Germany Team Selection Test 2014

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| – | VAIMO 1 |
| 1 | In Sikinia we only pay with coins that have a value of either 11 or 12 Kulotnik. In a burglary in one of Sikinia's banks, 11 bandits cracked the safe and could get away with 5940 Kulotnik. They tried to split up the money equally - so that everyone gets the same amount - but it just doesn't worked. After a while their leader claimed that it actually isn't possible.<br>Prove that they didn't get any coin with the value 12 Kulotnik. |
| 2 | Let $ABCD$ be a convex cyclic quadrilateral with $AD = BD$. The diagonals $AC$ and $BD$ intersect in $E$. Let the incenter of triangle $\triangle BCE$ be $I$. The circumcircle of triangle $\triangle BIE$ intersects side $AE$ in $N$.<br>Prove $$AN \cdot NC = CD \cdot BN.$$ |
| 3 | Let $a_1 \leq a_2 \leq \cdots$ be a non-decreasing sequence of positive integers. A positive integer $n$ is called *good* if there is an index $i$ such that $n = \dfrac{i}{a_i}$.<br>Prove that if 2013 is *good*, then so is 20. |
| – | VAIMO 2 |
| 1 | Let $n$ be an positive integer. Find the smallest integer $k$ with the following property; Given any real numbers $a_1, \cdots, a_d$ such that $a_1 + a_2 + \cdots + a_d = n$ and $0 \leq a_i \leq 1$ for $i = 1, 2, \cdots, d$, it is possible to partition these numbers into $k$ groups (some of which may be empty) such that the sum of the numbers in each group is at most 1. |
| 2 | Let $\mathbb{Z}_{>0}$ be the set of positive integers. Find all functions $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ such that $$m^2 + f(n) \mid mf(m) + n$$ for all positive integers $m$ and $n$. |
| 3 | In a triangle $ABC$, let $D$ and $E$ be the feet of the angle bisectors of angles $A$ and $B$, respectively. A rhombus is inscribed into the quadrilateral $AEDB$ |

(all vertices of the rhombus lie on different sides of $AEDB$). Let $\varphi$ be the non-obtuse angle of the rhombus. Prove that $\varphi \leq \max\{\angle BAC, \angle ABC\}$.