Art of Problem Solving

2011 Romania National Olympiad

Romania National Olympiad 2011

–	Grade level 7
–	Grade level 8
–	Grade level 9
–	Grade level 10
–	Grade level 11
–	April 18th
1	A row of a matrix belonging to $\mathcal{M}_n(\mathbb{C})$ is said to be *permutable* if no matter how we would permute the entries of that row, the value of the determinant doesn't change. Prove that if a matrix has two *permutable* rows, then its determinant is equal to 0 .
2	Let $u : [a,b] \to \mathbb{R}$ be a continuous function that has finite left-side derivative $u_l'(x)$ in any point $x \in (a,b]$. Prove that the function u is monotonously increasing if and only if $u_l'(x) \geq 0$, for any $x \in (a,b]$.
3	Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous and strictly decreasing function with $g(\mathbb{R}) = (-\infty, 0)$. Prove that there are no continuous functions $f : \mathbb{R} \to \mathbb{R}$ with the property that there exists a natural number $k \geq 2$ so that : $\underbrace{f \circ f \circ \ldots \circ f}_{k \text{ times}} = g$.
4	Let A , $B \in \mathcal{M}_2(\mathbb{C})$ so that : $A^2 + B^2 = 2AB$. **a)** Prove that : $AB = BA$. **b)** Prove that : $\mathrm{tr}\,(A) = \mathrm{tr}\,(B)$.
–	Grade level 12