Junior Balkan MO 2000

Ohrid, Macedonia

$\boxed{1}$ Let x and y be positive reals such that

$$x^3 + y^3 + (x+y)^3 + 30xy = 2000.$$

Show that $x + y = 10$.

$\boxed{2}$ Find all positive integers $n \geq 1$ such that $n^2 + 3^n$ is the square of an integer.

Bulgaria

$\boxed{3}$ A half-circle of diameter EF is placed on the side BC of a triangle ABC and it is tangent to the sides AB and AC in the points Q and P respectively. Prove that the intersection point K between the lines EP and FQ lies on the altitude from A of the triangle ABC.

Albania

$\boxed{4}$ At a tennis tournament there were $2n$ boys and n girls participating. Every player played every other player. The boys won $\frac{7}{5}$ times as many matches as the girls. It is knowns that there were no draws. Find n.

Serbia