Art of Problem Solving

2011 ISI B.Math Entrance Exam

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1 Given $a, x \in \mathbb{R}$ and $x \geq 0$,$a \geq 0$. Also $\sin(\sqrt{x+a}) = \sin(\sqrt{x})$. What can you say about a??? Justify your answer.

2 Given two cubes R and S with integer sides of lengths r and s units respectively . If the difference between volumes of the two cubes is equal to the difference in their surface areas , then prove that $r = s$.

3 For $n \in \mathbb{N}$ prove that

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{2n-1}{2n} \leq \frac{1}{\sqrt{2n+1}}.$$

4 Let $t_1 < t_2 < t_3 < \cdots < t_{99}$ be real numbers. Consider a function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = |x - t_1| + |x - t_2| + ... + |x - t_{99}|$. Show that $f(x)$ will attain minimum value at $x = t_{50}$.

5 Consider a sequence denoted by F_n of non-square numbers . $F_1 = 2$,$F_2 = 3$,$F_3 = 5$ and so on . Now , if $m^2 \leq F_n < (m+1)^2$. Then prove that m is the integer closest to $\sqrt{n}$.

6 Let $f(x) = e^{-x}\ \forall\ x \geq 0$ and let g be a function defined as for every integer $k \geq 0$, a straight line joining $(k, f(k))$ and $(k+1, f(k+1))$. Find the area between the graphs of f and g.

7 If $a_1, a_2, \cdots, a_7$ are not necessarily distinct real numbers such that $1 < a_i < 13$ for all i, then show that we can choose three of them such that they are the lengths of the sides of a triangle.

8 In a triangle ABC , we have a point O on BC . Now show that there exists a line l such that $l || AO$ and l divides the triangle ABC into two halves of equal area .