MATH 473/MTH 573 Assignment # 2

Due on October 9, 2025 (Thursday)

Instruction:

1. For questions solved by hand, please show middle steps. A simple final answer without necessary justification will receive no credit.

2. For questions involving coding, please include all the MATLAB/Python functions that you wrote for the problem, all the commands you typed with the inputs, and all the **required** numerical results. Please do NOT show intermediate outputs that are not required!

3. Please submit your solution as a single .pdf file on MyCourses. Homework late for more than 3 days will not be accepted.

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1. [7 pts] (solve by hand) Given the linear system

$$\begin{aligned} 6x_1 - 2\alpha x_2 &= 3, \\ \alpha x_1 - 3x_2 &= \frac{3}{2}. \end{aligned}$$

(a) Find value(s) of α for which the system has no solutions.
(b) Find value(s) of α for which the system has an infinite number of solutions.
(c) Assuming a unique solution exists for a given α, find the solution in terms of α.

2. [7 pts] (solve by hand) Given

$$A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 5 \\ 2 \\ 5 \end{pmatrix}$$

Find the LU factorization of A and use it to solve the linear system .

3. [8 pts] (solve by hand) Given

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ -1 & 2 & 3 \end{pmatrix} \qquad \text{and} \qquad \vec{b} = \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix},$$

(MTH 473:) use Gaussian Elimination with partial pivoting to solve $A\vec{x} = \vec{b}$.
(MTH 573:) find the $PA = LU$ factorization and use it to solve the linear system $A\vec{x} = \vec{b}$.

4. (For MTH 473) [8 pts]
(i) Write a MATLAB/Python code to implement the Gaussian Elimination without pivoting algorithm that we discussed in class.
(ii) Use your code from part **(i)** to solve the following linear systems

$$A\vec{x} = \vec{b}, \quad \text{where } A = (a_{ij}), \quad \vec{b} = (b_j)$$

with

$$a_{jj} = 3,\ a_{j,j-1} = 1,\ a_{j,j+1} = 1,\ b_{50} = 1,$$

and $a_{ij} = b_j = 0$, otherwise, with $1 \leq i, j \leq 100$. Plot the solution vector $\vec{x} = (x_j)$ as a function of j.

4. (For MTH 573) [8 pts]
(i) Write a MATLAB/Python function to implement the Gaussian Elimination with partial pivoting algorithm that we discussed in class.
(ii) Use your code from part **(a)** to solve the following linear system

$$A\vec{x} = \vec{b}, \quad \text{where } A = (a_{ij}), \quad \vec{b} = (b_j)$$

with

$$a_{jj} = \mathbf{1},\ a_{j,j-1} = 1,\ a_{j,j+1} = 1,\ b_{50} = 1,$$

and $a_{ij} = b_j = 0$, otherwise, with $1 \leq i, j \leq 100$. Plot the solution vector $\vec{x} = (x_j)$ as a function of j.