India

Regional Mathematical Olympiad

2010

$\boxed{1}$ Let $ABCDEF$ be a convex hexagon in which diagonals AD, BE, CF are concurrent at O. Suppose $[OAF]$ is geometric mean of $[OAB]$ and $[OEF]$ and $[OBC]$ is geometric mean of $[OAB]$ and $[OCD]$. Prove that $[OED]$ is the geometric mean of $[OCD]$ and $[OEF]$. (Here $[XYZ]$ denotes are of $\triangle XYZ$)

$\boxed{2}$ Let $P_1(x) = ax^2 - bx - c$, $P_2(x) = bx^2 - cx - a$, $P_3(x) = cx^2 - ax - b$ be three quadratic polynomials. Suppose there exists a real number α such that $P_1(\alpha) = P_2(\alpha) = P_3(\alpha)$. Prove that $a = b = c$.

$\boxed{3}$ Find the number of 4-digit numbers (in base 10) having non-zero digits and which are divisible by 4 but not by 8.

$\boxed{4}$ Find three distinct positive integers with the least possible sum such that the sum of the reciprocals of any two integers among them is an integral multiple of the reciprocal of the third integer.

$\boxed{5}$ Let ABC be a triangle in which $\angle A = 60^\circ$. Let BE and CF be the bisectors of $\angle B$ and $\angle C$ with E on AC and F on AB. Let M be the reflection of A in line EF. Prove that M lies on BC.

$\boxed{6}$ For each integer $n \geq 1$ define $a_n = \left[\frac{n}{[\sqrt{n}]}\right]$ (where $[x]$ denoted the largest integer not exceeding x, for any real number x). Find the number of all n in the set $\{1, 2, 3, \cdots, 2010\}$ for which $a_n > a_{n+1}$

