Source: All Russian 2015 Grade 9 Day 2 P 2

IstekOly...
495 posts

Dec 12, 2015, 10:24 pm PM #1

A field has a shape of checkboard $41x41$ square. A tank concealed in one of the cells of the field. By one shot, a fighter airplane fires one of the cells. If a shot hits the tank, then the tank moves to a neighboring cell of the field, otherwise it stays in its cell (the cells are neighbours if they share a side). A pilot has no information about the tank , one needs to hit it twice. Find the least number of shots sufficient to destroy the tank for sure. *(S.Berlov,A.Magazinov)*

Zipfer
12 posts

Dec 12, 2015, 10:37 pm PM #2

Color the grid like a chessboard, with the corners white squares. If you bomb all the black squares, followed by the white squares, followed by the black squares again this guarantees destroying the tank, for a total of 2521 shots. Not sure how to show this is the least.

Mellow...
5327 posts

Dec 12, 2015, 11:13 pm • 1 PM #3

To show this is best, note that given any pair of adjacent squares A, B, your shots must either fire on B then A then B (not necessarily consecutively) or A then B then A. This is the only way to account for both possibilities that the tank starts in A and is destroyed on B or starts in B and is destroyed on A. In particular, for any pair of adjacent squares, one must be shot twice. The least number of squares you can shoot twice while still hitting every adjacent pair of squares are the black squares in your coloring.

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anantmu...
839 posts

Apr 27, 2016, 12:06 am PM #4

The answer is $\frac{3 \cdot 41^2 - 1}{2} = 2521$

Indeed, this follows if we colour the chessboard alternatively (more white squares) and fire at all black squares, then all white ones and then again all black ones. This ensures that we win.

Now, otherwise, we can assume that each square has been hit once else there is the possibility of the tank being unharmed at all. Also, each square can't be hit at least 2 times since that would exceed the obvious bound of 2.41^2. Thus, there is some square which is hit only once. We mark any such square by an $\times$ and tile the rest of the board with dominoes. In each square we write the number of hits it received. If the sum of numbers in each domino is at least 3 then we have at least $\frac{3 \cdot (41^2 - 1)}{2} + 1$ shots fired. If not, then some domino has sum less or equal to 2 meaning both of its squares are fired exactly once. Since the pilot doesn't know the position of the tank, we may assume (since the pilot's strategy has to be deterministic) that the tank was placed at the square of the domino which was fired at later and after receiving the first hit, it immediately shifted to the other square of the domino. This proves the result.

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