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Applying multi-start simulated annealing to schedule a flowline manufacturing cell with sequence dependent family setup times

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ABSTRACT

Meta-heuristics that attempt to obtain (near) global optimal solutions of NP-hard combinatorial optimization problems generally require diversification to escape from local optimality. One way to achieve diversification is to utilize the multi-start hill climbing strategy. By combining the respective advantages of the multi-start hill climbing strategy and simulated annealing (SA), an effective multi-start simulated annealing (MSA) heuristic is proposed to minimize the makespan for a flowline manufacturing cell scheduling problem with sequence dependent family setup times. The heuristic performance is evaluated by comparing the results achieved by the proposed heuristic with those achieved by the existing meta-heuristics. The computational results show that following multi-start refinement the proposed MSA heuristic is more effective compared to the state-of the-art meta-heuristics on the same benchmark instances.

1. Introduction

Given ever-changing customer requirements, cellular manufacturing (CM) has been broadly applied in small-to-medium lot productions of highly customized and complex products (Safaei and Tavakkoli-Moghaddam, 2009). The extensive applications of CM make the manufacturing cell scheduling problem (MCSP) an area of active research in scheduling. MCSPs are mainly concerned with sequencing part families and sequencing parts within families, which may involve either single or multiple stages (Hendizadeh et al., 2008). When each part is processed on each machine in the same order of a multiple stages MCSP, it is called a flowline manufacturing cell scheduling problem (FMCSP; Schaller et al., 2000).

In practice, parts are usually assigned to part families based on similarities in characteristics and operation requirements, and the similarities among parts in a part family enable setup reductions that lead to reduced lot sizes (Ateme-Nguema and Dao, 2009; Lin et al., 2010; Ying et al., 2011). An explicit consideration of sequence dependent family setup time (SDFST) before processing each part family is usually required in a real-world CM system. This study thus focuses on a FMCSP with SDFSTs, which is quite common in PCB manufacturing, TFT-LCD manufacturing and in numerically controlled manufacturing.

Since the pioneering works of Burbidge (1971) and Mitrofanov (1996), FMCSP has inspired a significant number of researches, and some exact methods (Das and Canel, 2005; Gupta and Schaller, 2006) have been proposed for solving this problem. However, FMCSP with SDFSTs is widely acknowledged to be one of the computationally intractable NP-hard combinatorial optimization problems even for the two-machine case (Gupta and Darrow, 1986). Therefore, those exact methods require considerable computational efforts and thus cannot solve problems with realistic sizes. The formidable computational requirements have forced researchers and practitioners frequently seeking approximation algorithms that generate near-optimum solutions with relatively little computational expense. Several studies have experimentally compared various approximation algorithms (Frazier, 1996; Schaller et al., 2000). Recently, the interest in developing efficient approximation algorithms has turned to meta-heuristics, which are particularly attractive for large scale problems. The relevant literature indicates that meta-heuristics have growingly been applied to FMCSPs and recognized as the state-of-the-art methods. Currently available algorithms based on meta-heuristics for FMCSPs with SDFSTs include genetic algorithms (GA; França et al., 2005; Lin et al., 2009b), memetic algorithms (MA; França et al., 2005), Tabu searches (TS; Logendran et al., 2006; Hendizadeh et al., 2008; Lin et al., 2009b), simulated annealing (SA; Lin et al., 2009a, 2009b; Ying et al., 2010) and hybrid meta-heuristics (Zolfaghari and Liang, 1999).

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Among the meta-heuristics based algorithms listed above, the SA based meta-heuristic proposed by Lin et al. (2009a) has emerged as a highly effective and efficient algorithmic approach to this problem. The SA resembles the physical process of annealing (Kirkpatrick et al., 1983) and possesses a formal proof of convergence. From an algorithmic perspective, the SA generally comprises two processes; the first is a local search process, while the second uses a physical annealing mechanism that depends on a cooling procedure to avoid been trapped in local optimal solutions (Katayama and Narihisa, 2001; Wu et al., 2009). If a solution was found when the temperature is high, the neighborhood close to the best solution of the SA (which may contain a better solution) will never be explored (Oliveira et al., 2006). This phenomenon occurs because, when the temperature is high, the Metropolis Criteria tend to perform a drastic locomotion in the solution space to seek the solutions, and there is a high probability that the method may accept worse solutions. Consequently, SAs that aspire to obtain (near) global optima typically require some form of diversification to escape from local optimality. Without such diversification, SAs may be restricted in a small area of the solution space, losing the possibility of finding a global optimum (Martí, 2003). The multi-start hill climbing strategy can provide an effective way of avoiding the search being trapped in local optimum. Therefore, the combination of the advantages of SAs in effectively achieving convergence and the multi-start hill climbing strategy in escaping from local optimality provides the rationale for developing a multi-start simulated annealing (MSA) heuristic to solve combinatorial optimization problems.

By combining the respective advantages of the two paradigms, this paper develops an effective MSA heuristic for solving the FMCSP with SDFSTs. The remainder of this paper is organized as follows. Following the formulation of the problem in Section 2, the proposed multi-start SA heuristic is presented in Section 3. Using a benchmark problem data set, Section 4 then empirically evaluates the effectiveness and efficiency of the proposed multi-start SA heuristic by comparing its performance against the state-of-the-art meta-heuristics. Finally, Section 5 presents conclusions and gives recommendations for future research directions.

2. Problem formulation

The flowline manufacturing cell scheduling problem (FMCSP) with sequence dependent family setup times (SDFSTs) considered in this study can be formally formulated as follows. A set $N=\{P_1, P_2, \ldots, P_n\}$ of n given parts (jobs) is to be processed on m machines in the same technological order. Each part belongs to one of F mutually exclusive and collectively exhaustive part families $f=\{f_1, f_2, \ldots, f_F\}$. Let n_{fk} denote the number of parts in part family f_k ($k=1, 2, \ldots, F$) and let N_{fk} be the subset of parts in part family f_k. Furthermore, let the parts be numbered sequentially such that the first n_{f1} parts belong to the first family f_1, the next n_{f2} parts belong to f_2, and so on. Therefore, $n=n_{f1}+n_{f2}+\cdots+n_{fF}$. Each part P_i ($i=1, 2, \ldots, n$) at machine j has a processing time $p_{i,\,j}$. Besides, before processing each part family, the SDFSTs s^j_{xy} are incurred when part family f_y is processed immediately following part family f_x on machine j, where $s^j_{yy}=0$ for all f_y and j. The individual part setup times are very small compared to the family setup times; hence, are considered to be included in the part processing times. Furthermore, the FMCSP with SDFSTs considered in this paper satisfies the following assumptions of Hendizadeh et al. (2008):

- The ready time of each part is zero, meaning all parts are available for processing at the start time.
- The number of parts and their processing times, as well as the number of part families and their setup times are non-negative integers and are known in advance.
- The exhaustive rule is applied; that is, when a part family is being processed, it cannot be interrupted by a part from another family.

Based on these definitions, the objective is to identify a schedule σ for the part families and parts within each part family so as to minimize the makespan, $C(\sigma)=C(\sigma(n), m)$, where σ is the best among all possible permutations of n parts on m machines, in which all parts of the same part family are processed together. The completion time of part $\sigma(i)$ in a schedule $\sigma=(\sigma(1), \ldots,\sigma(n))$ on machine j, $C(\sigma(i), j)$, is provided by the following recursive relationship:

$$C(\sigma(i),j)=max\left\{C(\sigma(i),j-1);\ C(\sigma(i-1),j)+s^j_{xy}\right\}+p_{\sigma(i),j}$$

where $\sigma(i-1)\in N_{fx}$, $\sigma(i)\in N_{fy}$, and $C(\phi, j)=C(\sigma(i),0)=0$ for all i and j.

3. Proposed multi-start simulated annealing heuristic

The proposed multi-start simulated annealing (MSA) heuristic combines the advantages of SAs and multi-start hill climbing strategies for solving the FMCSP with SDFSTs. The following subsections further discuss the solution representation, initial solution, neighborhood, the parameters and the MSA procedures.

3.1. Solution representation

A solution to FMCSP consists of not only the operating sequence between part families, but also the sequences of parts within individual families. The solution representation in this study comprises 1+F segments, where F is the number of part families. The first segment represents the operating sequence of part families while each of the other F segments corresponds to the operating sequence of parts within a part family. For example, a solution encoded as {(3-1-4-2-5), (2-1), (6-4-7-3-5), (8-10-11-9-12), (15-14-13-16-17), (20-22-18-21-19)} can be decoded as follows. Five main part families require scheduling, and the operating sequence of them is 3-1-4-2-5. The operating sequences for parts in part families 1, 2, 3, 4 and 5 are 2-1, 6-4-7-3-5, 8-10-11-9-12, 15-14-13-16-17, and 20-22-18-21-19, respectively. Thus, the operating sequence for the 22 parts on all of the machines is 8-10-11-9-12-2-1-15-14-13-16-17-6-4-7-3-5-20-22-18-21-19.

3.2. Initial solution and neighborhood

The initial solutions are generated as multi-start points by randomly sequencing the part families and the parts in each family. Let S denote the set of feasible solutions and let X_i ($i=1, \ldots, P_{\text{size}}$) represent the current set of solutions, where $X_i\in S$, and P_{size} represents the number of starting points in the MSA approach. The sets $N(X_i)$ ($i=1, \ldots, P_{\text{size}}$) denote the sets of solutions neighboring X_i ($i=1, \ldots, P_{\text{size}}$). $N(X_i)$ can be obtained by applying either a swap or insertion operation to the sequence of part families and the sequences of the parts within part families. That is, for a randomly selected part family, a neighborhood solution $N(X_i)$ can be obtained by randomly picking up two parts in this family and then directly swapping them, or by choosing randomly one part and inserting it immediately before another part in the same family. Similarly, for the sequence of part families, $N(X_i)$ is obtained by randomly selecting two families and swapping them, or by randomly choosing one part family and inserting it immediately before another part family in the sequence. Since the solution representation comprises 1+F segments, the probabilities of changing the sequence of part families and of modifying the sequences within part families are $1/(1+F')$, where F' denotes the number of part families containing more than one part. Clearly, the swap and insertion operations within the part family cannot be performed for families containing only one part.

3.3. Parameters

The proposed MSA heuristic begins with five parameters, namely I_{iter}, T_0, T_F, α, and P_{size}, where I_{iter} denotes the maximum number of iterations performed by the search at a particular temperature, T_0 represents the initial temperature, T_F is the final temperature (the MSA procedure terminates when the current temperature is below T_F), and α denotes the coefficient controlling the cooling schedule.

3.4. MSA procedure

The proposed MSA heuristic procedure is depicted in Fig. 1. First, the current temperature T is set to T_0. Next, initial solutions X_i ($i=1, \ldots, P_{\text{size}}$) are randomly generated as multi-start points. For each iteration, the next solutions Y_i ($i=1, \ldots, P_{\text{size}}$) are chosen from their corresponding $N(X_i)$. Furthermore, let obj(X_i) denote the objective function value ($C_{\max}$) of X_i, and let Δ_i denote the difference between obj(X_i) and obj(Y_i); that is Δ_i=obj(Y_i) – obj(X_i). The probability of replacing X_i with Y_i, where X_i is the current solution and Y_i is the next solution, given that $\Delta_i > 0$, is $T/(T^2+\Delta_i^2)$. This process is achieved by generating a random number $r \in [0, 1]$ and replacing the solution X_i with Y_i if $r < T/(T^2+\Delta_i^2)$. As proposed by Tiwari et al. (2006) and Lin et al. (2009a), the Cauchy function can replace the Boltzmann function in the annealing process and provide the SA with more opportunities to escape from local minima. Thus, this study adopts the Cauchy function rather than the Boltzmann function. On the other hand, if $\Delta_i \leq 0$, the probability of replacing X_i with Y_i is 1.

T is decreased after running I_{iter} iterations from the previous temperature decrease, according to the formula $T \leftarrow \alpha T$, where $0 < \alpha < 1$. If T is less than T_F, the algorithm is terminated. X_{best} records the best solution as the proposed approach progresses. If a new X_{best} solution is obtained, then all the current solutions X_i ($j=1, \ldots, P_{\text{size}}$) are set to be the same as this new X_{best} solution and the MSA procedure is continued. Following the termination of the MSA procedure, the final schedule can be derived by X_{best}.

4. Computational results

The proposed approach was implemented using C language and run on a PC with an Intel Pentium 4 (2.4 GHz) CPU and 512 MB memory. The performance of the proposed MSA heuristic was compared with that of eight existing approaches. These existing approaches include CMD_{SGV}, MA_{FGMMV}, TS_{HFMGE}, $\text{TS/SA}_{\text{HFMGE}}$, $\text{TS/SA+Elitism}_{\text{HFMGE}}$, $\text{TS-LSM (MIN)}_{\text{HFMGE}}$, MA_{HFMGE}, and SA_{LGYL}. CMD_{SGV} is a composite two-stage scheduling method developed by Schaller et al. (2000), who demonstrated that CMD_{SGV} outperformed the other 11 heuristics. Meanwhile, MA_{FGMMV} is a memetic algorithm (MA) with a population structure proposed by França et al. (2005). The experimental results of França et al. demonstrated that MA_{FGMMV} is superior to CMD_{SGV}. TS_{HFMGE}, $\text{TS/SA}_{\text{HFMGE}}$, $\text{TS/SA+Elitism}_{\text{HFMGE}}$, $\text{TS-LSM (MIN)}_{\text{HFMGE}}$ and MA_{HFMGE} are five tabu search based meta-heuristics developed by Hendizadeh et al. (2008). Hendizadeh et al. demonstrated that all five of these meta-heuristics outperform CMD_{SGV}, although they need more CPU times and the best among them has the same performance as the MA_{FGMMV} algorithm. SA_{LGYL} is a

MSA(T_0, T_F, α, I_{iter}, P_{size})
Step 1: Generating the initial solutions X_i randomly, i=1, 2, …, P_{size};
Step 2: Let $T=T_0$; N=0; X_{best}= the best X_i among the P_{size} solutions; C_{best}=obj(X_{best});
Step 3: For i=1 to P_{size} {
    Step 3.1 If i=1 {N=N+1;}
    Step 3.2 Generating a solution Y_i based on X_i;
    Step 3.3 If Δ_i=obj(Y_i)-obj(X_i)≤ 0 {Let X_i=Y_i;}
        Else {
            Generate $r \sim U(0,1)$;
            If $r < \frac{T}{T^2+\Delta_i^2}$ {Let X_i=Y_i;}
        }
    Step 3.4 If obj(X_i)<C_{best} {
        X_{best}= X_i ;
        C_{best}=obj(X_i);
        Let X_j = X_{best} (j=1, 2, ..., P_{size})
    }
}
Step 4: If N=I_{iter} { T=$T\times\alpha$; N =0;}
    Else {Go to Step 3;}
Step 5: If T<T_f {Terminate the MSA procedure;}
    Else {Go to Step 3;}

Fig. 1. The pseudo-code of proposed MSA Approach.

simulated annealing approach proposed by Lin et al. (2009a). All experiments of SA_{LGYL} were terminated when one of the following conditions was met: (1) after 44 ($1000 \times 0.9^{44} < 10$) temperature reductions and (2) the solution reached Lower Bound. Lin et al. (2009a) showed that SA_{LGYL} performs extremely well compared to CMD_{SGV}, MA_{FGMMV}, TS_{HFMGE}, TS/SA_{HFMGE}, $TS/SA+Elitism_{HFMGE}$, TS-LSM $(MIN)_{HFMGE}$, and MA_{HFMGE}.

4.1. Test problems

Two test problem data sets were used in this study. The first is an existing benchmark problem data set which was used in earlier studies (Schaller et al., 2000; França et al., 2005; Hendizadeh et al., 2008; Lin et al., 2009a). Briefly, the processing times at each stage were randomly generated using a uniform distribution in the range of [1, 10]. The number of machines varies between 3 and 10, the number of parts varies between 1 and 10, and the number of families varies between 3 and 10. Since computational expenses are likely to depend on whether there was a balance between average processing times and average setup times, three different classes of the family setup times were randomly generated as follows:

- *Small Set-Ups* (*SSU*): *U*(1, 20), implies that the ratio of mean family setup time to mean part processing time is approximately 2:1.
- *Medium Set-Ups* (*MSU*): *U*(1, 50), implies that the ratio of mean family setup time to mean part processing time is approximately 5:1.
- *Large Set-Ups* (*LSU*): *U*(1, 100), implies that the ratio of mean family setup time to mean part processing time is approximately 10:1.

Thirty problem sets were generated for each combination of problem parameters, and 30 problem instances were generated for each problem set. Thus, 900 different problem instances of the first benchmark problem set were tested. As an example of the notation, LSU34 set of problems consists of 30 instances generated with *Large Set-Ups*, three families and four machines.

To investigate whether the performance of the SA based meta-heuristic can be improved by incorporating the multi-start hill climbing strategy, the proposed MSA heuristic was further compared with the SA heuristic (i.e., $P_{size}=1$) using an expanded problem set. In this problem set, the job processing times at each stage were generated as random integers from the uniform distribution $U(1, UP)$, where $UP=100$ and 1000 (Note that the first problem set mentioned above was generated using $UP=10$). The family setup times were randomly generated from the uniform distribution $U(1, US)$ in three categories: *Small Set-Ups* (SSU) $US=2UP$, *Medium Set-Ups* (MSU) $US=5UP$ and *Large Set-Ups* (LSU) $US=10UP$. The number of machines, the number of parts, and the number of families were the same as those in the first problem set. For each class of *UP*, thirty problem sets were generated for each combination of problem parameters, and 30 problem instances were generated for each problem set. Thus, 1800 different problem instances are included in the second test problem set. Consequently, the test bed comprised a total of 2700 problem instances (recall that the first problem set has 900 instances). These test problems ranged from loosely to tightly constrained problem instances.

4.2. Results and discussion

Parameter settings may influence algorithm performance. The initial experiments which were conducted for determining most appropriate parameter values applied the following parameter settings: $T_0=1000$, $I_{iter}=(F+n)\times 1000/P_{size}$, $\alpha=0.90$, $T_F=10$, where P_{size} ranges from 1 to 10, where F denotes the number of families and n represents the number of parts to be processed. Note that if $P_{size}=1$, the proposed MSA heuristic is equivalent to the conventional SA meta-heuristic. All experiments were terminated after 44 ($1000 \times 0.9^{44} < 10$) temperature reductions. Since the number of iterations (I_{iter}) is inversely proportion to the value of P_{size}, the total number of solutions evaluated is the same for all problems with different P_{size} values, providing a fair basis for comparison.

The makespan obtained by all algorithms was compared with the corresponding Lower Bound (*LB*), proposed by Schaller et al. (2000), using the following relative error rate (*RER*):

$$RER=\left[(M^h - LB)/LB\right]100\%$$

where M^h is the makespan obtained by different algorithms.

To determine the best values of P_{size} for the experiments on the first and second test problem sets, each problem instance was solved five times under different P_{size} values. The average minimum *RER* (Min. *RER*), total average *RER* (Ave. *RER*), and average maximum *RER* (Max. *RER*) obtained for different P_{size} values are presented in Table 1. In order to be consistent in selecting appropriate P_{size} values for different problem sets (i.e., different values of *UP*), the best-2-of-3 criterion (Min. *RER*, Ave. *RER*, Max.

Table 2
Paired *t*-tests between means of $P_{size}=1$–5 and means of $P_{size}=6$–10 on the Min. *RER*, Ave. *RER* and Max. *RER*.

$P_{size}=1$–5 *vs.* $P_{size}=6$–10	Test on Min. *RER*	Test on Ave. *RER*	Test on Max. *RER*
Difference	0.00185	0.005019	0.02495
Degree of freedom	89	89	89
t-value	2.07403	3.21386	2.29297
One-tailed significant	0.02048	0.00091	0.01210

Table 1
Computational results of Min. *RER*, Ave. *RER*, and Max. *RER* for different P_{size} values.

Problem set	Index	P_{size}									
		1	2	3	4	5	6	7	8	9	10
$UP=10$	Min. *RER*	0.0561	0.0573	0.0541	0.0559	0.0533	0.0571	0.0539	0.0533	0.0555	0.0518
	Ave. *RER*	0.7178	0.7156	0.7147	0.7145	0.7142	0.7142	0.7141	0.7131	0.7134	0.7151
	Max. *RER*	2.2825	2.2658	2.2747	2.2672	2.2670	2.2737	2.2701	2.2642	2.2691	2.2732
$UP=100$	Min. *RER*	0.0731	0.0743	0.0731	0.0743	0.0743	0.0743	0.0731	0.0743	0.0731	0.0731
	Ave. *RER*	0.7727	0.7730	0.7711	0.7691	0.7722	0.7705	0.7723	0.7694	0.7693	0.7697
	Max. *RER*	2.7107	2.7365	2.7226	2.7301	2.7188	2.7034	2.7360	2.7041	2.6992	2.7185
$UP=1000$	Min. *RER*	0.0929	0.0997	0.0925	0.0859	0.0919	0.0950	0.0842	0.0840	0.0863	0.0914
	Ave. *RER*	1.0053	0.9996	0.9835	0.9746	1.0016	0.9838	0.9723	0.9761	0.9871	0.9835
	Max. *RER*	3.5932	3.5329	3.4558	3.4485	3.4912	3.4686	3.4224	3.4309	3.4419	3.4481

Table 3
Comparison of objective function values ($UP=10$).

Instance	MSA with $P_{size}=8$					SA_{LGYL}					CMD_{SGV}					MA_{FGMMV}				
	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*[a]	*OPT*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*[a]	*OPT*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*[a]	*OPT*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*[a]	*OPT*
LSU33	0.000	0.077	1.115	1.48	27	0.000	0.078	1.128	0.06	27	0.00	0.91	8.41	0.04	21	0.00	0.07	1.12	3.1	27
LSU34[b]	0.000	0.325	2.381	1.67	20	0.000	0.330	2.439	0.23	20	0.00	1.08	16.39	0.06	12	0.00	0.32	2.43	10.1	20
LSU44	0.000	0.201	1.087	3.06	20	0.000	0.202	1.099	0.33	20	0.00	1.95	10.27	0.12	8	0.00	0.20	1.09	10.1	20
LSU55	0.000	0.281	1.828	4.65	18	0.000	0.284	1.862	0.83	18	0.00	2.49	9.57	0.21	4	0.00	0.28	1.86	12.1	18
LSU56	0.000	0.505	2.370	5.27	9	0.000	0.510	2.427	1.66	9	0.00	3.37	17.07	0.26	4	0.00	0.51	2.42	21.1	9
LSU65	0.000	0.308	2.371	6.43	15	0.000	0.312	2.428	1.35	15	0.00	3.29	10.02	0.30	3	0.00	0.31	2.42	15.1	15
LSU66	0.000	0.197	1.351	8.06	15	0.000	0.198	1.370	1.87	15	0.00	3.03	10.47	0.44	3	0.00	0.19	1.36	15.3	15
LSU88	0.000	0.580	1.829	16.20	6	0.000	0.591	2.019	6.18	6	0.28	6.25	18.17	0.95	0	0.00	0.58	1.86	24.7	6
LSU108	0.000	0.397	0.766	27.20	1	0.000	0.413	0.874	12.94	1	0.12	6.22	11.25	1.82	0	0.00	0.47	1.19	29.8	1
LSU1010	0.000	0.571	1.540	31.70	2	0.000	0.613	1.448	18.16	2	0.53	6.30	11.42	2.37	0	0.00	0.77	2.27	29.5	1
MSU33	0.000	0.371	3.261	1.48	24	0.000	0.379	3.371	0.13	23	0.00	0.92	11.46	0.04	21	0.00	0.37	3.37	7.1	23
MSU34	0.000	0.560	2.247	1.68	17	0.000	0.569	2.299	0.40	17	0.00	2.00	16.28	0.05	11	0.00	0.56	2.29	13.1	17
MSU44	0.000	0.498	2.273	3.23	11	0.000	0.504	2.326	0.88	11	0.00	1.96	11.11	0.13	4	0.00	0.50	2.32	19.1	11
MSU55	0.000	0.446	2.055	4.61	15	0.000	0.451	2.098	1.19	15	0.00	3.10	8.48	0.19	4	0.00	0.45	2.09	15.1	15
MSU56	0.000	0.857	3.030	5.26	6	0.000	0.872	3.125	2.05	6	0.00	3.58	13.13	0.27	1	0.00	0.87	3.12	24.1	6
MSU65	0.000	0.367	1.214	6.63	14	0.000	0.370	1.229	1.55	14	0.00	3.68	8.88	0.30	3	0.00	0.36	1.22	16.2	14
MSU66	0.000	0.504	1.609	8.07	8	0.000	0.509	1.635	2.95	8	0.00	4.59	15.77	0.40	1	0.00	0.50	1.63	22.7	8
MSU88	0.000	0.973	2.893	16.24	3	0.000	0.981	2.979	7.81	3	0.63	5.68	12.68	0.97	0	0.00	0.99	2.98	27.3	3
MSU108	0.000	0.767	1.608	27.28	1	0.000	0.837	2.124	14.54	1	2.89	6.11	10.83	1.84	0	0.00	0.86	1.80	30.1	1
MSU1010	0.149	0.948	2.362	31.71	0	0.150	1.002	2.903	19.16	0	2.29	5.73	9.92	2.37	0	0.15	1.15	2.53	30.8	0
SSU33	0.000	0.308	2.419	1.48	23	0.000	0.313	2.479	0.20	23	0.00	0.67	4.13	0.04	18	0.00	0.31	2.47	7.1	23
SSU34	0.000	0.819	2.857	1.68	15	0.000	0.835	2.941	0.50	15	0.00	1.85	8.46	0.05	8	0.00	0.83	2.94	15.1	15
SSU44	0.000	0.569	2.825	3.15	15	0.000	0.578	2.907	0.87	15	0.00	1.94	8.33	0.12	8	0.00	0.57	2.90	15.0	15
SSU55	0.000	0.903	2.294	4.61	4	0.000	0.927	2.347	2.27	4	0.00	3.15	6.61	0.20	1	0.00	0.92	2.34	26.0	4
SSU56	0.000	1.532	2.996	5.29	3	0.000	1.564	3.089	3.04	3	0.00	4.02	8.93	0.25	1	0.00	1.56	3.08	27.4	3
SSU65	0.000	0.976	3.333	6.55	6	0.000	0.990	3.448	2.84	6	0.00	3.00	6.90	0.32	2	0.00	0.99	3.44	24.0	6
SSU66	0.000	1.230	2.655	8.11	1	0.000	1.262	2.727	4.61	1	1.24	4.06	9.16	0.44	0	0.00	1.28	2.72	29.1	1
SSU88	0.286	1.742	3.343	16.33	0	0.287	1.779	3.458	9.86	0	3.20	5.62	8.59	0.91	0	0.28	1.85	3.31	30.3	0
SSU108	0.480	1.465	2.637	27.82	0	0.482	1.519	2.708	16.97	0	2.58	5.63	8.96	1.82	0	0.72	1.77	2.90	30.7	0
SSU1010	0.591	2.073	3.125	31.57	0	0.594	2.190	3.441	22.70	0	4.07	6.86	9.11	2.21	0	0.59	2.33	3.65	30.6	0
Total Average	0.050	0.712	2.256	10.62	9.97	0.050	0.732	2.358	5.27	9.93	0.59	3.63	10.69	0.64	4.6	0.06	0.76	2.37	20.4	9.90

Instance	TS_{HFMGE}				TS/SA_{HFMGE}				$TS/SA+Elitism_{HFMGE}$				TS-LSM $(MIN)_{HFMGE}$				MA_{HFMGE}			
	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *Time*
LSU33	0.00	0.08	1.13	3.30	0.00	0.08	1.13	3.44	0.00	0.08	1.13	4.14	0.00	0.08	1.13	3.30	0.00	0.08	1.13	3.00
LSU34	0.00	0.33	2.44	10.20	0.00	0.33	2.44	10.40	0.00	0.33	2.44	11.11	0.00	0.33	2.44	10.50	0.00	0.33	2.44	10.00
LSU44	0.00	0.20	1.10	11.90	0.00	0.21	1.10	11.97	0.00	0.22	1.10	13.80	0.00	0.20	1.10	11.00	0.00	0.20	1.10	10.10
LSU55	0.00	0.29	1.86	13.00	0.00	0.29	1.86	12.69	0.00	0.30	1.86	13.67	0.00	0.30	1.86	12.70	0.00	0.28	1.86	12.10
LSU56	0.00	0.52	2.43	21.50	0.00	0.52	2.43	21.72	0.00	0.52	2.43	22.24	0.00	0.55	2.43	21.60	0.00	0.52	2.43	21.10
LSU65	0.00	0.31	2.43	16.50	0.00	0.32	2.43	16.26	0.00	0.32	2.43	17.83	0.00	0.33	2.43	17.00	0.00	0.31	2.43	15.10
LSU66	0.00	0.23	1.37	19.40	0.00	0.21	1.37	19.22	0.00	0.23	1.37	21.09	0.00	0.21	1.37	18.60	0.00	0.20	1.37	16.00
LSU88	0.00	0.65	2.05	25.40	0.00	0.65	2.05	26.41	0.00	0.64	2.05	27.41	0.00	0.63	2.02	26.60	0.00	0.61	1.86	24.60
LSU108	0.11	0.48	0.97	31.70	0.00	0.47	1.10	32.25	0.11	0.56	2.16	32.12	0.11	0.47	0.98	32.30	0.00	0.44	0.88	29.80
LSU1010	0.00	0.69	1.80	31.20	0.00	0.64	1.68	31.19	0.00	0.77	1.98	32.50	0.00	0.67	1.87	31.30	0.00	0.68	1.61	29.30
MSU33	0.00	0.38	3.37	7.30	0.00	0.38	3.37	7.54	0.00	0.38	3.37	8.32	0.00	0.38	3.37	7.40	0.00	0.36	3.37	6.50
MSU34	0.00	0.59	2.30	14.90	0.00	0.57	2.30	14.34	0.00	0.57	2.30	14.52	0.00	0.60	2.30	15.30	0.00	0.57	2.30	13.10
MSU44	0.00	0.58	2.51	20.40	0.00	0.55	2.51	19.87	0.00	0.52	2.71	20.74	0.00	0.57	2.33	21.40	0.00	0.50	2.33	19.10
MSU55	0.00	0.50	2.10	17.10	0.00	0.45	2.10	17.18	0.00	0.45	2.10	17.51	0.00	0.58	2.42	17.60	0.00	0.45	2.10	15.10
MSU56	0.00	0.90	3.13	25.10	0.00	0.87	3.13	24.56	0.00	0.91	3.13	26.06	0.00	0.93	3.13	25.80	0.00	0.87	3.13	24.20
MSU65	0.00	0.40	1.72	17.90	0.00	0.39	1.47	18.66	0.00	0.44	2.00	19.85	0.00	0.46	2.00	18.60	0.00	0.40	1.23	18.10

MSU66	0.00	0.52	1.63	23.80	0.00	0.59	1.63	26.03	0.00	0.60	1.63	25.63	0.00	0.61	1.63	26.00	0.00	0.51	1.63	22.30
MSU88	0.00	1.10	2.98	28.80	0.00	1.12	2.98	29.64	0.00	1.19	2.98	28.90	0.00	1.18	2.98	29.70	0.00	1.06	2.98	28.20
MSU108	0.00	1.17	3.05	31.30	0.00	1.07	3.10	30.90	0.00	1.13	2.45	32.88	0.00	1.08	2.29	31.60	0.00	1.05	2.29	29.50
MSU1010	0.15	1.22	3.71	31.90	0.15	1.20	3.39	32.33	0.19	1.35	3.16	32.71	0.15	1.18	2.90	33.00	0.15	1.21	3.06	30.30
SSU33	0.00	0.31	2.48	8.80	0.00	0.31	2.48	7.79	0.00	0.31	2.48	9.50	0.00	0.46	2.65	10.60	0.00	0.31	2.48	7.70
SSU34	0.00	0.96	6.62	16.90	0.00	0.83	2.94	16.55	0.00	0.85	2.94	17.56	0.00	1.13	6.62	18.00	0.00	0.83	2.94	15.10
SSU44	0.00	0.64	2.91	17.00	0.00	0.59	2.91	15.77	0.00	0.60	2.91	17.91	0.00	0.92	6.06	18.70	0.00	0.59	2.91	15.10
SSU55	0.00	0.94	2.35	26.80	0.00	0.95	2.35	27.05	0.00	0.97	2.35	27.40	0.00	1.04	2.44	26.70	0.00	0.93	2.35	26.10
SSU56	0.00	1.63	3.09	28.90	0.00	1.58	3.09	28.00	0.00	1.70	3.32	29.50	0.00	1.76	5.21	29.70	0.00	1.60	3.09	27.30
SSU65	0.00	1.03	3.45	24.60	0.00	1.02	3.45	25.02	0.00	1.13	3.45	25.56	0.00	1.23	3.45	25.20	0.00	1.01	3.45	24.20
SSU66	0.00	1.34	2.73	30.20	0.31	1.36	2.73	30.64	0.00	1.47	3.30	30.96	0.00	1.45	4.09	30.60	0.00	1.33	2.73	29.20
SSU88	0.29	2.14	4.53	30.90	0.29	2.21	3.58	31.39	0.29	2.29	4.06	31.96	0.29	2.35	3.82	31.20	0.29	2.02	3.82	30.10
SSU108	0.72	1.97	3.13	31.90	0.87	1.89	3.17	32.21	0.87	2.18	3.73	32.91	0.87	2.20	3.37	32.80	0.87	1.99	3.07	30.20
SSU1010	0.68	2.69	4.60	32.40	0.59	2.64	4.60	32.50	0.68	2.98	4.37	32.82	0.59	3.03	4.95	33.50	0.91	2.95	4.09	30.30
Total Average	0.06	0.83	2.67	21.70	0.07	0.81	2.50	21.80	0.07	0.87	2.59	22.60	0.07	0.90	2.85	22.30	0.07	0.81	2.41	20.40

[a] CPU time in seconds.
[b] LSU34 set of problems consist of 30 instances generated with Large Set-Ups, three families and four machines.

RER) was used; that is, the P_{size} value which gives the best values for at least two out of the three indices will be selected. This criterion results in the selection of $P_{\text{size}}=8$, 9 and 7 for $UP=10$, 100 and 1000, respectively. Thus, in the comparison with the other eight meta-heuristics, we set $P_{\text{size}}=8$ in the proposed MSA algorithm to obtain the solutions of the first benchmark problem set ($UP=10$). Meanwhile, the solutions of the second benchmark problem set in the comparison with SA (i.e., $P_{\text{size}}=1$) to further examine the performance improvement from incorporating the multi-start hill climbing strategy.

It makes intuitive sense that if P_{size} value increases, the possibility of getting a superior result improves. To examine this proposition, one-tailed paired *t*-tests between means of $P_{\text{size}}=1$–5 and means of $P_{\text{size}}=6$–10 on the Min. *RER*, Ave. *RER* and Max. *RER*, respectively, were performed. As shown in Table 2, at significance level $\alpha=0.05$, the results of one-tailed paired *t*-tests reveal that MSA with $P_{\text{size}}=6$–10 is superior to that with $P_{\text{size}}=1$–5 on the Min. *RER*, Ave. *RER* and Max. *RER*.

Table 3 lists Min. *RER*, Ave. *RER*, Max. *RER*, average computational time (Ave. *Time*), and the number of optimal solutions (*OPT*) (i.e., the values of makespan that reached their *LBs*) as obtained using the proposed MSA algorithms with $P_{\text{size}}=8$ and using the other three meta-heuristics: SA_{LGYL}, CMD_{SGV}, and MA_{FGMMV}, for the 30 problem instances of each problem set. The results obtained by the other five meta-heuristics: TS_{HFMGE}, TS/SA_{HFMGE}, $TS/SA+Elitism_{HFMGE}$, TS-LSM $(MIN)_{HFMGE}$, and MA_{HFMGE}, are cited from the original paper (Hendizadeh et al., 2008).

As seen in Table 3, 0.050%, 0.712%, and 2.256% were obtained by the proposed MSA algorithm for the Min. *RER*, Ave. *RER* and Max. *RER*, respectively. Based on the comparison with the other 8 heuristics, the proposed MSA algorithm outperforms all the other approaches. Furthermore, the number of optimal solutions (*OPT*) found per problem set was 9.97 for the proposed MSA algorithm, whereas for SA_{LGYL}, CMD_{SGV}, and MA_{FGMMV}, the numbers were 9.93, 4.6, and 9.90, respectively. Because the original study (Hendizadeh et al., 2008) did not provide the number of optimal solutions (*OPT*) per problem set obtained by TS_{HFMGE}, TS/SA_{HFMGE}, $TS/SA+Elitism_{HFMGE}$, $TS-LSM(MIN)_{HFMGE}$, and MA_{HFMGE}, further comparisons could not be made.

The CMD_{SGV} algorithm was coded in Pascal language, and the tests were performed on an HP Vectra VL series 4 personal computer (Schaller et al., 2000). The MA_{FGMMV} algorithm was coded and run on a PC that has a Pentium II 266 MHz processor, with 128 Mb RAM, using the Sun Java 2 JDK compiler (França et al., 2005). The TS_{HFMGE}, TS/SA_{HFMGE}, $TS/SA+Elitism_{HFMGE}$, TS-LSM $(MIN)_{HFMGE}$, and MA_{HFMGE} algorithms were coded in C++ and run on a 2.6 GHz Pentium 4 PC with 512 MB RAM under Windows XP OS (Hendizadeh et al., 2008). Because different programming languages, development tools, and execution platforms were used for these algorithms, this study did not make a direct comparison of the computational efficiencies of these algorithms. Nevertheless, the proposed MSA heuristic was found to be able to obtain better solutions with reasonable computational expenses.

To verify the effectiveness of the proposed MSA algorithm, one-tailed paired *t*-tests were performed on the Min. *RER*, Ave. *RER*, Max. *RER* and *OPT* to compare the proposed MSA algorithm with SA_{LGYL}, CMD_{SGV}, MA_{FGMMV}, TS_{HFMGE}, TS/SA_{HFMGE}, $TS/SA+Elitism_{HFMGE}$, $TS-LSM(MIN)_{HFMGE}$, and MA_{HFMGE} algorithms. Table 4 lists the test results.

As shown in Table 4, at significance level $\alpha=0.05$, the one-tailed paired *t*-tests performed on the average Min. *RER* in overall test problems of each problem set indicates that the proposed MSA heuristic significantly outperformed the SA_{LGYL}, CMD_{SGV}, TS_{HFMGE}, TS/SA_{HFMGE}, $TS/SA+Elitism_{HFMGE}$, $TS-LSM(MIN)_{HFMGE}$, and MA_{HFMGE} algorithms. Furthermore, the test results also show

Table 4
Paired *t*-tests on Min. *RER*, Ave. *RER*, Max. *RER*, and *OPT* ($UP=10$).

MSA vs.	LSU ($df=9$)	MSU ($df=9$)	SSU ($df=9$)	Overall test problems ($df=29$)
Test on Min. *RER*	SA_{LGYL}: (0.000, 0.500)[a] CMD_{SGV}: (−1.646, 0.067) MA_{FGMMV}: (0.000, 0.500) TS_{HFMGE}: (−1.000, 0.172) TS/SA_{HFMGE}: (0.000, 0.500) $TS/SA+Elitism_{HFMGE}$: (−1.000, 0.172) TS-LSM $(MIN)_{HFMGE}$: (−1.000, 0.172) MA_{HFMGE}: (0.000, 0.500)	SA_{LGYL}: (−1.000, 0.172) CMD_{SGV}: (−1.688, 0.063) MA_{FGMMV}: (−1.000, 0.172) TS_{HFMGE}: (−1.000, 0.172) TS/SA_{HFMGE}: (−1.000, 0.172) $TS/SA+Elitism_{HFMGE}$: (−1.000, 0.172) TS-LSM $(MIN)_{HFMGE}$: (−1.000, 0.172) MA_{HFMGE}: (−1.000, 0.172)	SA_{LGYL}: (−1.765, 0.056) CMD_{SGV}: (−1.646, 0.067) MA_{FGMMV}: (−2.235, 0.026) TS_{HFMGE}: (−1.354, 0.104) TS/SA_{HFMGE}: (−1.496, 0.084) $TS/SA+Elitism_{HFMGE}$: (−1.239, 0.123) TS-LSM $(MIN)_{HFMGE}$: (−1.009, 0.170) MA_{HFMGE}: (−1.501, 0.084)	SA_{LGYL}: (−1.882, 0.035) CMD_{SGV}: (−2.866, 0.004) MA_{FGMMV}: (−0.974, 0.169) TS_{HFMGE}: (−2.898, 0.004) TS/SA_{HFMGE}: (−3.346, 0.001) $TS/SA+Elitism_{HFMGE}$: (−4.468, 0.000) TS-LSM $(MIN)_{HFMGE}$: (−3.315, 0.000) MA_{HFMGE}: (−3.659, 0.000)
Test on Ave. *RER*	SA_{LGYL}: (−2.235, 0.026) CMD_{SGV}: (−5.093, 0.000) MA_{FGMMV}: (−1.238, 0.122) TS_{HFMGE}: (−2.538, 0.016) TS/SA_{HFMGE}: (−2.949, 0.008) $TS/SA+Elitism_{HFMGE}$: (−2.385, 0.020) TS-LSM $(MIN)_{HFMGE}$: (−3.120, 0.006) MA_{HFMGE}: (−1.919, 0.044)	SA_{LGYL}: (−2.456, 0.018) CMD_{SGV}: (−6.046, 0.000) MA_{FGMMV}: (−1.517, 0.082) TS_{HFMGE}: (−2.601, 0.014) TS/SA_{HFMGE}: (−2.606, 0.014) $TS/SA+Elitism_{HFMGE}$: (−2.633, 0.014) TS-LSM $(MIN)_{HFMGE}$: (−4.261, 0.001) MA_{HFMGE}: (−1.968, 0.040)	SA_{LGYL}: (−3.293, 0.005) CMD_{SGV}: (−5.585, 0.000) MA_{FGMMV}: (−2.250, 0.025) TS_{HFMGE}: (−2.935, 0.008) TS/SA_{HFMGE}: (−2.538, 0.016) $TS/SA+Elitism_{HFMGE}$: (−2.816, 0.010) TS-LSM $(MIN)_{HFMGE}$: (−4.515, 0.000) MA_{HFMGE}: (−2.112, 0.032)	SA_{LGYL}: (−4.338, 0.000) CMD_{SGV}: (−9.711, 0.000) MA_{FGMMV}: (−2.959, 0.003) TS_{HFMGE}: (−3.920, 0.000) TS/SA_{HFMGE}: (−3.557, 0.001) $TS/SA+Elitism_{HFMGE}$: (−3.772, 0.000) TS-LSM $(MIN)_{HFMGE}$: (−4.482, 0.000) MA_{HFMGE}: (−2.734, 0.005)
Test on Max. *RER*	SA_{LGYL}: (−1.993, 0.038) CMD_{SGV}: (−10.416, 0.000) MA_{FGMMV}: (−1.799, 0.053) TS_{HFMGE}: (−3.127, 0.006) TS/SA_{HFMGE}: (−2.832, 0.010) $TS/SA+Elitism_{HFMGE}$: (−1.701, 0.062) TS-LSM $(MIN)_{HFMGE}$: (−2.903, 0.009) MA_{HFMGE}: (−4.741, 0.001)	SA_{LGYL}: (−2.432, 0.019) CMD_{SGV}: (−11.620, 0.000) MA_{FGMMV}: (−4.063, 0.001) TS_{HFMGE}: (−2.296, 0.024) TS/SA_{HFMGE}: (−2.161, 0.029) $TS/SA+Elitism_{HFMGE}$: (−2.945, 0.008) TS-LSM $(MIN)_{HFMGE}$: (−3.048, 0.007) MA_{HFMGE}: (−2.212, 0.027)	SA_{LGYL}: (−4.381, 0.000) CMD_{SGV}: (−10.801, 0.000) MA_{FGMMV}: (−2.583, 0.016) TS_{HFMGE}: (−1.984, 0.039) TS/SA_{HFMGE}: (−2.007, 0.038) $TS/SA+Elitism_{HFMGE}$: (−3.083, 0.007) TS-LSM $(MIN)_{HFMGE}$: (−3.397, 0.004) MA_{HFMGE}: (−2.599, 0.014)	SA_{LGYL}: (−4.171, 0.000) CMD_{SGV}: (−13.290, 0.000) MA_{FGMMV}: (−3.763, 0.000) TS_{HFMGE}: (−2.898, 0.004) TS/SA_{HFMGE}: (−3.346, 0.001) $TS/SA+Elitism_{HFMGE}$: (−4.468, 0.000) TS-LSM $(MIN)_{HFMGE}$: (−3.415, 0.000) MA_{HFMGE}: (−3.659, 0.001)
Test on *OPT*	SA_{LGYL}: (0.000, 0.500) CMD_{SGV}: (5.432, 0.000) MA_{FGMMV}: (1.000, 0.172)	SA_{LGYL}: (1.000, 0.172) CMD_{SGV}: (4.521, 0.001) MA_{FGMMV}: (1.000, 0.172)	SA_{LGYL}: (0.000, 0.500) CMD_{SGV}: (3.314, 0.005) MA_{FGMMV}: (0.000, 0.500)	SA_{LGYL}: (1.000, 0.163) CMD_{SGV}: (7.058, 0.000) MA_{FGMMV}: (1.439, 0.080)

[a] SA_{LGYL} (*t*-value, *p*-value).

that the proposed MSA heuristic markedly outperformed all other approaches on the total Ave. *RER* and the average Max. *RER* in overall test problems. Meanwhile, when paired *t*-tests were performed on the *OPT*, the proposed MSA heuristic considerably outperformed CMD_{SGV}.

To examine the effect of incorporating the multi-start hill climbing strategy, the proposed MSA heuristic was further compared with the SA heuristic (i.e., $P_{size}=1$) on the first and second problem data sets. Each one of the 2700 problem instances was replicated five times. Table 5 summarizes the experimental results of Min. *RER*, Ave. *RER*, Max. *RER*, the average number of optimal solutions (Ave. *OPT*), and Ave. *Time*. As seen in Table 5, 0.053%, 0.713%, 2.264%, 9.95 and 10.63 for $UP=10$ problem data set, 0.073%, 0.769%, 2.699%, 9.66 and 10.41 for $UP=100$ problem data set, and 0.084%, 0.972%, 3.422%, 8.41 and 10.73 for $UP=1000$ problem data set were obtained by the proposed MSA algorithm for the total average of Min. *RER*, Ave. *RER*, Max. *RER*, Ave. *OPT* and Ave. *Time*, respectively. However, for SA heuristic, the performance measures were 0.056%, 0.718%, 2.282%, 9.95 and 10.62 for $UP=10$ problem set, 0.073%, 0.773%, 2.711%, 9.63 and 10.40 for $UP=100$ problem set, and 0.093%, 1.005%, 3.593%, 8.35 and 10.58 for $UP=1000$ problem set, respectively. These results show that the proposed MSA heuristic outperformed the SA algorithms on the total Ave. *RER* for all problem data sets.

The paired *t*-tests results listing in Table 6 verify again the effectiveness of incorporating the multi-start hill climbing strategy. At significance level $\alpha=0.05$, the one-tailed paired *t*-tests performed on the average Min. *RER*, Ave. *RER*, Max. *RER*, Ave. *OPT* and Ave. *Time* for overall test problems of each problem set show that the proposed MSA heuristic outperformed the SA heuristic on the total Ave. *RER* for all problem data sets.

In summary, these statistical test results show that the performance of the SA meta-heuristic was improved by employing the multi-start hill climbing strategy, and that the proposed MSA heuristic is relatively more effective in minimizing makespan than the best known meta-heuristic algorithms.

5. Conclusions and recommendations for future studies

The flowline manufacturing cell scheduling problem (FMCSP) with sequence dependent family setup times (SDFSTs) is one of the most computationally intractable NP-hard combinatorial optimization problems. This study presented a high-performance and efficient multi-start simulated annealing (MSA) heuristic which combines a SA technique with a multi-start hill climbing strategy to minimize makespan for FMCSP with SDFSTs. The use of the MSA heuristic takes advantage of the main properties of the SAs (e.g. effective convergence, small population, efficient use of memory, and easy implementation) and those of multi-start hill climbing strategies (e.g. sufficient diversification, excellent capability to escape from local optimality, and efficient sampling of the neighborhood solution space). Experimental investigations and numerical calculations proved that the use of the proposed MSA heuristic for solving FMCSPs with SDFSTs resulted in superior performance in terms of makespan criterion. Given that the FMCSP with SDFSTs is an extremely challenging NP-hard optimization problem, the results obtained by the proposed MSA heuristic clearly indicate

Table 5
The comparison between MSA and SA (UP=10, 100 and 1000 with 5 runs).

Instance	UP=10									
	MSA (P_{size}=8)					SA (P_{size}=1)				
	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *OPT*	Ave. *Time*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *OPT*	Ave. *Time*
LSU33	0.000	0.077	1.115	27.00	1.36	0.000	0.077	1.115	27.00	1.36
LSU34	0.000	0.325	2.381	20.00	1.60	0.000	0.325	2.381	20.00	1.61
LSU44	0.000	0.201	1.087	20.00	2.93	0.000	0.201	1.087	20.00	2.94
LSU55	0.000	0.281	1.828	18.00	4.55	0.000	0.281	1.828	18.00	4.55
LSU56	0.000	0.505	2.370	9.00	5.28	0.000	0.505	2.370	9.00	5.26
LSU65	0.000	0.308	2.371	15.00	6.38	0.000	0.308	2.371	15.00	6.36
LSU66	0.000	0.197	1.351	15.00	8.08	0.000	0.197	1.351	15.00	8.07
LSU88	0.000	0.580	1.919	6.00	16.41	0.000	0.581	1.949	6.00	16.41
LSU108	0.000	0.400	0.788	1.00	27.12	0.000	0.403	0.809	1.00	27.11
LSU1010	0.000	0.570	1.442	2.00	32.21	0.000	0.573	1.418	2.00	32.15
MSU33	0.000	0.361	3.261	23.60	1.38	0.000	0.361	3.261	23.60	1.36
MSU34	0.000	0.560	2.247	17.00	1.61	0.000	0.560	2.247	17.00	1.59
MSU44	0.000	0.498	2.273	11.00	2.97	0.000	0.498	2.273	11.00	2.96
MSU55	0.000	0.446	2.055	15.00	4.57	0.000	0.446	2.055	15.00	4.57
MSU56	0.000	0.857	3.030	6.00	5.30	0.000	0.857	3.030	6.00	5.26
MSU65	0.000	0.367	1.214	14.00	6.41	0.000	0.367	1.214	14.00	6.39
MSU66	0.000	0.504	1.609	8.00	8.07	0.000	0.504	1.609	8.00	8.03
MSU88	0.000	0.966	2.893	3.00	16.53	0.000	0.968	2.893	3.00	16.50
MSU108	0.000	0.771	1.608	1.00	27.40	0.000	0.795	1.923	1.00	27.21
MSU1010	0.149	0.952	2.362	0.00	32.01	0.149	0.966	2.393	0.00	32.28
SSU33	0.000	0.308	2.419	23.00	1.38	0.000	0.308	2.419	23.00	1.37
SSU34	0.000	0.819	2.857	15.00	1.63	0.000	0.819	2.857	15.00	1.61
SSU44	0.000	0.569	2.825	15.00	2.98	0.000	0.569	2.825	15.00	2.97
SSU55	0.000	0.903	2.294	4.00	4.58	0.000	0.910	2.294	4.00	4.56
SSU56	0.000	1.537	2.996	3.00	5.32	0.000	1.537	2.996	3.00	5.30
SSU65	0.000	0.976	3.333	6.00	6.46	0.000	0.973	3.333	6.00	6.43
SSU66	0.000	1.228	2.655	1.00	8.09	0.000	1.226	2.655	1.00	8.12
SSU88	0.286	1.746	3.388	0.00	16.62	0.286	1.748	3.365	0.00	16.60
SSU108	0.575	1.487	2.637	0.00	27.31	0.658	1.529	2.637	0.00	27.26
SSU1010	0.591	2.094	3.321	0.00	32.47	0.591	2.140	3.518	0.00	32.46
Total Ave.	0.053	0.713	2.264	9.95	10.63	0.056	0.718	2.282	9.95	10.62

Instance	UP=100									
	MSA (P_{size}=9)					SA (P_{size}=1)				
	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *OPT*	Ave. *Time*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *OPT*	Ave. *Time*
LSU33	0.000	0.027	0.627	28.00	1.60	0.000	0.027	0.627	28.00	1.38
LSU34	0.000	0.264	2.070	21.00	1.88	0.000	0.264	2.070	21.00	1.85
LSU44	0.000	0.275	2.053	19.00	3.07	0.000	0.275	2.053	19.00	3.06
LSU55	0.000	0.239	1.871	16.00	4.55	0.000	0.239	1.871	16.00	4.52
LSU56	0.000	0.449	1.456	11.00	5.43	0.000	0.449	1.456	11.00	5.40
LSU65	0.000	0.215	0.943	12.00	6.95	0.000	0.215	0.943	12.00	6.87
LSU66	0.000	0.517	2.515	8.00	7.16	0.000	0.517	2.515	8.00	7.09
LSU88	0.000	0.596	1.313	3.00	16.36	0.000	0.595	1.391	3.00	16.28
LSU108	0.000	0.580	1.916	4.80	23.10	0.000	0.561	1.467	4.60	25.59
LSU1010	0.000	0.663	1.736	1.00	34.04	0.000	0.717	2.269	1.00	33.86
MSU33	0.000	0.214	1.841	23.00	1.53	0.000	0.214	1.841	23.00	1.48
MSU34	0.000	0.732	4.631	17.00	1.82	0.000	0.732	4.631	17.00	1.77
MSU44	0.000	0.417	2.424	16.00	2.78	0.000	0.417	2.424	16.00	2.69
MSU55	0.000	0.485	2.190	12.00	4.64	0.000	0.485	2.190	12.00	4.64
MSU56	0.000	0.602	2.383	9.00	5.61	0.000	0.602	2.383	9.00	5.62
MSU65	0.000	0.283	1.320	16.00	6.36	0.000	0.283	1.320	16.00	6.15
MSU66	0.000	0.732	2.897	5.00	7.42	0.000	0.732	2.897	5.00	7.33
MSU88	0.000	0.749	2.107	1.00	17.54	0.000	0.758	2.107	1.00	17.68
MSU108	0.000	0.726	2.407	3.00	25.95	0.000	0.727	2.362	2.80	25.63
MSU1010	0.295	1.220	2.425	0.00	31.95	0.295	1.241	2.639	0.00	31.19
SSU33	0.000	0.411	4.440	21.00	1.50	0.000	0.411	4.440	21.00	1.45
SSU34	0.000	1.019	4.493	11.00	1.67	0.000	1.019	4.493	11.00	1.63
SSU44	0.000	0.739	4.780	16.00	2.74	0.000	0.739	4.780	15.60	2.69
SSU55	0.000	1.198	3.910	8.00	4.78	0.000	1.198	3.910	8.00	4.72
SSU56	0.000	1.632	4.122	2.00	5.17	0.000	1.633	4.122	2.00	5.08
SSU65	0.000	0.971	3.219	4.00	6.89	0.000	0.971	3.219	4.00	6.83
SSU66	0.000	1.458	4.575	2.00	7.68	0.000	1.459	4.575	2.00	7.58
SSU88	0.374	1.865	3.429	0.00	16.76	0.374	1.867	3.429	0.00	16.56
SSU108	0.300	1.556	3.387	0.00	24.91	0.300	1.579	3.387	0.00	25.12
SSU1010	1.223	2.240	3.498	0.00	30.60	1.223	2.258	3.515	0.00	30.28
Total Ave.	0.073	0.769	2.699	9.66	10.41	0.073	0.773	2.711	9.63	10.40

Instance	UP=1000									
	MSA (P_{size}=7)					SA (P_{size}=1)				
	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *OPT*	Ave. *Time*	Min. *RER*	Ave. *RER*	Max. *RER*	Ave. *OPT*	Ave. *Time*
LSU33	0.000	0.130	2.133	24.60	1.56	0.000	0.136	2.133	24.20	1.36
LSU34	0.000	0.473	3.467	13.00	2.01	0.000	0.473	3.467	13.00	1.82
LSU44	0.000	0.218	1.999	19.60	3.19	0.000	0.190	1.903	20.00	3.01
LSU55	0.000	0.328	2.726	16.80	5.13	0.000	0.360	3.103	16.80	4.71
LSU56	0.000	0.548	3.302	5.80	6.05	0.000	0.533	3.302	6.00	5.61
LSU65	0.000	0.254	2.420	17.20	6.09	0.000	0.196	1.636	18.60	5.86
LSU66	0.000	0.535	2.852	11.00	8.14	0.000	0.598	3.326	10.80	8.14
LSU88	0.000	0.643	2.822	5.60	17.57	0.000	0.723	3.193	5.00	17.48
LSU108	0.000	1.305	5.116	3.40	25.43	0.000	1.432	5.198	2.20	25.13
LSU1010	0.078	1.409	3.615	0.00	31.92	0.086	1.749	5.514	0.00	31.87
MSU33	0.000	0.147	1.686	24.00	1.65	0.000	0.147	1.686	24.00	1.63
MSU34	0.000	0.477	4.334	20.00	1.96	0.000	0.475	4.334	20.00	1.95
MSU44	0.000	0.711	5.146	14.00	3.21	0.000	0.687	5.146	14.00	3.20
MSU55	0.000	0.593	2.862	9.80	4.79	0.000	0.604	2.853	10.00	4.76
MSU56	0.000	1.072	3.001	6.00	5.38	0.000	1.086	3.001	5.80	5.36
MSU65	0.000	0.529	3.691	7.60	6.81	0.000	0.555	3.537	7.20	6.81
MSU66	0.000	1.031	3.103	4.00	7.29	0.000	1.081	3.801	4.00	7.26
MSU88	0.000	1.368	3.497	1.20	18.49	0.000	1.526	4.356	1.00	18.35
MSU108	0.000	1.460	4.155	1.80	26.40	0.000	1.575	4.689	1.60	26.01
MSU1010	0.415	2.045	4.479	0.00	30.12	0.398	2.131	5.212	0.00	29.81
SSU33	0.000	0.470	3.299	17.00	1.71	0.000	0.470	3.299	17.00	1.62
SSU34	0.000	0.907	3.303	13.00	1.94	0.000	0.907	3.303	13.00	1.84
SSU44	0.000	0.750	2.280	6.00	3.06	0.000	0.739	2.238	6.00	3.01
SSU55	0.000	1.177	4.029	4.00	4.76	0.000	1.181	4.029	4.00	4.75
SSU56	0.000	1.902	4.475	2.00	5.45	0.000	1.924	4.475	1.80	5.38
SSU65	0.000	0.981	3.925	3.00	6.75	0.000	0.942	3.856	2.80	6.58
SSU66	0.000	1.185	3.141	2.00	7.71	0.000	1.182	3.141	1.80	7.66
SSU88	0.434	1.873	3.887	0.00	17.20	0.434	1.918	3.990	0.00	17.18
SSU108	0.191	1.824	3.521	0.00	27.09	0.311	1.835	3.620	0.00	26.75
SSU1010	1.408	2.828	4.408	0.00	33.05	1.560	2.806	4.459	0.00	32.55
Total Ave.	0.084	0.972	3.422	8.41	10.73	0.093	1.005	3.593	8.35	10.58

Table 6
Paired *t*-tests on *RER* for the comparison between MSA and SA (*UP*=10, 100 and 1000).

	LSU (df=49)	MSU (df=49)	SSU (df=49)	Overall test problems (df=149)
UP=10, MSA (P_{size}=8) vs. SA (P_{size}=1)				
Min. *RER*	t-value=0.000, p-value=0.500	t-value=0.000, p-value=0.500	t-value=−0.864, p-value=0.196	t-value=−0.865, p-value=0.194
Ave. *RER*	t-value=−1.552, p-value=0.064	t-value=−2.635, p-value=0.006	t-value=−2.750, p-value=0.004	t-value=−3.703, p-value=0.000
Max. *RER*	t-value=−0.392, p-value=0.348	t-value=−2.193, p-value=0.017	t-value=−1.119, p-value=0.135	t-value=−2.348, p-value=0.010
Ave. *OPT*	t-value=0.000, p-value=0.500	t-value=0.000, p-value=0.500	t-value=0.000, p-value=0.500	t-value=0.000, p-value=0.500
Ave. *Time*	t-value=1.513, p-value=0.932	t-value=0.324, p-value=0.626	t-value=1.591, p-value=0.941	t-value=1.045, p-value=0.851
UP=100, MSA (P_{size}=9) vs. SA (P_{size}=1)				
Min. *RER*	t-value=0.000, p-value=0.500	t-value=0.000, p-value=0.500	t-value=0.000, p-value=0.500	t-value=0.000, p-value=0.500
Ave. *RER*	t-value=−1.188, p-value=0.120	t-value=−1.467, p-value=0.321	t-value=−1.867, p-value=0.034	t-value=−1.854, p-value=0.033
Max. *RER*	t-value=−0.244, p-value=0.404	t-value=−0.654, p-value=0.258	t-value=−0.450, p-value=0.326	t-value=−0.494, p-value=0.311
Ave. *OPT*	t-value=1.000, p-value=0.161	t-value=1.000, p-value=0.161	t-value=1.429, p-value=0.080	t-value=2.020, p-value=0.023
Ave. *Time*	t-value=−0.063, p-value=0.475	t-value=2.052, p-value=0.970	t-value=2.359, p-value=0.989	t-value=0.172, p-value=0.586
UP=1000, MSA (P_{size}=7) vs. SA (P_{size}=1)				
Min. *RER*	t-value=−0.704, p-value=0.242	t-value=−0.667, p-value=0.254	t-value=−1.415, p-value=0.500	t-value=−1.573, p-value=0.059
Ave. *RER*	t-value=−2.441, p-value=0.009	t-value=−2.210, p-value=0.016	t-value=−0.060, p-value=0.476	t-value=−3.126, p-value=0.001
Max. *RER*	t-value=−1.256, p-value=0.107	t-value=−2.188, p-value=0.017	t-value=−0.456, p-value=0.325	t-value=−2.291, p-value=0.012
Ave. *OPT*	t-value=0.280, p-value=0.390	t-value=1.158, p-value=0.126	t-value=1.769, p-value=0.042	t-value=1.117, p-value=0.133
Ave. *Time*	t-value=1.278, p-value=0.897	t-value=0.598, p-value=0.724	t-value=0.744, p-value=0.770	t-value=1.514, p-value=0.934

that such an approach contributes significantly to the research of heuristic techniques for solving this problem.

This study has made progress towards establishing an effective MSA heuristic for FMCSPs with SDFSTs. Future studies could build upon this research in several ways. First, further study may focus on developing other efficient and effective meta-heuristics for this problem. Second, it is possible to extend the proposed MSA heuristic to solve more complex FMCSPs such as those involving multiple machines at various stages in a multiple stage hybrid flowshop. Third, extensions of the proposed MSA heuristic to solve this problem with other performance criteria or with a secondary criterion deserve further investigation. Finally, multi-FMCSPs, while practical, are more complex problems requiring further study.

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