Tournament Of Towns 1998

1 A $20 \times 20 \times 20$ block is cut up into 8000 non-overlapping unit cubes and a number is assigned to each. It is known that in each column of 20 cubes parallel to any edge of the block, the sum of their numbers is equal to 1. The number assigned to one of the unit cubes is 10. Three $1 \times 20 \times 20$ slices parallel to the faces of the block contain this unit cube. Find the sume of all numbers of the cubes outside these slices.

2 The units-digit of the square of an integer is 9 and the tens-digit of this square is 0. Prove that the hundreds-digit is even.

3 In a triangle ABC the points A', B' and C' lie on the sides BC, CA and AB, respectively. It is known that $\angle AC'B' = \angle B'A'C$, $\angle CB'A' = \angle A'C'B$ and $\angle BA'C' = \angle C'B'A$. Prove that A', B' and C' are the midpoints of the corresponding sides.

4 Twelve candidates for mayor participate in a TV talk show. At some point a candidate said: "One lie has been told." Another said: "Now two lies have been told". "Now three lies," said a third. This continued until the twelfth said: "Now twelve lies have been told". At this point the moderator ended the discussion. It turned out that at least one of the candidates correctly stated the number of lies told before he made the claim. How many lies were actually told by the candidates?

5 Let n and m be given positive integers. In one move, a chess piece called an (n, m)-crocodile goes n squares horizontally or vertically and then goes m squares in a perpendicular direction. Prove that the squares of an infinite chessboard can be painted in black and white so that this chess piece always moves from a black square to a white one or vice-versa.