

China Girls Math Olympiad 2004

### Day 1

- 1 We say a positive integer  $n$  is *good* if there exists a permutation  $a_1, a_2, \dots, a_n$  of  $1, 2, \dots, n$  such that  $k + a_k$  is perfect square for all  $1 \leq k \leq n$ . Determine all the good numbers in the set  $\{11, 13, 15, 17, 19\}$ .

- 2 Let  $a, b, c$  be positive reals. Find the smallest value of

$$\frac{a+3c}{a+2b+c} + \frac{4b}{a+b+2c} - \frac{8c}{a+b+3c}.$$

- 3 Let  $ABC$  be an obtuse inscribed in a circle of radius 1. Prove that  $\triangle ABC$  can be covered by an isosceles right-angled triangle with hypotenuse of length  $\sqrt{2} + 1$ .

- 4 A deck of 32 cards has 2 different jokers each of which is numbered 0. There are 10 red cards numbered 1 through 10 and similarly for blue and green cards. One chooses a number of cards from the deck. If a card in hand is numbered  $k$ , then the value of the card is  $2^k$ , and the value of the hand is sum of the values of the cards in hand. Determine the number of hands having the value 2004.

### Day 2

- 5 Let  $u, v, w$  be positive real numbers such that  $u\sqrt{vw} + v\sqrt{wu} + w\sqrt{uv} \geq 1$ . Find the smallest value of  $u + v + w$ .

- 6 Given an acute triangle  $ABC$  with  $O$  as its circumcenter. Line  $AO$  intersects  $BC$  at  $D$ . Points  $E, F$  are on  $AB, AC$  respectively such that  $A, E, D, F$  are concyclic. Prove that the length of the projection of line segment  $EF$  on side  $BC$  does not depend on the positions of  $E$  and  $F$ .

- 7 Let  $p$  and  $q$  be two coprime positive integers, and  $n$  be a non-negative integer. Determine the number of integers that can be written in the form  $ip + jq$ , where  $i$  and  $j$  are non-negative integers with  $i + j \leq n$ .



# Art of Problem Solving

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When the unit squares at the four corners are removed from a three by three squares, the resulting shape is called a cross. What is the maximum number of non-overlapping crosses placed within the boundary of a  $10 \times 11$  chessboard? (Each cross covers exactly five unit squares on the board.)

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