MATH 473/573 Assignment # 4

Due on November 4, 2025 (Tuesday)

Instruction:

1. For questions solved by hand, please show middle steps. A simple final answer without necessary justification will receive no credit.

2. For questions involving coding, please include all the MATLAB/Python functions that you wrote for the problem, all the commands you typed with the inputs, and all the **required** numerical results. Please do NOT show intermediate outputs that are not required!

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1. [14 pts] Consider the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

(i) Use the Gram-Schmidt Orthogonalization to find the reduced QR factorization $A = \hat{Q}\hat{R}$.
(ii) Find the orthogonal projector P_1 onto range(A).
(iii) Use the reduced QR factorization from part (i) to find the least squares solution to $A\vec{x} = \vec{b}$.

2. [16 pts]
(i) Using MATLAB or Python to implement the Classical Gram-Schmidt orthogonalization algorithm that we discussed in class. Your code should take an $m \times n$ matrix A as input, and return a matrix $Q \in \mathbb{R}^{m\times n}$ with orthonormal columns and an upper triangular matrix $R \in \mathbb{R}^{n\times n}$, such that $A = QR$.
(ii) Implement the modified Gram-Schmidt orthogonalization algorithm that we discussed in class. Again, A is the input, and Q and R are outputs.
(iii) Use your code in parts (i) and (ii) to compute the reduced QR factorizations of the matrix

$$A = \begin{pmatrix} 1 & 1+10^{-8} & 1 \\ 1 & 1 & 1+10^{-8} \\ 1 & 1 & 1 \end{pmatrix}.$$

For each resulting Q, check whether the second column is orthogonal to the third column. Compare the results from Classical and Modified Gram-Schmidt.

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