Canada National Olympiad 2003

1 Consider a standard twelve-hour clock whose hour and minute hands move continuously. Let m be an integer, with $1 \leq m \leq 720$. At precisely m minutes after 12:00, the angle made by the hour hand and minute hand is exactly 1°. Determine all possible values of m.

2 Find the last three digits of the number $2003^{2002^{2001}}$.

3 Find all real positive solutions (if any) to

$$x^3 + y^3 + z^3 = x + y + z, \text{ and}$$
$$x^2 + y^2 + z^2 = xyz.$$

4 Prove that when three circles share the same chord AB, every line through A different from AB determines the same ratio $XY : YZ$, where X is an arbitrary point different from B on the rst circle while Y and Z are the points where AX intersects the other two circles (labeled so that Y is between X and Z).

5 Let S be a set of n points in the plane such that any two points of S are at least 1 unit apart.
Prove there is a subset T of S with at least $\frac{n}{7}$ points such that any two points of T are at least $\sqrt{3}$ units apart.