# India
## National Olympiad
### 2008

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$\boxed{1}$ Let $ABC$ be triangle, $I$ its in-center; $A_1, B_1, C_1$ be the reflections of $I$ in $BC, CA, AB$ respectively. Suppose the circum-circle of triangle $A_1B_1C_1$ passes through $A$. Prove that $B_1, C_1, I, I_1$ are concylic, where $I_1$ is the in-center of triangle $A_1, B_1, C_1$.

$\boxed{2}$ Find all triples $(p, x, y)$ such that $p^x = y^4 + 4$, where $p$ is a prime and $x$ and $y$ are natural numbers.

$\boxed{3}$ Let $A$ be a set of real numbers such that $A$ has at least four elements. Suppose $A$ has the property that $a^2 + bc$ is a rational number for all distinct numbers $a, b, c$ in $A$. Prove that there exists a positive integer $M$ such that $a\sqrt{M}$ is a rational number for every $a$ in $A$.

$\boxed{4}$ All the points with integer coordinates in the $xy$-Plane are coloured using three colours, red, blue and green, each colour being used at least once. It is known that the point $(0, 0)$ is red and the point $(0, 1)$ is blue. Prove that there exist three points with integer coordinates of distinct colours which form the vertices of a right-angled triangle.

$\boxed{5}$ Let $ABC$ be a triangle; $\Gamma_A, \Gamma_B, \Gamma_C$ be three equal, disjoint circles inside $ABC$ such that $\Gamma_A$ touches $AB$ and $AC$; $\Gamma_B$ touches $AB$ and $BC$; and $\Gamma_C$ touches $BC$ and $CA$. Let $\Gamma$ be a circle touching circles $\Gamma_A, \Gamma_B, \Gamma_C$ externally. Prove that the line joining the circum-centre $O$ and the in-centre $I$ of triangle $ABC$ passes through the centre of $\Gamma$.

$\boxed{6}$ Let $P(x)$ be a polynomial with integer coefficients. Prove that there exist two polynomials $Q(x)$ and $R(x)$, again with integer coefficients, such that **(i)** $P(x) \cdot Q(x)$ is a polynomial in $x^2$ , and **(ii)** $P(x) \cdot R(x)$ is a polynomial in $x^3$.

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