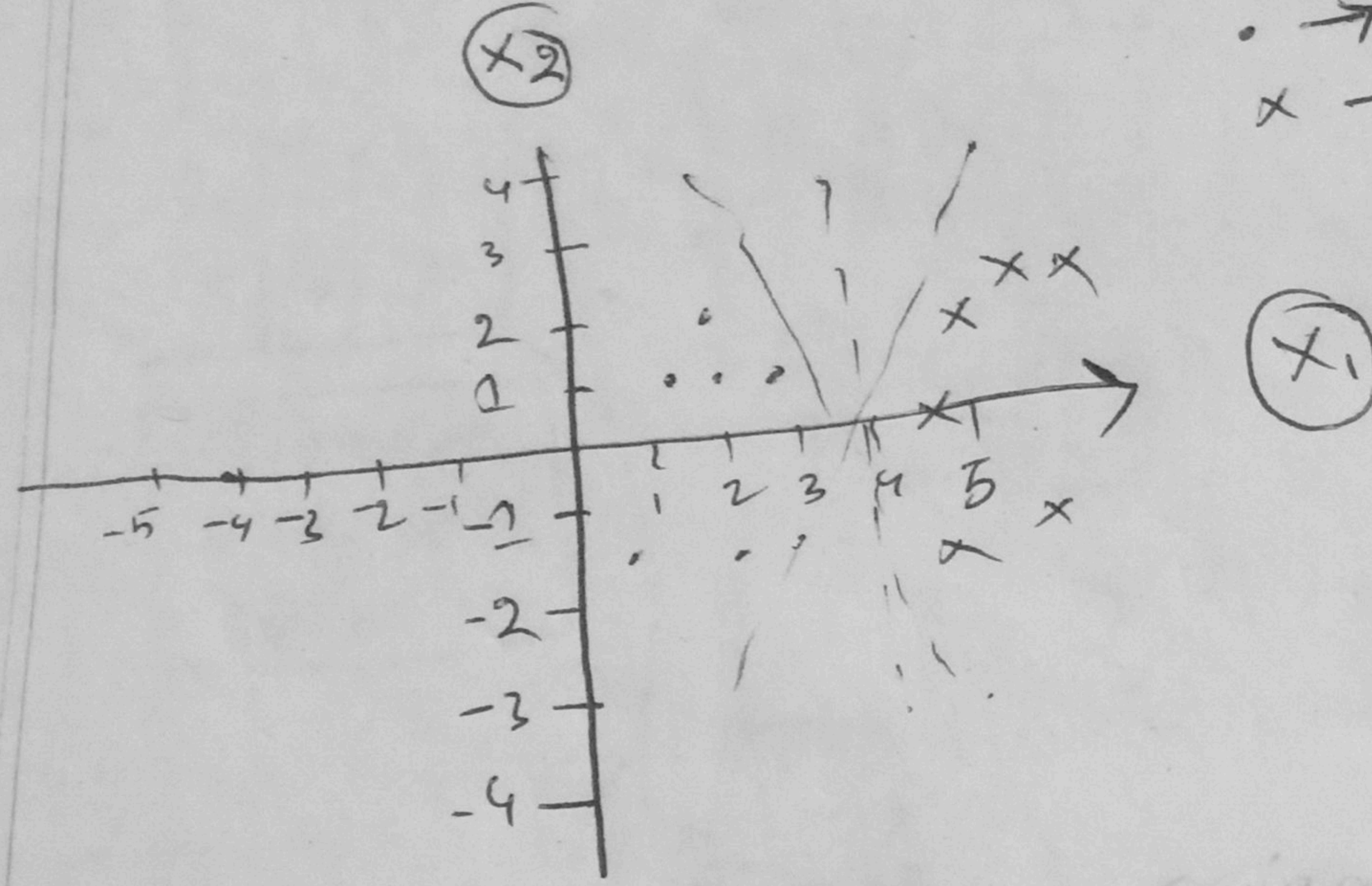
#Support Vector Machine

· → -ve
x → +ve



→ # How can we draw linear plane

Support vector, $(s_1, s_2, s_3) \rightarrow$ ~~closest~~ Nearest point

$$s_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \tilde{s}_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$s_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \qquad \tilde{s}_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$s_3 = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \qquad \tilde{s}_3 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

for 3 support vectors, we will have 3 parameters

$\alpha_1 \; \alpha_2 \; \alpha_3$

$$\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

$$\tilde{S}\,\tilde{S}^T \alpha$$

$$\tilde{S} = \begin{pmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \end{pmatrix} \quad (3 \times 1) \qquad \tilde{S}^T = \begin{pmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3 \end{pmatrix} \quad (1 \times 3)$$

$$\tilde{S}\tilde{S}^T = \begin{bmatrix} \tilde{S}_1\tilde{S}_1 & \tilde{S}_1\tilde{S}_2 & \tilde{S}_1\tilde{S}_3 \\ \tilde{S}_2\tilde{S}_1 & \tilde{S}_2\tilde{S}_2 & \tilde{S}_2\tilde{S}_3 \\ \tilde{S}_3\tilde{S}_1 & \tilde{S}_3\tilde{S}_2 & \tilde{S}_3\tilde{S}_3 \end{bmatrix}$$

(3×3)

$$\alpha_1 \tilde{S}_1\tilde{S}_1 + \alpha_2 \tilde{S}_1\tilde{S}_2 + \alpha_3 \tilde{S}_1\tilde{S}_3 = -1$$
$$\alpha_1 \tilde{S}_2\tilde{S}_1 + \alpha_2 \tilde{S}_2\tilde{S}_2 + \alpha_3 \tilde{S}_2\tilde{S}_3 = -1$$
$$\alpha_1 \tilde{S}_3\tilde{S}_1 + \alpha_2 \tilde{S}_3\tilde{S}_2 + \alpha_3 \tilde{S}_3\tilde{S}_3 = +1$$

$$\alpha_1 \begin{pmatrix}2\\1\\1\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\1\\1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}2\\1\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix}2\\-1\\1\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\-1\\1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}2\\-1\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\0\\1\end{pmatrix}\begin{pmatrix}4\\0\\1\end{pmatrix} = 1$$

$$\vec{A} = \begin{pmatrix}a_1\\b_1\\c_1\end{pmatrix} \quad \vec{B} = \begin{pmatrix}a_2\\b_2\\c_2\end{pmatrix}$$

$$\vec{A}\cdot\vec{B} = a_1a_2 + b_1b_2 + c_1c_2$$

$$\boxed{\begin{aligned} &\alpha_1(4+1+1) + \alpha_2(4) + \alpha_3(9) = -1 \\ &\alpha_1(4) + \alpha_2(6) + \alpha_3(9) = -1 \\ &9\alpha_1 + 9\alpha_2 + 17\alpha_3 = 1 \end{aligned}}$$

$$\boxed{\begin{aligned} 6\alpha_1 + 4\alpha_2 + 9\alpha_3 &= -1 \\ 4\alpha_1 + 6\alpha_2 + 9\alpha_3 &= -1 \\ 9\alpha_1 + 9\alpha_2 + 17\alpha_3 &= 1 \end{aligned}}$$

var $\leq$ eq

param

$\alpha_1 = -3.25$

$\alpha_2 = -3.25$

$\alpha_3 = 3.5$

Decision boundary → linear (SVM)

$y = mx + c$ → Math

$y = wx + b$ → ML

max $D = D_1 + D_2$

$\sim$ → Augmented

$$\tilde{w} = \sum \alpha_i s_i$$

$$= \alpha_1 \tilde{s}_1 + \alpha_2 \tilde{s}_2 + \alpha_3 \tilde{s}_3$$

$$\tilde{w} = \alpha_1 \tilde{s}_1 + \alpha_2 \tilde{s}_2 + \alpha_3 \tilde{s}_3$$

$$= -3.25\begin{pmatrix}2\\1\\1\end{pmatrix} + (-3.25)\begin{pmatrix}2\\-1\\1\end{pmatrix} + 3.5\begin{pmatrix}4\\0\\1\end{pmatrix}$$

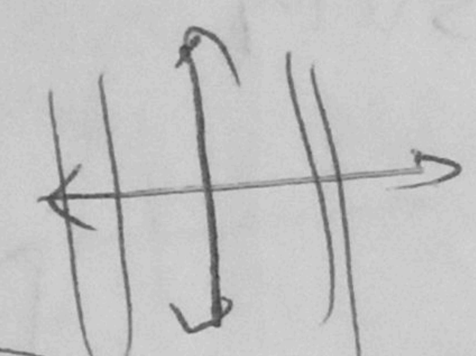
$$= \begin{pmatrix}-6.50\\-3.25\\-3.25\end{pmatrix} + \begin{pmatrix}-6.50\\3.25\\-3.25\end{pmatrix} + \begin{pmatrix}14.0\\0\\3.5\end{pmatrix} = \begin{pmatrix}1\\0\\-3\end{pmatrix}$$

$$= \begin{pmatrix}1\\0\\-3\end{pmatrix}$$

$\tilde{w} = \begin{pmatrix}1\\0\\-3\end{pmatrix}$

bias

$w = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$



(parallel to y axis.)

$b = -3 \longrightarrow x = 3$

$w = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$

$\Downarrow$

(parallel to x axis)

$y = wx + b$

$= \begin{pmatrix}1\\0\end{pmatrix}x + -3$

$x = 3$ is the hyperplane

$\boxed{y = \begin{pmatrix}1\\0\end{pmatrix}x - 3}$

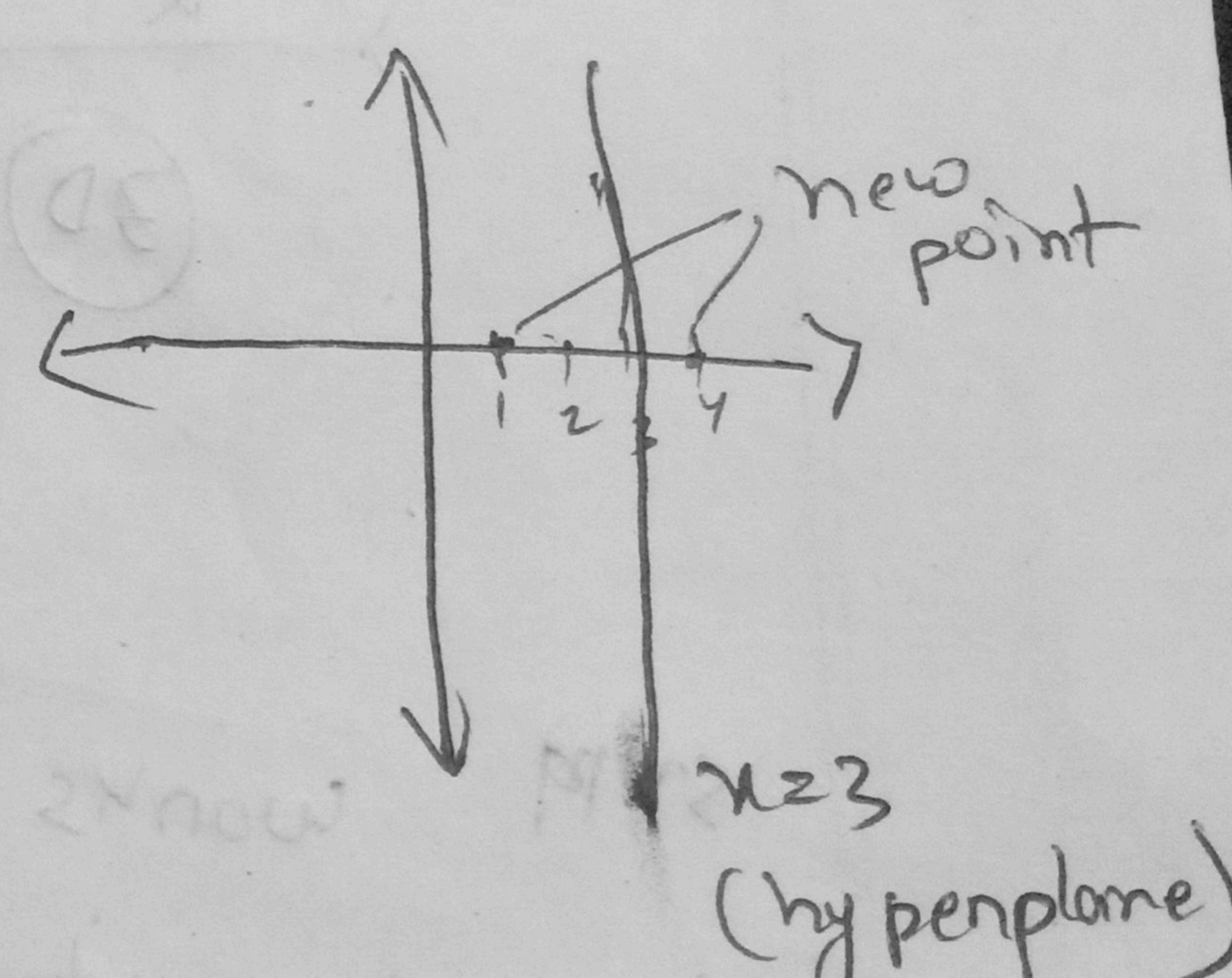
new $\begin{pmatrix}1\\0\end{pmatrix}$

(LSVM)

$y = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} - 3$

$= 1 - 3 = -2$

$y = -2$ belongs to $-ve$



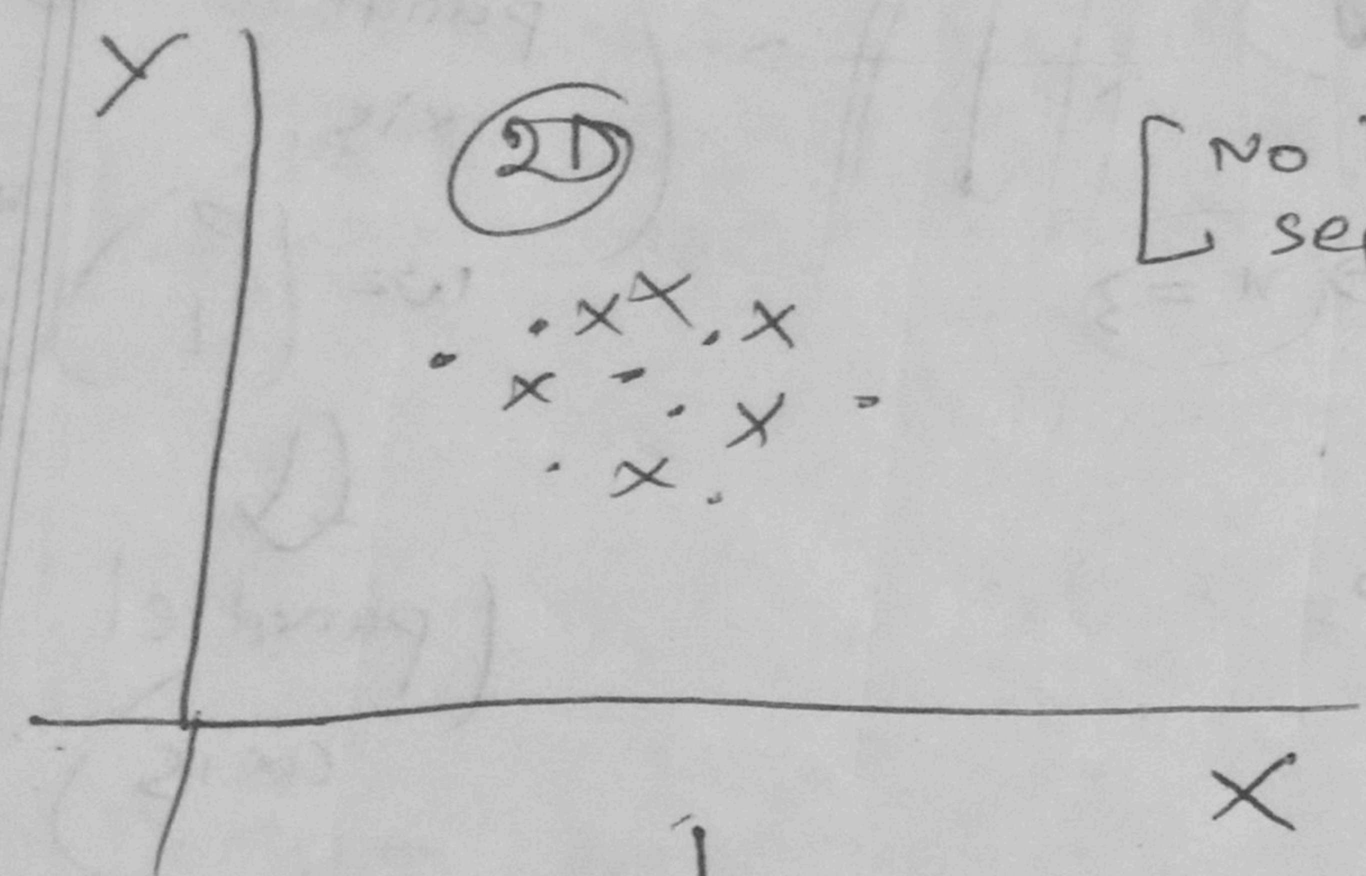
new $\begin{pmatrix}4\\0\end{pmatrix}$

$y = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}4\\0\end{pmatrix} - 3$

$= 4 - 3 = 1$

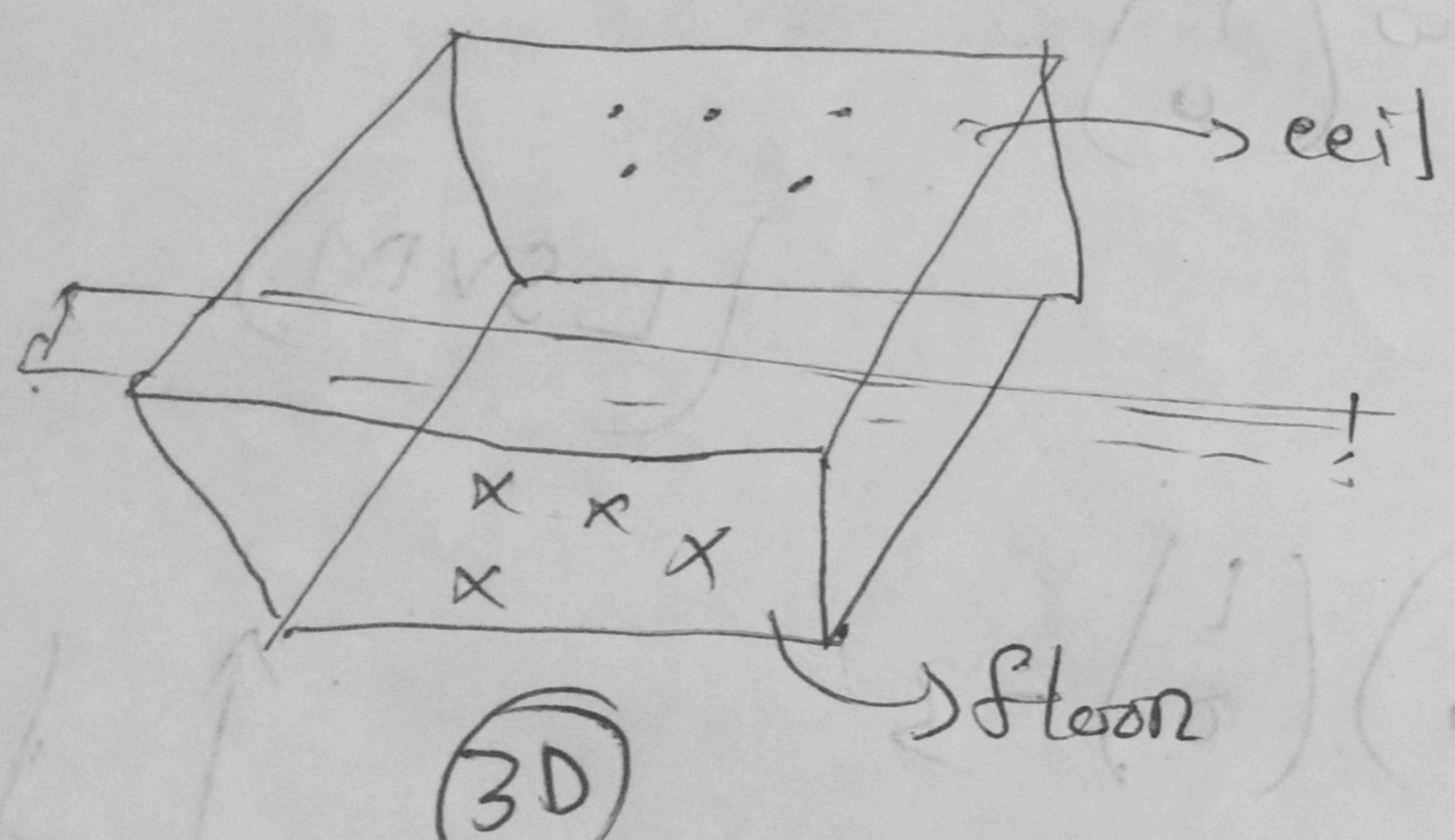
$\boxed{y = 1 \text{ belongs to } +ve}$

Non linear SVM



[No linear separable]

↓ (Make it in 3D)



} Now linearly separable through surface ( )

kernel trick

SVM works well in Non linear Data set. Comparing to other classifier (~~Non~~ a) because it has flexibility.

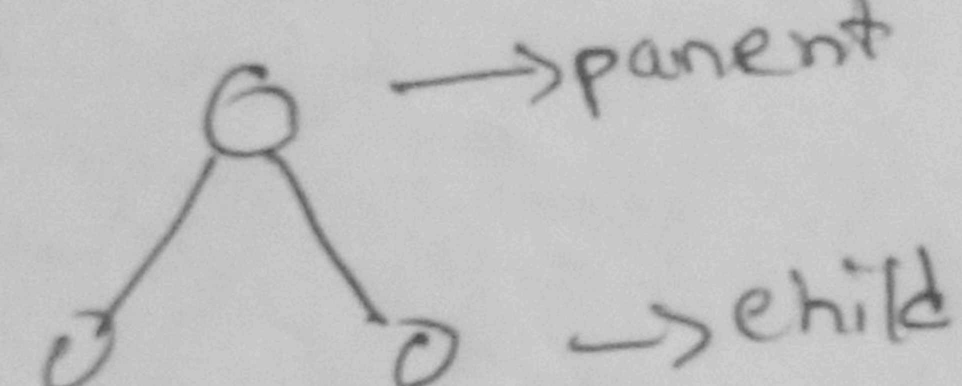
Deision Tree

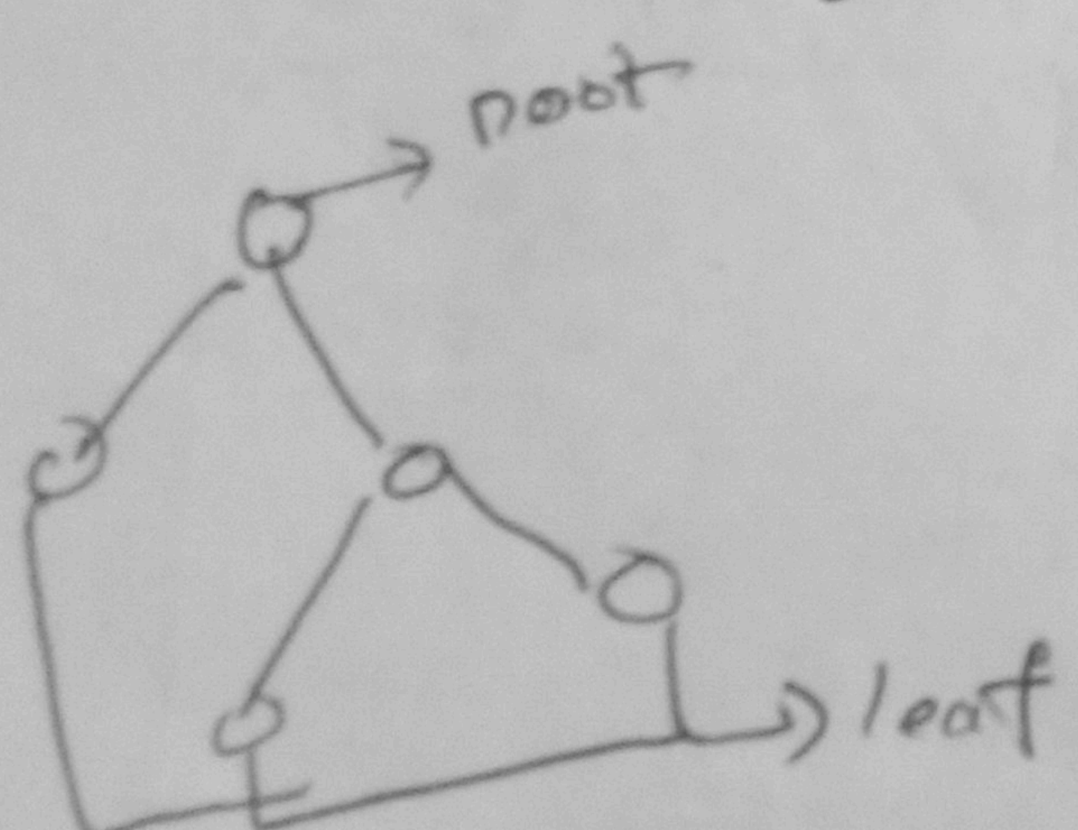
chest pain	good blood circulation	Blocked arteries	Heart disease
No	No	No	No
Yes	Yes	Yes	Yes
Yes	Yes	No	No
Yes	No	Yes	Yes

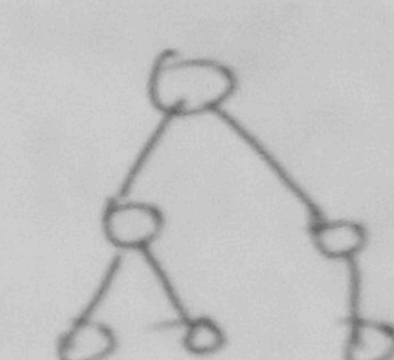
Weight	Heart disease
220	Yes
180	Yes
225	Yes
190	No
155	No

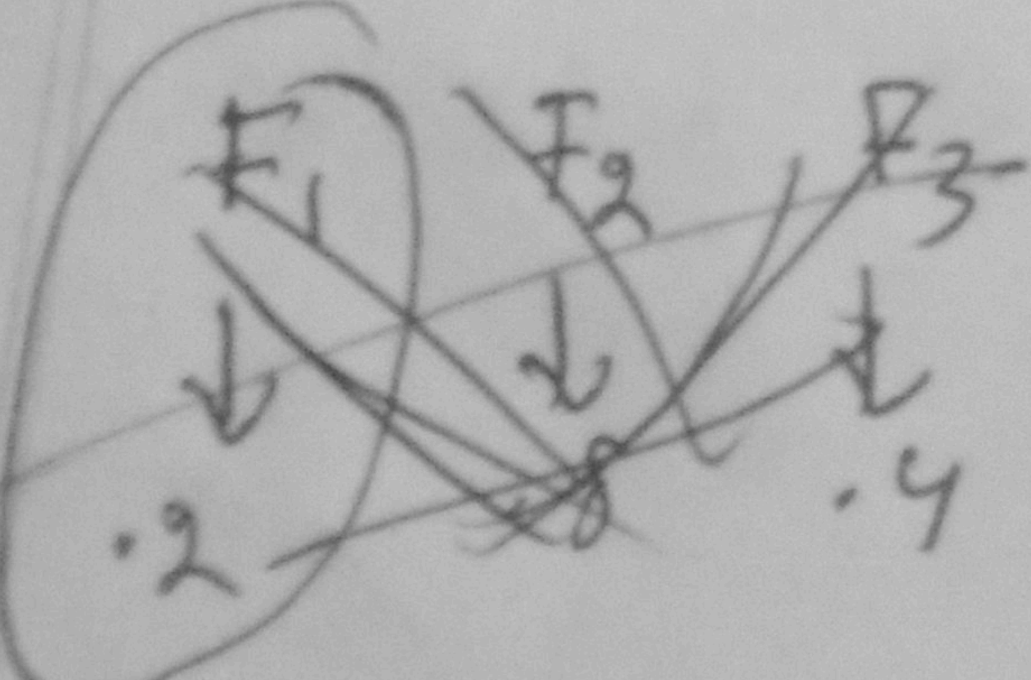
~~Dec~~

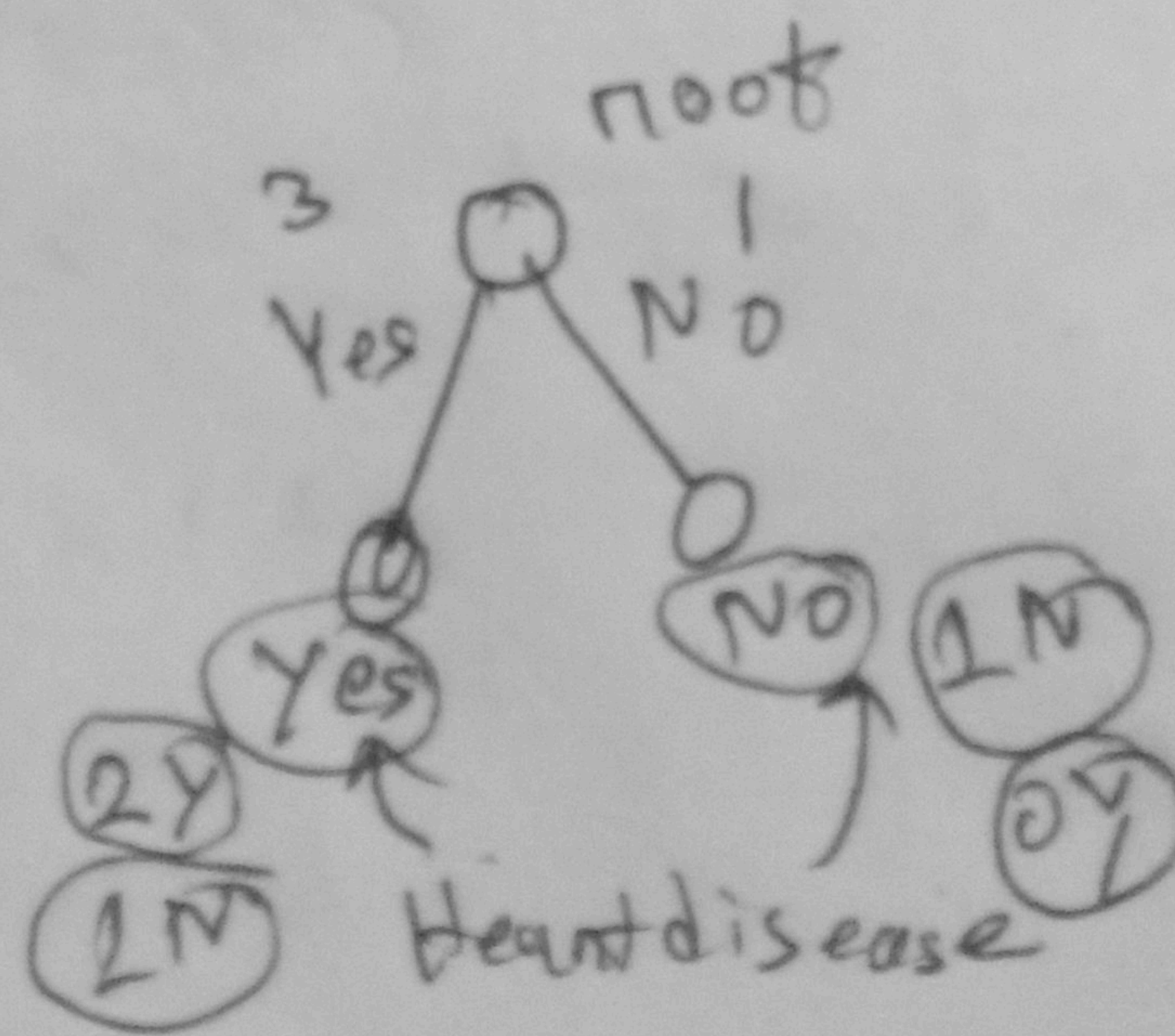
Decision Tree











$\boxed{\text{chest pain}}$

4 → 3✓✓

1X

misprediction — 1

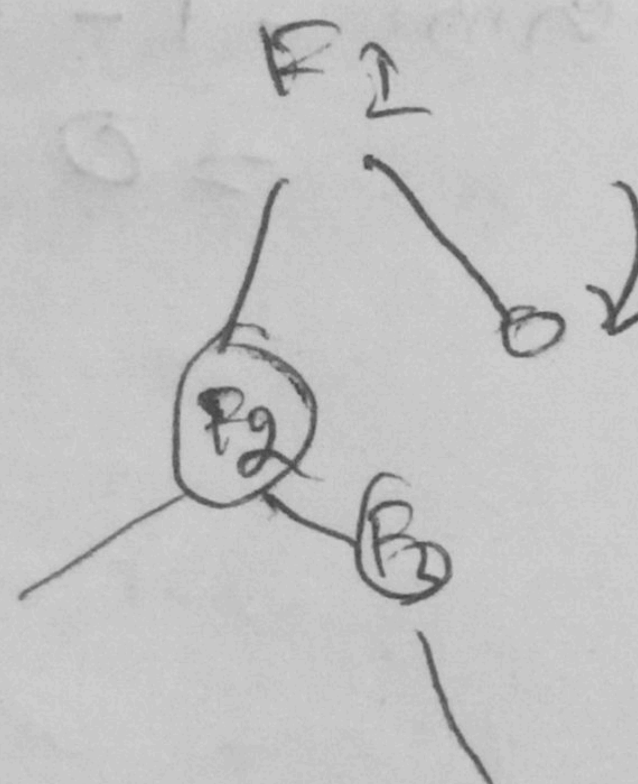
Impurity ⇒ For each feature

lowest impurity feature

$$Gini = 1 - \sum P(i)^2$$

$$Entropy = -\sum P_i \log_2(P_i)$$

F1	F2	F3
↓	↓	↓
0.2	0.8	0.4
	↓	↓
	.3	.4



Example (How it works)

Right

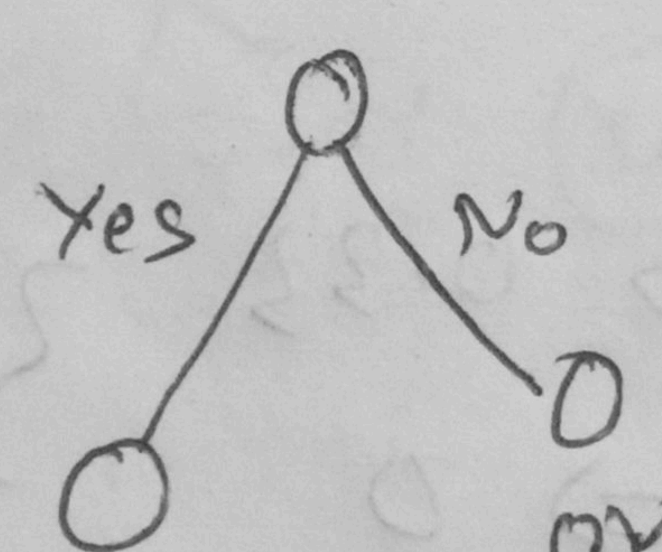
$P_{yes} = \frac{0}{1} = 0$

$P_{No} = \frac{1}{1} = 1$

$Gini = 1 - 0^2 - 1^2$

$= 0$

Chest pain



2Y
1N
(Left)
↓
0.455

0Y
1N
(Right)
↓
0

Left

$Gini = 1 - \sum P(i)^2$

$P_{yes} = \frac{2}{3} = 0.66$

$P_{No} = \frac{1}{3} = 0.33$

$Gini = 1 - (0.66)^2 - (0.33)^2$

≈ 0.455

~~Gini = Gini (left) + Gini (Right)~~

~~= 0.455 + 0~~

~~≈ 0.455~~

we will weighted average

$$\text{Gini} = \frac{3}{4} \times 0.455 + \frac{1}{4} \times 0$$

$= \boxed{\ }$ penfect 0.34125

Bloeked antanies

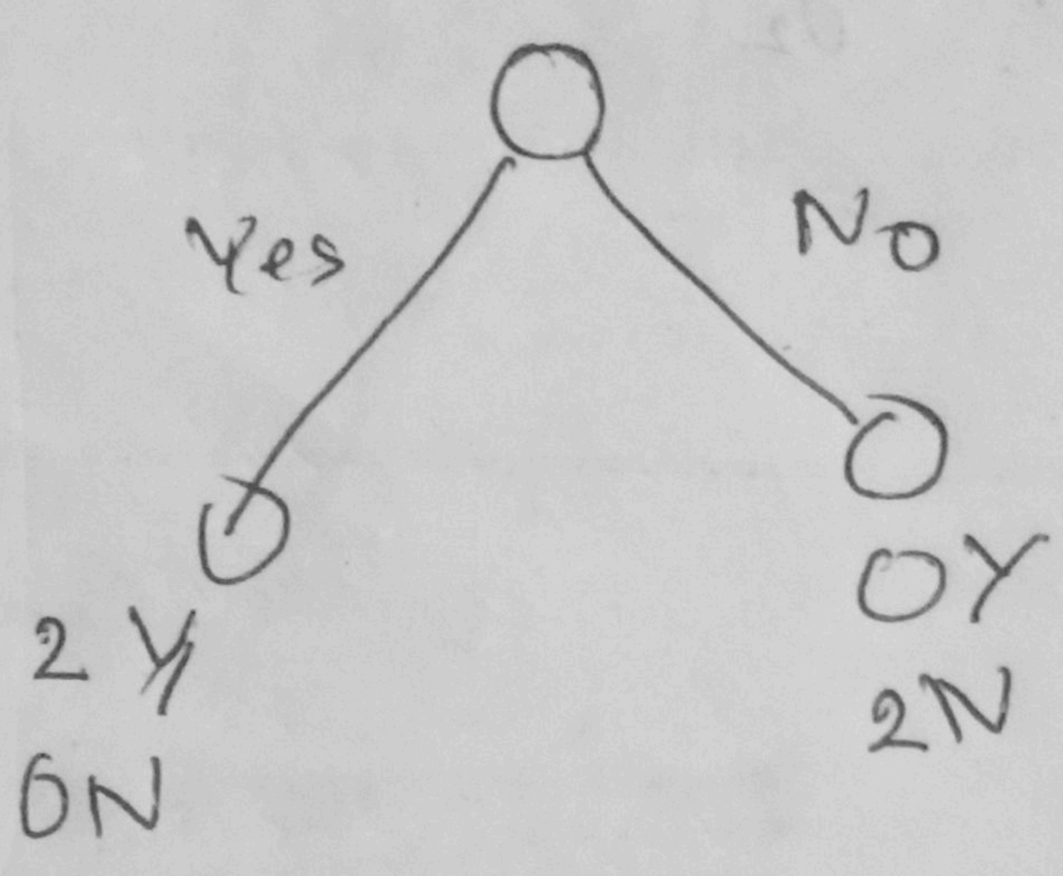
left

$P_{yes} = \frac{2}{2} = 1$

$P_{No} = \frac{0}{2} = 0$

$\text{Gini} = 1 - (1)^2 - 0^2$

$= 0$



Right

$P_{yes} = \frac{0}{2} = 0$

$P_{No} = \frac{2}{2} = 1$

$\text{Gini} = 1 - 0^2 - 1^2$

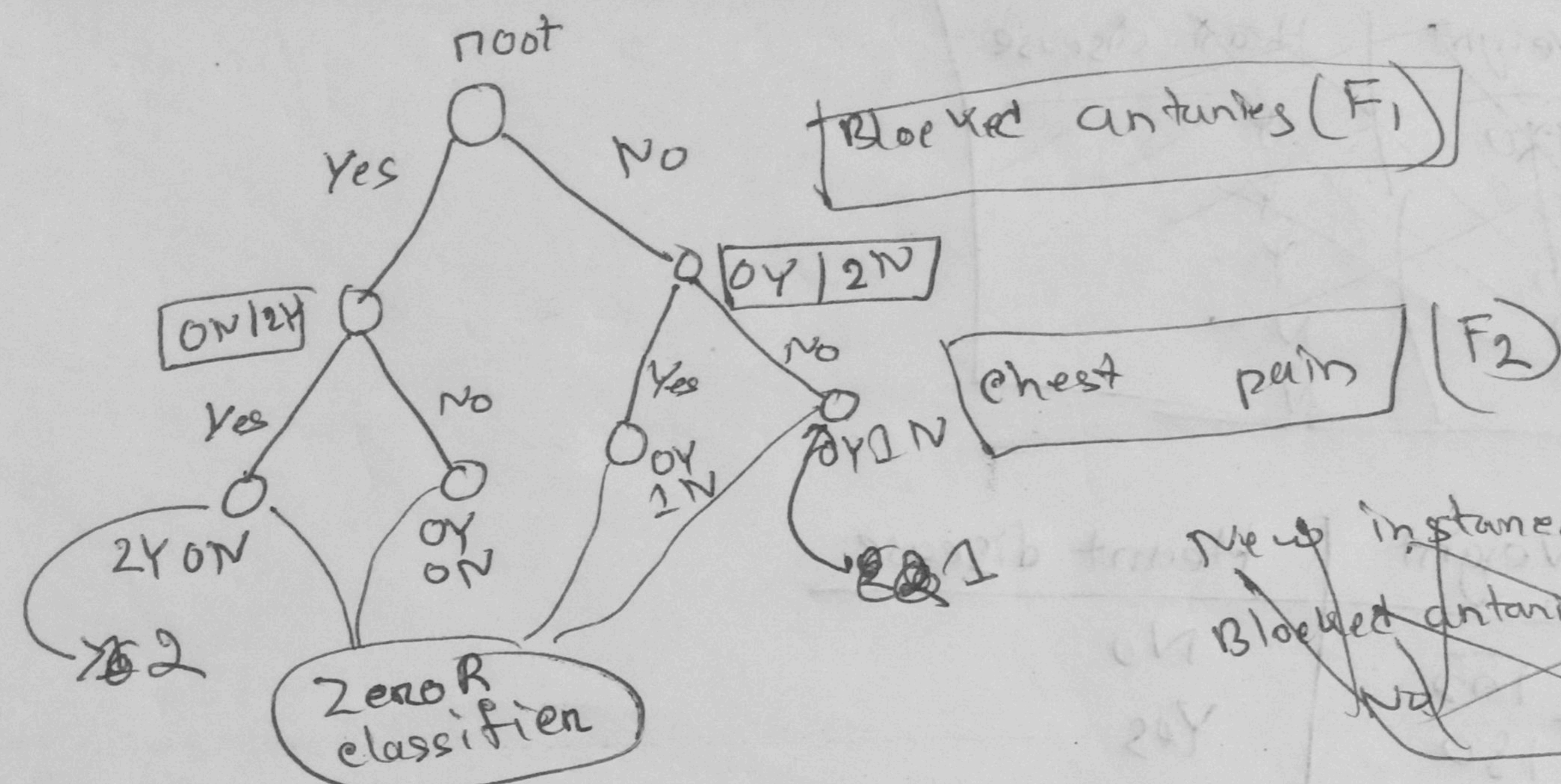
$= 0$

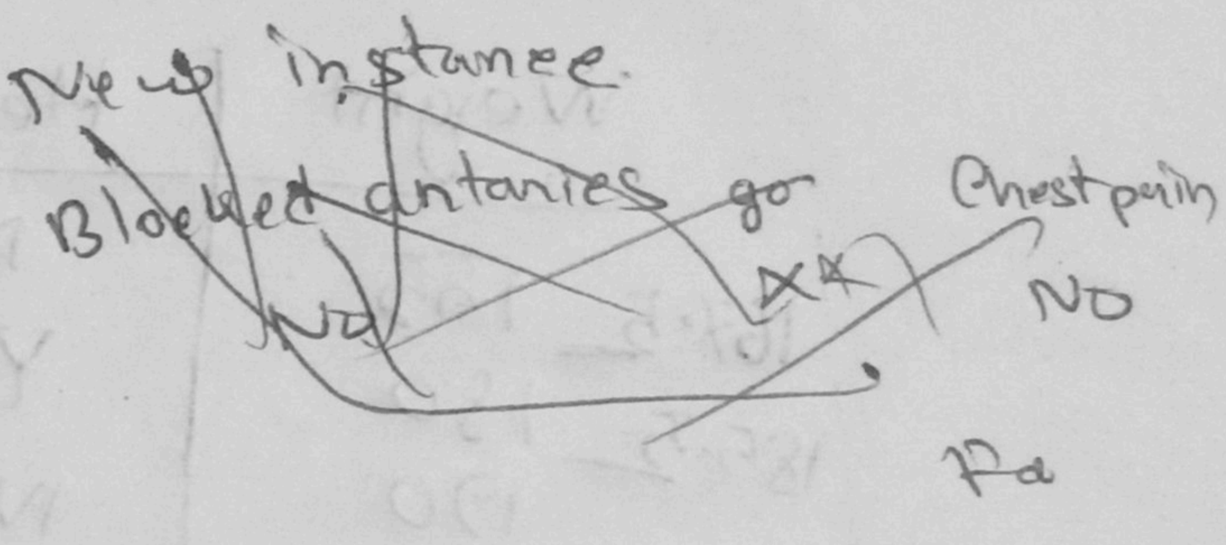
$$\text{Gini} = \frac{2}{4}(0) + \frac{2}{4}(0)$$

$= 0$

Chest pain $\Rightarrow$ 0.33
Bloeked antenies $\Rightarrow$ 0

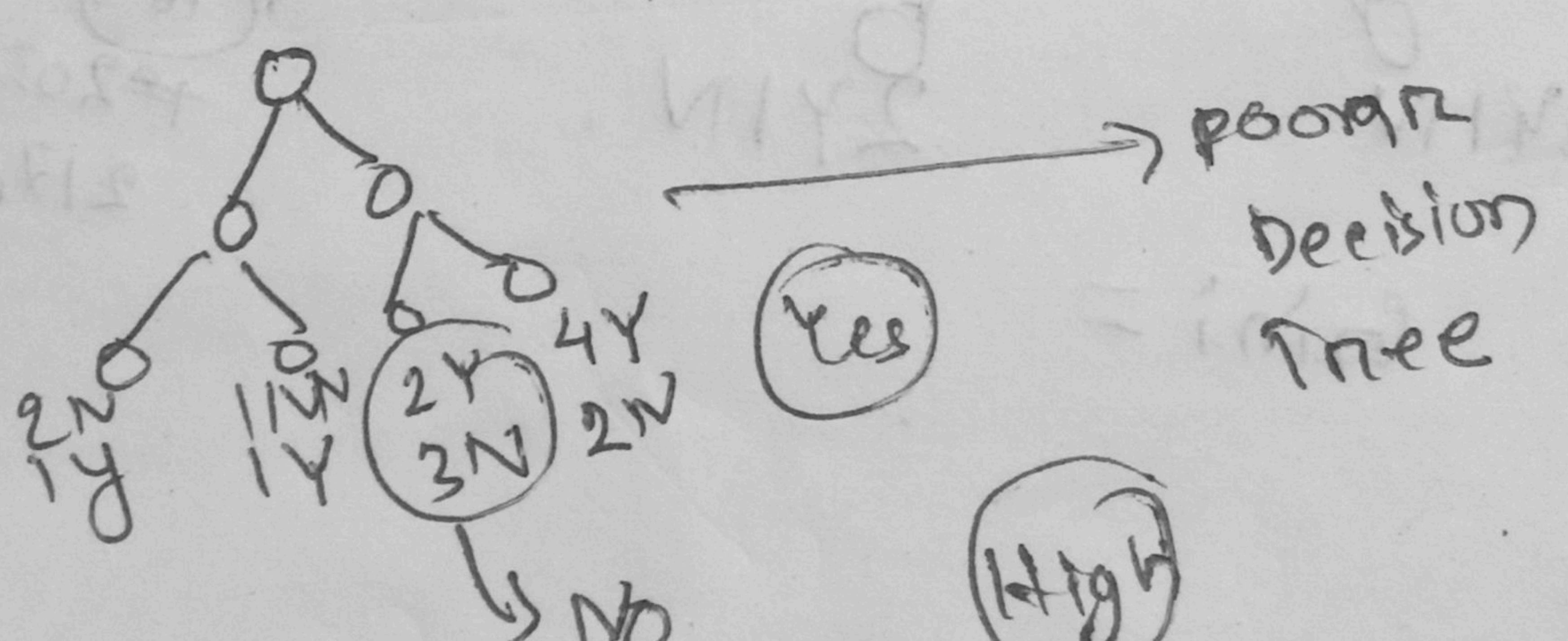
} So root will be Bloeked antenies





New instances

Blocked anteries	Chest pain	Heart diseases
1 → No	No	?? → No
2 → Yes	Yes	?? → Yes



Yes

High

low

Information gain = Gini (Parent) − Gini (Child)

$= 0.2 - 0.2$

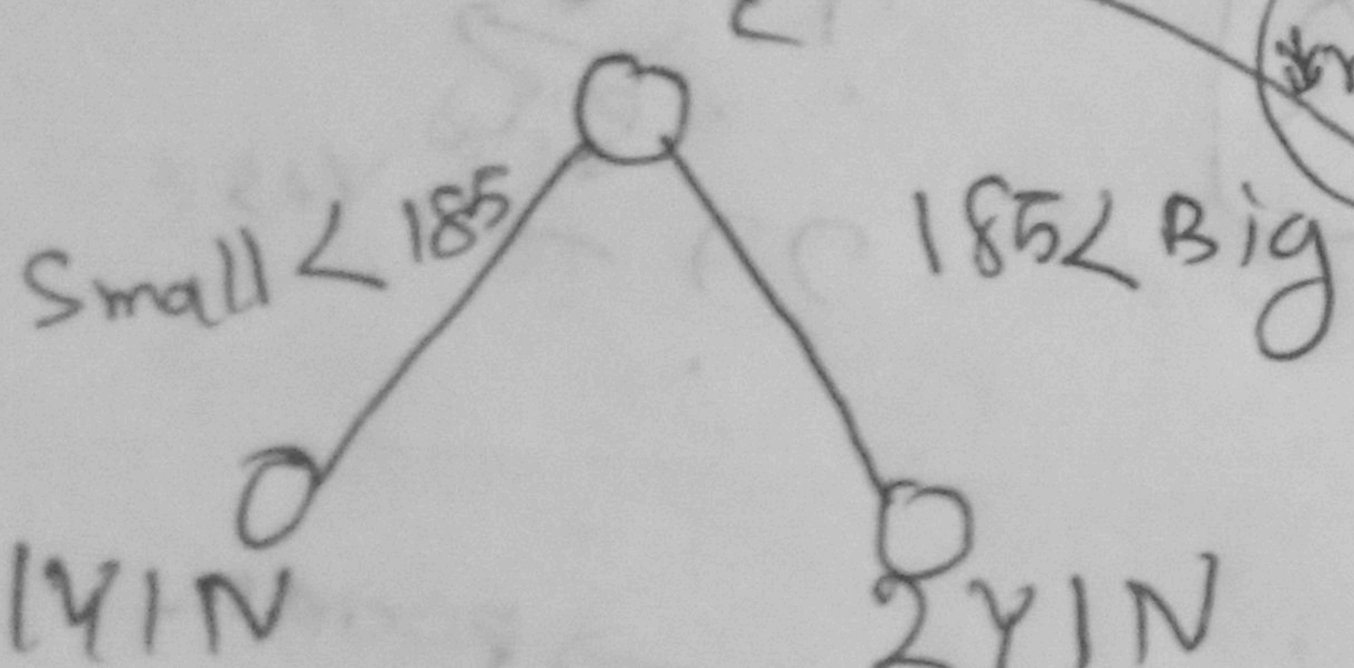
$= 0$

~~If Inf~~

| If IG is very high → split it |
| " " " very low → Do not split |

~~Weight~~	~~Heart disease~~
~~220~~	~~Y~~
~~180~~	~~Y~~
~~225~~	~~Y~~
~~190~~	~~N~~
~~155~~	~~N~~

	Weight	Heant disease
167.5	155	No
185.5	180	Yes
	190	No
205	220	Yes
217.5	225	Yes



min Gini value

167.5
185
~~1~~205
217.5

Gini =

Take thien value and select ~~to~~ the minimum Gini value to split. OR High Information gain to split