MATH 473/MTH 573 Assignment # 3

Due on October 16, 2025 (Thursday)

Instruction:

1. For questions solved by hand, please show middle steps. A simple final answer without necessary justification will receive no credit.

2. For questions involving coding, please include all the MATLAB/Python functions that you wrote for the problem, all the commands you typed with the inputs, and all the **required** numerical results. Please do NOT show intermediate outputs that are not required!

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1. [8 pts]
(i) Write a MATLAB/Python function to implement the Cholesky factorization $A = R^T R$ that we discussed in class.
(ii) Implement the forward substitution and the backward substitution for solving triangular systems.
(iii) Use your MATLAB/Python functions from part **(i)** and part **(ii)** to solve the following linear system

$$\begin{aligned} 4x_1 + x_2 \quad + x_3 + x_4 &= 0.6, \\ x_1 + 3x_2 \; - x_3 + x_4 &= 0, \\ x_1 - x_2 \; + 2x_3 &= 0, \\ x_1 + x_2 \qquad + 2x_4 &= 0.4. \end{aligned}$$

2. [5 pts] **(For MTH 473)** Consider

$$\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

Find the orthogonal projector onto the space $S = \text{span}\{\vec{v}\}$ and the orthogonal projector onto the space that is orthogonal to S.

2. [5 pts] **(For MTH 573)** Consider

$$\vec{v} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}.$$

Find the orthogonal projector onto the space $S = \text{span}\{\vec{v}\}$ and the orthogonal projector onto the space that is orthogonal to S.

3. [**7 pts**] Consider the matrix

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(i) Verify that A has orthonormal columns.
(ii) Find the orthogonal projector P onto range(A).
(iii) Find range(P) and null(P). Are they orthogonal to each other?