

- [1]  $AB$  is a chord (not a diameter) of a circle with centre  $O$ . Let  $T$  be a point on segment  $OB$ . The line through  $T$  perpendicular to  $OB$  meets  $AB$  at  $C$  and the circle at  $D$  and  $E$ . Denote by  $S$  the orthogonal projection of  $T$  onto  $AB$ . Prove that  $AS \cdot BC = TE \cdot TD$ .
- [2] Find all positive integers  $m$  and  $n$  such that  $n^m - m$  divides  $m^2 + 2m$ .
- [3] Find all real solutions  $x$  to the equation  $\lfloor x^2 - 2x \rfloor + 2\lfloor x \rfloor = \lfloor x \rfloor^2$ .
- [4] The numbers  $\frac{1}{1}, \frac{1}{2}, \dots, \frac{1}{2012}$  are written on the blackboard. Acha chooses any two numbers from the blackboard, say  $x$  and  $y$ , erases them and she writes instead the number  $x + y + xy$ . She continues to do this until only one number is left on the board. What are the possible values of the final number?
- [5] Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x^2 - y^2) = (x + y)(f(x) - f(y))$  for all real numbers  $x$  and  $y$ .
- [6] (i) Find the angles of  $\triangle ABC$  if the length of the altitude through  $B$  is equal to the length of the median through  $C$  and the length of the altitude through  $C$  is equal to the length of the median through  $B$ .
- (ii) Find all possible values of  $\angle ABC$  of  $\triangle ABC$  if the length of the altitude through  $A$  is equal to the length of the median through  $C$  and the length of the altitude through  $C$  is equal to the length of the median through  $B$ .