**Fubini定理**

**Thm1 可积情形的Fubini定理**

设C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\11EA9935.tmp，则

1. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\699F960B.tmpfor C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\E3C76751.tmp
2. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\E895B687.tmp
3. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\670F692D.tmp

**Remark**

C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\9FEA00C3.tmp可积则累次积分可交换次序

**Thm2 非负可测的Tonelli定理**

设C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\5981FAC9.tmp非负可测

1. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\B80030BF.tmpfor C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\5C863825.tmp
2. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\EBF9C27B.tmp
3. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\21C6FD41.tmp

证明thm2需要以下引理  
令C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\CB1F1F7.tmp，这里C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\738EE61D.tmp满足上述1，2，3

**Lemma**

1. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\9579BB33.tmp,C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\F4E44EB9.tmp
2. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\8E93DA2F.tmp
3. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\35655315.tmp
4. C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\B970CAEB.tmp

1,2显然，4用Levi定理可证，看3  
C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\74E3CF31.tmp在C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\C2AAC967.tmp上几乎处处有限，则C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\32C15F0D.tmp几乎处处有限，  
C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\F5C1D1A3.tmp

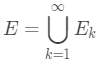
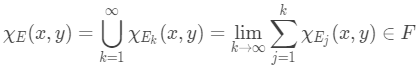
**证明Thm2**

只要证明C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\650B5EA9.tmp

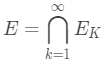
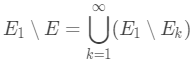
(i) 若C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\AE979F9F.tmp

C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\F356EA05.tmp满足Tonelli的1

C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\84ABAF5B.tmp满足2，这个积分等于C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\D15CDD21.tmp，满足3

(ii) 若C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\EC173CD7.tmp开集  
  
这里C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\C8494413.tmp是C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\26162A99.tmp的互不相交的半开立方体，则有  


(iii) 若C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\DCE9FCF5.tmp为有界闭集，存在开球C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\46116FCB.tmp，由C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\37312711.tmp知C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\2C4E4C47.tmp，而这两个集合都是开集，由Lemma的3以及(ii)知C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\73BB44ED.tmp

(iv) C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\12571283.tmp为C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\37E3B289.tmp型  
  
这里C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\186D8BE5.tmp渐缩，C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\ABC90C3B.tmp，C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\A21FAD01.tmp，则有  
  
而C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\FE20B1DD.tmp渐张，  
C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\46F23CF3.tmp

(v) C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\E12F679.tmp  
这是存在渐缩开集C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\E4A997EF.tmp，满足C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\9BF096D5.tmp，且C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\89B984AB.tmp。令C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\F9A76EF1.tmp，由(ii)和(iv)知C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\E7383F27.tmp，因此  
C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\2C751ACD.tmp  
由C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\B6E1C363.tmp，C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\8002F669.tmp，知C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\E893CD5F.tmp  
即C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\1D421DC5.tmp满足Tonelli的3，同时上式表明C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\4989D91B.tmp，从而证明满足Tonelli的1和2

(vi) 对一般可测集C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\1076CE1.tmp  
由于C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\61592297.tmp，其中C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\6E677FBD.tmp是有界闭集，C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\ACA5D3.tmp，C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\B8D2B259.tmp。  
令C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\BDC11ECF.tmp，不妨设C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\1F120B5.tmp渐张，则C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\5AA1098B.tmp，从而C:\Users\mahinmy\AppData\Local\Microsoft\Windows\INetCache\Content.MSO\473EA6D1.tmp