Source: All Russian 2014 Grade 9 Day 2 P2

mathuz
1229 posts

May 3, 2014, 8:53 pm PM #1

Let $ABCD$ be a trapezoid with $AB \parallel CD$ and Ω is a circle passing through A, B, C, D. Let ω be the circle passing through C, D and intersecting with CA, CB at A_1, B_1 respectively. A_2 and B_2 are the points symmetric to A_1 and B_1 respectively, with respect to the midpoints of CA and CB. Prove that the points A, B, A_2, B_2 are concyclic.

I. Bogdanov

Luis Gonz...

May 9, 2014, 4:32 am • 1 PM #2

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respectively. A_2 and B_2 are the points symmetric to A_1 and B_1 respectively, with respect to the midpoints of CA and CB. Prove that the points A, B, A_2, B_2 are concyclic.

Typo corrected in red color. This is proved in the solution of the problem All Russian-2014, Grade 11, day 2, P2.

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mathuz
1229 posts

May 17, 2014, 2:08 am PM #3

you are right!
Thank you Luis.

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nima1376
111 posts

May 18, 2014, 12:29 pm PM #4

D is a center of spiral similar which goes BB_1 to $AA_1 \Rightarrow \dfrac{AA_1}{BB_1} = \dfrac{AD}{BD} = \dfrac{BC}{CA}$

$AA_1.AC = BB_1.BC \Rightarrow CA_2.AC = CB_2.BC$
so A_2B_2BA is cycle.
done

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saturzo
54 posts

May 19, 2014, 5:02 pm • 1 PM #5

$ABCD$ is cyclic in Ω. So, $\measuredangle BAC = \measuredangle DCA \Rightarrow BC = AD$
Similarly $BD = AC$.
Now let $\{D, D'\} = AD \cap \omega$. And by symmetry, $AD' = BB_1$
Now A_1CDD' is cyclic(in ω) and $A_1C \cap DD' = A$. So(using power of point),
$AA_1.AC = AD'.AD$.
$\therefore CA_2/CB_2 = AA_1/BB_1 = AA_1/AD' = AD/AC = CB/CA \Rightarrow CA_2.CA = CB_2.CB$
$\therefore A_2, B_2, A, B$ are concyclic.

$[QED]$

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thecmd999
2874 posts

Sep 23, 2014, 12:50 am PM #6

Solution

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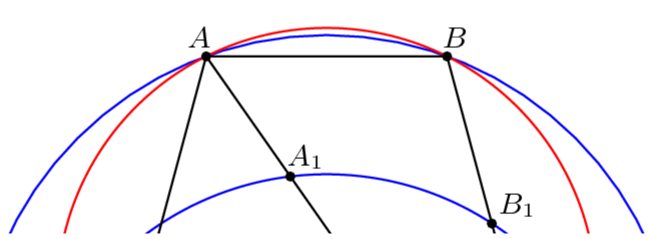
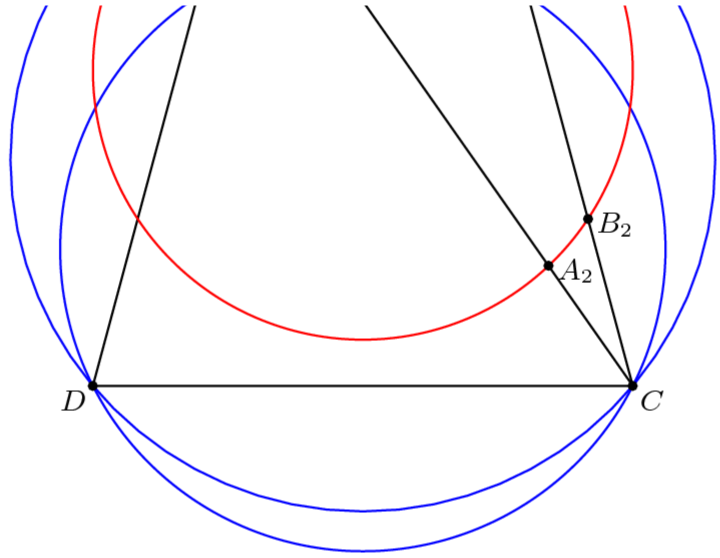
v_Enhance
4253 posts

Dec 1, 2014, 7:36 am PM #7

What a nice illustration of spiral similarity. Though I would have just said "isosceles trapezoid" in the problem statement.





We have $\triangle DAA_1 \sim \triangle DBB_1$, but $DA = CB$ and $DB = AC$. So $AA_1 \cdot AC = BB_1 \cdot BC$, implying that $CA_2 \cdot CA = CB_2 \cdot CB$.

$BB_1 \cdot BC = BA' \cdot BD$
$\implies BB_1 \cdot CB = AA_1 \cdot CA$
$\implies CB_2 \cdot CB = CA_2 \cdot CA$
QED

aditya21
697 posts

Mar 22, 2015, 1:55 pm PM #9

easy!! but still posting!

let ω intersect AD in K
than quite easily $\angle AKB_1 = \angle ACD = 180 - \angle ABB_1$
and hence ABB_1K is isosceles trapezium.
now by POP
we have $AD.AK = AA_1.AC = CA_2.AC$
on other note $AD.AK = BC.AK = BC.BB_1 = BC.BB_2$
and hence $BC.BB_2 = CA_2.CA$
and hence by POP we have ABB_2A_2 is cyclic quad.
thus we are done

anantmu...
839 posts

Oct 23, 2015, 6:46 pm PM #10

Another solution:

Let the circle AA_2B intersect AB again at B'.

Now, AB is the radical axis of $(ABCD); (AA_2B)$ and CD is the radical axis of $(ABCD); (DCA_1B_1)$.

Now, $AB \parallel CD$ and so $AB \parallel CD \parallel l$ where l is the radical axis of $(AA_2B); (DCA_1B_1)$

Let M and N be the mid points of CA, CB respectively. It is evident that $MN \parallel l$ and also,

$MA_1.MC = MA_2.MA$

so M lies on l. Therefore, N lies on l too and so by power of a point $B_2 \equiv B'$ thus, the result holds.

bobaboby1
8 posts

Jul 10, 2016, 9:35 pm PM #11

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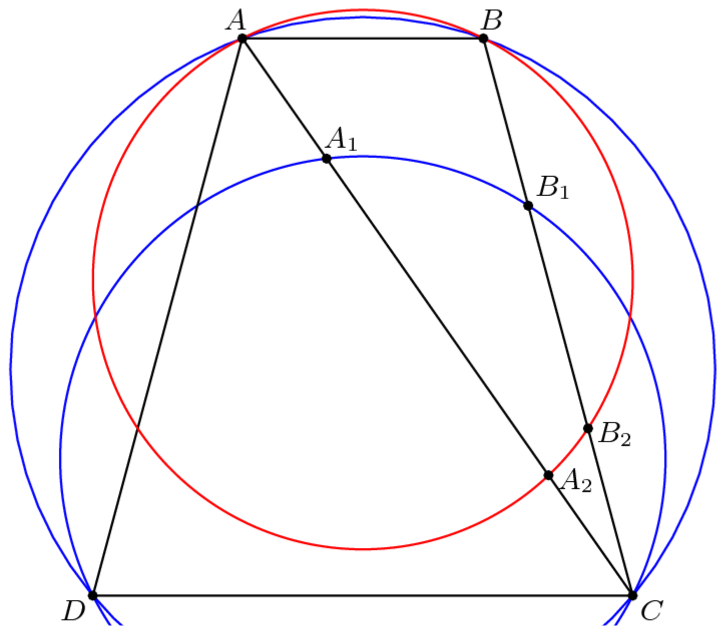

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Can we just use symmetry to prove BB1×CB=AA1×CA

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