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1.a) Since Π_1 and Π_2 are two probability dist. over the utility matrix that satisfy the following inequality, both will yield the same amount of payoff for each player.

$$\forall i: \sum_A \Pi_1 \times u_i(A) \geq \sum_A \Pi' \times u_i(A) \quad \forall \Pi \in \Delta(A)$$

Therefore, any convex combination of Π_1 and Π_2 has the same utility for each player.

$$\forall i \sum_A \Pi_1 \times u_i(A) = \sum_A \Pi_2 u_i(A) \Rightarrow$$

$$\forall i \quad \sum_A \Pi_i u_i(A) = M \sum \Pi_1 \times u_i(A) + (1-M) \sum \Pi_2 \times u_i(A))$$

1.b

	A	B
A	(1,2)	(0,0)
B	(0,0)	(2,1)

$AA = \frac{1}{2}$ $AB = BA = 0$ $BB = \frac{1}{2}$

$\Rightarrow u_1 = \frac{3}{2}, u_2 = \frac{3}{2}$

	q	$1-q$
p	(1,2)	(0,0)
$1-p$	(0,0)	(2,1)

$u_1 = pq + 2(1-p)(1-q)$

$u_2 = 2pq + (1-p)(1-q)$

$$\left.\begin{array}{l} \arg\max_p u_1 \Rightarrow p = \frac{1}{3} \\ \arg\max_q u_2 \Rightarrow q = \frac{2}{3} \end{array}\right\} \Rightarrow \begin{array}{l} u_1 = \frac{2}{3} \\ u_2 = \frac{2}{3} \end{array}$$

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2 b) Find a k such that the following procedure yields the maximum amount of payoff for each player.

- randomly chose k agents and label as "ON",

From all of the 2^n strategies, $\binom{n}{k}$ of them gets the probability of $\frac{1}{\binom{n}{k}}$ and the other is equal to 0.

Therefore, for an agent i, it gets labeled as "ON" $\frac{k}{n}$ of the times and recieves $u_{ON}(k-1)$ and is labeled as "OFF" $\frac{n-k}{n}$ of the times are recieves $u_{OFF}(k)$.

$\Rightarrow$ the proper k has to maximize $\frac{k}{n} u_{ON}(k-1) + \frac{n-k}{n} u_{OFF}(k)$.

$$\Rightarrow k^* = \underset{k}{\arg\max}\ \frac{k}{n} * u_{ON}(k-1) + \frac{n-k}{n} u_{OFF}(k)$$

Finding k^* takes $O(n)$ and the rest can be done in poly(n).

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2a) This strategy will provide each player a payoff of $\frac{n}{12}$.
(the chance that they get picked)

However, if the referee chooses exactly six and send them to A and rest to B: each get 1.

$\Rightarrow$ it is not CNE!

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3. Agreement Point = (0, 6)

$p = (0, \frac{1}{14}, \frac{13}{14}, 0)$ $q = (0, 0, \frac{4}{7}, \frac{3}{7})$ Treat Strategy

$D_1 - D_2 = \frac{21}{50}$ $\left(\frac{6 + 0.42}{2}, \frac{6 - 0.42}{2} \right)$

$$\begin{array}{c|c} & \begin{array}{cccc} 0 & 0 & \frac{4}{7} & \frac{3}{7} \end{array} \\ \begin{array}{c} 0 \\ \frac{1}{14} \\ \frac{13}{14} \\ 0 \end{array} & \begin{bmatrix} 1 & 3 & -6 & 0 \\ 2 & 0 & -3 & 3 \\ 5 & -6 & 0 & 4 \\ -7 & 7 & -1 & -4 \end{bmatrix} \end{array} = (A - B)$$

$D_1 = \mathrm{val}(A)$ $D_2 = \mathrm{val}(B)$

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4.a) $y = 4 - x^2$ $\qquad \underset{x,y}{\operatorname{argmax}}\ (y)(x) = \underset{x}{\operatorname{argmax}}\ (4-x^2)(x)$

$f(x) = 4x - x^3 \rightarrow f'(x) = 0 \rightarrow 4 - 3x^2 = 0$

$\rightarrow x = \frac{\pm 2}{\sqrt{3}}$

$x = \frac{-2}{\sqrt{3}}$ is not acceptable since y is ≥ 0 and $\frac{2}{\sqrt{3}} > \frac{-2}{\sqrt{3}}$

$x = \frac{2}{\sqrt{3}}, \ y = \frac{8}{3}$

4.b) $y = 4 - x^2$ $\qquad \underset{x,y}{\operatorname{argmax}}\ (y-1)(x)$ $\qquad y = \frac{c}{x} + 1$

Another way of looking at it

$$\begin{cases} 4 - x^2 = \frac{c}{x} + 1 \\ -2x = \frac{-c}{x^2} \end{cases} \rightarrow 4x - x^3 - 1 - 2x^3 = 0$$

$\rightarrow -3x^3 + 4x - 1$

$\rightarrow x = 1$

$y = 3$

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5) a) $v(\{1\}) = a_1$ $v(\{2\}) = 0$ $v(\{3\}) = 0$

$v(\{1,2\}) = a_2$ $v(\{1,3\}) = a_3$ $v(\{2,3\}) = 0$

$v(\{1,2,3\}) = a_3$

Imputation Vector = $x_1 + x_2 + x_3 = a_3$ $x_1 \geq a_1$ $x_3 \geq 0$

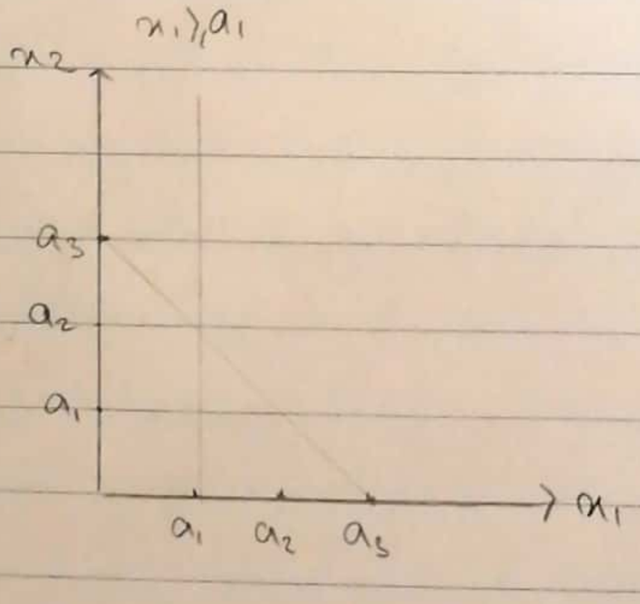
$x_2 \geq 0$

Core Description = $x_1 + x_2 \geq a_2$ (*) $x_1 + x_3 \geq a_3$ (*')

$x_2 + x_3 \geq 0$

* $\Rightarrow x_1 + a_3 - x_2 - x_1 \geq a_3 \Rightarrow x_2 \leq 0$

*' $\Rightarrow x_2 + a_3 - x_1 - x_2 \geq a_2 \Rightarrow x_1 \leq a_3 - a_2$



$a_3 - (x_1 + x_2) \geq 0 \Rightarrow$

$a_3 - x_2 \geq x_1$

Core $\begin{cases} x_2 \leq 0 \\ x_1 \leq a_3 - a_2 \end{cases}$

if $a_3 - a_2 \geq a_1$

Core is $a_1 \leq x_1 \leq a_3 - a_2$

else

Core is empty

SEPEHR

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5) b

Permutation	1	2	3
1 2 3	0	30	10
1 3 2	0	0	0
2 1 3	30	0	10
2 3 1	40	0	0
3 1 2	40	0	0
3 2 1	40	0	0
Shapley Value	25	5	10

Since $x_2 > 0 \Rightarrow$ Shapley is not in Core

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6) a) $v(S) = -\sum_{r \in} c_r + \sum_{i \in} [i]$

shortest path excluding S_{-i}

$v(\{1\}) = -1$ $v(\{2\}) = -1$

$v(\{3\}) = 0$ $v(\{1,2\}) = -1$

$v(\{1,3\}) = -1$ $v(\{2,3\}) = 6$

$v(\{1,2,3\}) = 8$

6) b)

	A	B	C
ABC	-1	+2	7
ACB	-1	9	0
BAC	2	-1	7
BCA	2	-1	7
CAB	-1	9	0
CBA	2	6	0
shapley value	$\frac{3}{2}$	4	$\frac{7}{2}$

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7.a) we set $p(a) = \Pi_a \times e^{-\delta a}$

Suppose $a_i, a'_i \in A_i$ and $a_{-i} \in A_{-i}$

if $u_i(a_i, a_{-i}) \geqslant u_i(a'_i, a_{-i}) \Leftrightarrow \Pi_a e^{-\delta a} - \frac{\Pi a}{a_i} \geqslant \Pi_{a'} e^{-\delta a'} - \frac{\Pi_{a'}}{a'_i}$

$\xleftrightarrow{\frac{\Pi a}{a_i} = \frac{\Pi a'}{a'_i}} \Pi_a e^{-\delta a} \geqslant \Pi_{a'} e^{-\delta a'} \Leftrightarrow p(a) \geqslant p(a')$

7.b For player i, $a_i = \underset{a_i}{\operatorname{argmax}} \; \Pi_a \times e^{-\delta a} - \frac{\Pi a}{a_i}$

$$\left(\Pi_a \times e^{-\delta a} - \frac{\Pi a}{a_i}\right)' = \frac{\Pi a}{a_i} \times e^{-\delta a} + (-1) \quad \Pi_a e^{-\delta a} = 0$$

$$\Rightarrow \frac{\Pi a}{a_i} \times e^{-(\delta a - a_i)} (1 - (a_i)) = 0 \Rightarrow a_i = 1$$

Since the game is symetric, strategy profile for NE $(1, 1, 1, \dots, 1)$

7.c) Mycpic best response will converge to NE in a potential game $\Rightarrow$ result for MBR algorithm is $(1, 1, 1, \dots, 1)$

(However, the number of strategies is not finite therefore, the algorithm might never halt)

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8)a) Consider the following function

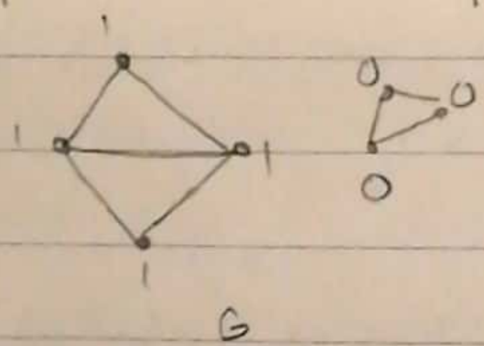
$$P(S) = -|\{(v,w) \in E(G) : S_v \neq S_w\}|$$

meaning counting the number of conflicts in the labeled graph.

if by changing strategy, every player has an increse in utility : the number of conflicts has decreased $\Leftrightarrow$ p(a) increase

8)b) The best response for a player to every other player chosing either 1 or 0, is choosing the same label.

Also, it will maximize the pay off for each player if only a connected component has same label for its player.



Meaning, the best respose for a player that every other reachable player has the same label is choosing that label.

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8.c) If by change the strategy, all players get to increase their payoff, then one player has to have consistency in its in-neighbors and its out-neighbors. this is unnecessary of this game, without knowing the structure of the G.

9.)a)

$$S_i = \begin{cases} \text{private road} & i \leq \frac{n}{2} \\ \text{public} & i \geq \frac{n}{2} \end{cases}$$

public is better for agent #n at most n
private is better for agent #1 at most 1
by induction public is better for n-i $i \leq \frac{n}{2}$
private is better for i $i \leq \frac{n}{2}$

$$\text{Social Cost} = \sum_{i=1}^{\frac{n}{2}} + \left(\frac{n}{2}\right)^2 = \frac{n^2}{8} + \frac{n}{4} + \frac{n^2}{4} = \frac{3n^2+2n}{8}$$

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)b. General Social Cost $= \sum_{i \in S} i + (n-|S|)^2$

the previous strategy is PO since no other coalition can decrease their cost by changing their strategy.

SNE $\Rightarrow$ PO

$$\min_{K} \text{Social Cost} = \sum_{i=1}^{K} i + (n-K)^2 = K = \frac{2n - \frac{1}{2}}{3} \approx \Rightarrow K = \frac{2}{3}n$$

$$\text{PoA} = \frac{\frac{3n^4 + 2n}{8}}{\frac{(\frac{2}{3}n)(\frac{2}{3}n+1)}{2} + (\frac{2}{3}n)^2}$$