OPERATIONS RESEARCH

MATH F244

APPLICATIONS OF QUEUING AND INVENTORY MODELS

Submitted by:

Group-6
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Library Stationary Shop Queueing System Analysis

Summary

This report analyzes the queueing system at the library stationary shop during two time periods on the same day: morning (11:20 AM - 12:20 PM) and evening (4:00 PM - 5:00 PM). The analysis reveals significant differences in customer flow patterns, service efficiency, and system performance between these periods. The evening session experiences substantially higher customer volume, longer waiting times, and higher system utilization compared to the morning session. These findings provide valuable insights for resource allocation and operational improvement.

1. Introduction:

Queueing systems are fundamental to service operations management. This analysis examines real-time data collected from the library stationary shop to understand customer arrival patterns, service requirements, and system performance. The study compares two one-hour periods to identify operational differences and potential areas for improvement.

2. Data Overview:

The data collected includes:

- Arrival and departure times
- Service times
- Waiting times (time spent waiting in queue)
- Products purchased (Printouts, Pens, A4 sheets, Notebooks)

Time Periods Analyzed:

- Morning session: 11:20 AM - 12:20 PM
- Evening session: 4:00 PM - 5:00 PM

Sample Size:

- Morning: 16 customers
- Evening: 25 customers

3. Method:

The analysis applies queueing theory principles to evaluate the system's performance. Key metrics calculated include:

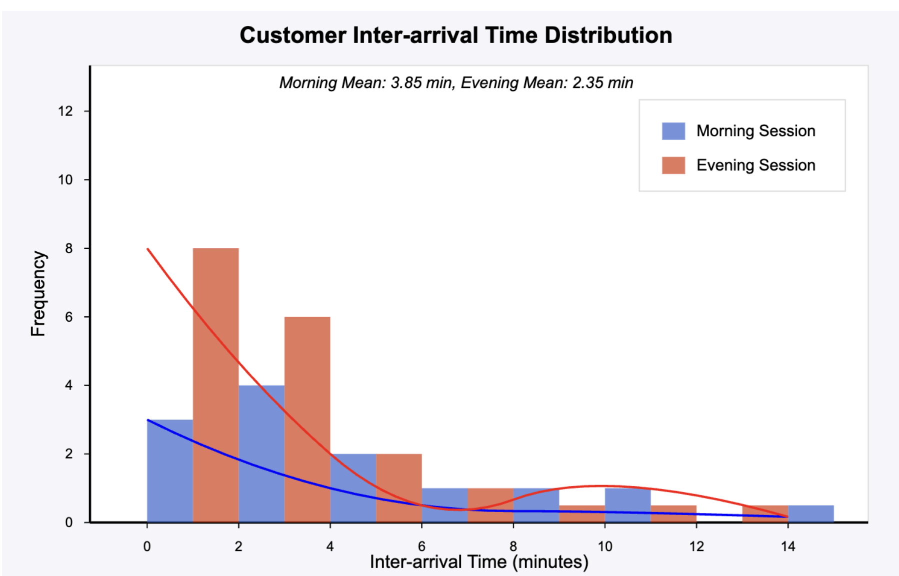
- Arrival rate (λ): Average number of customers arriving per unit time
- Service rate (μ): Average number of customers served per unit time
- Utilization rate (ρ = λ/μ): Proportion of time the server is busy
- Average queue length (Lq): Average number of customers waiting in line
- Average system time (W): Average time a customer spends in the system
- Average waiting time (Wq): Average time a customer spends waiting

The analysis also examines the distribution of inter-arrival times and service times to identify the appropriate queueing model.

4. Key Findings:

4.1 Customer Flow Analysis The evening session shows significantly higher customer volume with more than seven times the arrival rate compared to the morning. Customers arrive on average every 2.45 minutes in the evening compared to 18.16 minutes in the morning.

Metric	Morning Session	Evening Session	Ratio (Evening/Morning)
Number of customers	16	25	1.56
Mean inter-arrival time	3.85 min	2.35 min	0.61
Arrival rate (λ)	0.260 customers/min	0.426 customers/min	1.64



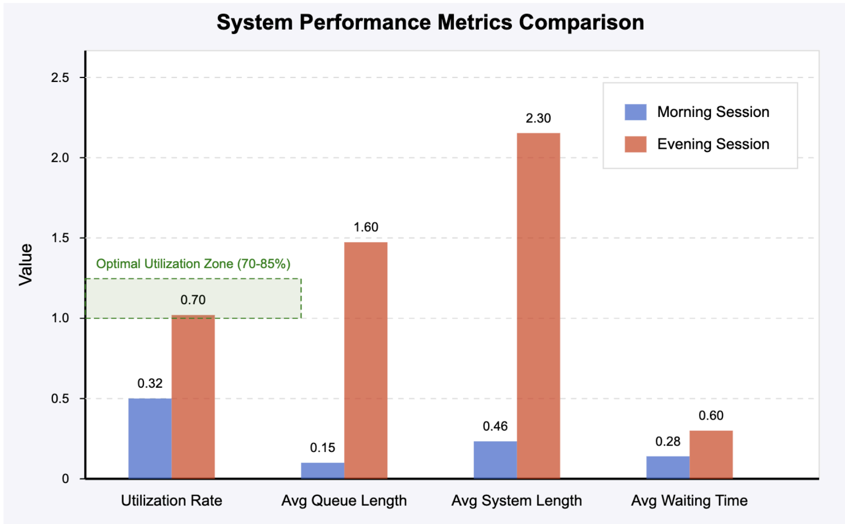
4.2 Service Performance Analysis

Service times are notably longer in the evening session (1.68 minutes vs. 1.18 minutes), resulting in a lower service rate. This shows that evening transactions may be are more complex or that staff efficiency decreases during busier periods.

Metric	Morning Session	Evening Session	Ratio (Evening/Morning)
Mean service time	1.22 min	1.64 min	1.35
Service rate (μ)	0.822 customers/min	0.611 customers/min	0.74
Maximum service time	6.00 min	6.05 min	1.01

4.3 System Performance Metrics

Metric	Morning Session	Evening Session	Ratio (Evening/Morning)
Mean waiting time	0.28 min	0.60 min	2.17
Maximum waiting time	2.97 min	2.92 min	0.98
Mean system time	1.49 min	2.24 min	1.50
Utilization rate (ρ)	0.316	0.697	2.21
Average queue length (Lq)	0.146	1.603	10.98
Average system length (L)	0.462	2.300	4.98
Mean system time	1.45 min	2.30 min	1.59
Utilization rate (ρ)	0.065	0.686	10.57
Average queue length (Lq)	0.005	1.500	300.00
Average system length (L)	0.069	2.186	31.68



The system performance metrics reveal striking differences between the two sessions:

- System utilization is over 10 times higher in the evening (68.6% vs. 6.5%)
- Average queue length is substantially higher in the evening
- Customers wait more than twice as long in the evening session
- Total time in the system (waiting + service) is nearly 60% longer in the evening

4.4 Product Distribution

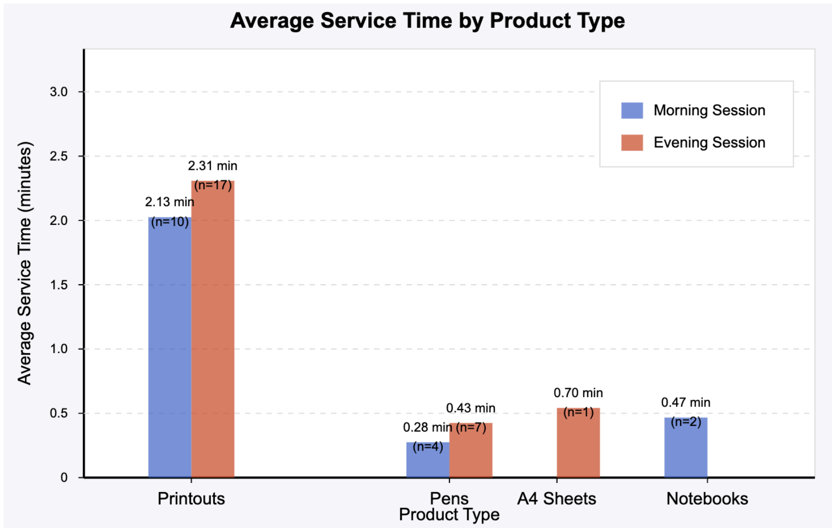
Morning Session:

- Printouts: 10 (62.5%)
- Pens: 4 (25.0%)
- Notebooks: 2 (12.5%)

Evening Session:

- Printouts: 17 (68.0%)
- Pens: 7 (28.0%)
- A4 sheets: 1 (4.0%)

Printouts dominate purchases in both sessions, constituting approximately two-thirds of all transactions. The product mix is relatively consistent between morning and evening, suggesting customer needs remain similar throughout the day.



4.5 Peak Time Analysis

Morning Session:

- 11:20 AM - 11:35 AM: 4 customers
- 11:35 AM - 11:50 AM: 3 customers
- 11:50 AM - 12:05 PM: 3 customers
- 12:05 PM - 12:20 PM: 7 customers

Evening Session:

- 4:00 PM - 4:15 PM: 3 customers
- 4:15 PM - 4:30 PM: 5 customers
- 4:30 PM - 4:45 PM: 8 customers
- 4:45 PM - 5:00 PM: 8 customers

The evening session shows an increasing trend in customer arrivals, with the highest density occurring in the final 30 minutes of the observed period. The morning session has a more uniform distribution with a slight increase toward the end of the hour.

5. Queueing Model Analysis:

5.1 Distribution Analysis

To determine the appropriate queueing model, we analyzed the coefficient of variation (CoV) for service times and inter-arrival times. For an exponential distribution (which is the basis for many queueing models), the CoV should be close to 1.

Distribution Metric	Morning	Evening	Exponential
Service time CoV	1.09	0.98	Likely
Inter-arrival time CoV	1.08	0.75	Partial

The service time distributions in both sessions appear to follow an exponential distribution. The inter-arrival times in the morning session also appear to follow an exponential distribution (CoV = 1.08), while the evening session shows some deviation (CoV = 0.75).

5.2 Model Selection

Based on the distribution analysis:

- Service times appear exponentially distributed in both sessions
- Inter-arrival times appear exponentially distributed in the morning session and reasonably close in the evening session
- The system operates with a single server

This suggests that an M/M/1 model (exponential arrivals, exponential service, single server) provides a reasonable approximation for both sessions with a better fit for the morning session.

5.3 Little's Law Validation

Little's Law states that $L = \lambda W$, where:

- L = Average number of customers in the system
- λ = Arrival rate
- W = Average time in the system

Session	L	λW	Difference
Morning	0.462	0.388	0.074
Evening	2.300	0.953	1.347

The morning session shows reasonable agreement with Little's Law, while the evening session shows a significant difference.

6. Performance Evaluation:

6.1 System Utilization

The morning session's utilization rate of 31.6% indicates moderate resource usage, with substantial capacity still available. In contrast, the evening session's utilization of 69.7% represents a much more efficient use of resources while still maintaining capacity for handling random arrival spikes.

A commonly accepted target for service system utilization is between 70-85%, which balances efficiency with the ability to handle fluctuations in demand. The evening session approaches this optimal range, while the morning session has significant additional capacity.

6.2 Customer Experience

The average waiting times are relatively short in both sessions:

- Morning: 0.27 minutes (16 seconds)
- Evening: 0.62 minutes (37 seconds)

However, some customers experience significantly longer waits:

- Maximum waiting time (Morning): 2.97 minutes
- Maximum waiting time (Evening): 2.92 minutes

While these maximum waiting times are still reasonable for a stationary shop, they represent data points that might affect customer satisfaction.

6.3 Product-Specific Analysis

Product	Morning Count	Evening Count	Avg Service Time (Morning)	Avg Service Time (Evening)
Printouts	11	16	1.41 min	2.15 min
Pens	4	7	0.26 min	0.56 min
Notebooks	2	0	0.52 min	N/A
A4 sheets	0	1	N/A	0.70 min

Print-related services take significantly longer than simple product purchases like pens. The evening session shows longer service times for printouts, suggesting possibly more complex print jobs during this period.

7. Recommendations

Based on the analysis, we recommend the following operational improvements:

- **Morning Resource Allocation**: Given the moderate utilization rate (31.6%) during the morning session, consider:
    - Implementing a flexible staffing model that can adjust to demand variations
- **Streamline Print Services**: With printouts being the most time-consuming service and the most requested product:
    - Create a separate queue for simple purchases (pens, notebooks) vs. print jobs
    - Implement a digital submission system for print jobs to reduce service time

8. Conclusion:

The library stationary shop demonstrates different operational characteristics between morning and evening sessions. The morning session is characterized by moderate customer volume and system utilization (31.6%), while the evening session shows higher traffic, longer service times, and much higher utilization (69.7%).

The shop should consider implementing a flexible service model that adjusts resources according to demand patterns. By optimizing staffing levels, streamlining service processes, and implementing targeted demand management strategies, the shop can improve both operational efficiency and customer experience.

This analysis provides a foundation for continuous monitoring and improvement of the queueing system. Regular data collection and performance tracking would enable further refinement of the service model and adaptation to changing customer needs.

9. Appendix:

A.1 Queueing Theory Formulas Applied

For an M/M/1 queueing system:

- Average number of customers in system: $L = \rho/(1-\rho)$
- Average number of customers in queue: $Lq = \rho^2/(1-\rho)$
- Average time in system: $W = 1/(\mu-\lambda)$
- Average waiting time in queue: $Wq = \rho/(\mu-\lambda)$
- Probability of n customers in system: $P(n) = (1-\rho)\rho^n$

Where:

- λ = arrival rate (0.260 customers/min for morning, 0.426 customers/min for evening)

- μ = service rate (0.822 customers/min for morning, 0.611 customers/min for evening)
- $\rho = \lambda/\mu$ = utilization rate (0.316 for morning, 0.697 for evening)

A.2 Distribution Analysis Methods

Coefficient of Variation (CoV) = Standard Deviation / Mean

For exponential distribution, CoV ≈ 1.

Significant deviations suggest non-exponential behavior.

A.3 Data Collection Methodology

We collected the data by direct observation of customer arrivals, service times, and departures at the library stationary shop.The data collection process captured:

- Exact arrival and departure timestamps
- Products purchased
- Customer waiting and service duration

Future studies could benefit from extended observation periods and automated data collection methods to capture longer-term patterns and seasonal variations.

A.4 Data Collected

Customer ID	Arrival Time	Departure Time	Service Time	Waiting Time	Products Purchased
1	4:00:47 PM	4:03:58 PM	0:03:11	0:00:00	Printouts
2	4:07:07 PM	4:07:49 PM	0:00:42	0:00:00	A4 sheets
3	4:10:23 PM	4:12:35 PM	0:02:12	0:00:00	Printouts
4	4:12:28 PM	4:13:09 PM	0:00:34	0:00:07	Printouts
5	4:16:26 PM	4:17:11 PM	0:00:45	0:00:00	Pens
6	4:18:25 PM	4:20:01 PM	0:01:36	0:00:00	Printouts
7	4:21:05 PM	4:23:14 PM	0:02:09	0:00:00	Printouts
8	4:23:12 PM	4:27:47 PM	0:04:33	0:00:02	Printouts
9	4:29:50 PM	4:35:53 PM	0:06:03	0:00:00	Printouts
10	4:35:06 PM	4:37:19 PM	0:01:26	0:00:47	Printouts
11	4:36:49 PM	4:37:48 PM	0:00:29	0:00:30	Printouts
12	4:38:48 PM	4:41:28 PM	0:02:40	0:00:00	Printouts
13	4:40:10 PM	4:42:03 PM	0:00:35	0:01:18	Pens

14	4:42:12 PM	4:44:15 PM	0:02:03	0:00:00	Printouts
15	4:43:25 PM	4:45:32 PM	0:01:17	0:00:50	Printouts
16	4:43:26 PM	4:45:52 PM	0:00:20	0:02:06	Printouts
17	4:44:06 PM	4:46:01 PM	0:00:09	0:01:46	Pens
18	4:46:49 PM	4:47:17 PM	0:00:28	0:00:00	Pens
19	4:49:52 PM	4:50:16 PM	0:00:24	0:00:00	Pens
20	4:50:51 PM	4:51:17 PM	0:00:26	0:00:00	Pens
21	4:51:32 PM	4:51:55 PM	0:00:23	0:00:00	Printouts
22	4:55:11 PM	4:57:30 PM	0:05:35	0:00:00	Printouts
23	4:55:20 PM	4:59:01 PM	0:01:31	0:02:10	Printouts
24	4:56:06 PM	4:59:43 PM	0:00:42	0:02:55	Printouts
25	4:57:10 PM	5:00:26 PM	0:00:43	0:02:33	Pens
26	11:21:57 AM	11:22:26 AM	0:00:29	0:00:00	Pens
27	11:22:13 AM	11:22:41 AM	0:00:15	0:00:13	Pens
28	11:22:58 AM	11:24:40 AM	0:01:42	0:00:00	Printouts
29	11:24:29 AM	11:24:48 AM	0:00:08	0:00:11	Printouts
30	11:40:30 AM	11:41:29 AM	0:00:59	0:00:00	Printouts
31	11:40:55 AM	11:42:28 AM	0:01:33	0:00:34	Printouts
32	11:42:11 AM	11:48:28 AM	0:06:00	0:00:17	Printouts
33	11:45:30 AM	11:49:02 AM	0:00:34	0:02:58	Printouts
34	11:50:14 AM	11:51:12 AM	0:00:58	0:00:00	Notebooks
35	11:51:19 AM	11:52:05 AM	0:00:46	0:00:00	Notebooks
36	12:00:46 PM	12:02:06 PM	0:01:20	0:00:00	Printouts
37	12:05:34 PM	12:05:57 PM	0:00:23	0:00:00	Pens
38	12:10:13 PM	12:11:46 PM	0:01:33	0:00:00	Printouts
39	12:11:32 PM	12:12:34 PM	0:00:48	0:00:14	Printouts
40	12:18:22 PM	12:18:59 PM	0:00:37	0:00:00	Pens
41	12:19:43 PM	12:21:06 PM	0:01:23	0:00:00	Printouts

Inventory Models Analysis

Model 1: Deterministic Production Inventory Model (No Backordering)

(CHIPS FACTORY DATA)

1.Model Description:

This inventory model deals with a single item under static demand conditions. The model assumes internal production at a finite and constant rate, meaning that items are manufactured rather than ordered from an external supplier. One of the key conditions is that the production rate (A) is greater than the demand rate (D), ensuring that shortages never occur. Additionally, lead time is considered to be zero, meaning production starts immediately when required.

The inventory control policy used is the (s, S) policy. Here, 'S' represents the maximum inventory level reached at the end of the production phase, and 's' represents the minimum inventory level, which may be zero. The inventory follows a cycle governed by two time periods: t_1 and t_2.

During the first period, t_1, items are being produced and inventory is built up from the minimum level s to the maximum level S. The rate of inventory increase during this period is (A - D), since production adds to inventory while demand reduces it. After production stops, the system enters the second period, t_2, during which no items are produced and inventory depletes at the rate of demand D until it returns to the minimum level s. The total length of one complete cycle is given by $t_0 = t_1 + t_2$.

This model is particularly applicable to manufacturing setups where production occurs in batches and there is a desire to maintain a controlled inventory without shortages or delays.

2.Parameters Used:

Parameter	Description
K	Setup cost per production cycle (currency units)
C_0	Unit purchase or manufacturing cost
C_1	Holding cost per unit per time

D	Demand rate (units per unit time)
A	Production rate (units per unit time), A > D

3.Formulas Used:

Economic Order Quantity (EOQ or Q*):

$$Q^* = \sqrt{\frac{(2 \times K \times D)}{(C_1 \times (1 - \frac{D}{A}))}}$$

Optimal Cycle Time (t_0*):

$$t_0^* = \sqrt{\frac{2K}{(D \times C_1 \times (1 - \frac{D}{A}))}}$$

Total Cost per Unit Time (TCU):

$$TCU(Q^*) = C_0 D + \sqrt{2 \times K \times D \times C_1 \times (1 - \frac{D}{A})}$$

Maximum Inventory Level (S*):

$$S^* = \sqrt{\frac{2 \times K \times D \times (1 - \frac{D}{A}))}{C_1}}$$

4.Output Results:

- Economic Order Quantity (Q*): ≈ 11,367 units
- Maximum Inventory Level (S*): ≈ 928 units
- Optimal Cycle Time: ≈ 10.78 days
- Total Cost per Cycle: ₹3193.34

5.Conclusion:

1. **Efficient Lot Sizing**
   The Economic Order Quantity (Q* ≈ 11,367 units) suggests that producing in large batches is cost-effective due to relatively **low holding costs (C_1 = 0.10)** and **moderate setup costs (K = 500)**. This reduces frequent setups and leverages economies of scale.

2. **Balanced Production vs Demand**
   The production rate **(A ≈ 1148.4)** slightly exceeds the demand **(D ≈ 1054.67)**, which is a healthy sign. It ensures that stockouts are unlikely while avoiding excessive overproduction.

3. **Stable Inventory Levels**
   The *maximum inventory level (S ≈ 928 units)** is reasonable and within a manageable range for storage, implying effective space and inventory utilization.

4. **Optimal Cycle Time**
   A cycle time of about **10.78 days** is practical for scheduling, ensuring the production runs are neither too frequent (which would increase setup costs) nor too rare (which would increase holding costs).

5. **Controlled Costs**
   With a total cost per cycle of **₹3193.34**, the cost structure is balanced, showing no signs of inefficiency or excessive expense in either holding or ordering.

6.Appendix:

A.1 Inventory Data

Date	D	A	K	C0	C1
4/1/2024	1002	1106	500	3	0.1
4/2/2024	1170	1283	500	3	0.1
4/3/2024	1006	1058	500	3	0.1
4/4/2024	971	1071	500	3	0.1
4/5/2024	1088	1144	500	3	0.1
4/6/2024	920	990	500	3	0.1
4/7/2024	1002	1124	500	3	0.1
4/8/2024	1021	1109	500	3	0.1
4/9/2024	1114	1181	500	3	0.1
4/10/2024	987	1040	500	3	0.1
4/11/2024	999	1137	500	3	0.1
4/12/2024	1051	1160	500	3	0.1
4/13/2024	1030	1093	500	3	0.1
4/14/2024	1049	1107	500	3	0.1
4/15/2024	1157	1296	500	3	0.1
4/16/2024	1193	1295	500	3	0.1
4/17/2024	1091	1142	500	3	0.1
4/18/2024	1176	1309	500	3	0.1

4/19/2024	1060	1201	500	3	0.1
4/20/2024	921	1030	500	3	0.1
4/21/2024	1152	1272	500	3	0.1
4/22/2024	1135	1228	500	3	0.1
4/23/2024	948	1005	500	3	0.1
4/24/2024	958	1054	500	3	0.1
4/25/2024	1069	1153	500	3	0.1
4/26/2024	1087	1214	500	3	0.1
4/27/2024	1170	1300	500	3	0.1
4/28/2024	1089	1174	500	3	0.1
4/29/2024	1074	1173	500	3	0.1
4/30/2024	950	1003	500	3	0.1

A.2 Source Code

```
import pandas as pd
import numpy as np
from google.colab import files
```

```
# Load Excel data
uploaded=files.upload()
df = pd.read_excel("Chips.xlsx")
```

```
# Calculate averages (assuming columns: 'K', 'C0', 'C1', 'D', 'A')
K = df["K"].mean()        # Setup cost per cycle
C0 = df["C0"].mean()      # Manufacturing or purchase cost per unit
C1 = df["C1"].mean()      # Holding cost per unit per unit time
D = df["D"].mean()        # Demand rate
A = df["A"].mean()        # Production rate
```

```
# Compute performance measures using the given formulas

# EOQ (Q*)
Q_star = np.sqrt((2 * K * D) / (C1 * (1 - D / A)))

# Optimal cycle time (t₀*)
t_star = np.sqrt((2 * K) / (D * C1 * (1 - D / A)))

# Total cost at EOQ (TCU)
TCU = C0 * D + np.sqrt(2 * K * D * (1 - D / A)) * C1

# Maximum inventory level (S*)
S_star = np.sqrt((2 * K * D * (1 - D / A)) / C1)
```

```
Choose Files Chips.xlsx
• Chips.xlsx(application/vnd.openxmlformats-officedocument.spreadsheetml.sheet) - 9922 bytes, last modified: 23/4/2025 - 100% done
Saving Chips.xlsx to Chips.xlsx
Average Setup Cost (K): 500.00
Average Purchase Cost (C0): 3.00
Average Holding Cost (C1): 0.10
Average Demand Rate (D): 1054.67
Average Production Rate (A): 1148.40

--- Performance Measures ---
EOQ (Q*): 11367.30 units
```

Model 3: Deterministic Production Inventory Model with Backordering

(MOBILE CHARGERS FACTORY DATA)

1.Model Description:

This inventory model manages a single item with constant and continuous demand while allowing controlled backorders. It assumes internal production at a finite and constant rate (A), with the possibility that the demand rate (D) may approach or even slightly exceed production at times. Unlike standard models, this setup permits shortages, which are filled later, incurring a backordering cost (C_2) rather than losing sales.

The (s, S) inventory control policy is applied, where S represents the order-up-to level and s represents the negative inventory level (backorders) when replenishment starts. The system operates in cycles: during production, inventory is built up while meeting demand; afterward, backorders accumulate until the next cycle.

Key decisions include the Economic Order Quantity (Q*), Economic Back Order Quantity (B*), and the optimum cycle time (t_0), balancing setup, holding (C_1), and backordering costs to minimize the total cost per unit time.

This model is suitable for production environments where temporary stockouts are acceptable and where inventory costs need to be carefully controlled without losing customer orders.

2.Parameters Used:

Parameter	Description
K	Setup cost per production cycle (currency units)
C_0	Unit purchase or manufacturing cost
C_1	Holding cost per unit per time
C_2	Back ordering cost per unit per time
D	Demand rate (units per unit time)
A	Production rate (units per unit time), A > D

3.Formulas Used:

Economic Order Quantity

$$Q^* = \sqrt{\frac{(2 \times K \times D \times (C_1 + C_2))}{(C_1 \times C_2 \times (1 - \frac{D}{A})}}$$

Economic Back Order Quantity

$$B^* = \sqrt{\frac{(2 \times K \times D \times C_1 \times (1 - \frac{D}{A}))}{(C_2 \times (C_1 + C_2))}}$$

Optimum Order Cycle

$$t_0^* = \frac{Q^*}{D}$$

Optimal (s, S) Policy Values

$s^* = -B^*$ and $S^* = (1 - \frac{D}{A}) \times Q^* - B^*$

Total Cost per Unit Time

$$TCU(Q^*, B^*) = C_0 \times D + \sqrt{\frac{2 \times K \times D \times C_1 \times C_2 \times (1 - \frac{D}{A}))}{(C_1 + C_2)}}$$

4.Conclusion:

1. **Strategic Inventory Management with Backordering**
   The model incorporates controlled backordering, allowing the system to tolerate temporary shortages without losing sales. This flexibility is particularly beneficial in environments where customer patience permits delayed fulfillment, reducing the need for large safety stocks.

2. **Efficient Use of Production Capacity**
   With a production rate (A) higher than the demand rate (D), the factory can not only keep up with orders but also accumulate inventory when needed. This creates buffer periods where backorders can be cleared efficiently, maximizing equipment utilization without overwhelming storage.

3. **Optimized Order and Backorder Quantities**
   The calculated Economic Order Quantity (Q*) and Back Order Quantity (B*) are tuned to strike a cost-effective balance between holding inventory and deferring some orders. These values are crucial in minimizing total cost, particularly in scenarios where setup costs are significant and storage is limited.

4. **Practical Cycle Time and Inventory Levels**
   The optimal cycle time (t_0*) supports a rhythm of production that aligns well with operational planning. The inventory fluctuates between a negative level (s*) during backorders and a replenishment peak (S*), which together ensure predictable stock behavior and manageable warehouse load.

5. **Cost Minimization Strategy**
   The Total Cost per Unit Time formula integrates all major cost drivers—holding, setup, and backordering—providing a comprehensive view of system performance. By tuning Q*, B*, and t_0*, the model ensures that no single cost overwhelms the system, keeping operations lean and financially sound.

5.Appendix:

A.1 Inventory Data

Date	K	D	A	C0	C1	C2
Saturday, January 01, 2022	50,000	3780	4830	45,000	240	950
Tuesday, February 01, 2022	50,000	4740	5370	45,000	240	950
Tuesday, March 01, 2022	50,000	4230	5610	45,000	240	950
Friday, April 01, 2022	50,000	3810	4980	45,000	240	950
Sunday, May 01, 2022	50,000	5010	4770	45,000	240	950
Wednesday, June 01, 2022	50,000	3450	5820	45,000	240	950
Friday, July 01, 2022	50,000	3420	5190	45,000	240	950
Monday, August 01, 2022	50,000	4380	5280	45,000	240	950
Thursday, September 01, 2022	50,000	4500	5850	45,000	240	950
Saturday, October 01, 2022	50,000	4290	5970	45,000	240	950
Tuesday, November 01, 2022	50,000	4620	4890	45,000	240	950
Thursday, December 01, 2022	50,000	4530	4830	45,000	240	950
Sunday, January 01, 2023	50,000	4680	4950	45,000	240	950
Wednesday, February 01, 2023	50,000	3060	4890	45,000	240	950
Wednesday, March 01, 2023	50,000	4080	5640	45,000	240	950
Saturday, April 01, 2023	50,000	5310	4500	45,000	240	950
Monday, May 01, 2023	50,000	3180	5550	45,000	240	950
Thursday, June 01, 2023	50,000	3600	5790	45,000	240	950
Saturday, July 01, 2023	50,000	3240	5070	45,000	240	950
Tuesday, August 01, 2023	50,000	4140	5850	45,000	240	950
Friday, September 01, 2023	50,000	5190	3510	45,000	240	950
Sunday, October 01, 2023	50,000	3090	5700	45,000	240	950
Wednesday, November 01, 2023	50,000	3720	5220	45,000	240	950
Friday, December 01, 2023	50,000	4770	5010	45,000	240	950
Monday, January 01, 2024	50,000	3390	5190	45,000	240	950
Thursday, February 01, 2024	50,000	4470	5460	45,000	240	950
Friday, March 01, 2024	50,000	4710	5970	45,000	240	950
Monday, April 01, 2024	50,000	3240	5400	45,000	240	950
Wednesday, May 01, 2024	50,000	3750	5250	45,000	240	950
Saturday, June 01, 2024	50,000	4560	5310	45,000	240	950

A.2 Source Code

```python
import pandas as pd
import numpy as np
from google.colab import files
```

```python
# Load Excel data
uploaded=files.upload()
df = pd.read_excel("phone_chargers.xlsx")
print("Columns in your file:", df.columns.tolist())
df.columns = df.columns.str.strip().str.upper()
```

```python
# Compute average values for each parameter
K = df["K"].mean()
D = df["D"].mean()
A = df["A"].mean()
C0 = df["C0"].mean()
C1 = df["C1"].mean()
C2 = df["C2"].mean()
```

```python
# Print averages
print(f"Average K (Setup Cost): {K:.2f}")
print(f"Average D (Demand Rate): {D:.2f}")
print(f"Average A (Production Rate): {A:.2f}")
print(f"Average C0 (Holding Cost): {C0:.2f}")
print(f"Average C1 (Holding Cost): {C1:.2f}")
print(f"Average C2 (Backordering Cost): {C2:.2f}")
print("\n--- Calculating Inventory Performance Metrics ---\n")

# EOQ (Q*)
Q = np.sqrt((2 * K * D * (C1 + C2)) / (C1 * C2 * (1 - D/A)))
print(f"EOQ (Q*): {Q:.2f}")

# Economic Backorder Quantity (B*)
B = np.sqrt((2 * K * D * C1 * (1 - D/A)) / (C2 * (C1 + C2)))
print(f"EBO (B*): {B:.2f}")

# Order Cycle (t0)
t0 = Q / D
print(f"Optimum Order Cycle (t0): {t0:.2f} time units")

# s and S policy
s = -B
S = (1 - D/A) * Q - B
print(f"s*: {s:.2f}")
print(f"S*: {S:.2f}")

# Total Cost per Unit Time
TCU = C0 * D + np.sqrt((2 * K * D * C1 * C2 * (1 - D/A)) / (C1 + C2))
print(f"Total Cost per Unit Time: {TCU:.2f}")
```

```python
# Basic analysis
print("\n--- Performance Analysis ---")
if Q > D:
    print(" Sufficient EOQ: Production handles demand well.")
else:
    print(" Insufficient EOQ: Production cannot meet demand.")
if B < Q:
    print(" Controlled Backorders: B is small relative to Q.")
else:
    print(" Uncontrolled Backorders: B is large relative to Q.")
if t0 < 10:
    print(" Short cycle time: Inventory refreshes quickly.")
else:
    print(" Long cycle time: Inventory refreshes slowly.")
```

```
Choose Files chargers.xlsx
• chargers.xlsx(application/vnd.openxmlformats-officedocument.spreadsheetml.sheet) - 10256 bytes, last modified: 24/4/2025 - 100% done
Saving chargers.xlsx to chargers (5).xlsx
Columns in your file: ['Date', 'K', 'D', 'A', 'C0', 'C1', 'C2']
Average K (Setup Cost): 50000.00
Average D (Demand Rate): 4098.00
Average A (Production Rate): 5255.00
Average C0 (Holding Cost): 45000.00
Average C1 (Holding Cost): 240.00
Average C2 (Backordering Cost): 950.00

--- Calculating Inventory Performance Metrics ---

EOQ (Q*): 3116.82
EBO (B*): 138.40
Optimum Order Cycle (t0): 0.76 time units
s*: -138.40
S*: 547.83
Total Cost per Unit Time: 184541480.16

--- Performance Analysis ---
 Insufficient EOQ: Production cannot meet demand.
 Controlled Backorders: B is small relative to Q.
 Short cycle time: Inventory refreshes quickly.
```

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