

Cono Sur Olympiad 2007

Day 1

- 1 Find all pairs (x, y) of nonnegative integers that satisfy

$$x^3y + x + y = xy + 2xy^2.$$

- 2 Given are 100 positive integers whose sum equals their product. Determine the minimum number of 1s that may occur among the 100 numbers.

- 3 Let ABC be an acute triangle with altitudes AD , BE , CF where D , E , F lie on BC , AC , AB , respectively. Let M be the midpoint of BC . The circumcircle of triangle AEF cuts the line AM at A and X . The line AM cuts the line CF at Y . Let Z be the point of intersection of AD and BX . Show that the lines YZ and BC are parallel.

Day 2

- 1 Some cells of a 2007×2007 table are colored. The table is *charrua* if none of the rows and none of the columns are completely colored. (a) What is the maximum number k of colored cells that a *charrua* table can have? (b) For such k , calculate the number of distinct *charrua* tables that exist.

- 2 Let $ABCDE$ be a convex pentagon that satisfies all of the following:
 -There is a circle Γ tangent to each of the sides.
 -The lengths of the sides are all positive integers.
 -At least one of the sides of the pentagon has length 1.
 -The side AB has length 2.
 Let P be the point of tangency of Γ with AB .
 (a) Determine the lengths of the segments AP and BP .
 (b) Give an example of a pentagon satisfying the given conditions.

- 3 Show that for each positive integer n , there is a positive integer k such that the decimal representation of each of the numbers $k, 2k, \dots, nk$ contains all of the digits $0, 1, 2, \dots, 9$.