High School Olympiads

Product of two consecutive integers

number theory ✏

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Source: All Russian MO 2015, grade 10, problem 1

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silouan
3804 posts

Aug 7, 2015, 5:12 pm

👁 ✏PM #1

We say that a positive integer is an *almost square*, if it is equal to the product of two consecutive positive integers. Prove that every almost square can be expressed as a quotient of two almost squares.
V. Senderov

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huricane
610 posts

Aug 7, 2015, 5:23 pm

👁 ✏PM #2

Note that
$$n(n + 1) \cdot (n + 1)(n + 2) = n(n + 2) \cdot (n + 1)^2 = [(n + 1)^2 - 1] \cdot (n + 1)^2,$$so
$$n(n + 1) = \frac{[(n + 1)^2 - 1] \cdot (n + 1)^2}{(n + 1)(n + 2)}.$$Thus every almost square can be expressed as a quotients as two almost squares.

K.N
466 posts

Jun 18, 2016, 12:30 pm

👁 ✏PM #4

> ❝ *silouan wrote:*
>
> We say that a positive integer is an *almost square*, if it is equal to the product of two consecutive positive integers. Prove that every almost square can be expressed as a quotient of two almost squares.
> V. Senderov

What an easy problem for ARMO
$$a(a + 1) = \frac{(a - 1)(a + 1)a^2}{(a - 1)a}$$

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