MATH 473/MTH 573 Assignment # 5

Due on November 18, 2025 (Tuesday)

Instruction:

1. For questions to be solved by hand, please show middle steps. A simple final answer without necessary justification will receive no credit.

2. For questions involving coding, Please show all functions that you defined, all the commands you typed with inputs, and all the **required** numerical results.

---

1. (solve by hand) Given a vector

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix},$$

use Householder reflector to find a matrix F so that $F\vec{x} = (-3, 0, 0)^T$.

2. (solve by hand) Given a set of data points $\{(0,1), (1,2), (2,2), (3,3)\}$, we want to find the linear function $P(x) = c_0 + c_1 x$ that fitts the data points in the least square sense.

(i) Find a matrix A and a vector $\vec{b}$ so that $\vec{c} = (c_0, c_1)^T$, where c_0 and c_1 are coefficents in $P(x)$, is the solution to the least squares problem

$$\min_{\vec{c} \in \mathbb{R}^2} \|A\vec{c} - \vec{b}\|_2.$$

(ii) Solve the least square problem using the normal equation.

3. (a) Write a MATLAB or Python function that computes an implicit representation of a full QR factorization $A = QR$ of an $m \times n$ matrix A with $m \geq n$ using the Householder reflection algorithm that we discussed in class. The output variables are a lower triangular matrix $V \in \mathbb{C}^{m \times n}$ whose columns in the lower triangular part are the vectors v_k defining the Householder reflections, and a triangular matrix $R \in \mathbb{C}^{m \times n}$.

(b) Write a MATLAB or Python function that takes the matrix V produced by your function in part **(a)** as input and generates a corresponding $m \times m$ orthogonal matrix Q.

(c) Use your code in part **(a)** **(b)** to compute the full QR factorization of the matrix A in problem 2. Output the matrices V, R and Q.

4. (Required for MTH 573, Optional for MTH 473)
Suppose $m = 50, n = 10$. We want to use a polynomial of degree $n-1$ (with n coefficients) for the least squares fitting of the function $\cos(4t)$ on m equally spaced grid points from 0 to 1. The least squares problem is in the form of $A\mathbf{x} = \mathbf{b}$, where A is the $m \times n$ Vandermonde matrix and $\mathbf{b}$ is the column vector containing the values of the function $\cos(4t)$ evaluated on the m grid points.

Now form the matrix A and the vector $\mathbf{b}$. Then calculate and print (to sixteen-digit precision) the least squares coefficient vector x by the following methods:
(a) solving the normal equation by Choleskey factorization,
(b) reduced QR factorization computed by the modified Gram-Schmidt code in Homework 3,
(c) full QR factorization computed by Householder triangularization in problem 3 of this homework,
(d) using built-in functions, \ in MATLAB or solve() in Python.

The calculations above will produce four lists of ten coefficients. In each list, mark the digits that appear to be wrong (affected by rounding error). Comment on what differences you observes.