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(a) $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$

we know,

$$x \equiv \sum a_i N_i y_i \pmod N$$

where,

$$N_i = \frac{N}{n_i} \quad ; \quad N = \prod n_i$$

y_i = N_i-এর ইনভার্স মড n_i

$$N = 3.5.7 = 105$$

$$N_1 = \frac{105}{3} = 35 \; ; \; N_2 = \frac{105}{5} = 21 \; ; \; N_3 = \frac{105}{7} = 15$$

$\therefore 35 \equiv 2 \pmod 3 \rightarrow 2^{-1} \equiv 2 \pmod 3$

$\therefore 21 \equiv 1 \pmod 5 \rightarrow 1^{-1} \equiv 1 \pmod 5$

$\therefore 15 \equiv 1 \pmod 7 \rightarrow 1^{-1} \equiv 1 \pmod 7$

Now,

$$x \equiv 1 \cdot 35 \cdot 2 + 2 \cdot 21 \cdot 1 + 3 \cdot 15 \cdot 1$$
$$= 70 + 42 + 45$$
$$= 157$$

$\therefore 157 \equiv 52 \pmod{105}$

$\therefore x \equiv 52 \pmod{105}$ Ans:-

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(b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$

We know,

$$x \equiv \sum a_i N_i y_i \pmod N$$

Where,

$$N_i = \frac{N}{n_i} \; ; \quad N = \prod n_i$$

$y_i = N_i$ - এর ইনভার্স মড n_i

$$N = 11 \cdot 29 \cdot 31 = 9889$$

$$N_1 = \frac{9889}{11} = 899 \; ; \quad N_2 = \frac{9889}{29} = 341 \; ; \quad N_3 = \frac{9889}{31} = 319$$

$\therefore 899 \equiv 8 \pmod{11} \longrightarrow 8^{-1} \equiv 7 \pmod{11}$

$\therefore 341 \equiv 22 \pmod{29} \rightarrow 22^{-1} \equiv 4 \pmod{29}$

$\therefore 319 \equiv 9 \pmod{31} \rightarrow 9^{-1} \equiv 7 \pmod{31}$

Now,

$$x \equiv 5 \cdot 899 \cdot 7 + 14 \cdot 341 \cdot 4 + 15 \cdot 319 \cdot 7$$
$$= 31465 + 19096 + 33495$$
$$= 84056$$

$\therefore 84056 \equiv 4944 \pmod{9889}$

$\therefore x \equiv 4944 \pmod{9889}$ Ans.

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(c) $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$

we know,

$$x \equiv \sum a_i N_i y_i \pmod N$$

where, $N_i = \frac{N}{n_i}$; $N = \prod n_i$

$y_i = N_i$ - এর ইনভার্স এর n_i

$$N = 6 \cdot 11 \cdot 17 = 1122$$

$N_1 = \frac{1122}{6} = 187$; $N_2 = \frac{1122}{11} = 102$;

$N_3 = \frac{1122}{17} = 66$

$\therefore 187 \equiv 1 \pmod 6 \rightarrow 1^{-1} \equiv 1 \pmod 6$

$\therefore 102 \equiv 3 \pmod{11} \rightarrow 3^{-1} \equiv 4 \pmod{11}$

$\therefore 66 \equiv 15 \pmod{17} \rightarrow 15^{-1} \equiv 8 \pmod{17}$

Now,
$$x \equiv 5 \cdot 187 \cdot 1 + 4 \cdot 102 \cdot 4 + 3 \cdot 66 \cdot 8$$
$$= 935 + 1632 + 1584 = 4151$$

$\therefore 4151 \equiv 785 \pmod{1122}$

$\therefore x \equiv 785 \pmod{1122}$

Ans:-