India
National Olympiad
2004

---

1 $ABCD$ is a convex quadrilateral. K, L, M, N are the midpoints of the sides AB, BC, CD, DA. BD bisects KM at Q. $QA = QB = QC = QD$, and $\frac{LK}{LM} = \frac{CD}{CB}$. Prove that $ABCD$ is a square

2 $p > 3$ is a prime. Find all integers a, b, such that $a^2 + 3ab + 2p(a + b) + p^2 = 0$.

3 If a is a real root of $x^5 - x^3 + x - 2 = 0$, show that $[a^6] = 3$

4 ABC is a triangle, with sides a, b, c , circumradius R, and exradii r_a, r_b, r_cIf $2R \leq r_a$, show that $a > b$, $a > c$, $2R > r_b$, and $2R > r_c$.

5 S is the set of all (a, b, c, d, e, f) where a, b, c, d, e, f are integers such that $a^2+b^2+c^2+d^2+e^2 = f^2$. Find the largest k which divides abcdef for all members of S.

6 Show that the number of 5-tuples (a, b, c, d, e) such that $abcde = 5(bcde + acde + abde + abce + abcd)$ is odd