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Day 1

$\boxed{1}$ Prove that from a set of ten distinct two-digit numbers, it is always possible to find two disjoint subsets whose members have the same sum.

$\boxed{2}$ Given $n > 4$, prove that every cyclic quadrilateral can be dissected into n cyclic quadrilaterals.

$\boxed{3}$ Prove that $(2m)!(2n)!$ is a multiple of $m!n!(m+n)!$ for any non-negative integers m and n.

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Day 2

$\boxed{1}$ Find all positive real solutions to:

$$\begin{aligned}
(x_1^2 - x_3x_5)(x_2^2 - x_3x_5) &\leq 0(x_2^2 - x_4x_1)(x_3^2 - x_4x_1) \leq \\
0(x_3^2 - x_5x_2)(x_4^2 - x_5x_2) &\leq 0(x_4^2 - x_1x_3)(x_5^2 - x_1x_3) \leq \\
0(x_5^2 - x_2x_4)(x_1^2 - x_2x_4) &\leq 0
\end{aligned}$$

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f and g are real-valued functions defined on the real line. For all x and y, $f(x + y) + f(x - y) = 2f(x)g(y)$. f is not identically zero and $|f(x)| \leq 1$ for all x. Prove that $|g(x)| \leq 1$ for all x.

Given four distinct parallel planes, prove that there exists a regular tetrahedron with a vertex on each plane.