Aviaroutes in country

Source: All russian olympiad 2016,Day2,grade 11,P6

RagvaloD
376 posts

May 5, 2016, 3:40 pm PM #1

There are $n>1$ cities in the country, some pairs of cities linked two-way through straight flight. For every pair of cities there is exactly one aviaroute (can have interchanges).
Major of every city X counted amount of such numberings of all cities from 1 to n , such that on every aviaroute with the beginning in X, numbers of cities are in ascending order. Every major, except one, noticed that results of counting are multiple of 2016.
Prove, that result of last major is multiple of 2016 too.

This post has been edited 1 time. Last edited by RagvaloD, May 6, 2016, 2:42 pm

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kreegyt
10 posts

May 6, 2016, 2:29 pm PM #2

I'm sorry but I have some (stupid) questions:
-are all the pairs of cities linked two-way? since you wrote that "some" of them are, and then for "every pair" there's one aviaroute
-what is an interchange
-in which order do you visit the cities since you tlak about ascending order. You start from X, you go to one city Y linked to X by an aviaroute, and then?

I'm really sorry if these questions are stupid, and thanks for the translations.

Misha57
394 posts

May 6, 2016, 2:40 pm • 1 PM #3

It should be flight in first sentence not aviaroute.Flight is an edge in graph.Aviaroute is a path in graph.

RagvaloD
376 posts

May 6, 2016, 2:40 pm • 1 PM #4

Let A,B,C are some cities and (A,B) and (B,C) - two-way flights.
Then if we want fly from A to C we should fly from A to B , then change airliner (it is interchange, maybe I chose wrong word for it) and fly from B to C.

PS. I make some changes in problem, maybe problem became more clear.

This post has been edited 1 time. Last edited by RagvaloD, May 6, 2016, 2:44 pm

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kreegyt
10 posts

May 6, 2016, 2:56 pm PM #5

Thanks a lot for your two answers, it's really clear now!

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aleksam
72 posts

May 6, 2016, 6:20 pm PM #6

I will prove it for prime p instead of 2016, but it can be easily generalized. The solution will be splitted in three parts:two lemmas and their application to this problem. Some notation at the begining: let $f(X)$ be the nubmer of numerations counted by the major of X. Let $s_X(A)$ be the number of cities in the country such that the A is in the route from X to that city(maybe as an endpoint, so for example $s_X(X)=n$...).

Lemma 1: $f(X)=\frac{(n-1)!}{\prod_{v\neq X} s_X(v)}$.

Proof:The proof is inductive. Root the tree in X, and let it's childrens be $A_1,A_2,\cdots,A_n$.
Now, proceed to computing $f(X)=f(A_1)\cdots f(A_n)\frac{n!}{s_X(A_1)!\cdots s_X(A_n)!}$, and after the easy computation, we get the desired result.

Lemma 2:$v_p(n!)>\sum_{v\in G} v_p(s_X(v))$ for $n>p$

Proof: The proof goes again by induction. Base:$n=p$, is obvious. Then, let again X be the root and A's his children. By a well-known result(multinomial coeficents are integers...) $v_p((n-1)!)>v_p(s_X(A_i)!)>$(By summing the inductive hypothesis for all A_i's)$\sum_{v\neq X} v_p(s_X(v))$. Done.

By combining these two lemmas we easily get the required result, because $n>p$ from $p|f(A)$, for $A\neq X$.

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dgrozev
760 posts

May 7, 2016, 1:49 pm PM #7

> aleksam wrote:

> Lemma 1: $f(X)=\dfrac{(n-1)!}{\prod_{v\neq X} s_X(v)}$.
> Proof:The proof is inductive. Root the tree in X, and let it's childrens be $A_1,A_2,\cdots,A_n$. Now, proceed to computing $f(X)=f(A_1)\cdots f(A_n)\dfrac{n!}{s_X(A_1)!\cdots s_X(A_n)!}$, and after the easy computation, we get the desired result.

I think there is something wrong about this claim. For example could you explain why:

$$f(X)=f(A_1)\cdots f(A_n)\frac{n!}{s_X(A_1)!\cdots s_X(A_n)!}$$

It would be true if instead of $f(A_i)$ we put $f'(A_i)$, where $f'(A_i)$ means the number of ways the branch of the tree (a subtree), initiated from A_i and away from X, can be enumerated. As you see, it's something different than $f(A_i)$.

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toto123456...
622 posts

Jun 6, 2016, 9:33 pm PM #8

There is a very easy solution actually. :)
Let's color the cities in two colors: white and black so that no two linked cities have the same color.
Then, we can easily know that the sum of the result of countings for the same colored cities are the same.
Which means that the result of the last city is also a multiple of 2016. :)

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dgrozev
760 posts

Jun 7, 2016, 1:28 am PM #9

Yes, it was the official solution. The problem was proposed by Fedor Petrov.

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toto123456...
622 posts

Jun 7, 2016, 6:14 pm PM #10

I got a strange result while solving this problem. For adjacent two cities u,v and the number of cities connected to u and v are
$x-1,y-1$. (I mean the number of cities that we can travel from u without touching v and the same for u.) Then, I got
$f(u):f(v)=x:y$. :?

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roughlife
7 posts

Aug 12, 2016, 7:34 pm PM #11

> *toto1234567890 wrote:*
> There is a very easy solution actually. :)
> Let's color the cities in two colors: white and black so that no two linked cities have the same color.
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> Which means that the result of the last city is also a multiple of 2016. :)

please let me know how to prove it

This post has been edited 1 time. Last edited by roughlife, Aug 12, 2016, 7:36 pm

Quick Reply