## **EEE269-Electrical Drives and Instrumentation**

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1. Ideal Transformer

a. 
$$a = \frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{I_S}{I_P}$$

- 2. Real Transformer
  - a. Basic

i. 
$$E = 4.44Nf\phi_{max}$$

ii. 
$$\phi_{max} = B_{max} \times A$$

1. B = Magnetic Flux density

iii. 
$$H = \frac{N_p I_M}{l}$$

- 1. H = magnetic field intensity (A-t/m)
- 2.  $N_P$  = Turns in primary winding
- 3.  $I_M$  = Magnetizing current (A)
- 4. l = Mean length of core
- b. Determining components
  - i. Open circuit test
    - 1. Performed on low voltage side in rated voltage

$$2. PF = \frac{P_{OC}}{V_{OC}I_{OC}}$$

3. 
$$Y_E = \frac{I_{OC}}{V_{OC}} \angle - \cos^{-1} PF = \frac{1}{R_C} - j\frac{1}{X_M}$$

- ii. Short circuit test
  - 1. Performed on high voltage side in rated current

$$2. PF = \frac{P_{SC}}{V_{SC}I_{SC}}$$

3. 
$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \cos^{-1} PF = R_{eq,s} + jX_{eq,s}$$

c. Voltage regulation

i. 
$$VR = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\%$$

ii. 
$$VR = \frac{\left|\frac{V_P}{a}\right| - \left|V_{S,fl}\right|}{\left|V_{S,fl}\right|} \times 100\%$$
 (Referred to Secondary)

1. 
$$\frac{V_P}{a} = V_S + \left(R_{eq,S} + jX_{eq,S}\right) \times I_S$$

iii. 
$$VR = \frac{V_P - aV_{S,fl}}{aV_{S,fl}} \times 100\%$$
 (Referred to Primary)

1. 
$$V_P = aV_S + (R_{eq,p} + jX_{eq,p}) \times \frac{I_S}{a}$$

d. Transformer efficiency

i. 
$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

ii. 
$$P_{in} = P_{out} + P_{Cu} + P_{core} = V_P I_P \cos \theta$$

iii. 
$$P_{out} = V_s I_s \cos \theta$$

iv. 
$$P_{core} = \frac{\left(\frac{V_P}{a}\right)^2}{R_{c,p}} = \frac{V_P^2}{R_{c,p}}$$

v. 
$$P_{Cu} = I_S^2 R_{eq,S}$$

vi. 
$$I_P = I_{EX} + \frac{I_S}{a} = \frac{V_P}{R_C} + \frac{V_P}{X_M} + \frac{I_S}{a}$$

## 3. Induction motor

a. Slip

i. 
$$n_{slip} = n_{sync} - n_m$$

ii. 
$$s = \frac{n_{slip}}{n_{sync}} \times 100\% = \frac{n_{sync} - n_m}{n_{sync}} \times 100\% = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} \times 100\%$$

b. Speed

i. 
$$n_m = (1 - s) n_{sync}$$

ii. 
$$\omega_m = (1-s)\omega_{sync}$$

iii. 
$$\omega_m = \frac{2\pi}{60} n_m$$

c. Frequency

i. 
$$n_{sync} = \frac{120f_{se}}{p}$$

ii. 
$$f_{re} = sf_{se}$$

- d. Determine parameters
  - i. No load test (Open circuit)

1. 
$$\frac{V_{\phi,nl}}{I_{1,nl}} = X_1 + X_M$$

2. Y connected stator, 
$$V_{\phi} = \frac{V_T}{\sqrt{3}}$$
,  $I_1 = I_L \rightarrow \frac{V_T}{\sqrt{3}I_L} = X_1 + X_M$ 

3. 
$$\Delta$$
 connected stator,  $V_{\phi} = V_T$ ,  $I_1 = \frac{I_L}{\sqrt{3}} \rightarrow \frac{\sqrt{3}V_T}{I_L} = X_1 + X_M$ 

ii. DC test

1. Y connected stator, 
$$R_1 = \frac{V_{DC}}{2I_{DC}}$$

2. 
$$\Delta$$
 connected stator,  $R_1 = 3 \frac{V_{DC}}{2I_{DC}}$ 

iii. Locked rotor test (Short circuit)

1. 
$$|Z_{LR}| = \frac{V_{\phi}}{I_1}$$

2. 
$$Z_{LR} = R_{LR} + jX'_{LR} = |Z_{LR}|\cos\theta + j|Z_{LR}|\sin\theta$$

3. 
$$R_{LR} = R_1 + R_2 = |Z_{LR}| \cos \theta$$

4. 
$$X'_{LR} = X'_1 + X'_2 = |Z_{LR}| \sin \theta$$

5. 
$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

6. PF = 
$$\cos \theta = \frac{P_{in}}{\sqrt{3}V_TI_L}$$

e. Torque

i. 
$$\tau_{load} = \frac{P_{out}}{\omega_m}$$

ii. 
$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{AG}}{\omega_{sync}}$$

iii. At max torque

1. 
$$s_{max} = \frac{R_2}{|Z_{TH} + X_2|}$$

2. 
$$I_2 = \frac{V_{TH}}{Z_{TH} + X_2 + \frac{R_2}{s_{max}}}$$

3. 
$$P_{AG} = 3I_2^2 \frac{R_2}{s_{max}}$$

3. 
$$P_{AG} = 3I_2^2 \frac{R_2}{s_{max}}$$
4.  $s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$ 

5. 
$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \frac{R_2}{s_{max}}\right]}$$

f. Power

i. 
$$P_{in} = \sqrt{3}V_TI_A\cos\theta = P_{out} + P_{RCL} + P_{F\&W} + P_{misc} + P_{SCL} + P_{core}$$
 
$$1. \ I_A = \frac{V_\phi}{Z_{eq}}$$

ii. 
$$P_{SCL} = 3I_1^2 R_1$$

iii. 
$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

iv. 
$$P_{RCL} = 3I_2^2 R_2 = s P_{AG}$$

v. 
$$P_{conv} = (1 - s)P_{AG}$$

vi. 
$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$

g. Efficiency

i. 
$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

4. Synchronous generator

a. Frequency

i. 
$$f_{se} = \frac{n_m P}{120}$$

b. Internal generated voltage

i. 
$$E_A = k\phi\omega$$

c. Equivalent

i. 
$$V_{\phi} = E_A - jX_SI_A - R_AI_A$$

ii. 
$$\Delta$$
 connected,  $V_{\phi} = V_T$ ,  $I_A = \frac{I_L}{\sqrt{3}}$ 

iii. Y connected, 
$$V_{\phi} = \sqrt{3}V_T$$
,  $I_A = I_L$ 

d. Torque

i. 
$$\tau_{app} = \frac{P_{in}}{\omega_m}$$

e. Power

i. 
$$P_{out} = \sqrt{3}V_T I_L \cos \theta$$

ii. 
$$P_{in} = P_{out} + P_{elec} + P_{core} + P_{mech} + P_{stray}$$

iii. Ignoring  $R_A$ 

1. 
$$P_{conv} = P_{out} = \frac{3V_{\phi}E_A}{X_S}\sin\delta$$

2. 
$$P_{out} = P_{conv} = \sqrt{3}V_T I_L \cos \theta$$

3. Static stability limit, 
$$P_{max} = \frac{3V_{\phi}E_A}{X_S}$$

4. 
$$E_A \sin \delta = X_S I_A \cos \theta$$

5. 
$$\pi_{ind} = \frac{3V_{\phi}E_A}{\omega_m X_S} \sin \delta$$

f. Voltage regulation

i. 
$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

g. Efficiency

i. 
$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

5. Synchronous generator

a. Frequency

i. 
$$f_{se} = \frac{n_m P}{120}$$

b. Internal generated voltage

i. 
$$E_A = k\phi\omega$$

c. Equivalent

i. 
$$V_{\phi} = E_A - jX_SI_A - R_AI_A$$

ii. 
$$\Delta$$
 connected,  $V_{\phi} = V_T$ ,  $I_A = \frac{I_L}{\sqrt{3}}$ 

iii. Y connected, 
$$V_{\phi} = \sqrt{3}V_T$$
,  $I_A = I_L$ 

d. Torque

i. 
$$au_{app} = \frac{P_{in}}{\omega_m}$$

e. Power

i. 
$$P_{out} = \sqrt{3}V_T I_L \cos \theta$$

ii. 
$$P_{in} = P_{out} + P_{elec} + P_{core} + P_{mech} + P_{stray}$$

iii. Ignoring  $R_A$ 

1. 
$$P_{conv} = P_{out} = \frac{3V_{\phi}E_A}{X_S}\sin\delta$$

2. 
$$P_{conv} = \sqrt{3}V_T I_L \cos \theta$$

3. Static stability limit, 
$$P_{max} = \frac{3V_{\phi}E_A}{X_S}$$

4. 
$$E_A \sin \delta = X_S I_A \cos \theta$$

5. 
$$\pi_{ind} = \frac{3V_{\phi}E_A}{\omega_m X_S} \sin \delta$$

f. Voltage regulation

i. 
$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

g. Efficiency

i. 
$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

- 6. DC Motor
  - a. Separately excited

i. 
$$I_F = \frac{V_F}{R_F}$$

ii. 
$$I_A = I_L$$

b. Shunt

i. 
$$V_T = E_A + I_A R_A$$

ii. 
$$I_F = \frac{V_T}{R_F}$$

iii. 
$$I_F^* = I_F - \frac{F_{AR}}{N_F}$$

iv. 
$$E_A = k\phi\omega_m$$

- v. Linear magnetization curve (Assume when curve is not given)
  - 1. Flux is proportional to field current

$$2. \ \frac{\phi_1}{\phi_2} = \frac{I_{F1}}{I_{F2}}$$

c. Series

i. 
$$V_T = E_A + I_A (R_A + R_S)$$

ii. 
$$I_s = I_A = I_L$$

d. Compounded

i. 
$$I_F = \frac{V_T}{R_F}$$

ii. 
$$I_L = I_F + I_A$$

iii. 
$$I_A = I_S$$

iv. Cumulatively Compounded

1. 
$$I_F^* = I_F + \frac{N_{SE}}{N_F} I_A - \frac{F_{AR}}{N_F}$$

2. 
$$I_F^* = I_F + \frac{N_{SE}}{N_F} I_A$$
 (Ignoring AR)

v. Differentially Compounded

1. 
$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A - \frac{F_{AR}}{N_F}$$

2. 
$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A$$
 (Ignoring AR)

- 7. Measurement
  - a. Differential Amplifier

i. 
$$V_o = \frac{R_2}{R_1} (V_1 - V_2)$$

ii. 
$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

iii. 
$$CMMR(in \ dB) = 20 \log_{10} \frac{|A_d|}{|A_{cm}|}$$

iv. 
$$A_d = Gain in differential mode$$

v. 
$$A_{cm} = Gain in common mode = \frac{V_o}{V_{i(cm)}}$$

b. Instrumentation Amplifier

i. 
$$A = \frac{V_0}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}}\right) \frac{R_3}{R_2} = 1 + \frac{2}{a} \left[R_2 = R_3 \text{ and } R_1 = R, R_{gain} = aR\right]$$

ii. Gain from buffer = 
$$\left(1 + \frac{2R_1}{R_{gain}}\right)$$

iii. Gain from differential amplifier = 
$$\frac{R_3}{R_2}$$

c. Logarithmic Amplifier

i. 
$$V_o = -nV_T \ln \frac{V_i}{RI_S}$$

- 1.  $I_S$  = Reverse saturation current
- 2. n = Ideality factor

3. 
$$V_T = Thermal\ voltage = \frac{KT}{q} [depends\ on\ temparature]$$

## d. Transducer

- i. Thermoelectrical transducer
  - 1. Thermistor

a. 
$$R_1 = R_2 e^{\beta \left[ \frac{1}{T_1} - \frac{1}{T_2} \right]}$$

i. 
$$R_1$$
 = Resistance at  $T_1$ ° $K$ 

ii. 
$$R_2$$
 = Resistance at  $T_2$ ° $K$ 

iii. 
$$\beta$$
 = Temperature constant

b. 
$$R_{\theta} = R_{\theta_o} [1 + \alpha_{\theta_o} \Delta \theta]$$

i. 
$$\alpha_{\theta_0}$$
 = Resistance temperature coefficient

2. Thermocouple

a. 
$$E \approx a(\Delta \theta)$$

ii. Piezoelectric transducer

1. Charge, 
$$Q = d \times F = dAE\left(\frac{\Delta t}{t}\right)$$

2. Output voltage, 
$$E_o = \frac{Q}{C_P} = \frac{dF}{\epsilon_r \epsilon_0 A/t} = \frac{dt}{\epsilon_r \epsilon_0} \frac{F}{A} = gtP$$

a. Capacitance between electrodes, 
$$C_P = \epsilon_r \epsilon_0 A/t$$

b. Voltage sensitivity, 
$$g=d/\epsilon_r\epsilon_0$$

c. 
$$t = thickness$$

- d.  $\epsilon_r\epsilon_0=$  Permittivity of quartz 3. d= charge sensitivity of the crystal
- 4. F = applied force