India
National Olympiad
1996

$\boxed{1}$ a) Given any positive integer n, show that there exist distint positive integers x and y such that $x + j$ divides $y + j$ for $j = 1, 2, 3, \ldots, n$;

b) If for some positive integers x and y, $x + j$ divides $y + j$ for all positive integers j, prove that $x = y$.

$\boxed{2}$ Let C_1 and C_2 be two concentric circles in the plane with radii R and $3R$ respectively. Show that the orthocenter of any triangle inscribed in circle C_1 lies in the interior of circle C_2. Conversely, show that every point in the interior of C_2 is the orthocenter of some triangle inscribed in C_1.

$\boxed{3}$ Solve the following system for real a, b, c, d, e:

$$\begin{cases} 3a &=& (b + c + d)^3 \\ 3b &=& (c + d + e)^3 \\ 3c &=& (d + e + a)^3 \\ 3d &=& (e + a + b)^3 \\ 3e &=& (a + b + c)^3. \end{cases}$$

$\boxed{4}$ Let X be a set containing n elements. Find the number of ordered triples (A, B, C) of subsets of X such that A is a subset of B and B is a proper subset of C.

$\boxed{5}$ Define a sequence $(a_n)_{n \geq 1}$ by $a_1 = 1$ and $a_2 = 2$ and $a_{n+2} = 2a_{n+1} - a_n + 2$ for $n \geq 1$. prove that for any m , $a_m a_{m+1}$ is also a term in this sequence.

$\boxed{6}$ There is a $2n \times 2n$ array (matrix) consisting of $0's$ and $1's$ and there are exactly $3n$ zeroes. Show that it is possible to remove all the zeroes by deleting some n rows and some n columns.