

**IMO 2001**  
Washington DC, USA

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**Day 1 - 06 July 2001**

- [1] Consider an acute-angled triangle  $ABC$ . Let  $P$  be the foot of the altitude of triangle  $ABC$  issuing from the vertex  $A$ , and let  $O$  be the circumcenter of triangle  $ABC$ . Assume that  $\angle C \geq \angle B + 30^\circ$ . Prove that  $\angle A + \angle COP < 90^\circ$ .
- [2] Prove that for all positive real numbers  $a, b, c$ ,

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1.$$

- [3] Twenty-one girls and twenty-one boys took part in a mathematical competition. It turned out that each contestant solved at most six problems, and for each pair of a girl and a boy, there was at least one problem that was solved by both the girl and the boy. Show that there is a problem that was solved by at least three girls and at least three boys.

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**Day 2 - 07 July 2001**

- [4] Let  $n$  be an odd integer greater than 1 and let  $c_1, c_2, \dots, c_n$  be integers. For each permutation  $a = (a_1, a_2, \dots, a_n)$  of  $\{1, 2, \dots, n\}$ , define  $S(a) = \sum_{i=1}^n c_i a_i$ . Prove that there exist permutations  $a \neq b$  of  $\{1, 2, \dots, n\}$  such that  $n!$  is a divisor of  $S(a) - S(b)$ .
- [5] Let  $ABC$  be a triangle with  $\angle BAC = 60^\circ$ . Let  $AP$  bisect  $\angle BAC$  and let  $BQ$  bisect  $\angle ABC$ , with  $P$  on  $BC$  and  $Q$  on  $AC$ . If  $AB + BP = AQ + QB$ , what are the angles of the triangle?
- [6] Let  $a > b > c > d$  be positive integers and suppose that

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that  $ab + cd$  is not prime.