India
National Olympiad
1987

1 Given m and n as relatively prime positive integers greater than one, show that

$$\frac{\log_{10} m}{\log_{10} n}$$

is not a rational number.

2 Determine the largest number in the infinite sequence

$$1, \sqrt[2]{2}, \sqrt[3]{3}, \sqrt[4]{4}, \ldots, \sqrt[n]{n}, \ldots$$

3 Let T be the set of all triplets (a, b, c) of integers such that $1 \leq a < b < c \leq 6$ For each triplet (a, b, c) in T, take number $a \cdot b \cdot c$. Add all these numbers corresponding to all the triplets in T. Prove that the answer is divisible by 7.

4 If x, y, z, and n are natural numbers, and $n \geq z$ then prove that the relation $x^n + y^n = z^n$ does not hold.

5 Find a finite sequence of 16 numbers such that: (a) it reads same from left to right as from right to left. (b) the sum of any 7 consecutive terms is -1, (c) the sum of any 11 consecutive terms is $+1$.

6 Prove that if coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers, then the roots of the equation cannot be rational numbers.

7 Construct the $\triangle ABC$, given h_a, h_b (the altitudes from A and B) and m_a, the median from the vertex A.

8 Three congruent circles have a common point O and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the incentre and the circumcentre of the triangle and the common point O are collinear.

9 Prove that any triangle having two equal internal angle bisectors (each measured from a vertex to the opposite side) is isosceles.