India
National Olympiad
2009

Day 1

1 Let ABC be a tringle and let P be an interior point such that $\angle BPC = 90, \angle BAP = \angle BCP$.Let M, N be the mid points of AC, BC respectively.Suppose $BP = 2PM$.Prove that A, P, N are collinear.

2 Define a a sequence $< a_n >_{n=1}^{\infty}$ as follows

$a_n = 0$, if number of positive divisors of n is *odd* $a_n = 1$, if number of positive divisors of n is *even*

(The positive divisors of n include 1 as well as n.)Let $x = 0.a_1a_2a_3.....$ be the real number whose decimal expansion contains a_n in the n-th place,$n \geq 1$.Determine,with proof,whether x is rational or irrational.

3 Find all real numbers x such that: $[x^2 + 2x] = [x]^2 + 2[x]$

(Here $[x]$ denotes the largest integer not exceeding x.)

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Day 2

4 All the points in the plane are colored using three colors.Prove that there exists a triangle with vertices having the same color such that *either* it is isosceles *or* its angles are in geometric progression.

5 Let ABC be an acute angled triangle and let H be its ortho centre.Let h_{max} denote the largest altitude of the triangle ABC.Prove that:

$AH + BH + CH \leq 2h_{max}$

6 Let a, b, c be positive real numbers such that $a^3 + b^3 = c^3$.Prove that:

$a^2 + b^2 - c^2 > 6(c - a)(c - b)$.