

Question:-

See the practice problems and Answers.  
Justify with explanations. (Any 6),

Ans:-

problem-1:- Let  $G$  be a group of orders  $pq$ , where  $p$  and  $q$  are distinct primes. Prove that  $G$  is abelian.

Ans:- Abelian of  $G$ .

problem-2:- Prove that in any group  $G$ , the set of elements of finite orders form a subgroup of  $G$ .

Ans:- True of  $G$ .

problem-3:- Let  $G$  be a group and  $a, b \in G$ . Prove that if  $a^4 = b^2$  and  $ab = ba$ , then  $(ab)^6 = e$ .

Ans:- True of  $(ab)^6 = e$ .

problem-4:-

Let  $G$  be a group and  $H$  be a subgroup of  $G$ . prove that if  $[G:H] = n$ , then for any  $x \in G$ ,  $x^n \in H$ .

Ans:- True of  $x \in G$ ,  $x^n \in H$ .

problem-5:-

Let  $G$  be a finite group and  $H$  be a proper subgroup of  $G$ . prove that the union of all conjugates of  $H$  cannot be equal to  $G$ .

Ans:- false.

problem-6:-

Let  $G$  be a finite group and  $H$  be a subgroup of  $G$ . prove that if  $|G| = p^m$  where  $p$  is prime,  $p$  does not divide  $m$ , and  $|H| = p^n$ , then  $H$  is normal in  $G$ .

Ans:- True of equations.