China National Olympiad 2002

**Day 1**

**1** the edges of triangle $ABC$ are $a,b,c$ respectively,$b<c$,$AD$ is the bisector of $\angle A$,point $D$ is on segment $BC$.
(1)find the property $\angle A$,$\angle B$,$\angle C$ have,so that there exists point $E,F$ on $AB,AC$ satisfy $BE=CF$,and $\angle NDE=\angle CDF$
(2)when such $E,F$ exist,express $BE$ with $a,b,c$

**2** Given the polynomial sequence $(p_n(x))$ satisfying $p_1(x)=x^2-1$, $p_2(x)=2x(x^2-1)$, and $p_{n+1}(x)p_{n-1}(x)=(p_n(x))^2-(x^2-1)^2$, for $n\geq 2$, let $s_n$ be the sum of the absolute values of the coefficients of $p_n(x)$. For each $n$, find a non-negative integer $k_n$ such that $2^{-k_n}s_n$ is odd.

**3** In a competition there are 18 teams and in each round 18 teams are divided into 9 pairs where the 9 matches are played coincidentally. There are 17 rounds, so that each pair of teams play each other exactly once. After $n$ rounds, there always exists 4 teams such that there was exactly one match played between these teams in those $n$ rounds. Find the maximum value of $n$.

**Day 2**

**1** For every four points $P_1,P_2,P_3,P_4$ on the plane, find the minimum value of $\frac{\sum_{1\leq\ i<j\leq\ 4}P_iP_j}{\min_{1\leq\ i<j\leq\ 4}(P_iP_j)}$.

**2** Suppose that a point in the plane is called *good* if it has rational coordinates. Prove that all good points can be divided into three sets satisfying:
1) If the centre of the circle is good, then there are three points in the circle from each of the three sets.
2) There are no three collinear points that are from each of the three sets.

**3** Suppose that $c\in\left(\frac{1}{2},1\right)$. Find the least $M$ such that for every integer $n\geq 2$ and real numbers $0<a_1\leq a_2\leq\ldots\leq a_n$, if $\frac{1}{n}\sum_{k=1}^{n}ka_k=c\sum_{k=1}^{n}a_k$, then we always have that $\sum_{k=1}^{n}a_k\leq M\sum_{k=1}^{m}a_k$ where $m=[cn]$