Junior Balkan MO 2010

$\boxed{1}$ The real numbers a, b, c, d satisfy simultaneously the equations

$$abc - d = 1, \quad bcd - a = 2, \quad cda - b = 3, \quad dab - c = -6.$$

Prove that $a + b + c + d \neq 0$.

$\boxed{2}$ Find all integers n, $n \geq 1$, such that $n \cdot 2^{n+1} + 1$ is a perfect square.

$\boxed{3}$ Let AL and BK be angle bisectors in the non-isosceles triangle ABC (L lies on the side BC, K lies on the side AC). The perpendicular bisector of BK intersects the line AL at point M. Point N lies on the line BK such that LN is parallel to MK. Prove that $LN = NA$.

$\boxed{4}$ A 9×7 rectangle is tiled with tiles of the two types: L-shaped tiles composed by three unit squares (can be rotated repeatedly with $90°$) and square tiles composed by four unit squares. Let $n \geq 0$ be the number of the 2×2 tiles which can be used in such a tiling. Find all the values of n.