

Ans1 Asymptotic notations is used to maintain & describe the running time of an algorithm. i.e. how much time an algorithm takes with a given input.

These tools are mathematical representation to represent the complexities

→ Big(O) notation.

It gives us upper bound. for a function $f(n)$ to write within a upper bound.

→ Omega notation (Ω)

Gives us lower bound for a function $f(n)$ to within a constant factor

→ Big theta (Θ)

It gives bound for a function $f(n)$ to within a constant factor

Ans2 $for(i=1; i \leq n; i++)$

$$\text{Iter 1} = i = 1 = 2^0$$

$$2 \quad i = 2 = 2^1$$

$$3 \quad i = 4 = 2^2$$

$$4 \quad i = 8 = 2^3$$

⋮

$$\text{Iter } p = i = 2^{p-1} = n$$

$$2^{p-1} = n$$

$$p-1 = \log_2 n$$

$$p = \log n + 1$$

$$O(\log n)$$

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$$T(n) = 3T(n-1) \quad \text{if } n > 0 \quad \text{otherwise}$$

$$T(n) = 3T(n-1)$$

$$T(1) = 3T(1-1)$$

$$T(1) = 3T(0)$$

$$T(1) = 3$$

$$T(2) = 3T(2-1)$$

$$= 3T(1)$$

$$= 3 \times 3 = 9$$

$$T(3) = 3T(3-1)$$

$$= 3T(2)$$

$$= 3 \times 9$$

$$= 27$$

$$T(4) = 3T(4-1)$$

$$= 3T(3)$$

$$= 3 \times 27$$

$$= 81$$

$$T(n) = 3 + 9 + 27 + \dots$$

$$T(n) \approx 3^n$$

Hence. time complexity
 $O(n)$

4 $T(n) = 2T(n-1) - 1$ if $n > 0$ otherwise 1

$$\begin{aligned} T(1) &= 2T(1-1) - 1 \\ &= 2T(0) - 1 \\ &= 2 \times 1 - 1 = 1 \end{aligned}$$

$$\begin{aligned} T(2) &= 2T(2-1) - 1 \\ &= 2T(1) - 1 \\ &= 2 \times 1 - 1 = 1 \end{aligned}$$

$$\begin{aligned} T(3) &= 2T(3-1) - 1 \\ &= 2T(2) - 1 \\ &= 2 \times 1 - 1 = 1 \end{aligned}$$

$$\begin{aligned} T(n) &= 1 \\ O(1) \end{aligned}$$

⑤ Time complexity

iter 1 $S = 1 + 2$

iter 2 $S = 1 + 2 + 3$

iter 3 = $S = 1 + 2 + 3 + 4$

iter K = $\sum_{i=1}^K i = n$

$$\sum_{i=1}^K i = \frac{K(K+1)}{2} = O(K^2)$$

$$O(K^2) = n$$

$$O(\sqrt{n})$$

6

iter 1 = $i = 1$

iter 2 $i = 2$

\vdots

iter k = $\frac{n}{i}$

$$k^2 = n$$

$$k = \sqrt{n}$$

$$O(\sqrt{n})$$

7)

iter 1 $i = n/2 + 0$

iter 2 $i = n/2 + 1$

iter 3 $i = n/2 + 2$

iter k $\Rightarrow i = n/2 + k - 1 = n$

$$\frac{n}{2} + k - 1 = n$$

$$k = n - \frac{n}{2} + 1$$

$$= \frac{2n - n + 2}{2} = \frac{n + 2}{2}$$

$$= \frac{n}{2} + 1$$

$$O(n)$$

iter 1 $j = 1 = 2^0$

iter 2 $j = 2 = 2^1$

3 $j = 4 = 2^2$

$j = 8 = 2^3$

\vdots

iter k $j = 2^{k-1} = n$

$$k - 1 = \log n$$

$$K = \log n + 1$$

$$= \Theta(\log n)$$

iter 1 $K=1$
 iter 2 $K=2$
 iter 3 $K=3$
 iter 4 $K=4$

$$1 \times n \times \log n \times \log n$$

$$\Theta(n(\log n)^2)$$

iter 1 $K = 2^{K-1} = n$
 $2^{p-1} = n$

$$p-1 = \log n$$

$$p = \log n + 1$$

$$\Theta(\log n)$$

Q cannot be find.
 because to find the time complexity
 it must be algorithm as there is no
 terminating point so it is not a
 algorithm

Q iter 1 $\Rightarrow i=1 \rightarrow d(1)$

iter 1 $j=1$

iter 2 $j=2$

same for this also.