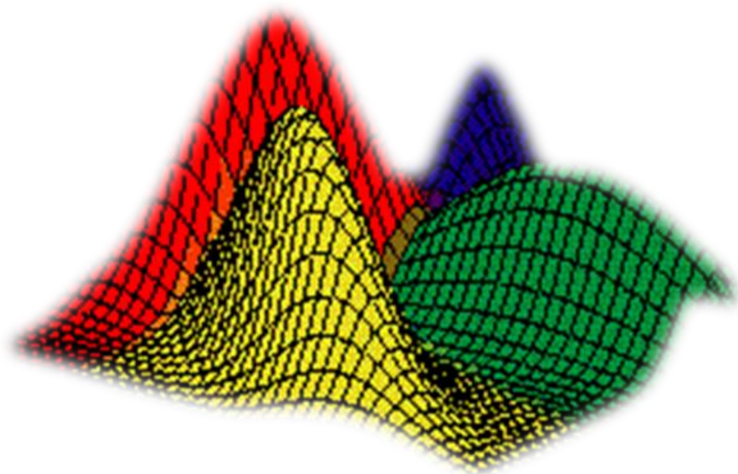
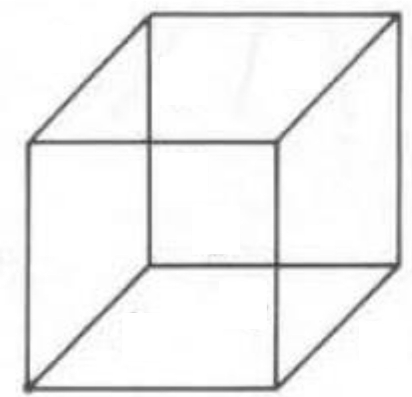


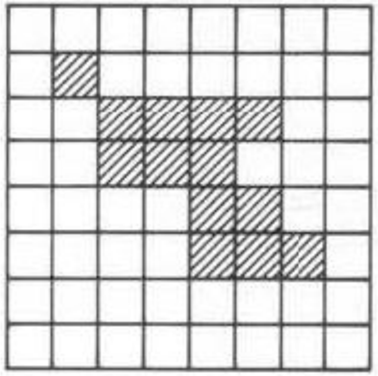
CSE 473: Pattern Recognition

Syntactic Pattern Recognition

Capability of String Grammar

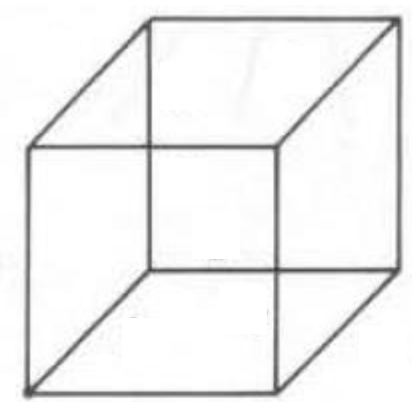
- able to represent 1D pattern
- difficult to classify patterns like these:

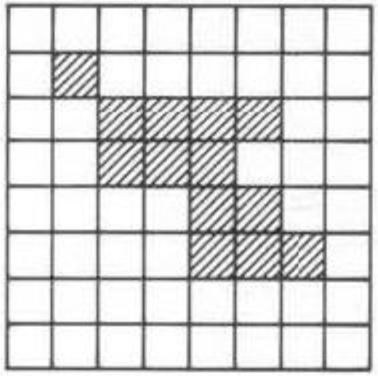




Capability of String Grammar

- These structure contains
  - information in both direction
  - hierarchical structure





Capability of String Grammar

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  - information in both direction
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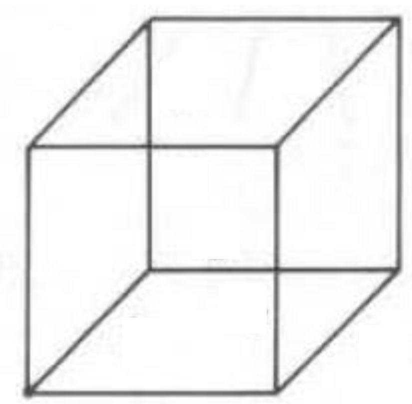
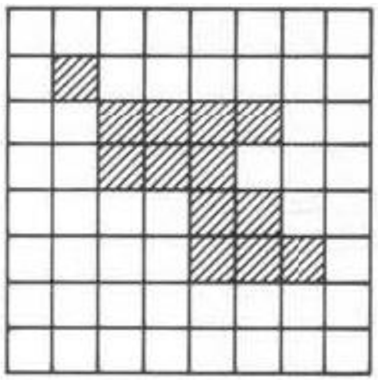
Solution: Higher Order Grammar

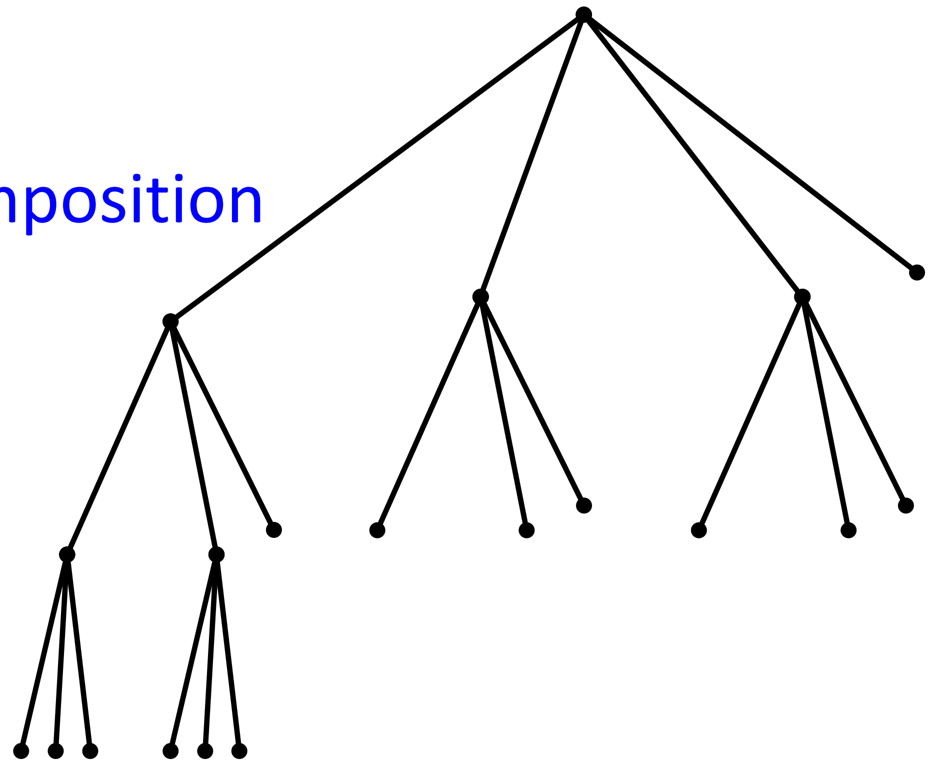




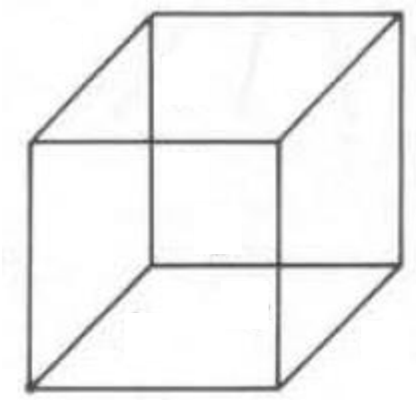
Higher Order Grammar: Tree Grammar

- Information is stored in
  - *nodes*: as primitives or sub-structures
  - *edges*: relation between primitives or sub-structures

- Allows hierarchical decomposition of a complex structure



Representation using Tree (1)



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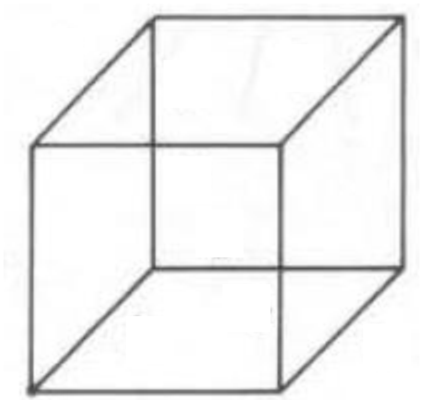

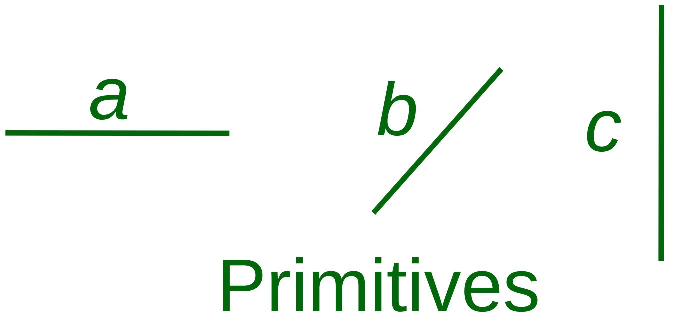

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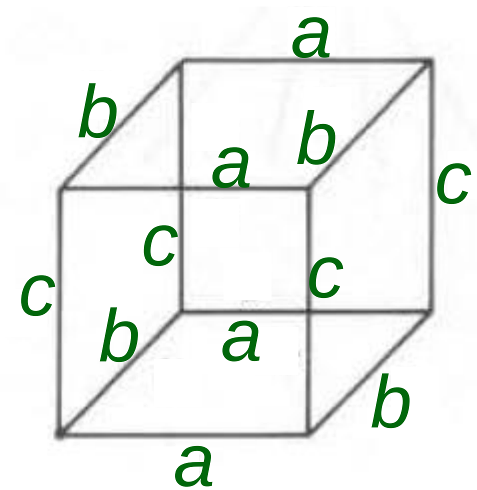

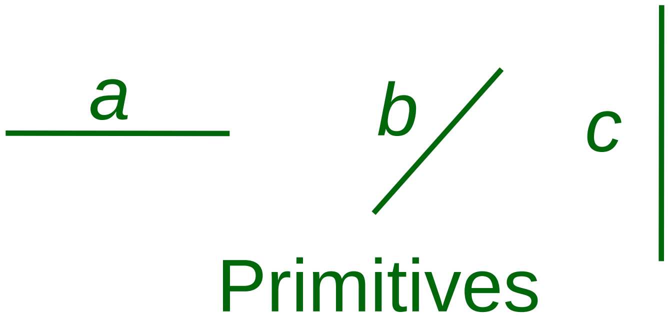

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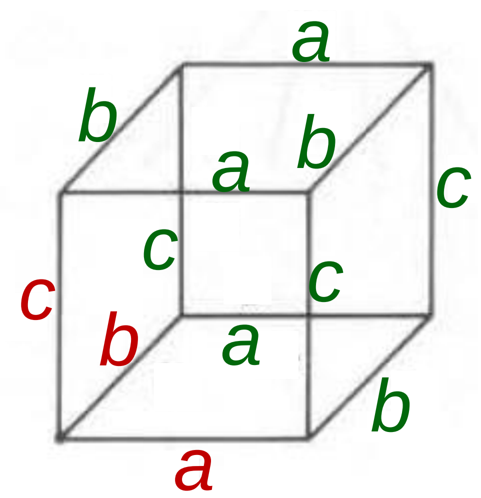

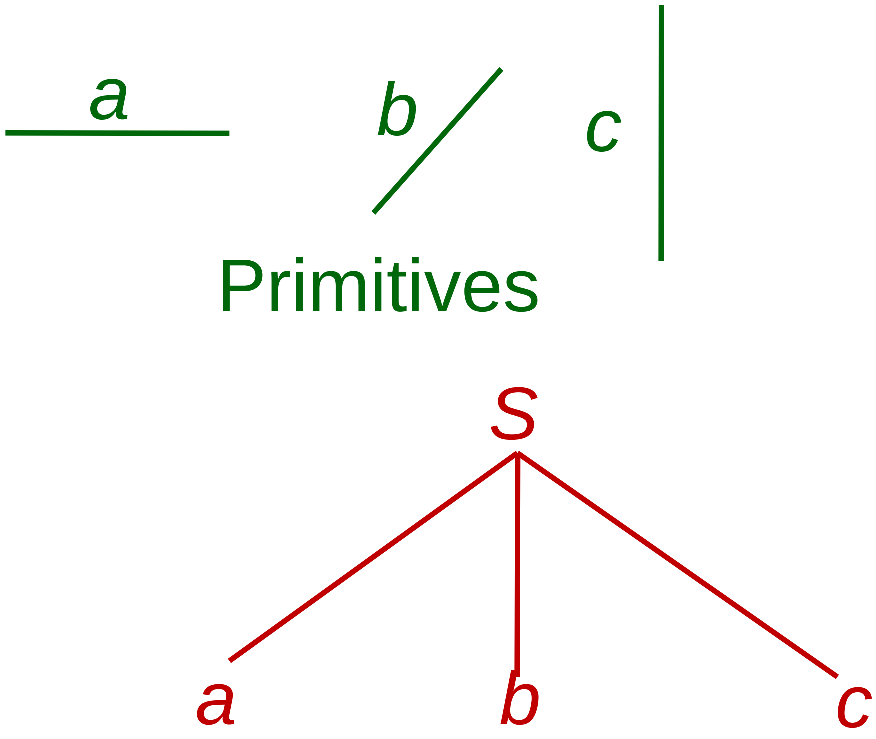

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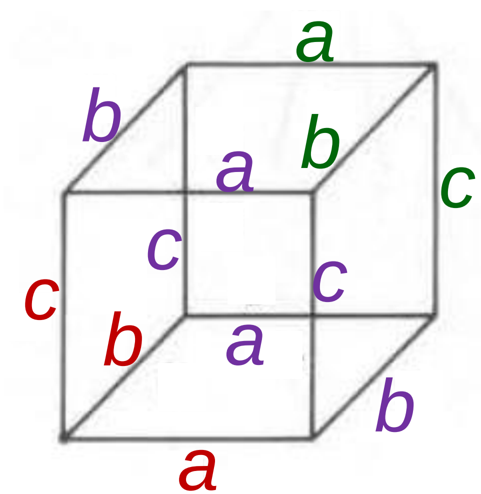
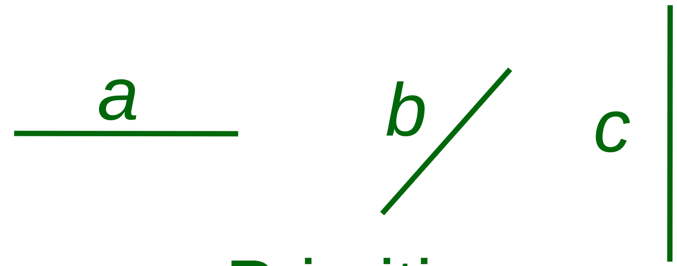
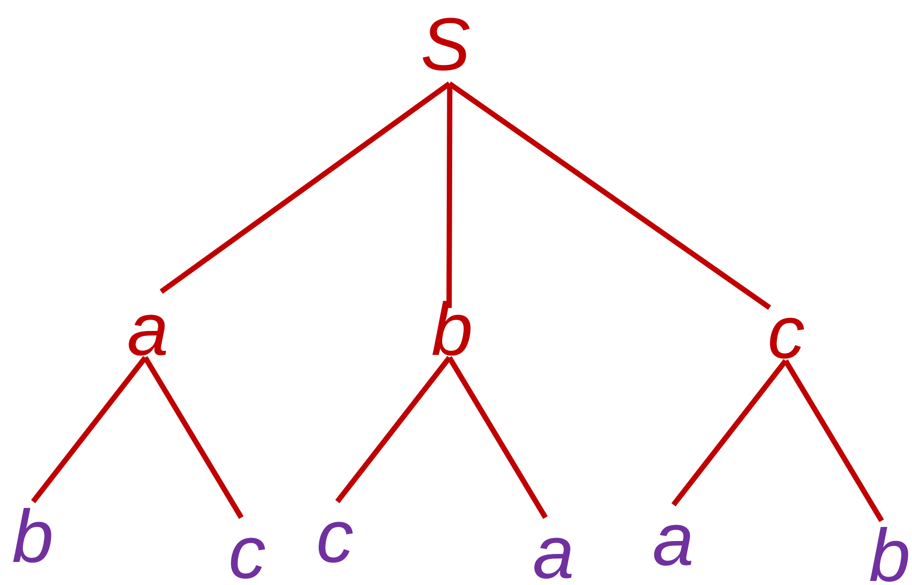




Primitives



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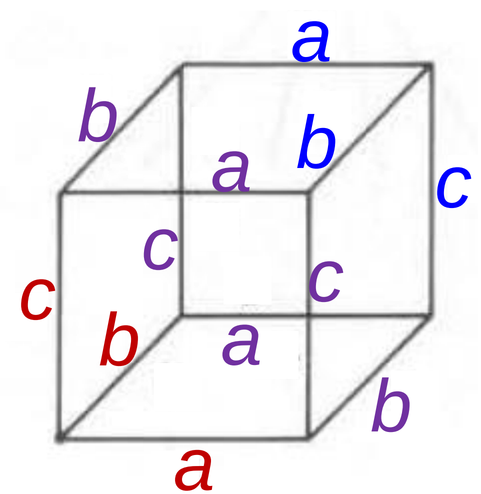
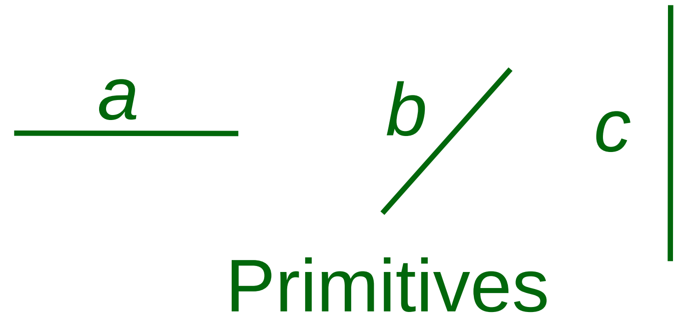
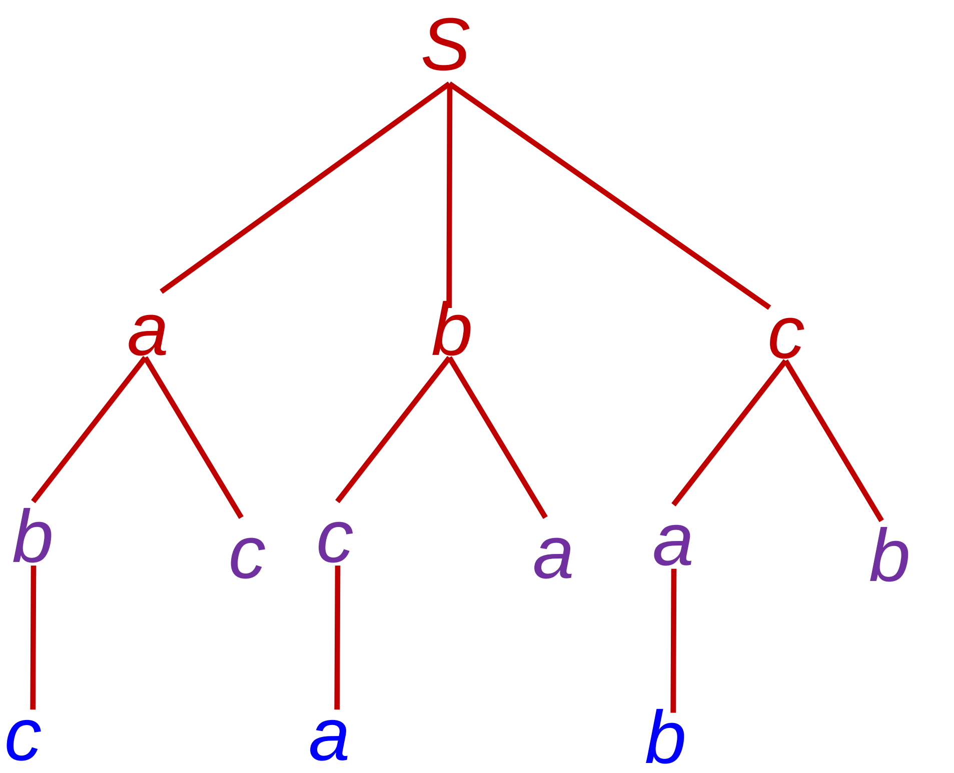
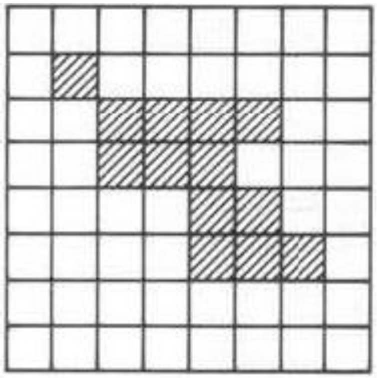


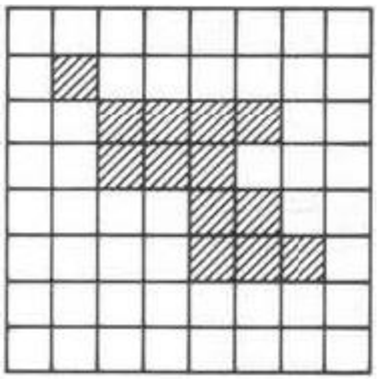


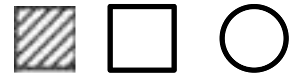


Representation using Tree (2)



Representation using Tree (2)





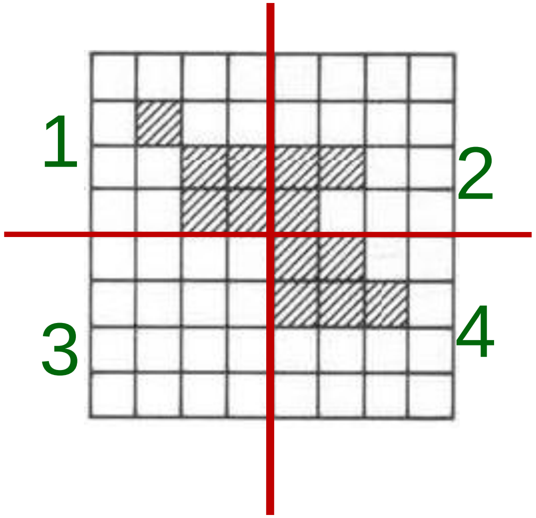
Primitives

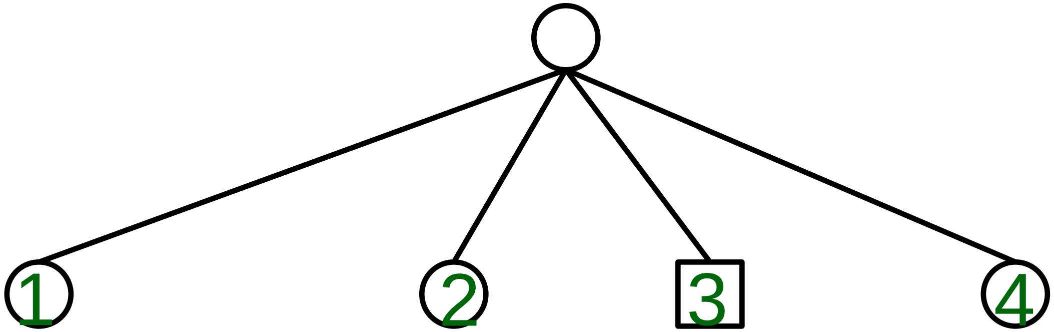
Black region

White region

Gray region

Representation using Tree (2)





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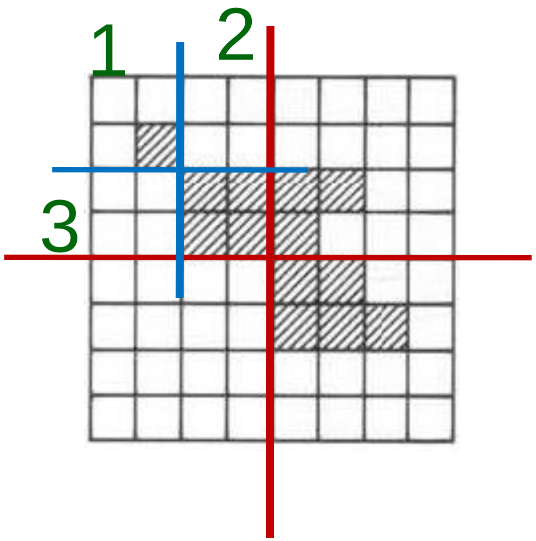
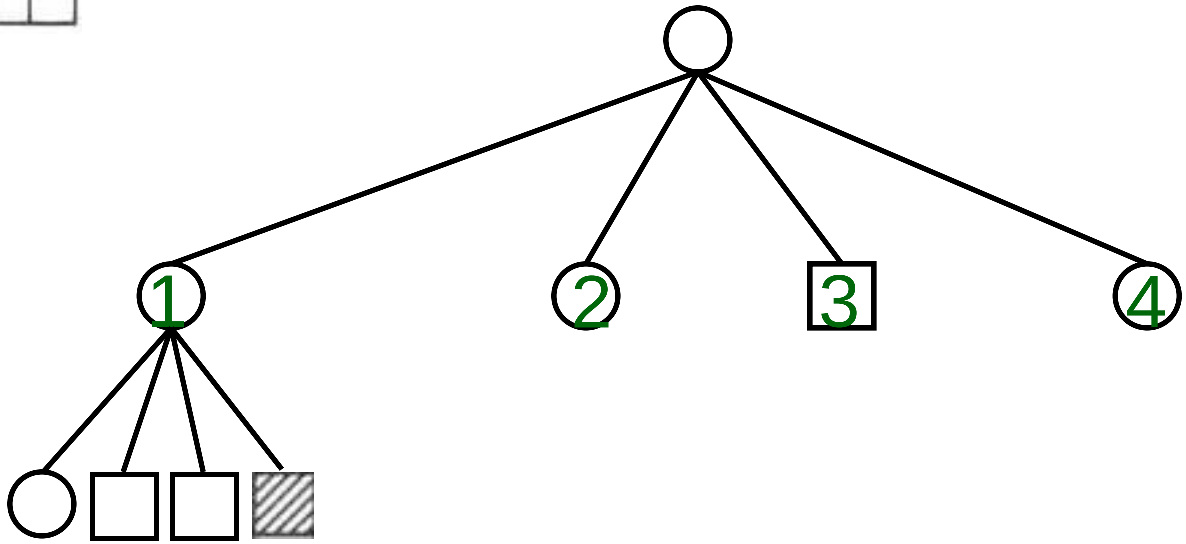
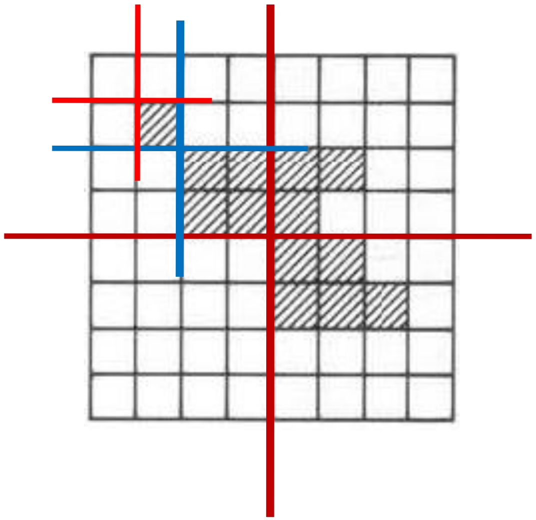
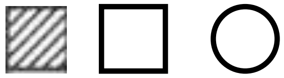


Primitives

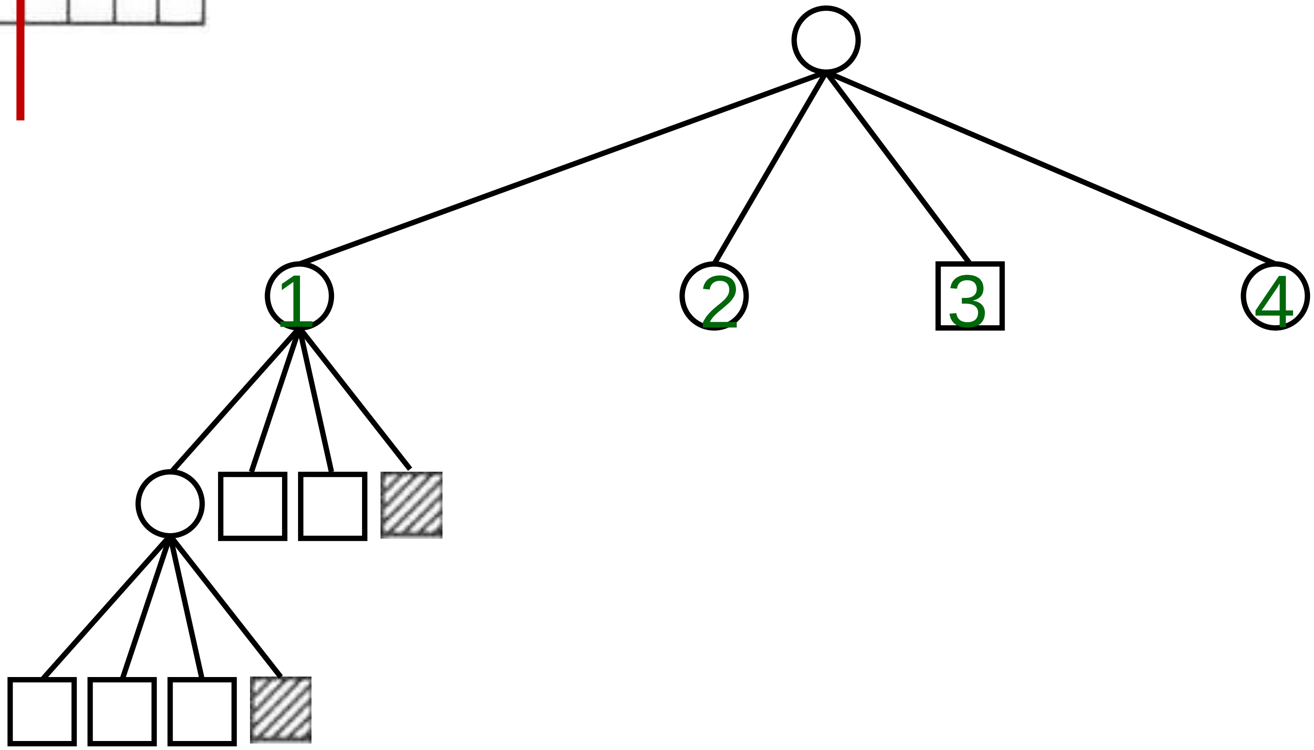


Representation using Tree (2)

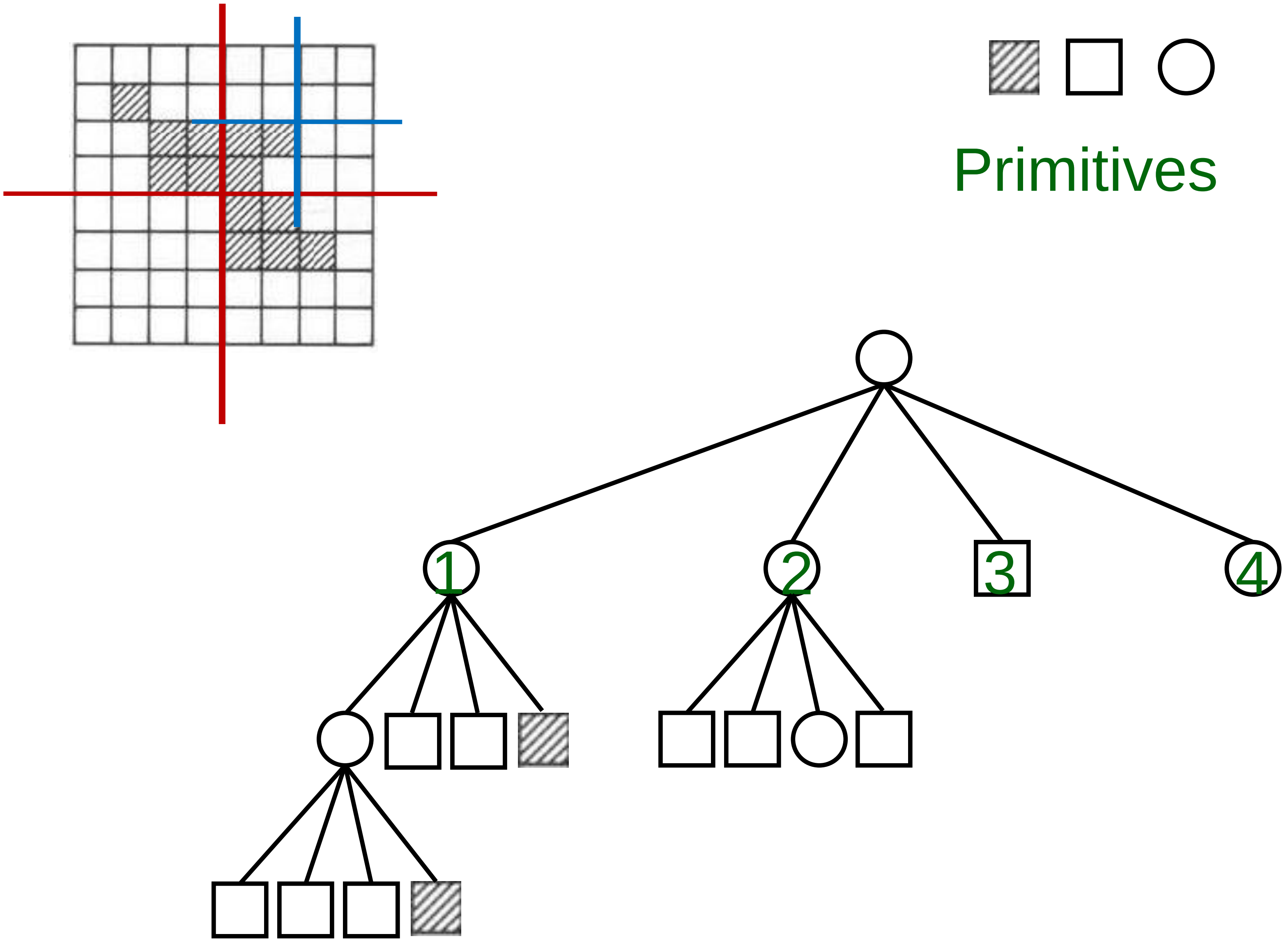




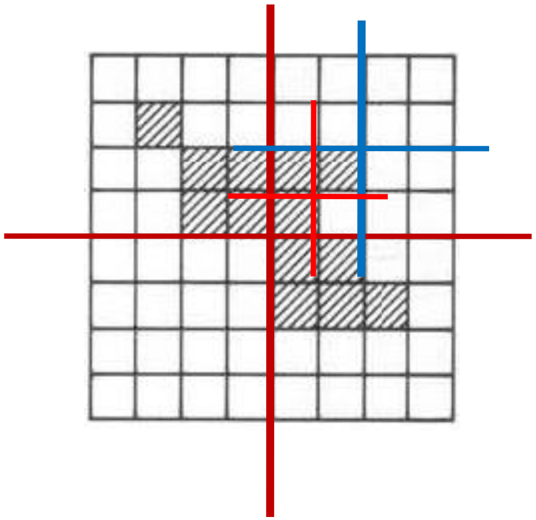
Primitives

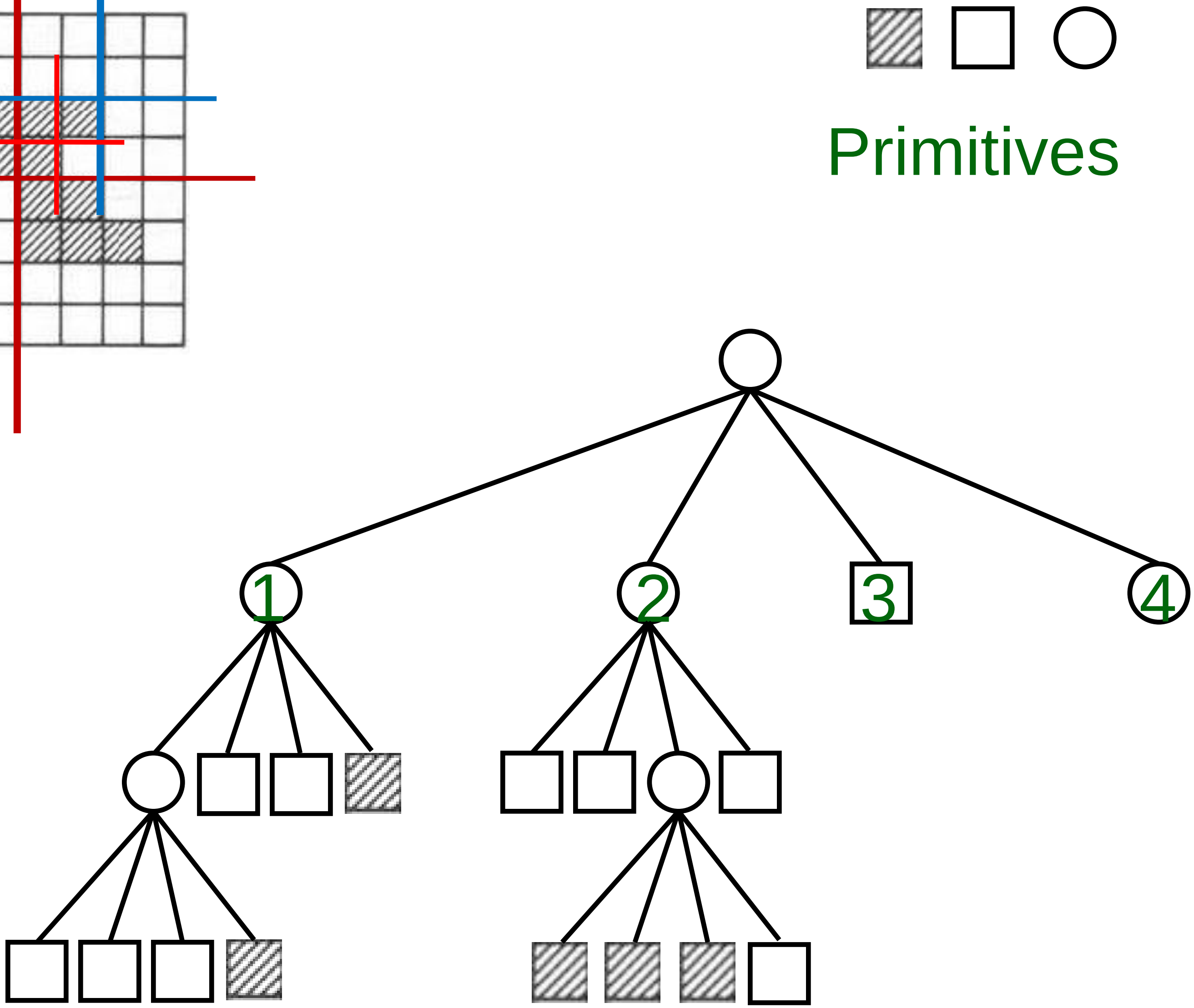


Representation using Tree (2)



Representation using Tree (2)





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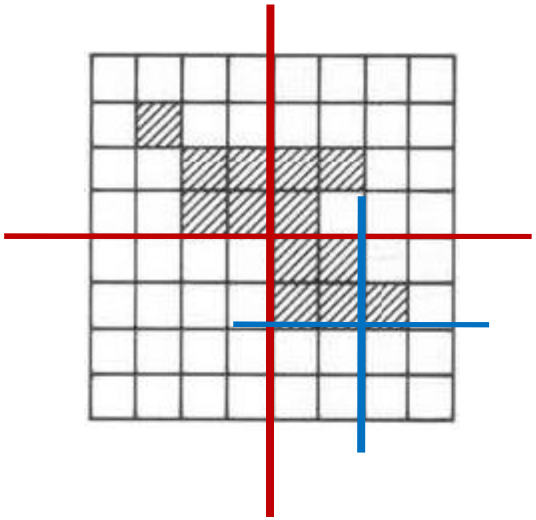

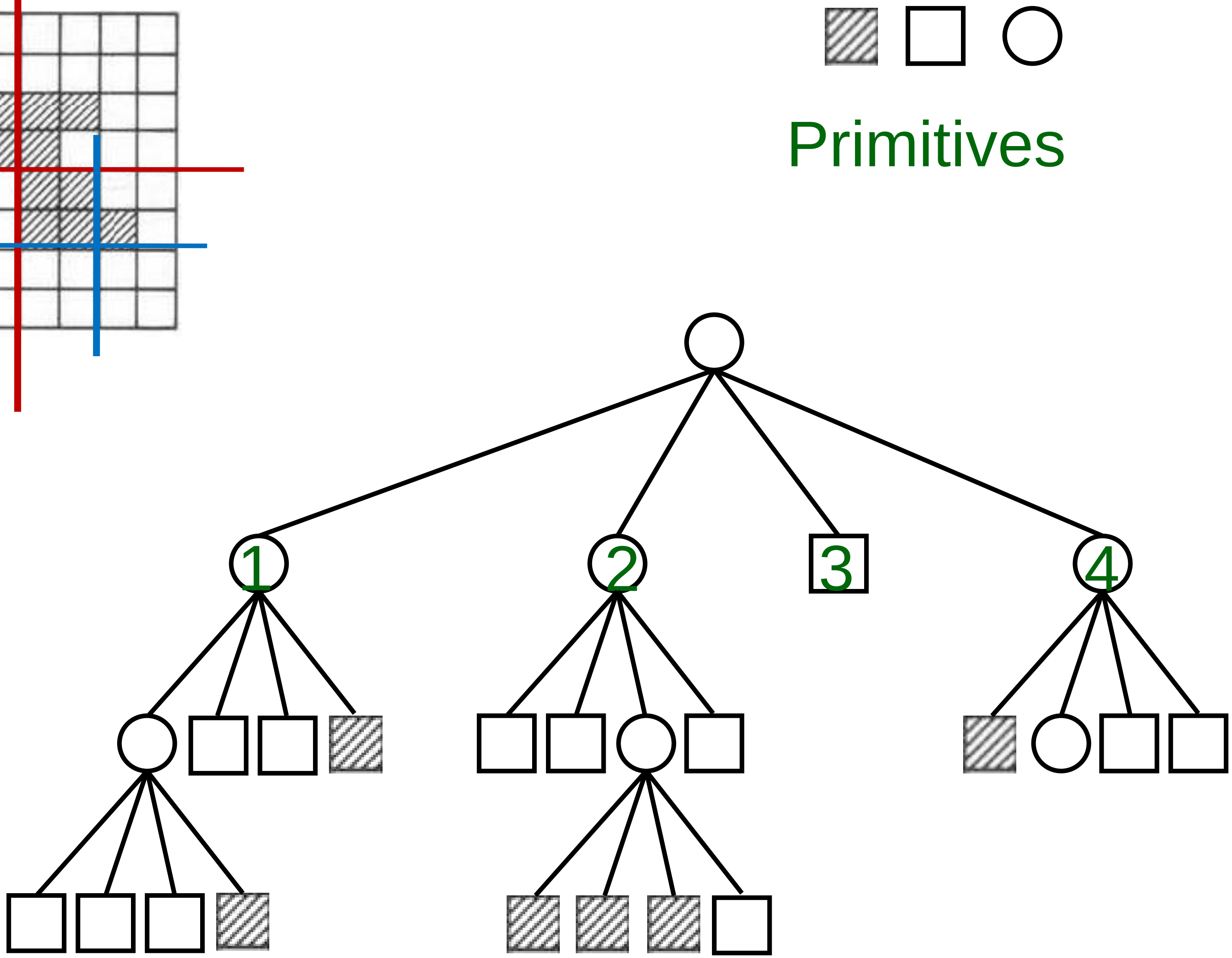

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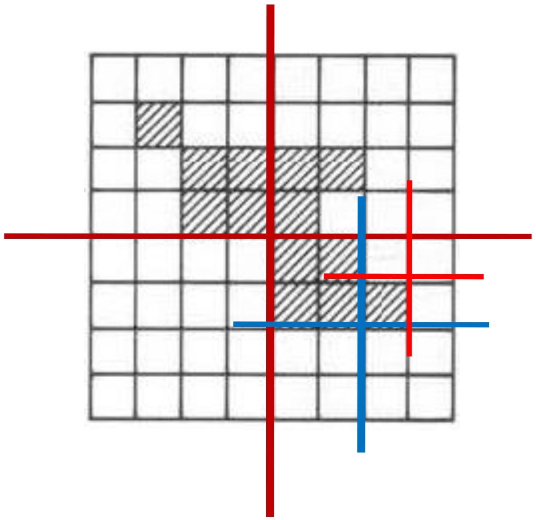

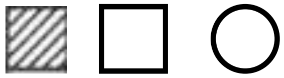

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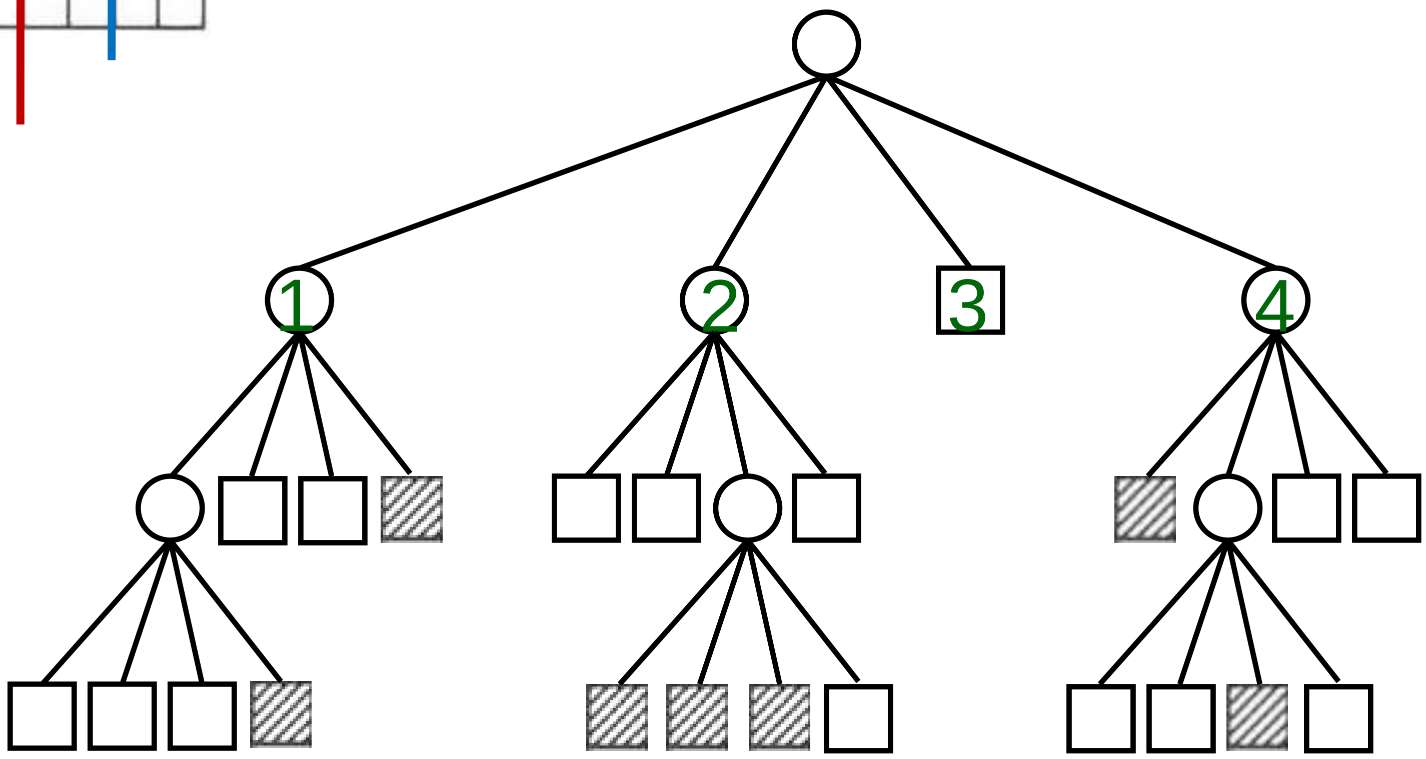

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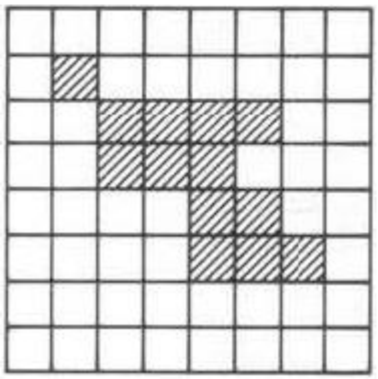

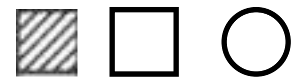

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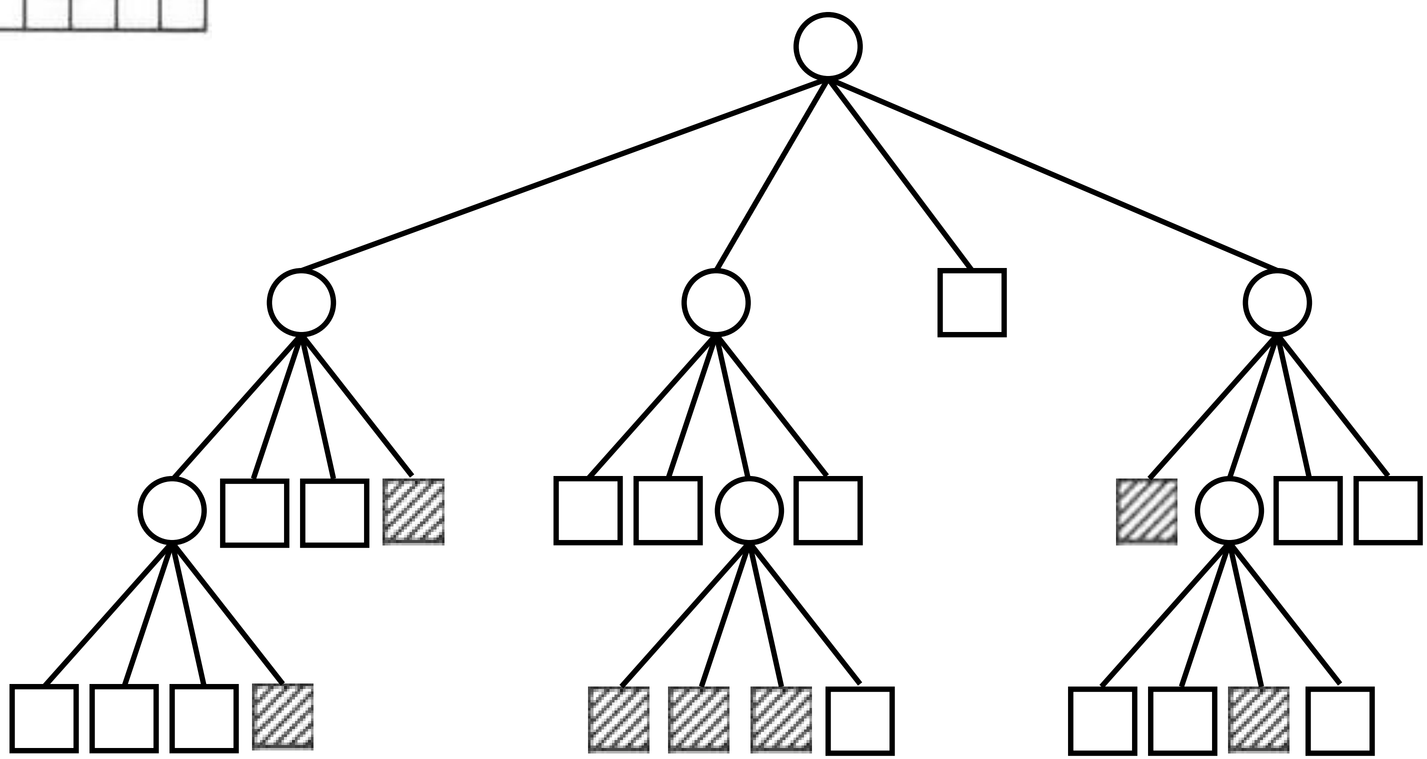
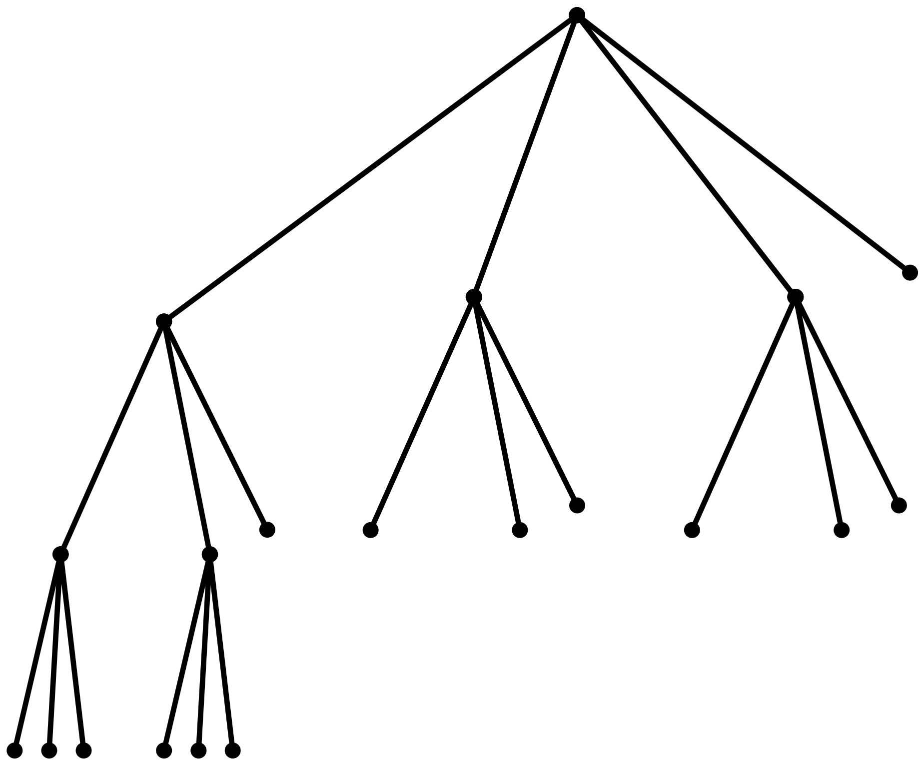


Tree Traversal and Representation

- Many methodologies
  - DFS, BFS, . . .



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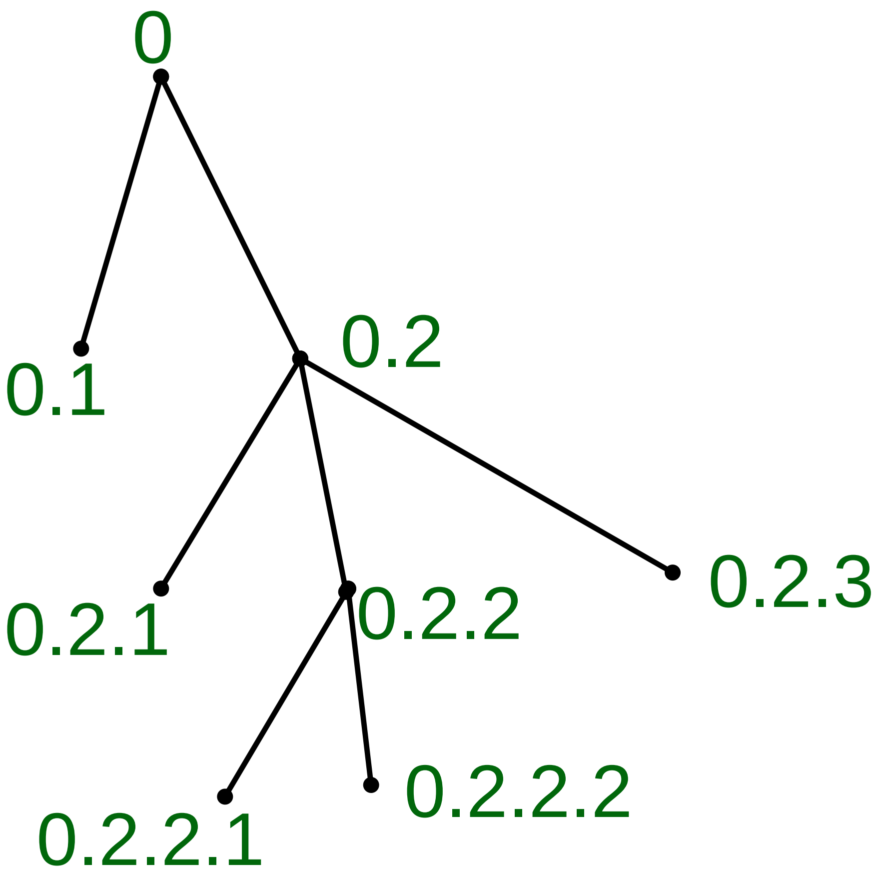
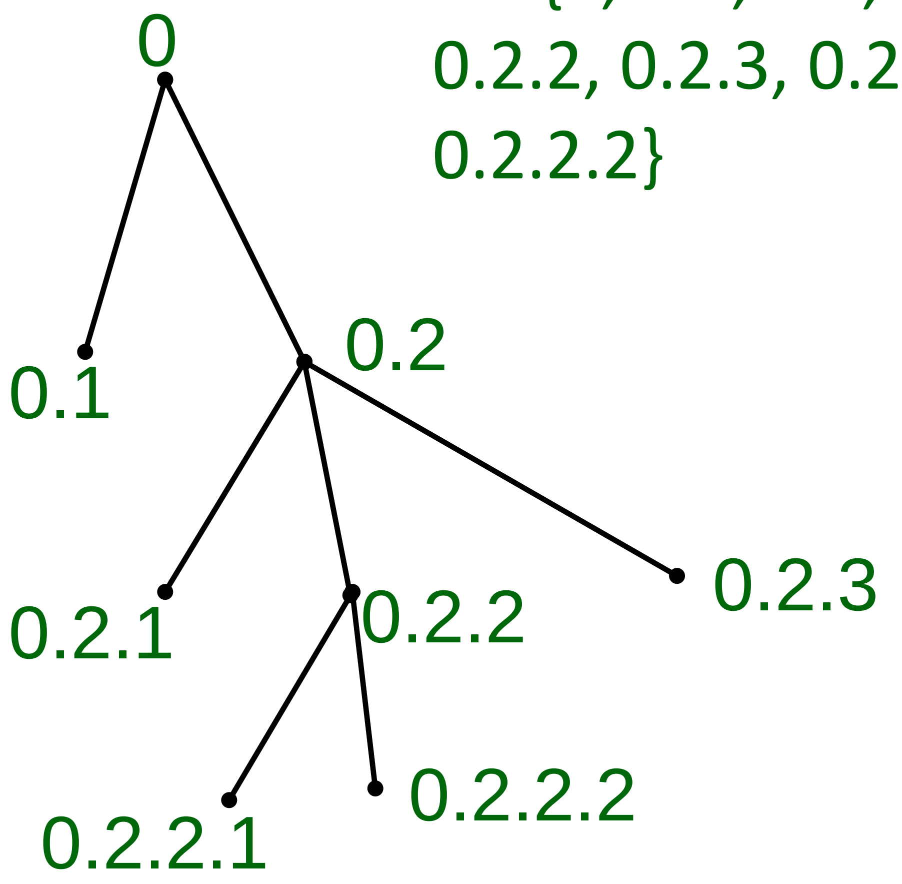


Tree Traversal and Representation

- Many methodologies
  - DFS, BFS, . . .

- T = {0, 0.1, 0.2, 0.2.1, 0.2.2, 0.2.3, 0.2.2.1, 0.2.2.2}



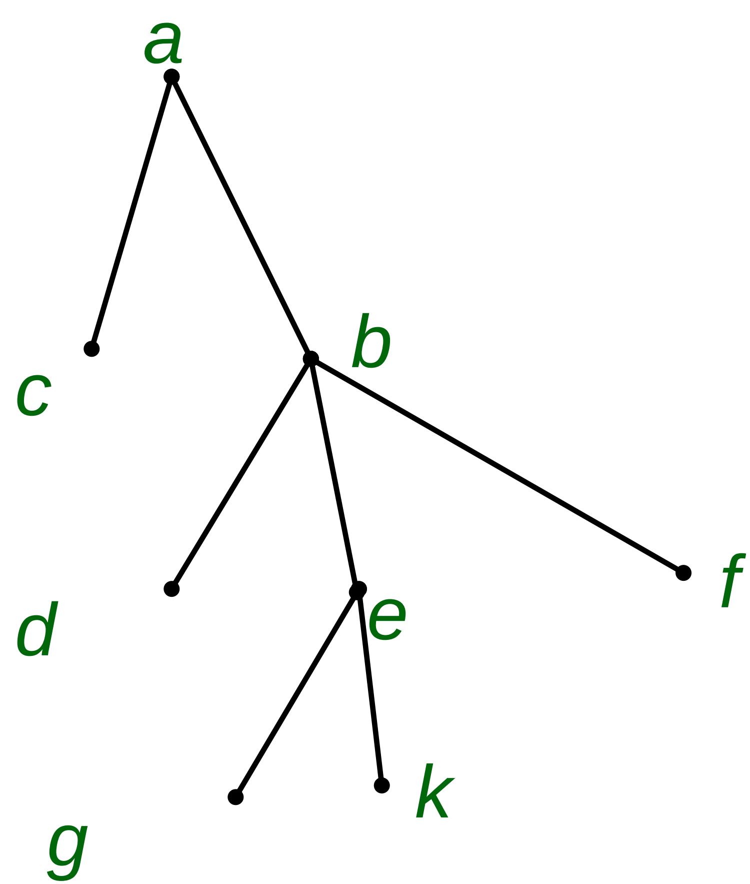
Tree Grammar

- A tree grammar G_T is defined as
  - $G_T = \{V, r, P, S\}$
  - where
    - P = set of productions involving trees
    - $V = V_T \cup V_N$
    - S= starting or root trees, T_V
    - *r = rank of a node*

Tree Grammar

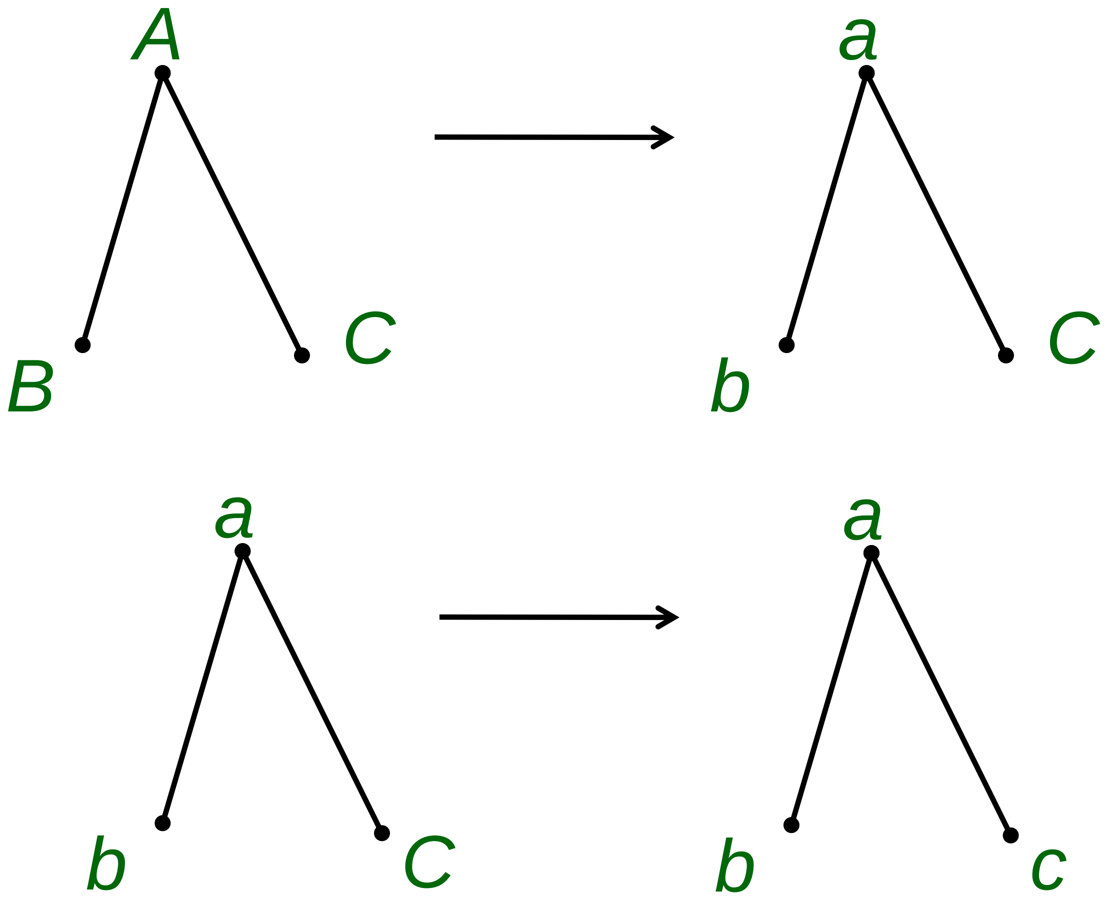
- Rank of a node
  - $r(a) = 2$
  - $r(b) = 3$

  - Therefore, this is out-degree of a node



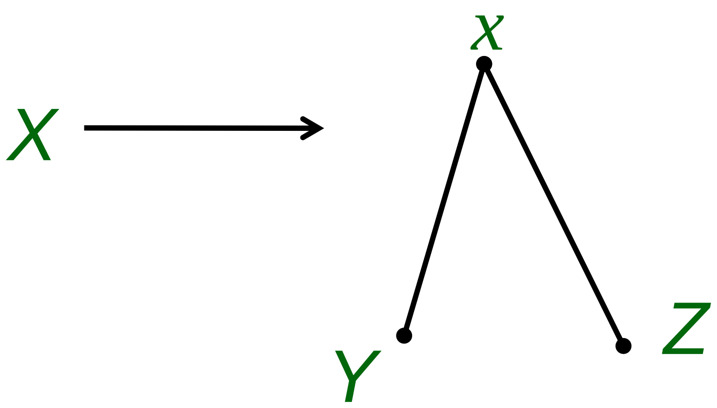
Productions of Tree Grammar

- Preserve the same structure but replace some terminal and/or non-terminals



Productions of Tree Grammar

- Expansive form:
  - $X \text{ -> } xX_1 . . . X_n$



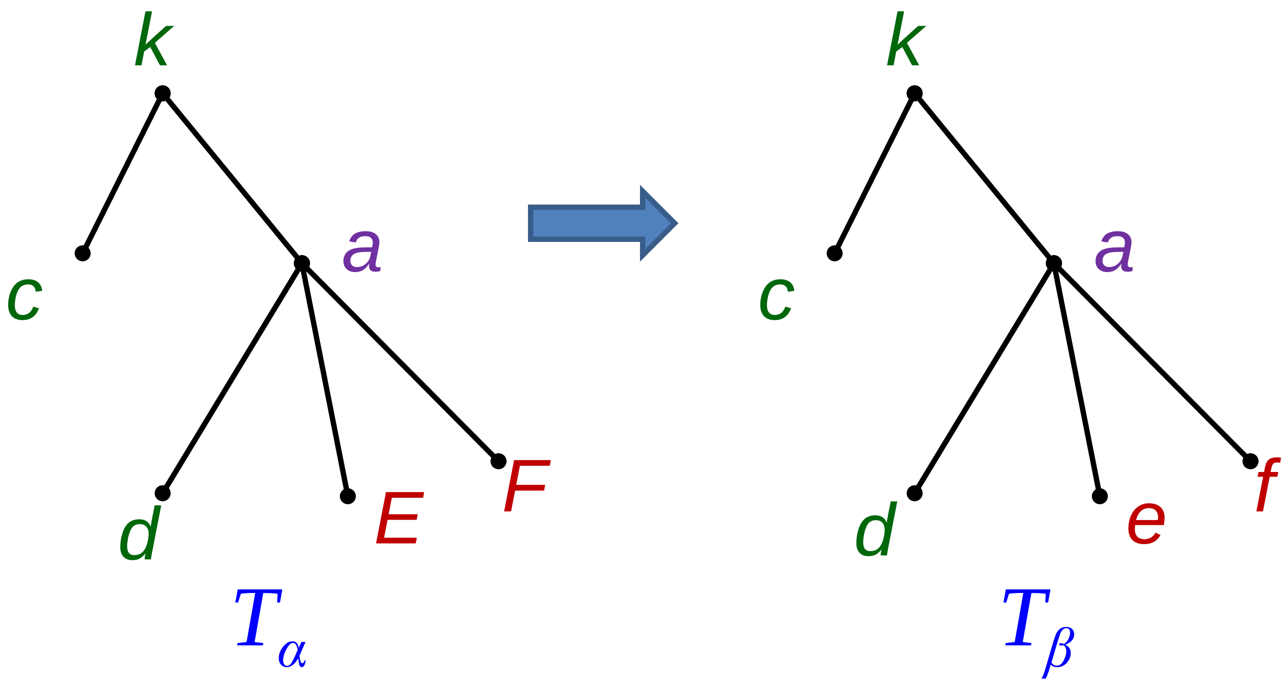
$r(x) = 2$

$A \longrightarrow b \qquad r(b) = 0$

$C \longrightarrow c \qquad r(c) = 0$

Derivation in a Tree Grammar

- A derivation $T_\alpha \overset{a}{\Rightarrow} T_\beta$ means:



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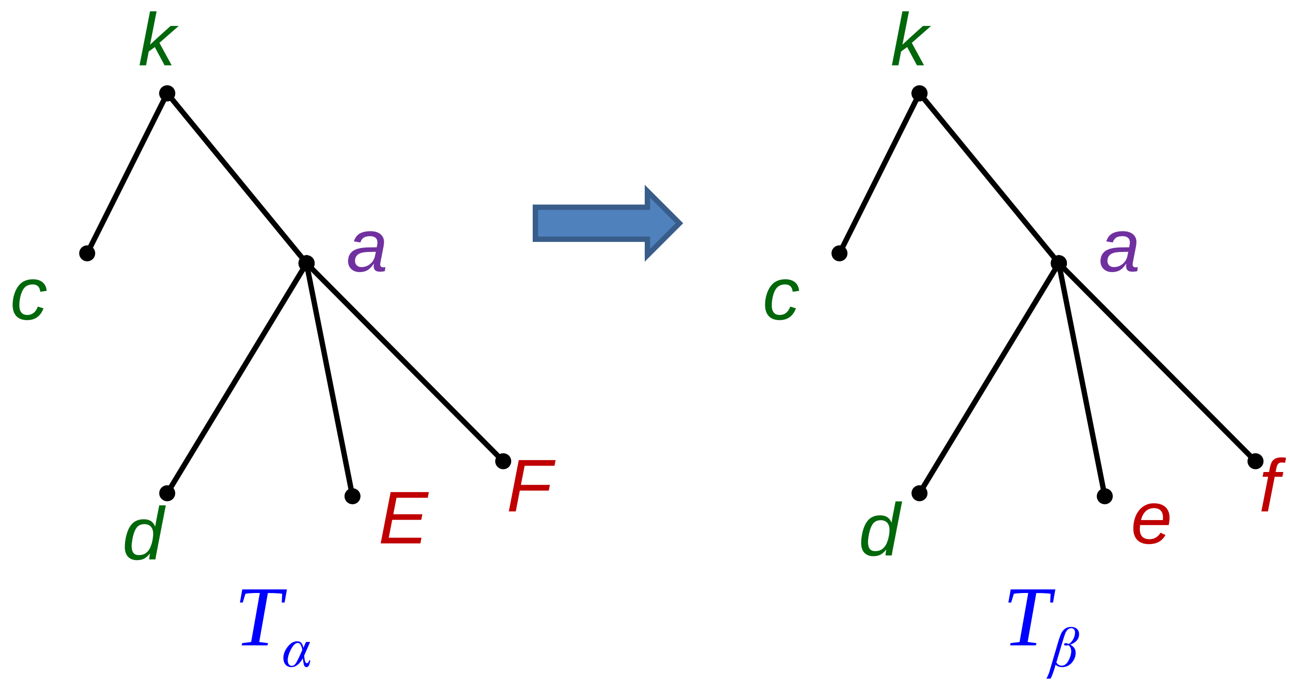
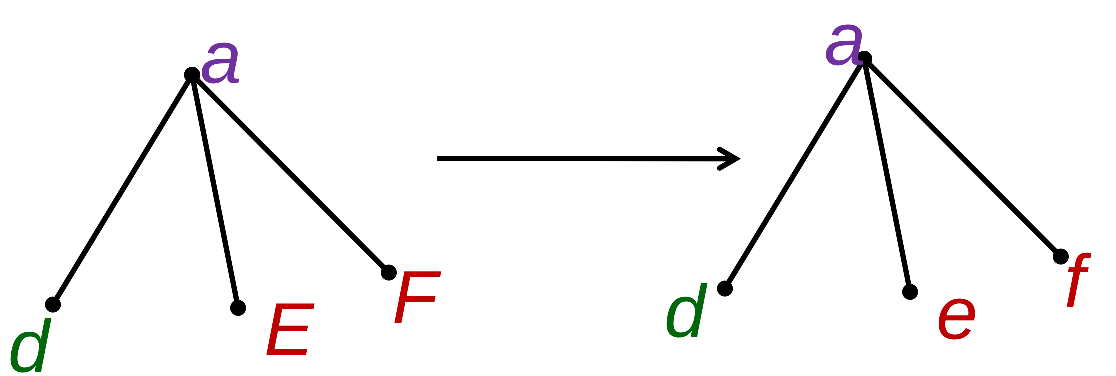


- The production rule applied:



Language of A Tree Grammar

$$L(T_G) = \{T \mid T \in T_{V_T} \cap T_i \Rightarrow T \qquad T_i \in S\}$$

Stochastic Grammar

- Formal grammars allow
  - no error
  - no ambiguity
  - no structural deformation

Stochastic Grammar

- Formal grammars allow
  - no error
  - no ambiguity
  - no structural deformation
- In formal grammar,

$$L(G_1) \cap L(G_2) = \phi$$

- Often this may be unrealistic

Stochastic Grammar

- Stochastic grammar is defined as

$$G_s = \{V_N, V_T, P_s, S_s\}$$

- The Production P_s is of the form

$$\alpha_i \xrightarrow{p_{ij}} \beta_j$$

Stochastic Grammar

- Stochastic grammar is defined as

$$G_s = \{V_N, V_T, P_s, S_s\}$$

- The Production P_s is of the form

$$\alpha_i \xrightarrow{p_{ij}} \beta_j$$

because, there are multiple probabilistic consequents

Proper Stochastic Grammar

$$\alpha_i \xrightarrow{p_{ij}} \beta_j$$

- A stochastic grammar is proper if

$$\alpha_i \in V_N$$

$$\beta_j \in (V_T \cup V_N)^+$$

- There exist n_i no. of β_j so that

$$\boxed{\sum_{j=1}^{n_i} p_{ij} = 1}$$

Characteristic Stochastic Grammar

- A stochastic grammar is a characteristic grammar
  - when probabilities are removed from the productions

that is, $\alpha_i \xrightarrow{p_{ij}} \beta_j$ replaces $\alpha_i \rightarrow \beta_j$

Classification of Stochastic Grammars

- A stochastic grammar is classified based on its corresponding characteristic grammar:
  - Type 0
  - Type 1
  - Type 2
  - Type 3

Derivation in Stochastic Grammar

- Let there are *m* production rules, p^i, 1 <= *i* <= *m*
- we need to derive:

$$S \Rightarrow x$$

in a number of *n* steps

$$S = \alpha_0 \Rightarrow \alpha_1 \Rightarrow \alpha_2 \cdots \Rightarrow \alpha_n = x$$

Derivation in Stochastic Grammar

- Let there are m production rules, p^i, 1 <= i <= m

- we need to derive:

$$S \Rightarrow x$$

in a number of n steps

$$S = \alpha_0 \stackrel{t_{0,1}}{\Rightarrow} \alpha_1 \stackrel{t_{1,2}}{\Rightarrow} \alpha_2 \cdots \stackrel{t_{n-1,n}}{\Rightarrow} \alpha_n = x$$

where, t_{ij} is a production rule applied in the step

$$\alpha_i \stackrel{t_{i,j}}{\Rightarrow} \alpha_j$$

Derivation in Stochastic Grammar

- The probability of generating

$$S = \alpha_0 \overset{t_{0,1}}{\Longrightarrow} \alpha_1 \overset{t_{1,2}}{\Longrightarrow} \alpha_2 \cdots \overset{t_{n-1,n}}{\Longrightarrow} \alpha_n = x$$

is

$$P(t_{0,1} \cap t_{1,2} \cap \cdots \cap t_{n-1,n})$$

Derivation in Stochastic Grammar

- The probability of generating

$$S = \alpha_0 \overset{t_{0,1}}{\Rightarrow} \alpha_1 \overset{t_{1,2}}{\Rightarrow} \alpha_2 \cdots \overset{t_{n-1,n}}{\Rightarrow} \alpha_n = x$$

is

$$P(t_{0,1} \cap t_{1,2} \cap \cdots \cap t_{n-1,n})$$
$$= P(t_{0,1}, t_{1,2}, \cdots, t_{n-2,n-1}, t_{n-1,n})$$
$$= P(t_{n-1,n}, t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

Derivation in Stochastic Grammar

$$P(t_{n-1,n}, t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n} \mid t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1}) \times P(t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

Derivation in Stochastic Grammar

$$P(t_{n-1,n}, t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n} \mid t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1}) \times P(t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n}) \times P(t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

- Assumes that the probability of applying the next production is independent of previously applied productions

Derivation in Stochastic Grammar

$$P(t_{n-1,n}, t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n} \mid t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1}) \times P(t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n}) \times P(t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n}) \times P(t_{n-2,n-1}) \times P(t_{n-3,n-2} \cdots, t_{1,2}, t_{0,1})$$

- Applying the same approach

Derivation in Stochastic Grammar

$$P(t_{n-1,n}, t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n} \mid t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1}) \times P(t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n}) \times P(t_{n-2,n-1}, \cdots, t_{1,2}, t_{0,1})$$

$$= P(t_{n-1,n}) \times P(t_{n-2,n-1}) \times P(t_{n-3,n-2} \cdots, t_{1,2}, t_{0,1})$$

$$= \prod_{q=1}^{n} P(t_{q-1,q})$$

Stochastic Language

- A stochastic language is of the form

$$L(G_s) = \{(x, p(x) \mid x \in V_T^+, S_s \overset{p_j}{\Rightarrow} x, j = 1,2,\cdots k, \text{and } p(x) = \sum_{j=1}^{k} p_j\}$$

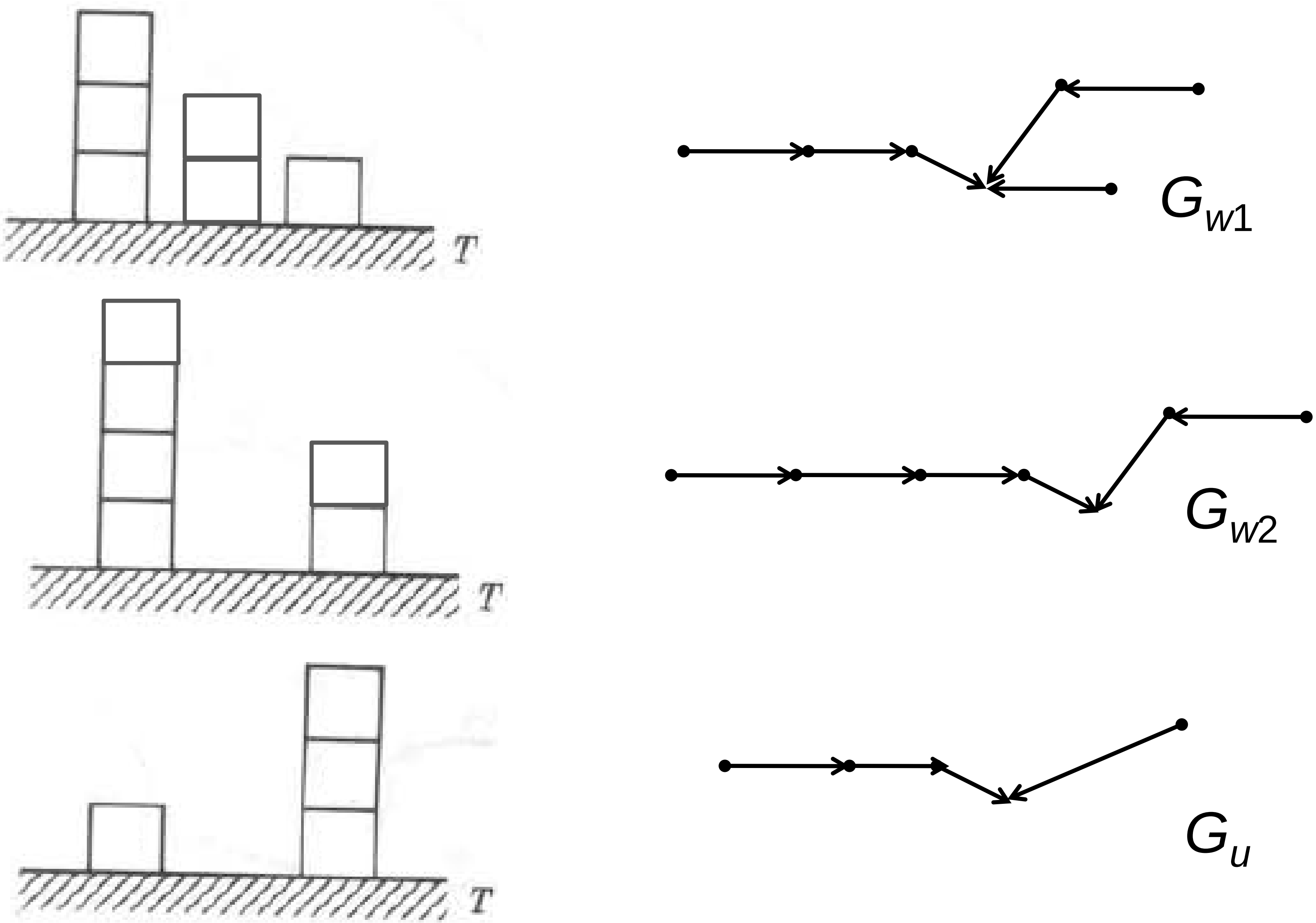
Syntactic Pattern Recognition using Graph Theory

Graphical Approaches to SyntPR

- Graphical alternatives to represent structures or relational information

- Natural extensions of higher dimensional grammars

Graphical Description of Patterns



Graph Representation

- A graph $G = \{N, R\}$ is an ordered pair represented using:
  - a set of nodes (vertices), N,
  - a set of edges (arcs), $R \subseteq N \times N$

Graph Representation

- A sub-graph $G_s = \{N_s, R_s\}$ is itself a graph, where
  - $N_s \subseteq N$
  - $R_s \subseteq R$, however, R_s consists of edgese that connects only the nodes in N_s.

Graph Representation

- A graph is *connected* if there is a path between all pairs of its nodes.
- A graph is *complete* if there is an edge between all pairs of its nodes.

Other definitions related to Graph

- In *directional graphs* (digraphs), edges have directional significance, i.e., $(a, b) \in R$ means there is an edge from node a to node b.
- When the direction of edges in a graph is not important, i.e., specification of either (a, b) or $(b, a) \in R$ is acceptable, the graph is an *undirected graph*.

Other definitions related to Graph

- A *relational graph* represents a particular relation graphically using arrows to show this relation between the elements as a directed graph.

Classification Task by Comparing Relational Graph Descriptions

Naïve approach:

- Represent each class as a *prototypical relational graph*
- Convert unknown pattern to *relational graph*
- Compare with the library of prototypes

Classification Task by Comparing Relational Graph Descriptions

- The observed data rarely matches a stored relational representation "exactly",

- We need other approaches
  - graph similarity should be measured.

Classification Task by Comparing Relational Graph Descriptions

- One approach is to check whether the observed data match a “portion” of a relational model.
  - Case 1: Any relation not present in both graphs is a failure.
  - Case 2: Any single match of a relation is a success.
  - A realistic strategy is somewhere in between these extremes.

Comparing Relational Graph Descriptions

- We define some terms for graph similarity
  - Use graph *homomorphism* and/or *isomorphism*

Graph Homomorphism

- Consider two graphs $G_1 = \{N_1, R_1\}$ and $G_2 = \{N_2, R_2\}$.
- A *homomorphism* from G_1 to G_2 is a function f from N_1 to N_2:

such that

$$(v_1, w_1) \in R_1 \Rightarrow (f(v_1), f(w_1)) \in R_2$$

Graph Isomorphism

- A stricter term

► A stricter test is that of *isomorphism*, where f is required to be 1:1 and onto:

$$(v_1, w_1) \in R_1 \Leftrightarrow (f(v_1), f(w_1)) \in R_2$$

► Isomorphism simply states that relabeling of nodes yields the same graph structure.

Graph Isomorphism Test: *Adjaceny Matrix*

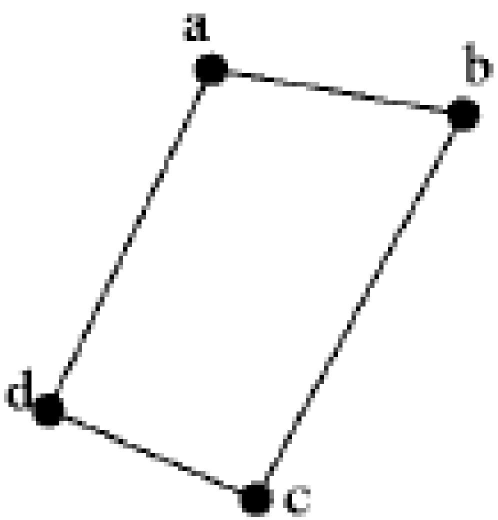
- A digraph G with p nodes can be converted to an *adjacency matrix*:
  - Number each node by an index $\{1, \ldots, p\}$.
  - Represent the existence or absence of an edge as

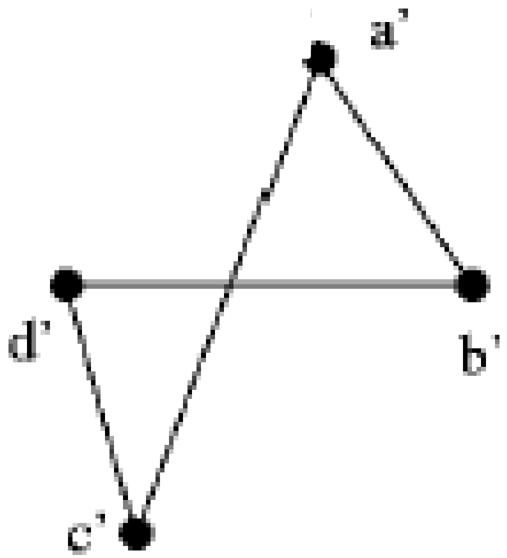
$$\text{Adj}(i, j) = \begin{cases} 1 & \text{if } G \text{ contains an edge from node } i \text{ to node } j, \\ 0 & \text{otherwise.} \end{cases}$$

Graph Isomorphism Test

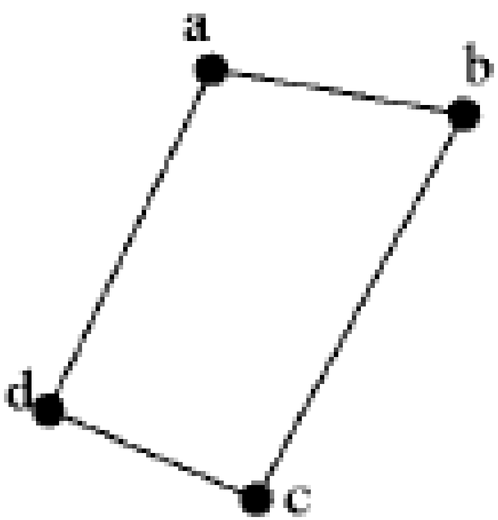
- Given two graphs G_1 and G_2 each with p nodes, to determine isomorphism:
  - Label the nodes of each graph with labels $1, \ldots, p$.
  - Form the adjacency matrices M_1 and M_2 for both graphs.
  - If $M_1 = M_2$, G_1 and G_2 are isomorphic.
  - Otherwise, consider all the $p!$ possible labelings on G_2.

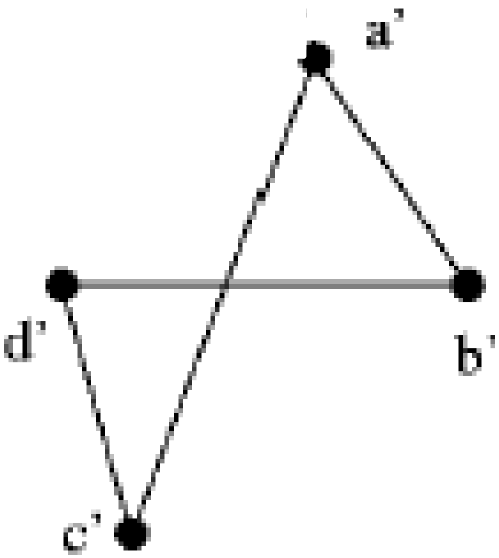
Graph Isomorphism Test





Graph Isomorphism Test

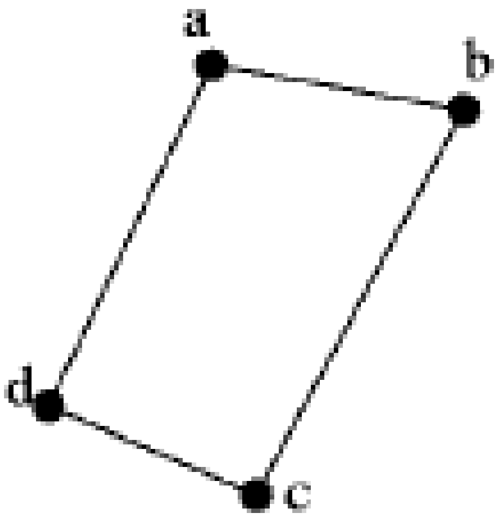




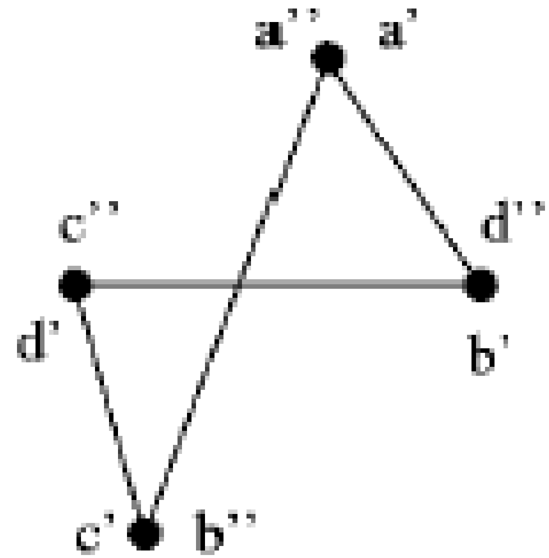
	a	b	c	d
a	0	1	0	1
b	1	0	1	0
c	0	1	0	1
d	1	0	1	0

	a'	b'	c'	d'
a'	0	1	1	0
b'	1	0	0	1
c'	1	0	0	1
d'	0	1	1	0

Graph Isomorphism Test



$$f(a') = a''$$
$$f(b') = d''$$
$$f(c') = b''$$
$$f(d') = c''$$



	a	b	c	d
a	0	1	0	1
b	1	0	1	0
c	0	1	0	1
d	1	0	1	0

	a"	b"	c"	d"
a"	0	1	0	1
b"	1	0	1	0
c"	0	1	0	1
d"	1	0	1	0

Graph Isomorphism Test

- Computationally expensive
- Allows only exact measure, not realistic
  - Many relations may not be observed in practical examples

Graph Isomorphism Test: Quick Reject

- Check for following properties
  - Number of nodes
  - Number of edges

Graph Isomorphism Test: Quick Reject

- These are useful, too:
  - in-degree *i* of a node
  - out-degree *j* of a node
  - degree *k* of a node (for undirected graph)
  - closed path of length *l*

Alternate Graph Isomorphism Test: Sub-isomorphism

- G_1 and G_2 are called *subisomorphic* if a subgraph of G_1 is isomorphic to a subgraph of G_2.

- Clearly, this is a less restrictive structural match than that of isomorphism.

Alternate Graph Isomorphism Test: Sub-isomorphism

- G_1 and G_2 are called *subisomorphic* if a subgraph of G_1 is isomorphic to a subgraph of G_2.
- Clearly, this is a less restrictive structural match than that of isomorphism.
- However, determining subisomorphism is also computationally expensive.

Extension to Graph Matching Approach

- To allow structural deformations, numerous extensions to graph matching have been proposed.
  - Extract features from graphs G_1 and G_2 to form feature vectors x_1 and x_2, respectively, and use statistical pattern recognition techniques to compare x_1 and x_2.
  - Use a matching metric as the minimum number of transformations necessary to transform G_1 into G_2.

Extension to Graph Matching Approach

- Common transformations include:
  - Node insertion,
  - Node deletion,
  - Node splitting,
  - Node merging,
  - Edge insertion,
  - Edge deletion.

Extension to Graph Matching Approach

- $\boxed{\text{Common transformations}}$ include:
  - Node insertion,
  - Node deletion,
  - Node splitting,
  - Node merging,
  - Edge insertion,
  - Edge deletion.

- Computation is still high
- Difficult to design a suitable distance measure to distinguish structural deformations

Extension to Graph Matching Approach

- An *attributed graph* $G = \{N, P, R\}$ is a 3-tuple where
  - N is a set of nodes,
  - P is a set of properties of these nodes,
  - R is a set of relations between nodes.

Matching Through Attributed Graph

- Let $p_q^i(n)$ denote the value of the q'th property of node n of graph G_i.

- Nodes $n_1 \in N_1$ and $n_2 \in N_2$ are said to form an assignment (n_1, n_2) if

$$p_q^1(n_1) \sim p_q^2(n_2)$$

where "$\sim$" denotes similarity.

Matching Through Attributed Graph

- Let $r^i_j(n_x, n_y)$ denote the j'th relation involving nodes $n_x, n_y \in N_i$.

- Two assignments (n_1, n_2) and (n'_1, n'_2) are considered *compatible* if

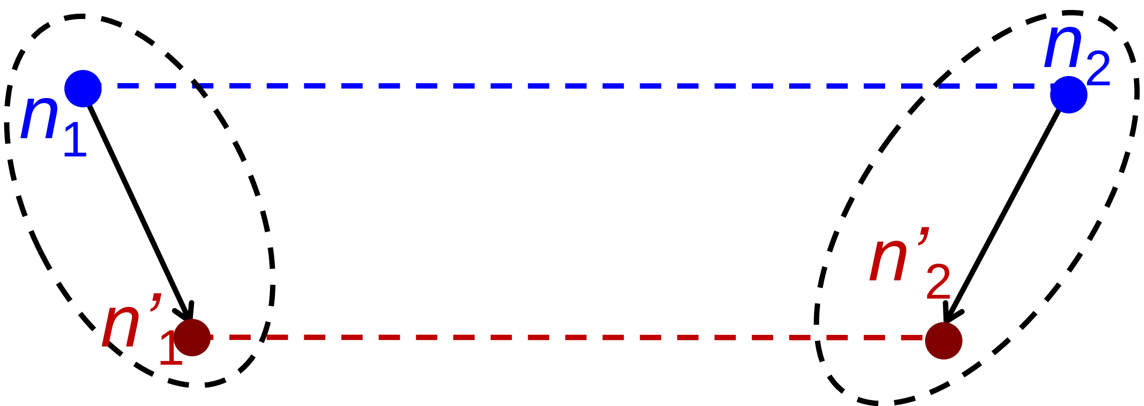
$$r^1_j(n_1, n'_1) \sim r^2_j(n_2, n'_2) \quad \forall j.$$

Matching Through Attributed Graph

- Let $r^i_j(n_x, n_y)$ denote the j'th relation involving nodes $n_x, n_y \in N_i$.

- Two assignments (n_1, n_2) and (n'_1, n'_2) are considered *compatible* if

$$r^1_j(n_1, n'_1) \sim r^2_j(n_2, n'_2) \quad \forall j.$$



Matching Through Attributed Graph

- Let $r_j^i(n_x, n_y)$ denote the j'th relation involving nodes $n_x, n_y \in N_i$.

- Two assignments (n_1, n_2) and (n_1', n_2') are considered *compatible* if

$$r_j^1(n_1, n_1') \sim r_j^2(n_2, n_2') \quad \forall j.$$

- Two attributed graphs G_1 and G_2 are isomorphic if there exists a set of 1:1 assignments of nodes in G_1 to nodes in G_2 such that all assignments are compatible.

Matching Through Attributed Graph

- A strategy for measuring the similarity between two attributed graphs is to find node pairings using the cliques of a match graph.

G_1 G_2

↓

Match Graph

↓

Maximal Clique

↓

Calculate Measure of Graph Similarity

Matching Through Attributed Graph

- A *match graph* is formed from two graphs G_1 and G_2 as follows:
  - Nodes of the match graph are assignments from G_1 to G_2.
  - An edge in the match graph exists between two nodes if the corresponding assignments are compatible.

- A *clique* of a graph is a totally connected subgraph.
- A *maximal clique* is not included in any other clique.

Approaches in Matching Through Attributed Graph

- Find
  - the *attributed graphs* from the patterns
  - the *match graphs* from the attributed graph
  - the *maximum clique* from the match graph
  - the *similarity*

Recursive Procedure to Find Cliques

Input:

- *X*: an initial clique (possibly empty)
- *Y*: the graph

Output:

- The set of all maximal cliques

Recursive Procedure to Find Cliques

Procedure *clique* (X, Y)

  Form $Y - X$;

  If a node y in $Y - X$ is connected to all nodes of X,

  Then return *cliques* ($X \cup \{y\}, Y$) U *cliques* ($X, Y - \{y\}$)

  Else return X

End

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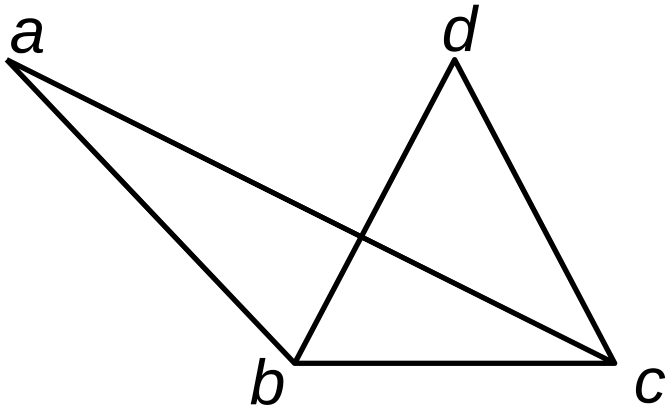

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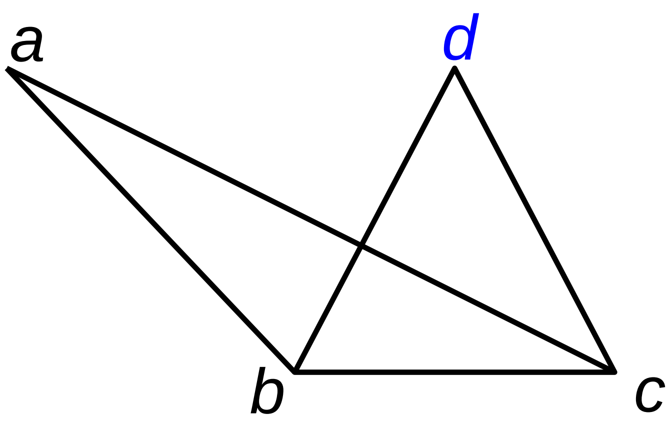
Else return X

End

Find *clique* (d, Y)?



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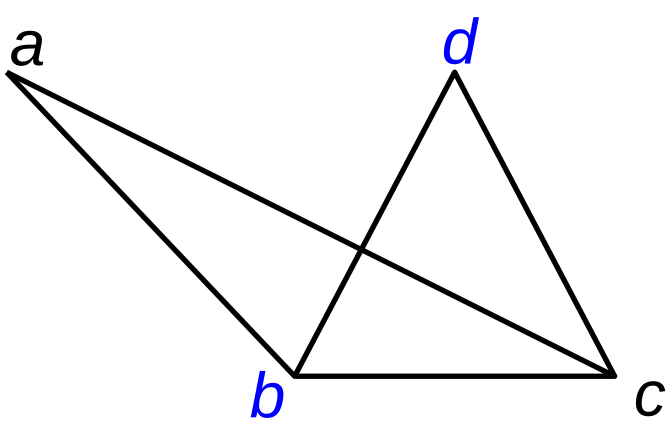
End

clique (d, Y) = *clique* ($\{d, b\}$, Y) U *clique* (d, $\{a, c, d\}$)



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End

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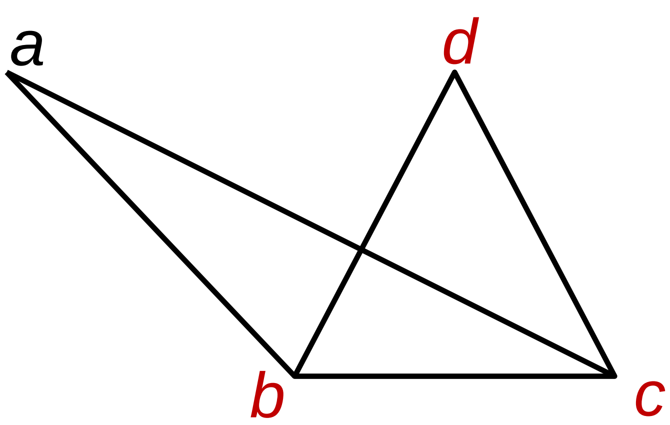
clique ($\{d, b\}$, Y) = *clique* ($\{d, b, c\}$, Y) U *clique* ($\{d, b\}$, $\{a, b, d\}$)



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Procedure *clique* (*X*, *Y*)

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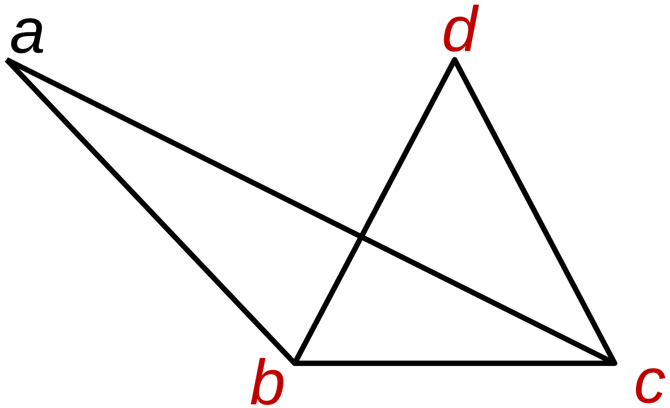
Else return *X*

End



clique (*d*, *Y*) = *clique* ({*d*, *b*}, *Y*) U *clique* (*d*, {*a*, *c*, *d*})

clique ({*d*, *b*}, *Y*) = *clique* ({*d*, *b*, *c*}, *Y*) U *clique* ({*d*, *b*}, {*a*, *b*, *d*})

⇓

{*d*, *b*}

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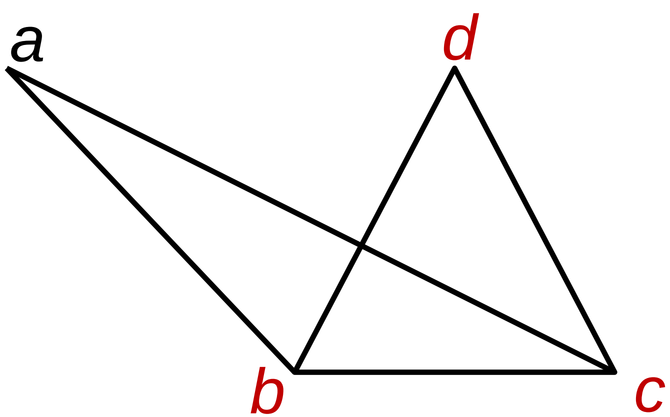

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clique ({*d*, *b*}, *Y*) = *clique* ({*d*, *b*, *c*}, *Y*) U {*d*, *b*}

{*d*, *b*, *c*}

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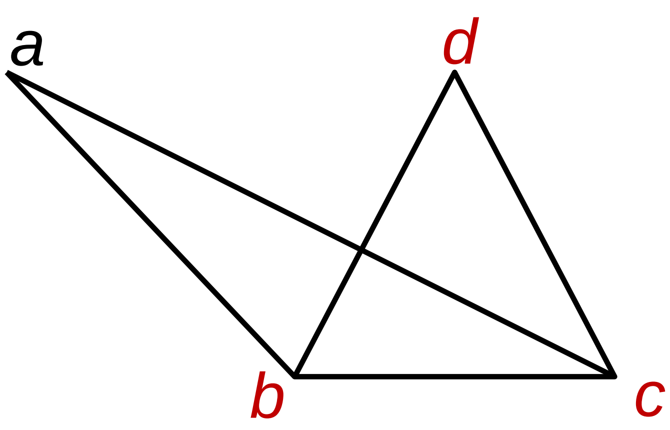
clique ({*d*, *b*}, *Y*) = {*d*, *b*, *c*} U {*d*, *b*}



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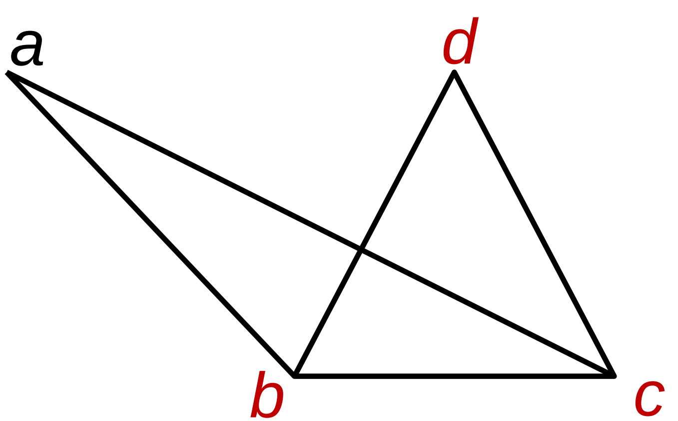
End



clique (d, Y) = *clique* ($\{d, b\}$, Y) U *clique* (d, $\{a, c, d\}$)

clique ($\{d, b\}$, Y) = $\{d, b, c\}$ U $\{d, b\}$

= $\{d, b, c\}$

Recursive Procedure to Find Cliques

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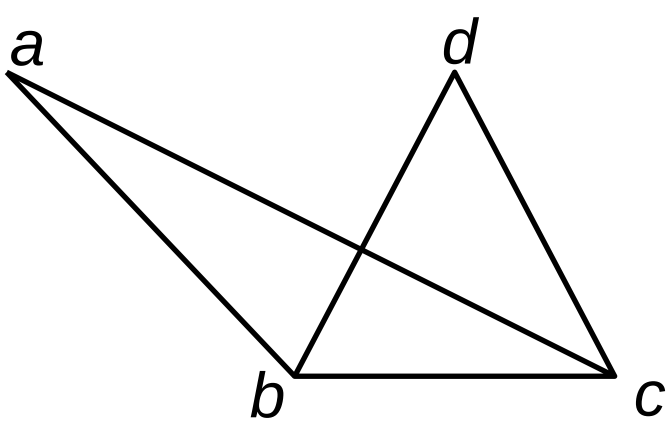

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Else return X

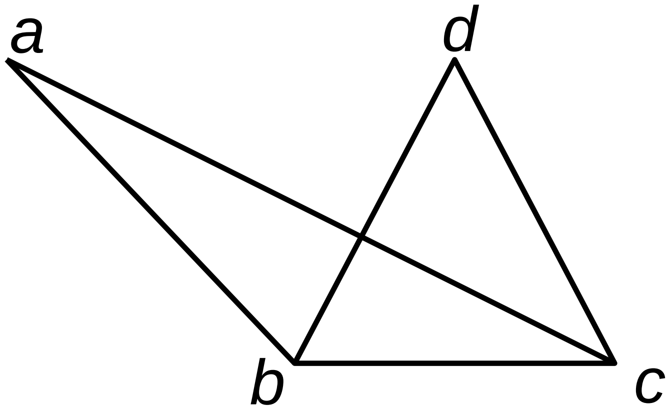
End

clique (d, Y) = $\{d, b, c\} \cup \{d, b\} \cup$ *clique* (d, $\{a, c, d\}$)

clique (d, $\{a, c, d\}$)

= *clique* ($\{d, c\}$, $\{a, c, d\}$)

$\cup$ *clique* (d, $\{a, d\}$)



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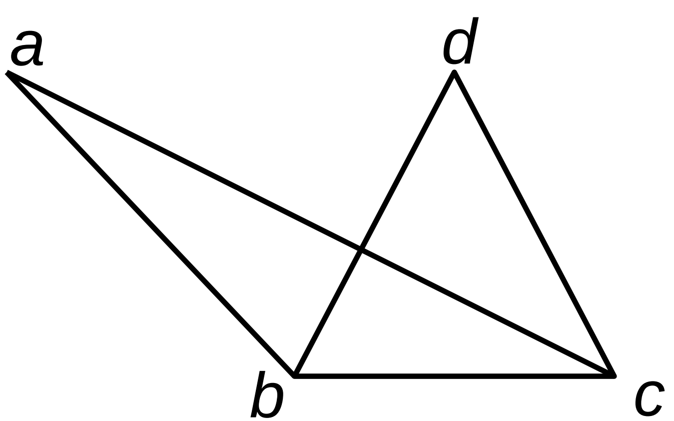
End



clique (d, Y) = $\{d, b, c\} \cup \{d, b\} \cup$ *clique* (d, $\{a, c, d\}$)

clique (d, $\{a, c, d\}$)

= *clique* ($\{d, c\}$, $\{a, c, d\}$)

$\cup$ *clique* (d, $\{a, d\}$)

$=\{d, c\} \cup \{d\}$

Recursive Procedure to Find Cliques

Procedure *clique* (X, Y)

  Form $Y - X$;

  If a node y in $Y - X$ is connected to all nodes of X,

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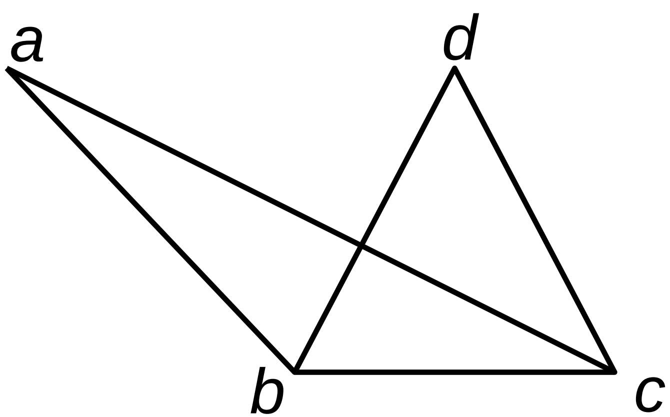
  Else return X

End

clique (d, Y) = $\{d, b, c\} \cup \{d, b\}$
$\cup \{d, c\} \cup \{d\}$



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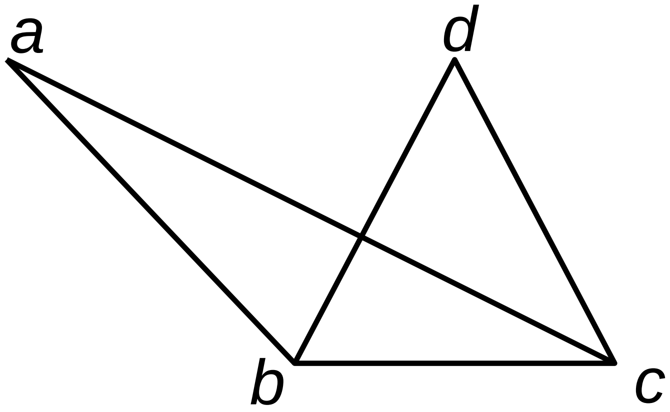
Else return *X*

End

The maximal clique is = {*d*, *b*, *c*}



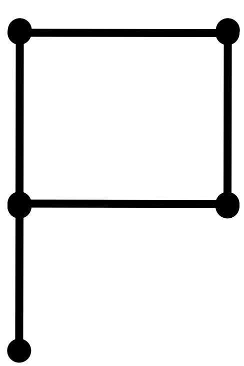
Example: Pattern Matching Using Attributed Graph

P T

Example: Pattern Matching Using Attributed Graph

P

Reference



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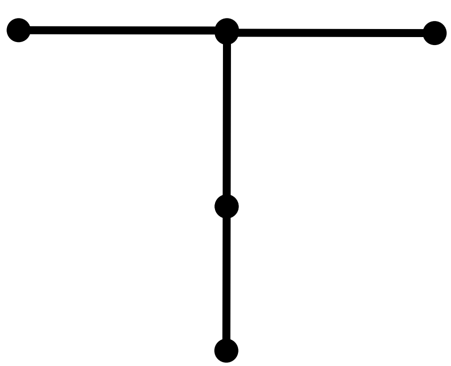
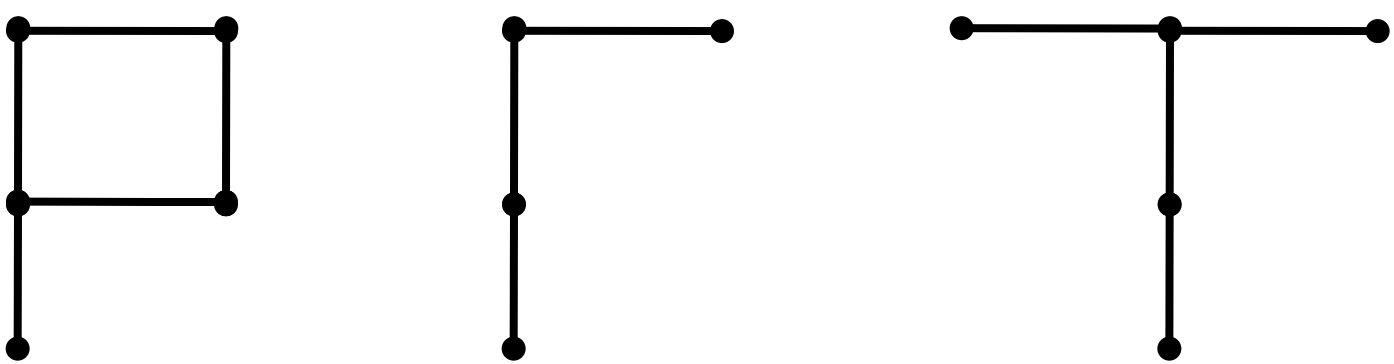


Example: Pattern Matching Using Attributed Graph

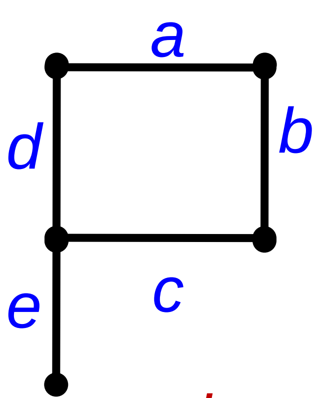
P T

Reference Test Reference

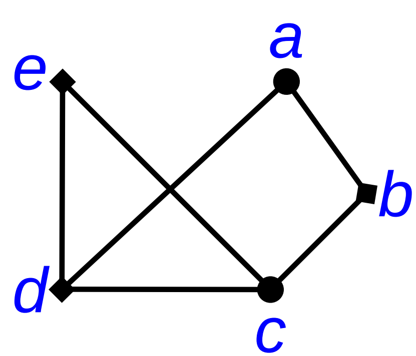


Example: Pattern Matching Using Attributed Graph

Find Attributed Graph for all patterns



I_1



AG_{I_1}

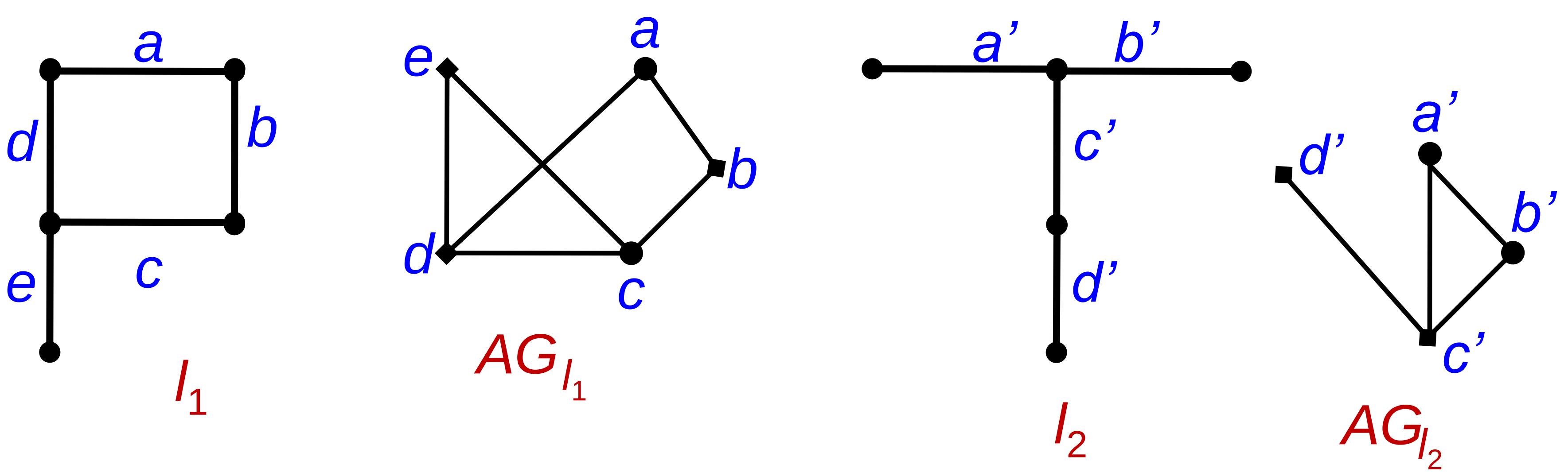
● Horizontal line

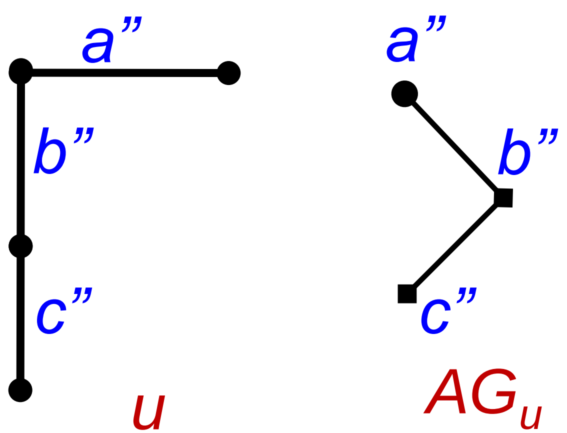
◊ Vertical line

Relation: —— connected

Example: Pattern Matching Using Attributed Graph

Find Attributed Graph for all patterns





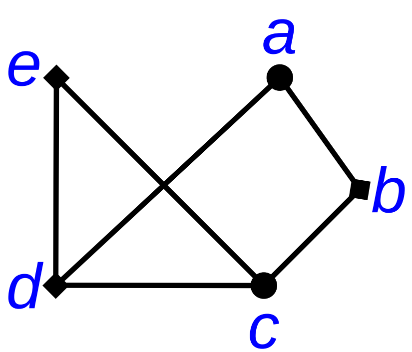
● Horizontal line

◊ Vertical line

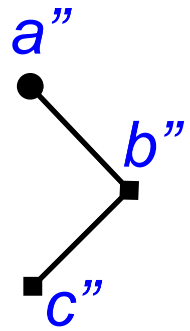
Relation: —— connected

Example: Pattern Matching Using Attributed Graph

Find Matched Graph between AGl_1 and AG_u



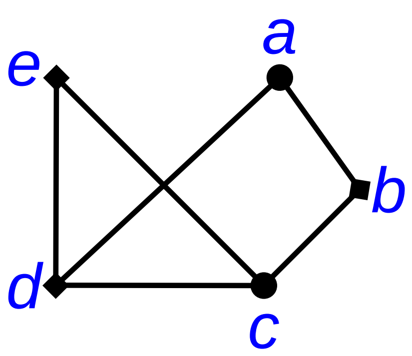
AG_{l_1}



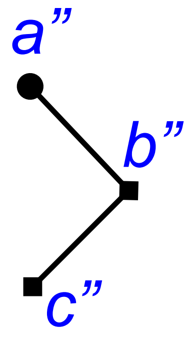
AG_u

Example: Pattern Matching Using Attributed Graph

Find Matched Graph between AGl_1 and AG_u



AG_{l_1}



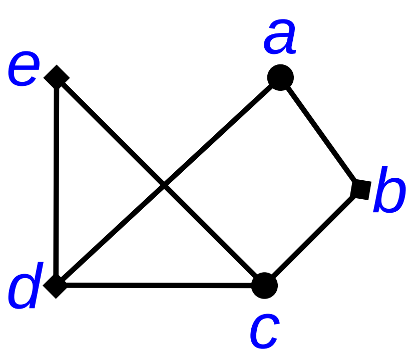
AG_u

Find all assignments between AGl_1 and AG_u:

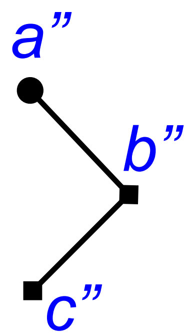
(a, a”), (b, b”), (b, c”), (c, a”), (d, b”), (d, c”), (e, b”), (e, c”)

Example: Pattern Matching Using Attributed Graph

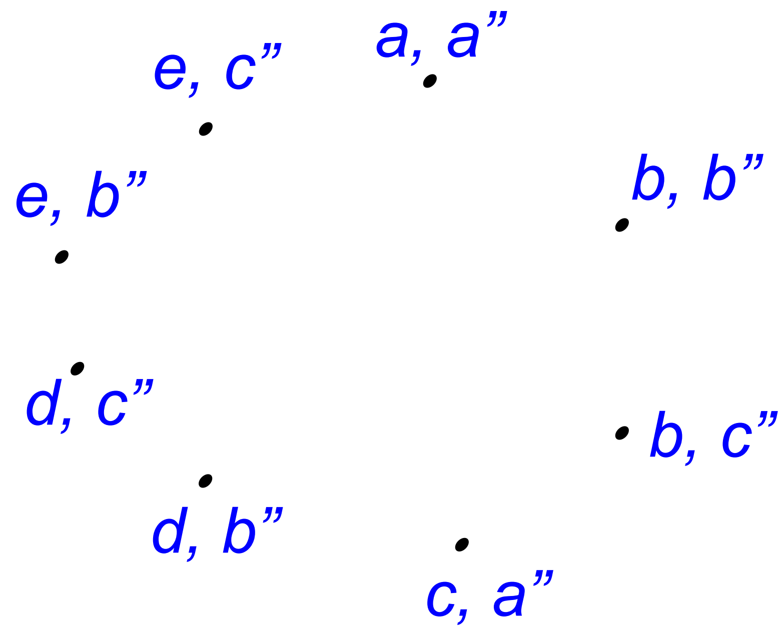
Find Matched Graph between AGl_1 and AG_u



AG_{l_1}



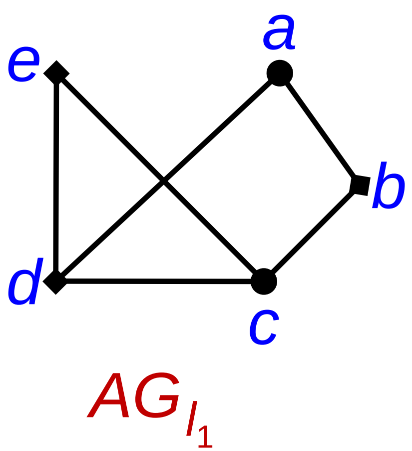
AG_u

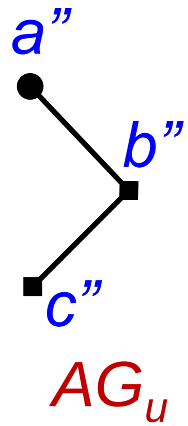


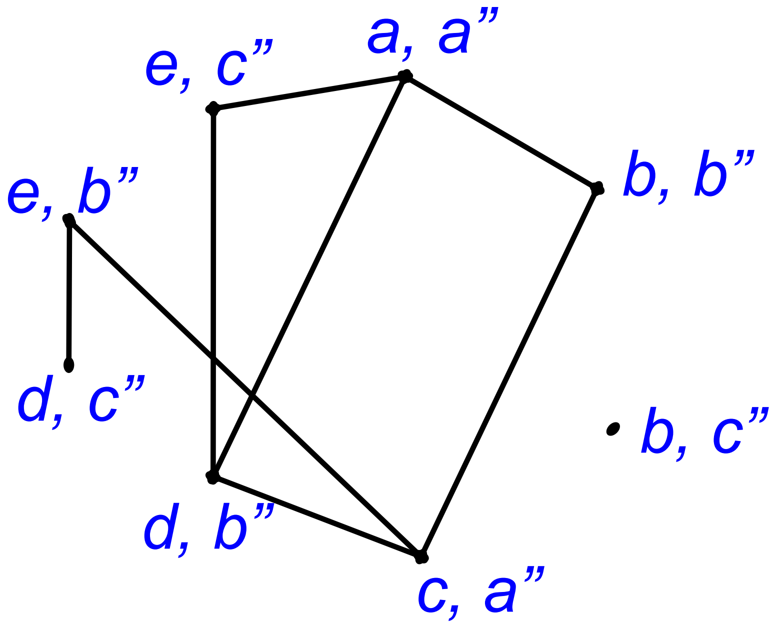
Find all assignments between AGl_1 and AG_u

Example: Pattern Matching Using Attributed Graph

Find Matched Graph between AGl_1 and AG_u



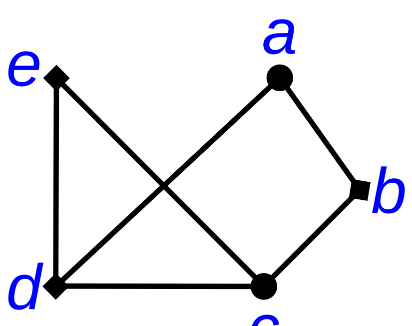




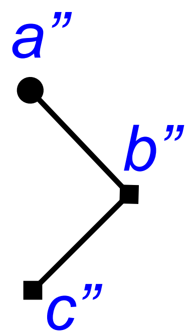
Connect the compatible assignments

Example: Pattern Matching Using Attributed Graph

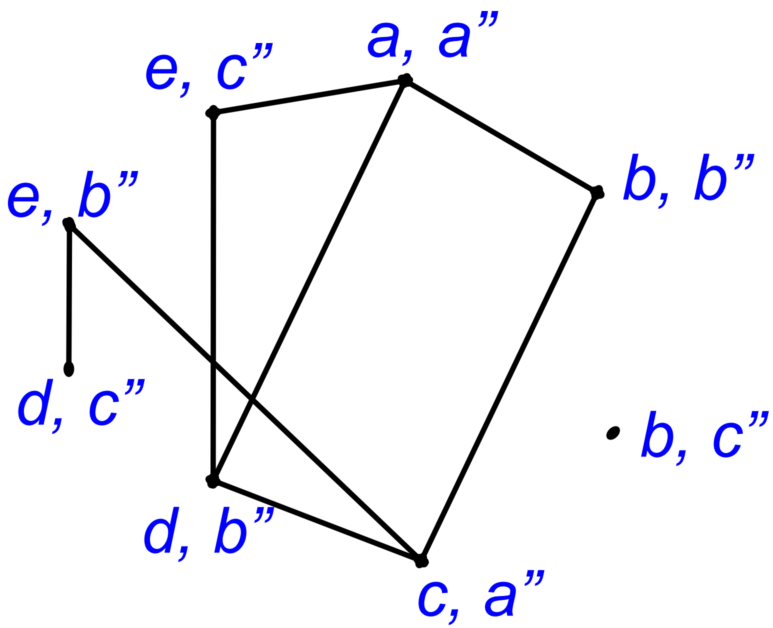
Find Matched Graph between AGl_1 and AG_u

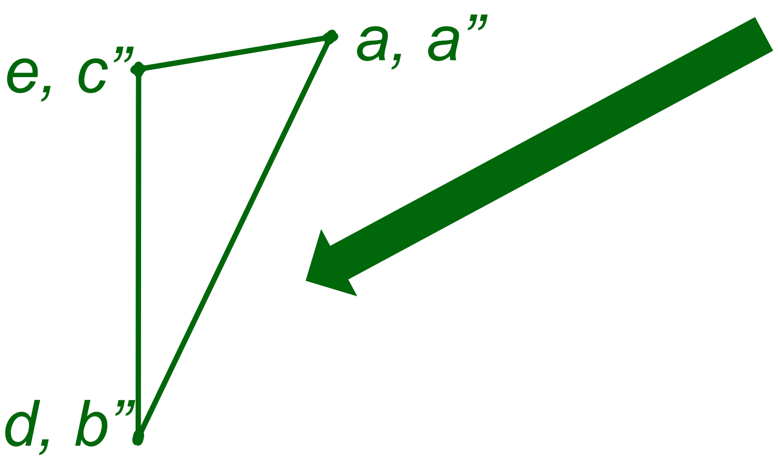


AG_{l_1}



AG_u

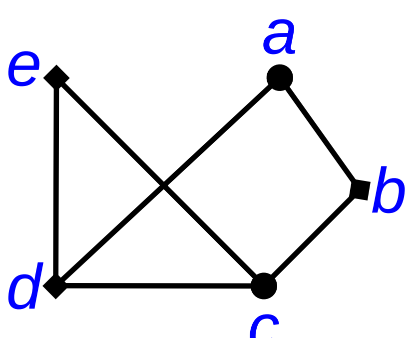




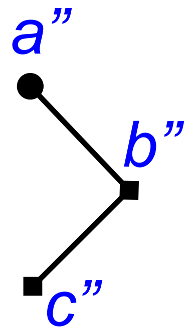
Find the maximal clique:
{(*a*, *a*”),(*d*, *b*”), (*e*, *c*”)}

Example: Pattern Matching Using Attributed Graph

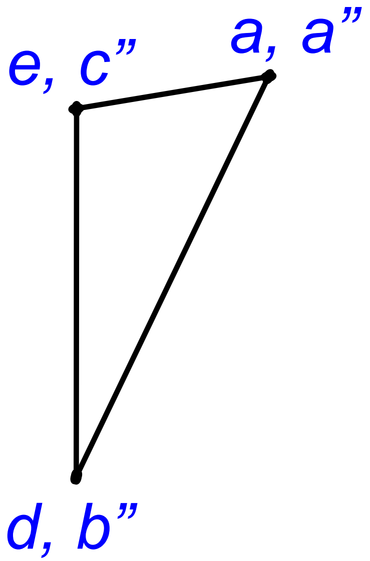
Find Matched Graph between AGl_1 and AG_u

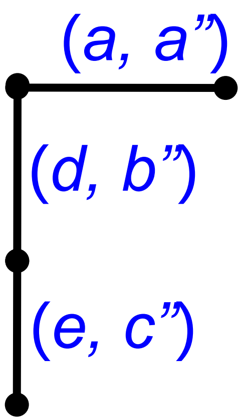


AG_{l_1}



AG_u

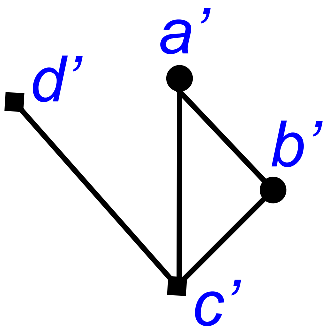




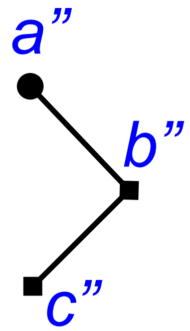
Visual Representation

Example: Pattern Matching Using Attributed Graph

Find Matched Graph between AGl_1 and AG_u



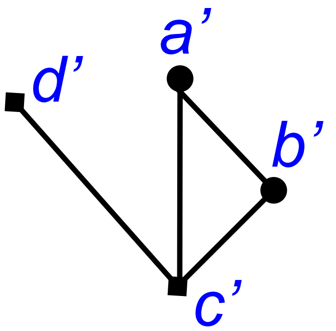
AG_{l_2}



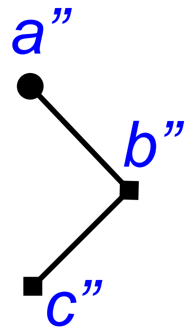
AG_u

Example: Pattern Matching Using Attributed Graph

Find Matched Graph between AGl_1 and AG_u



AG_{l_2}



AG_u

Find all assignments between AGl_2 and AG_u:

(*a*', *a*"), (*b*', *a*"), (*c*', *b*"), (*c*', *c*"), (*d*', *b*"), (*d*', *c*")