

1.2 Непрограммируемое 1

Построение полинома Жегалкина 15 различными функциями от четырех переменных.

$$1. f = (X_1 \vee X_2) \wedge (X_3 \vee X_4)$$

X_1	X_2	X_3	X_4	$X_1 \vee X_2$	$X_3 \vee X_4$	f
0	0	0	0	0	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	1	0	0
0	1	0	1	1	1	1
0	1	1	0	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	1	0	0
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

$$f(0,0,0,0) = a_0 = 0$$

$$f(0,0,0,1) = a_0 \oplus a_4 = 0$$

$$f(0,0,1,0) = a_0 \oplus a_3 = 0$$

$$f(0,0,1,1) = a_0 \oplus a_3 \oplus a_4 \oplus a_{34} = 0$$

$$f(0,1,0,0) = a_0 \oplus a_2 = 0$$

$$f(0,1,0,1) = a_0 \oplus a_2 \oplus a_4 \oplus a_{24} = 1$$

$$f(0,1,1,0) = a_0 \oplus a_2 \oplus a_3 \oplus a_{23} = 1$$

$$f(0,1,1,1) = a_0 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_{23} \oplus a_{34} \oplus a_{24} \oplus a_{234} = 1$$

$$f(1,0,0,0) = a_0 \oplus a_1 = 1$$

$$f(1,0,0,1) = a_0 \oplus a_1 \oplus a_4 \oplus a_{14} = 1$$

$$f(1,0,1,0) = a_0 \oplus a_1 \oplus a_3 \oplus a_{13} = 1$$

$$f(1,0,1,1) = a_0 \oplus a_1 \oplus a_3 \oplus a_4 \oplus a_{13} \oplus a_{14} \oplus a_{34} \oplus a_{134} = 1$$

$$f(1,1,0,0) = a_0 \oplus a_1 \oplus a_2 \oplus a_{12} = 0$$

$$f(1,1,0,1) = a_0 \oplus a_1 \oplus a_2 \oplus a_4 \oplus a_{12} \oplus a_{24} \oplus a_{14} \oplus a_{124} = 1$$

$$f(1,1,1,0) = a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_{12} \oplus a_{13} \oplus a_{23}a_{123} = 1$$

$$f(1,1,1,1) = a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_{12} \oplus a_{13} \oplus a_{14} \oplus a_{24} \oplus a_{23} \oplus$$

$$\oplus a_{34} \oplus a_{123} \oplus a_{124} \oplus a_{134} \oplus a_{234} \oplus a_{1234} = 1$$

$$f = X_1X_2X_3 \oplus X_1X_3 \oplus X_1X_4 \oplus X_2X_3 \oplus X_2X_4 \oplus X_1X_3X_4 \oplus X_2X_3X_4 \oplus$$

$\oplus X_1 X_2 X_4 \oplus X_1 X_2 X_3 X_4$ – полином Жегалкина

2. $f = \overline{X_1} \oplus X_2 \oplus X_3 \oplus \overline{X_4}$

X_1	X_2	X_3	X_4	$\overline{X_1}$	$\overline{X_1} \oplus X_2$	$\overline{X_1} + X_2 \oplus X_3$	$\overline{X_4}$	f
0	0	0	0	1	1	1	1	0
0	0	0	1	1	1	1	0	1
0	0	1	0	1	1	0	1	1
0	0	1	1	1	1	0	0	0
0	1	0	0	1	0	0	1	1
0	1	0	1	1	0	0	0	0
0	1	1	0	1	0	1	1	0
0	1	1	1	1	0	1	0	1
1	0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	1	1	0
1	0	1	1	0	0	1	0	1
1	1	0	0	0	1	1	1	0
1	1	0	1	0	1	1	0	1
1	1	1	0	0	1	0	1	1
1	1	1	1	0	1	0	0	0

Решение методом треугольника:

[illegible]

$f = X_1 \oplus X_2 \oplus X_3 \oplus X_4$ – полином Жегалкина

3.

Таблица истинности:

X_1	X_2	X_3	X_4	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

Решение:

$$f(0,0,0,0) = a_0 = 0$$

$$f(0,0,0,1) = a_0 \oplus a_4 = 0$$

$$f(0,0,1,0) = a_0 \oplus a_3 = 0$$

$$f(0,0,1,1) = a_0 \oplus a_3 \oplus a_4 \oplus a_{34} = 0$$

$$f(0,1,0,0) = a_0 \oplus a_2 = 0$$

$$f(0,1,0,1) = a_0 \oplus a_2 \oplus a_4 \oplus a_{24} = 1$$

$$f(0,1,1,0) = a_0 \oplus a_2 \oplus a_3 \oplus a_{23} = 0$$

$$f(0,1,1,1) = a_0 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_{23} \oplus a_{24} \oplus a_{34} \oplus a_{234} = 1$$

$$f(1,0,0,0) = a_0 \oplus a_1 = 0$$

$$f(1,0,0,1) = a_0 \oplus a_1 \oplus a_4 \oplus a_{14} = 0$$

$$f(1,0,1,0) = a_0 \oplus a_1 \oplus a_3 \oplus a_{13} = 1$$

$$f(1,0,1,1) = a_0 \oplus a_1 \oplus a_3 \oplus a_4 \oplus a_{13} \oplus a_{14} \oplus a_{134} = 1$$

$$f(1,1,0,0) = a_0 \oplus a_{12} \oplus a_1 \oplus a_2 = 0$$

$$f(1,1,0,1) = a_0 \oplus a_1 \oplus a_2 \oplus a_4 \oplus a_{12} \oplus a_{14} \oplus a_{24} \oplus a_{124} = 1$$

$$f(1,1,1,0) = a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_{12} \oplus a_{13} \oplus a_{23}a_{123} = 0$$

$$f(1,1,1,1) = a_0 \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_4 \oplus a_{12} \oplus a_{13} \oplus a_{14} \oplus a_{23} \oplus a_{24} \oplus$$

$$\oplus a_{34} \oplus a_{134} \oplus a_{123} \oplus a_{124} \oplus a_{234} \oplus a_{1234} = 1$$

$$f = X_1X_2X_3 \oplus X_2X_4 \oplus X_1X_3 \text{ -полином Жегалкина}$$

4. $f = (X_1 \wedge X_3) \wedge (X_2 \vee X_4)$

X_1	X_2	X_3	X_4	$X_1 \wedge X_3$	$X_2 \vee X_4$	f
0	0	0	0	0	0	0
0	0	0	1	0	1	1
0	0	1	0	0	0	0
0	0	1	1	0	1	1
0	1	0	0	0	1	1
0	1	0	1	0	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1
1	0	0	0	0	0	0
1	0	0	1	0	1	1
1	0	1	0	1	0	1
1	0	1	1	1	1	0
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	1	1	0
1	1	1	1	1	1	0

$f = X_4 \oplus X_2 \oplus X_2 X_4 \oplus X_1 X_3$ — полином Жегалкина

$$5. f = ((\overline{X_1} \oplus X_2) \vee (\overline{X_3} \oplus X_4)) | X_1$$

X_1	X_2	X_3	X_4	$\overline{X_1}$	$\overline{X_3}$	$\overline{X_1} \oplus X_2$	$\overline{X_3} \oplus X_4$	$(\overline{X_1} \oplus X_2) \vee (\overline{X_3} \oplus X_4)$	f
0	0	0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	0	1	1
0	0	1	0	1	0	1	0	1	1
0	0	1	1	1	0	1	1	1	1
0	1	0	0	1	1	0	1	1	1
0	1	0	1	1	1	0	0	0	1
0	1	1	0	1	0	0	0	0	1
0	1	1	1	1	0	0	1	1	1
1	0	0	0	0	1	0	1	1	0
1	0	0	1	0	1	0	0	0	1
1	0	1	0	0	0	0	0	0	1
1	0	1	1	0	0	0	1	1	0
1	1	0	0	0	1	1	1	1	0
1	1	0	1	0	1	1	0	1	0
1	1	1	0	0	0	1	0	1	0
1	1	1	1	0	0	1	1	1	0

$f = 1 \oplus X_1 \oplus X_1X_4 \oplus X_1X_3 \oplus X_1X_2X_4 \oplus X_1X_2X_3$ — полином
Жегалкина

6. $f = (\overline{X_1} \vee X_2) \wedge (\overline{X_3} \vee X_4)$

X_1	X_2	X_3	X_4	$\overline{x_1}$	$\overline{x_1} \vee x_2$	$\overline{X_3}$	$\overline{X_3} \vee X_4$	f
0	0	0	0	1	1	1	1	1
0	0	0	1	1	1	1	1	1
0	0	1	0	1	1	0	0	0
0	0	1	1	1	1	0	1	1
0	1	0	0	1	1	1	1	1
0	1	0	1	1	1	1	1	1
0	1	1	0	1	1	0	0	0
0	1	1	1	1	1	0	1	1
1	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	1	0
1	0	1	0	0	0	0	0	0
1	0	1	1	0	0	0	1	0
1	1	0	0	0	1	1	1	1
1	1	0	1	0	1	1	1	1
1	1	1	0	0	1	0	0	0
1	1	1	1	0	1	0	1	1

[illegible]

$$f = X_3 \oplus X_3X_4 \oplus X_1 \oplus X_1X_3 \oplus X_1X_3X_4 \oplus X_1X_2 \oplus X_1X_2X_3 \oplus$$

$\oplus X_3 X_3 X_3 X_3 \oplus 1$ – полином Жегалкина

7. $f = X_1 \wedge (X_2 \vee (X_3 \wedge X_4))$

X_1	X_1	X_1	X_1	$X_3 \wedge x_4$	$X_2 \vee (X_3 \wedge X_4)$	f
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	1	1	0
0	1	0	0	0	1	0
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	1	1	0
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	1	1	1
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	1	1

8. $X_1 \oplus (X_2 \wedge X_3) \oplus X_4$

X_1	X	X_1	X_1	$X_2 \wedge X_3$	$X_1 \oplus (X_2 \wedge X_3)$	f
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	0
0	0	1	1	0	0	1
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1
0	1	1	1	1	1	0
1	0	0	0	0	1	1
1	0	0	1	0	1	0
1	0	1	0	0	1	1
1	0	1	1	0	1	0
1	1	0	0	0	1	1
1	1	0	1	0	1	0
1	1	1	0	1	0	0
1	1	1	1	1	0	1

[illegible]

$f = X_1 \oplus X_4 \oplus X_2X_3$ –полиом Жегалкина

9. $f = (X_1/X_2) \wedge (X_3/X_4)$

X_1	X_2	X_3	X_4	X_1/X_2	X_3/X_4	f
0	0	0	0	1	1	1
0	0	0	1	1	1	1
0	0	1	0	1	1	1
0	0	1	1	1	0	0
0	1	0	0	1	1	1
0	1	0	1	1	1	1
0	1	1	0	1	1	1
0	1	1	1	1	0	0
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	0	0
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	0	0	0

[illegible]

$$f = X_1X_2 \oplus X_3X_4 \oplus X_1X_2X_3X_4 \oplus 1$$
 —полином Жегалкина

10. $f = (X_1 \downarrow \overline{X_2}) \vee (\overline{X_3} | X_4)$

X_1	X_2	X_3	X_4	$\overline{X_2}$	$\overline{X_3}$	$X_1 \downarrow \overline{X_2}$	$\overline{X_3} X_4$	f
0	0	0	0	1	1	0	1	1
0	0	0	1	1	1	0	0	0
0	0	1	0	1	0	0	1	1
0	0	1	1	1	0	0	1	1
0	1	0	0	0	1	1	1	1
0	1	0	1	0	1	1	0	1
0	1	1	0	0	0	1	1	1
0	1	1	1	0	0	1	1	1
1	0	0	0	1	1	0	1	1
1	0	0	1	1	1	0	0	0
1	0	1	0	1	0	0	1	1
1	0	1	1	1	0	0	1	1
1	1	0	0	0	1	0	1	1
1	1	0	1	0	1	0	0	0
1	1	1	0	0	0	0	1	1
1	1	1	1	0	0	0	1	1

11. $f = ((X_1 \wedge X_2) | (X_3 \vee X_4)) \downarrow X_4$

X_1	X_2	X_3	X_4	$X_1 \wedge X_2$	$X_3 \vee X_4$	$(X_1 \wedge X_2) (X_3 \vee X_4)$	f
0	0	0	0	0	0	1	0
0	0	0	1	0	1	1	0
0	0	1	0	0	1	1	0
0	0	1	1	0	1	1	0
0	1	0	0	0	0	1	0
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	0
1	0	0	0	0	0	1	0
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	0
1	0	1	1	0	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	1	0	0
1	1	1	0	1	1	0	1
1	1	1	1	1	1	0	0

$f = X_1X_2X_3 \oplus X_1X_2X_3X_4$ – полином Жегалкина

12. $f = ((X_1 \wedge X_4) \downarrow (\overline{X_2}|X_3)) \vee (\overline{X_1} \wedge X_2)$

[illegible]

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13. $f = \overline{(X_1 \vee X_3)} \vee (\overline{X_4} \wedge X_3)$

X_1	X_2	X_3	X_4	$X_1 \vee X_3$	$\overline{X_1 \vee X_3}$	$\overline{X_2}$	$\overline{X_2} \wedge X_4$	f
0	0	0	0	0	1	1	0	1
0	0	0	1	0	1	1	1	1
0	0	1	0	1	0	1	0	0
0	0	1	1	1	0	1	1	1
0	1	0	0	0	1	0	0	1
0	1	0	1	0	1	0	1	1
0	1	1	0	1	0	0	0	0
0	1	1	1	1	0	0	0	0
1	0	0	0	1	0	1	0	0
1	0	0	1	1	0	1	1	1
1	0	1	0	1	0	1	0	0
1	0	1	1	1	0	1	1	1
1	1	0	0	1	0	0	0	0
1	1	0	1	1	0	0	0	0
1	1	1	0	1	0	0	0	0
1	1	1	1	1	0	0	0	0

14. $f = (X_1 \oplus X_2) \wedge (\overline{X_3} \oplus X_4)$

X_1	X_2	X_3	X_4	$\overline{X_3}$	$\overline{X_3} \oplus X_4$	$X_1 \oplus X_2$	f
0	0	0	0	1	1	0	0
0	0	0	1	1	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	1	0	0
0	1	0	0	1	1	1	1
0	1	0	1	1	0	1	0
0	1	1	0	0	0	1	0
0	1	1	1	0	1	1	1
1	0	0	0	1	1	1	1
1	0	0	1	1	0	1	0
1	0	1	0	0	0	1	0
1	0	1	1	0	1	1	1
1	1	0	0	1	1	0	0
1	1	0	1	1	0	0	0
1	1	1	0	0	0	0	0
1	1	1	1	0	1	0	0

[illegible]

$$f = X_2 \oplus X_2X_4 \oplus X_2X_3 \oplus X_1 \oplus X_1X_4 \oplus X_1X_3 \text{ —полином Жегалкина}$$

15. $f = (\overline{X_1} \wedge (X_2 \downarrow X_3)) | (X_4 \oplus X_1)$

X_1	X_1	X_1	X_1	$X_2 \downarrow X_3$	$\overline{X_3}$	$\overline{X_1} \wedge (X_2 \downarrow X_3)$	$X_4 \oplus X_1$	f
0	0	0	0	1	1	1	0	1
0	0	0	1	1	1	1	1	0
0	0	1	0	0	1	0	0	1
0	0	1	1	0	1	0	1	1
0	1	0	0	0	1	0	0	1
0	1	0	1	0	1	0	1	1
0	1	1	0	0	1	0	0	1
0	1	1	1	0	1	0	1	1
1	0	0	0	1	0	0	1	1
1	0	0	1	1	0	0	0	1
1	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	1
1	1	0	0	0	0	0	1	1
1	1	0	1	0	0	0	0	1
1	1	1	0	0	0	0	1	1
1	1	1	1	0	0	0	0	1

