All-Russian Olympiad 2016

Grade 11 Day 1

1 There are 30 teams in NBA and every team play 82 games in the year. Bosses of NBA want to divide all teams on Western and Eastern Conferences (not necessary equally), such that number of games between teams from different conferences is half of number of all games. Can they do it?

2 In the space given three segments A_1A_2, B_1B_2 and C_1C_2, do not lie in one plane and intersect at a point P. Let O_{ijk} be center of sphere that passes through the points A_i, B_j, C_k and P. Prove that $O_{111}O_{222}, O_{112}O_{221}, O_{121}O_{212}$ and$O_{211}O_{122}$ intersect at one point. (P.Kozhevnikov)

3 We have sheet of paper, divided on 100×100 unit squares. In some squares we put rightangled isosceles triangles with leg =1 (Every triangle lies in one unit square and is half of this square). Every unit grid segment(boundary too) is under one leg of triangle. Find maximal number of unit squares, that don't contains triangles.

4 There is three-dimensional space. For every integer n we build planes $x \pm y \pm z = n$. All space is divided on octahedrons and tetrahedrons.
Point (x_0, y_0, z_0) has rational coordinates but not lies on any plane. Prove, that there is such natural k , that point (kx_0, ky_0, kz_0) lies strictly inside the octahedron of partition.

Grade 11 Day 2

5 Let $n \in \mathbb{N}$. $k_0, k_1, ..., k_{2n}$ - nonzero integers, and $k_0 + ... + k_{2n} \neq 0$.

Can we always find such permutation $(a_0, ...a_{2n})$ of $(k_0, k_1, ..., k_{2n})$, that equation $a_{2n}x^{2n} + a_{2n-1}x^{2n-1} + ... + a_0 = 0$ has not integer roots?

6 There are $n > 1$ cities in the country, some pairs of cities linked two-way through straight flight. For every pair of cities there is exactly one aviaroute (can have interchanges).
Major of every city X counted amount of such numberings of all cities from 1 to n , such that on every aviaroute with the beginning in X, numbers of cities are in ascending order. Every major, except one, noticed that results of counting are multiple of 2016.
Prove, that result of last major is multiple of 2016 too.

7 All russian olympiad 2016,Day 2 ,grade 9,P8 :
Let a, b, c, d be are positive numbers such that $a+b+c+d=3$ .Prove that

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{d^2}\le\frac{1}{a^2b^2c^2d^2}$$

All russian olympiad 2016,Day 2,grade 11,P7 :
Let a, b, c, d be are positive numbers such that $a+b+c+d=3$ .Prove that

$$\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}+\frac{1}{d^3}\le\frac{1}{a^3b^3c^3d^3}$$

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8 Medians AM_A, BM_B, CM_C of triangle ABC intersect at M.Let Ω_A be circumcircle of triangle passes through midpoint of AM and M_B, M_C.Define Ω_B and Ω_C analogusly.Prove that Ω_A, Ω_B and Ω_C intersect at one point.(A.Yakubov)