Junior Balkan MO 1997

Belgrad, Yugoslavia

$\boxed{1}$ Show that given any 9 points inside a square of side 1 we can always find 3 which form a triangle with area less than $\frac{1}{8}$.

Bulgaria

$\boxed{2}$ Let $\frac{x^2+y^2}{x^2-y^2} + \frac{x^2-y^2}{x^2+y^2} = k$. Compute the following expression in terms of k:

$$E(x,y) = \frac{x^8+y^8}{x^8-y^8} - \frac{x^8-y^8}{x^8+y^8}.$$

Ciprus

$\boxed{3}$ Let ABC be a triangle and let I be the incenter. Let N, M be the midpoints of the sides AB and CA respectively. The lines BI and CI meet MN at K and L respectively. Prove that $AI + BI + CI > BC + KL$.

Greece

$\boxed{4}$ Determine the triangle with sides a, b, c and circumradius R for which $R(b+c) = a\sqrt{bc}$.

Romania

$\boxed{5}$ Let n_1, n_2, …, n_{1998} be positive integers such that

$$n_1^2 + n_2^2 + \cdots + n_{1997}^2 = n_{1998}^2.$$

Show that at least two of the numbers are even.