IMO 1962

Ceske Budejovice, Czechoslovakia

Day 1

$\boxed{1}$ Find the smallest natural number n which has the following properties:

a) Its decimal representation has a 6 as the last digit.

b) If the last digit 6 is erased and placed in front of the remaining digits, the resulting number is four times as large as the original number n.

$\boxed{2}$ Determine all real numbers x which satisfy the inequality:

$$\sqrt{3-x}-\sqrt{x+1}>\frac{1}{2}$$

$\boxed{3}$ Consider the cube $ABCDA'B'C'D'$ ($ABCD$ and $A'B'C'D'$ are the upper and lower bases, repsectively, and edges AA', BB', CC', DD' are parallel). The point X moves at a constant speed along the perimeter of the square $ABCD$ in the direction $ABCDA$, and the point Y moves at the same rate along the perimiter of the square $B'C'CB$ in the direction $B'C'CBB'$. Points X and Y begin their motion at the same instant from the starting positions A and B', respectively. Determine and draw the locus of the midpionts of the segments XY.

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Day 2

$\boxed{4}$ Solve the equation $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$

$\boxed{5}$ On the circle K there are given three distinct points A, B, C. Construct (using only a straightedge and a compass) a fourth point D on K such that a circle can be inscribed in the quadrilateral thus obtained.

$\boxed{6}$ Consider an isosceles triangle. let R be the radius of its circumscribed circle and r be the radius of its inscribed circle. Prove that the distance d between the centers of these two circle is

$$d = \sqrt{R(R-2r)}$$

$\boxed{7}$ The tetrahedron $SABC$ has the following property: there exist five spheres, each tangent to the edges SA, SB, SC, BC, CA, AB, or to their extensions.

a) Prove that the tetrahedron $SABC$ is regular.

b) Prove conversely that for every regular tetrahedron five such spheres exist.