India
Regional Mathematical Olympiad
2003

---

1 Let ABC be a triangle in which $AB = AC$ and $\angle CAB = 90°$. Suppose that M and N are points on the hypotenuse BC such that $BM^2 + CN^2 = MN^2$. Prove that $\angle MAN = 45°$.

2 If n is an integer greater than 7, prove that $\binom{n}{7} - \left[\frac{n}{7}\right]$ is divisible by 7.

3 Let a, b, c be three positive real numbers such that $a + b + c = 1$. prove that among the three numbers $a - ab, b - bc, c - ca$ there is one which is at most $\frac{1}{4}$ and there is one which is at least $\frac{2}{9}$.

4 Find the number of ordered triples (x, y, z) of non-negative integers satisfying (i) $x \leq y \leq z$ (ii) $x + y + z \leq 100$.

5 Suppose P is an interior point of a triangle ABC such that the ratios

$$\frac{d(A, BC)}{d(P, BC)}, \frac{d(B, CA)}{d(P, CA)}, \frac{d(C, AB)}{d(P, AB)}$$

are all equal. Find the common value of these ratios. $d(X, YZ)$ represents the perpendicular distance fro X to the line YZ.

6 Find all real numbers a for which the equation $x^2 a - 2x + 1 = 3|x|$ has exactly three distinct real solutions in x.

7 Consider the set $X = \{1, 2 \ldots 10\}$. Find two disjoint nonempty sunsets A and B of X such that

a) $A \cup B = X$;

b) $\prod_{x \in A} x$ is divisible by $\prod_{x \in B} x$, where $\prod_{x \in C} x$ is the product of all numbers in C;

c) $\frac{\prod\limits_{x \in A} x}{\prod\limits_{x \in B} x}$ is as small as possible.