

India
National Olympiad
2003

- [1] Let P be an interior point of an acute-angled triangle ABC . The line BP meets the line AC at E , and the line CP meets the line AB at F . The lines AP and EF intersect each other at D . Let K be the foot of the perpendicular from the point D to the line BC . Show that the line KD bisects the angle $\angle EKF$.
- [2] Find all primes p, q and even $n > 2$ such that $p^n + p^{n-1} + \dots + 1 = q^2 + q + 1$.
- [3] Show that $8x^4 - 16x^3 + 16x^2 - 8x + k = 0$ has at least one real root for all real k . Find the sum of the non-real roots.
- [4] Find all 7-digit numbers which use only the digits 5 and 7 and are divisible by 35.
- [5] Let a, b, c be the sidelengths and S the area of a triangle ABC . Denote $x = a + \frac{b}{2}$, $y = b + \frac{c}{2}$ and $z = c + \frac{a}{2}$. Prove that there exists a triangle with sidelengths x, y, z , and the area of this triangle is $\geq \frac{9}{4}S$.
- [6] Each lottery ticket has a 9-digit numbers, which uses only the digits 1, 2, 3. Each ticket is colored red, blue or green. If two tickets have numbers which differ in all nine places, then the tickets have different colors. Ticket 122222222 is red, and ticket 222222222 is green. What color is ticket 123123123 ?