India
Regional Mathematical Olympiad
1994

$\boxed{1}$ A leaf is torn from a paperback novel. The sum of the numbers on the remaining pages is 15000. What are the page numbers on the torn leaf?

$\boxed{2}$ In a triangle ABC, the incircle touches the sides BC, CA, AB at D, E, F respectively. If the radius if the incircle is 4 units and if BD, CE, AF are consecutive integers, find the sides of the triangle ABC.

$\boxed{3}$ Find all 6-digit numbers $a_1a_2a_3a_4a_5a_6$ formed by using the digits $1, 2, 3, 4, 5, 6$ once each such that the number $a_1a_2a_2 \dots a_k$ is divisible by k for $1 \leq k \leq 6$.

$\boxed{4}$ Solve the system of equations for real x and y:

$$5x\left(1+\frac{1}{x^2+y^2}\right) = 125y\left(1-\frac{1}{x^2+y^2}\right) = 4.$$

(0)

Let A be a set of 16 positive integers with the property that the product of any two distinct members of A will not exceed 1994. Show that there are numbers a and b in the set A such that the gcd of a and b is greater than 1.

Let AC and BD be two chords of a circle with center O such that they intersect at right angles inside the circle at the point M. Suppose K and L are midpoints of the chords AB and CD respectively. Prove that $OKML$ is a parallelogram.

Find all rational numbers $\frac{m}{n}$ such that
(i) $0 < \frac{m}{n} < 1$;
(ii) m and n are relatively prime;
(iii) $mn = 25!$.

If a, b, c are positive real numbers such that $a + b + c = 1$, prove that

$$(1+a)(1+b)(1+c) \geq 8(1-a)(1-b)(1-c).$$