

# CSI - 3105 Design & Analysis of Algorithms

## Course 15

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Fall 2019

## §5.3 Longest Common Subsequence

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$$Y = (b, d, c, a, b, a).$$

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The sequence  $Z = (b, c, d, b)$  is a subsequence of  $X$ , but not of  $Y$ .

$LCS(X, Y)$  is the longest common subsequence of  $X$  and  $Y$ :

$$(b, c, b, a) \quad \text{or} \quad (b, d, a, b).$$

Both have length 4.

# Problem:

**Input** : Sequences

- $X = (x_1, x_2, \dots, x_m)$
- $Y = (y_1, y_2, \dots, y_n)$

**Output** : A longest sequence that is a subsequence of  $X$  and a subsequence of  $Y$ , i.e.,  $Z = LCS(X, Y)$ .

## Step 1: Structure of the Optimal Solution

$$X = (x_1, x_2, \dots, x_m)$$

$$Y = (y_1, y_2, \dots, y_n)$$

Consider

$$Z = (z_1, z_2, \dots, z_k) = LCS(X, Y)$$

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Case 2 b):  $z_k \neq y_n$

$$(z_1, z_2, \dots, z_k) = LCS(x_1x_2 \cdots x_m, y_1y_2 \cdots y_{n-1})$$

But we do not know if we are in Case 2 a) or 2 b).

If  $x_m \neq y_n$ , then  $(z_1, z_2, \dots, z_k)$  is the longest of

$$LCS(x_1x_2 \cdots x_{m-1}, y_1y_2 \cdots y_n)$$

and

$$LCS(x_1x_2 \cdots x_m, y_1y_2 \cdots y_{n-1}).$$

## Step 2: Set Up a recurrence for the Optimal Solution

For  $0 \leq i \leq m$  and  $0 \leq j \leq n$ , define

$$c(i, j) = \text{length of } LCS(x_1x_2 \cdots x_i, y_1y_2 \cdots y_j).$$

We want to compute  $c(m, n)$ .

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$$c(i, j) = \max \{c(i - 1, j), c(i, j - 1)\}$$

## Step 3: Solve the Recurrence Bottom-Up

Fill in the matrix  $c(i, j)$  for  $0 \leq i \leq m$  and  $0 \leq j \leq n$ .

First row:

$$c(0, 0) = c(0, 1) = \dots = c(0, n) = 0$$

First column:

$$c(0, 0) = c(1, 0) = \dots = c(m, 0) = 0$$

Then fill in the matrix, row by row, in each row from left to right.

# Algorithm

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## Algorithm Longest Common Subsequence

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```
1: for  $i = 0$  to  $m$  do
2:    $c(i, 0) = 0$ 
3: end for
4: for  $j = 0$  to  $n$  do
5:    $c(0, j) = 0$ 
6: end for
7: for  $i = 1$  to  $m$  do
8:   for  $j = 1$  to  $n$  do
9:     if  $x_i = y_j$  then
10:       $c(i, j) = 1 + c(i - 1, j - 1)$ 
11:     else
12:       $c(i, j) = \max \{c(i - 1, j), c(i, j - 1)\}$ 
13:     end if
14:   end for
15: end for
16: return  $c(m, n)$ 
```

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0							
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2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2			
4	$B$	0							
5	$D$	0							
6	$A$	0							
7	$B$	0							

$X = ABCBDAB$

$Y = BDCABA$

$$c(i, j) = \begin{cases} 0 & i = 0 \text{ or } j = 0 \\ 1 + c(i-1, j-1) & i \geq 1, j \geq 1 \text{ and } x_i = y_j \\ \max\{c(i, j-1), c(i-1, j)\} & i \geq 1, j \geq 1 \text{ and } x_i \neq y_j \end{cases}$$

		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2		
4	$B$	0							
5	$D$	0							
6	$A$	0							
7	$B$	0							

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0							
5	$D$	0							
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1						
5	$D$	0							
6	$A$	0							
7	$B$	0							

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1					
5	$D$	0							
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2				
5	$D$	0							
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2			
5	$D$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3		
5	$D$	0							
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0							
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1						
6	$A$	0							
7	$B$	0							

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2					
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2				
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2			
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	0
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3		
6	$A$	0							
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0							
7	$B$	0							

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1						
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2				
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2	3			
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
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7	$B$	0							

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2	3	3	4	
7	$B$	0							



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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2	3	3	4	
7	$B$	0	1						

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2	3	3	4	
7	$B$	0	1	2					

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2	3	3	4	
7	$B$	0	1	2	2				

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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2	3	3	4	
7	$B$	0	1	2	2	3			

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$$c(i,j) = \begin{cases} 0 & i = 0 \text{ or } j = 0 \\ 1 + c(i-1, j-1) & i \geq 1, j \geq 1 \text{ and } x_i = y_j \\ \max\{c(i, j-1), c(i-1, j)\} & i \geq 1, j \geq 1 \text{ and } x_i \neq y_j \end{cases}$$

		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
4	$B$	0	1	1	2	2	3	3	
5	$D$	0	1	2	2	2	3	3	
6	$A$	0	1	2	2	3	3	4	
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
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$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$
0	$x_i$	0	0	0	0	0	0	0
1	$A$	0	0	0	0	1	1	1
2	$B$	0	1	1	1	1	2	2
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$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
3	$C$	0	1	1	2	2	2	2	
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		$j$	0	1	2	3	4	5	6
		$i$	$y_j$	$B$	$D$	$C$	$A$	$B$	$A$
0	$x_i$		0	0	0	0	0	0	0
1	$A$		0	0	0	0	1	1	1
2	$B$		0	1	1	1	1	2	2
3	$C$		0	1	1	2	2	2	2
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$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$		0	0	0	0	0	0	
1	$A$		0	0	0	0	1	1	1
2	$B$		0	1	1	1	1	2	2
3	$C$		0	1	1	2	2	2	2
4	$B$		0	1	1	2	2	3	3
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$		0	0	0	0	0	0	
1	$A$		0	0	0	0	1	1	1
2	$B$		0	1	1	1	1	2	2
3	$C$		0	1	1	2	2	2	2
4	$B$		0	1	1	2	2	3	3
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$		0	0	0	0	0	0	
1	$A$		0	0	0	0	1	1	1
2	$B$		0	1	1	1	1	2	2
3	$C$		0	1	1	2	2	2	2
4	$B$		0	1	1	2	2	3	3
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$		0	0	0	0	0	0	
1	$A$		0	0	0	0	1	1	1
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		$j$	0	1	2	3	4	5	6
		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	0
1	$A$	0	0	0	0	1	1	1	
2	$B$	0	1	1	1	1	2	2	
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		$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$	
0	$x_i$	0	0	0	0	0	0	0	
1	$A$	0	0	0	0	1	1	1	
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	$j$	0	1	2	3	4	5	6
$i$		$y_j$	$B$	$D$	$C$	$A$	$B$	$A$
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Running time:  $O(mn)$

Space:  $O(mn)$

But if we only want to compute  $C(m, n)$ , we only need the current row and the previous row. Hence,

Space:  $O(m + n)$ .