Balkan MO 2009

1 Solve the equation

$$3^x - 5^y = z^2.$$

in positive integers.

Greece

2 Let MN be a line parallel to the side BC of a triangle ABC, with M on the side AB and N on the side AC. The lines BN and CM meet at point P. The circumcircles of triangles BMP and CNP meet at two distinct points P and Q. Prove that $\angle BAQ = \angle CAP$.

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3 A 9×12 rectangle is partitioned into unit squares. The centers of all the unit squares, except for the four corner squares and eight squares sharing a common side with one of them, are coloured red. Is it possible to label these red centres $C_1, C_2, \ldots, C_{96}$ in such way that the following to conditions are both fulfilled
i) the distances $C_1C_2, \ldots, C_{95}C_{96}, C_{96}C_1$ are all equal to $\sqrt{13}$,
ii) the closed broken line $C_1C_2 \ldots C_{96}C_1$ has a centre of symmetry?

Bulgaria

4 Denote by S the set of all positive integers. Find all functions $f : S \to S$ such that

$$f(f^2(m) + 2f^2(n)) = m^2 + 2n^2$$

for all $m, n \in S$.

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