

India
Regional Mathematical Olympiad
1995

- [1] In triangle ABC , K and L are points on the side BC (K being closer to B than L) such that $BC \cdot KL = BK \cdot CL$ and AL bisects $\angle KAC$. Show that $AL \perp AB$.
- [2] Call a positive integer n *good* if there are n integers, positive or negative, and not necessarily distinct, such that their sum and products are both equal to n . Show that the integers of the form $4k + 1$ and $4l$ are good.
- [3] Prove that among any 18 consecutive three digit numbers there is at least one number which is divisible by the sum of its digits.
- [4] Show that the quadratic equation $x^2 + 7x - 14(q^2 + 1) = 0$, where q is an integer, has no integer root.
- [5] Show that for any triangle ABC , the following inequality is true:

$$a^2 + b^2 + c^2 > \sqrt{3} \max\{|a^2 - b^2|, |b^2 - c^2|, |c^2 - a^2|\}.$$

- [6] Let $A_1 A_2 A_3 \dots A_{21}$ be a 21-sided regular polygon inscribed in a circle with centre O . How many triangles $A_i A_j A_k$, $1 \leq i < j < k \leq 21$, contain the centre point O in their interior?
- [7] Show that for any real number x :

$$x^2 \sin x + x \cos x + x^2 + \frac{1}{2} > 0.$$