India
ISI Entrance Examination
2012

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1 **i)** If X, Y, Z be the angles of a triangle then show that

$$\tan\frac{X}{2}\tan\frac{Y}{2}+\tan\frac{Y}{2}\tan\frac{Z}{2}+\tan\frac{Z}{2}\tan\frac{X}{2}=1$$

ii) Prove using **(i)** or otherwise that

$$\tan\frac{X}{2}\tan\frac{Y}{2}\tan\frac{Z}{2}\leq\frac{1}{3\sqrt{3}}$$

2 Consider the following function

$$g(x)=(\alpha+|x|)^2e^{(5-|x|)^2}$$

i) Find all the values of α for which $g(x)$ is continuous for all $x\in\mathbb{R}$

ii) Find all the values of α for which $g(x)$ is differentiable for all $x\in\mathbb{R}$.

3 Consider the numbers arranged in the following way:

1 3 6 10 15 21......

2 5 9 14 20 ....

4 8 13 19 ....

7 12 18 ....

11 17 ......

16 .....

......

Find the row number and the column number in which the the number 20096 occurs.

4 Prove that the polynomial equation $x^8-x^7+x^2-x+15=0$ has no real solution.

5 Let m be a number containing only 0 and 6 as its digits.Show that m can't be a perfect square.

6 **i)** Let $0<a<b$.Prove that amongst all triangles having base a and perimeter $a+b$ the triangle having two sides(other than the base) equal to $\frac{b}{2}$ has the maximum area.

ii) Using i) or otherwise, prove that amongst all quadrilateral having give perimeter the square has the maximum area.

7 Let Γ_1,Γ_2 be two circles centred at the points $(a,0),(b,0);0<a<b$ and having radii a,b respectively.Let Γ be the circle touching Γ_1 externally and Γ_2 internally. Find the locus of the centre of of Γ

$\boxed{8}$ Let $S = \{1, 2, 3, \ldots, n\}$. Consider a function $f \colon S \to S$. A subset D of S is said to be invariant if for all $x \in D$ we have $f(x) \in D$. The empty set and S are also considered as invariant subsets. By $\deg(f)$ we define the number of invariant subsets D of S for the function f.

i) Show that there exists a function $f \colon S \to S$ such that $\deg(f) = 2$.

ii) Show that for every $1 \leq k \leq n$ there exists a function $f \colon S \to S$ such that $\deg(f) = 2^k$.