China National Olympiad 2009

**Day 1**

**1** Given an acute triangle $PBC$ with $PB \neq PC$. Points $A, D$ lie on $PB, PC$, respectively. $AC$ intersects $BD$ at point $O$. Let $E, F$ be the feet of perpendiculars from $O$ to $AB, CD$, respectively. Denote by $M, N$ the midpoints of $BC, AD$. (1): If four points $A, B, C, D$ lie on one circle, then $EM \cdot FN = EN \cdot FM$. (2): Determine whether the converse of (1) is true or not, justify your answer.

**2** Find all the pairs of prime numbers $(p, q)$ such that $pq|5^p + 5^q$.

**3** Given two integers $m, n$ satisfying $4 < m < n$. Let $A_1A_2 \cdots A_{2n+1}$ be a regular $2n+1$ polygon. Denote by $P$ the set of its vertices. Find the number of convex $m$ polygon whose vertices belongs to $P$ and exactly has two acute angles.

**Day 2**

**1** Given an integer $n > 3$. Let $a_1, a_2, \cdots, a_n$ be real numbers satisfying $min|a_i - a_j| = 1, 1 \leq i \leq j \leq n$. Find the minimum value of $\sum_{k=1}^{n} |a_k|^3$.

**2** Let $P$ be a convex $n$ polygon each of which sides and diagnoals is colored with one of $n$ distinct colors. For which $n$ does: there exists a coloring method such that for any three of $n$ colors, we can always find one triangle whose vertices is of $P$' and whose sides is colored by the three colors respectively.

**3** Given an integer $n > 3$. Prove that there exists a set $S$ consisting of $n$ pairwisely distinct positive integers such that for any two different non-empty subset of $S$:$A, B$, $\frac{\sum_{x\in A} x}{|A|}$ and $\frac{\sum_{x\in B} x}{|B|}$ are two composites which share no common divisors.