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- For output, look into the next state and output column of the newly constructed Mealy machine. For a state as a next state  $(letq_i)$  in the new constructed Mealy Machine if output is 0 then for the state  $(q_i)$  as a present state in the constructing Moore Machine; the output will be 0.
- For the Moore machine, the output depends only on the present state. This means that from the beginning state for  $\land$  input, we can get an output. If the output is 1, the newly constructed Moore machine can accept zero length string, which was not accepted by the given Mealy machine. To make the Moore machine not to accept  $\land$  string, we have to add an extra state  $q_b$  (new beginning state), whose state transactions will be identical with those of the existing beginning state, but with output as 0.

(For a Mealy machine with m number of states and n number of outputs, the Moore machine will be of n number of outputs but not more than mn+1 number of states. The following examples (Example 3.18 to 3.20) describe the previous method.

**Example 3.18** Convert the given Mealy machine to an equivalent Moore machine.

	I/P =	0	I/P =	1
Present State	Next State	O/P	Next State	O/P
$\longrightarrow$ $q_0$	$q_0$	1	$\mathbf{q}_{\mathbf{i}}$	0
$q_1$	$q_3$	1	$q_3$	1
$\mathbf{q}_2$	$\mathbf{q}_{_{1}}$	1	$\mathbf{q}_2$	1
$q_3$	$\mathbf{q}_{_{2}}$	0	$q_0$	1

**Solution:** Look into the next state and output columns of the given Mealy machine. For I/P 0 for  $q_1$  as a next state, the output is 1. For I/P 1 for  $q_1$  as a next state, the output is 0. The same thing happens for  $q_2$  as a next state for input 0 and input 1. So the state  $q_1$  is broken as  $q_10$  and  $q_11$ , and the state  $q_2$  is broken as  $q_20$  and  $q_21$ . After breaking, the modified Mealy machine becomes

	I/P = 0		I/P =	1
Present State	Next State	O/P	Next State	O/P
$\longrightarrow$ $q_0$	${\bf q_0}$	1	$q_{10}$	0
$q_{10}$	$q_3$	1	$q_3$	1
$q_{11}$	$q_3$	1	$q_3$	1
$q_{20}$	$q_{11}$	1	$q_{21}$	1
$q_{21}$	$q_{11}$	1	$q_{21}$	1
$q_3$	$q_{20}$	0	$q_0$	1

For the present state  $q_0$  for input 1, the next state is  $q_10$ , because there is no  $q_1$  in the modified Mealy machine. It has been broken into  $q_10$  and  $q_11$  depending on the output 0 and 1, respectively. For the present state  $q_0$  for input 1, the output is 0. So the next state is  $q_10$ . The same cases occur for the others also. For the broken states, the next states and outputs are the same as the original, from where the broken states have come.

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From this, the Moore machine becomes

	Next		
Present State	I/P = 0	I/P = 1	O/P
<b>→</b> q <sub>0</sub>	$\mathbf{q}_{\mathrm{o}}$	$\mathbf{q}_{10}$	1
$q_{10}$	$q_3$	$q_3$	0
$q_{11}$	$q_3$	$q_3$	1
$q_{20}$	$q_{11}$	$q_{21}$	0
$\mathbf{q}_{21}$	$q_{11}$	$q_{21}$	1
$\mathbf{q}_3$	$q_{20}$	$q_0$	1

For Moore machine the beginning state is  $q_0$  and the corresponding output is 1. That means, with null length input (no input), we are getting an output of 1. That is, the Moore machine accepts 0 length sequence [because here output depends only on the present state] which is not acceptable for the Mealy machine. To overcome this situation, we must add a new beginning state  $q_b$  with the same transactions as  $q_0$ , but with output as 0. By including the new state, the Moore machine is

	Next	State	
Present State	I/P = 0	I/P = 1	O/P
— <b>▶</b> q <sub>b</sub>	$q_0$	$q_{10}$	0
$\mathbf{q}_{\mathrm{o}}$	$q_0$	$q_{10}$	1
$\mathbf{q}_{10}$	$q_3$	$q_3$	0
$\boldsymbol{q}_{11}$	$q_3$	$q_3$	1
$\mathbf{q}_{20}$	$q_{11}$	$\boldsymbol{q}_{21}$	0
$\mathbf{q}_{21}$	$q_{11}$	$q_{21}$	1
$q_3$	$\boldsymbol{q}_{20}$	$q_0$	1

**Example 3.19** Convert the given Mealy machine to an equivalent Moore machine.

	I/P = 0		I/P = 1	
Present State	Next State	O/P	Next State	O/P
—► q <sub>0</sub>	$\boldsymbol{q}_2$	$Z_0$	$\boldsymbol{q}_1$	$z_1$
${\bf q}_1$	$\mathbf{q}_{0}$	$Z_0$	${\bf q}_2$	$Z_0$
$\mathbf{q}_{2}$	$\mathbf{q}_{\mathrm{o}}$	$z_1$	${\bf q}_2$	$z_1$

**Solution:** For  $q_0$  for input 0, the output differs. For  $q_2$  for inputs 0 and 1, the output differs. So the states are broken as  $q_00$ ,  $q_01$  and  $q_20$ ,  $q_21$ . According to the new states, the modified Mealy machine becomes

	I/P = 0		I/P =	1
Present State	Next State	O/P	Next State	O/P
— <b>►</b> q <sub>00</sub>	$\mathbf{q}_{20}$	$Z_0$	$\mathbf{q}_{1}$	$Z_1$
$q_{01}$	$q_{20}$	$Z_0$	$\mathbf{q}_{_{1}}$	$Z_1$

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	I/P = 0		I/P = 1	
Present State	Next State	O/P	Next State	O/P
${\bf q}_1$	$\mathbf{q}_{00}$	$Z_0$	$\mathbf{q}_{20}$	$Z_0$
$\mathbf{q}_{20}$	$\mathbf{q}_{01}$	$z_1$	$\mathbf{q}_{21}$	$z_1$
$\boldsymbol{q}_{21}$	$\mathbf{q}_{01}$	$z_1$	$\mathbf{q}_{21}$	$z_1$

From this modified Mealy machine, the Moore machine is

	Next		
Present State	I/P = 0	I/P = 1	O/P
— <b>▶</b> q <sub>00</sub>	$\boldsymbol{q}_{20}$	$\boldsymbol{q}_1$	$Z_0$
$\mathbf{q}_{\mathtt{01}}$	${\bf q}_{20}$	${\bf q_1}$	$z_1$
$\mathbf{q}_{1}$	$\boldsymbol{q}_{00}$	$\boldsymbol{q}_{20}$	$z_1$
$\mathbf{q}_{20}$	$\boldsymbol{q}_{01}$	$\boldsymbol{q}_{21}$	$Z_0$
$q_{21}$	$q_{01}$	$\boldsymbol{q}_{21}$	$z_1$

**Example 3.20** Convert the given Mealy machine to an equivalent Moore machine.

	I/P = 0		I/P = 1	
Present State	Next State	O/P	Next State	O/P
$\longrightarrow \mathbf{q_0}$	$\boldsymbol{q}_2$	0	$\boldsymbol{q}_1$	0
$\boldsymbol{q}_1$	$\mathbf{q}_0$	1	$q_3$	0
${\bf q}_2$	$\boldsymbol{q}_1$	1	$\mathbf{q}_{0}$	1
$q_3$	$\mathbf{q}_3$	1	$\boldsymbol{q}_2$	0

**Solution:** In the next state column of the given Mealy machine, the output differs for  $q_1$  and  $q_3$  as the next state. So, the states are divided as  $q_10$ ,  $q_11$  and  $q_30$ ,  $q_31$ , respectively. After dividing the states, the modified Mealy machine becomes

	I/P = 0		I/P =	1
Present State	Next State	O/P	Next State	O/P
— <b>▶</b> q <sub>0</sub>	${\boldsymbol q}_2$	0	$\mathbf{q}_{10}$	0
$\mathbf{q}_{10}$	$q_0$	1	$q_{30}$	0
$\mathbf{q}_{11}$	$\mathbf{q}_{\mathrm{o}}$	1	$q_{30}$	0
${\bf q}_2$	$\mathbf{q}_{11}$	1	$q_0$	1
$q_{30}$	$q_{31}$	1	$\mathbf{q}_2$	0
$q_{31}$	$\mathbf{q}_{31}$	1	${\bf q}_2$	0

In the next state column of the modifi ed Mealy machine, when  $q_0$  is a next state, the output is 0. So, in the constructing Moore machine, for the present state  $q_0$ , the output is also 0. Similarly, for the present

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state  $q_2$ , the output is 0. For the divided states like  $q_10$ ,  $q_11$ , there is no need to mention the output as they were divided according to the distinguished output. So, the constructing Moore machine is

	Next		
Present State	I/P = 0	I/P = 1	O/P
<b>→</b> q <sub>0</sub>	$\boldsymbol{q}_2$	$\boldsymbol{q}_{10}$	1
$\mathbf{q}_{10}$	$q_0$	$q_{30}$	0
$q_{11}$	$q_0$	$q_{30}$	1
${\bf q}_2$	$q_{11}$	$\mathbf{q}_0$	0
$q_{30}$	$q_{31}$	$\boldsymbol{q}_2$	0
$q_{31}$	$\boldsymbol{q}_{31}$	$\boldsymbol{q}_2$	1

To get rid of the problem of the occurrence of null string, we need to include another state,  $q_a$ , with same transactions as of  $q_0$  but with output 0. The modified final Moore machine equivalent to the given Mealy machine becomes

	Next		
Present State	I/P = 0	I/P = 1	O/P
— <b>▶</b> q_a	$\boldsymbol{q}_2$	$\boldsymbol{q}_{10}$	0
$\mathbf{q}_{0}$	$\mathbf{q}_2$	$\mathbf{q}_{10}$	1
$\mathbf{q}_{10}$	$\mathbf{q}_{\scriptscriptstyle 0}$	$q_{30}$	0
$q_{11}$	$q_0$	$q_{30}$	1
${\bf q}_2$	$q_{11}$	$\mathbf{q}_{\mathrm{o}}$	0
$q_{30}$	$q_{31}$	$\boldsymbol{q}_2$	0
$\mathbf{q}_{31}$	$\boldsymbol{q}_{31}$	${\bf q}_2$	1

**3.13.2** Transitional Format From the transitional diagram of a Moore or Mealy machine, conversion can be done easily. That process is described here.

## textit3.13.2.1 Moore Machine to Mealy Machine

Let us assume that a Moore machine Mo is to be converted to an equivalent Mealy machine MC. There are certain steps for this conversion.

Step I: For a Moore machine, each state is labelled with the output, because for a Moore machine, output depends only on the present state. For the conversion, let us take a state S, which is labelled with output O. Look into the incoming edges to the state S. These incoming edges are labelled by the input alphabets of the Moore machine. These incoming edges are relabelled by the input alphabet as well as the output of the state S. The output for the state must be removed.

Step II: Keep the outgoing edges from the state S as it was. (The outgoing edge of a state must be the incoming edge of some other state.)