Computational Complexity

Exam #3-Take Home
Start Date: 16/4/1398 13:00
Due Date: 18/4/1398 15:00

Some Rules.
1. Please hand in your worksheet prior to 15:00 Tuesday 18 Tir 1398. Overdue papers are not acceptable at all.
2. You are allowed to use the Internet and the textbooks. You are NOT allowed to talk with your classmates about the problems and the solutions.
3. Partial answers are appreciated, so write down your thinking even though you may think it is insufficient or imperfect.

Problem 1.
we are given a family F of subsets of a universe U and a positive integer k, and the goal is to test whether there exists a coloring of U with two colors such that at least k sets in F contain members of both colors. Show that the problem admits a kernel with at most $2k$ subsets and $O(k^2)$ universe size.

Problem 2.
Given a graph G and an integer ℓ, the INDUCED MATCHING PROBLEM asks whether there is a subset of 2ℓ vertices in G inducing a matching (i.e. l nonadjacent edges such no two of them have a common adjacent edge). Show that this problem can be solved in $2^{\mathcal{O}(k)}n^{\mathcal{O}(1)}$ where $k = tw(G)$.

Problem 3.
Let P be a maximization problem and for instance I, let $OPT_P(I)$ be the optimal solution for I. Its corresponding decision problem $\hat{P}$ is defined as follows:
Input: Instance I and integer k.
Query: Is $OPT_P(I) \geq k$?
We say that Problem P admits an FPTAS if for every $\epsilon > 0$ there is an ϵ-approximation algorithm for P, i.e. for instance I, the algorithm outputs $ALG_P(I)$ such that
(1) $1 \leq \frac{OPT_P(I)}{ALG_P(I)} \leq 1 + \epsilon$.
(2) the running time of the algorithm is bounded by a polynomial of $|I|$ and $\frac{1}{\epsilon}$.
Prove that if P admits an FPTAS, then $\hat{P}$ is fixed-parameter tractable with parameter k.

Good Luck!