5-th Czech–Polish–Slovak Match 2005

Zwardoń, Poland
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1. Let n be a given positive integer. Solve the system

$$\begin{aligned} x_1 + x_2^2 + x_3^3 + \cdots + x_n^n &= n, \\ x_1 + 2x_2 + 3x_3 + \cdots + nx_n &= \tfrac{n(n+1)}{2} \end{aligned}$$

in the set of nonnegative real numbers.

2. A convex quadrilateral $ABCD$ is inscribed in a circle with center O and circumscribed to a circle with center I. Its diagonals meet at P. Prove that points O, I and P lie on a line.

3. Find all integers $n \geq 3$ for which the polynomial

$$W(x) = x^n - 3x^{n-1} + 2x^{n-2} + 6$$

can be written as a product of two non-constant polynomials with integer coefficients.

4. We distribute $n \geq 1$ labelled balls among nine persons $A, B, C, \ldots, I$. How many ways are there to do this so that A gets the same number of balls as B, C, D and E together?

5. Given a convex quadrilateral $ABCD$, find the locus of the points P inside the quadrilateral such that

$$S_{PAB} \cdot S_{PCD} = S_{PBC} \cdot S_{PDA}$$

(where S_X denotes the area of triangle X).

6. Determine all pairs of integers (x, y) satisfying the equation

$$y(x + y) = x^3 - 7x^2 + 11x - 3.$$

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