CONTROL SYSTEMS

G. MAHIMA
EE19BTECH11048
Electrical Engineering
IIT Hyderabad

September 4, 2020

1. Question

2. Answer

Chapter 2 - Question 50

Consider the differential equation

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = f(x) \tag{1.1}$$

where f(x) is the input and is a function of the output, x. If f(x) = sinx, linearize the differential equation for small excursions.

a. x = 0

b. x = π

Answer:

- The presence of sinx makes the equation non-linear.

a: Since we want to linearize the equation about x = 0, we let
$x = \delta x \ + 0,$
where
δx *is the small excursion about* 0, *and substitute x in equation* : 1.1

$$\frac{d^2(\delta x + 0)}{dt^2} + 3\frac{d(\delta x + 0)}{dt} + 2(\delta x) = sin(\delta x + 0) \tag{2.1}$$

Since,

$$sin(\delta x + 0) - sin(0) = \frac{dsinx}{dx}|_{x=0}\ \delta x \tag{2.2}$$

$$=>\ sin(\delta x + 0) = 0 + cosx|_{x=0}\ \delta x = \delta x \tag{2.3}$$

Continuation..

Therefore,

$$\frac{d^2(\delta x)}{dt^2} + 3\frac{d(\delta x)}{dt} + 2(\delta x) = \delta x \tag{2.4}$$

$$=> \frac{d^2(\delta x)}{dt^2} + 3\frac{d(\delta x)}{dt} + \delta x = 0 \tag{2.5}$$

Therefore, the linearized differential equation is:

$$\frac{d^2(\delta x)}{dt^2} + 3\frac{d(\delta x)}{dt} + \delta x = 0 \tag{2.6}$$

Continuation..

b: Since we want to linearize the equation about $x = \pi$, *we let* $x = \delta x \ + \pi$,
where
δx *is the small excursion about* π, *and substitute* x *in equation* : 1.1

$$\frac{d^2(\delta x + \pi)}{dt^2} + 3\frac{d(\delta x + \pi)}{dt} + 2(\delta x) = sin(\delta x + \pi) \tag{2.7}$$

Since,

$$sin(\delta x + \pi) - sin(\pi) = \frac{dsinx}{dx}|_{x=\pi} \ \delta x \tag{2.8}$$

$$=> \ sin(\delta x + \pi) = 0 + cosx|_{x=\pi} \ \delta x = -\delta x \tag{2.9}$$

Continuation..

Therefore,

$$\frac{d^2(\delta x)}{dt^2} + 3\frac{d(\delta x)}{dt} + 2(\delta x) = -\delta x \tag{2.10}$$

$$=> \frac{d^2(\delta x)}{dt^2} + 3\frac{d(\delta x)}{dt} + 3(\delta x) = 0 \tag{2.11}$$

Therefore, the linearized differential equation is:

$$\frac{d^2(\delta x)}{dt^2} + 3\frac{d(\delta x)}{dt} + 3(\delta x) = 0 \tag{2.12}$$