STRUKTURE ZA IMPLEMENTACIJU VREMENSKI-DISKRETNIH LTI SITEMA

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-Digitalna obrada signala-

Uvod

- Ekvivalentne karakterizacije LTI sistema:
  - Diferencna jednačina;
  - Impulsni odziv h[n];
  - Prenosna funkcija H(z).
- Ovakve predstave LTI sistema je potrebno prevesti u strukturu koja se implementira na nekoj digitalnoj platformi.
- Ove strukture sadrže osnovne operacije sabiranja, množenja sa konstantom i kašnjenja.

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - az^{-1}}, |z| > |a|$$

$$y[n] - ay[n-1] = b_0 x[n] + b_1 x[n-1]$$

$$\Downarrow$$

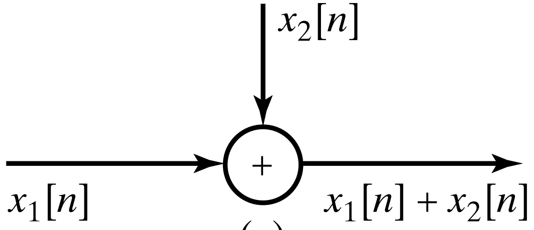
$$h[n] = b_0 a^n u[n] + b_1 a^{n-1} u[n-1]$$

$$y[n] = ay[n-1] + b_0 x[n] + b_1 x[n-1]$$

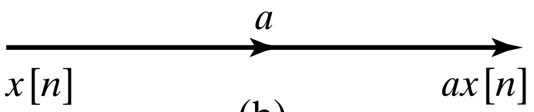
Sistem ima impulsni odziv beskonačno dugog trajanja i nije moguća njegova implementacija preko konvolucione sume.

Predstavljanje diferencinih jednačina sa konstantnim koeficijentima pomoću blok dijagrama

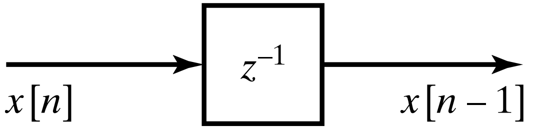
- Implementacija LTI sistema na osnovu rekurentnih formula dobijenih iz diferencnih jednačina zahtjeva zakašnjele ulazne uzorke izlaza i ulaza, kao i nekih pomoćnih sekvenci.



(a)



(b)

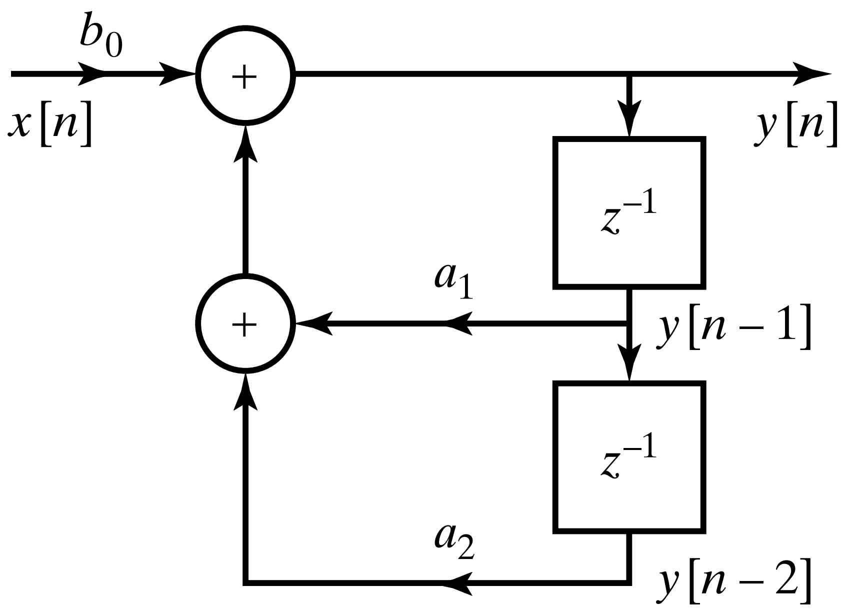


(c)

Primjer

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}}$$



Generalizacija prethodnog primjera

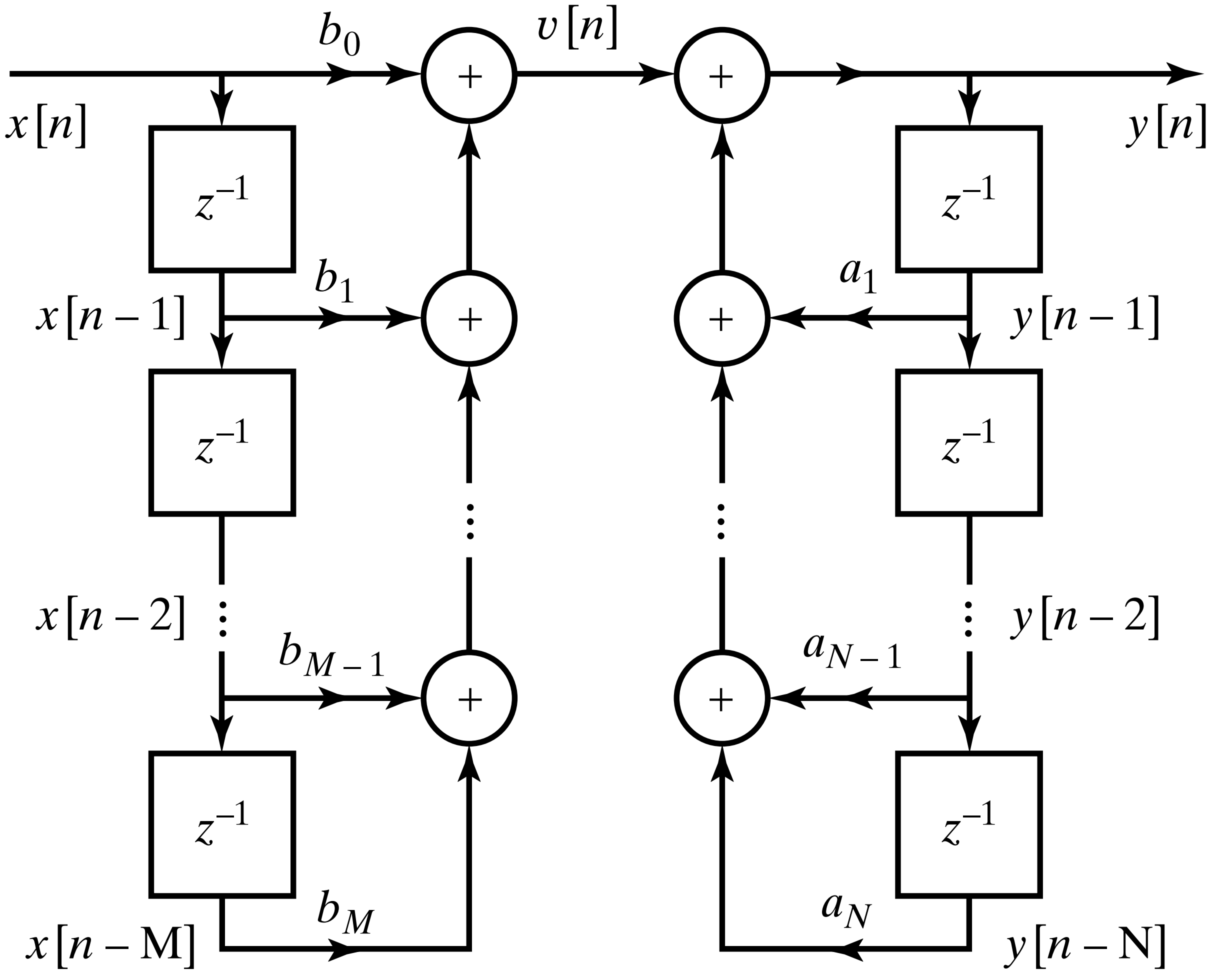
DIREKTNA FORMA I

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] - \sum_{k=1}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$



Prethodna struktura kao kaskada dva sistema

$$v[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + v[n]$$

$$H(Z) = H_2(Z)H_1(Z) = \left(\frac{1}{1-\sum_{K=1}^{N} a_k z^{-k}}\right)\left(\sum_{k=0}^{M} b_k z^{-k}\right)$$

$$V(z) = H_1(z)X(z) = \left(\sum_{k=0}^{M} b_k z^{-k}\right)X(z)$$

$$Y(z) = H_2(z)V(z) = \left(\frac{1}{1-\sum_{k=1}^{N} a_k z^{-k}}\right)V(z)$$

$$w[n] = \sum_{k=1}^{N} a_k w[n-k] + x[n]$$

$$y[n] = \sum_{k=0}^{M} b_k w[n-k]$$

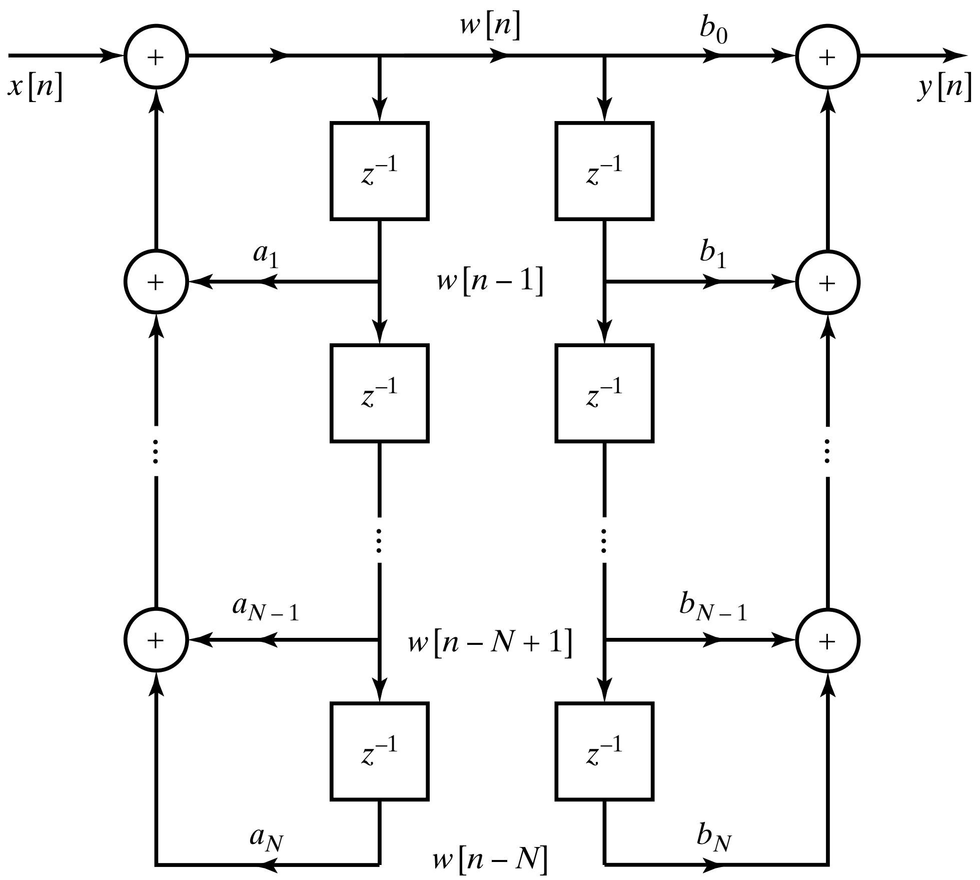
$$H(Z) = H_1(Z)H_2(Z) = \left(\sum_{k=0}^{M} b_k z^{-k}\right)\left(\frac{1}{1-\sum_{K=1}^{N} a_k z^{-k}}\right)$$

$$\mathrm{W}(z) = H_2(z)X(z) = \left(\frac{1}{1-\sum_{k=1}^{N} a_k z^{-k}}\right)X(z)$$

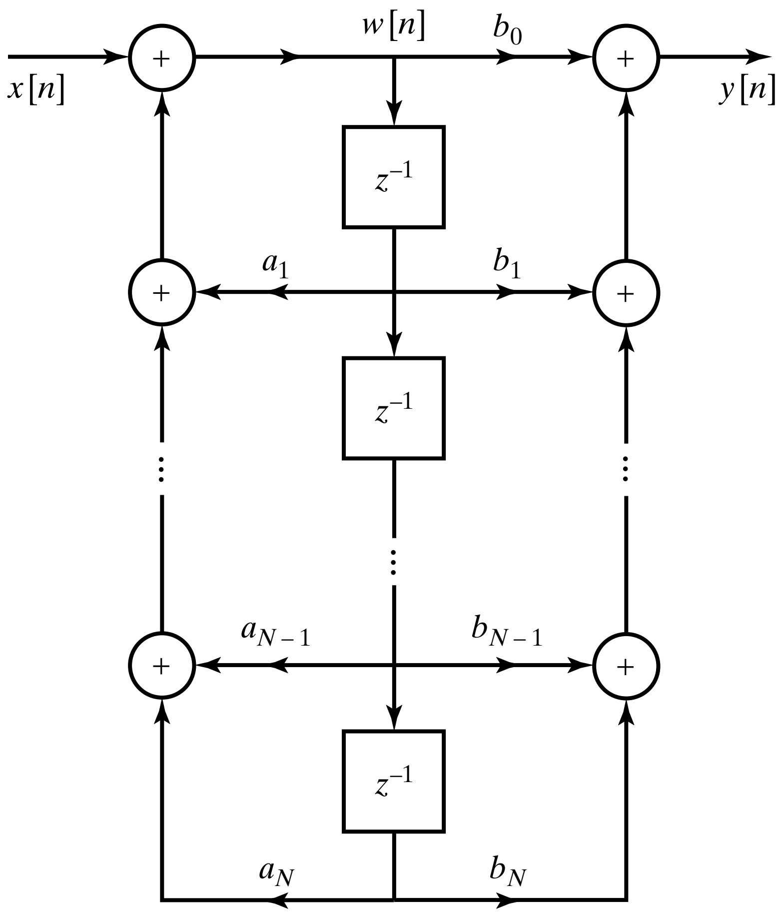
$$Y(z) = H_1(z)X(z) = \left(\sum_{k=0}^{M} b_k z^{-k}\right)W(z)$$

DIREKTNA FORMA II

Reorganizirana prethodna struktura
(kaskadna forma)



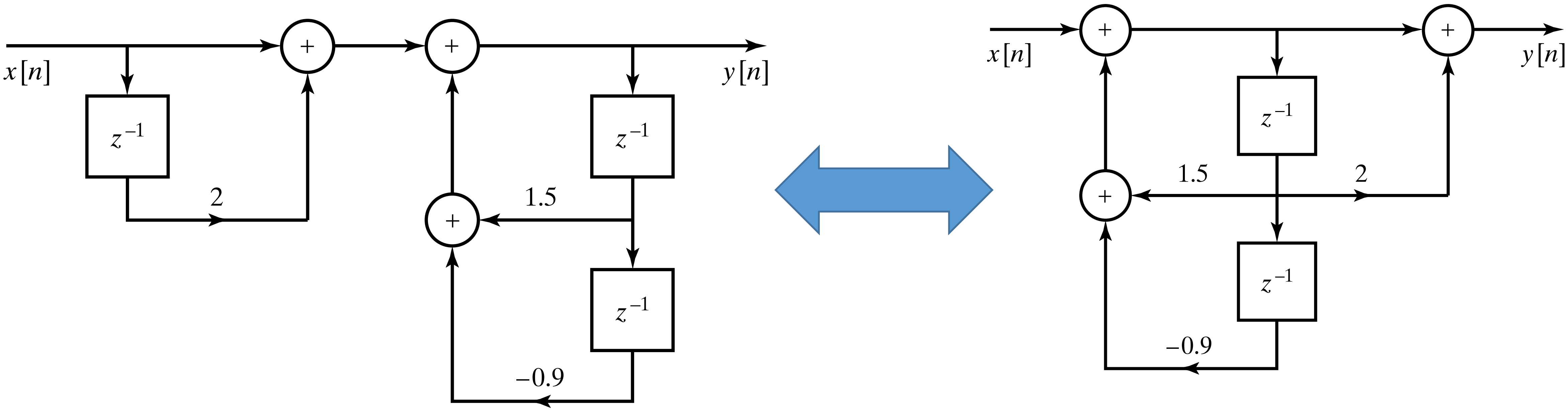
DIREKTNA FORMA II ili **KANONSKA FORMA**



Implementacija sa minimalnim brojem elemanata za kašnjenje!

Primjer implementacije IIR sistema

$$H(z) = \frac{1 + 2z^{-1}}{1 - 1{,}5z^{-1} + 0{,}9z^{-2}}$$



Strukture FIR sistema

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n}$$

$$h[n] = \begin{cases} b_n, & n = 0,1,\dots M \\ 0, & ostalo\ n \end{cases}$$

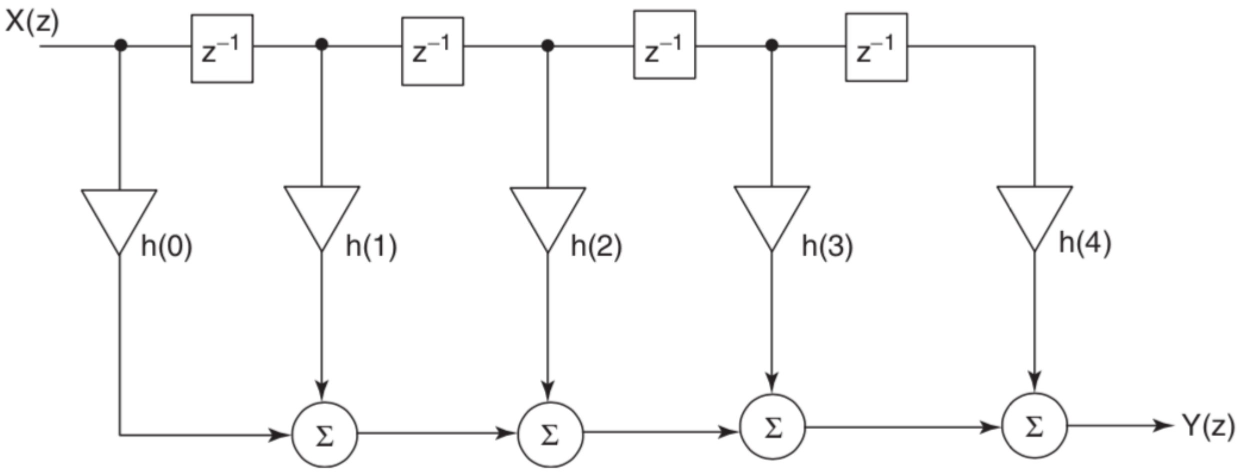
$$h[M-n] = h[n], n = 0,1,\dots M$$

Za kauzalne FIR sisteme.

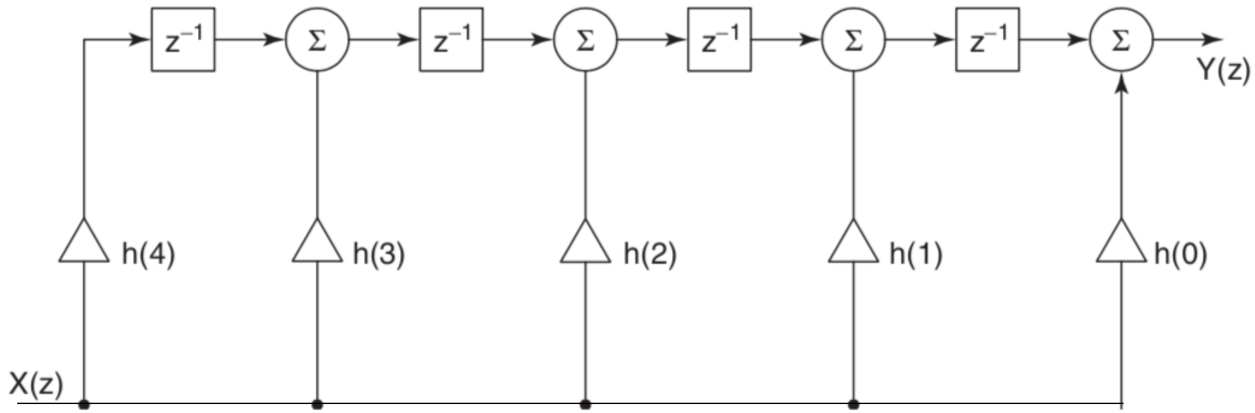
$$y[n] = \sum_{k=0}^{(M-1)/2} h[k](x[n-k] + x[n-M+k])$$

M je neparan broj!

Direktna forma I



Transponovana forma



Literatura

- A.V. Oppenheim, R.W. Schaffer, J.R. Buck, “Discrete-time signal processing”, *Prentice-Hall*, 1999.