right angle

geometry circumcircle geometry proposed

Source: AllRussian-2014, Grade 9, day1, P4

mathuz
1229 posts

May 3, 2014, 8:42 pm PM #1

Let M be the midpoint of the side AC of acute-angled triangle ABC with $AB > BC$. Let Ω be the circumcircle of ABC. The tangents to Ω at the points A and C meet at P, and BP and AC intersect at S. Let AD be the altitude of the triangle ABP and ω the circumcircle of the triangle CSD. Suppose ω and Ω intersect at $K \neq C$. Prove that $\angle CKM = 90^\circ$.

V. Shmarov

Tangents
37 posts

May 4, 2014, 8:52 am • 4 PM #2

We have $KDSC$ is cyclic so $\angle KDP = \angle KCS = 180^\circ - \angle KDS$.
On the other hand, $\angle KCS = \angle KAP$ in Ω. Hence, $\angle KAP = \angle KDP$. We obtain $PADK$ is cyclic. It follows that $\angle PKA = \angle PDA = 90^\circ$.
We also have $\angle PDA = \angle PMA = 90^\circ$ then $PDMA$ is cyclic. But $PKDA$ is cyclic so $PKMA$ is cyclic. We get
$\angle PKM = 180^\circ - \angle PAM = 180^\circ - \angle ABC = \angle AKC$. Thus,
$\angle PKM = \angle AKC$ or $\angle PKA = \angle MKC = 90^\circ$. $\blacksquare$

The problem still true for any triangle ABC.

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jayme
5236 posts

May 5, 2014, 12:47 pm PM #3

Dear Mathlinkers,
are you sure that KDSM is cyclic?
Sincerely
Jean-Louis

Tangents
37 posts

May 8, 2014, 3:09 pm PM #4

Sorry, my mistake. It is $KDSC$.

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nima1376
111 posts

May 18, 2014, 12:56 pm • 1 PM #5

$\widehat{KDS} = \widehat{KCA} = \widehat{KAP} \Rightarrow PDKMA$is
cycle$\Rightarrow$ $\widehat{AKM} = \widehat{APM} = 90 - \widehat{B} \Rightarrow \widehat{CKM} = 90$
done

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saturzo
54 posts

May 19, 2014, 3:44 pm • 2 PM #6

$CSKD$ cyclic.
So, $\measuredangle KDP = \measuredangle KDS = \measuredangle KCS = \measuredangle KCA = \measuredangle KAP[\because PA$ touches $\Omega]$
So, $KPAD$ is cyclic and thus, $\measuredangle AKP = \measuredangle ADP = 90^0$
Now, using Zhao's Lemma No-1, we get KP is the symmedian of $\triangle KAC$.
So, $\measuredangle MKC = \measuredangle AKP = 90^0$
And so, obviously $\angle CKM = 90^0$
$[QED]$

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junioragd
277 posts

Jul 30, 2014, 12:39 am PM #7

First,notice PDMA is cyclic,now let circumcircles of PDMA and ABC intersect at T.We have <PAM=<ABC,so <PTM=<ATC and <TPM=<TAC and from this two we have <PMT=<ACT=<PDT from whic we have KCST is a cyclic,so K=T.Now,<CKM=180-<KCA-(90-<KMP)=90-(<KCA-<KCA)=90 so we are finished

<KCA-(90-<KMP)=90-(<KCA-<KCA)=90,so we are finished

thecmd999
2874 posts

Sep 23, 2014, 12:44 am • 1 PM #8

Solution

anantmu...
839 posts

Oct 23, 2015, 7:18 pm • 1 PM #9

Her is a bash(without calculations) approach:

Let the feet of altitude from M to BC be X and let Y, Z be points on line BS such that $\angle CYM = \angle CZM = 90\circ$.

Now, all we have to do is prove that $(CSD), (ABC), (CYMZ)$ are co-axial circles for which we apply

the Lemma in Mathematical Reflections Issue 5,2015 (see awesomemath.org)

So, we need to prove that

$\frac{SM}{SA} = \frac{DY.DZ}{DB.DT}$ where T is the point at which the line BS meets (ABC) again.

This is just(?) length chasing and fairly nasty but computationally feasible.

anantmu...
839 posts

Aug 12, 2016, 11:32 pm PM #10

Double-posting because it's just so nice!

Suppose that the points A' and C' are opposite to A and C, respectively, in the circumcircle of triangle ABC. Clearly, points C', M, K are collinear. Since A' is symmetric to C' in line PM (perpendicular bisector of AC) and $C'K$ is a median; we have $A'K$ is a symmedian in triangle $AA'C$ and the points P, A', K are collinear. Moreover, points A, M, D, K, P lie on a circle with diameter AP. Therefore, $\angle MAK = \angle MKD$ and since we have the relations $\angle MSD = 90^\circ - \angle MAK = 90^\circ - \angle MKD = \angle DKC$, we conclude that the points C, S, D, K lie on a circle.

Comment. 800th post!

This post has been edited 2 times. Last edited by anantmudgal09, Aug 12, 2016, 11:33 pm

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