RM 294 – Optimization I

Project 1 – Linear Programming

Group Members:
Mahika Bansal (mb62835), Shawn Kalish (smk3874), Vishu Agarwal (va7729), Xiaohan Sun (xs3236)

----- Please note that all the relevant code chunks have been placed in the appendix ----

1) Assume that your company is deciding how to spend a marketing budget of $10M. You work in the marketing department as a data scientist and the chief marketing officer has asked you write a report recommending how to spread this budget among several marketing mediums. Your department has employed an outside consulting firm to estimate the return on investment (ROI) of each marketing medium under consideration. The results are in the table below, and in a CSV attached to this assignment.

Our company decided a total marketing budget of $10M, which could be invested into 1o different platforms. Our goal is to propose the best budget allocation to maximize the return on investment. First, we have known the estimate ROI of each marketing medium under consideration based on the file provided by outside consulting firm. Here are the details about the ROI data:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	0.031	0.049	0.024	0.039	0.016	0.024	0.046	0.026	0.033	0.044

---------- End of Question 1 ----------

2) The boss also required us to constrain the budget by following these three rules:

a. **Investment $_{\text{print \& TV}} \leq$ Investment $_{\text{FB \& Email}}$**
b. **Investment $_{\text{FB + LinkedIn + IG + Snapchat + Twitter}} \geq 2*$(Investment $_{\text{SEO + AdWords}}$)**
c. **Investment for each platform should be no more than $3M**

As we mentioned in question 1, the total marketing budget has been limited as $10M.

We have set a total of 13 constraints as shown in the code. These constraints are then used to optimize the objective function which provides the ROI.

---------- End of Question 2 ----------

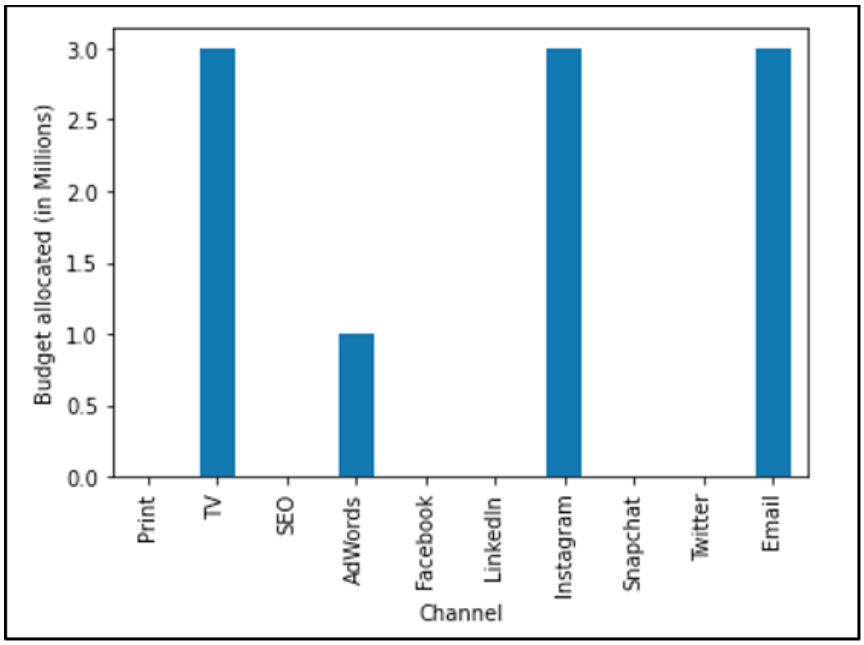
3) Formulate the marketing budget allocation problem as a linear program. Use Gurobi to find the optimal budget allocation.

We formulated this problem as linear equations and used Gurobi to solve it. We set the Gurobi model, put all objectives into the model, and used the MAXIMIZE function to find the optimal budget allocation. Finally, the results indicate that the optimal return on investment would be $456,000.

```
ROI obtained from first consulting firm's estimates is $0.45600000000000007 (in Millions).
```

For the specific allocations, we would suggest our boss to put $3M on each of the following platform: TV, Instagram, and Email, and the last $1M should be invested in AdWords.

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Budget allocation using Firm 1's estimations (in Millions)	0.0	3.0	0.0	1.0	0.0	0.0	3.0	0.0	0.0	3.0



---------- **End of Question 3** ----------

4) Your boss is happy to see the promising results presented by the marketing department. However, your boss is also very concerned because your boss recalls being somewhat disappointed after following such recommendations in the past. To be cautious about the decision, your team has decided to get another opinion about the ROI data and rerun the analysis. The second consulting firm returns the estimates of the ROI data in the table below (also in the CSV file mentioned above). You are asked to compare the two optimal allocations from these two ROI estimates.

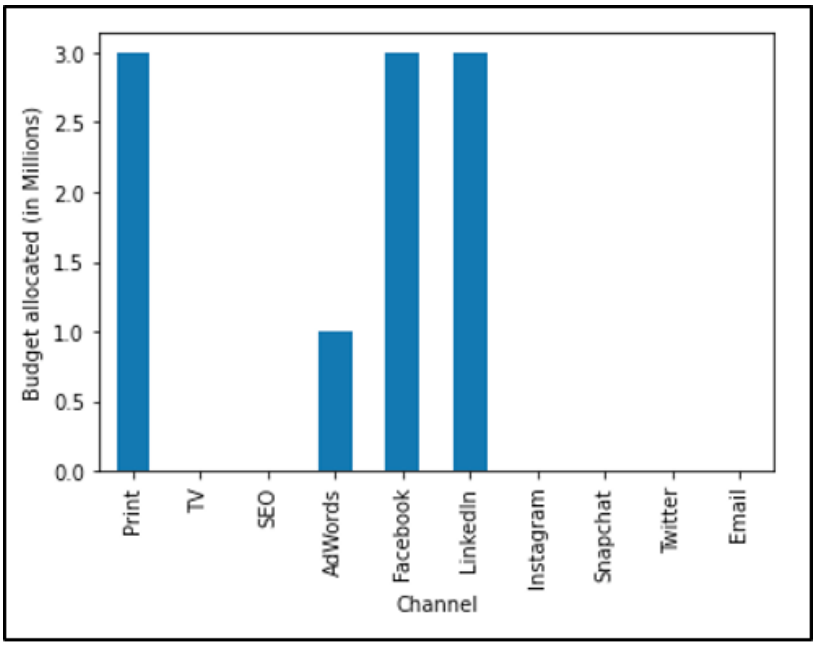
Since our boss concerned about the previous ROI data, we got a new estimate about ROI data as shown below:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Second Firms ROI Estimate	0.049	0.023	0.024	0.039	0.044	0.046	0.026	0.019	0.037	0.026

Next, we kept the constraints unchanged, but replaced the ROI data with the new one. After re-running the analysis, we got the following ROI and allocation:

```
ROI obtained from second consulting firm's estimates is $0.45600000000000007 (in Millions).
```

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Budget allocation using Firm 2's estimates (in Millions)	3.0	0.0	0.0	1.0	3.0	3.0	0.0	0.0	0.0	0.0

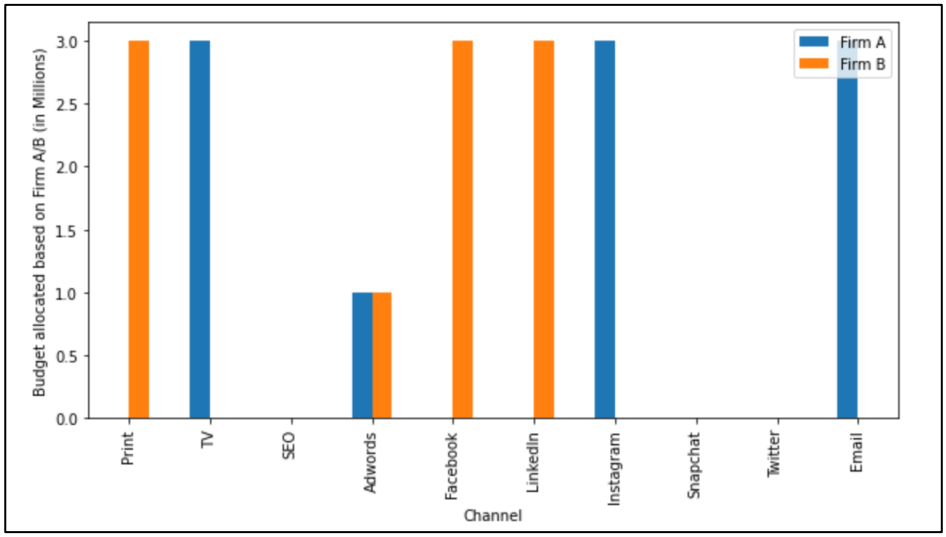


The new allocation shows that the optimal ROI is still $456,000. And we should invest $3M in each of these channels: Print, Facebook, and LinkedIn, and invest $1M in AdWords.

---------- End of Question 4 ----------

5) Are the allocations the same? Assuming the first ROI data is correct, if you were to use the second allocation (the allocation that assumed the second ROI data was correct) how much lower would the objective be relative to the optimal objective (the one that uses the first ROI data and the first allocation)? Assuming the second ROI data is correct, if you used the first allocation how much lower would the objective be relative to the optimal objective? Do you think the third constraint above, based on your boss' experience, is useful?

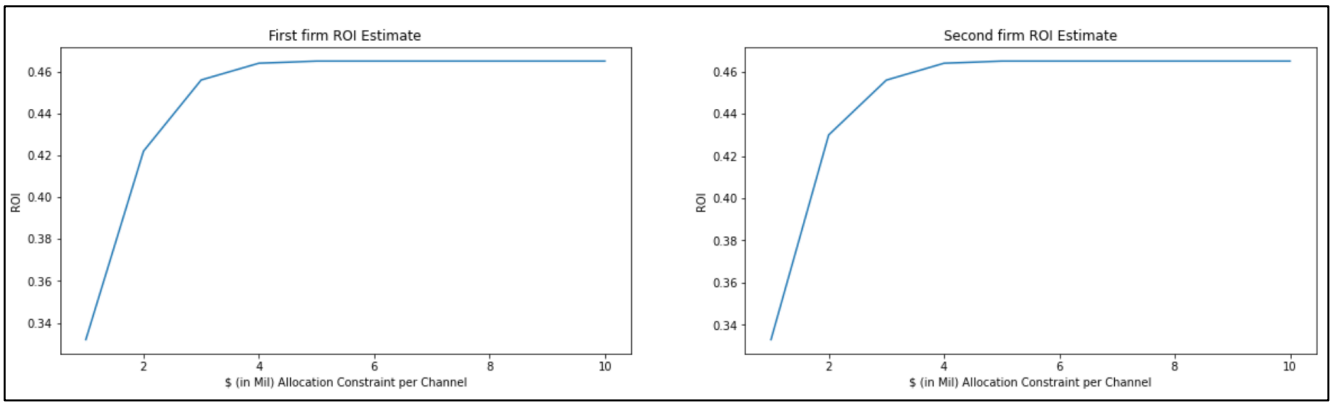
The allocations obtained based on the two ROI estimates are not same, but the optimal ROI obtained is same.



Assuming the first ROI data is correct, if we were to use the second allocation, objective would be lower by $0.204 Million than the optimum value of $0.456 Million. Conversely, assuming second ROI is correct and if were to use the first allocation, objective would be lower by $0.192 Million than the optimal value of $0.456 Million. About the third constraint used by our boss based on his experience, it has both pros and cons.

Cons

We tried varying the value of third constraint from 1 - 10 and analyzed its effect on the optimal objective. Optimal value of objective increases till 5 (i.e., maximum amount that can be invested in a single channel is 5) and then stays constant. Hence, we are losing out on ROI due to the third constraint suggested by the boss



Pros

If we look at the allocation of funds for all 10 iterations, we observe that on increasing the value of third constraint, majority of the funds are invested in very few channels (50% of the funds are invested in 1 channel) which is a bit risky. To be risk averse, their investment should be diversified across different channels. Hence, boss's suggestion kind of takes care of the diversification of funds across different channels.

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	ROI
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.332
2	0.0	2.0	0.0	2.0	0.0	0.0	2.0	0.0	2.0	2.0	0.422
3	0.0	3.0	0.0	1.0	0.0	0.0	3.0	0.0	0.0	3.0	0.456
4	0.0	4.0	0.0	0.0	0.0	0.0	2.0	0.0	0.0	4.0	0.464
5	0.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	0.465
6	0.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	0.465
7	0.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	0.465
8	0.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	0.465
9	0.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	0.465
10	0.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	0.465

Overall, if we are certain about the ROI estimates, we can do without the third constraint suggested by the boss.

---------- End of Question 5 ----------

6) To explore this further perform some analysis of how your optimal allocation would change based on changes in the ROI data. Use the first ROI data as your starting point. By how much could each advertising medium's ROI increase or decrease and still result in the same optimal allocation you found in step (3)?

We used Gurobi's inbuilt functions to extract the minimum and maximum ROI of each advertising medium such that it still results in the same optimal allocation we obtained in step 3.

Channel	Initial ROI	Lower ROI	Upper ROI
Print	0.049	-inf	0.049
TV	0.023	0.039	0.062
SEO	0.024	-inf	0.039
AdWords	0.039	0.033	0.046
Facebook	0.044	-inf	0.029
LinkedIn	0.046	-inf	0.039
Instagram	0.026	0.039	inf
Snapchat	0.019	-inf	0.039
Twitter	0.037	-inf	0.039
Email	0.026	0.029	inf

In the case of lower bound, it is only for the strategies which received any funds now that the allocation matters. But for upper bounds, the max cap of 3 M for any strategy makes it critical for the strategies allocated funds now to have upper bounds too.

---------- End of Question 6 ----------

7) Your boss has gained permission to reinvest half of the return. For example, if the marketing obtains a 4% return in January, the budget of February will be $10M + $10M × 4% × 50% = $10.2M. The monthly ROI for next year is given in Project1.Rdata. The three constraints given by your boss are still in place for each month. What is the optimal allocation for each month?

We have the ROI estimates per channel from the first firm [(1)]. We also have the actual monthly ROI per channel [(2)]. Hence, we will optimize the model to provide the best allocation of funds based on the ROI estimates of the first firm, and apply these allocations on the actual monthly ROI to obtain the optimal allocation in each month [(3)]. This will provide us the budget for the next month as per the conditions mentioned above.

(1) ROI estimate per channel

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	0.031	0.049	0.024	0.039	0.016	0.024	0.046	0.026	0.033	0.044

(2) Actual monthly ROI

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
January	4.0	3.6	2.4	3.9	3.0	3.5	3.6	2.25	3.5	3.5
February	4.0	3.9	2.7	3.8	4.3	3.2	2.7	1.80	3.7	3.5
March	3.5	2.9	3.1	3.8	2.4	4.1	3.7	2.60	4.2	2.5
April	3.8	3.1	2.4	4.4	2.4	3.8	3.7	2.50	3.6	2.9
May	3.5	3.2	1.9	3.4	2.7	2.7	3.9	2.20	4.5	3.9
June	4.0	3.2	2.7	3.4	3.4	3.0	4.5	2.10	3.8	4.1
July	3.9	3.6	2.0	4.4	3.9	3.7	4.3	1.80	4.0	3.8
August	4.2	3.3	2.8	4.2	2.0	3.7	3.6	1.50	4.4	4.3
September	4.1	2.8	2.5	4.2	2.9	3.7	2.8	2.50	4.0	3.4
October	3.0	3.0	3.1	4.6	3.1	3.3	3.2	2.30	2.5	3.2
November	4.8	3.3	2.7	4.1	2.9	3.6	4.2	3.00	3.1	4.1
December	4.8	4.0	1.9	3.7	4.2	3.6	2.6	2.90	3.6	3.7

(3) Optimal allocation per month

Month	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Budget	ROI
January	0.0	3.0	0.0	1.0	0.0	0.0	3.0	0.0	0.0	3.0	10.0	0.36
February	0.0	3.0	0.0	1.18	0.0	0.0	3.0	0.0	0.0	3.0	10.18	0.34784
March	0.0	3.0	0.0	1.35392	0.0	0.0	3.0	0.0	0.0	3.0	10.35392	0.324449
April	0.0	3.0	0.0	1.505381	0.0	0.0	3.0	0.0	0.010763	3.0	10.516144	0.357624
May	0.0	3.0	0.0	1.564986	0.0	0.0	3.0	0.0	0.129971	3.0	10.694957	0.389058
June	0.0	3.0	0.0	1.629829	0.0	0.0	3.0	0.0	0.259657	3.0	10.889486	0.419281
July	0.0	3.0	0.0	1.699709	0.0	0.0	3.0	0.0	0.399418	3.0	11.099126	0.441764
August	0.0	3.0	0.0	1.773336	0.0	0.0	3.0	0.0	0.546672	3.0	11.320008	0.434534
September	0.0	3.0	0.0	1.845758	0.0	0.0	3.0	0.0	0.691517	3.0	11.537275	0.375183
October	0.0	3.0	0.0	1.908289	0.0	0.0	3.0	0.0	0.816578	3.0	11.724866	0.390196
November	0.0	3.0	0.0	1.973321	0.0	0.0	3.0	0.0	0.946643	3.0	11.919964	0.458252
December	0.0	3.0	0.0	2.049697	0.0	0.0	3.0	0.0	1.099393	3.0	12.14909	0.424417

---------- **End of Question 7** ----------

8) A stable budget is defined as a monthly allocation such that for each platform the monthly change in spend is no more than $1M. Is the allocation you found stable? If it isn't, you do not need to solve a new optimization model. Describe how my might model this?

Based on the allocation mentioned below in the table, we can see that monthly change in spend is no more than $1M for any channel. Hence, our allocation is stable! If this was unstable, we could have added more constraints to limit the increase for each entity.

Since our allocation is stable, we can choose not to change the allocation every month and keep it same since that would be more convenient.

---------- **End of Question 7** ----------

Appendix (Codes)

Question 1

```
#reading the estimated roi csv file
df_roi = pd.read_csv('ROI_data.csv', index_col = 0)
df_roi.iloc[[0]]
```

Question 2

```
# objective vector
obj = objective_1

# initialize constraint matrix
A = np.zeros((13,10))
A[0,:] = 1 # total amount invested cannot exceed $10M
A[1:11,:] = np.diag(np.ones(10)) # the amount invested should not be more than $3M
A[11,:] = [0,0,2,2,-1,-1,-1,-1,-1,0] # social media budget constraint
A[12,:] = [1,1,0,0,-1,0,0,0,0,-1] # print and TV budget constaint
b = np.array([10,3,3,3,3,3,3,3,3,3,0,0]) # constraint values
sense = np.array(['<']*13)
```

Question 3

```
# creating LP model
budgetMod_1 = gp.Model()

# tell the model how many variables there are
budgetx_1 = budgetMod_1.addMVar(len(obj))

# define the variables before adding constraints
budgetModCon_1 = budgetMod_1.addMConstrs(A, budgetx_1, sense, b)
budgetMod_1.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE)

#optimizing the LP
budgetMod_1.Params.OutputFlag = 0
budgetMod_1.optimize()
```

```
#optimal ROI
roi_1 = budgetMod_1.objVal
print("ROI obtained from first consulting firm's estimates is ${} (in Millions).".format(bud
```

```
#optimal budget allocation
budget_alloc_1 = budgetx_1.x
df_budget_alloc_1 = pd.DataFrame(budget_alloc_1).T
df_budget_alloc_1.columns = list(df_roi.columns)
df_budget_alloc_1.index = ["Budget allocation using Firm 1's estimations (in Millions)"]
df_budget_alloc_1
```

Question 4

```
# objective vector
obj = objective_2

# initialize constraint matrix
A = np.zeros((13,10))
A[0,:] = 1 # total amount invested cannot exceed $10M
A[1:11,:] = np.diag(np.ones(10)) # the amount invested should not be more than $3M
A[11,:] = [0,0,2,2,-1,-1,-1,-1,-1,0] # social media budget constraint
A[12,:] = [1,1,0,0,-1,0,0,0,0,-1] # print and TV budget constaint
b = np.array([10,3,3,3,3,3,3,3,3,3,0,0]) # constraint values
sense = np.array(['<']*13)
```

```
# creating model
budgetMod_2 = gp.Model()

# tell the model how many variables there are
budgetx_2 = budgetMod_2.addMVar(len(obj))

# define the variables before adding constraints
budgetModCon_2 = budgetMod_2.addMConstrs(A, budgetx_2, sense, b)
budgetMod_2.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE)

#optimizing the LP
budgetMod_2.Params.OutputFlag = 0
budgetMod_2.optimize()
```

```
#optimal ROI
roi_2 = budgetMod_2.objVal
print("ROI obtained from second consulting firm's estimates is ${} (in Millions).".format(bu
```

```
#optimal budget allocation
budget_alloc_2 = budgetx_2.x
df_budget_alloc_2 = pd.DataFrame(budget_alloc_2).T
df_budget_alloc_2.columns = list(df_roi.columns)
df_budget_alloc_2.index = ["Budget allocation using Firm 2's estimates (in Millions)"]
df_budget_alloc_2
```

Question 6

```
#Sensitivity analysis
df_sensitivity = pd.DataFrame({'Channel': list(df_roi.columns),
                               'Initial ROI': obj,
                               'Lower ROI': budgetx_1.SAObjLow,
                               'Upper ROI': budgetx_1.SAObjUp }).set_index('Channel')

df_sensitivity
```

Question 7

```
a = list(df_roi.columns)
a.append('Budget')
a.append('ROI')
months =  ['January','February','March','April','May','June','July','August','September','October','November','Decem
df = pd.DataFrame(columns = a, index = months)
df.index.names = ['Month']
total_budget = 10
counter = 0
for i in df.index.values:

    # objective vector
    obj = objective_1
    # initialize constraint matrix
    A = np.zeros((13,10))
    A[0,:] = 1 # indivudial channel budge constriant
    A[1:11,:] = np.diag(np.ones(10)) # combined budget constraint
    A[11,:] = [0,0,2,2,-1,-1,-1,-1,-1,0] # social media budget constraint
    A[12,:] = [1,1,0,0,-1,0,0,0,0,-1] # print and TV budget constaint
    b = np.array([total_budget,3,3,3,3,3,3,3,3,3,0,0]) # constraint values
    sense = np.array(['<']*13)
    # creating model
    budgetMod = gp.Model()
    # tell the model how many variables there are
    budgetx = budgetMod.addMVar(len(obj))
    # define the variables before adding constraints
    budgetModCon = budgetMod.addMConstrs(A, budgetx, sense, b)
    budgetMod.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE)
    budgetMod.Params.OutputFlag = 0
    budgetMod.optimize()
    #this month's allocation and roi
    a = np.append(np.append(budgetx.x, total_budget),
                  sum(df_acutal_roi.loc[i]*budgetx.x*0.01))
    df.loc[i] = a
    #updating the budget for next month ROI
    total_budget += sum(df_acutal_roi.loc[i]*budgetx.x*0.01)*0.5
df
```