# Art of Problem Solving
2015 Germany Team Selection Test

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**1** Find the least positive integer $n$, such that there is a polynomial

$$P(x) = a_{2n}x^{2n} + a_{2n-1}x^{2n-1} + \cdots + a_1x + a_0$$

with real coefficients that satisfies both of the following properties:
- For $i = 0, 1, \dots, 2n$ it is $2014 \leq a_i \leq 2015$.
- There is a real number $\xi$ with $P(\xi) = 0$.

**2** A positive integer $n$ is called *naughty* if it can be written in the form $n = a^b + b$ with integers $a, b \geq 2$.
Is there a sequence of 102 consecutive positive integers such that exactly 100 of those numbers are naughty?

**3** Let $ABC$ be an acute triangle with $|AB| \neq |AC|$ and the midpoints of segments $[AB]$ and $[AC]$ be $D$ resp. $E$. The circumcircles of the triangles $BCD$ and $BCE$ intersect the circumcircle of triangle $ADE$ in $P$ resp. $Q$ with $P \neq D$ and $Q \neq E$.
Prove $|AP| = |AQ|$.

[i](Notation: $|\cdot|$ denotes the length of a segment and $[\cdot]$ denotes the line segment.)[/i]

**4** Determine all pairs $(x, y)$ of positive integers such that

$$\sqrt[3]{7x^2 - 13xy + 7y^2} = |x - y| + 1.$$

*Proposed by Titu Andreescu, USA*

**5** Let $ABC$ be an acute triangle with the circumcircle $k$ and incenter $I$. The perpendicular through $I$ in $CI$ intersects segment $[BC]$ in $U$ and $k$ in $V$. In particular $V$ and $A$ are on different sides of $BC$. The parallel line through $U$ to $AI$ intersects $AV$ in $X$.
Prove: If $XI$ and $AI$ are perpendicular to each other, then $XI$ intersects segment $[AC]$ in its midpoint $M$.

[i](Notation: $[\cdot]$ denotes the line segment.)[/i]

**6** Construct a tetromino by attaching two $2 \times 1$ dominoes along their longer sides such that the midpoint of the longer side of one domino is a corner of the other domino. This construction yields two kinds of tetrominoes with opposite orientations. Let us call them $S$- and $Z$-tetrominoes, respectively.

Assume that a lattice polygon $P$ can be tiled with $S$-tetrominoes. Prove that no matter how we tile $P$ using only $S$- and $Z$-tetrominoes, we always use an even number of $Z$-tetrominoes.

*Proposed by Tamas Fleiner and Peter Pal Pach, Hungary*