IMO 1981

Day 1

1 Consider a variable point P inside a given triangle ABC. Let D, E, F be the feet of the perpendiculars from the point P to the lines BC, CA, AB, respectively. Find all points P which minimize the sum

$$\frac{BC}{PD} + \frac{CA}{PE} + \frac{AB}{PF}.$$

2 Take r such that $1 \le r \le n$, and consider all subsets of r elements of the set $\{1, 2, \ldots, n\}$. Each subset has a smallest element. Let $F(n, r)$ be the arithmetic mean of these smallest elements. Prove that:

$$F(n, r) = \frac{n+1}{r+1}.$$

3 Determine the maximum value of m^2+n^2, where m and n are integers in the range $1, 2, \ldots, 1981$ satisfying $(n^2 - mn - m^2)^2 = 1$.

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Day 2

$\boxed{1}$ **a.)** For which $n > 2$ is there a set of n consecutive positive integers such that the largest number in the set is a divisor of the least common multiple of the remaining $n-1$ numbers?

b.) For which $n > 2$ is there exactly one set having this property?

$\boxed{2}$ Three circles of equal radius have a common point O and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the incenter and the circumcenter of the triangle are collinear with the point O.

$\boxed{3}$ The function $f(x,y)$ satisfies: $f(0,y) = y+1$, $f(x+1,0) = f(x,1)$, $f(x+1,y+1) = f(x, f(x+1,y))$ for all non-negative integers x, y. Find $f(4,1981)$.