

Junior Balkan MO 2005

Veria, Greece

- 1 Find all positive integers x, y satisfying the equation

$$9(x^2 + y^2 + 1) + 2(3xy + 2) = 2005.$$

- 2 Let ABC be an acute-angled triangle inscribed in a circle k . It is given that the tangent from A to the circle meets the line BC at point P . Let M be the midpoint of the line segment AP and R be the second intersection point of the circle k with the line BM . The line PR meets again the circle k at point S different from R .

Prove that the lines AP and CS are parallel.

- 3 Prove that there exist

(a) 5 points in the plane so that among all the triangles with vertices among these points there are 8 right-angled ones;

(b) 64 points in the plane so that among all the triangles with vertices among these points there are at least 2005 right-angled ones.

- 4 Find all 3-digit positive integers \overline{abc} such that

$$\overline{abc} = abc(a + b + c),$$

where \overline{abc} is the decimal representation of the number.