IT-24666

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Question:- prove that, The set of rational numbers Q, equipped with the two binary operations of Addition and multiplication, forms a field.,

Ans:- Let $Q = \frac{a}{b} \mid a, b \in Z, b \neq 0$

1. closure: for any $\frac{a}{b}, \frac{c}{d} \in Q$:
   - $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \in Q$.
   - $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \in Q$.

2. Axioms of addition and multiplication:
   - Both operations are associative & commutative
   - Additive identity is $0 = \frac{0}{1}$.
   - multiplicative identity is $1 = \frac{1}{1}$.
   - Each $\frac{a}{b}$ has additive Inverse $-\frac{a}{b}$.
   - Each nonzero $\frac{a}{b}$ has multipliative inverse $\frac{b}{a}$

3. Distributivity: $\frac{a}{b}\left(\frac{c}{d} + \frac{e}{f}\right) = \frac{ac}{bd} + \frac{ae}{bf}$

Hence, $(Q, +, \cdot)$ satisfies all the field is an axioms. (proved).