Winter Camp 2008 Mock Olympiad

Monday, January 7, 2008

1. Find all triples of positive integers x, y, z satisfying

$$1 + 2^x 3^y = z^2.$$

2. For positive real numbers a, b, c such that $abc \leq 1$, prove that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq a + b + c.$$

3. In acute triangle ABC, $\angle A < 45^\circ$. Point D lies in the interior of triangle ABC such that $BD = CD$ and $\angle BDC = 4\angle A$. Point E is the reflection of C across line AB, and point F is the reflection of B across line AC. Prove that $AD \perp EF$.

4. Let m, n be two positive integers with $m \geq n$. Prove that

$$\sum_{k=0}^{n} (-1)^k \binom{m-k}{n} \binom{n}{k} = 1.$$