India
Regional Mathematical Olympiad
1992

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1 Determine the set of integers n for which $n^2 + 19n + 92$ is a square.

2 If $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, where a, b, c are positive integers with no common factor, prove that $(a + b)$ is a square.

3 Determine the largest 3 digit prime number that is a factor of $\binom{2000}{1000}$.

4 $ABCD$ is a cyclic quadrilateral with $AC \perp BD$; AC meets BD at E. Prove that

$$EA^2 + EB^2 + EC^2 + ED^2 = 4R^2$$

where R is the radius of the circumscribing circle.

5 $ABCD$ is a quadrilateral and P, Q are the midpoints of CD, AB, AP, DQ meet at X and BP, CQ meet at Y. Prove that $A[ADX] + A[BCY] = A[PXOY]$.

6 Prove that

$$1 < \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \cdots + \frac{1}{3001} < 1\frac{1}{3}.$$

7 Solve the system

$$\begin{aligned} (x+y)(x+y+z) &= 18(y+z)(x+y+\cancel{z}) \\ 30(x+z)(x+y+z) &= 2A \end{aligned}$$

(0)

in terms of the parameter A.

The cyclic octagon $ABCDEFGH$ has sides a, a, a, a, b, b, b, b respectively. Find the radius of the circle that circumscribes $ABCDEFGH$.