

India
Regional Mathematical Olympiad
1997

- [1] Let P be an interior point of a triangle ABC and let BP and CP meet AC and AB in E and F respectively. If $S_{BPF} = 4, S_{BPC} = 8$ and $S_{CPE} = 13$, find S_{AFPE} .
- [2] For each positive integer n , define $a_n = 20 + n^2$ and $d_n = \gcd(a_n, a_{n+1})$. Find the set of all values that are taken by d_n and show by examples that each of these values is attained.
- [3] Solve for real x :

$$\frac{1}{[x]} + \frac{1}{[2x]} = x - [x] + \frac{1}{3}.$$

- [4] In a quadrilateral $ABCD$, it is given that AB is parallel to CD and the diagonals AC and BD are perpendicular to each other. Show that (a) $AD \cdot BC \geq AB \cdot CD$ (b) $AD + BC \geq AB + CD$.
- [5] Let x, y, z be three distinct real positive numbers, Determine whether or not the three real numbers

$$\left| \frac{x}{y} - \frac{y}{x} \right|, \left| \frac{y}{z} - \frac{z}{y} \right|, \left| \frac{z}{x} - \frac{x}{z} \right|$$

can be the lengths of the sides of a triangle.

- [6] Find the number of unordered pairs $\{A, B\}$ of subsets of an n -element set X that satisfies the following: (a) $A \neq B$ (b) $A \cup B = X$