IMO 1972

Day 1

1. Prove that from a set of ten distinct two-digit numbers, it is always possible to find two disjoint subsets whose members have the same sum.

2. Given $n > 4$, prove that every cyclic quadrilateral can be dissected into n cyclic quadrilaterals.

3. Prove that $(2m)!(2n)!$ is a multiple of $m!n!(m+n)!$ for any non-negative integers m and n.

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Day 2

$\boxed{1}$ Find all positive real solutions to:

$$\begin{array}{rcl}(x_1^2 - x_3x_5)(x_2^2 - x_3x_5) & \leq & 0(x_2^2 - x_4x_1)(x_3^2 - x_4x_1) \\ 0(x_3^2 - x_5x_2)(x_4^2 - x_5x_2) & \leq & 0(x_4^2 - x_1x_3)(x_5^2 - x_1x_3) \\ 0(x_5^2 - x_2x_4)(x_1^2 - x_2x_4) & \leq & 0\end{array}$$

(0)

f and g are real-valued functions defined on the real line. For all x and $y, f(x+y) + f(x-y) = 2f(x)g(y)$. f is not identically zero and $|f(x)| \leq 1$ for all x. Prove that $|g(x)| \leq 1$ for all x.

Given four distinct parallel planes, prove that there exists a regular tetrahedron with a vertex on each plane.