2010 Sharygin Geometry Olympiad

Sharygin Geometry Olympiad 2010

1	Does there exist a triangle, whose side is equal to some of its altitudes, another side is equal to some of its bisectors, and the third is equal to some of its medians?	Amir Hossein view topic
2	Bisectors AA_1 and BB_1 of a right triangle ABC $(\angle C = 90^\circ)$ meet at a point I. Let O be the circumcenter of triangle CA_1B_1. Prove that $OI \perp AB$.	Amir Hossein view topic
3	Points A', B', C' lie on sides BC, CA, AB of triangle ABC. for a point X one has $\angle AXB = \angle A'C'B' + \angle ACB$ and $\angle BXC = \angle B'A'C' + \angle BAC$. Prove that the quadrilateral $XA'BC'$ is cyclic.	Amir Hossein view topic
4	The diagonals of a cyclic quadrilateral $ABCD$ meet in a point N. The circumcircles of triangles ANB and CND intersect the sidelines BC and AD for the second time in points A_1, B_1, C_1, D_1. Prove that the quadrilateral $A_1B_1C_1D_1$ is inscribed in a circle centered at N.	Amir Hossein view topic
5	A point E lies on the altitude BD of triangle ABC, and $\angle AEC = 90^\circ$. Points O_1 and O_2 are the circumcenters of triangles AEB and CEB; points F, L are the midpoints of the segments AC and O_1O_2. Prove that the points L, E, F are collinear.	Amir Hossein view topic
6	Points M and N lie on the side BC of the regular triangle ABC (M is between B and N), and $\angle MAN = 30^\circ$. The circumcircles of triangles AMC and ANB meet at a point K. Prove that the line AK passes through the circumcenter of triangle AMN.	Amir Hossein view topic
7	The line passing through the vertex B of a triangle ABC and perpendicular to its median BM intersects the altitudes dropped from A and C (or their extensions) in points K and N. Points O_1 and O_2 are the circumcenters of the triangles ABK and CBN respectively. Prove that $O_1M = O_2M$.	Amir Hossein view topic
8	Let AH be the altitude of a given triangle ABC. The points I_b and I_c are the incenters of the triangles ABH and ACH respectively. BC touches the incircle of the triangle ABC at a point L. Find $\angle LI_bI_c$.	Amir Hossein view topic
9	A point inside a triangle is called "*good*" if three cevians passing through it are equal. Assume for an isosceles triangle ABC $(AB = BC)$ the total number of "*good*" points is odd. Find all possible values of this number.	Amir Hossein view topic
10	Let three lines forming a triangle ABC be given. Using a two-sided ruler and drawing at most eight lines construct a point D on the side AB such that $\frac{AD}{BD} = \frac{BC}{AC}$.	Amir Hossein view topic
11	A convex $n-$gon is split into three convex polygons. One of them has n sides, the second one has more than n sides, the third one has less than n sides. Find all possible values of n.	Amir Hossein view topic
12	Let AC be the greatest leg of a right triangle ABC, and CH be the altitude to its hypotenuse. The circle of radius CH centered at H intersects AC in point M. Let a point B' be the reflection of B with respect to the point H. The perpendicular to AB erected at B' meets the circle in a point K. Prove that a) $B'M \parallel BC$ b) AK is tangent to the circle.	Amir Hossein view topic
13	Let us have a convex quadrilateral $ABCD$ such that $AB = BC$. A point K lies on the diagonal BD, and $\angle AKB + \angle BKC = \angle A + \angle C$. Prove that $AK \cdot CD = KC \cdot AD$.	Amir Hossein view topic
14	We have a convex quadrilateral $ABCD$ and a point M on its side AD such that CM and BM are parallel to AB and CD respectively. Prove that $S_{ABCD} \geq 3S_{BCM}$. *Remark.* S denotes the area function.	Amir Hossein view topic
15	Let AA_1, BB_1 and CC_1 be the altitudes of an acute-angled triangle ABC. AA_1 meets $B_1C_$ in a point K. The circumcircles of triangles A_1KC_1 and A_1KB_1 intersect the lines AB and AC for the second time at points N and L	Amir Hossein

respectively. Prove that

a) The sum of diameters of these two circles is equal to BC,

b) $\frac{A_1N}{BB_1} + \frac{A_1L}{CC_1} = 1.$

view topic

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A circle touches the sides of an angle with vertex A at points B and $C.$ A line passing through A intersects this circle in points D and $E.$ A chord BX is parallel to $DE.$ Prove that XC passes through the midpoint of the segment $DE.$

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Construct a triangle, if the lengths of the bisectrix and of the altitude from one vertex, and of the median from another vertex are given.

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A point B lies on a chord AC of circle $\omega.$ Segments AB and BC are diameters of circles ω_1 and ω_2 centered at O_1 and O_2 respectively. These circles intersect ω for the second time in points D and E respectively. The rays O_1D and O_2E meet in a point $F,$ and the rays AD and CE do in a point $G.$ Prove that the line FG passes through the midpoint of the segment $AC.$

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A quadrilateral $ABCD$ is inscribed into a circle with center $O.$ Points P and Q are opposite to C and D respectively. Two tangents drawn to that circle at these points meet the line AB in points E and $F.$ (A is between E and B, B is between A and F). The line EO meets AC and BC in points X and Y respectively, and the line FO meets AD and BD in points U and V respectively. Prove that $XV = YU.$

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The incircle of an acute-angled triangle ABC touches AB, BC, CA at points C_1, A_1, B_1 respectively. Points A_2, B_2 are the midpoints of the segments B_1C_1, A_1C_1 respectively. Let P be a common point of the incircle and the line CO, where O is the circumcenter of triangle $ABC.$ Let also A' and B' be the second common points of PA_2 and PB_2 with the incircle. Prove that a common point of AA' and BB' lies on the altitude of the triangle dropped from the vertex $C.$

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A given convex quadrilateral $ABCD$ is such that $\angle ABD + \angle ACD > \angle BAC + \angle BDC.$ Prove that

$$S_{ABD} + S_{ACD} > S_{BAC} + S_{BDC}.$$

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A circle centered at a point F and a parabola with focus F have two common points. Prove that there exist four points A, B, C, D on the circle such that the lines AB, BC, CD and DA touch the parabola.

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A cyclic hexagon $ABCDEF$ is such that $AB \cdot CF = 2BC \cdot FA, CD \cdot EB = 2DE \cdot BC$ and $EF \cdot AD = 2FA \cdot DE.$ Prove that the lines AD, BE and CF are concurrent.

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Let us have a line ℓ in the space and a point A not lying on $\ell.$ For an arbitrary line ℓ' passing through A, XY (Y is on ℓ') is a common perpendicular to the lines ℓ and $\ell'.$ Find the locus of points $Y.$

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For two different regular icosahedrons it is known that some six of their vertices are vertices of a regular octahedron. Find the ratio of the edges of these icosahedrons.

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