Source: All Russian Olympiad 2015 11.3

utkarshg...
1931 posts

Dec 11, 2015, 9:55 pm • 1 PM #1

110 teams participate in a volleyball tournament. Every team has played every other team exactly once (there are no ties in volleyball). Turns out that in any set of 55 teams, there is one which has lost to no more than 4 of the remaining 54 teams. Prove that in the entire tournament, there is a team that has lost to no more than 4 of the remaining 109 teams.

JackXD
85 posts

Dec 24, 2015, 10:54 pm PM #2

Is it correct??

Consider the team T_i with the minimum no. of losses.Let this minimal no be atleast 5.Take any 5 teams which has won with T_i along with T_i and form a subset of 55 teams taking 49 other teams.In this subset there exists a team other than T_i with losses ≤ 4.It follows that that team will have a corresponding team not in the subset with which it has lost.Include the team originally outside the subset in the subset by eliminating another team.Continuing this way we can eventually find a subset of 55 teams with all losing ≥ 5 in the subset,a,c ontradiction

This post has been edited 2 times. Last edited by JackXD, Dec 24, 2015, 10:56 pm
Reason: xx

JackXD
85 posts

Dec 25, 2015, 12:09 am PM #3

Seems that my solution is not correct.Any solutions please??

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utkarshg...
1931 posts

Apr 15, 2016, 11:03 pm PM #4

Bump...
Anyone ?

Complex...
74 posts

Apr 17, 2016, 10:52 am • 1 PM #5

Claim: For $k \geq 55,$ if in any set of k teams, there is one which has lost to no more than 4 of the remaining $k-1$ teams, then in any set of $k+1$ teams, there is one which has lost to no more than 4 of the remaining k teams as well.

Proof. Suppose contrary, there exists a set $M=\{C_1,C_2,\ldots,C_{k+1}\}$ such that each C_i has lost to at least 5 teams among $M-\{C_i\}$. By our induction hypothesis there exist C_j such that C_j has lost to no more than 4 in $M-\{C_i\}$, i.e. C_j has lost to exactly 5 teams and C_i is one of them. Hence we consider a mapping

$$f:M\mapsto M \mid f(C_i)\stackrel{\text{def}}{=}C_j.$$

Clearly $s=\mid f(M)\mid\geq\lceil\frac{k+1}{5}\rceil\geq 12.$ Consider the tournament among that s teams, by PHP one of them has lost to at least $\frac{\binom{s}{2}}{s}=\frac{s-1}{2}>5,$ which contradicts the result above(they has lost to exactly 5 teams). $\square$

This post has been edited 1 time. Last edited by Complex2Liu, Apr 17, 2016, 10:54 am
Reason: typo

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viperstrike
1111 posts

May 28, 2016, 2:29 am PM #6

> *Complex2Liu wrote:*
> **Claim:** For $k \geq 55,$ if in any set of k teams, there is one which has lost to no more than 4 of the remaining $k-1$ teams, then in any set of $k+1$ teams, there is one which has lost to no more than 4 of the remaining k teams as well.
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Nice solution.

I would just point out that for a given C_i, the C_j might not be unique, so the mapping is not well-defined. This is hardly a problem though because you could just consider the C_j with j minimal.

viperstrike
1111 posts

May 28, 2016, 2:36 am PM #7

> *JackXD wrote:*
> Seems that my solution is not correct.Any solutions please??

> *JackXD wrote:*
> Is it correct??
>
> Consider the team T_i with the minimum no. of losses.Let this minimal no be atleast 5.Take any 5 teams which has won with T_i along with T_i and form a subset of 55 teams taking 49 other teams.In this subset there exists a team other than T_i with losses ≤ 4.It follows that that team will have a corresponding team not in the subset with which it has lost.Include the team originally outside the subset in the subset by eliminating another team.Continuing this way we can eventually find a subset of 55 teams with all losing ≥ 5 in the subset,a,c ontradiction

This doesn't work because you include the team originally outside the subset by eliminating another team, and then you continue like that; what if at some step you have remove this team that you just added...

v_Enhance
4253 posts

Jul 7, 2016, 8:34 pm PM #8

We prove the result is true if 110 is replaced by any $N \geq 55$. To see this, we proceed by induction on N, with the base case given.

Assume for contradiction that we have $N+1$ people such that each team lost at least five times, but within every N some team lost at most four times. In that case, a *champion* is a team which lost exactly five times.

Lemma: There are at most 11 champions.
Proof: Otherwise, note that when 12 champions play each other, they each lose on average 5.5 games to each other, contradiction.

On the other hand, note that for any person p, some champion lost to p (by inductive hypothesis). Thus there are at least $\frac{1}{5}(N+1)$ champions. For $N \geq 55$ this is a contradiction.

Quick Reply