India
Regional Mathematical Olympiad
2009

---

1 Let ABC be a triangle in which $AB = AC$ and let I be its in-centre. Suppose $BC = AB + AI$. Find $\angle BAC$

2 Show that there is no integer a such that $a^2 - 3a - 19$ is divisible by 289.

3 Show that $3^{2008} + 4^{2009}$ can be written as product of two positive integers each of which is larger than 2009^{182}.

4 Find the sum of all 3-digit natural numbers which contain at least one odd digit and at least one even digit.

5 A convex polygon is such that the distance between any two vertices does not exceed 1. (i) Prove that the distance between any two points on the boundary of the polygon does not exceed 1. (ii) If X and Y are two distinct points inside the polygon, prove that there exists a point Z on the boundary of the polygon such that $XZ + YZ \leq 1$.

6 In a book with page numbers from 1 to 100 some pages are torn off. The sum of the numbers on the remaining pages is 4949. How many pages are torn off?