IMO 1975

Day 1

$\boxed{1}$ We consider two sequences of real numbers $x_1 \geq x_2 \geq \ldots \geq x_n$ and $y_1 \geq y_2 \geq \ldots \geq y_n$. Let $z_1, z_2, \ldots, z_n$ be a permutation of the numbers $y_1, y_2, \ldots, y_n$. Prove that $\sum\limits_{i=1}^{n} (x_i - \ y_i)^2 \leq \sum\limits_{i=1}^{n} (x_i - z_i)^2$.

$\boxed{2}$ Let $a_1, \ldots, a_n$ be an infinite sequence of strictly positive integers, so that $a_k < a_{k+1}$ for any k. Prove that there exists an infinity of terms m, which can be written like $a_m = x \cdot a_p + y \cdot a_q$ with x, y strictly positive integers and $p \neq q$.

$\boxed{3}$ In the plane of a triangle ABC, in its exterior, we draw the triangles ABR, BCP, CAQ so that $\angle PBC = \angle CAQ = 45^\circ$, $\angle BCP = \angle QCA = 30^\circ$, $\angle ABR = \angle RAB = 15^\circ$.

Prove that

a.) $\angle QRP = 90^\circ$, and

b.) $QR = RP$.

IMO 1975

Day 2

$\boxed{4}$ When 4444^{4444} is written in decimal notation, the sum of its digits is A. Let B be the sum of the digits of A. Find the sum of the digits of B. (A and B are written in decimal notation.)

$\boxed{5}$ Can there be drawn on a circle of radius 1 a number of 1975 distinct points, so that the distance (measured on the chord) between any two points (from the considered points) is a rational number?

$\boxed{6}$ Determine the polynomials P of two variables so that:

a.) for any real numbers t, x, y we have $P(tx, ty) = t^n P(x, y)$ where n is a positive integer, the same for all t, x, y;

b.) for any real numbers a, b, c we have $P(a + b, c) + P(b + c, a) + P(c + a, b) = 0$;

c.) $P(1, 0) = 1$.