India
Regional Mathematical Olympiad
2008

$\boxed{1}$ Let ABC be an acute angled triangle; let D, F be the midpoints of BC, AB respectively. Let the perpendicular from F to AC and the perpendicular from B ti BC meet in N: Prove that ND is the circumradius of ABC.

[15 points out of 100 for the 6 problems]

$\boxed{2}$ Prove that there exist two infinite sequences $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ of positive integers such that the following conditions hold simultaneously: (i) $0 < a_1 < a_2 < a_3 < \cdots$; (ii) $a_n < b_n < a_n^2$, for all $n \geq 1$; (iii) $a_n - 1$ divides $b_n - 1$, for all $n \geq 1$ (iv) $a_n^2 - 1$ divides $b_n^2 - 1$, for all $n \geq 1$

[19 points out of 100 for the 6 problems]

$\boxed{3}$ Suppose a and b are real numbers such that the roots of the cubic equation $ax^3 - x^2 + bx - 1$ are positive real numbers. Prove that:

$$(i)\ 0 < 3ab \leq 1 \text{ and } (i)\ b \geq \sqrt{3}$$

[19 points out of 100 for the 6 problems]

$\boxed{4}$ Find the number of all 6-digit natural numbers such that the sum of their digits is 10 and each of the digits $0, 1, 2, 3$ occurs at least once in them.

[14 points out of 100 for the 6 problems]

$\boxed{5}$ Three nonzero real numbers a, b, c are said to be in harmonic progression if $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$. Find all three term harmonic progressions a, b, c of strictly increasing positive integers in which $a = 20$ and b divides c.

[17 points out of 100 for the 6 problems]

$\boxed{6}$ Find the number of all integer-sided *isosceles obtuse-angled* triangles with perimeter 2008.

[16 points out of 100 for the 6 problems]