IMO 1989

Day 1

$\boxed{1}$ Prove that in the set $\{1, 2, \ldots, 1989\}$ can be expressed as the disjoint union of subsets $A_i, \{i = 1, 2, \ldots, 117\}$ such that

i.) each A_i contains 17 elements

ii.) the sum of all the elements in each A_i is the same.

$\boxed{2}$ ABC is a triangle, the bisector of angle A meets the circumcircle of triangle ABC in A_1, points B_1 and C_1 are defined similarly. Let AA_1 meet the lines that bisect the two external angles at B and C in A_0. Define B_0 and C_0 similarly. Prove that the area of triangle $A_0B_0C_0 = 2\cdot$ area of hexagon $AC_1BA_1CB_1 \geq 4\cdot$ area of triangle ABC.

$\boxed{3}$ Let n and k be positive integers and let S be a set of n points in the plane such that

i.) no three points of S are collinear, and

ii.) for every point P of S there are at least k points of S equidistant from P.

Prove that:

$$k < \frac{1}{2} + \sqrt{2 \cdot n}$$

Day 2

4 Let $ABCD$ be a convex quadrilateral such that the sides AB, AD, BC satisfy $AB = AD+BC$. There exists a point P inside the quadrilateral at a distance h from the line CD such that $AP = h + AD$ and $BP = h + BC$. Show that:

$$\frac{1}{\sqrt{h}} \geq \frac{1}{\sqrt{AD}} + \frac{1}{\sqrt{BC}}$$

5 Prove that for each positive integer n there exist n consecutive positive integers none of which is an integral power of a prime number.

6 A permutation $\{x_1, \ldots, x_{2n}\}$ of the set $\{1, 2, \ldots, 2n\}$ where n is a positive integer, is said to have property T if $|x_i - x_{i+1}| = n$ for at least one i in $\{1, 2, \ldots, 2n-1\}$. Show that, for each n, there are more permutations with property T than without.