Triangles on the paper

combinatorics

Source: All russian olympiad 2016,Day1,grade 11,P3

RagvaloD
376 posts

May 5, 2016, 2:51 pm PM #1

We have sheet of paper, divided on 100×100 unit squares. In some squares we put rightangled isosceles triangles with leg $=1$ (Every triangle lies in one unit square and is half of this square). Every unit grid segment(boundary too) is under one leg of triangle. Find maximal number of unit squares, that don`t contains triangles.

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kimthoon1
18 posts

Jun 12, 2016, 7:11 pm PM #2

i can't understand the question ...

what is 'unit grid segment' and
under one leg of triangle?

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kimthoon1
18 posts

Jun 12, 2016, 10:51 pm PM #3

Can I know the answer?
Maybe I can guess what question means from the answer ...

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dgrozev
760 posts

Jun 13, 2016, 11:04 pm PM #4

In Russian, what is translated here as "Every unit grid segment(boundary too) is under one leg of triangle" means:
Every segment with length 1 connecting neighbouring knots of the grid(including those on the boundary) is covered by exactly one leg of some triangle.
I hope it's clearer.

MellowM...
5327 posts

Jun 13, 2016, 11:28 pm PM #5

Can two triangles be put in the same square?

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dgrozev
760 posts

Jun 14, 2016, 12:18 am • 1 PM #6

The original text only says the triangles are non overlapping, so I suppose - yes, it's permitted to put two triangles into one square.

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anantmudg...
839 posts

Jun 14, 2016, 12:32 am PM #7

Is my interpretation correct?

We have a 100×100 board and we place some right-angled isosceles triangles in it such that each grid-segment (a line segment connecting two adjacent knots on the grid, including the boundary) is a leg of exactly one triangle. What maximal number of unit squares do not have a triangle placed inside them?

MellowM...
5327 posts

Jun 14, 2016, 1:50 am • 1 PM #8

My question was kind of dumb since any tiling requires some squares to have two triangles.... Anyways, here's a full solution.

Upper bound

> Each triangle covers a vertical and horizontal segment. Consider the triangles in each row. There are 101 vertical segments and only triangles in the row cover them, so every row has 101 triangles. In the top row, there are 100 horizontal segments above it which must be covered by triangles in this row, so 100 of the top row's triangles cover a horizontal segment above, meaning exactly one covers

a horizontal segment below. Then in the second row form the top, there are 99 horizontal segments above remaining to be covered, so 99 cover a top segment and 2 cover a segment below. This continues until we show that in row k from the top ($1 \leq k \leq 100$), $101 - k$ triangles cover a horizontal segment above the row and k triangles cover a segment below the row.

If a square has no triangles in it, this means the segment above the square and the segment below the square are not covered by triangles in that row. Therefore, in row k, we can have up to $\min(100 - (101 - k), 100 - k)$ squares that are blank before we run out of segments above or segments below that are not covered by triangles in that row. Summing this over all k from 1 to 100 gives at most

$$0 + 1 + 2 + \cdots + 48 + 49 + 49 + 48 + \cdots + 2 + 1 + 0 = 49 \cdot 50$$

blank squares.

Construction

This obtains $49 \cdot 50$ blank squares, the upper bound shown above. Assuming rows and columns are numbered 1 to 100 and squares are identified by ordered pairs (column #, row #),
1. place upper-left oriented triangles in squares (x, y) with $x + y \leq 51$,
2. place lower-left oriented triangles in squares (x, y) with $y \geq 51 + x$,
3. place upper-right oriented triangles in squares (x, y) with $x \geq 51 + y$,
4. place lower-right oriented triangles in squares (x, y) with $x + y \geq 151$, and
5. in the remaining squares (x, y) without any triangles yet, if $x + y$ is even, then place two triangles, and if $x + y$ is odd, leave it blank.

This is easily generalized to $2k \times 2k$ obtaining $k(k - 1)$ blank squares. Here's what the pattern looks like for 10 by 10, where slashes are squares with one triangle, Os are squares with two triangles, and spaces are empty squares (there are $4 \cdot 5 = 20$ of them):

```
/////0\\\\
////0 0\\\
///0 0 0\\
//0 0 0 0\
/0 0 0 0 0
0 0 0 0 0/
\0 0 0 0//
\\0 0 0///
\\\0 0////
\\\\0/////
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This post has been edited 1 time. Last edited by MellowMelon, Jun 14, 2016, 3:15 am
Reason: fix misstated value

RagvaloD
376 posts

Jun 14, 2016, 1:11 pm PM #9

Right answer

$49 * 50 = 2450$

kimthoon1
18 posts

Jun 14, 2016, 11:09 pm PM #10

Oh thank you guys
I thought that I can put 1 or 0 triangles in one unit square ...
So I got contradiction and confused
Anyways, thanks!!!

(Sorry for my English)

BUT... isn't it possible?

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□ □ □ □ □ □ □ □ □ □ (at 10×10)

(□ have triangles , ○ do not have triangle)

Do I misunderstood the question?

This post has been edited 1 time. Last edited by kimthoon1, Jun 14, 2016, 11:23 pm
Reason: to post my answer

MellowM...
5327 posts

Jun 14, 2016, 11:41 pm PM #11

You have not obeyed the "exactly one leg of some triangle" constraint. Some segments of that grid are covered by two triangles.

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kimthoon1
18 posts

Jun 16, 2016, 11:31 pm PM #12

OH...

thanks!

Quick Reply