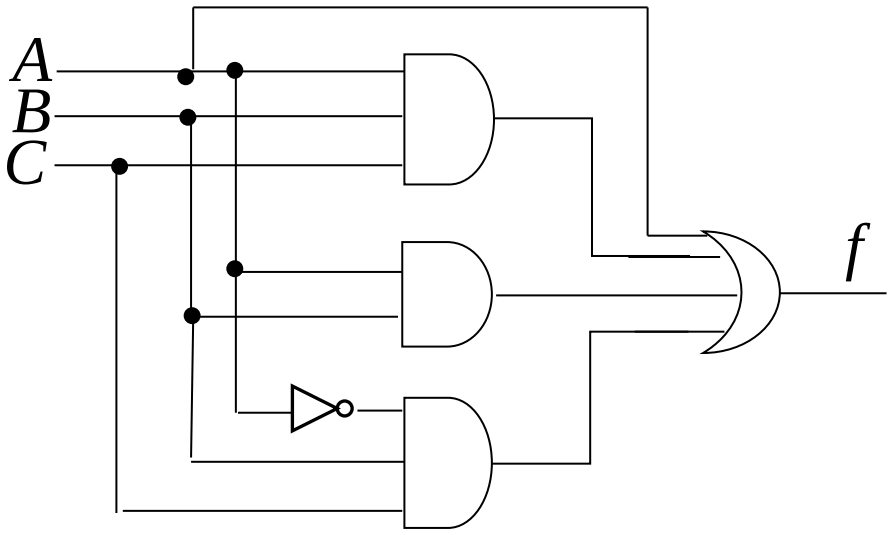
Minimizacija logičkih funkcija

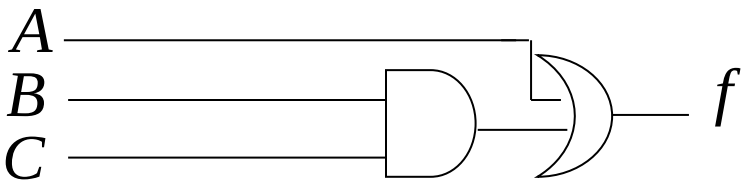
a) algebarska metoda

$$f = A + ABC + AB + \overline{A}BC$$

$$= A\underbrace{(1 + B)}_{1} + BC\underbrace{(A + \overline{A})}_{1}$$

$$= A + BC$$





Karnaugh-ove tablice (mape)
Funkcija od 2 varijable

A	B	f	P_i
0	0	v_0	$\overline{A}\overline{B} = P_0$
0	1	v_1	$\overline{A}B = P_1$
1	0	v_2	$A\overline{B} = P_2$
1	1	v_3	$AB = P_3$

A \ B	0	1
0	00	01
1	10	11

A \ B	0	1
0	v_0	v_1
1	v_2	v_3

Primjeri

A \ B	0	1
0		
1		1

$$f = AB$$

A \ B	0	1
0		1
1	1	

$$f = A\overline{B} + \overline{A}B = A \oplus B$$

A \ B	0	1
0		1
1		1

$$f = B$$

$$f = \overline{A}B + AB = B(\overline{A} + A) = B$$

A \ B	0	1
0		1
1	1	1

$$f = B + A$$

$$f = \overline{A}B + AB + A\overline{B}$$

$$= \overline{A}B + AB + AB + A\overline{B}$$

$$= B(A + \overline{A}) + A(B + \overline{B})$$

$$= B + A$$

Karnaugh-ova mapa za funkciju od 3 varijable

A \ BC	00	01	11	10
0	$\overline{A}\overline{B}\overline{C}$	$\overline{A}\overline{B}C$	$\overline{A}BC$	$\overline{A}B\overline{C}$
1	$A\overline{B}\overline{C}$	$A\overline{B}C$	ABC	$AB\overline{C}$

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

$$f(A,B,C) = \overline{A}\overline{B}C + \overline{A}BC + A\overline{B}\overline{C} + AB\overline{C}$$

A \ BC	00	01	11	10
0		1	1	
1	1			1

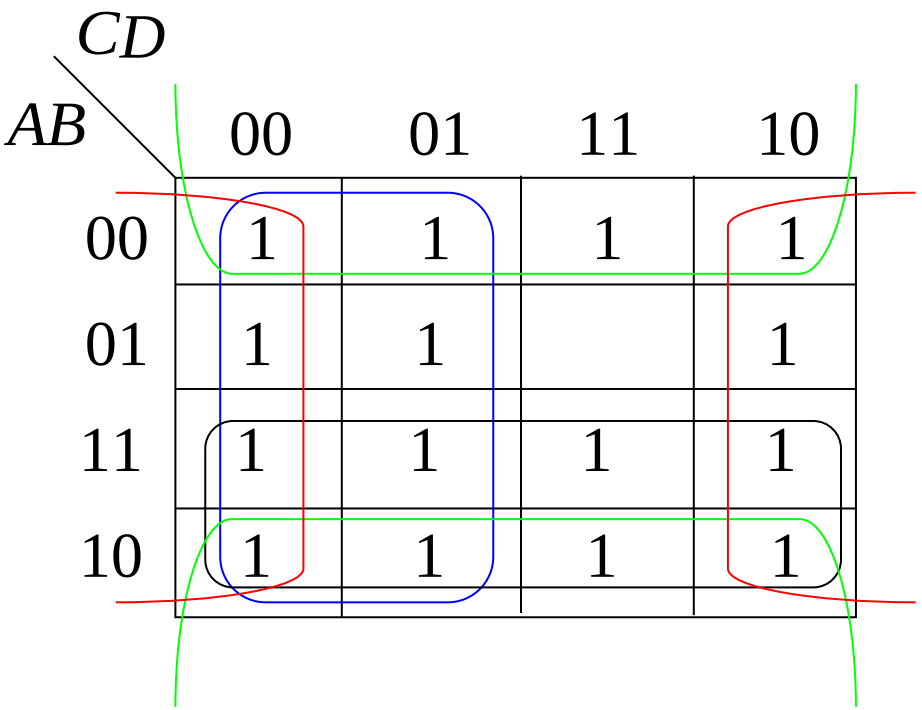
AB \ C	0	1
00		1
01		1
11	1	
10	1	

$$f_{\min} = \overline{A}C + A\overline{C} = A \oplus C$$

$$f_{\min} = \overline{A}C + A\overline{C}$$

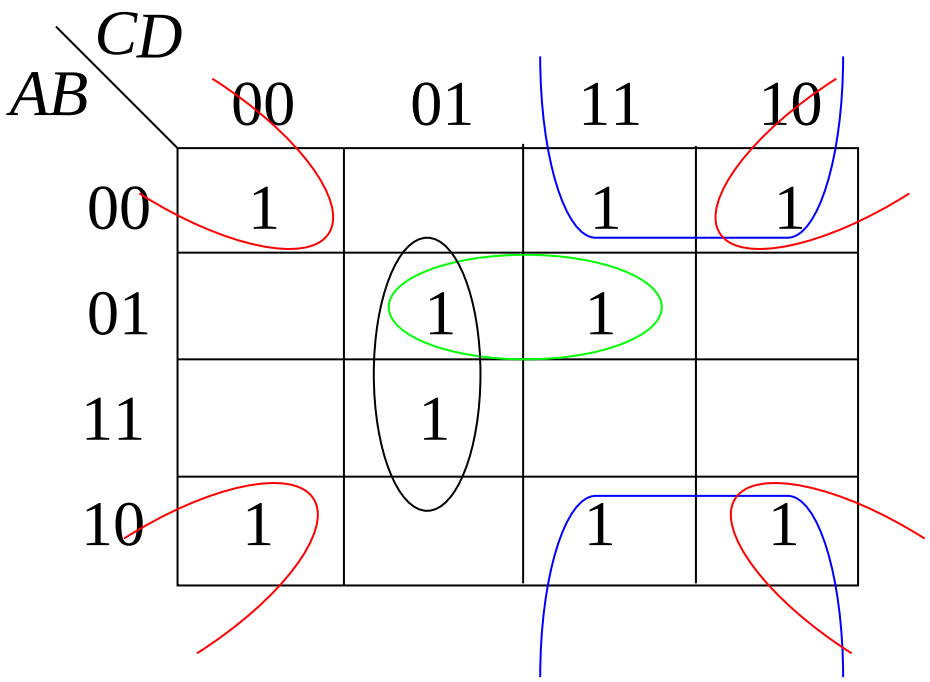
Karnaugh-ova mapa za funkciju od 4 varijable

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}



$$f_{\min} = A + \overline{B} + \overline{C} + \overline{D}$$

Primjer



$$f_{\min} = \overline{B}C + \overline{B}\,\overline{D} + \overline{A}BD + B\overline{C}D$$

Karnaugh-ova mapa za funkciju od 5 varijabli

$$f = \sum(2,3,4,7,10,11,13,16,18,19,23,24,26,27,29,31) = f(A,B,C,D,E)$$

BC \ DE	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

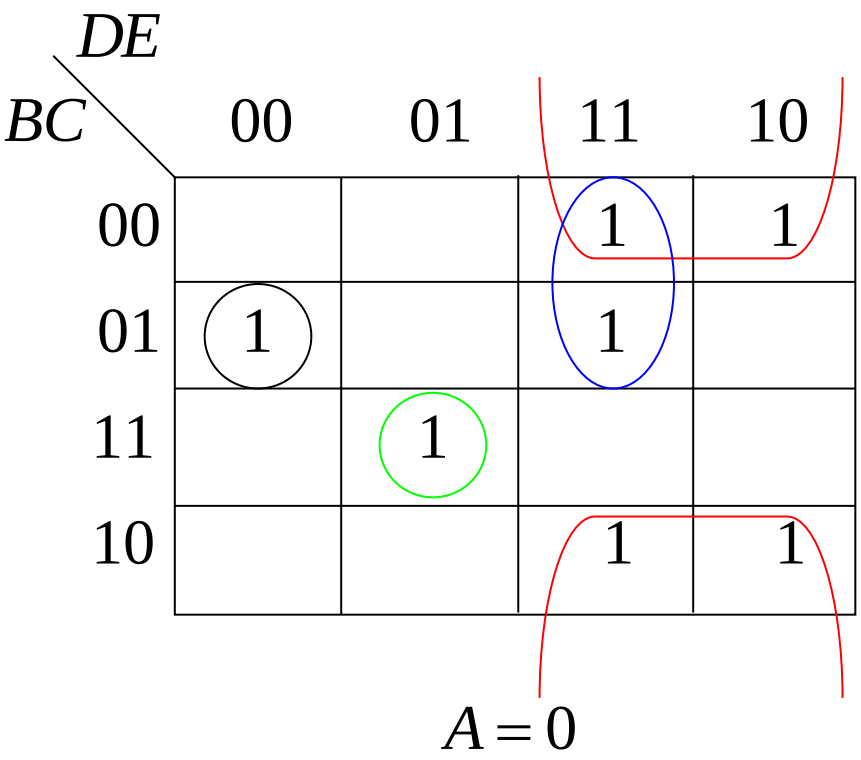
$A = 0$

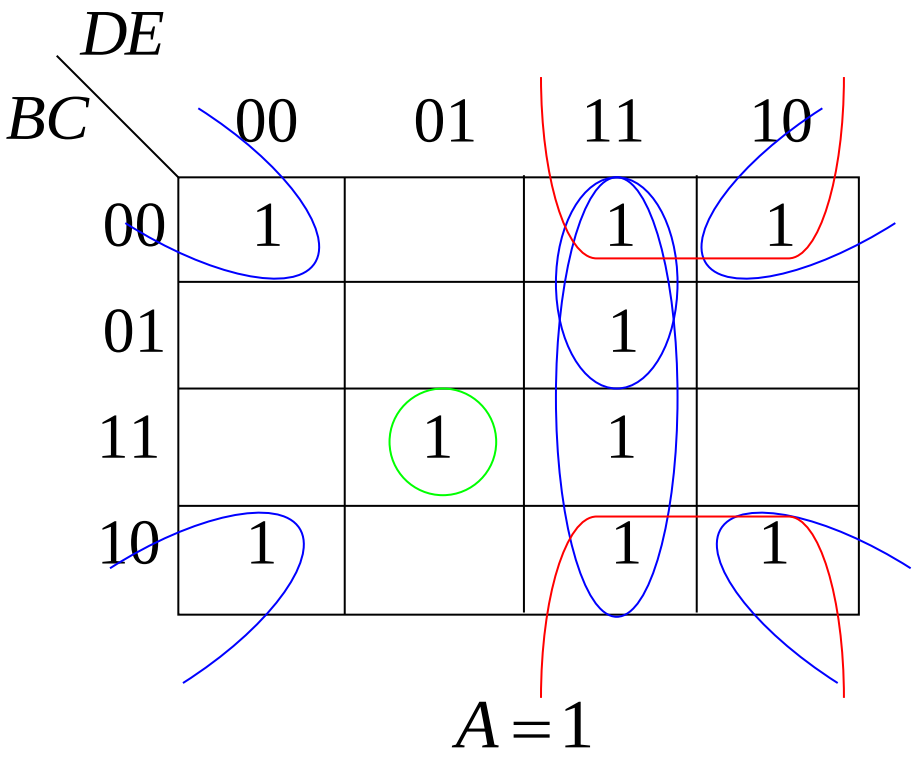
BC \ DE	00	01	11	10
00	m_{16}	m_{17}	m_{19}	m_{18}
01	m_{20}	m_{21}	m_{23}	m_{22}
11	m_{28}	m_{29}	m_{31}	m_{30}
10	m_{24}	m_{25}	m_{27}	m_{26}

$A = 1$

Karnaugh-ova mapa za funkciju od 5 varijabli - primjer

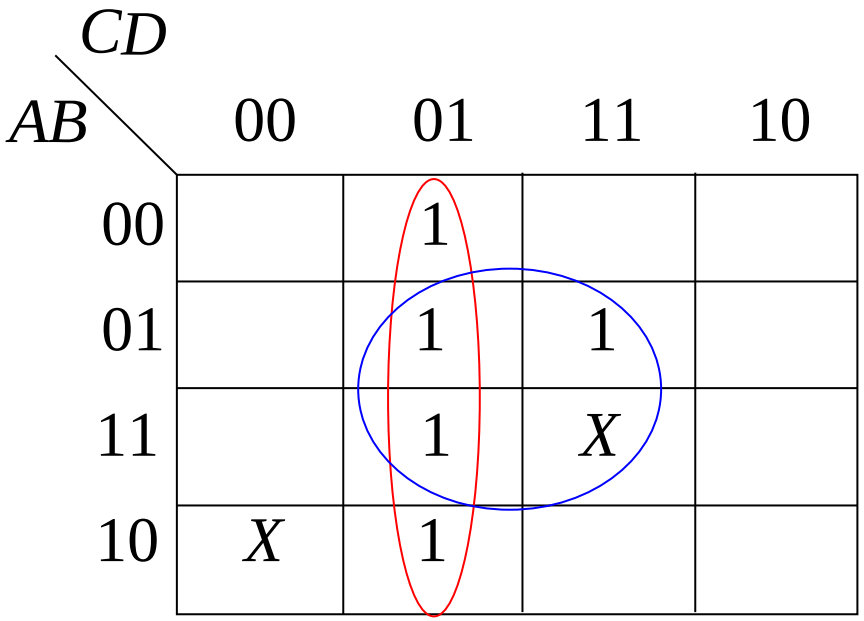
$$f = \sum(2,3,4,7,10,11,13,16,18,19,23,24,26,27,29,31) = f(A,B,C,D,E)$$





$$f_{\min} = \overline{C}D + ADE + A\overline{C}\,\overline{E} + \overline{B}DE + BC\overline{D}E + \overline{A}\,\overline{B}C\overline{D}\,\overline{E}$$

Nepotpuno specificirane funkcije



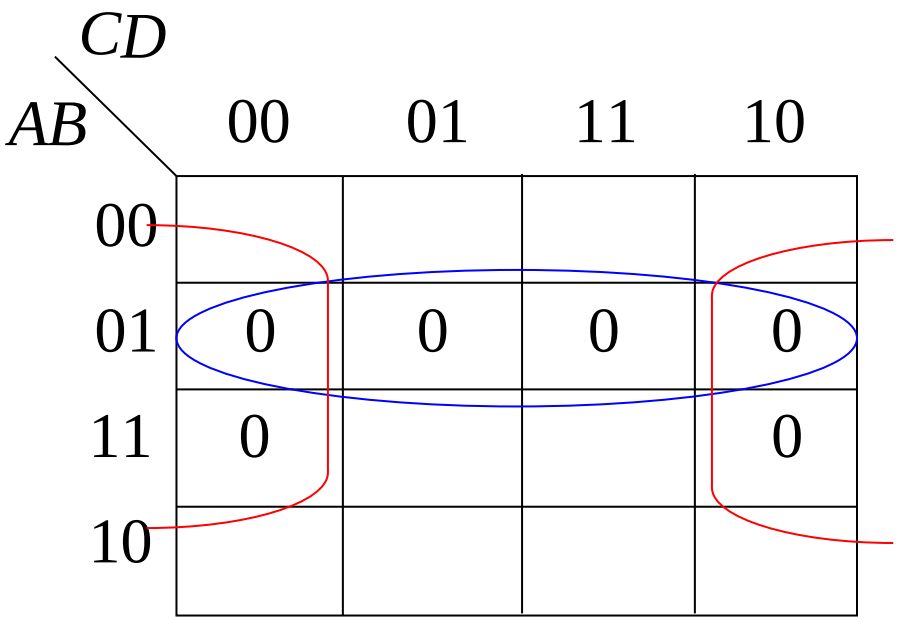
$$f_{\min} = BD + \overline{C}D$$

$$f = \sum(\ \ ) + X(\ \ )$$

$$f = \prod(\ \ ) + X(\ \ )$$

Minimizacija proizvoda makstermi

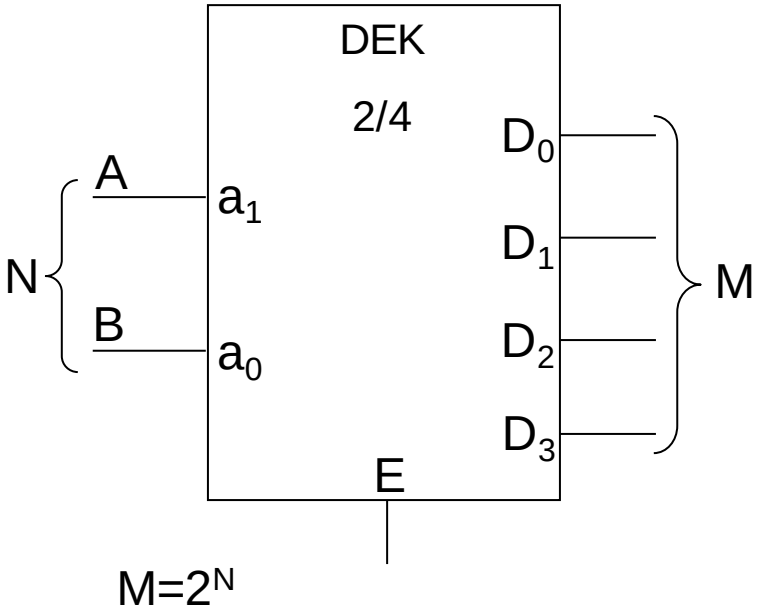
$$f = \prod(4,5,6,7,12,14)$$



$$f_{\min} = (A + \overline{B})(\overline{B} + D)$$

Kombinacijski moduli

Dekoder

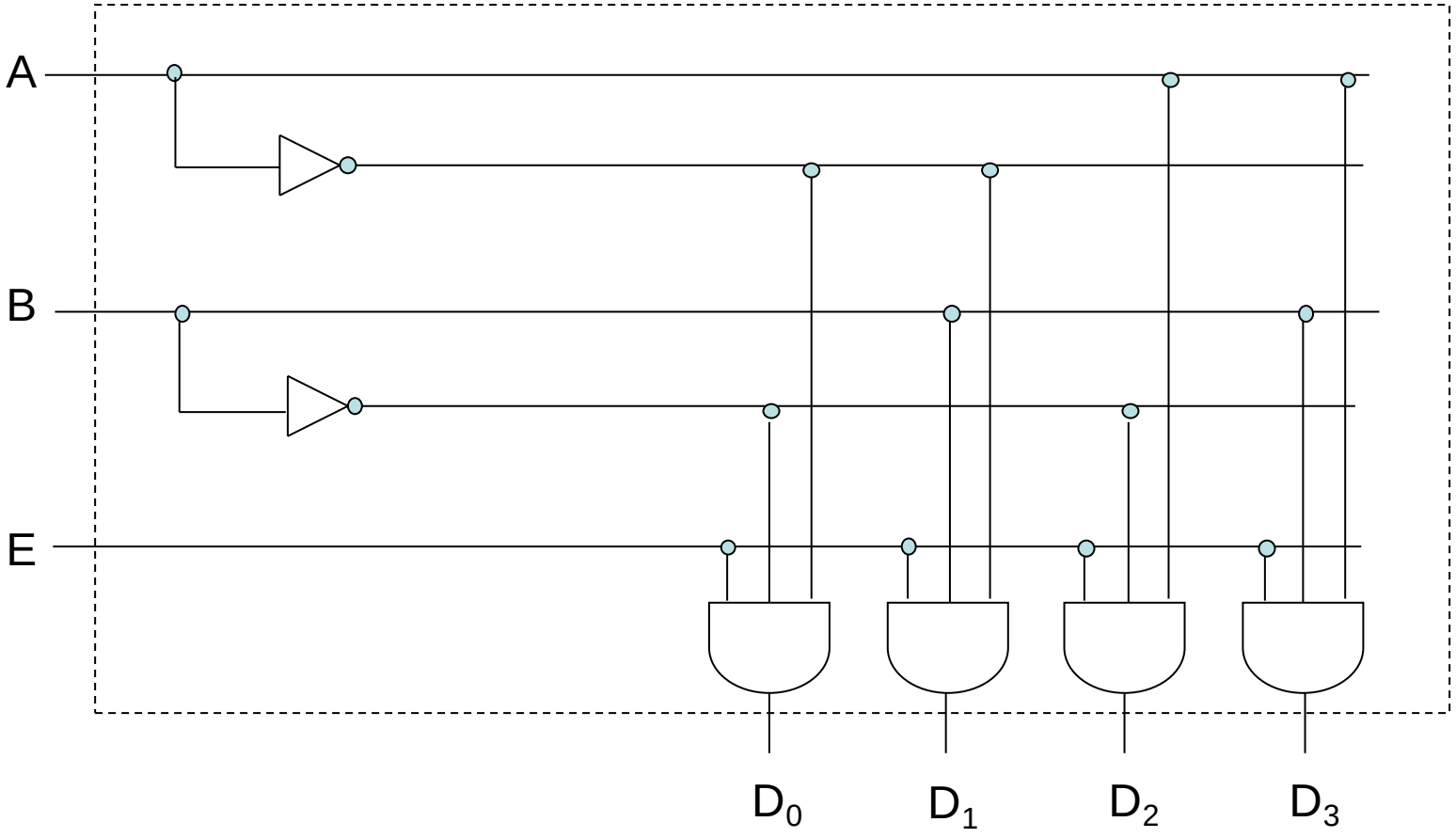


E	A	B	D_0	D_1	D_2	D_3
0	X	X	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

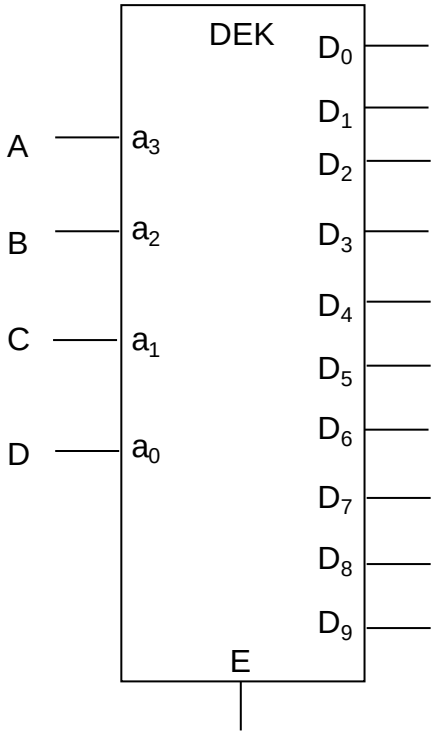
$$D_0 = E(\overline{A}\,\overline{B}) \qquad D_2 = E(A\overline{B})$$

$$D_1 = E(\overline{A}B) \qquad D_3 = E(AB)$$

Dekoder 2/4 - realizacija

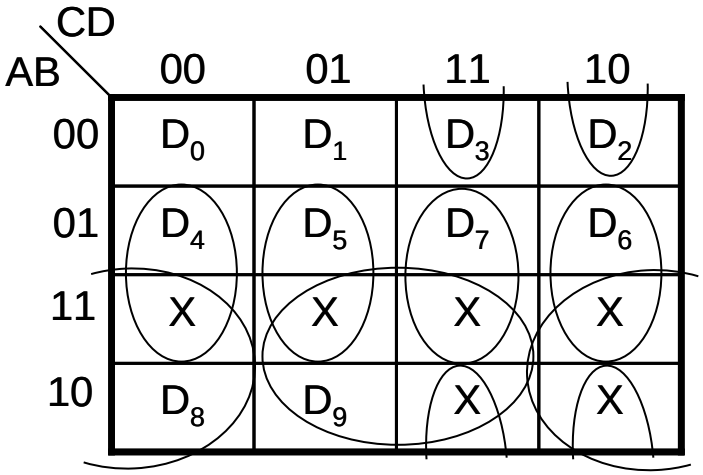


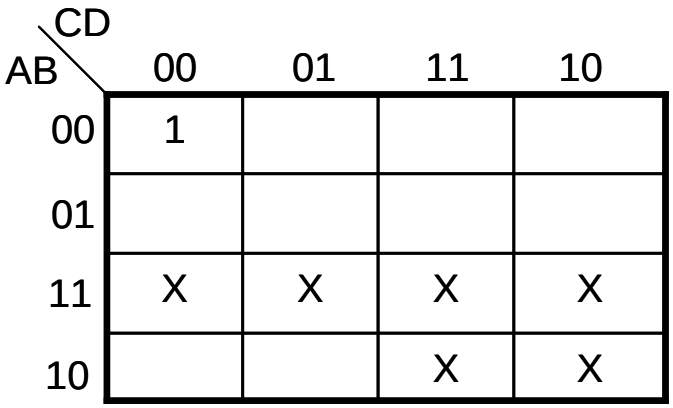
BCD-dekadski dekoder



E	A	B	C	D	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
0	X	X	X	X	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1									
1	0	0	0	1		1								
1	0	0	1	0			1							
1	0	0	1	1				1						
1	0	1	0	0					1					
1	0	1	0	1						1				
1	0	1	1	0							1			
1	0	1	1	1								1		
1	1	0	0	0									1	
1	1	0	0	1										1
1	1	0	1	0										
1	1	0	1	1										
1	1	1	0	0										
1	1	1	0	1										
1	1	1	1	0										
1	1	1	1	1										

BCD-dekadski dekoder

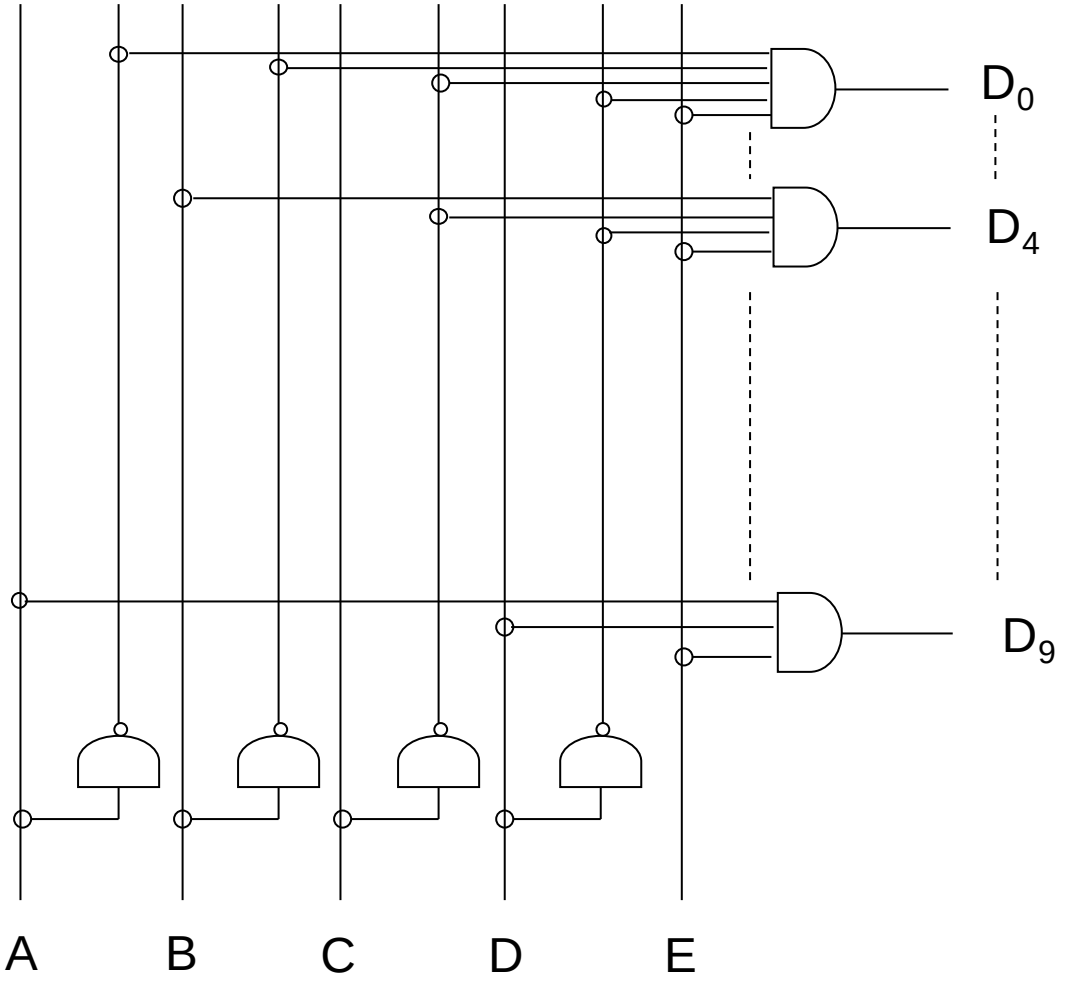




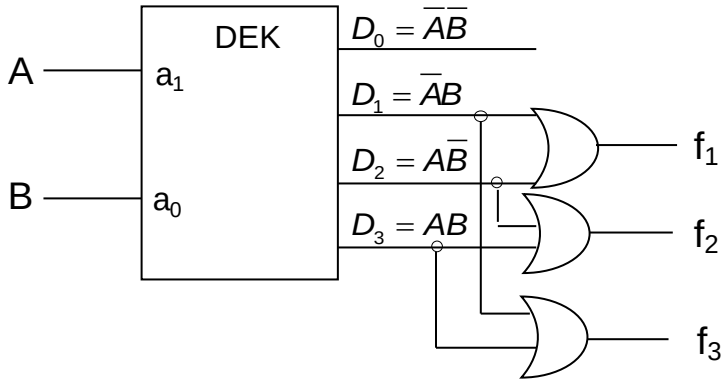
D_0

$$D_0 = E(\overline{A}\overline{B}\overline{C}\overline{D})$$
$$D_1 = E(\overline{A}\overline{B}\overline{C}D)$$
$$D_2 = E(\overline{B}C\overline{D})$$
$$D_3 = E(\overline{B}CD)$$
$$D_4 = E(B\overline{C}\overline{D})$$
$$D_5 = E(B\overline{C}D)$$
$$D_6 = E(BC\overline{D})$$
$$D_7 = E(BC\overline{D})$$
$$D_8 = E(A\overline{D})$$
$$D_9 = E(AD)$$

BCD-dekadski dekoder - realizacija



Ostvarivanje logičkih funkcija pomoću dekodera

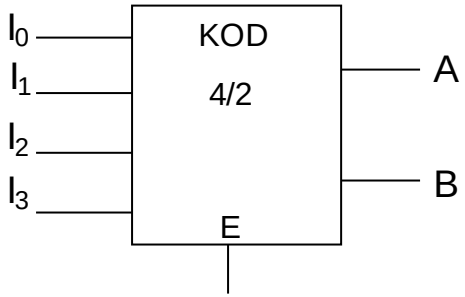


$$f_1 = A\overline{B} + \overline{A}B$$

$$f_2 = A\overline{B} + AB$$

$$f_3 = B = B(A + \overline{A}) = AB + \overline{A}B$$

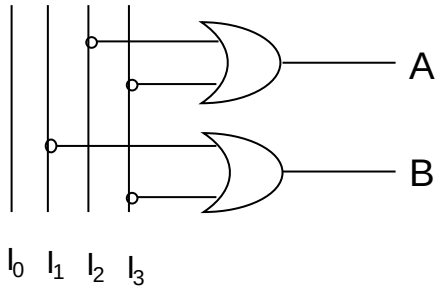
Koder



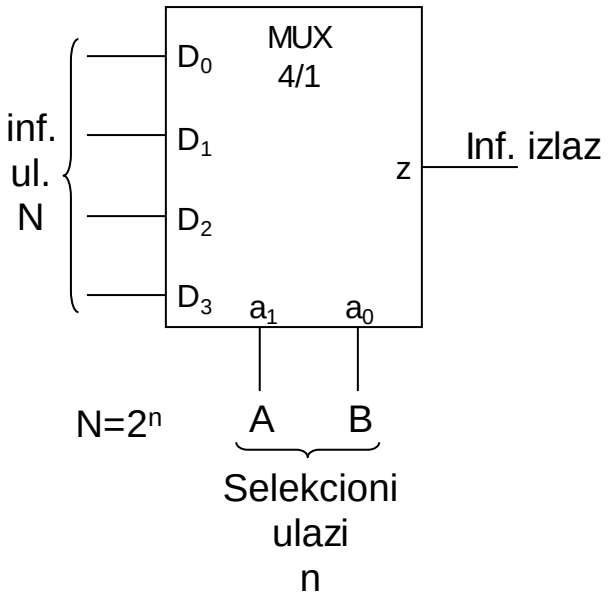
E	I_0	I_1	I_2	I_3	A	B
0	X	X	X	X	0	0
1	1	0	0	0	0	0
1	0	1	0	0	0	1
1	0	0	1	0	1	0
1	0	0	0	1	1	1

$A = I_2 + I_3$

$B = I_1 + I_3$



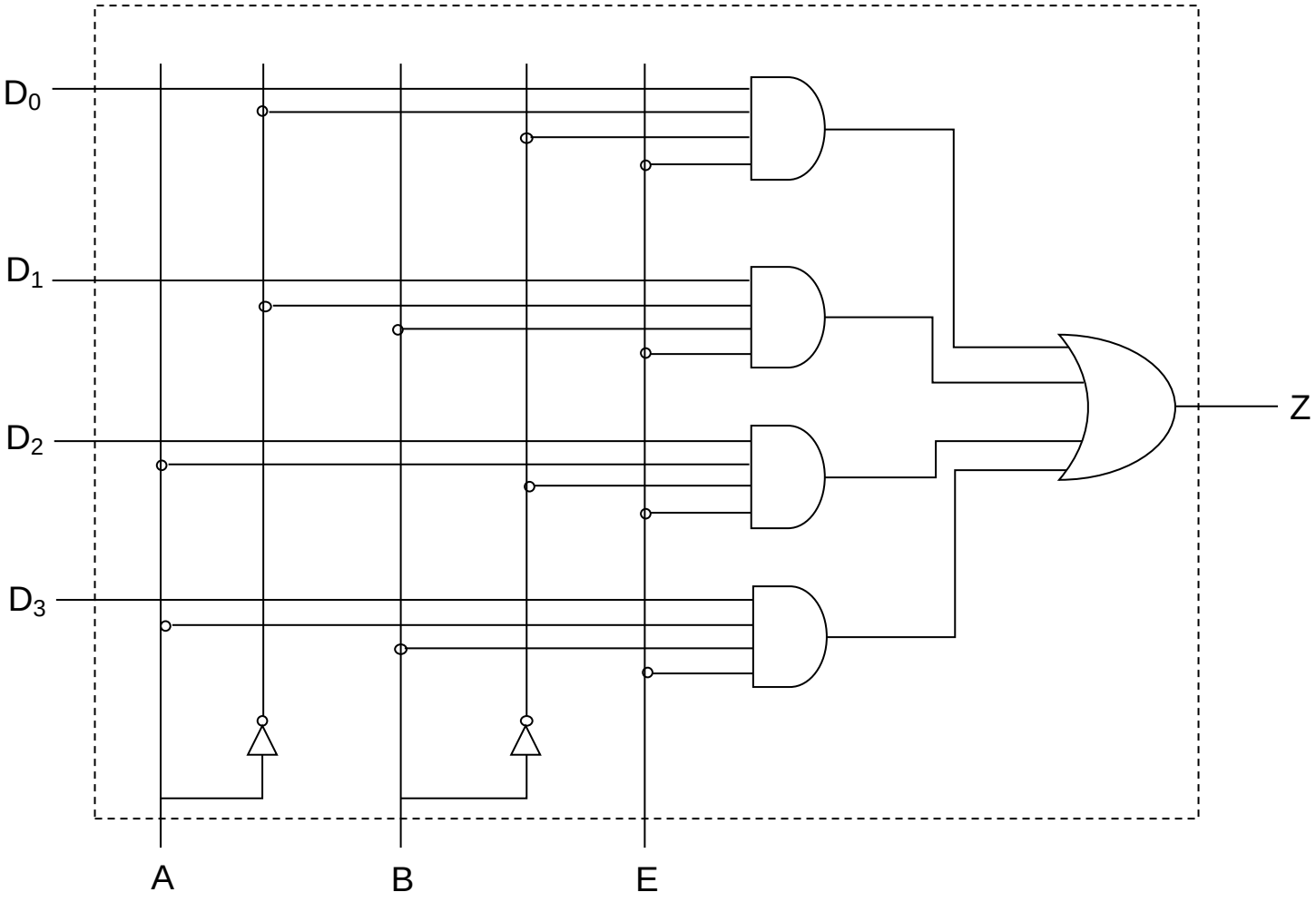
Multiplekser



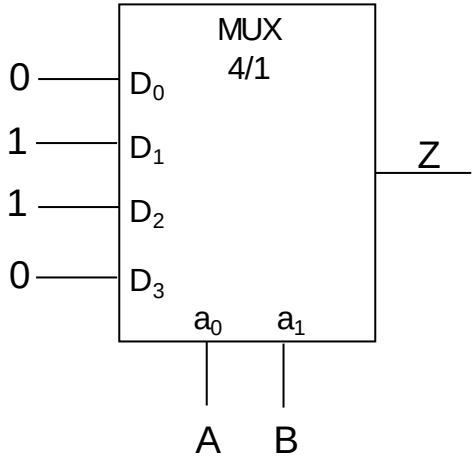
E	A	B	Z
0	X	X	0
1	0	0	D_0
1	0	1	D_1
1	1	0	D_2
1	1	1	D_3

$$Z = (D_0\overline{A}\,\overline{B} + D_1\overline{A}B + D_2A\overline{B} + D_3AB)E$$

Multiplekser - realizacija



Ostvarivanje logičkih funkcija pomoću multipleksera

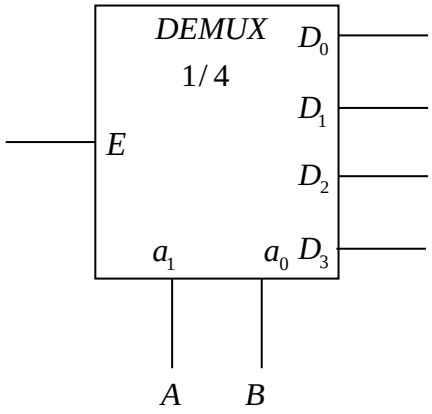


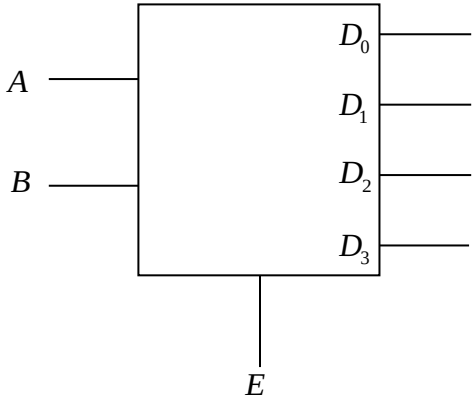
$$f = \overline{A}B + A\overline{B}$$

$$f = D_0\overline{A}\,\overline{B} + D_1\overline{A}B + D_2A\overline{B} + D_3AB$$

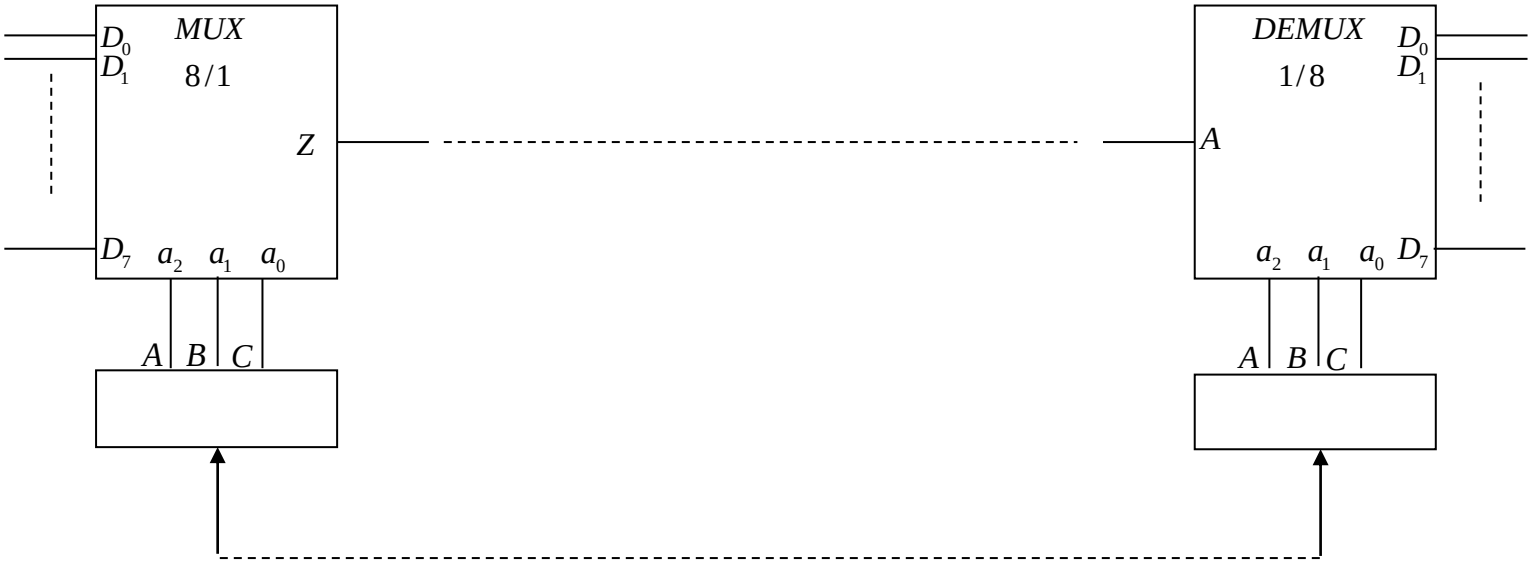
$$0 \qquad 1 \qquad 1 \qquad 0$$

Demultiplekser - DEMUX





Prijenos podataka pomoću MUX-a

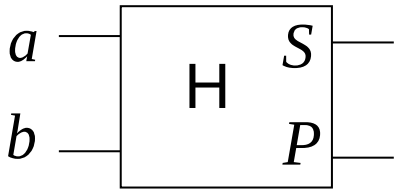


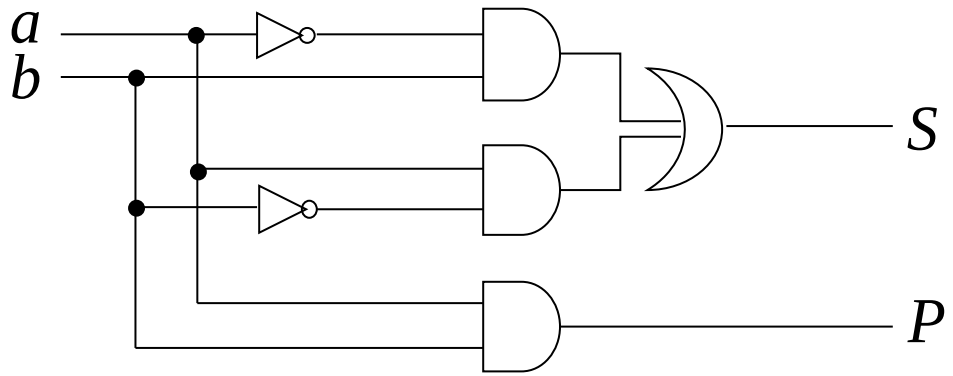
Sklop za sabiranje dvije binarne cifre - polusabirač

a	b	S	P
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

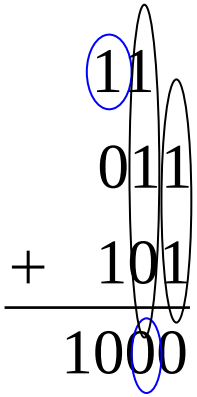
$$S = \bar{a}b + a\bar{b}$$

$$P = ab$$



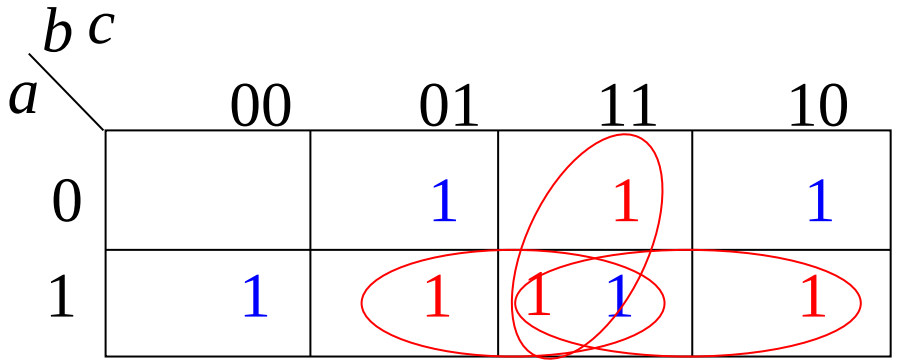


Sklop za sabiranje 3 binarne cifre



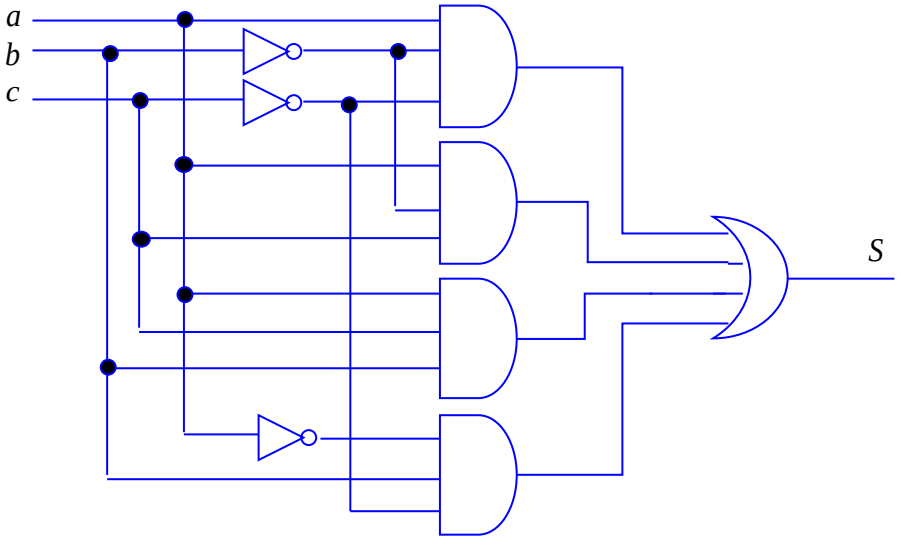
prijenos sa pozicije ispred

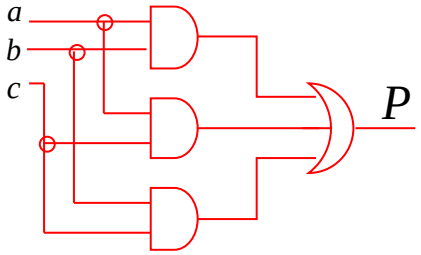
a	b	c	S	P
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



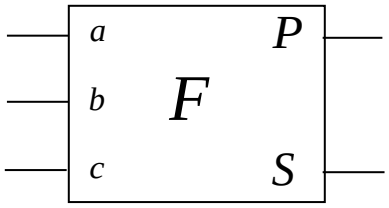
$$S = \bar{a}b\bar{c} + \bar{a}\bar{b}c + abc + \bar{a}bc$$

$$P = ac + bc + ab$$





Sklop za sabiranje tri binarne cifre – potpuni sabirač



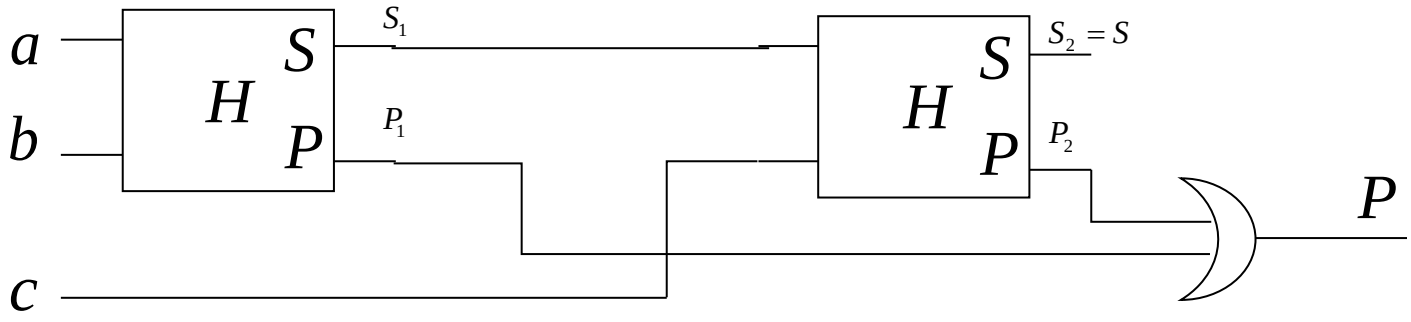
$$S_1 = a\bar{b} + \bar{a}b$$

$$P_1 = ab$$

$$S_2 = S = \bar{S}_1 c + S_1 \bar{c} =$$

$$= (\overline{a\bar{b} + \bar{a}b})c + \bar{a}\bar{b}\bar{c} + ab\bar{c}$$

$$= a\bar{b}c + \bar{a}\bar{b}\bar{c} + \bar{a}bc + abc$$



$$P_2 = S_1 c = (a\bar{b} + \bar{a}b)c$$

$$= a\bar{b}c + \bar{a}bc$$

$$P = P_1 + P_2 = ab + a\bar{b}c + \bar{a}bc$$

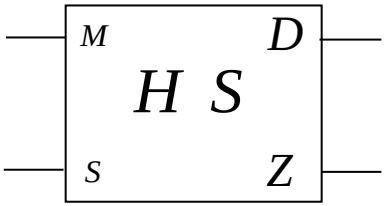
$$= ac + ab + bc$$

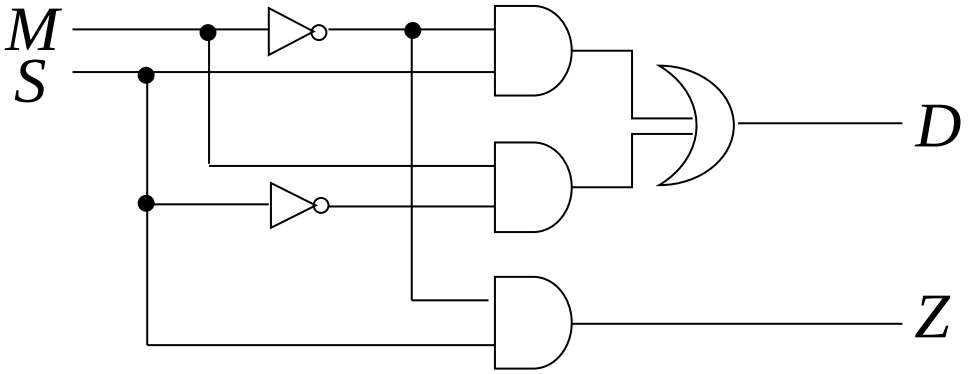
Sklop za oduzimanje - poluodbijalo

M	S	D	Z
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = \overline{M}S + M\overline{S}$$

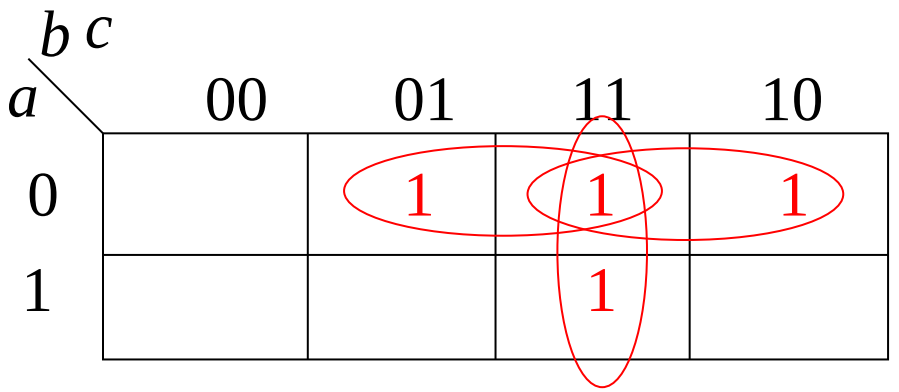
$$Z = \overline{M}S$$





Potpuno odbijalo

M	S	Z_{k-1}	D	Z_{k+1}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



$$D = \overline{M}\,\overline{S}Z_{k-1} + M\,\overline{S}\,\overline{Z}_{k-1} + MSZ_{k-1} + \overline{M}S\,\overline{Z}_{k-1}$$

$$Z_{k+1} = \overline{M}Z_{k-1} + \overline{M}S + SZ_{k-1}$$

Potpuno odbijalo realizovano sa dva poluodbijala