

Normalization

Defn

The Normal form of a relation refers to the highest Normal form condition that it meets, and hence indicates the degree to which it has been normalized.

Normalization of data can be considered a process of analysing the given relation schemas based on their FDs and primary keys to achieve the desirable properties of (1) minimizing redundancy and (2) minimizing the insertion, deletion and update anomalies.

Two properties must hold

→ (1) Lossy Join

(2) Dependency preservation.

Denormalization is the process of doing the join of higher normal form relations as a base relⁿ.

Attribute is considered as prime attribute of R if it is member of some candidate key

Attribute not member of candidate key are Non-Prime Attribute.

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Guidelines for Database Design

- (1) Design a relation schema so that it is easy to explain its meaning. Do not combine attributes from multiple entity types and relationship types into a single relation.
- (2) Design the base relation schemas so that no insertion, deletion, or modification anomalies are present in the relations.
- (3) Avoid placing attributes in a base relation whose values may frequently be NULL. If NULLs are unavoidable, make sure that they apply in exceptional cases only and do not apply to majority of tuples in the relation.
- (4) Design relation schemas so that they can be joined with equality conditions on attributes that are appropriately related pairs in a way that guarantees that no spurious tuples are generated.

Anomalies

update anomalies

- Insertion anomalies
- Deletion anomalies
- Modification anomalies

combine
it in

①

Functional Dependencies

A functional dependency is a constraint between two sets of attributes from the database. let $R = \{A_1, A_2, \dots, A_n\}$

A functional dependency, denoted by $X \rightarrow Y$ between two sets of attributes X and Y that are subsets of R specifies a constraint on the possible tuples that can form a relation state r of R . For any two tuples t_1 and t_2 in r that have $t_1[A] = t_2[A]$ they must also have $t_1[B] = t_2[B]$.

$r(A, B, C, G, H, I)$

- $A \rightarrow B$
- $A \rightarrow C$
- $CG \rightarrow H$
- $CG \rightarrow I$
- $B \rightarrow H$

$$X \rightarrow Y$$

a	1
a	2 X
d	4
c	3

X	Y
a	1
a	2
c	3
d	4

Type of FD

Trivial

$AB \rightarrow A$

$Y \subseteq X$

Y is a proper subset of X

→ valid,

→ no new info found

Non Trivial

$Y \not\subseteq X$

$A \rightarrow BC$
 $DE \rightarrow C$
 $C \rightarrow DE$
 $BC \rightarrow A$

R

A	B	C	D	E
a	2	3	4	5
2	a	3	4	5
a	2	3	6	5
a	2	3	6	6

$A \rightarrow C$
 $A \rightarrow BC$
 $A \rightarrow BC$
 if all the values
 different then
 valid

F.D does not depend on data. Data depends on F.D.

Check whether two values of A are giving same value or not.

for eg $a \rightarrow 2, 3$

$\therefore A \rightarrow BC$ is valid

$DE \rightarrow C$ ✓

$4, 5 \rightarrow 3$

$6, 5 \rightarrow 3$

Now all values on RHS are same \therefore rule is valid

$C \rightarrow DE$

$3 \rightarrow 4, 5$

$3 \rightarrow 6, 5$

two different values for same value

$C \therefore$ it is invalid

$BC \rightarrow A$

$2, 3 \rightarrow a$

$a, 3 \rightarrow 2$

$$\begin{aligned}
 A &\rightarrow B \quad \checkmark \\
 A &\rightarrow CD \quad \checkmark \\
 A &\rightarrow BCDE \quad \checkmark
 \end{aligned}$$

If all the values on L.H.S are different then all the F.D are valid

$$X \rightarrow Y$$

$$\begin{aligned}
 A &\rightarrow C \quad \checkmark \\
 BD &\rightarrow C \quad \checkmark \\
 ABDE &\rightarrow C \quad \checkmark
 \end{aligned}$$

If all the values on R.H.S are same that is shown in above example the all the rules are valid

(2)

A	B	C	D	E
a	2	3	4	5
b	a	3	4	5
c	2	3	6	5
d	2	3	6	6

CLOSURE SET of F.D

Given a relational schema $\mathcal{R}(R)$, a FD, f on R is logically implied by a set of functional dependencies, F on R or if every instance of $\mathcal{R}(R)$ that satisfies F also satisfies f .

$$F = \left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ CG \rightarrow H \\ CH \rightarrow I \\ B \rightarrow H \end{array} \right\}$$

$$F^+ = \left\{ \begin{array}{l} A \rightarrow H \\ A \rightarrow A \\ B \rightarrow B \\ CG \rightarrow C \\ CG \rightarrow G \end{array} \right\}$$

Computing f^+ is very lengthy and difficult. We can only check whether a FD belongs to F closure or not by applying certain Axiom or rules of inference.

By using these rules, we can find f^+ given F .

The collection of rules is called Armstrong's Axiom. It is sound because it doesn't generate incorrect FD. They are complete because they allow to compute f^+ if F is given.

(1) Reflexivity :- If $\beta \subseteq \alpha$ then $\alpha \rightarrow \beta$

(2) Augmentation :- If $\alpha \rightarrow \beta$ then $\gamma \alpha \rightarrow \gamma \beta$

(3) Transitivity :- If $\alpha \rightarrow \beta$, $\beta \rightarrow \gamma$ then $\alpha \rightarrow \gamma$

Some additional rules to simplify the computation of f^+ .

(1) Union rule: If $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$ then $\alpha \rightarrow \beta \gamma$

(2) Decomposition :- If $\alpha \rightarrow \beta \gamma$ then $\alpha \rightarrow \beta$, $\alpha \rightarrow \gamma$

(3) Pseudotransitivity :- If $\alpha \rightarrow \beta$, $\gamma \beta \rightarrow \delta$ then $\alpha \gamma \rightarrow \delta$

Composition: If $\alpha \rightarrow \beta$, $\gamma \rightarrow \delta$ then $\alpha \gamma \rightarrow \beta \delta$

Let us apply these rules

(3)

$R = (A, B, C, G, H, I)$

$$F = \left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ CG \rightarrow H \\ CG \rightarrow I \\ B \rightarrow H \end{array} \right\}$$

$$F^+ = \left\{ \begin{array}{l} A \rightarrow H \\ CG \rightarrow HI \\ AG \rightarrow I \end{array} \right\} \begin{array}{l} \text{: Transitivity holds} \\ \text{: Union rule holds} \\ \text{: pseudotransitivity} \end{array}$$

Closure of Attribute Set

Size of R , n (attribute in relⁿ)

No of subset = 2^n

$$F^+ = \left\{ \begin{array}{l} 2^n \times 2^n \\ 2^{2n} \end{array} \right\} \text{ possible FD}$$

Initially result = $AG(A, G, AG)$

result = result \cup B

$AGBC$

$AGBCI$

$AGBCIH$

Compute $(AG)^+$

$A \rightarrow B$

$A \rightarrow C$

$(G \rightarrow I)$

$B \rightarrow H$

\therefore

\therefore

\therefore

\therefore

There are several uses of attribute closure

Equivalent

→ Superkey: we compute α^+ and check if it contains all attribute in R .

→ we can check if a FD $\alpha \rightarrow \beta$ holds by checking if $\beta \subseteq \alpha^+$ that is we compute α^+ by using attribute closure, and then check if it contains β .

→ It's an alternative way to calculate F^+ .
for each $Z \subseteq R$ we find closure Z^+ and for each $S \subseteq Z^+$ we compute FD $Z \rightarrow S$.

Q1

$R(A, B, C)$

$A^+ = \{A, B, C\}$
 $B^+ = \{B, C\}$
 $C^+ = \{C\}$

Q2 $R(A, B, C, D, E, F, G)$

$\neg A \rightarrow B$
 $\neg B \rightarrow DE$

$\times AEG \rightarrow G$

find AC^+

$= \{A, C\}$

$\{A, B, C\} : A \rightarrow B$

$\{A, B, C, D, E\} : B \rightarrow DE$

if we use normalization the AC can determine only ABC, DE

$A \rightarrow BC$
 $CD \rightarrow E$
 $\neg B \rightarrow D$
 $E \rightarrow A$

$B^+ = \{B\}$
 $\{B, D\}$

$AB \rightarrow C$
 $BC \rightarrow AD$
 $D \rightarrow E$
 $CF \rightarrow B$

$(AB)^+$
 $= \{A, B\}$
 $= \{A, B, C\} : AB \rightarrow C$
 $\{A, B, C, D\} : BC \rightarrow AD$
 $\{A, B, C, D, E\} : D \rightarrow E$

$A \rightarrow BC$
 $CD \rightarrow E$
 $E \rightarrow C$
 $D \rightarrow AEF$
 $AB \rightarrow BD$
 $DM \rightarrow BC$

find if $BCD \rightarrow n?$
find $(BCD)^+$
 $BCDE$
 $AB CDE$

Equivalence of FD

(4)

$R(A B C D E H)$

$F: A \rightarrow C$

$AC \rightarrow D$

$E \rightarrow AD$

$E \rightarrow H$

$G: A \rightarrow CD$

$E \rightarrow AH$

$F \subseteq G$

$G \subseteq F$

$(A)^+ \quad ACD$

$(AC)^+ \quad ACD$

$(E)^+ \quad EADCH$

$(A)^+ \quad \{ACD\}$

$(E)^+ \quad \{EADCH\}$

It means $F = G$

There are two Set of FD but they are not at same start point. Take both Set of FD to same start point by find closure of each left hand side of f.d by using FD set of opposite Set. Similarly for Set G.

Now check whether all the ~~At~~ FD in set F are valid already to the attribute closure computed. If all the FD are valid then Both sets are equal.

Canonical Cover: Whenever we update a relation, the DB must ensure that update does not violate any FD. i.e all FD are satisfied in a new DB state. So verification of simpler FD set is easier because both sets would give same closure.

Minimal Set of FD

[Canonical Form]

Irreducible Set of FD

$R(WXYZ)$

$X \rightarrow W$

$WZ \rightarrow XY$

$Y \rightarrow WXZ$

$\alpha \rightarrow \beta$

Extraneous attribute: An attribute is said to be extraneous, if we can remove it w/o changing the closure set of FD

i) $AB \rightarrow C, A \rightarrow C$

the B is extraneous

ii) $AB \rightarrow CD, A \rightarrow C$

the C is extraneous in RHS

iii) $AB \rightarrow CD, A \rightarrow E, E \rightarrow C$

compute $(AB)^+ = \{AB^+CD^+E^+\}$

R $X \rightarrow W$

$X^+ = XW$

Now compute X^+ w/o $X \rightarrow W$ which is $X^+ = X$

Since both X^+ and X^{+1} are not same \therefore it is essential

$X \rightarrow W$

$WZ \rightarrow X$

$WZ \rightarrow Y$

$Y \rightarrow W$

$Y \rightarrow X$

$Y \rightarrow Z$

$(WZ)^+ = WZXY$

$(WZ)^{+1} = \{WZXY\}$

$\therefore WZ \rightarrow X$ is not essential

$Y^+ = \{YW, X, Z\}$

$Y^{+1} = \{W, X, Y, Z\}$ \therefore it is not essential

$Y^+ = XYW, Z$

$Y^{+1} = \{Y, Z\}$ it is essential

Y is essential

$(WZ)^+ = \{WZXY\}$

$(W)^+ = W$

$(Z)^+ = Z$

} L.H.S

Canonical cover F_c for F is a set of dependencies such that F logically implies all dependencies in F_c , and F_c logically implies all dependencies in F .

→ No functional dependency in F_c contains extraneous Attribute

→ Each LHS of F.D in F_c is Unique.

$A \rightarrow BC$
 $B \rightarrow C$
 $A \rightarrow B$
 $AB \rightarrow C$

Find F_c

Combine ~~$A \rightarrow BC$~~ , ~~$A \rightarrow B$~~
 With all the FD in compute form
 $A \rightarrow BC \Rightarrow \left. \begin{array}{l} A \rightarrow B \\ A \rightarrow C \end{array} \right\} \text{Factor}$
 Rewrite
 $A \rightarrow B \checkmark$
 $A \rightarrow C \times$
 $B \rightarrow C \checkmark$
 $AB \rightarrow C \times$

Now check one by one each FD
 $A \rightarrow B$ find closure of $(A^+) = \{A, B\}$ Not equal
 Now excluding this FD find $A^+ = \{A, C\}$
 \therefore it is essential.

$A \rightarrow C$ find $A^+ = \{A, B, C\}$, $(A^+)' = \{AB\}$
 \therefore it is not essential

$B \rightarrow C$ find $(B^+) = \{B, C\}$, $(B^+)' = \{B\}$ Not equal essential

$AB \rightarrow C$ find $(AB)^+ = \{A, B, C\}$
 $(AB^+)' = \{A, B\}$ both are equal
 \therefore not essential.

$A \rightarrow B$
 $\rightarrow C \} = F_c$

for removing extraneous Attribute on LHS
FD then find the closure of LHS of
1st $AB \rightarrow C$ $(AB)^+ = ABC$

Key

Super Key

Now find the closure of individual attribute
on LHS

$$(A^+) = \{A, B, C\}$$

$$(B^+) = \{A, B, C\}$$

So in above case, if closure of (AB) w/o B
i.e. (A^+) is equivalent to $(AB)^+$ then B is
extraneous.

So in above case if closure of AB w/o A
i.e. $(B^+) = \{A, B\}$ which is not
equivalent to $(AB)^+$ \therefore A is
essential Attribute on LHS.

Keys

(6)

Super Key

: It is a key which can uniquely identify a row in a table. It consists of any no. of attributes.

Candidate Key

A
B
C
AB
BC
CD
DA
AC

A → BCD
AB → CD
ABC → D
BD → AB
C → AD

R(A, B, C, D) (Minimal Superkey)

SK	Candidate Key	PK
✓	✓	
✓	x	
✓	x	
✓	✓	
x		

All the attributes are not included in closure ∴ it is not super key.

If proper subset of Superkey is not super key then you are candidate key.

If there is not proper subset of super key exist then that is candidate key.

Primary Key : Any one among the set of candidate key one is used as Primary key.

R(A B C D E F G H)

AB → C

A → DE

B → F

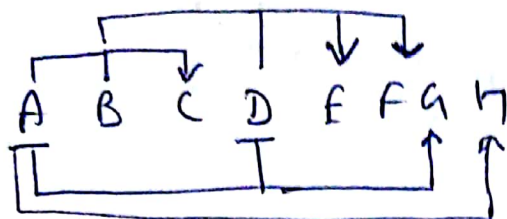
F → GH

$(AB)^+ = (AB C D E F G H) ✓$

$(B^+) = (B F G H) x$ $(A^+) = (A D E) x$

(2) $R(AB C D F G H)$

$AB \rightarrow C$
 $BD \rightarrow EF$
 $AD \rightarrow G$
 $A \rightarrow H$



Find the attributes which have

no incoming edge. which means are essential attributes and whatever be the candidate key must contain ABD.

Step 1. ABD

Step 2. $(ABD)^+ = \{A, B, C, D, E, F, G, H\}$

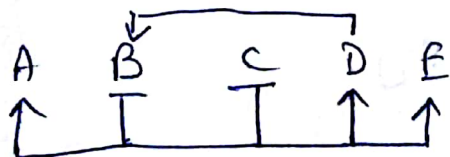
$\therefore ABD$ is candidate

It is the only candidate key because whatever be the candidate must be minimal and if an attribute is added to it then it becomes superkey not candidate key.

(3) $R(A B C D E)$

$BC \rightarrow A D E$

$D \rightarrow B$



Step 1 $\therefore C$

Step 2: $(C)^+ = \{C\}$ C alone can't find all the attributes

$\therefore C$ must be present in any of the candidate

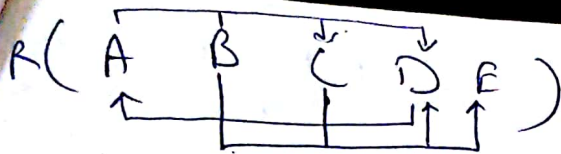
$(AC)^+ = \{AC\} \times$

$(BC)^+ = \{BCADE\} \checkmark$

$(CD)^+ = \{CDBAE\} \checkmark$

$(CE)^+ = \{CE\} \times$

Key joined
 for eg $AC, (BC), (CD), CE$
 Any further combination $(CAE)^+ = \{ACE\} \times$



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$$AB \rightarrow CD$$

$$D \rightarrow A$$

$$BC \rightarrow DE$$

In this case only 'B' is the attribute with no incoming edge
 Steps $(B)^+ = B$ B itself can't find all the attribute \therefore check it with combinations

AB, BC, BD, BE

$$(AB)^+ = (AB C D E) \checkmark$$

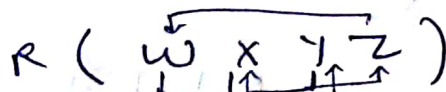
$$(BC)^+ = (A B C D E) \checkmark$$

$$(BD)^+ = (A B C D E) \checkmark$$

$$(BE)^+ = (B E) \times$$

ABE cannot be candidate key because AB is already
 Candidate key

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$$Z \rightarrow W$$

$$Y \rightarrow XZ$$

$$WX \rightarrow Y$$

Attribute w/o any incoming edge. In this case no attribute is found. so check every possible combination

$$(W)^+ = \{W\} \times$$

$$(X)^+ = \{X\} \times$$

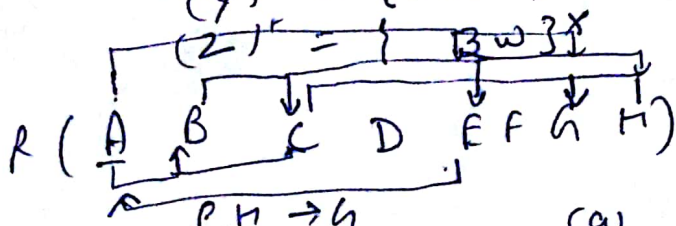
$$(Y)^+ = \{X Y Z W\} \checkmark$$

$$(Z)^+ = \{W\} \times$$

$$(WX)^+ = \{W X Y Z\} \checkmark$$

$$XZ = \{X W Y Z\} \checkmark$$

$$WZ = \{W\} \times$$



$$EH \rightarrow H$$

$$A \rightarrow BC$$

$$B \rightarrow CFH$$

$$E \rightarrow A$$

$$A \rightarrow EH$$

(a) $(DF)^+ = DF$

6

2
3
4
5
Cardinality

$R(A B C D E F G H)$

$$C h \rightarrow G$$

$$A \rightarrow BC$$

$$B \rightarrow CFH$$

$$E \rightarrow A$$

$$A \rightarrow FH$$

$(D)^+ \rightarrow D$ This is essential attribute

$$(AD)^+ = \{A B C D E F G H\} \checkmark$$

$$(BD)^+ = \{B D C F H G\} \times$$

$$(CD)^+ = \{C D\} \times$$

$$(ED)^+ = \{A E D B C F G\} \checkmark$$

$$(FD)^+ = \{F D\} \times$$

$$(GD)^+ = \{G D\} \times$$

$$(HD)^+ = \{H D\} \times$$

$$(BCD)^+ = \{B C D, F H G\} \times$$

$$(BFD)^+ = \{B, F, D, C H G\} \times$$

$$(BGD)^+ = \{B, G, D, C F H\} \times$$

$$(BHD)^+ = \{B, H, D, C F G\} \times$$

$$(CFD)^+ = \{C F D\} \checkmark$$

$$(GCD)^+ = \{G C D\} \times$$

$$(HCD)^+ = \{H C D G\} \checkmark$$

$$(GFD)^+ = \{G F D\} \times$$

$$(HFD)^+ = \{H F D\} \times$$

$$(GHD)^+ = \{G H D\} \times$$

$$BCFD \quad B$$

$$BCHD$$

$$BCHD$$

$$BCFD$$