India
Regional Mathematical Olympiad
1999

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1. Prove that the inradius of a right angled triangle with integer sides is an integer.

2. Find the number of positive integers which divide 10^{999} but not 10^{998}.

3. Let $ABCD$ be a square and M, N points on sides AB, BC respectively such that $\angle MDN = 45°$. If R is the midpoint of MN show that $RP = RQ$ where P, Q are points of intersection of AC with the lines MD, ND.

4. If p, q, r are the roots of the cubic equation $x^3 - 3px^2 + 3q^2x - r^3 = 0$, then show that $p = q = r$.

5. If a, b, c are sides of a triangle, prove that

$$\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a} \geq 3.$$

6. Find all solutions in integers m, n of the equation

$$(m-n)^2 = \frac{4mn}{m+n-1}.$$

7. Find the number of quadratic polynomials $ax^2 + bx + c$ which satisfy the following:

(a) a, b, c are distinct;

(b) $a, b, c \in \{1, 2, 3, \cdots 1999\}$;

(c) $x + 1$ divides $ax^2 + bx + c$.