Fixed domination of coins forming values

algorithm induction algebra binomial theorem combinatorics proposed combinatorics

Source: AllRussian-2014, Grade 9, day2, P3

61plus
223 posts

May 17, 2014, 11:34 pm PM #1

In a country, mathematicians chose an $\alpha > 2$ and issued coins in denominations of 1 ruble, as well as α^k rubles for each positive integer k. α was chosen so that the value of each coins, except the smallest, was irrational. Is it possible that any natural number of rubles can be formed with at most 6 of each denomination of coins?

---

ksun48
1515 posts

Jun 7, 2014, 7:14 am PM #2

Let's consider the quantity of 7 rubles. Obviously, the fourth coin cannot be used to make 7 rubles, since it is worth more than 8. It cannot be made with only 1's, because at most 6 of each denomination can be used. Since each power is irrational, both the second and third coin must be used. These are at more than 2 and 4 rubles, respectively. Thus $7 = \alpha + \alpha^2$, so $\alpha = \frac{\sqrt{29}-1}{2}$. However, the quantity of 12 rubles cannot be formed, either using the fourth coins or just combinations of the second and third. Thus not all natural numbers of rubles can be formed.

---

viperstrike
1111 posts

Jun 21, 2014, 10:34 pm • 1 PM #3

Ksun48, actually $12 = 5 + a + a^2$.

Correct Solution

This post has been edited 1 time. Last edited by viperstrike, May 31, 2016, 1:54 am

---

va2010
1147 posts

Apr 10, 2016, 10:09 pm PM #4

Here's a shorter solution. Again, take $\alpha = \frac{\sqrt{29}-1}{7}$ and use induction. Assume that we had a representation $c_0(\alpha)^0 + c_1(\alpha)^1 \cdots + c_n(\alpha)^n$. We do the following after adding one; change every instance of a number with an index larger than 6 from $c(\alpha)^n$ to $(c-7)(\alpha)^n + \alpha^{n+1} + \alpha^{n+2}$. Observe that this reduces the sum of the coefficients after every operation, so it must eventually terminate, since the coefficients stay positive. Hence, the algorithm terminates and we may induct.

Quick Reply