Pan African 2001

Day 1

$\boxed{1}$ Find all positive integers n such that:

$$\frac{n^3+3}{n^2+7}$$

is a positive integer.

$\boxed{2}$ Let n be a positive integer. A child builds a wall along a line with n identical cubes. He lays the first cube on the line and at each subsequent step, he lays the next cube either on the ground or on the top of another cube, so that it has a common face with the previous one. How many such distinct walls exist?

$\boxed{3}$ Let ABC be an equilateral triangle and let P_0 be a point outside this triangle, such that $\triangle AP_0C$ is an isoscele triangle with a right angle at P_0. A grasshopper starts from P_0 and turns around the triangle as follows. From P_0 the grasshopper jumps to P_1, which is the symmetric point of P_0 with respect to A. From P_1, the grasshopper jumps to P_2, which is the symmetric point of P_1 with respect to B. Then the grasshopper jumps to P_3 which is the symmetric point of P_2 with respect to C, and so on. Compare the distance P_0P_1 and P_0P_n. $n \in N$.

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Day 2

$\boxed{1}$ Let n be a positive integer, and let $a > 0$ be a real number. Consider the equation:

$$\sum_{i=1}^{n}(x_i^2 + (a - x_i)^2) = na^2$$

How many solutions $(x_1, x_2 \cdots, x_n)$ does this equation have, such that:

$$0 \leq x_i \leq a, i \in N^+$$

$\boxed{2}$ Find the value of the sum:

$$\sum_{i=1}^{2001}[\sqrt{i}]$$

where $[x]$ denotes the greatest integer which does not exceed x.

$\boxed{3}$ Let S_1 be a semicircle with centre O and diameter AB.A circle C_1 with centre P is drawn, tangent to S_1, and tangent to AB at O. A semicircle S_2 is drawn, with centre Q on AB, tangent to S_1 and to C_1. A circle C_2 with centre R is drawn, internally tangent to S_1 and externally tangent to S_2 and C_1. Prove that $OPRQ$ is a rectangle.