IMO 2004

Athens, Greece

Day 1 - *12 July 2004*

$\boxed{1}$ 1. Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N respectively. Denote by O the midpoint of the side BC. The bisectors of the angles $\angle BAC$ and $\angle MON$ intersect at R. Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the side BC.

$\boxed{2}$ Find all polynomials f with real coefficients such that for all reals a, b, c such that $ab+bc+ca = 0$ we have the following relations

$$f(a-b) + f(b-c) + f(c-a) = 2f(a+b+c).$$

$\boxed{3}$ Define a "hook" to be a figure made up of six unit squares as shown below in the picture, or any of the figures obtained by applying rotations and reflections to this figure. [img]http://www.mathlinks.ro/Forum/files/hook.png[/img] Determine all $m \times n$ rectangles that can be covered without gaps and without overlaps with hooks such that

- the rectangle is covered without gaps and without overlaps - no part of a hook covers area outside the rectagle.

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Day 2 - *13 July 2004*

$\boxed{4}$ Let $n \geq 3$ be an integer. Let t_1, t_2, ..., t_n be positive real numbers such that

$n^2 + 1 > (t_1 + t_2 + ... + t_n)\left(\frac{1}{t_1} + \frac{1}{t_2} + ... + \frac{1}{t_n}\right).$

Show that t_i, t_j, t_k are side lengths of a triangle for all i, j, k with $1 \leq i < j < k \leq n$.

$\boxed{5}$ In a convex quadrilateral $ABCD$, the diagonal BD bisects neither the angle ABC nor the angle CDA. The point P lies inside $ABCD$ and satisfies

$\angle PBC = \angle DBA$ and $\angle PDC = \angle BDA.$

Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$.

$\boxed{6}$ We call a positive integer *alternating* if every two consecutive digits in its decimal representation are of different parity.

Find all positive integers n such that n has a multiple which is alternating.