

Lecture 15: Matrix Factorization

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Factorization

- Latent factor e.g. genre *thriller*, *fantasy*

		Item			
		W	X	Y	Z
User	A		4.5	2.0	
	B	4.0		3.5	
	C		5.0		2.0
	D		3.5	4.0	1.0

Rating Matrix

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A	1.2	0.8
B	1.4	0.9
C	1.5	1.0
D	1.2	0.8

User Matrix

X

	W	X	Y	Z
	1.5	1.2	1.0	0.8
	1.7	0.6	1.1	0.4

Item Matrix

Formulation

- Factorization of data matrix

$$\mathbf{X}_{N \times M} \approx \mathbf{U}_{N \times K} \times \mathbf{V}_{K \times M}^T$$
$$x_{n,m} = \mathbf{u}_n^T \mathbf{v}_m = \sum_{k=1}^K u_{n,k} v_{m,k}$$

- Objective function

$$\mathcal{L} = \sum_{(m,n) \in \Omega} (x_{n,m} - \mathbf{u}_n^T \mathbf{v}_m)^2$$
$$\mathcal{L}^{reg} = \sum_{(m,n) \in \Omega} (x_{n,m} - \mathbf{u}_n^T \mathbf{v}_m)^2 + \sum_{n=1}^N \lambda_u \|\mathbf{u}_n\|_2^2 + \sum_{m=1}^M \lambda_v \|\mathbf{v}_m\|_2^2$$

Alternating Least Square

- Alternate among factors

$$\arg \min_{\mathbf{v}_m} \sum_{n \in \Omega_{cm}} (x_{n,m} - \mathbf{u}_n^T \mathbf{v}_m)^2 + \lambda_v \|\mathbf{v}_m\|_2^2$$

$$\arg \min_{\mathbf{u}_n} \sum_{m \in \Omega_{rn}} (x_{n,m} - \mathbf{u}_n^T \mathbf{v}_m)^2 + \lambda_u \|\mathbf{u}_n\|_2^2$$