Junior Balkan MO 2011

$\boxed{1}$ Let a, b, c be positive real numbers such that $abc = 1$. Prove that:
$\prod(a^5 + a^4 + a^3 + a^2 + a + 1) \geq 8(a^2 + a + 1)(b^2 + b + 1)(c^2 + c + 1)$

$\boxed{2}$ Find all primes p such that there exist positive integers x, y that satisfy $x(y^2 - p) + y(x^2 - p) = 5p$

$\boxed{3}$ Let $n > 3$ be a positive integer. Equilateral triangle ABC is divided into n^2 smaller congruent equilateral triangles (with sides parallel to its sides). Let m be the number of rhombuses that contain two small equilateral triangles and d the number of rhombuses that contain eight small equilateral triangles. Find the difference $m - d$ in terms of n.

$\boxed{4}$ Let $ABCD$ be a convex quadrilateral and points E and F on sides AB, CD such that

$$\frac{AB}{AE} = \frac{CD}{DF} = n$$

If S is the area of $AEFD$ show that $S \leq \frac{AB \cdot CD + n(n-1)AD^2 + n^2 DA \cdot BC}{2n^2}$