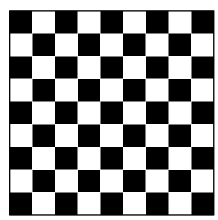
Canada National Olympiad 2004

**1** Find all ordered triples $(x, y, z)$ of real numbers which satisfy the following system of equations:

$$\begin{cases} xy &= z - x - y \\ xz &= y - x - z \\ yz &= x - y - z \end{cases}$$

**2** How many ways can 8 mutually non-attacking rooks be placed on the $9 \times 9$ chessboard (shown here) so that all 8 rooks are on squares of the same color?

(Two rooks are said to be attacking each other if they are placed in the same row or column of the board.)



**3** Let $A, B, C, D$ be four points on a circle (occurring in clockwise order), with $AB < AD$ and $BC > CD$. The bisectors of angles $BAD$ and $BCD$ meet the circle at $X$ and $Y$, respectively. Consider the hexagon formed by these six points on the circle. If four of the six sides of the hexagon have equal length, prove that $BD$ must be a diameter of the circle.

**4** Let $p$ be an odd prime. Prove that:

$$\sum_{k=1}^{p-1} k^{2p-1} \equiv \frac{p(p+1)}{2} \pmod{p^2}$$

**5** Let $T$ be the set of all positive integer divisors of $2004^{100}$. What is the largest possible number of elements of a subset $S$ of $T$ such that no element in $S$ divides any other element in $S$?