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Minimizing makespan and total flowtime in permutation flowshops by a bi-objective multi-start simulated-annealing algorithm

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ABSTRACT

In this study, a bi-objective multi-start simulated-annealing algorithm (BMSA) is presented for permutation flowshop scheduling problems with the objectives of minimizing the makespan and total flowtime of jobs. To evaluate the performance of the BMSA, computational experiments were conducted on the well-known benchmark problem set provided by Taillard. The non-dominated sets obtained from each of the existing benchmark algorithms and the BMSA were compared, and then combined to form a net non-dominated front. The computational results show that more than 64% of the solutions in the net non-dominated front are contributed by the proposed BMSA. It is believed that these solutions can serve as new benchmarks for future research.

1. Introduction

The permutation flowshop scheduling problem (PFSP) is one of the best known production scheduling problems, one found in many sectors. Since finding optimal solution may involve formidable computational requirements even for a moderate size problem, efficient scheduling of the PFSP has remained a topic of interest for researchers and practitioners for the last few decades [1–4]. A basic PFSP can be described as finding a sequence of n jobs to be processed on m machines in which each job has one operation on each machine and all jobs have the same ordering sequence on every machine. Over these years, many variants of the basic PFSP have been formulated and investigated by differentiating between side-constraints and objective functions. However, previous studies on the scheduling of PFSPs have generally focused on the optimization of individual performance measures [5].

To avoid unbalanced outcomes, in many practical situations, scheduling problems generally involve multiple objectives [6]. As a result of wide applications in practice, a few attempts have been made recently to consider multi-objective PFSPs, particularly, bi-objective PFSPs, to address the gap between scheduling research and practice. For further extensive reviews on multi-objective algorithms for the flowshop scheduling problem, the reader is referred to Minella et al. [7]. To help bridge this gap, this study tackles the PFSP with the objective of minimizing the makespan and total flowtime of jobs, and presents a highly effective bi-objective multi-start simulated-annealing algorithm (BMSA) to yield a set of non-dominated solutions (also called Pareto-optimal solutions in the case of Pareto-optimality) for this problem. As per the three-field notation introduced by Graham et al. [8], the problem under study is denoted as $F|prmu|C_{\max}, \sum C_i$ throughout this paper.

Minimization of the makespan and total flowtime are two important performance criteria that are well-justified in practice. Minimizing makespan (i.e., the completion time of the last job) is the most useful criterion adopted by the majority studies on PFSPs, as it implies the maximization of the resource utilization and the throughput of a production line [9]. On the other hand, the objective of minimizing total flowtime or, equivalently, total completion time if all jobs are available for processing at the start time, can lead to stable consumption of resources, rapid cycle time of jobs, and minimizing Work-In-Process inventory [10]. In practice, it is desirable to achieve both these two objectives simultaneously to reduce production costs. However, the $F|prmu|C_{\max}, \sum C_i$ problem is not a trivial issue. Even a simplified PFSP with either the makespan or total flowtime minimization as the objective is NP-hard if the number of machines is more than two [11]. Consequently, the $F|prmu|C_{\max}, \sum C_i$ problem is also NP-hard. While there is a great deal of research devoted to developing exact algorithms for individual optimization criterion PFSPs with respect to makespan or total flowtime, it appears that only few research has dealt with the $F|prmu|C_{\max}, \sum C_i$ problem by using exact methods [12]. For a problem of such complexity, the computational requirements for obtaining optimal solutions by exact methods (e.g. complete enumeration, dynamic programming, branch-and-bound, integer programming, elimination approaches and row generation algorithms) are severe even for problems of

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moderate size. Even in the state-of-art exact method proposed by Lemesre et al. [12], which can precisely solve some instances with a maximum problem size of 20 jobs and 20 machines, the computational time is about 7 days, for experiments carried out on 4 parallel computers with microprocessors between 2.6 and 3 GHz.

The formidable computational requirements have resulted in several attempts to develop multi-objective metaheuristic algorithms, which occupy a growing space and constitute the best-so-far approaches to the $F|prmu|C_{\max}, \sum C_i$ problem. One of the pioneering works is that of Chang et al. [13], who presented a gradual priority weighting approach based on the genetic algorithm (GPWGA) to address the problem under consideration. The fitness selection of the GPWGA is a linear combination of both objective function values. By gradually varying the priority weights of objectives in this linear combination, Chang et al. show that the GPWGA outperformed other GAs taken from related scheduling problems.

A representative study is that of Varadharajan and Rajendran [14], who presented a multi-objective simulated-annealing algorithm (MOSA) to obtain a set of non-dominated solutions for the $F|prmu|C_{\max}, \sum C_i$ problem. The performance of the MOSA was evaluated by two different parameter settings, called MOSA-I and MOSA-II, with respect to the temperature and epoch length. The authors show that both the MOSA-I and the MOSA-II outperform four existing multi-objective flowshop scheduling algorithms, including the MOGLS algorithm of Ishibuchi and Murata [15], the ENGA of Bagchi [16], the GPWGA of Chang et al. [13], and the PH of Framinan et al. [17]. It is worth noting that the MOSA-II can yield better results than the MOSA-I, as it uses a higher number of iterations and a lower termination temperature. Minella et al. [7] reported an extensive computational evaluation of more than twenty existing multi-objective algorithms available up to 2007 and concluded that the MOSA is the best performing multi-objective metaheuristic algorithm for the $F|prmu|C_{\max}, \sum C_i$ problem.

Later, Pasupathy et al. [18] proposed a Pareto genetic algorithm with an archive of non-dominated solutions subjected to a local search (PGA-ALS). The PGA-ALS algorithm preserves a set of non-dominated solutions obtained at the end of every generation that are being updated and improved through the implementation of local search techniques. A relative evaluation of PGA-ALS and GPWGA [13], MOGLS [15] and ENGA [16] shows that most of the solutions in the net non-dominated front are yielded by the PGA-ALS algorithm. Meanwhile, we found that some of the generated solutions of the PGA-ALS have very high qualities by comparison with those of the MOSA-I and the MOSA-II.

A new metaheuristic algorithm, named multi-objective iterated greedy search (MOIGS), soon rivaled the MOSA. It was originally proposed by Framinan and Leisten [5]. The MOIGS algorithm mainly iterates over a multi-criteria constructive heuristic approach, which is embedded in a greedy local search approach to yield a set of non-dominated solutions. The computational results of Framinan and Leisten showed that the MOIGS performs better than the MOSA in terms of generating more heuristically non-dominated solutions. In addition, it is compared against the MOSA for a wide number of multi-criteria performance measures. It is worth noting that in contrast to the several explicit and implicit parameters employed by the MOSA, the salient feature of the MOIGS algorithm is that it uses only one parameter, which is not critical for the final results.

More recently, Rajendran and Ziegler [19] proposed a multi-objective ant-colony algorithm (MOACA) for the $F|prmu|C_{\max}, \sum C_i$ problem. Twenty variants of the MOACA are derived by changing the settings of the parameters and the concatenation of improvement schemes of this algorithm. To prove the effectiveness of the MOACA, Rajendran and Ziegler formed a net set of non-dominated solutions by consolidating the solutions obtained by the twenty variants of the MOACA, the MOIGS and all of the multi-objective algorithms available up to 2007 that were evaluated by Minella et al. [7]. The authors show that most of the non-dominated solutions on the net non-dominated front are yielded by the variants of the MOACA. Further, the 20 variants of the MOACA contributed more solutions to the net non-dominated front than the MOIGS. After an extensive literature survey, to the best of current knowledge, this net non-dominated front can be retained as the best-so-far solution sets for the benchmark problem instances provided by Taillard [20] with respect to the $F|prmu|C_{\max}, \sum C_i$ problem.

The relevant literature indicates that several algorithms of these pioneering studies have emerged as acclaimed approaches for the $F|prmu|C_{\max}, \sum C_i$ problem. Among these acclaimed approaches, the MOSA algorithm proposed by Varadharajan and Rajendran [14] has emerged as a highly effective and efficient algorithmic approach for the $F|prmu|C_{\max}, \sum C_i$ problem. However, the search procedures of simulated-annealing (SA)-based algorithm that attempt to obtain (near) global optimal solutions typically require some form of diversification to escape from local optimality. Without such diversification, SA-based algorithms may become localized in a small area of the solution space, eliminating the possibility of finding a global optimum [21]. One way to achieve diversification is to use the multi-start hill climbing strategy, which can provide an appropriate framework for preventing the search from being trapped in a local optimal. To help develop more efficient algorithms for this problem, in this study, we proposed a bi-objective multi-start simulated-annealing algorithm (BMSA), which combines the advantages of SA algorithms in effectively achieving search convergence and the multi-start hill climbing strategy in escaping local optimality.

The remainder of the paper is divided into four sections. After formulating the $F|prmu|C_{\max}, \sum C_i$ problem in Section 2, the proposed BMSA is elaborated in Section 3. Using a famous benchmark problem set provided by Taillard [20], in Section 4, we empirically evaluate the effectiveness of the proposed BMSA by comparing the non-dominated sets obtained from it with those of existing benchmark algorithms and the best-so-far net non-dominated front. Meanwhile, a new complete set of net non-dominated front for the benchmark instances of Taillard is provided so that researchers can use them as benchmarks for additional research attempts. Conclusions are finally drawn in Section 5, along with recommendations for future research.

2. Problem formulation

To give a formal definition of the $F|prmu|C_{\max}, \sum C_i$ problem, consider a set $N=\{1, 2, \ldots, n\}$ of n independent jobs to be scheduled on a set $M=\{1, 2, \ldots, m\}$ of m machines, where all jobs follow the same machine routing and the sequence of jobs on every machine is identical. Each job $i \in N$ comprises exactly one operation on each machine j ($j \in M$) which requires a processing time p_{ij}. Moreover, the following assumptions are invoked for the $F|prmu|C_{\max}, \sum C_i$ problem considered in this study:

- Each machine can process no more than one job at the same time and each job can be processed on only one machine at any given time.
- The schedule is non-preemptive, meaning once a job starts to be processed on a machine, the process cannot be interrupted before completion.
- The number of jobs and their processing times on each machine are deterministic non-negative integers.
- The number of machines is known in advance, and all machines are persistently available to process all scheduled jobs as required.

- All jobs are ready for processing at the beginning, and have no precedence constraints among them.

With the above definitions, the $F|prmu|C_{\max}, \sum C_i$ problem addressed in this study is that of finding a set of non-dominated solutions (sequences) to simultaneously minimize the makespan and the total flowtime such that at least one such solution in the set is not inferior to any other given solution not contained in the set, and the solutions in the set do not dominate each other. If we express a solution for the $F|prmu|C_{\max}, \sum C_i$ problem as a vector $\Pi:=(\pi_1, \pi_2, \ldots, \pi_n)$, where π_i denotes the job that is to be processed on all machine in the ith order, then the corresponding values of makespan and total flowtime of Π can be calculated by $C_{\max}(\Pi)=C_{\pi_n,m}(\Pi)$ and $TFT(\Pi)=\sum_{i=1}^{n} C_{\pi_i,m}(\Pi)$, respectively, where $C_{\pi_i,m}(\Pi)$, $(i=1,\ldots,n)$ is the completion time of each job π_i on machine m that can be calculated by the following recursive equation:

$$C_{\pi_i,j}(\Pi) := \max\{C_{\pi_i,j-1}(\Pi), C_{\pi_{i-1},j}(\Pi)\}+p_{\pi_i,j}, \quad j=1,\ldots,m,$$

In the above equation $p_{\pi_i,j}$ denotes the processing time of job π_i on machine j and $C_{\pi_i,0}(\Pi):=C_{\phi,j}(\Pi):=0$ for all i and j; ϕ is the initial null sequence.

3. Proposed bi-objective multi-start simulated-annealing algorithm

The proposed BMSA is based on SA algorithms, the multi-start hill climbing strategy, and the characteristics of bi-objectives. In the following subsections, we describe the details of solution representation, initial solutions, parameters, neighborhoods and procedures of the proposed BMSA.

3.1. Solution representation and initial solutions

In this study, a sequence of jobs is represented by a string of numbers consisting of a permutation of n jobs. For example, a solution is represented as [2 3 7 4 1 6 5 8], which represents that the operation sequence of eight jobs on all machines is 2-3-7-4-1-6-5-8. In addition, the job-sequences of initial solutions are generated by random in the BMSA. The main reason that we generated initial solutions by random is that the random approach offers more opportunities to obtain diversified solutions than using specific constructive heuristics which are restricted to a small area of the solution space, hence leading to better results.

3.2. Parameters

The BMSA have seven parameters, namely I_{iter}, T_0, T_F, $S_{\max}$, K, α and P_{size}, where I_{iter} denotes the number of iterations performed by the search at a particular temperature; T_0 represents the initial temperature, T_F is the final temperature; $S_{\max}$ is the maximal solution evaluated; K stands for the Boltzmann constant influencing the probability of accepting a worse solution; α denotes the coefficient controlling the cooling schedule, and P_{size} is the number of multi-start points in the BMSA.

3.3. Neighborhoods

Let S denotes the set of feasible solutions and let Π_k $(k=1,\ldots,P_{size})$ represents the current solution. The set $N(\Pi_k)$ $(k=1, \ldots, P_{size})$ is then the set of solutions neighboring Π_k, which is generated by either a swap or insertion operation in the BMSA. Briefly, each $N(\Pi_k)$ is sampled by randomly selecting a pair of jobs of Π_k in order to swap them directly; or by randomly selecting one job of Π_k and inserting the chosen job immediately before another job. The probabilities of performing the swap and insertion operations were fixed at 0.5 and 0.5, respectively.

3.4. BMSA procedures

The procedures of the proposed BMSA are depicted in Fig. 1. First, a set of initial solutions Π_k $(k=1,\ldots,P_{size})$ is randomly generated as multi-start points. Next, the current temperature T is set to T_0. During each iteration, a new solution Π'_k is chosen from its corresponding $N(\Pi_k)$. Then, the difference between Π'_k and Π_k of the first and second objective function values, i.e., $\Delta_k^{C_{\max}}=C_{\max}(\Pi'_k)-C_{\max}(\Pi_k)$ and $\Delta_k^{TFT}=TFT(\Pi'_k)-TFT(\Pi_k)$, are calculated. In view of the fact that a bi-objective is being considered, there are five conditions for handling the new neighbor solution Π'_k:

(1) $\Delta_k^{C_{\max}} \le 0$ and $\Delta_k^{TFT} \le 0$: the transition is accepted;
(2) $\Delta_k^{C_{\max}} > 0$ and $\Delta_k^{TFT} \le 0$: the transition is accepted only if $r < \text{Exp}(-\Delta_k^{C_{\max}}/KT)$;
(3) $\Delta_k^{C_{\max}} \le 0$ and $\Delta_k^{TFT} > 0$, the transition is accepted only if $r < \text{Exp}(-\Delta_k^{TFT}/KT)$;
(4) $\Delta_k^{C_{\max}} > 0$, $\Delta_k^{TFT} > 0$ and $\Delta_k^{C_{\max}}/C_{\max}(\Pi'_k) < \Delta_k^{TFT}/TFT(\Pi'_k)$, the transition is accepted only if $r < \text{Exp}(-\Delta_k^{TFT}/KT)$;
(5) $\Delta_k^{C_{\max}} > 0$, $\Delta_k^{TFT} > 0$ and $\Delta_k^{C_{\max}}/C_{\max}(\Pi'_k) \ge \Delta_k^{TFT}/TFT(\Pi'_k)$, the transition is accepted only if $r < \text{Exp}(-\Delta_k^{C_{\max}}/KT)$.

where r is a randomly generated number from the uniform distribution and the Boltzmann constant $K=1$ for the conditions (2) and (5), $K=n \times 0.6$ for the conditions (3) and (4). As the BMSA progresses, if Π_k dominates all solutions in the non-dominated solution set P_S, then all of the solutions Π_k $(k=1,\ldots, P_{size})$ are set to be the same as Π_k and the BMSA procedure is continued.

After updating the current solutions and P_S, once I_{iter} iterations from the previous temperature decrease were run, the temperature T is decreased according to the formula $T \leftarrow \alpha T$, where $0 < \alpha < 1$. When T is decreased, a local search procedure which sequentially performs swap and insertion operations is used to improve Π_k $(k=1, \ldots, P_{size})$. If T is less than T_F or the evaluated solutions exceeds $S_{\max}$, the algorithm is terminated. Following the termination of search procedures, a non-dominated solutions set may be obtained.

4. Computational results and discussion

In this section, we describe the computational tests used to evaluate the performance of the proposed BMSA. The details of the test problems, the parameters selection, and the computational results of the proposed BMSA against the best-so-far algorithms are depicted in the following subsections.

4.1. Test problems

To evaluate the performance of the proposed BMSA, as used by the best-so-far algorithms for the $F|prmu|C_{\max}, \sum C_i$ problem, computational experiments are conducted on 90 benchmark instances from the well-known benchmark problem set of Taillard [20]. These benchmark instances consisted of nine different sizes with the number of jobs varying from 20 to 100, and the number of machines varying from 5 to 20. Briefly, these benchmark instances were randomly generated as follows [20]: for each job i $(i=1, \ldots, n)$ on each machine j $(j=1, \ldots, m)$, an integer processing time p_{ij} was generated from the uniform distribution [1,99]. In order to propose instances that were as hard as possible, Taillard generated many instances of problems, then 10 instances

BMSA (I_{iter}, T_0, T_F, $S_{\max}$, K, α and P_{size})

Step 1: Generate the initial solutions Π_k ($k=1, \ldots, P_{size}$) randomly;
Step 2: Let $T=T_0$; $Iter=0$; S=0; Update non-dominated front P_S;
Step 3: $Iter = Iter+1$;
Step 4: For $k=1$ to P_{size} {
  Step 4.1: Randomly generate a neighbor Π'_k of Π_k by a/an swap/insertion procedure for the inner level; S= S+1;
  Step 4.2: $\Delta_k^{C_{\max}} = C_{\max}(\Pi'_k) - C_{\max}(\Pi_k)$; $\Delta_k^{TFT} = TFT(\Pi'_k) - TFT(\Pi_k)$;
    Generate $r \sim U(0,1)$;
    IF $\Delta_k^{C_{\max}} \le 0$ and $\Delta_k^{TFT} \le 0$ THEN $\Pi_k = \Pi'_k$;
    ELSE_IF $\Delta_k^{C_{\max}} > 0$ AND $\Delta_k^{TFT} \le 0$ {
      IF $r < Exp(-\Delta_k^{C_{\max}}/KT)$ THEN $\Pi_k = \Pi'_k$;
    }
    ELSE_IF $\Delta_k^{C_{\max}} \le 0$ AND $\Delta_k^{TFT} > 0$ {
      IF $r < Exp(-\Delta_k^{TFT}/KT)$ THEN $\Pi_k = \Pi'_k$;
    }
    ELSE_IF $\Delta_k^{C_{\max}} > 0$, $\Delta_k^{TFT} > 0$ AND $\Delta_k^{C_{\max}}/C_{\max}(\Pi'_k) < \Delta_k^{TFT}/TFT(\Pi'_k)$ {
      IF $r < Exp(-\Delta_k^{TFT}/KT)$; THEN $\Pi_k = \Pi'_k$;
    }
    ELSE_IF $\Delta_k^{C_{\max}} > 0$, $\Delta_k^{TFT} > 0$ AND $\Delta_k^{C_{\max}}/C_{\max}(\Pi'_k) \ge \Delta_k^{TFT}/TFT(\Pi'_k)$ {
      IF $r < Exp(-\Delta_k^{C_{\max}}/KT)$ THEN $\Pi_k = \Pi'_k$;
    }
  Step 4.3: IF (Π_k dominates all solution in P_S) THEN $\Pi_j = \Pi_k, \forall j$;
  Step 4.4: Update non-dominated front P_S;
}
Step 5: IF ($Iter = Iter_{\max}$)
  {
    $T = \alpha T$;
    $Iter = 0$;
    Perform Local Search on Π_k and update S value, $\forall k$;
    IF (Π_k dominates all solution in P_S) THEN $\Pi_j = \Pi_k, \forall j$;
  }
Step 6: IF $T < T_{final}$ or $S_{\max} < S$ THEN terminate the BMSA procedure;
  Else go to Step 3;

Fig. 1. The pseudo-code of proposed BMSA heuristic.

of each size of problems that appeared to be the hardest ones were selected to form a basic problem set. The test problem files are available via Taillard's web site (URL: http://www.idsia.ch/~eric) or can be downloaded from the OR-Library web site (URL: http://people.brunel.ac.uk/~mastjjb/jeb/orlib/flowshopinfo.html).

4.2. Performance measures

To measure the performance of the proposed BMSA, four pairwise comparison measures [13,15,17,22] with respect to the number of solutions in the non-dominated solution front, i.e., A, B, B/A and C, were used. The indicator A denotes the number of efficient solutions found by each algorithm, B is the number of non-dominated solutions in the net non-dominated front found by each algorithm, B/A denotes the percentage of non-dominated solutions (the quotient between B and A) found by each algorithm, and C is a coverage set measure that can be defined in the following manner [22]: let X and Y be two sets of decision vectors representing the sets of solutions found by two algorithms. Then, C maps the ordered pair (X,Y) into the interval [0,1]:

$$C(X,Y) = \frac{|x \in X / \exists y \in Y : x \succeq y|}{|Y|}$$

Clearly, the larger the value of C the better the performance of X. Especially, all solutions in Y are dominated by X in the case of $C(X,Y)=1$, and none of the solutions in Y are dominated by X in the case of $C(X,Y)=0$.

4.3. Algorithm parameters selection

To determine the appropriate values of experimental parameters, an extensive computational testing was performed. To compare the proposed BMSA with different existing algorithms on a fair basis, two better sets of parameters are then adopted for the subsequent experiments in this study: (1) $I_{iter}=n \times 1500/P_{size}$, $T_0=7.5$, $T_F=0.0$, $S_{\max}=1.875(n-1)(6n+15{,}380)$ and $\alpha=0.90$; (2) $I_{iter}=n \times 4500/P_{size}$, $T_0=7.5$, $T_F=0.3$, $S_{\max}=\infty$ and $\alpha=0.90$, where P_{size} is an integer ranges from 1 to 9. It is clear that the

Table 1
Number of non-dominated solutions contributed by each algorithm to the net non-dominated front.

		Problem number										
Problem size	**Algorithm**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Average**
(a)												
20×5	BMSA	1	6	12	8	11	9	11	9	7	15	8.9
	MOSA-I	1	1	0	3	6	3	0	3	1	0	1.8
	MOSA-II	1	0	1	4	9	3	5	6	4	0	3.3
	GPWGA	0	0	0	0	0	0	0	2	0	0	0.2
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	0	2	1	3	1	1	0	0	2	0	1.0
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		2	9	14	16	19	7	16	17	13	15	12.8
20×10	BMSA	7	14	4	14	6	8	11	5	1	9	7.9
	MOSA-I	4	2	3	2	12	11	12	5	6	3	6.0
	MOSA-II	9	6	2	8	6	6	5	5	8	8	6.3
	GPWGA	0	0	0	5	0	0	0	0	0	0	0.5
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	0	1	1	0	3	3	4	1	7	0	2.0
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		16	8	10	26	12	28	13	15	20	19	16.7
20×20	BMSA	5	11	15	3	3	5	4	18	6	14	8.4
	MOSA-I	9	5	10	2	1	6	14	3	4	5	5.9
	MOSA-II	4	8	10	14	13	8	5	10	12	4	8.8
	GPWGA	0	0	0	0	0	0	0	0	0	0	0.0
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	0	1	2	6	1	2	0	5	2	0	1.9
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		15	25	32	18	18	21	19	31	22	20	22.1
(b)												
50×5	BMSA	8	6	7	10	6	6	7	3	13	9	7.5
	MOSA-I	1	2	1	0	1	1	1	0	1	1	0.9
	MOSA-II	1	2	0	0	0	2	2	1	2	0	1.0
	GPWGA	2	0	0	0	1	0	0	0	0	0	0.3
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	0	1	0	0	1	0	0	0	0	0	0.2
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		12	11	8	10	9	9	10	4	15	10	9.8
50×10	BMSA	4	10	14	23	13	20	10	17	10	11	13.2
	MOSA-I	0	11	4	0	0	0	0	2	0	2	1.9

Table 1 (*continued*)

Problem size	**Algorithm**	**Problem number**										
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Average**
	MOSA-II	6	2	9	0	4	0	0	1	1	4	2.7
	GPWGA	0	0	0	0	0	0	0	0	1	0	0.1
	PH	0	0	0	0	0	0	0	0	1	0	0.1
	MOGLS	0	0	0	0	0	0	0	0	1	0	0.1
	ENGA	0	0	0	0	0	0	0	0	1	0	0.1
Number of solutions in the net front		10	23	27	23	17	20	10	20	11	17	17.8
50×20	BMSA	13	21	13	15	26	14	20	12	13	13	16.0
	MOSA-I	0	1	5	0	0	0	0	0	0	2	0.8
	MOSA-II	0	4	11	0	0	12	1	0	6	6	4.0
	GPWGA	0	0	0	0	0	0	0	0	0	0	0.0
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	0	0	0	0	0	0	0	0	0	0	0.0
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		13	27	29	15	26	26	21	12	19	22	21.0
(c)												
100×5	BMSA	4	10	7	4	12	6	5	10	7	3	6.8
	MOSA-I	0	0	5	6	1	1	0	0	1	2	1.6
	MOSA-II	1	0	8	3	2	1	1	6	1	0	2.3
	GPWGA	0	2	2	0	1	0	0	0	0	2	0.7
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	0	0	0	0	0	0	0	0	0	0	0.0
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		5	12	22	13	16	8	6	16	9	7	11.4
100×10	BMSA	9	14	16	9	9	9	6	3	12	1	8.8
	MOSA-I	1	6	2	12	5	3	1	10	1	5	4.6
	MOSA-II	4	10	3	0	0	10	4	1	8	8	4.8
	GPWGA	0	0	0	0	0	0	1	0	0	0	0.1
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	1	0	0	0	0	0	0	0	0	0	0.1
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		15	28	21	21	14	22	12	14	21	14	18.2
100×20	BMSA	20	7	19	15	2	9	10	20	17	14	13.3
	MOSA-I	0	7	4	0	0	0	0	1	0	0	1.2
	MOSA-II	0	11	3	21	11	8	13	0	0	4	7.1
	GPWGA	0	0	0	0	0	0	0	7	0	0	0.7
	PH	0	0	0	0	0	0	0	0	0	0	0.0
	MOGLS	0	0	0	0	0	0	0	0	0	0	0.0
	ENGA	0	0	0	0	0	0	0	0	0	0	0.0
Number of solutions in the net front		20	25	26	36	13	17	23	28	17	18	22.3

computational time required for the first parameter set is much shorter than that of the second parameter set. Since the number of iterations (I_{iter}) is inversely proportion to the value of P_{size}, the number of solutions evaluated by the proposed BMSA for the same problem are almost the same for different P_{size} values, making the comparison fair. Meanwhile, the pilot experimental results reveal that the best performance of the BMSA was obtained when P_{size} is set to around 3. Therefore, this parameter setting is adopted by the BMSA for further comparison with the best-so-far algorithms.

4.4. Results and discussion

The proposed BMSA was implemented using the C language, and run on a PC with an Intel Pentium 4 (1.8 GHz) CPU and 1024 MB memory. After an extensive literature survey, the algorithmic performance of the proposed BMSA was compared with the following eight existing benchmark algorithms that executed on the same benchmark problem set: GPWGA [13], MOGLS [15], ENGA [16], PH [17], MOSA-I [14], MOSA-II [14], MOIGS [5], and MOACA [19].

For comparing with MOSA-I, MOSA-II, GPWGA, PH, MOGLS, and ENGA algorithms on the same basis, the computational results of one run of BMSA with the first parameter set were used. These are done so as to evaluate the relative effectiveness of BMSA with the same number of enumerated solutions in the course of the entire search process as done by MOSA-II algorithm, i.e., 562020, 1458240 and 2975940 enumerated solutions for the instances with $n=20$, 50, and 100, respectively. The number of non-dominated solutions contributed by BMSA and these six existing benchmark algorithms to the non-dominated front are shown in Table 1a–c. Each cell in the table denotes the number of non-dominated solutions obtained by each algorithm for each problem instance. The results from these tables show that the proposed BMSA yields the most non-dominated solutions in most of the test instances. For the 90 benchmark instances, in totally, the proposed BMSA obtained the most non-dominated solutions in 78 instances.

The complete set of heuristically net non-dominated front for the ninety problem instances of Taillard (by consolidating the solutions obtained by the proposed BMSA and the best-so-far net non-dominated front (denoted by B_{RZ}) provided by Rajendran and Ziegler [19]) for each problem instance are given in Appendixs A–I. It is noted that the net non-dominated front provided by Rajendran and Ziegler are from the consolidation with the solutions obtained by 20 variants of the MOACA, the solutions presented by Varadharajan and Rajendran [14], and the solutions reported by Framinan and Leisten [5] through the execution of MOIGS with eight different values of the parameter d. Further, to compare the solutions obtained by the proposed BMSA with those of B_{RZ} on a fair basis, the computational results of the proposed BMSA that executed four runs with the second parameter set were used. It is believed that the new non-dominated front listed in Appendixs A–I can possibly serve as benchmark for future research attempts.

Table 2 listed the net non-dominated solutions found by the proposed BMSA corresponding to the number of solutions in the net non-dominated front. Each cell in the table denotes the non-dominated solutions found by BMSA and the number of solutions in the net non-dominated front. In total, BMSA found 1429 non-dominated solutions, while the number of solutions in the net non-dominated front is 2217. That is, the proposed BMSA

Table 2
Non-dominated solutions found by BMSA corresponding to the number of solutions in the net non-dominated front.

Problem size	Problem number										
	1	2	3	4	5	6	7	8	9	10	Total
20 × 5	1/5[a]	7/9	12/16	15/17	18/20	24/25	11/13	16/21	17/18	13/14	134/158
20 × 10	13/15	23/28	10/15	23/34	13/14	25/38	17/17	14/16	17/20	20/24	175/221
20 × 20	20/30	18/28	34/45	17/20	14/30	13/24	13/23	26/44	13/28	19/27	187/299
50 × 5	10/16	8/18	7/12	14/20	4/7	6/14	6/15	3/9	14/27	12/17	84/155
50 × 10	28/36	17/42	22/31	13/32	10/19	21/21	20/23	17/40	14/19	18/25	180/288
50 × 20	15/30	12/29	3/26	17/18	27/49	23/39	20/34	30/32	15/33	12/13	174/303
100 × 5	5/9	12/12	10/25	7/12	5/19	13/18	8/14	12/17	12/18	4/13	88/157
100 × 10	23/26	5/36	17/23	20/41	13/31	16/34	18/30	12/24	15/28	6/23	145/296
100 × 20	27/41	30/37	29/36	22/24	15/21	36/48	35/44	23/28	28/28	17/33	262/340

[a] Non-dominated solutions found by BMSA/Number of solutions in the net non-dominated front.

Table 3
Comparison of BMSA with B_{RZ} for pairwise comparison measures.

Problem size	Performance measure									
	C		*A*		*B*		*B/A*		*Ave. CPU* (s)	
	BMSA, B_{RZ}	B_{RZ}, BMSA	BMSA	B_{RZ}	BMSA	B_{RZ}	BMSA	B_{RZ}	BMSA	MOIGS
20 × 5	0.27	0.14	14.90	14.60	13.40	10.30	0.85	0.71	4.98	12.42
20 × 10	0.08	0.07	19.40	20.90	17.50	18.40	0.92	0.91	9.03	15.37
20 × 20	0.08	0.17	23.50	29.20	18.70	26.70	0.78	0.91	15.65	21.50
50 × 5	0.67	0.18	11.00	18.80	8.40	7.10	0.77	0.42	24.75	74.06
50 × 10	0.67	0.19	25.90	32.20	18.00	10.80	0.71	0.32	49.34	94.21
50 × 20	0.64	0.27	24.50	31.30	17.40	12.90	0.72	0.42	95.35	134.88
100 × 5	0.59	0.22	12.20	16.40	8.80	6.90	0.75	0.43	81.76	295.43
100 × 10	0.74	0.15	19.00	35.80	14.50	15.10	0.81	0.43	174.24	372.42
100 × 20	0.88	0.06	28.70	48.80	26.20	7.80	0.92	0.16	353.20	530.22
Average	0.51	0.16	19.90	27.56	15.88	12.89	0.80	0.52	89.81	172.28

contributed more than 64% of the solutions to the net non-dominated front, thereby demonstrating its effectiveness. The main reason that BMSA performs better than other algorithmic approaches in the $F|prmu|C_{\max}, \sum C_i$ problem is because the BMSA provides an appropriate framework that combines the respective advantages of SAs in effectively achieving search convergence and the multi-start hill climbing strategy in escaping local optimality.

In order to further compare the proposed BMSA with B_{RZ}, four pairwise comparison measures (C, A, B, and B/A) and the computational time (CPU time in second) required are listed in Table 3. As shown in Table 3, the total average values of indicators C, A, B, and B/A for BMSA is 0.51, 19.90, 15.88, and 0.80, respectively, while the corresponding values are 0.16, 27.56, 12.89, and 0.52 for B_{RZ}, respectively. It can be found that the proposed BMSA is superior to B_{RZ} across most problem sizes with respect to the four indicators. The analytical results of these four indicators also reveal that the majority of the solutions obtained by B_{RZ} are dominated by those of BMSA. Since the computational expenses may vary with hardware, software and programming skills, we did not directly compare the computational efficiency in this study. However, the average computational time of the proposed BMSA and the MOIGS algorithm listed in Table 3 show that the proposed BMSA is quite effective in discovering more non-dominated solutions.

5. Conclusions and future research

Multi-objective metaheuristic algorithms are a well-established research field for combinatorial optimization problems. In this paper, we consider the PFSP of developing heuristically non-dominated sequences with the objectives of minimizing makespan

Table A1
Net non-dominated front obtained for the problem size (20 × 5).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*
1278	14064(1)(2)	1359	15749(1)	1081	13813(1)	1293	16619(2)	1235	14163(1)(2)
1313	14058(2)	1361	15567(2)	1083	13793(1)	1299	16342(2)	1239	14151(1)(2)
1315	14048(2)	1364	15548(2)	1084	13771(1)	1301	15983(1)(2)	1243	14047(1)(2)
1324	14041(2)	1368	15522(1)	1085	13759(1)(2)	1303	15963(1)	1244	14002(1)(2)
1339	14033(2)	1369	15500(1)	1086	13666(1)(2)	1304	15925(1)(2)	1250	13943(1)(2)
		1371	15493(1)	1095	13665(2)	1306	15852(1)(2)	1254	13927(1)(2)
		1372	15448(1)	1096	13587(1)(2)	1307	15844(1)(2)	1264	13890(1)(2)
		1383	15156(1)(2)	1097	13545(1)	1309	15828(1)(2)	1266	13885(1)(2)
		1385	15151(1)(2)	1099	13524(2)	1311	15819(1)(2)	1278	13875(1)(2)
				1100	13505(1)(2)	1312	15784(1)	1285	13872(2)
				1107	13496(1)(2)	1313	15755(1)	1289	13834(1)(2)
				1111	13418(1)(2)	1316	15748(1)	1291	13823(1)
				1122	13400(1)(2)	1319	15716(1)	1293	13816(1)
				1140	13358(1)(2)	1320	15587(1)(2)	1301	13811(2)
				1183	13347(2)	1328	15555(1)	1305	13757(1)
				1289	13301(2)	1329	15484(1)(2)	1311	13732(1)(2)
						1354	15447(1)(2)	1328	13668(1)(2)
								1338	13619(1)(2)
								1360	13552(1)(2)
								1387	13529(1)(2)
Problem 6		**Problem 7**		**Problem 8**		**Problem 9**		**Problem 10**	
$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*
1195	14737(1)	1239	14069(1)	1206	14488(1)	1230	14977(1)(2)	1108	13649(1)(2)
1198	14719(1)	1241	13972(1)	1208	14444(1)	1232	14936(1)(2)	1112	13584(1)(2)
1199	14713(1)	1245	13870(1)	1211	14429(2)	1239	14935(1)	1113	13514(1)(2)
1200	14711(1)	1246	13855(1)	1212	14345(2)	1240	14924(1)	1115	13429(1)(2)
1202	14637(1)	1247	13807(1)	1213	14302(2)	1246	14902(2)	1126	13392(1)
1203	14624(1)	1249	13804(1)	1214	14253(2)	1247	14756(1)(2)	1134	13330(1)
1204	14619(1)	1252	13774(1)	1217	14157(1)(2)	1248	14729(1)(2)	1138	13173(1)(2)
1210	14346(2)	1253	13692(1)	1222	14153(1)(2)	1249	14715(1)(2)	1144	13129(1)
1213	14179(1)	1265	13629(1)	1225	14151(1)	1253	14505(1)(2)	1150	13126(1)(2)
1217	14045(1)	1269	13627(1)	1226	14129(1)	1255	14493(1)(2)	1151	13122(1)(2)
1218	13973(1)	1270	13612(1)	1229	14128(1)(2)	1256	14485(1)(2)	1153	13026(1)(2)
1220	13958(1)	1278	13578(2)	1230	14112(1)	1259	14449(1)(2)	1163	12999(1)(2)
1221	13945(1)	1283	13548(2)	1233	14093(1)(2)	1271	14446(1)(2)	1179	12981(2)
1222	13940(1)			1234	14080(1)(2)	1272	14386(1)(2)	1184	12943(1)(2)
1224	13608(1)(2)			1240	14068(1)	1281	14367(1)(2)		
1233	13583(1)(2)			1245	14067(1)	1284	14329(1)(2)		
1241	13581(1)(2)			1247	14059(2)	1336	14317(1)(2)		
1245	13332(1)			1252	14051(1)(2)	1337	14295(1)(2)		
1252	13280(1)(2)			1254	13994(1)(2)				
1255	13274(1)(2)			1320	13987(1)(2)				
1256	13212(1)(2)			1329	13948(1)(2)				
1257	13171(1)(2)								
1260	13160(1)(2)								
1261	13139(1)(2)								
1266	13123(1)(2)								

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

Table B1
Net non-dominated front obtained for the problem size (20 × 10).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(1)									
1582	22121(2)	1664	23888(2)	1496	20905(1)(2)	1377	19738(2)	1419	19277(1)(2)
1583	21731(1)(2)	1666	23877(2)	1501	20672(2)	1380	19721(2)	1420	19205(1)(2)
1590	21706(1)(2)	1667	23527(1)(2)	1508	20433(1)(2)	1381	19608(1)	1422	19203(1)(2)
1592	21421(1)(2)	1668	23525(1)(2)	1515	20364(1)(2)	1384	19563(1)	1432	18966(1)(2)
1595	21420(1)(2)	1671	23519(1)(2)	1521	20118(1)(2)	1385	19556(1)	1435	18952(1)(2)
1608	21385(1)(2)	1672	23399(1)(2)	1534	20061(1)(2)	1386	19533(1)(2)	1446	18873(1)(2)
1629	21337(2)	1676	23375(2)	1546	20036(1)(2)	1387	19523(1)(2)	1455	18855(1)
1640	21284(1)(2)	1678	23230(1)	1547	20003(1)(2)	1389	19499(1)	1459	18846(1)
1641	21204(1)(2)	1694	23166(1)(2)	1577	19962(2)	1390	19462(1)	1463	18829(1)(2)
1656	21122(1)(2)	1698	23157(1)	1589	19958(1)(2)	1392	19431(1)(2)	1466	18798(1)(2)
1685	21025(1)(2)	1699	23156(1)(2)	1615	19927(1)(2)	1393	19343(1)	1473	18794(1)(2)
1686	21011(1)(2)	1700	23112(1)(2)	1624	19917(1)(2)	1399	19280(1)(2)	1476	18766(1)(2)
1698	21003(1)(2)	1701	22999(1)(2)	1650	19877(2)	1400	19237(1)	1484	18754(1)
1705	20957(1)(2)	1706	22995(1)(2)	1693	19861(2)	1402	19214(1)	1486	18641(2)
1707	20911(1)(2)	1708	22853(1)(2)	1703	19833(2)	1406	19177(1)(2)		
		1728	22807(2)			1409	19149(1)(2)		
		1737	22726(1)(2)			1414	19120(1)		
		1744	22720(1)(2)			1416	19094(1)(2)		
		1758	22697(1)			1421	19091(1)		
		1781	22617(1)(2)			1424	19082(1)(2)		
		1782	22608(1)(2)			1425	19044(1)(2)		
		1818	22606(1)(2)			1431	19024(1)		
		1827	22559(2)			1432	19020(1)(2)		
		1831	22524(1)(2)			1437	18992(1)(2)		
		1841	22492(1)(2)			1443	18987(1)(2)		
		1847	22473(1)(2)			1445	18948(2)		
		1872	22446(1)(2)			1451	18908(2)		
		1893	22440(1)(2)			1473	18893(2)		
						1476	18852(2)		
						1493	18828(2)		
						1494	18800(2)		
						1509	18792(2)		
						1525	18751(2)		
						1558	18750(2)		

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(2)									
1397	20725(1)(2)	1484	19232(1)(2)	1544	22075(2)	1593	20779(1)(2)	1591	22719(1)(2)
1402	20612(1)(2)	1489	19217(1)	1545	21827(2)	1597	20765(1)(2)	1595	22575(1)(2)

Table B1 (*continued*)

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*
1403	20512(1)(2)	1492	19166(1)(2)	1546	20927(1)(2)	1602	20763(1)(2)	1598	22334(2)
1404	20374(1)(2)	1498	19159(1)(2)	1552	20682(1)	1607	20761(1)(2)	1603	21987(1)
1407	20264(1)	1500	18894(1)(2)	1556	20674(1)(2)	1608	20725(1)(2)	1604	21945(1)(2)
1410	20262(1)	1510	18846(1)(2)	1561	20489(1)(2)	1612	20651(1)(2)	1608	21930(1)(2)
1413	20127(1)(2)	1525	18765(1)(2)	1570	20480(1)(2)	1616	20592(1)(2)	1612	21882(1)(2)
1418	20053(1)	1526	18658(1)(2)	1573	20471(1)(2)	1622	20591(1)(2)	1630	21872(1)(2)
1423	20044(1)	1533	18598(1)(2)	1577	20466(1)(2)	1627	20564(1)(2)	1631	21756(1)
1424	20027(1)(2)	1540	18584(1)(2)	1578	20381(1)(2)	1635	20538(1)(2)	1632	21675(1)(2)
1427	20024(2)	1543	18526(1)(2)	1579	20374(1)(2)	1648	20487(1)(2)	1642	21662(1)(2)
1429	19896(1)(2)	1550	18476(1)(2)	1589	20358(1)(2)	1656	20483(1)	1647	21659(1)(2)
1436	19862(1)(2)	1562	18445(1)(2)	1594	20350(1)	1657	20454(1)(2)	1652	21627(1)
1440	19856(1)(2)	1579	18409(1)(2)	1598	20347(1)(2)	1668	20421(1)(2)	1657	21553(1)
1441	19838(1)(2)	1594	18377(1)(2)	1608	20288(1)(2)	1669	20419(1)(2)	1671	21497(1)(2)
1442	19775(2)	1600	18376(1)(2)	1641	20241(1)(2)	1676	20412(1)(2)	1681	21462(2)
1448	19735(1)	1617	18363(1)(2)			1677	20374(1)(2)	1684	21459(1)(2)
1451	19721(1)(2)					1685	20356(2)	1685	21453(1)(2)
1455	19711(1)(2)					1749	20347(2)	1712	21418(1)(2)
1462	19687(1)					1762	20330(2)	1735	21405(2)
1466	19621(1)							1768	21402(2)
1467	19598(1)							1770	21359(1)(2)
1477	19543(2)							1774	21352(1)(2)
1483	19522(1)(2)							1778	21320(1)(2)
1488	19500(1)								
1522	19495(2)								
1523	19473(2)								
1526	19464(1)								
1529	19416(1)								
1536	19395(1)								
1551	19382(2)								
1554	19357(2)								
1568	19340(2)								
1594	19331(2)								
1635	19290(2)								
1655	19283(2)								
1659	19249(2)								
1696	19245(2)								

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

Table C1
Net non-dominated front obtained for the problem size (20 × 20).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(1)									
2297	35831(2)	2099	33261(1)(2)	2328	36809(2)	2223	33282(2)	2294	36054(1)(2)
2298	35764(2)	2100	32912(1)(2)	2329	36746(1)	2224	32841(2)	2296	36044(1)
2299	35724(2)	2104	32874(1)(2)	2332	36578(1)(2)	2225	32546(2)	2300	36040(1)(2)
2300	35665(2)	2105	32786(1)(2)	2336	35985(1)(2)	2233	32516(1)(2)	2305	35960(1)
2301	35623(2)	2111	32769(1)(2)	2353	35829(1)(2)	2234	32231(1)(2)	2309	35834(1)(2)
2302	35384(1)(2)	2118	32708(1)	2363	35821(2)	2249	32124(1)(2)	2314	35608(2)
2303	35358(1)(2)	2120	32684(2)	2366	35739(1)(2)	2251	32121(1)(2)	2322	35528(1)(2)
2310	35322(1)(2)	2125	32681(2)	2369	35363(1)(2)	2253	32025(1)(2)	2336	35451(1)(2)
2313	35274(1)(2)	2126	32673(1)	2373	35251(1)(2)	2260	31993(1)(2)	2337	35440(2)
2317	35237(1)(2)	2128	32630(1)	2383	35243(1)(2)	2261	31928(1)(2)	2343	35365(1)(2)
2324	35195(1)(2)	2132	32489(2)	2385	35217(1)(2)	2263	31855(1)(2)	2345	35215(1)(2)
2325	34965(2)	2145	32482(1)(2)	2388	35120(1)(2)	2264	31826(1)(2)	2390	35214(2)
2341	34961(1)(2)	2146	32475(1)	2393	35035(1)	2265	31804(1)(2)	2399	35154(2)
2344	34954(2)	2147	32462(2)	2397	34995(1)	2276	31753(1)(2)	2401	35131(2)
2345	34738(1)(2)	2149	32360(1)(2)	2399	34991(1)(2)	2289	31726(1)(2)	2402	35076(1)(2)
2346	34581(1)(2)	2153	32339(2)	2400	34959(1)(2)	2296	31714(1)(2)	2411	34942(1)(2)
2351	34533(1)(2)	2154	32316(1)(2)	2402	34917(2)	2301	31708(1)(2)	2434	34805(2)
2352	34467(1)(2)	2163	32205(2)	2407	34840(1)(2)	2311	31690(1)(2)	2488	34765(1)
2355	34374(1)(2)	2166	32089(1)(2)	2414	34783(1)(2)	2387	31677(1)(2)	2519	34710(2)
2363	34220(1)(2)	2196	31906(2)	2422	34732(1)(2)	2405	31661(1)(2)	2537	34705(1)
2380	34139(1)(2)	2206	31826(2)	2426	34707(1)(2)			2538	34667(2)
2386	34126(1)(2)	2214	31777(1)(2)	2429	34703(1)(2)			2551	34662(1)
2388	34026(1)(2)	2238	31730(1)	2430	34679(1)(2)			2560	34659(2)
2391	33998(2)	2254	31716(2)	2433	34614(1)(2)			2564	34649(2)
2392	33901(1)(2)	2259	31713(2)	2435	34480(1)(2)			2570	34645(2)
2412	33827(2)	2261	31612(1)(2)	2449	34400(1)(2)			2571	34616(2)
2418	33799(2)	2275	31597(1)(2)	2453	34388(1)(2)			2607	34605(2)
2427	33742(1)(2)	2334	31587(1)(2)	2456	34385(1)(2)			2613	34602(2)
2434	33735(1)(2)			2465	34377(1)(2)			2617	34590(2)
2437	33623(1)(2)			2466	34364(1)(2)			2622	34557(2)
				2474	34232(1)(2)				
				2484	34127(1)(2)				
				2508	34125(2)				
				2526	34110(1)(2)				
				2535	34107(2)				
				2547	34101(2)				
				2549	34084(2)				
				2554	34082(2)				
				2555	34072(2)				
				2557	34055(2)				
				2564	34051(1)(2)				
				2567	34016(1)(2)				
				2578	33977(1)(2)				
				2579	33932(1)(2)				
				2608	33920(2)				

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(2)									
2230	34231(2)	2276	34517(2)	2200	34792(2)	2237	34532(1)(2)	2179	33545(1)
2232	33853(1)(2)	2278	33756(1)(2)	2202	34758(2)	2243	34516(1)(2)	2180	33462(2)

Table C1 (*continued*)

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
2234	33652(1)(2)	2282	33438(1)(2)	2205	34712(2)	2248	34363(1)(2)	2183	33264(1)(2)
2239	33557(2)	2292	33436(1)(2)	2209	34612(2)	2253	34360(1)(2)	2191	33240(2)
2242	33407(2)	2299	33425(1)(2)	2210	34555(2)	2258	34338(1)(2)	2196	33125(1)(2)
2250	33291(1)	2305	33390(1)(2)	2212	34129(1)(2)	2260	34183(1)(2)	2201	33106(1)
2257	33268(1)	2307	33353(1)(2)	2221	34123(1)(2)	2281	34178(1)(2)	2202	32937(1)(2)
2260	33160(2)	2320	33325(1)(2)	2222	33931(1)(2)	2289	34138(2)	2217	32909(1)
2263	32876(1)(2)	2324	33295(1)(2)	2224	33882(1)(2)	2290	34127(1)	2222	32900(1)
2270	32853(1)(2)	2326	33254(1)	2232	33691(1)	2297	34077(1)(2)	2224	32859(1)
2281	32810(1)(2)	2336	33253(2)	2238	33661(1)(2)	2308	34065(1)(2)	2225	32822(1)
2284	32778(1)(2)	2340	33221(1)(2)	2242	33640(2)	2310	34062(1)(2)	2231	32805(2)
2292	32758(1)(2)	2343	33211(1)(2)	2243	33420(1)(2)	2316	34035(1)	2238	32764(1)(2)
2299	32722(1)	2350	33206(1)(2)	2257	33267(1)(2)	2318	34001(1)	2241	32661(1)
2301	32695(1)	2353	33184(2)	2266	33107(1)(2)	2343	33959(2)	2245	32654(1)(2)
2320	32693(2)	2356	33178(2)	2273	33068(1)(2)	2356	33900(2)	2246	32583(1)(2)
2324	32656(1)(2)	2359	33139(2)	2284	33045(1)(2)	2360	33847(2)	2249	32497(1)(2)
2334	32655(1)(2)	2368	33107(2)	2294	32990(1)(2)	2372	33805(2)	2250	32477(1)(2)
2358	32652(2)	2389	33083(1)	2297	32975(1)(2)	2379	33772(2)	2270	32423(1)(2)
2359	32650(2)	2407	32987(2)	2299	32943(1)(2)	2418	33734(2)	2287	32383(1)(2)
2360	32625(2)	2415	32970(2)	2311	32921(2)	2419	33729(2)	2308	32375(2)
2365	32616(2)	2453	32951(2)	2312	32909(1)(2)	2425	33727(2)	2309	32331(2)
2369	32604(2)	2466	32922(2)	2314	32897(1)(2)	2427	33722(2)	2329	32310(2)
2372	32564(2)			2318	32880(1)(2)	2428	33641(2)	2338	32299(1)(2)
				2323	32865(1)(2)	2448	33634(2)	2339	32292(2)
				2329	32854(1)	2455	33625(2)	2345	32269(1)(2)
				2331	32814(1)(2)	2458	33623(2)	2365	32262(2)
				2341	32803(1)(2)	2486	33612(2)		
				2351	32793(2)				
				2353	32783(1)				
				2360	32775(1)(2)				
				2373	32679(1)(2)				
				2380	32663(1)(2)				
				2391	32642(2)				
				2393	32629(2)				
				2394	32603(2)				
				2396	32552(2)				
				2408	32524(2)				
				2415	32509(2)				
				2433	32506(2)				
				2470	32499(2)				
				2476	32494(2)				
				2478	32485(2)				
				2492	32444(1)(2)				

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

and total flowtime of jobs, i.e. the $F|prmu|C_{\max}, \sum C_i$ problem. A BMSA combining the respective advantages of the multi-start hill climbing strategy and SA is presented for this problem. The effectiveness and efficiency of the proposed BMSA is established by comparing it with the best-so-far algorithms. The computational results show that BMSA is more effective than the best-so-far algorithms on the same benchmark instances. These results indicate that following multi-start refinement the proposed BMSA heuristic can prevent the search from being trapped in a local optimum and increased the possibility of finding a global optimum. Further, we also provide a new complete set of heuristically net non-dominated front for the ninety problem instances of Taillard so that researchers can use them as benchmarks for additional research. In terms of both solution quality and computational effort, this study successfully reinforces the development of high-performance metaheuristic algorithms for the $F|prmu|C_{\max}, \sum C_i$ problem.

Since the $F|prmu|C_{\max}, \sum C_i$ problem represents a very important but largely unresearched problem, this topic deserves further attention. The following recommendations are made for future studies. First, it would be worthwhile to apply the proposed BMSA to other multi-objective PFSPs. Second, extension of the proposed BMSA to solve other multi-objective scheduling problems may prove both interesting and useful. Third, more studies are necessary to develop additional metaheuristics for this argument. Fourth, it would be worthwhile to develop exact methods for this problem. Finally, PFSPs with sequence dependent setup times and job release date constraints, while practical, are complex problems requiring further investigation.

Table D1
Net non-dominated front obtained for the problem size (50×5).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*
2724	67351(2)	2838	70392(1)	2621	65944(2)	2753	71062(1)	2863	71446(1)
2728	67344(2)	2843	69837(1)	2622	65612(1)	2757	70199(2)	2864	70577(2)
2729	66986(1)	2848	69791(2)	2630	65278(2)	2758	70088(2)	2865	70558(2)
2731	66590(1)	2849	69708(2)	2641	65081(2)	2759	70045(1)	2886	70214(1)
2735	65782(1)	2853	69693(2)	2642	64846(1)	2764	70036(2)	2887	70036(2)
2744	65776(2)	2854	69656(2)	2660	64817(2)	2765	69937(1)	2904	69738(1)
2745	65752(2)	2857	69522(2)	2663	64492(1)	2767	69633(2)	2916	69719(1)
2746	65726(2)	2859	68779(1)	2665	64232(2)	2768	69613(2)		
2747	65698(2)	2862	68774(1)	2667	63762(1)	2775	69586(2)		
2752	65109(1)	2871	68726(1)	2671	63652(1)	2777	69566(1)		
2764	65103(1)	2873	68554(1)	2733	63636(1)	2778	69406(1)		
2770	65062(1)	2884	68522(1)	2766	63628(1)	2782	69049(1)		
2774	65038(1)	2885	68512(1)			2819	69044(1)		
2790	64964(1)	2951	68507(2)			2822	69019(1)		
2795	64962(1)	2954	68491(2)			2828	68895(1)		
2807	64940(1)	2957	68457(2)			2864	68857(1)		
		2960	68415(2)			2875	68854(1)		
		2967	68413(2)			2894	68828(1)		
						2930	68800(1)		
						2931	68738(1)		

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*
2829	72618(2)	2725	71670(1)	2683	70478(2)	2554	72756(2)	2782	71801(2)
2832	69148(1)	2732	70120(2)	2686	69432(2)	2560	72368(2)	2783	70644(2)
2839	68900(2)	2736	69580(2)	2694	68338(2)	2561	67091(2)	2784	70372(1)
2841	68787(2)	2737	69491(2)	2697	68208(2)	2564	65484(1)	2786	70339(1)
2845	68346(2)	2741	68586(2)	2703	68033(2)	2565	65376(1)	2788	70310(1)
2846	68343(2)	2743	68487(2)	2704	67890(2)	2566	65371(1)	2789	69960(1)
2847	68095(2)	2745	67533(2)	2705	64780(1)	2568	65242(1)	2792	69813(1)
2864	67908(1)	2746	67482(2)	2799	64755(1)	2570	65196(2)	2794	69620(2)
2882	67833(2)	2758	67380(2)	2831	64736(1)	2571	64442(2)	2796	69597(2)
2886	67424(2)	2760	67212(2)			2572	64360(2)	2802	69518(1)
2887	67316(1)	2767	66600(1)			2573	64313(2)	2820	69517(1)
2888	67243(1)	2769	66526(1)			2577	64303(2)	2831	69499(1)
2919	67236(1)	2782	66523(1)			2581	64190(2)	2841	69495(1)
2921	67190(1)	2795	66514(1)			2584	64143(2)	2844	69489(2)
		2936	66497(1)			2589	64122(2)	2866	69462(1)
						2590	64100(2)	2891	69448(1)
						2595	64072(2)	2895	69414(1)
						2596	63721(1)		
						2599	63558(1)		
						2600	63538(1)		
						2620	63475(1)		
						2627	63449(1)		
						2631	63433(1)		
						2664	63306(1)		
						2699	63216(1)		
						2702	63212(1)		
						2710	63183(1)		

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

Table E1
Net non-dominated front obtained for the problem size (50×10).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(1)									
3027	97529(2)	2911	87898(1)	2878	86459(2)	3064	93360(2)	3012	92036(2)
3031	97447(2)	2912	87306(1)	2879	86207(2)	3065	92283(2)	3013	92013(2)
3033	96868(2)	2913	87240(1)	2883	85762(2)	3067	91415(2)	3014	91151(1)
3034	93859(2)	2919	87044(1)	2885	85740(2)	3071	90659(1)	3019	90642(1)
3037	90826(1)	2920	87010(1)	2887	85112(2)	3072	90400(1)	3020	90588(1)
3038	90814(1)	2921	87008(1)	2891	85071(1)	3075	90275(1)	3021	90063(1)
3039	90683(1)	2923	86823(1)	2892	85008(1)	3076	90103(1)	3026	89928(1)
3042	90657(1)	2926	86816(2)	2893	84799(1)	3077	89791(1)	3027	89862(1)
3043	90574(1)	2927	86753(2)	2909	84724(1)	3089	89786(1)	3030	89137(1)
3053	90551(1)	2928	86468(1)	2910	84533(1)	3095	89716(1)	3042	88694(2)
3059	90415(2)	2929	86399(2)	2914	84439(1)	3107	89633(1)	3045	88455(2)
3061	90342(1)	2931	85928(2)	2915	83844(1)	3112	89534(1)	3061	88131(2)
3062	90080(1)	2940	85913(2)	2916	83750(1)	3114	89403(2)	3063	88124(2)
3063	90057(1)	2947	85904(2)	2917	83316(2)	3115	89260(2)	3065	88044(2)
3065	89894(2)	2948	85781(2)	2919	83196(1)	3116	89053(2)	3079	87838(1)
3069	89312(2)	2949	85716(2)	2944	82822(1)	3123	89005(2)	3085	87668(2)
3072	89137(1)	2950	85285(2)	2947	82782(1)	3127	88736(2)	3107	87667(2)
3093	89127(1)	2952	85240(2)	2966	82282(1)	3128	88720(2)	3110	87615(1)
3095	89025(1)	2953	85189(2)	2977	82198(1)	3130	88483(2)	3119	87227(1)
3097	88992(1)	2958	85140(2)	2979	82154(2)	3139	88434(2)		
3100	88874(1)	2967	85087(1)	2981	81978(1)	3143	88209(2)		
3116	88859(1)	2975	85017(2)	2983	81943(2)	3150	88083(2)		
3119	88669(1)	2976	84876(1)	2986	81849(1)	3168	88066(2)		
3123	88644(1)	2978	84803(1)	2993	81605(1)	3200	87941(1)		
3127	88524(1)	2981	84790(1)	3004	81479(1)	3202	87932(1)		
3130	88523(2)	2982	84786(1)	3005	81460(1)	3212	87913(1)		
3145	88475(1)	2984	84585(1)	3009	81096(1)	3246	87903(1)		
3167	88401(1)	3000	84571(1)	3028	80888(2)	3262	87892(2)		
3171	88301(1)	3008	84495(1)	3050	80873(1)	3264	87692(2)		
3176	88295(1)	3020	84469(2)	3092	80577(1)	3274	87574(2)		
3178	88279(1)	3033	84428(1)	3103	80559(1)	3287	87509(2)		
3182	88244(1)	3034	84332(2)			3289	87321(2)		
3203	88139(1)	3059	84284(2)						
3256	88130(1)	3083	84262(2)						
3258	88124(1)	3085	84206(2)						
3266	88113(1)	3090	84198(2)						
		3095	84099(2)						
		3100	84084(2)						
		3106	83844(2)						
		3108	83812(2)						
		3116	83808(2)						
		3136	83722(2)						

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(2)									
3043	90182(1)	3115	99295(2)	3043	99269(2)	2908	91310(2)	3099	94227(1)
3055	89631(1)	3124	97762(2)	3045	98936(2)	2909	91160(2)	3104	93469(1)
3064	89467(1)	3126	96113(2)	3046	98444(2)	2910	89958(2)	3107	93458(1)
3065	89446(1)	3127	92200(1)	3048	97405(2)	2920	89740(2)	3108	93220(1)
3068	89431(1)	3128	92199(1)	3050	97236(2)	2923	89384(2)	3116	93192(1)
3073	89371(1)	3131	92164(1)	3052	97222(2)	2944	88442(1)	3121	92620(1)
3075	88352(1)	3133	91958(1)	3055	96960(2)	2962	88167(1)	3122	92528(1)
3079	88347(1)	3140	91472(1)	3056	95518(2)	2965	88097(1)	3123	91586(1)
3103	88239(1)	3144	91282(1)	3057	92513(1)	2966	88071(1)	3126	91536(1)
3108	88228(1)	3148	90183(1)	3058	92504(1)	2967	87991(1)	3127	91530(1)
3116	88172(1)	3165	90042(1)	3066	92496(2)	2989	87764(1)	3129	91194(1)
3117	88110(1)	3180	90027(1)	3067	91428(2)	2993	87761(1)	3138	91188(1)
3126	88092(1)	3182	89967(1)	3069	91420(2)	2994	87738(1)	3140	91132(1)
3144	87651(1)	3188	89949(1)	3072	91389(2)	2997	87679(1)	3147	91075(1)
3154	87640(1)	3197	89921(1)	3074	91327(2)	3007	87655(1)	3152	90861(1)
3155	87474(1)	3201	89895(1)	3077	89908(2)	3012	87362(1)	3157	90839(2)
3167	87464(1)	3244	89868(1)	3078	89746(2)	3014	87337(1)	3158	90081(2)
3177	87232(1)	3251	89847(1)	3082	89709(2)	3017	86761(1)	3164	89992(2)
3185	87224(1)	3271	89818(1)	3083	89595(2)	3025	86585(1)	3192	89946(2)
3200	87184(1)	3313	89795(1)	3086	89553(2)			3198	89804(2)
3205	87174(1)	3331	89767(1)	3087	89541(2)			3204	89709(2)
		3352	89549(1)	3091	89504(2)			3208	89535(2)
		3372	89488(1)	3092	89474(2)			3241	88892(1)
				3093	89416(2)			3333	88864(1)
				3101	89336(2)			3449	88770(1)
				3112	89256(1)				
				3119	88752(1)				
				3126	88731(1)				
				3137	88644(1)				
				3138	88639(1)				
				3141	88190(1)				
				3152	88082(1)				
				3155	88000(1)				
				3170	87997(1)				
				3175	87987(1)				
				3195	87974(1)				
				3218	87919(1)				
				3225	87906(1)				
				3289	87849(1)				
				3291	87792(1)				

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

Table F1
Net non-dominated front obtained for the problem size (50×20).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(1)									
3908	132472(1)	3744	125643(1)	3698	119742(1)	3785	127750(2)	3668	125806(2)
3912	130906(2)	3746	125551(1)	3700	119675(1)	3786	125480(1)	3672	125107(2)
3915	130783(2)	3748	124871(1)	3720	118930(2)	3790	125437(1)	3675	124900(2)
3921	130093(1)	3750	124868(1)	3760	118911(2)	3791	125177(1)	3681	124023(1)
3930	129745(1)	3754	124814(1)	3761	118906(2)	3792	124957(1)	3682	123966(1)
3933	129685(2)	3769	123577(1)	3763	118893(2)	3793	123754(1)	3683	123942(1)
3934	129376(2)	3776	123548(1)	3767	118857(1)	3820	123434(1)	3686	123933(1)
3941	129338(2)	3781	123397(1)	3773	118850(2)	3821	123334(1)	3687	123925(1)
3943	128374(1)	3782	123396(1)	3788	118817(2)	3826	123320(1)	3688	123743(1)
3944	128358(1)	3788	123327(1)	3794	118769(2)	3834	123103(1)	3689	123605(1)
3946	128272(1)	3798	123284(1)	3796	118739(2)	3835	123091(1)	3691	123554(1)
3947	128199(1)	3829	123201(2)	3799	118647(2)	3838	122931(1)	3694	122330(2)
3949	128051(1)	3831	123103(2)	3804	118593(2)	3841	122660(1)	3701	122102(2)
3953	127936(1)	3833	123078(2)	3809	118525(2)	3850	122635(1)	3708	122094(2)
3960	127919(1)	3841	122770(1)	3831	118517(2)	3851	122473(1)	3713	121949(1)
3996	127736(1)	3847	122626(2)	3834	118464(2)	3857	122276(1)	3736	121771(2)
4000	127723(1)	3852	122559(2)	3845	118269(2)	3861	122117(1)	3754	121673(2)
4043	127661(2)	3853	122208(2)	3937	118086(2)	3863	122098(1)	3762	121393(2)
4045	127655(2)	3862	121942(2)	3946	118083(2)			3764	121339(2)
4051	127573(1)	3888	121916(2)	4004	118036(2)			3777	121152(2)
4087	127518(2)	3894	121903(2)	4005	117926(2)			3789	121066(2)
4089	127332(1)	3895	121898(2)	4020	117636(2)			3797	120628(1)
4092	127325(1)	3915	121895(2)	4068	117619(2)			3832	120545(2)
4109	127308(2)	3920	121881(2)	4072	117600(2)			3833	120490(1)
4140	127302(2)	3921	121345(2)	4096	117556(2)			3842	120342(2)
4146	127040(2)	3961	121169(2)	4106	117500(2)			3850	120329(2)
4170	127037(2)	3962	121050(2)					3856	120256(2)
4177	126861(2)	3964	120562(2)					3859	119831(1)
4184	126846(2)	4099	120486(2)					3867	119830(1)
4202	126713(2)							3869	119728(1)
								3872	119700(1)
								3873	119697(1)
								3884	119610(1)
								3887	119519(1)
								3889	119494(1)
								3892	119462(1)
								3894	119396(1)
								3903	119388(1)
								3907	119358(1)
								3909	119348(2)
								3917	119314(2)
								3923	119308(1)
								3924	119288(1)
								3934	119258(1)
								3938	119225(1)

								3954	119222(2)
								3960	119183(2)
								3969	119165(2)
								3989	119156(2)

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*	$C_{\max}$	*TFT*
(2)									
3745	128128(1)	3765	129180(2)	3752	126797(1)	3812	130514(2)	3820	128480(2)
3746	128080(1)	3771	128924(2)	3753	126745(1)	3813	129684(2)	3824	127625(1)
3747	127087(1)	3773	128327(2)	3754	126730(1)	3814	128674(1)	3825	125891(1)
3751	126155(2)	3774	128285(2)	3760	126686(1)	3815	128308(1)	3834	125830(1)
3755	125986(2)	3781	128062(1)	3769	126482(1)	3819	127860(1)	3841	125795(1)
3758	125764(2)	3783	127473(1)	3783	126230(1)	3821	127444(1)	3889	125757(1)
3763	125663(2)	3786	127461(1)	3803	125258(1)	3828	126339(2)	3908	125731(1)
3765	125601(2)	3790	127398(1)	3815	125011(1)	3831	126321(2)	3910	125691(1)
3769	125590(2)	3792	127374(1)	3834	124880(1)	3833	126188(2)	4047	125666(1)
3771	125569(2)	3794	127340(1)	3836	124852(1)	3842	125873(2)	4074	125661(1)
3781	125359(1)	3795	127332(1)	3837	124822(1)	3846	125577(2)	4087	125465(1)
3790	124864(1)	3800	127328(1)	3862	124811(1)	3850	125575(2)	4131	125449(1)
3801	124818(1)	3803	126944(1)	3876	124665(1)	3857	125573(2)	4135	125379(1)
3807	124481(1)	3807	126885(1)	3877	124649(1)	3860	125552(2)		
3808	124463(1)	3810	126793(1)	3884	124518(2)	3869	125522(2)		
3817	124368(1)	3811	126178(1)	3889	124512(2)	3887	125484(2)		
3820	124219(2)	3815	125987(1)	3890	124120(1)	3889	125429(2)		
3823	124052(2)	3816	125815(1)	3892	124084(1)	3896	124974(1)		
3827	123906(2)	3824	125814(1)	3910	124080(1)	3897	124962(1)		
3829	123782(1)	3827	125802(1)	3937	124061(1)	3898	124029(1)		
3831	123722(1)	3832	125528(1)	3942	124042(1)	3911	124028(1)		
3832	123642(1)	3844	125118(1)	3950	124035(1)	3919	124006(1)		
3836	123545(1)	3852	125114(1)	3953	124014(1)	3921	123977(1)		
3839	123436(1)	3853	125082(1)	3954	123992(1)	3925	123974(1)		
3841	123381(1)	3950	125058(2)	3956	123980(1)	3929	123379(1)		
3844	123355(1)	3965	125043(2)	3965	123952(1)	3936	123356(2)		
3848	123314(1)	3978	125033(2)	3972	123927(1)	3943	123349(1)		
3859	123229(1)	3994	124972(2)	3980	123916(1)	3944	123292(2)		
3864	123019(1)	4002	124959(2)	3988	123909(1)	3950	123262(1)		
3870	122795(1)	4039	124937(2)	4016	123894(1)	3974	123253(1)		
3875	122737(1)	4049	124894(2)	4018	123873(1)	3979	123081(2)		
3882	122655(1)	4051	124816(2)	4175	123868(1)	3992	122879(2)		
3898	122166(1)	4079	124725(2)			3998	122779(2)		
3904	122157(2)	4227	124706(2)						
3915	122114(2)								
3921	122054(2)								
4092	122032(2)								
4098	122012(2)								
4106	121895(2)								

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

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Appendix A

See Table A1.

Appendix B

See Table B1.

Appendix C

See Table C1.

Appendix D

See Table D1.

Appendix E

See Table E1.

Appendix F

See Table F1.

Table G1
Net non-dominated front obtained for the problem size (100 × 5).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
5493	262040(2)	5283	251404(1)	5175	271196(2)	5017	269625(2)	5250	255617(2)
5495	257719(2)	5284	246755(1)	5177	268438(2)	5018	264631(2)	5251	255583(2)
5500	256992(2)	5289	246706(1)	5183	267517(2)	5019	244402(2)	5252	255567(2)
5527	256068(2)	5290	245921(1)	5186	267344(2)	5021	234041(2)	5255	247072(2)
5563	255767(1)	5293	245432(1)	5193	248064(1)	5023	233700(1)	5256	246971(2)
5564	255317(1)	5298	245336(1)	5194	245931(1)	5029	232753(1)	5257	246765(2)
5595	255267(1)	5300	245311(1)	5199	245438(1)	5035	230917(2)	5259	246663(2)
5609	255239(1)	5302	245249(1)	5200	245398(1)	5042	230584(1)	5260	245660(2)
5674	255179(1)	5305	244591(1)	5206	244666(1)	5044	229751(1)	5261	244612(2)
		5306	244568(1)	5207	244532(1)	5056	229720(1)	5263	244514(2)
		5311	244535(1)	5208	243806(2)	5096	229418(1)	5264	244450(2)
		5313	244426(1)	5209	243748(2)	5171	229295(1)	5267	244135(2)
				5210	243693(1)			5272	244104(2)
				5212	242690(2)			5276	244011(2)
				5221	242187(2)			5294	242647(1)
				5239	241938(2)			5339	242510(1)
				5240	241564(2)			5366	242425(1)
				5241	240893(1)			5394	242423(1)
				5244	240708(2)			5419	242402(1)
				5250	240634(2)				
				5251	240594(2)				
				5262	240509(2)				
				5267	240412(2)				
				5285	240128(1)				
				5291	239907(1)				

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
5135	262520(2)	5246	249966(1)	5094	245367(1)	5448	280016(2)	5322	278222(2)
5139	238702(1)	5255	247615(1)	5098	240896(1)	5454	254294(1)	5328	257564(2)
5147	238312(1)	5256	246961(1)	5102	240741(1)	5455	253157(1)	5329	257174(2)
5156	237401(2)	5259	246546(1)	5104	240585(1)	5464	252860(1)	5330	255535(2)
5157	236718(1)	5266	246268(1)	5105	240226(1)	5465	252622(2)	5334	250817(2)
5158	236466(2)	5270	245742(1)	5116	238815(1)	5468	252069(1)	5342	247238(2)
5159	236098(2)	5271	245091(1)	5119	238550(1)	5469	252021(1)	5346	246752(2)
5161	236039(2)	5276	244786(2)	5122	237448(1)	5474	251859(2)	5348	246746(2)
5162	235826(1)	5277	244483(2)	5130	236569(2)	5477	251854(2)	5372	246241(2)
5163	235822(1)	5279	244321(2)	5132	235109(1)	5479	251584(2)	5386	245335(1)
5164	235697(1)	5282	243988(2)	5134	234849(1)	5481	251560(2)	5389	244908(1)
5173	235657(1)	5296	243522(2)	5150	234557(2)	5494	251498(1)	5514	244904(1)
5178	235125(1)	5298	243281(2)	5152	234189(2)	5502	251433(1)	5527	244882(1)
5180	235093(1)	5305	242132(1)	5155	233910(2)	5503	251359(1)		
5186	234899(1)			5159	233908(2)	5504	251356(1)		
5196	234734(1)			5171	233305(1)	5510	251296(1)		
5204	234729(1)			5230	233246(1)	5514	251219(1)		
5288	234664(1)					5527	251176(1)		

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

Table H1
Net non-dominated front obtained for the problem size (100×10).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(1)									
5781	339398(2)	5362	299003(2)	5679	301194(1)	5826	342759(2)	5501	326804(2)
5782	339268(2)	5364	297917(2)	5681	300566(1)	5828	342586(2)	5505	326777(2)
5785	316108(1)	5365	297799(2)	5683	300506(1)	5829	321123(1)	5507	301808(2)
5788	315945(1)	5367	297348(2)	5686	300267(1)	5831	320749(1)	5509	301785(2)
5789	314861(1)	5370	297159(2)	5690	299879(1)	5832	320718(1)	5512	295526(1)
5790	314459(1)	5372	297037(2)	5691	297957(1)	5850	320153(1)	5513	294863(1)
5792	311070(1)	5373	296576(2)	5703	297229(1)	5852	316359(1)	5517	294703(1)
5793	311051(1)	5375	296557(2)	5705	296260(2)	5853	315555(1)	5521	294138(1)
5810	310739(1)	5377	289483(2)	5720	295830(2)	5869	315152(1)	5522	293218(1)
5812	310172(1)	5380	287083(1)	5724	295394(2)	5871	315065(1)	5535	293206(1)
5821	310157(1)	5386	285263(1)	5726	295343(2)	5872	314741(1)	5571	292242(1)
5835	309329(1)	5387	283589(2)	5731	295299(2)	5873	314729(1)	5593	291526(1)
5837	308812(1)	5391	283584(2)	5732	295189(2)	5874	313804(1)	5594	291504(1)
5838	308661(1)	5394	283533(2)	5736	294108(1)	5876	313786(1)	5609	291058(1)
5873	308550(2)	5395	283258(2)	5738	294030(1)	5877	313260(1)	5615	291003(1)
5874	306289(1)	5403	282655(2)	5748	293538(1)	5878	313231(1)	5617	290395(1)
5877	305160(1)	5407	282501(2)	5754	293441(1)	5883	313202(2)	5623	290354(1)
5878	305139(1)	5410	282206(2)	5762	293390(1)	5884	311505(2)	5649	290309(2)
5907	304904(1)	5414	281538(2)	5804	293021(1)	5886	311480(2)	5650	290256(2)
5912	304207(1)	5418	281040(2)	5807	292979(1)	5891	310851(2)	5670	290138(2)
5913	303928(1)	5422	280611(2)	5832	292975(1)	5902	309761(2)	5672	290086(2)
5914	303885(1)	5425	280367(1)	5839	292971(1)	5907	309571(2)	5673	289995(2)
5915	303772(1)	5427	279674(1)	5850	292533(1)	5911	309244(2)	5676	289959(2)
5916	303418(1)	5428	279571(1)			5922	309212(2)	5769	289957(2)
5921	303406(1)	5447	279259(2)			5923	309156(2)	5797	289915(2)
6032	303385(1)	5449	278666(2)			5933	309090(2)	5799	289827(2)
		5450	278656(2)			5938	308370(2)	5821	289714(2)
		5452	278650(2)			5943	308319(2)	5831	289682(2)
		5462	278591(2)			5952	308301(2)	5835	289667(2)
		5464	278464(2)			5957	308212(2)	5836	289589(2)
		5465	278418(2)			5968	307835(1)	5869	289498(2)
		5575	278415(2)			5974	307821(1)		
		5593	278229(2)			5998	307799(1)		
		5648	278189(2)			6010	307675(2)		
		5653	278142(2)			6015	307674(1)		
		5661	278077(2)			6029	306313(1)		
						6030	306201(1)		
						6088	306165(2)		
						6105	306153(2)		
						6153	306137(2)		
						6157	306034(2)		

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(2)									
5308	310735(2)	5602	299370(2)	5653	313225(2)	5916	354188(2)	5881	328588(2)
5311	310461(2)	5603	298022(2)	5655	312447(2)	5918	353627(2)	5883	328000(2)

Table H1 (*continued*)

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
5314	309833(2)	5604	298018(2)	5657	308086(2)	5919	349544(2)	5889	327950(2)
5315	309763(2)	5611	296590(2)	5658	307545(2)	5920	348927(2)	5892	327706(2)
5316	309576(2)	5617	296255(2)	5668	306254(2)	5923	347760(2)	5897	327126(2)
5317	292932(2)	5619	296149(2)	5670	306097(2)	5928	319642(2)	5903	300298(1)
5318	292018(2)	5620	295646(2)	5672	306018(2)	5932	319589(2)	5909	300223(2)
5319	291083(2)	5622	292096(2)	5675	304991(2)	5935	319029(2)	5910	298010(1)
5321	282465(1)	5623	292018(2)	5677	303990(2)	5940	313635(2)	5911	297922(1)
5322	282455(1)	5632	290221(1)	5680	302147(1)	5941	312810(2)	5912	297904(1)
5323	281774(1)	5641	288400(1)	5693	301794(1)	5942	312755(2)	5914	297650(1)
5326	280835(1)	5662	287523(1)	5694	301378(2)	5955	312314(2)	5928	297554(1)
5331	280751(2)	5673	287045(2)	5695	298228(1)	5958	312309(2)	5932	297333(2)
5332	280337(2)	5676	286988(2)	5719	296783(1)	5961	310702(1)	5933	297131(2)
5334	280324(2)	5695	286579(1)	5724	296614(1)	5964	309912(1)	5938	296920(2)
5345	279544(2)	5701	285576(1)	5725	296461(1)	5965	309875(1)	5951	296736(2)
5347	279248(2)	5702	284972(2)	5736	296190(1)	5967	309778(1)	5961	296729(2)
5348	278753(2)	5708	284054(1)	5739	296151(1)	5979	308246(1)	5962	296575(2)
5350	278714(2)	5717	283938(1)	5743	296073(1)	5982	308147(1)	5978	296442(2)
5357	278121(1)	5718	283818(1)	5756	296023(1)	5985	307801(1)	6001	296406(2)
5358	277944(1)	5729	283731(1)	5760	295985(1)	5986	307533(1)	6002	296398(2)
5361	277590(2)	5739	283535(1)	5761	295885(1)	5987	306846(1)	6015	296343(2)
5397	276658(1)	5768	283405(1)	5915	295714(2)	6012	306128(1)	6023	296140(2)
5401	276617(1)	5787	283337(1)	5988	295638(2)	6022	306115(1)		
5403	276087(1)	5818	283324(1)			6026	306090(1)		
5406	275775(1)	5824	283299(1)			6027	305831(1)		
5407	275623(1)	5828	283241(1)			6031	305745(1)		
5408	275610(1)	5830	283239(1)			6049	305736(1)		
5410	275517(1)	5848	283207(1)						
5440	274952(1)	5884	283054(1)						
5454	274933(1)								
5462	274813(2)								
5465	274749(2)								
5467	274644(1)								

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

Table I1
Net non-dominated front obtained for the problem size (100×20).

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(1)									
6341	383274(1)	6302	392791(1)	6383	400928(2)	6363	403871(2)	6422	384925(1)
6348	383090(1)	6303	392670(1)	6389	400907(2)	6364	401043(2)	6444	384528(1)
6363	382931(1)	6306	392495(1)	6390	400820(2)	6369	392100(1)	6452	383867(1)
6365	382871(1)	6314	392067(1)	6391	400743(2)	6375	390910(1)	6465	383853(1)
6366	382761(1)	6318	392030(1)	6394	400720(2)	6376	390905(1)	6476	383139(1)
6369	381122(1)	6322	391227(1)	6395	392250(2)	6377	390896(1)	6481	383038(1)
6371	381091(1)	6327	390360(1)	6411	387694(1)	6402	390352(1)	6483	382875(1)
6419	378624(1)	6350	389085(1)	6412	387572(1)	6404	390196(1)	6498	382341(1)
6420	378556(1)	6351	388831(1)	6426	387453(2)	6407	390195(1)	6502	382010(1)
6478	378549(1)	6360	388249(1)	6436	386744(1)	6408	390048(1)	6504	381632(2)
6481	378534(1)	6361	388111(1)	6464	385314(1)	6415	386536(1)	6510	378543(1)
6484	378193(1)	6362	387918(1)	6472	385156(1)	6419	386518(1)	6532	377701(1)
6488	377804(1)	6366	387898(1)	6474	383433(1)	6500	386457(1)	6542	377695(1)
6492	376615(1)	6367	386759(1)	6476	383397(1)	6501	384850(1)	6544	376904(1)
6501	376190(1)	6413	386726(2)	6481	383394(1)	6502	383592(1)	6560	376714(1)
6514	375922(2)	6420	385568(2)	6485	383347(1)	6519	383555(1)	6691	376613(1)
6520	375833(2)	6423	384944(2)	6486	382588(1)	6522	383539(1)	6699	376432(2)
6521	375831(2)	6426	384892(2)	6489	382331(1)	6526	382130(1)	6701	376421(2)
6527	375070(1)	6429	383777(2)	6494	382313(1)	6548	381957(1)	6709	375943(2)
6536	375040(1)	6431	383224(2)	6495	382072(1)	6553	380282(1)	6732	375857(2)
6560	374983(1)	6433	383211(2)	6520	381843(1)	6578	379250(1)	6733	375346(2)
6567	374923(1)	6475	382027(1)	6522	381789(1)	6627	379228(1)		
6570	374916(1)	6477	381931(1)	6534	381449(1)	6631	379222(1)		
6574	374849(1)	6508	381910(1)	6556	380977(1)	6775	379172(1)		
6575	374429(1)	6557	381895(1)	6562	380616(1)				
6585	374385(1)	6558	381397(1)	6573	379823(1)				
6625	374382(1)	6561	379549(1)	6601	379661(1)				
6633	374364(1)	6570	379470(1)	6606	379043(1)				
6654	374274(1)	6614	379467(1)	6609	378824(1)				
6659	374254(1)	6620	379435(1)	6656	377532(1)				
6736	374010(2)	6623	379427(1)	6685	377168(1)				
6786	373687(2)	6654	379225(1)	6725	377160(1)				
6790	373563(2)	6665	379148(1)	6734	377152(1)				
6799	373534(2)	6673	379125(1)	6742	377052(1)				
6800	373462(2)	6677	379095(1)	6761	377007(1)				
6809	373250(2)	6684	379081(1)	6762	376938(1)				
6814	373218(2)	6685	379074(1)						
6838	373193(2)								
6840	373148(2)								
6902	373140(2)								
6915	373051(2)								

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
(2)									
6487	396342(1)	6394	403384(2)	6533	407390(1)	6377	391731(1)	6528	409325(2)
6488	392050(1)	6395	403352(2)	6534	407032(1)	6403	391642(1)	6529	409286(2)

Table I1 (*continued*)

Problem 6		Problem 7		Problem 8		Problem 9		Problem 10	
C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*	C_{max}	*TFT*
6501	392035(1)	6397	398870(2)	6535	404470(1)	6404	390907(1)	6541	398403(1)
6504	390572(1)	6398	398859(2)	6537	403488(1)	6405	389491(1)	6563	397161(1)
6506	390547(1)	6400	398775(2)	6540	403404(1)	6406	389178(1)	6564	395869(1)
6508	389874(1)	6409	392242(1)	6543	400108(1)	6414	387834(1)	6583	395563(2)
6512	389463(1)	6410	392137(1)	6562	398841(1)	6429	387597(1)	6585	395375(2)
6522	389461(1)	6417	390141(1)	6566	398240(1)	6432	387587(1)	6588	394639(2)
6523	388576(1)	6418	390064(1)	6655	396514(1)	6433	387459(1)	6603	393528(2)
6530	388307(1)	6422	389809(1)	6661	395981(1)	6437	387280(1)	6606	393255(2)
6533	388031(1)	6424	389709(1)	6671	395839(1)	6441	386850(1)	6624	392822(2)
6543	387696(1)	6439	389584(1)	6688	395824(1)	6457	386785(1)	6625	392802(2)
6545	385512(1)	6442	389402(1)	6706	395013(2)	6474	385697(1)	6626	392397(1)
6546	385506(1)	6465	388495(1)	6711	394922(2)	6475	385552(1)	6637	391442(2)
6547	385362(1)	6467	387319(1)	6721	394850(2)	6481	385488(1)	6644	391163(2)
6549	385182(1)	6468	387214(1)	6731	394669(2)	6484	384888(1)	6654	390946(2)
6550	385031(1)	6469	387201(1)	6733	393458(2)	6487	384083(1)	6663	389961(2)
6572	384995(1)	6473	387171(1)	6744	393386(1)	6494	383963(1)	6667	389957(2)
6573	384635(1)	6475	387052(1)	6758	393265(1)	6504	383750(1)	6672	389951(2)
6574	383889(1)	6479	386865(1)	6761	393211(1)	6514	383624(1)	6674	389815(2)
6575	383876(1)	6480	386506(1)	6778	393045(1)	6569	382537(1)	6676	389549(1)
6579	383856(1)	6483	386422(1)	6780	392888(1)	6589	382476(1)	6691	389484(1)
6590	383828(1)	6490	386412(1)	6804	392761(1)	6629	382459(1)	6692	388862(1)
6597	383099(1)	6497	386395(1)	6827	392744(1)	6662	382458(1)	6699	387422(1)
6600	383082(1)	6539	386312(1)	6851	392560(1)	6679	381612(1)	6703	386360(1)
6601	383070(1)	6543	386307(1)	6855	392480(1)	6705	381532(1)	6738	386228(1)
6604	383022(1)	6547	386298(1)	6889	392473(1)	6711	380860(1)	6751	386118(1)
6606	382023(1)	6549	386297(1)	6901	392449(1)	6727	380802(1)	6753	385948(1)
6608	381947(1)	6550	385858(1)					6764	385877(1)
6614	381906(1)	6554	384073(1)					6774	385691(1)
6670	381845(1)	6577	383549(1)					6844	385685(1)
6679	381706(1)	6579	382702(1)					6852	384756(1)
6688	381688(1)	6581	382614(1)					6865	384518(1)
6690	381671(1)	6582	382580(1)						
6705	381665(2)	6624	382436(1)						
6708	381565(1)	6641	382433(1)						
6722	381548(2)	6648	382416(1)						
6731	381444(2)	6649	382290(1)						
6736	381263(1)	6650	382227(1)						
6757	380832(2)	6651	382137(1)						
6786	380640(2)	6698	381782(2)						
6789	380601(2)	6699	381671(2)						
6790	380479(2)	6701	381579(2)						
6804	380466(2)	6824	381504(2)						
6816	380253(2)								
6827	380235(2)								
6829	379890(2)								
6864	379836(2)								

(1) Denotes the solution was obtained by BMSA.
(2) Denotes the solution was obtained by B_{RZ}.

Appendix G

See Table G1.

Appendix H

See Table H1.

Appendix I

See Table I1.

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