

Parallelogram to find angle



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Source: All Russian MO 2015, grade 10, problem 2

silouan
3804 posts

Aug 7, 2015, 5:38 pm [PM #1](#)

Given is a parallelogram $ABCD$, with $AB < AC < BC$. Points E and F are selected on the circumcircle ω of ABC so that the tangenst to ω at these points pass through point D and the segments AD and CE intersect.
It turned out that $\angle ABF = \angle DCE$. Find the angle $\angle ABC$.
A. Yakubov, S. Berlov

vanstraelen
2804 posts

Aug 7, 2015, 10:20 pm [PM #2](#)

“ silouan wrote:

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Can this text be read as

Points E and F are selected on the circumcircle ω of ABC so that the tangents to ω at these points pass through point D and through the intersection of the segments AD and CE ?

silouan
3804 posts

Aug 8, 2015, 4:26 am [PM #3](#)

No, the text is as it was given. The meaning of the condition that AD and CE intersect is to avoid different configurations.

Luis González
3883 posts

Aug 9, 2015, 7:36 am [PM #4](#)

Denote by O the center of ω . Parallel from B to AC cuts DC, DA at Y, Z . Since ω becomes 9-point circle of $\triangle DYZ$, then it cuts DC again at the projection P of Z on DC . Since $\angle ABF = \angle DCE \implies AF = EP \implies AEF P$ is isosceles trapexoid with bases $EF \parallel AP \implies DO \perp (EF \parallel AP) \implies DO$ is perpendicular bisector of $\overline{AP} \implies DP = DA = AZ = AP \implies \triangle DAP$ is equilateral $\implies \angle ABC = \angle ADP = 60^\circ$.

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