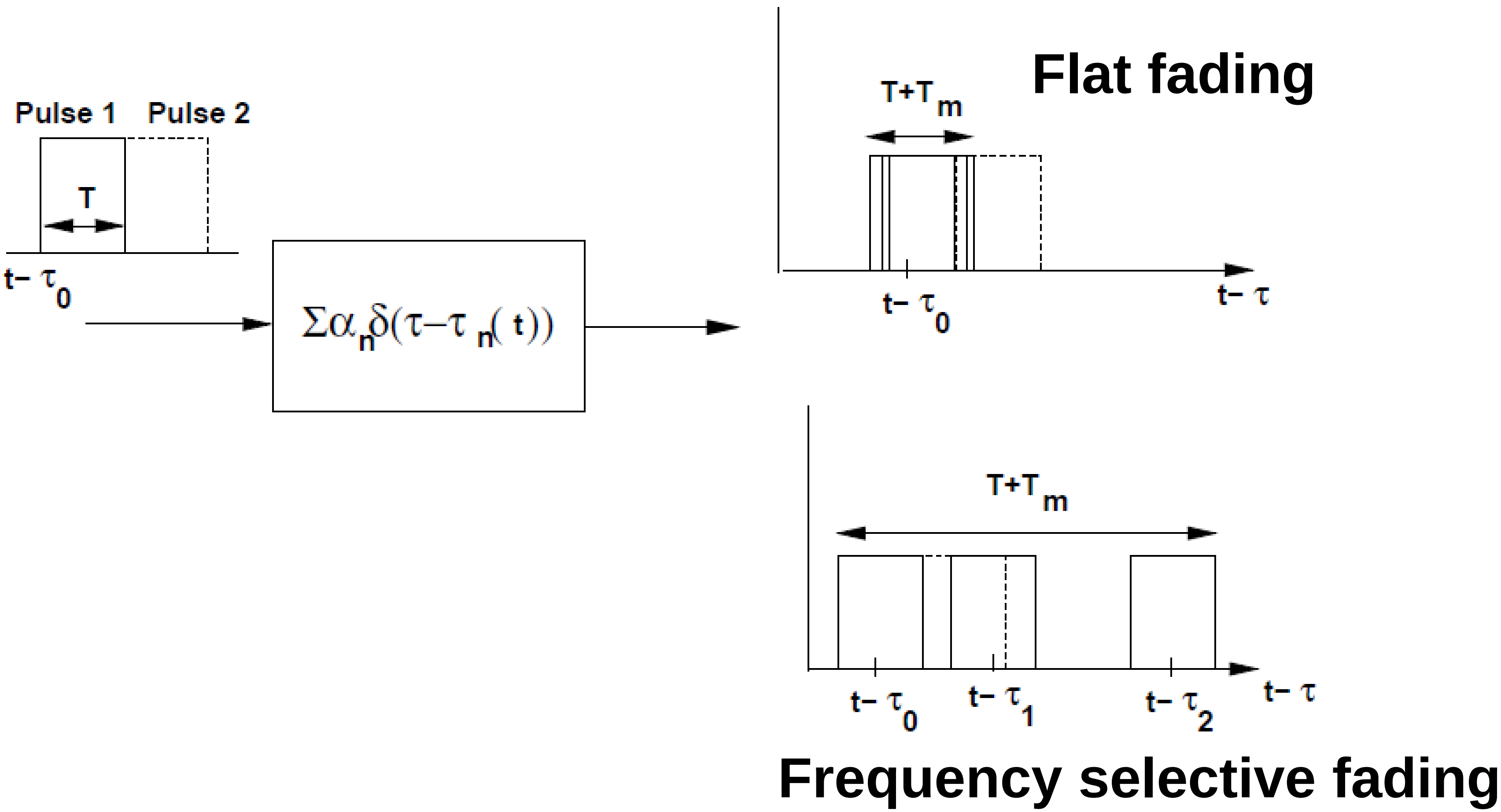
- 当信号带宽远小于相干带宽时，称信道为平坦衰落信道(Flat Fading)，此时，码元间隔T远大于时延扩展T_d，因此，利用单抽头滤波器就足以表示信道。

- 当信号带宽远大于相干带宽时，称信道为频率选择性衰落信道 (Frequency Selective Fading)，此时，必须利用多个信道滤波器的抽头表示信道。

- 平坦衰落或频率选择性衰落并不单纯是信道本身的属性，而是信号带宽和信道相干带宽之间关系的体现。

Flat fading and frequency-selective fading




Pulse 1
Pulse 2
T
t− τ0
Σαnδ(τ−τn(t))
Flat fading
T+Tm
t− τ0
t− τ
T+Tm
t− τ0
t− τ1
t− τ2
t− τ
Frequency selective fading

Fast fading and slow fading

- **Doppler spread (<<carrier frequency)**

$$\mathcal{D} = \max_j \mathcal{D}_j - \min_j \mathcal{D}_j$$

- **信道相干时间（channel coherence time）**

$$\mathcal{T}_{\mathrm{coh}} = \frac{1}{2\mathcal{D}},$$

- When channel coherence time decreases, the channel fading is faster, with shorter fade durations, and channel measurements become fast

Since $\mathcal{D}$ is typically less than 1000H, $\mathcal{T}_{\mathrm{coh}}$ is typically greater than 1/2 msec.

Fast fading and slow fading

- If the channel coherence time is much smaller than the transmission delay, the channel is fast fading. Otherwise, it is slow fading.

- Whether the channel is fast or slow fading, it not only depends on the environments, but also depends on the specific implementation.

快衰落和慢衰落

- 如果相干时间远小于符号的传输时延，则称信道是快衰落的 (Fast Fading)，反之称信道是慢衰落的 (Slow Fading)。
- 信道体现为快衰落还是慢衰落不仅取决于信道的传输环境，还与具体的应用有关。
- 例如，对于语音传输，它的传输时延通常小于100ms，而一些数据应用的传输时延则较为宽松，因此，对于语音应用来说是慢衰落的信道对于数据应用来说可能就体现为快衰落的。

Types of channel	Defining characteristic
Fast fading	
Slow fading	
Flat fading	
Frequency-selective fading	
Double selective fading	

Types of channel	Defining characteristic
Fast fading	$T_c \ll$ delay requirement
Slow fading	$T_c \gg$ delay requirement
Flat fading	$W \ll W_c$
Frequency-selective fading	$W \gg W_c$
Double selective fading	

Rice fading

- If the channel coefficient satisfies $h_l \sim CN(0,\sigma_l^2)$

  the channel is called **Rayleigh fading**

$$\boxed{\frac{1}{\sigma_\ell^2}\exp\left\{\frac{-x}{\sigma_\ell^2}\right\}, \quad x \geq 0.}$$

(Line-of-Sight, LOS)

- If there is line-of-sight component:

$$h_l = \sqrt{\frac{\kappa}{\kappa+1}}\sigma_l e^{j\theta} + \sqrt{\frac{1}{\kappa+1}}CN(0,\sigma_l^2)$$

The channel is called **Rice fading**

κ: a parameter which represents the energy ratio of the LOS path and the scatter paths

Coherent detection in fading channels

$$y = hx + w$$

$$w \sim \mathcal{CN}(0, N_0)$$

- About the CSI
  - For flat fading， CSI is a random variable
  - For Rayleigh fading， *h* is complex Gaussian with $h \sim \mathcal{CN}(0, 1)$
  - When CSI is available at the receiver, it is called CSIR
  - When CSI is available at the transmitter, it is called CSIT

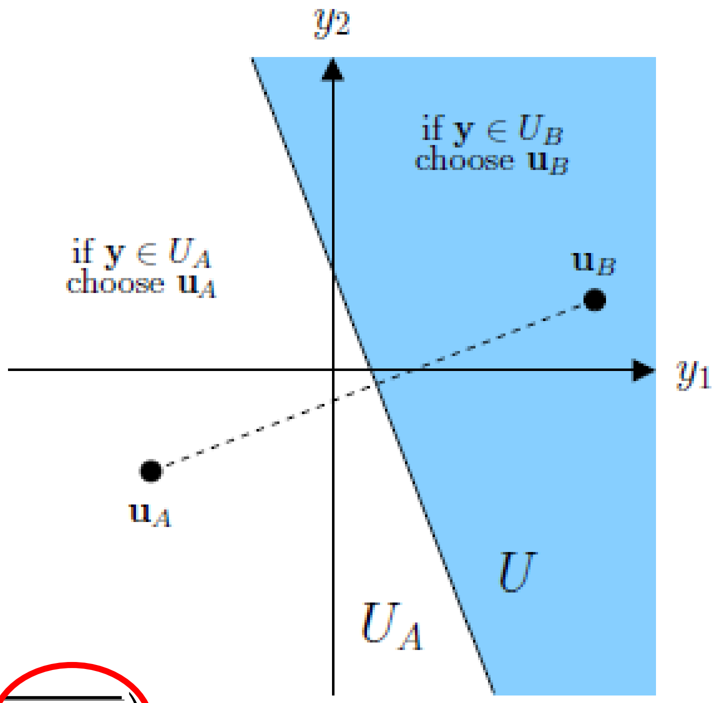
How?

Coherent detection for BPSK

$$y = hx + w \qquad x \in \{-a, +a\}$$

ML rule：

$$\| \mathbf{y} - \mathbf{x}_1 \| \underset{m1}{\overset{m2}{\gtrless}} \| \mathbf{y} - \mathbf{x}_2 \|$$



Error probability:

$$P_e = Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right)$$

$$\Rightarrow \quad P_e = Q\left(\frac{2a|h|}{\sqrt{2N_0}}\right) = Q\left(\sqrt{2|h|^2 \mathrm{SNR}}\right)$$

$$\mathrm{SNR} = a^2 / N_0$$

Coherent detection for BPSK

Conditional error probability when CSIR

$$P_e = Q\left(\sqrt{2|h|^2\,\mathrm{SNR}}\right)$$

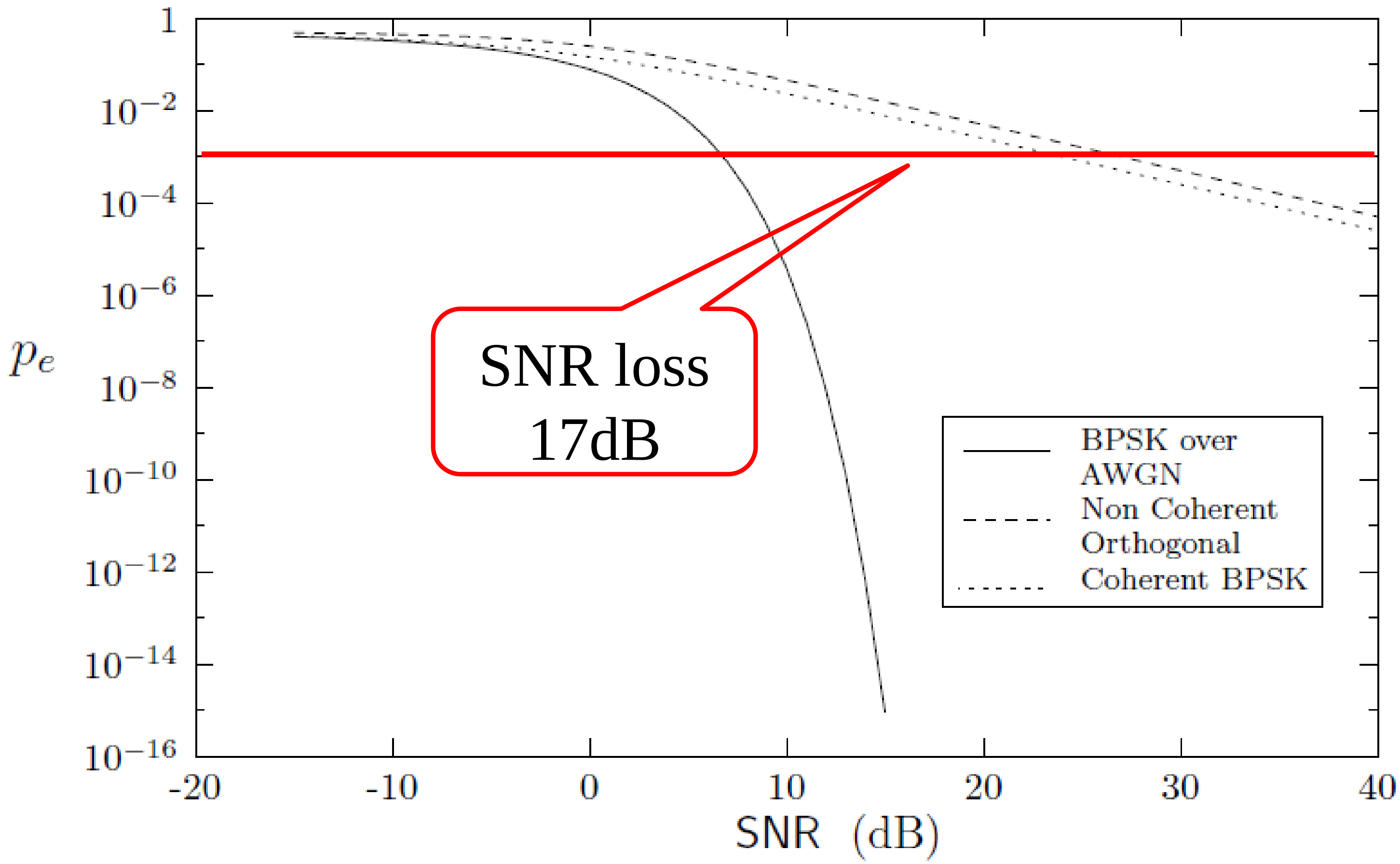
Thus, the average error probability of the system is

$$P_e = \mathrm{E}\left[Q\left(\sqrt{2|h|^2\,\mathrm{SNR}}\right)\right] = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNR}}{1+\mathrm{SNR}}}\right)$$

For high SNR :

$$\boxed{P_e \approx \frac{1}{4\mathrm{SNR}}}$$

Coherent detection for BPSK



Problems caused by fading

- Channel gain

$$G = |h|^2 \quad h \sim CN(0,1) \Longrightarrow p_G(g) = \mathrm{e}^{-g} I_{\{g \geq 0\}}.$$

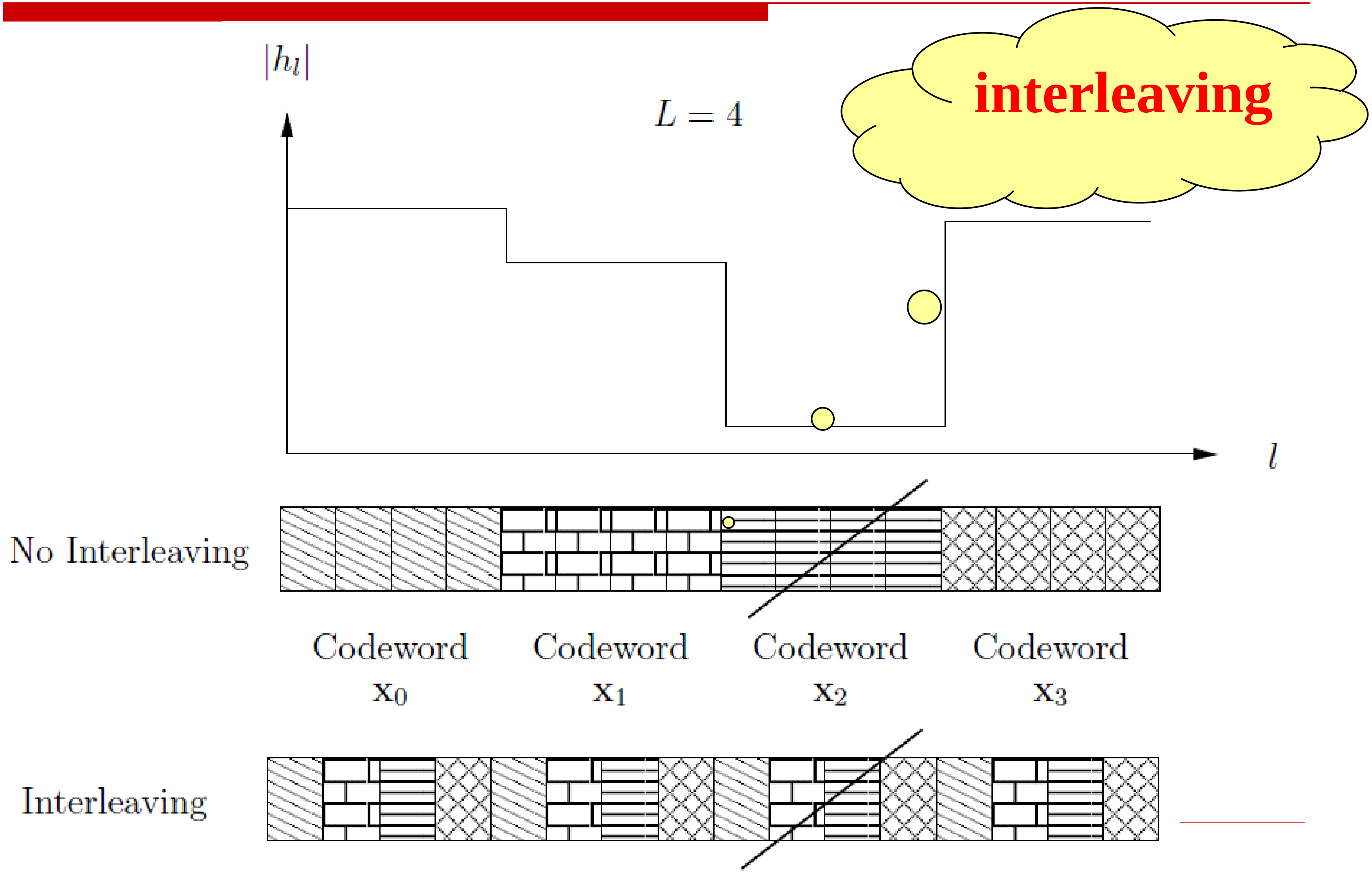
We define the average SNR parameter $\bar{S} = \bar{E}_b/N_0$, and note that the SNR $S = E_b/N_0 = G\bar{S}$. Rayleigh fading causes performance degradation because the power gain G has a high probability of taking on values significantly smaller than its r … The probability of a f …

How to deal with the fading?

$$P[G \leq \epsilon] = 1 - \mathrm{e}^{-\epsilon} \approx \epsilon, \quad \epsilon \ll 1.$$

Thus, the probability of a 10 dB fade ($\epsilon = 0.1$) is about 10%, and the probability of a 20 dB fade ($\epsilon = 0.01$) is 1%. The bits sent during fades are very likely to be wrong, and it turns out that these events dominate the average error probability (averaged over the fading distribution) in uncoded systems.

Diversity –time domain



Diversity

Using MRC (maximum ratio combining) at the receiver:

$$y_\ell = h_\ell x_\ell + w_\ell, \qquad \ell = 1, \ldots, L.$$

$$\mathbf{y} = \mathbf{h} x_1 + \mathbf{w}$$

$$\frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{y} = \|\mathbf{h}\| x_1 + \frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{w}$$

The statistic of the received SNR has been changed

$$\boxed{\|\mathbf{h}\|^2 = \sum_{\ell=1}^{L} |h_\ell|^2 \qquad f(x) = \frac{1}{(L-1)!} x^{L-1} e^{-x}, \qquad x \geq 0.}$$

Given h, the conditional error probability is

$$P_e = Q\left(\sqrt{2 \, \|\mathbf{h}\|^2 \, \mathrm{SNR}}\right)$$

Diversity

$$p_e = \int_0^\infty Q\left(\sqrt{2x\mathsf{SNR}}\right) f(x)\, dx,$$

$$\boxed{f(x) = \frac{1}{(L-1)!} x^{L-1} e^{-x}, \qquad x \geq 0.}$$

$$= \left(\frac{1-\mu}{2}\right)^L \sum_{\ell=0}^{L-1} \binom{L-1+\ell}{\ell} \left(\frac{1+\mu}{2}\right)^\ell$$

$$\mu := \sqrt{\frac{\mathsf{SNR}}{1+\mathsf{SNR}}}.$$

High SNR

$$\frac{1-\mu}{2} \approx \frac{1}{4\mathsf{SNR}} \qquad \frac{1+\mu}{2} \approx 1$$

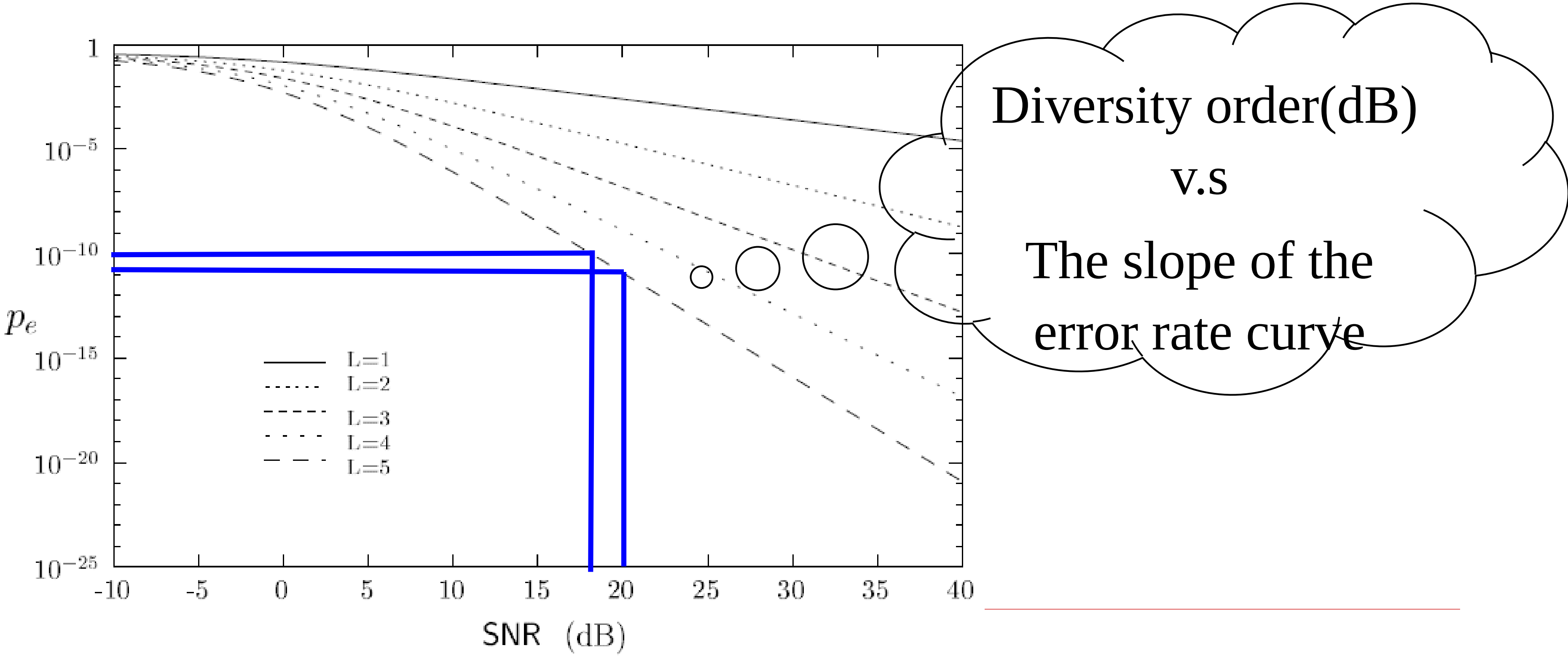
Average error probability at high SNR

$$p_e \approx \frac{1}{(4\mathsf{SNR})^L} \sum_{\ell=0}^{L-1} \binom{L-1+\ell}{\ell} \left(\frac{1+\mu}{2}\right)^\ell$$

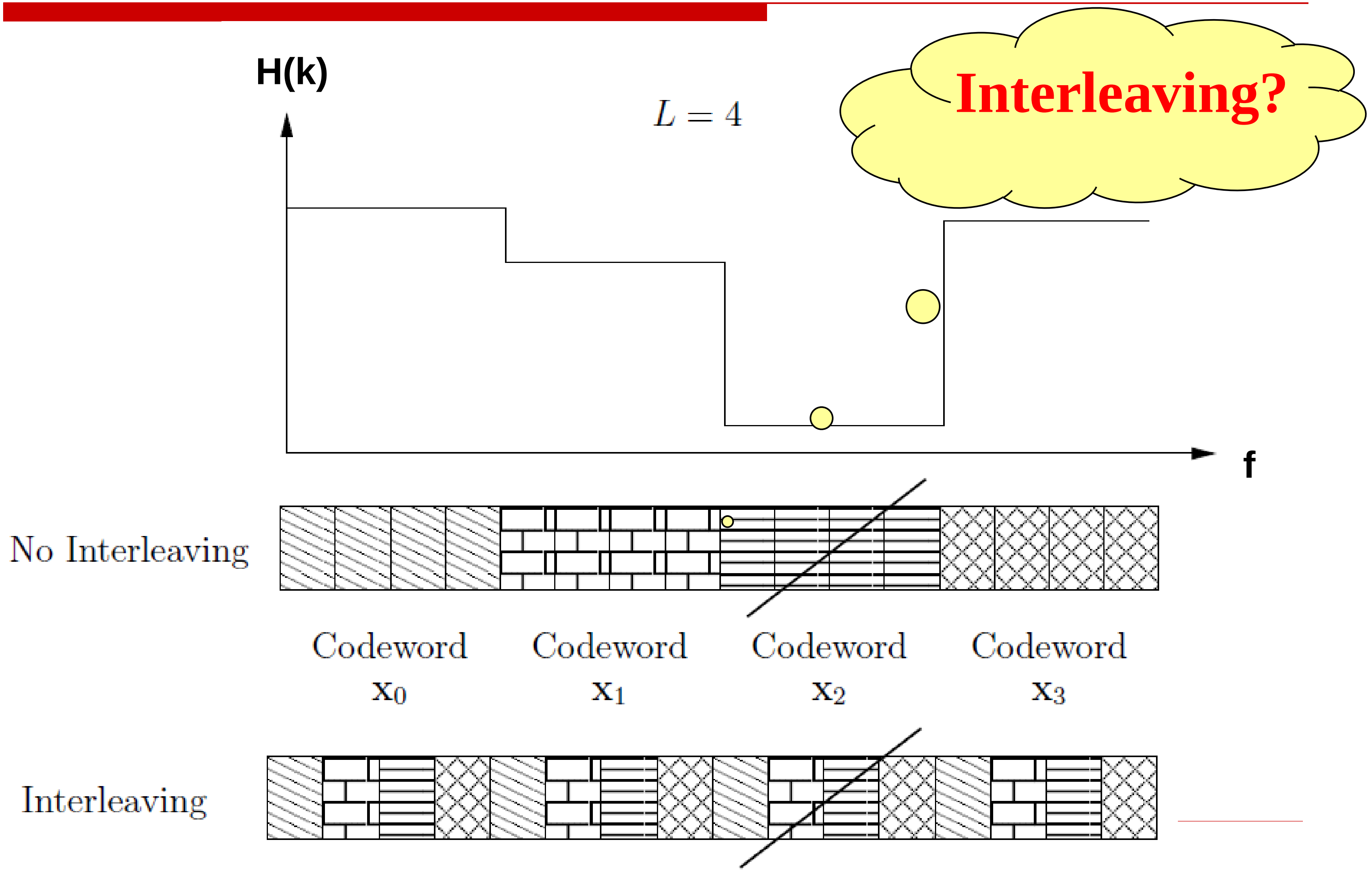
Diversity

Average error probability at high SNR:

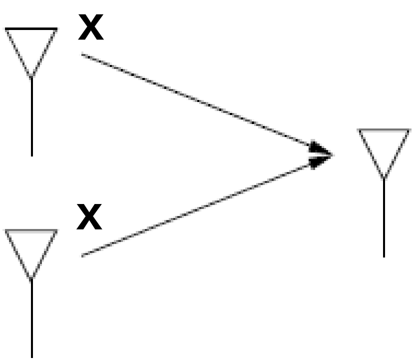
$$p_e \approx \binom{2L-1}{L} \frac{1}{(4\mathsf{SNR})^L}$$



Diversity –frequency domain

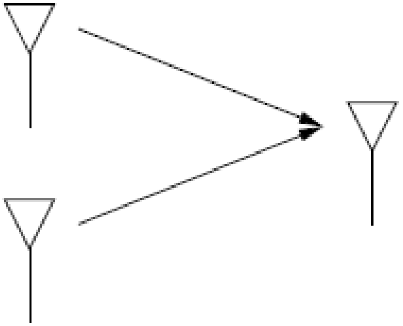


Diversity –multiple antennas



Can we transmit the same symbols on the two transmit antennas to achieve transmit diversity?

Diversity --Alamouti



$$\begin{bmatrix} y[1] & y[2] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix} + \begin{bmatrix} w[1] & w[2] \end{bmatrix}$$

$$\begin{bmatrix} y[1] \\ y[2]^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} w[1] \\ w[2]^* \end{bmatrix}$$

Reference

- David Tse and Pramod Viswanath, ***Fundamentals of Wireless Communication***, **The Press of University of Cambridge, 2005. Chapter 2.**
- **Andrea Goldsmith,** ***Wireless Communication***, **The Press of University of Cambridge, 2005. Chapters 2~3.**
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- **Theodore S. Rappaport,** ***Wireless Communications: Principles and Practice*** **(2nd ed.), Pearson Education, 2002. Chapters 4~5.**
- **John G. Proakis,** ***Digital Communications*** **(4th ed.), McGraw-Hill Education, 2001. Chapter 1.**