Gluing a 100*100 chessboard - who wins?

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Source: All-Russian Olympiad 2006 finals, problem 9.7

darij grinberg
6393 posts

May 8, 2006, 1:30 am • 1 PM #1

A 100×100 chessboard is cut into dominoes (1×2 rectangles). Two persons play the following game: At each turn, a player glues together two adjacent cells (which were formerly separated by a cut-edge). A player loses if, after his turn, the 100×100 chessboard becomes connected, i. e. between any two cells there exists a way which doesn't intersect any cut-edge. Which player has a winning strategy - the starting player or his opponent?

Summerburn
83 posts

Mar 21, 2007, 4:36 pm PM #2

Is the board cut in an arbitrary way, or is it cut in the "normal" way?

Erken
1363 posts

Aug 7, 2008, 6:45 pm PM #3

> *Summerburn wrote:*
> Is the board cut in an arbitrary way, or is it cut in the "normal" way?

What do you mean by "normal" way?

MBGO
315 posts

Feb 1, 2013, 10:54 pm • 1 PM #4

Second person has the winning strategy.
LEMMA : the maximum number of moves that are needed for the second person to punch all dominoes all arround the board so that raising one corner of the board makes all the border dominoes raised(call **A**), is less than needed moves for one to lose(call **B**).
▼ let second person starts punching all squares which are in the side of BIG square with all of their neighbours, it takes no more than 8*99 moves , summing up it with *first person's moves* it will get no more than 2*8*99 moves, for which any square belongs to exactly 1 domino , so the number of all dominoes punched together is no more than 2*2*8*99 which is clearly less than the whole number of dominoes;proved the Lemma □.
LEMMA : any connected set of dominoes with no square in its inner part not belonging to it, has an even number of segments as its sides and has an even number of small square's sides on its border.
▼ first part of Lemma caused by surrounding any horizontal segment by two vertical segments and vice versa so that the number of horizontal segments is equal to the number of vertical segments, the second part came from shadowing this shape so that the shape become a line parallel to x axes,it's obviouse that the thickness of this line is twice more than the thickness of the line which makes the border of the shape, same arguement for y axes will get the same result □

let the second person punchs all dominoes in the border of the BIG square, and then play illegantly; assume at one point, second player has no move to do i.e for any 2 squares which he punchs them,the BIG square becomes a connected shape. it means that the BIG square is now a collection of two connected shape,in which any two squares belong to one figure are punched to eachother and any two squares which do not belong to a same figure, are not connected, hence the number of all punchs which has been not uses untill now is even by the second Lemma, on the other hand the number of punchs needed to punch all the adjacent squares to eachothers, is even, following that the number of punchs used till now is even and so it's now the first player's turn. Contradiction.☐

MathPanda1
1013 posts

Sep 7, 2016, 9:55 am PM #5

Sorry, but what does it mean to punch all dominoes or raise a corner? Thanks!

MathPanda1
1013 posts

Sep 9, 2016, 6:23 am PM #6

Does anyone have a solution to this problem or an explanation of the solution above? Thank you so much for all your help!

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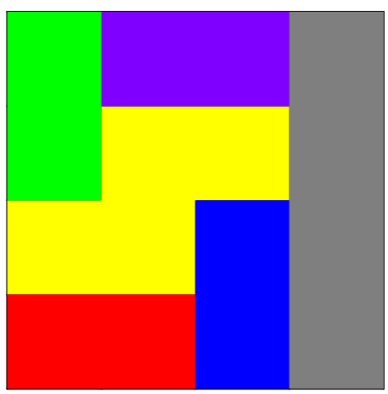
shinichiman
2655 posts

Today at 7:11 AM • 2 PM #7

We will prove for $n \times n$ chess board for sufficient large $2 \mid n$ that the second person has a winning strategy.

First, consider a tile X in $2n \times 2m$ (in here, we refer to a tile if there is no cut-edge in that tile), each cell in tile X can glue/connect to some different cells in a different tile, the sum of connection of all the cells in tile X is called *connected number*. We call a tile *odd tile* if it has odd connected number, otherwise *even tile*. For example, in the square 4×4 below: the red tile is *odd tile* because it has connected number of 3, yellow tile is *odd tile* with connected number of 7, purple tile is *even tile* with connected number of 4.



We start with 1×2 dominos. It not hard to observe that odd tile 1×2 can only appear at the border of the board (for example red tile, green, blue), all tiles not at the border has connected number of 6, so it's all *even tiles*. Not all 1×2 tiles at the border are odd tiles (e.g. purple tiles), we notice that odd tiles can be either at the conner of the board (green, red tiles) or connect to the border with its shorter edge (blue tile). Hence, it not hard to see that there are even number of odd tiles on the chess board (because the length of the chessboard are even).