Pan African 2005

Alger, Algeria

Day 1 - *01 August 2005*

$\boxed{1}$ For any positive real numbers a, b and c, prove:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{2}{a+b}+\frac{2}{b+c}+\frac{2}{c+a} \geq \frac{9}{a+b+c}$$

$\boxed{2}$ Let S be a set of integers with the property that any integer root of any non-zero polynomial with coefficients in S also belongs to S. If 0 and 1000 are elements of S, prove that -2 is also an element of S.

$\boxed{3}$ Let ABC be a triangle and let P be a point on one of the sides of ABC. Construct a line passing through P that divides triangle ABC into two parts of equal area.

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Day 2 - *02 August 2005*

$\boxed{1}$ Let $[x]$ be the greatest integer less than or equal to x, and let $\{x\} = x - [x]$. Solve the equation: $[x] \cdot \{x\} = 2005x$

$\boxed{2}$ Noah has to fit 8 species of animals into 4 cages of the Arc. He planes to put two species of animal in each cage. It turns out that, for each species of animal, there are at most 3 other species with which it cannot share a cage. Prove that there is a way to assign the animals to the cages so that each species shares a cage with a compatible species.

$\boxed{3}$ Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function such that: For all a and b in $\mathbb{Z} - \{0\}$, $f(ab) \geq f(a) + f(b)$. Show that for all $a \in \mathbb{Z} - \{0\}$ we have $f(a^n) = nf(a)$ for all $n \in \mathbb{N}$ if and only if $f(a^2) = 2f(a)$