

India
Regional Mathematical Olympiad
1990

- [1] Two boxes contain between them 65 balls of several different sizes. Each ball is white, black, red or yellow. If you take any five balls of the same colour, at least two of them will always be of the same size(radius). Prove that there are at least three ball which lie in the same box have the same colour and have the same size(radius).

- [2] For all positive real numbers a, b, c , prove that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.$$

- [3] A square sheet of paper $ABCD$ is so folded that B falls on the mid point of M of CD . Prove that the crease will divide BC in the ration 5 : 3.

- [4] Find the remainder when 2^{1990} is divided by 1990.

- [5] P is any point inside a triangle ABC . The perimeter of the triangle $AB + BC + Ca = 2s$. Prove that $s < AP + BP + CP < 2s$.

- [6] N is a 50-digit number (in decimal representation). All digits except the 26th digit (from the left) are 1. If N is divisible by 13, find its 26-th digit.

- [7] A census man on duty visited a house in which the lady inmates declined to reveal their individual ages, but said "We do not mind giving you the sum of the ages of any two ladies you may choose". Thereupon, the census man said, "In that case, please give me the sum of the ages of every possible pair of you". They gave the sums as: 30, 33, 41, 58, 66, 69. The census man took these figures and happily went away.

How did he calculate the individual ages?

- [8] If the circumcenter and centroid of a triangle coincide, prove that it must be equilateral.