Balkan MO 2000

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1 Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that

$$f(xf(x)+f(y))=f^2(x)+y$$

for all $x,y\in\mathbb{R}$.

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2 Let ABC be an acute-angled triangle and D the midpoint of BC. Let E be a point on segment AD and M its projection on BC. If N and P are the projections of M on AB and AC then the interior angule bisectors of $\angle NMP$ and $\angle NEP$ are parallel.

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3 How many $1\times 10\sqrt{2}$ rectangles can be cut from a 50×90 rectangle using cuts parallel to its edges?

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4 Show that for any n we can find a set X of n distinct integers greater than 1, such that the average of the elements of any subset of X is a square, cube or higher power.

Contributors: tranthanhnam, tchebytchev, Valentin Vornicu