

India
Regional Mathematical Olympiad
1991

- [1] Let P be an interior point of a triangle ABC and AP, BP, CP meet the sides BC, CA, AB in D, E, F respectively. Show that

$$\frac{AP}{PD} = \frac{AF}{FB} + \frac{AE}{EC}.$$

[hide="Remark"]This is known as *Van Aubel's Theorem*.

- [2] If a, b, c, d be any four positive real numbers, then prove that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} \geq 4.$$

- [3] A four-digit number has the following properties:

- (a) It is a perfect square;
- (b) Its first two digits are equal
- (c) Its last two digits are equal.

Find all such four-digit numbers.

- [4] There are two urns each containing an arbitrary number of balls. We are allowed two types of operations: (a) remove an equal number of balls simultaneously from both the urns, (b) double the number of balls in any of them Show that after performing these operations finitely many times, both the urns can be made empty.

- [5] Take any point P_1 on the side BC of a triangle ABC and draw the following chain of lines: P_1P_2 parallel to AC ; P_2P_3 parallel to BC ; P_3P_4 parallel to AB ; P_4P_5 parallel to CA ; and P_5P_6 parallel to BC . Here, P_2, P_5 lie on AB ; P_3, P_6 lie on CA and P_4 on BC . Show that P_6P_1 is parallel to AB .

- [6] Find all integer values of a such that the quadratic expression $(x + a)(x + 1991) + 1$ can be factored as a product $(x + b)(x + c)$ where b, c are integers.

- [7] Prove that $n^4 + 4^n$ is composite for all values of n greater than 1.

- [8] The 64 squares of an 8×8 chessboard are filled with positive integers in such a way that each integer is the average of the integers on the neighbouring squares. Show that in fact all the 64 entries are equal.