

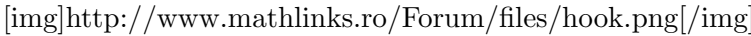
IMO 2004

Athens, Greece

Day 1 - 12 July 2004

- [1] 1. Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N respectively. Denote by O the midpoint of the side BC . The bisectors of the angles $\angle BAC$ and $\angle MON$ intersect at R . Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the side BC .
- [2] 2. Find all polynomials f with real coefficients such that for all reals a, b, c such that $ab+bc+ca = 0$ we have the following relations

$$f(a-b) + f(b-c) + f(c-a) = 2f(a+b+c).$$

- [3] 3. Define a "hook" to be a figure made up of six unit squares as shown below in the picture, or any of the figures obtained by applying rotations and reflections to this figure.  Determine all $m \times n$ rectangles that can be covered without gaps and without overlaps with hooks such that
- the rectangle is covered without gaps and without overlaps
 - no part of a hook covers area outside the rectangle.

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Day 2 - 13 July 2004

- [4] Let $n \geq 3$ be an integer. Let t_1, t_2, \dots, t_n be positive real numbers such that
- $$n^2 + 1 > (t_1 + t_2 + \dots + t_n) \left(\frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_n} \right).$$
- Show that t_i, t_j, t_k are side lengths of a triangle for all i, j, k with $1 \leq i < j < k \leq n$.
- [5] In a convex quadrilateral $ABCD$, the diagonal BD bisects neither the angle ABC nor the angle CDA . The point P lies inside $ABCD$ and satisfies
- $$\angle PBC = \angle DBA \quad \text{and} \quad \angle PDC = \angle BDA.$$
- Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$.
- [6] We call a positive integer *alternating* if every two consecutive digits in its decimal representation are of different parity.
- Find all positive integers n such that n has a multiple which is alternating.