Cono Sur Olympiad 2005

Day 1	May 24th	

1 Let a_n be the last digit of the sum of the digits of 20052005...2005, where the 2005 block occurs n times. Find $a_1 + a_2 + \cdots + a_{2005}$.

2 Let ABC be an acute-angled triangle and let AN, BM and CP the altitudes with respect to the sides BC, CA and AB, respectively. Let R, S be the pojections of N on the sides AB, CA, respectively, and let Q, W be the projections of N on the altitudes BM and CP, respectively.

(a) Show that R, Q, W, S are collinear.
(b) Show that $MP = RS - QW$.

3 The monetary unit of a certain country is called Reo, and all the coins circulating are integers values of Reos. In a group of three people, each one has 60 Reos in coins (but we don't know what kind of coins each one has). Each of the three people can pay each other any integer value between 1 and 15 Reos, including, perhaps with change. Show that the three persons together can pay exactly (without change) any integer value between 45 and 135 Reos, inclusive.

Day 2 May 25th

1 Let ABC be a isosceles triangle, with $AB = AC$. A line r that pass through the incenter I of ABC touches the sides AB and AC at the points D and E, respectively. Let F and G be points on BC such that $BF = CE$ and $CG = BD$. Show that the angle $\angle FIG$ is constant when we vary the line r.

2 We say that a number of 20 digits is *special* if its impossible to represent it as an product of a number of 10 digits by a number of 11 digits. Find the maximum quantity of consecutive numbers that are specials.

3 On the cartesian plane we draw circumferences of radii $1/20$ centred in each lattice point. Show that any circumference of radii 100 in the cartesian plane intersect at least one of the small circumferences.