India
National Olympiad
1995

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1. In an acute angled triangle ABC, $\angle A = 30^\circ$, H is the orthocenter, and M is the midpoint of BC. On the line HM, take a point T such that $HM = MT$. Show that $AT = 2BC$.

2. Show that there are infintely many pairs (a, b) of relatively prime integers (not necessarily positive) such that both the equations

$$x^2 + ax + b \quad = \quad 0x^2 + 2ax + b$$

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have integer roots.

Show that the number of 3−element subsets $\{a, b, c\}$ of $\{1, 2, 3, \ldots, 63\}$ with $a + b + c < 95$ is less than the number of those with $a + b + c \geq 95$.

Let ABC be a triangle and a circle Γ' be drawn lying outside the triangle, touching its incircle Γ externally, and also the two sides AB and AC. Show that the ratio of the radii of the circles Γ' and Γ is equal to $\tan^2\left(\frac{\pi - A}{4}\right)$.

Let $n \geq 2$. Let $a_1, a_2, a_3, \ldots a_n$ be n real numbers all less than 1 and such that $|a_k - a_{k+1}| < 1$ for $1 \leq k \leq n - 1$. Show that

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_4} + \ldots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1} < 2n - 1.$$

Find all primes p for which the quotient

$$\frac{2^{p-1} - 1}{p}$$

is a square.