India
National Olympiad
1989

$\boxed{1}$ Prove that the Polynomial $f(x) = x^4 + 26x^3 + 56x^2 + 78x + 1989$ can't be expressed as a product $f(x) = p(x)q(x)$, where $p(x)$ and $q(x)$ are both polynomial with integral coefficients and with degree at least 1.

$\boxed{2}$ Let a, b, c and d be any four real numbers, not all equal to zero. Prove that the roots of the polynomial $f(x) = x^6 + ax^3 + bx^2 + cx + d$ can't all be real.

$\boxed{3}$ Let A denote a subset of the set $\{1, 11, 21, 31, \dots, 541, 551\}$ having the property that no two elements of A add up to 552. Prove that A can't have more than 28 elements.

$\boxed{4}$ Determine all $n \in \mathbb{N}$ for which n is not the square of any integer, [/*:m] $\lfloor \sqrt{n} \rfloor^3$ divides n^2. [/*:m]

$\boxed{5}$ For positive integers n, define $A(n)$ to be $\frac{(2n)!}{(n!)^2}$. Determine the sets of positive integers n for which

(a) $A(n)$ is an even number,

(b) $A(n)$ is a multiple of 4.

$\boxed{6}$ Triangle ABC has incentre I and the incircle touches BC, CA at D, E respectively. Let BI meet DE at G. Show that AG is perpendicular to BG.

$\boxed{7}$ Let A be one of the two points of intersection of two circles with centers X, Y respectively.The tangents at A to the two circles meet the circles again at B, C. Let a point P be located so that $PXAY$ is a parallelogram. Show that P is also the circumcenter of triangle ABC.

