

India
Regional Mathematical Olympiad
1998

- [1] Let $ABCD$ be a convex quadrilateral in which $\angle BAC = 50^\circ$, $\angle CAD = 60^\circ$ and $\angle BDC = 25^\circ$. If E is the point of intersection of AC and BD , find $\angle AEB$.
- [2] Let n be a positive integer and $p_1, p_2, p_3, \dots, p_n$ be n prime numbers all larger than 5 such that 6 divides $p_1^2 + p_2^2 + p_3^2 + \dots + p_n^2$. prove that 6 divides n .
- [3] Prove that for every natural number $n > 1$

$$\frac{1}{n+1} \left(1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-1} \right) > \frac{1}{n} \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2n} \right).$$

- [4] Let ABC be a triangle with $AB = AC$ and $\angle BAC = 30^\circ$. Let A' be the reflection of A in the line BC ; B' be the reflection of B in the line CA ; C' be the reflection of C in line AB . Show that $A'B'C'$ is an equilateral triangle.
- [5] Find the minimum possible least common multiple of twenty natural numbers whose sum is 801.
- [6] Given the 7-element set $A = \{a, b, c, d, e, f, g\}$, find a collection T of 3-element subsets of A such that each pair of elements from A occurs exactly once on one of the subsets of T .