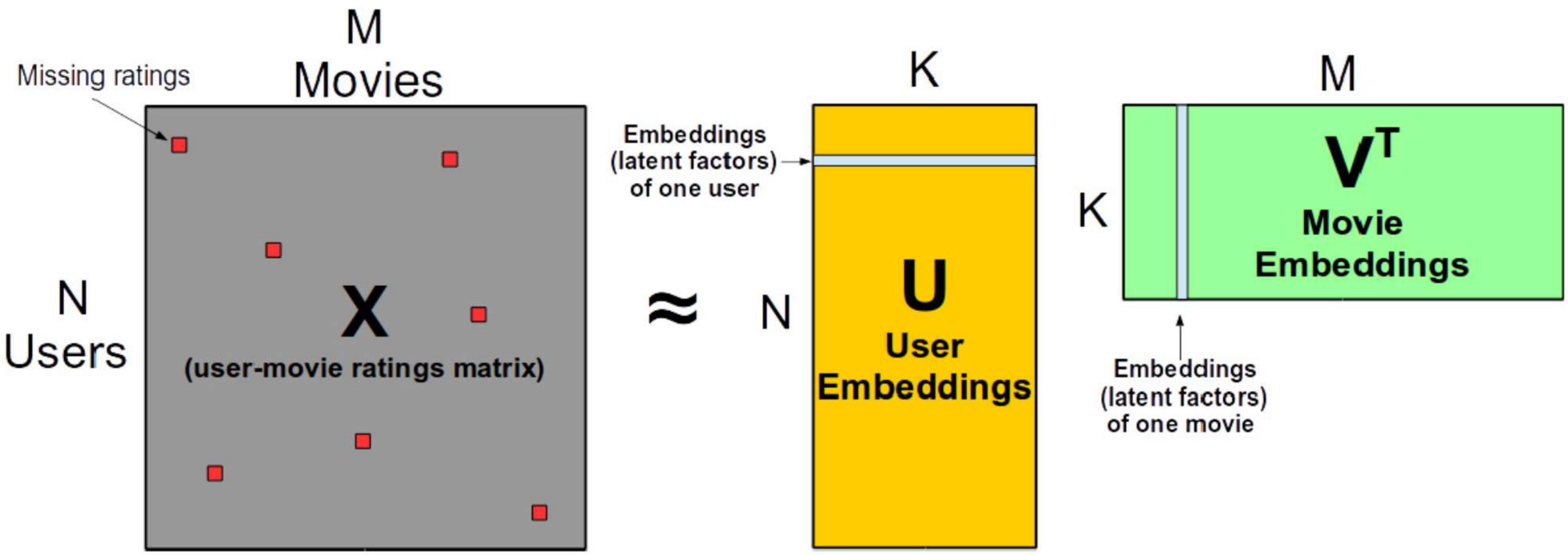
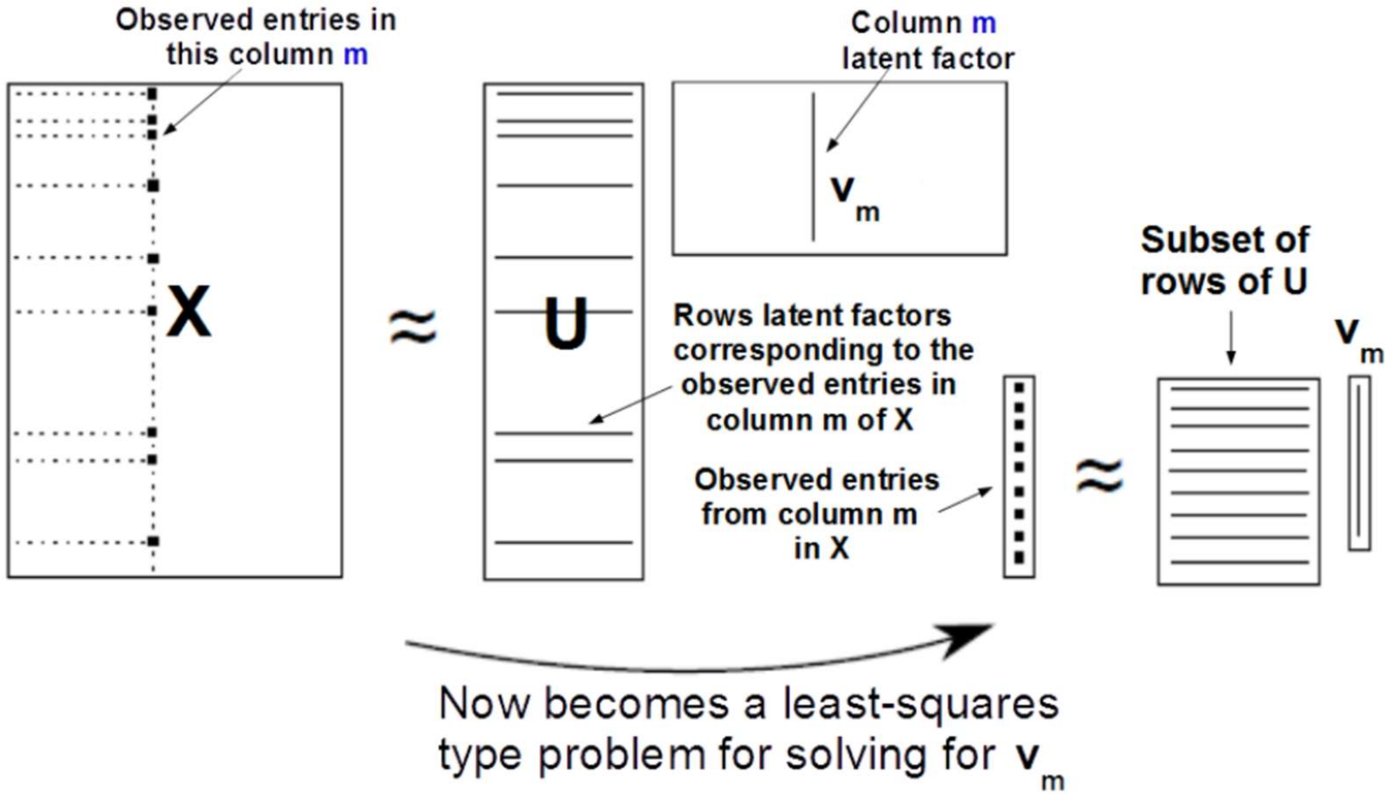
Lecture 16: Alternating Least Square

Course Teacher: Md. Shariful Islam Bhuyan

Matrix Factorization





$$\mathbf{w} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{X} + \lambda \mathbf{I}_{d+1}\right)^{-1} \mathbf{X}^{\mathrm{T}}\mathbf{y} = \left(\sum_{n=1}^{N} \mathbf{x}_n \mathbf{x}_n^T + \lambda \mathbf{I}_{d+1}\right)^{-1} \sum_{n=1}^{N} y_n \mathbf{x}_n$$

$$\mathbf{v}_m^* = \left(\sum_{n \in \Omega_{c_m}} \mathbf{u}_n \mathbf{u}_n^T + \lambda_v \mathbf{I}_K\right)^{-1} \sum_{n \in \Omega_{c_m}} x_{n,m} \mathbf{u}_n$$

Alternating Least Square

- Initialize the latent factors $\mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_N^T$ randomly
- Iterate until converge
  - Update each column latent factor $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_M$ (In parallel)

$$\mathbf{v}_m^* = \left(\sum_{n \in \Omega_{c_m}} \mathbf{u}_n \mathbf{u}_n^T + \lambda_v \mathbf{I}_K \right)^{-1} \sum_{n \in \Omega_{c_m}} x_{n,m} \mathbf{u}_n$$

  - Update each row latent factor $\mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_N^T$ (In parallel)

$$\mathbf{u}_n^* = \left(\sum_{m \in \Omega_{r_n}} \mathbf{v}_m \mathbf{v}_m^T + \lambda_u \mathbf{I}_K \right)^{-1} \sum_{m \in \Omega_{r_n}} x_{n,m} \mathbf{v}_m$$