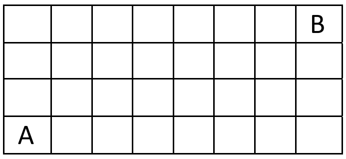
3rd Bangladesh IMO Team Selection Test (TST) - 2007

Day 1: Combinatory and Probability

1. 1st problem was from probability and big enough to type.....

2. A) you want to get from point **A** to Point **B.** If you can go only up ↑ and right → how many ways can you get to B from A



K-1 boxes

n Boxes

B) How many non-negative integer solutions does $x_1 + x_2 + x_3 + \cdots + x_k = n$ have?

3. We have n lamps $L_1, L_2, \ldots, L_n$ in a row. Each lamp is either on or of an n≥2 every second we change the state of the lamps in the rows as follows:

- If a lamp L_i and its neighbors are in the same state, then L_i is switched OFF.
- otherwise L_i is switched ON

Initially all the lamps are off except the leftmost one which is on. Prove the following:

a) There are infinitely many integers n for which all the lamps will eventually be off.

b) There are infinitely many integers n for which the lamps will never be all off.

Day 2: Geometry

1. Two circles with different radii are externally tangent, Lines PAB and PA'B' are common tangents with A and A' on the smaller circle and B and B' on the larger circle. If PA=PB=4. What is the area of the smaller circle?

2. In a cyclic quadrilateral ABCD, the diagonal AC bisects the angle $\angle DAB$. The side Ad is extended beyond D to point E. Show that CE=CA if and only if DE=AB.

3. Let ABCD be a trapezoid with parallel sides AB>CD. Points K and L lie on the line segments AB and CD, respectively, so that AK/KB=DL/LC. Suppose that there are points P and Q on the line segment KL satisfying $\angle APB = \angle BCD \text{ and } \angle CQD = \angle ABC$. Prove that the points P,Q,B,C are cyclic.

4. Let ABCDE be a convex pentagon such that $\alpha = \angle BAC = \angle CAD = \angle DAE \text{ and } \beta = \angle ABC = \angle ACD = \angle ADE$ .The diagonals BD and CE meets at P. Prove that AP bisects CD

Day 3: Algebra

1. a) Find the largest possible n for which 5^n divides 2007! b) how many trailing zeroes does 2007! have?

2. let x_1=97 and for n>1, let $x_n = \frac{n}{x_{n-1}}$ what is $x_1 x_2 \dots x_8$?

3: Find all real numbers x,y,z such that the following hold:

$$x^2 - 3y - z = -8$$

$$y^2 - 5z - x = -12$$

$$z^2 - x - y = 6$$

4: For all x,y,z>0 prove that

$$1 + \frac{3}{xy + yz + zx} \geq \frac{6}{x + y + z}$$

5: For all positive real number $a_1, a_2, \dots, a_n$ Prove the inequality:

$$\sum_{1 \leq i < j \leq n} \frac{a_i a_j}{a_i + a_j} \leq \frac{n}{2(a_1 + a_2 + \cdots + a_n)} \sum_{1 \leq i < j \leq n} a_i a_j$$

6: A sequence of real numbers $a_0, a_1, \dots$ is defined by a formula

$a_i = \lfloor a_i \rfloor \times \{a_i\}$ for $i \geq 0$.

here a_0 is any real number, $\lfloor a_i \rfloor$ *denotes the greatest integer not exceding* a_i *.and* $\{a_i\} = a_i - \lfloor a_i \rfloor$

Prove that $a_i = a_{i+2}$ if i is sufficiently large.

7: Let a,b,c be the sides of a triangle . prove that,

$$\frac{\sqrt{b + c - a}}{\sqrt{b} + \sqrt{c} - \sqrt{a}} + \frac{\sqrt{c + a - b}}{\sqrt{c} + \sqrt{a} - \sqrt{b}} + \frac{\sqrt{a + b - c}}{\sqrt{a} + \sqrt{b} - \sqrt{c}} \leq 3$$

Day 4 (Home Work): Number Theory

(***You should not send solution before team selection)

1:*is there any positive integer solutions* (x, y, z); $(2548)^x + (-2005)^y = (-543)^z$

2:*show that the equation has no integer solutions* (x, y) $\frac{x^7 - 1}{x - 1} = y^5 - 1$

- Problem Sets, made by: **Mahbub Majumdar, Coach, Bangladesh IMO Team**