

Junior Balkan MO 2012

- [1] Let a, b, c be positive real numbers such that $a + b + c = 1$. Prove that

$$\frac{a}{b} + \frac{a}{c} + \frac{c}{b} + \frac{c}{a} + \frac{b}{c} + \frac{b}{a} + 6 \geq 2\sqrt{2} \left(\sqrt{\frac{1-a}{a}} + \sqrt{\frac{1-b}{b}} + \sqrt{\frac{1-c}{c}} \right).$$

When does equality hold?

- [2] Let the circles k_1 and k_2 intersect at two points A and B , and let t be a common tangent of k_1 and k_2 that touches k_1 and k_2 at M and N respectively. If $t \perp AM$ and $MN = 2AM$, evaluate the angle NMB .
- [3] On a board there are n nails, each two connected by a rope. Each rope is colored in one of n given distinct colors. For each three distinct colors, there exist three nails connected with ropes of these three colors. a) Can n be 6 ? b) Can n be 7 ?
- [4] Find all positive integers x, y, z and t such that $2^x 3^y + 5^z = 7^t$.