Junior Balkan MO 2009

1 Let $ABCDE$ be a convex pentagon such that $AB+CD = BC+DE$ and k a circle with center on side AE that touches the sides AB, BC, CD and DE at points P, Q, R and S (different from vertices of the pentagon) respectively. Prove that lines PS and AE are parallel.

2 Solve in non-negative integers the equation $2^a 3^b + 9 = c^2$

3 Let x, y, z be real numbers such that $0 < x, y, z < 1$ and $xyz = (1-x)(1-y)(1-z)$. Show that at least one of the numbers $(1-x)y, (1-y)z, (1-z)x$ is greater than or equal to $\frac{1}{4}$

4 Each one of 2009 distinct points in the plane is coloured in blue or red, so that on every blue-centered unit circle there are exactly two red points. Find the gratest possible number of blue points.