India
Regional Mathematical Olympiad
2000

$\boxed{1}$ Let AC be a line segment in the plane and B a points between A and C. Construct isosceles triangles PAB and QAC on one side of the segment AC such that $\angle APB = \angle BQC = 120^\circ$ and an isosceles triangle RAC on the other side of AC such that $\angle ARC = 120^\circ$. Show that PQR is an equilateral triangle.

$\boxed{2}$ Solve the equation $y^3 = x^3 + 8x^2 - 6x + 8$, for positive integers x and y.

$\boxed{3}$ Suppose $\{x_n\}_{n\geq 1}$ is a sequence of positive real numbers such that $x_1 \geq x_2 \geq x_3 \ldots \geq x_n \ldots$, and for all n

$$\frac{x_1}{1} + \frac{x_4}{2} + \frac{x_9}{3} + \ldots + \frac{x_{n^2}}{n} \leq 1.$$

Show that for all k

$$\frac{x_1}{1} + \frac{x_2}{2} + \ldots + \frac{x_k}{k} \leq 3.$$

$\boxed{4}$ All the 7 digit numbers containing each of the digits $1, 2, 3, 4, 5, 6, 7$ exactly once , and not divisible by 5 are arranged in increasing order. Find the $200th$ number in the list.

$\boxed{5}$ The internal bisector of angle A in a triangle ABC with $AC > AB$ meets the circumcircle Γ of the triangle in D. Join$D$ to the center O of the circle Γ and suppose that DO meets AC in E, possibly when extended. Given that BE is perpendicular to AD, show that AO is parallel to BD.

$\boxed{6}$ (i) Consider two positive integers a and b which are such that $a^a b^b$ is divisible by 2000. What is the least possible value of ab? (ii) Consider two positive integers a and b which are such that $a^b b^a$ is divisible by 2000. What is the least possible value of ab?

$\boxed{7}$ Find all real values of a such that $x^4 - 2ax^2 + x + a^2 - a = 0$ has all its roots real.

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