IMO 1995

York University, North York, Ontario

Day 1 - *19 July 1995*

$\boxed{1}$ Let A, B, C, D be four distinct points on a line, in that order. The circles with diameters AC and BD intersect at X and Y. The line XY meets BC at Z. Let P be a point on the line XY other than Z. The line CP intersects the circle with diameter AC at C and M, and the line BP intersects the circle with diameter BD at B and N. Prove that the lines AM, DN, XY are concurrent.

$\boxed{2}$ Let a, b, c be positive real numbers such that $abc = 1$. Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

$\boxed{3}$ Determine all integers $n > 3$ for which there exist n points $A_1, \cdots, A_n$ in the plane, no three collinear, and real numbers $r_1, \cdots, r_n$ such that for $1 \leq i < j < k \leq n$, the area of $\triangle A_i A_j A_k$ is $r_i + r_j + r_k$.

Day 2 - *20 July 1995*

4 Find the maximum value of x_0 for which there exists a sequence $x_0, x_1 \cdots, x_{1995}$ of positive reals with $x_0 = x_{1995}$, such that

$$x_{i-1} + \frac{2}{x_{i-1}} = 2x_i + \frac{1}{x_i},$$

for all $i = 1, \cdots, 1995$.

5 Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$ and $DE = EF = FA$, such that $\angle BCD = \angle EFA = \frac{\pi}{3}$. Suppose G and H are points in the interior of the hexagon such that $\angle AGB = \angle DHE = \frac{2\pi}{3}$. Prove that $AG + GB + GH + DH + HE \geq CF$.

6 Let p be an odd prime number. How many p-element subsets A of $\{1, 2, \ldots, 2p\}$ are there, the sum of whose elements is divisible by p?

