

# LUIS GONZÁLEZ POSTS COLLECTION



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## High School Olympiads

Perpendicular lines X[Reply](#)

Source: Own

**buratinogigle**

#1 May 25, 2016, 8:50 am • 2

Let  $ABC$  be a triangle and  $P$  is any point. Let  $K, L$  be the orthocenters of triangles  $PAB, PAC$ .  $PA$  cuts  $(PBC)$  again at  $Q$ .  $H$  is the projection of  $Q$  on  $BC$ . Prove that  $AH \perp KL$ .

**TelvCohl**

#2 May 25, 2016, 10:02 am • 5

Let  $\triangle P_A P_B P_C$  be the antipedal triangle of  $P$  WRT  $\triangle ABC$  and  $H_A, H_P, H_Q$  be the orthocenter of  $\triangle P_A BC, \triangle P_A P_B P_C, \triangle QBC$ , respectively. Clearly,  $\triangle QBC$  and  $\triangle P_A P_B P_C$  are inversely similar, so notice  $QH_Q \parallel P_A H_A$  we get

$$\frac{QH}{QA} = \frac{\text{dist}(Q, BC)}{\text{dist}(P_A, P_B P_C)} = \frac{P_A H_A}{P_A H_P}.$$

Combining  $QA \parallel P_A H_P$  and  $QH \parallel P_A H_A$  we get  $\triangle AHQ$  and  $\triangle H_P H_A P_A$  are homothetic  $\implies AH \parallel H_A H_P$ .

Let  $B^*, C^*$  be the isotomic conjugate of  $B, C$  WRT  $(P_C, P_A), (P_A, P_B)$ , respectively. Since  $P_A B^* \parallel P_C B \parallel AK, P_A C^* \parallel P_B C \parallel AL$ , so  $\triangle AKL, \triangle P_A B^* C^*$  are homothetic (also congruent)  $\implies KL \parallel B^* C^*$ , hence  $KL$  is parallel to the Newton line of the complete quadrilateral  $Q$  formed by  $\triangle P_A P_B P_C, BC$ . Notice  $H_A H_P$  is the Steiner line of  $Q$  we get  $AH \perp KL$ .

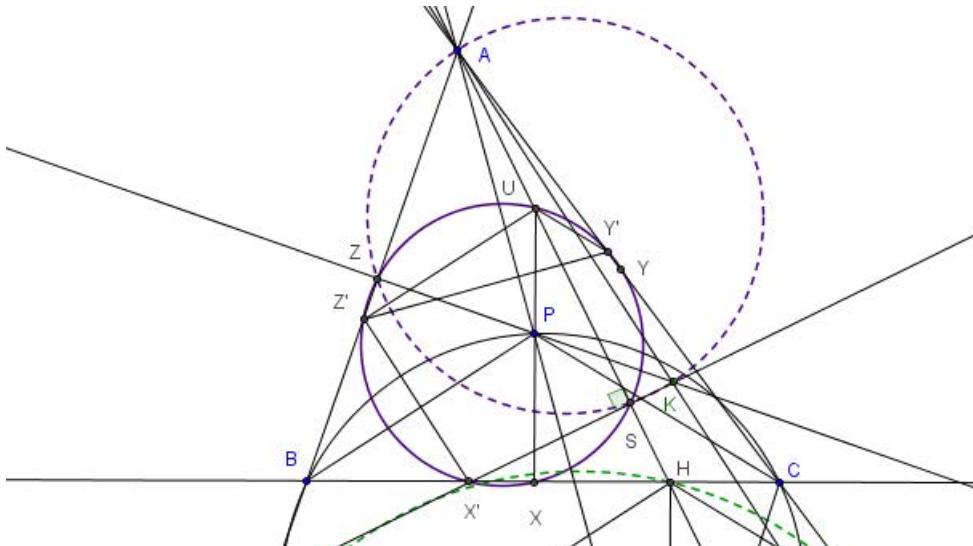
**Luis González**

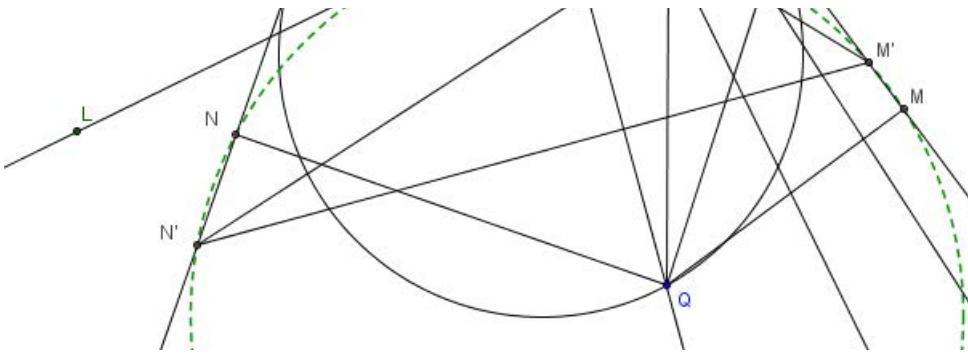
#3 Thursday at 9:14 AM • 6

Let  $\triangle XYZ$  and  $\triangle HMN$  be the pedal triangles of  $P$  and  $Q$  WRT  $\triangle ABC$ .  $\odot(XYZ)$  cuts  $BC, CA, AB$  again at  $X', Y', Z'$ , and  $\odot(HMN)$  cuts  $CA, AB$  again at  $M', N'$ .  $PX$  cuts  $\odot(XYZ)$  again at  $U$ . Clearly  $Y'Z' \parallel M'N'$  and  $\angle UZ'Y' = 90^\circ - \angle X'Z'Y' = \angle APB - 90^\circ = 90^\circ - \angle BCQ = \angle HQC = \angle HN'M' \implies UZ' \parallel HN'$  and similarly  $UY' \parallel HM' \implies \triangle UY'Z'$  and  $\triangle HM'N'$  are homothetic with center  $A \implies A, U, H$  are collinear.

Let  $AH$  cut  $\odot(XYZ)$  again at  $S$  and  $K^* \equiv PZ \cap SX'$ . Since  $AZSK^*$  is cyclic on account of the right angles at  $S, Z$ , then by Reim's theorem  $AK^* \parallel X'Z' \implies AK^* \perp PB \implies K \equiv K^*$ . Analogously,  $L \in SX' \implies ASH \perp KL$ .

Attachments:



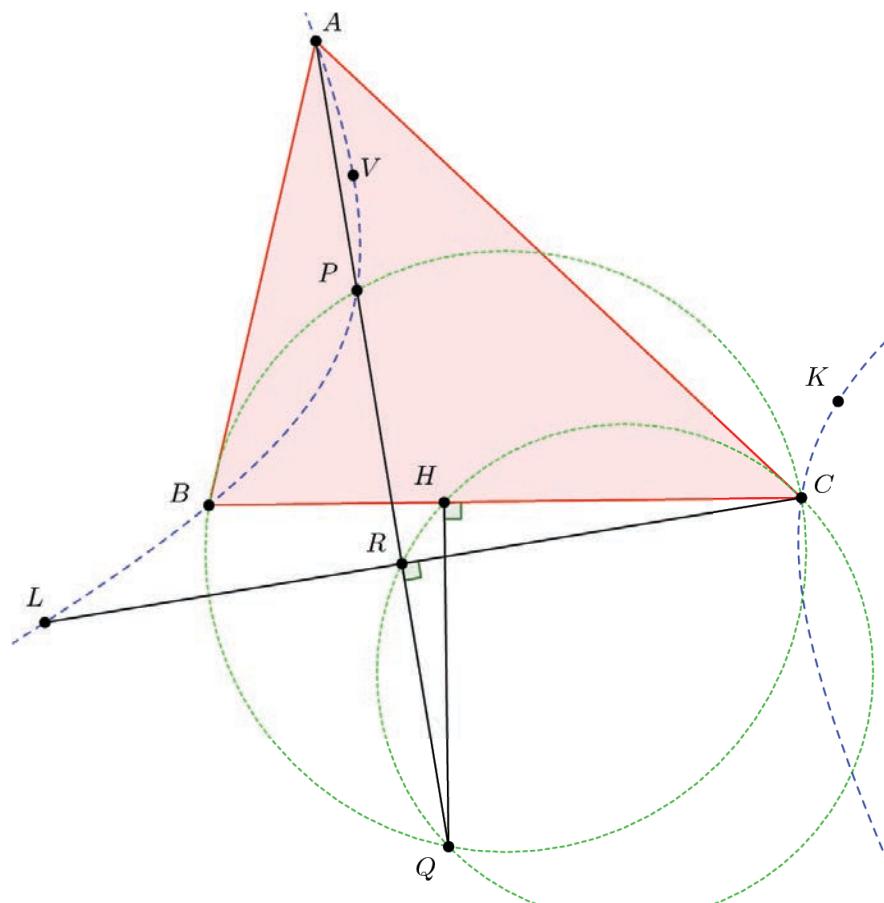


**Dukejukem**

#4 Thursday at 3:28 PM • 4

Let  $R$  be the projection of  $C$  onto  $AP$  and note that  $R \in CL$ . Let  $V$  be the orthocenter of  $\triangle AKL$  and denote  $H^* \equiv AV \cap BC$ .

From Reim's Theorem for  $\odot(BCPQ), \odot(CHQR)$  cut by  $BH, PR$ , we get  $BP \parallel HR$ .



On the other hand, if  $\mathcal{H}$  is the rectangular circumhyperbola of  $\triangle ABC$  passing through  $P$ , then then  $K, L \in \mathcal{H}$  on account of rectangularity, which in turn gives  $V \in \mathcal{H}$ . Then since  $(VL \parallel BP) \perp AK$ , Pascal's Theorem for  $AVLCBP$  gives  $BP \parallel H^*R$ . Hence,  $H^* \equiv H$ , so  $AH \perp KL$ .

This post has been edited 3 times. Last edited by Dukejukem Thursday at 3:51 PM  
Reason: Fix diagram



**jayme**

#5 Yesterday at 4:43 PM

Dear Mathlinkers,  
just to say that another synthetic proof is possible... next on my site...

Sincerely  
Jean-Louis

 Quick Reply

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## Concurrent lines with pedal circle



Reply



Source: Own



buratinogigle

#1 May 24, 2016, 1:20 am • 1

Let  $ABC$  be a triangle and  $DEF$  is pedal triangle of a point  $P$ . Circle diameter  $AD$  cuts  $(DEF)$  again at  $X$ . Similarly, we have  $Y, Z$ . Prove that  $AX, BY, CZ$  are concurrent.



Luis González

#2 May 24, 2016, 5:03 am • 2

If  $D', E', F'$  are the antipodes of  $D, E, F$  on  $\odot(DEF)$ , then obviously  $D' \in AX, F' \in BY$  and  $E' \in CZ$ . Now the configuration is a particular case of [Concurrent 14](#) (Problem b).



uraharakisuke\_hsgs

#3 May 24, 2016, 9:56 pm

### My soution

**Lemma** For any  $\triangle ABC$  with arbitrary point  $P$  in the triangle

Let  $D, E, F$  is projection of  $P$  on  $BC, CA, AB$  respectively

Let  $\omega$  is circumcircle of  $\triangle DEF$  and  $PD, PE, PF$  intersect  $\omega$  again at  $X, Y, Z$  respectively

Then  $AX, BY, CZ$  concurrent

**Prove** : this lemma is proved [here](#) at [post 2](#)

**Back to main problem** : Let  $(O)$  be the circumcenter of  $\triangle DEF$

$D'$  reflects with  $D$  about  $O$  then  $\angle DXD' = 90^\circ$ . It follows that  $A, D'$  and  $X$  are collinear

Similary with  $E'$  and  $F'$

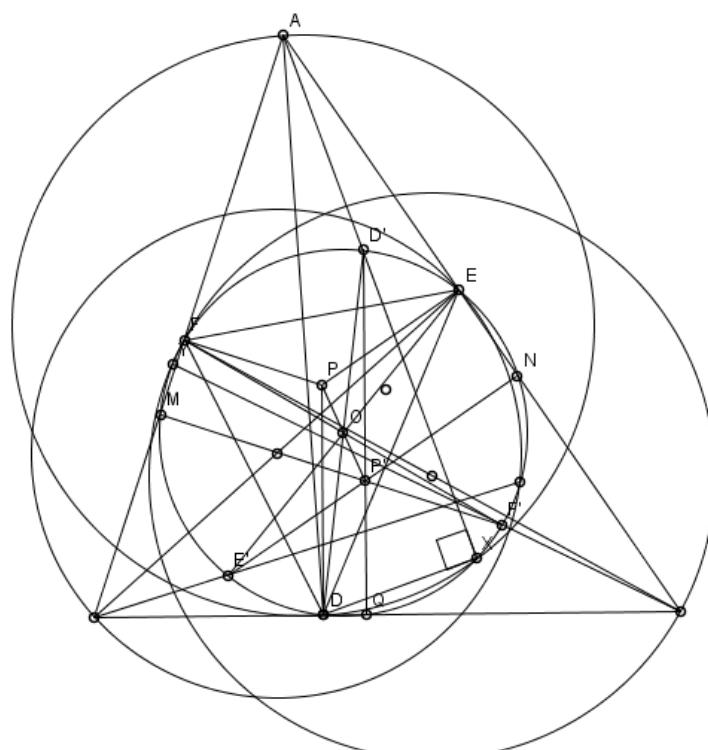
$Q, N, M$  are the projections of  $D', E', F'$  on  $BC, CA, AB$  respectively so  $Q, N, M$  lies on  $(O)$

$P'$  reflects with  $P$  about  $O$ , Obviously  $P' \in D'Q$

$\Rightarrow D'Q, E'N, F'M$  are concurrent at  $P'$ .

By applying the lemma we have  $AX, BY, CZ$  are concurrent

Attachments:



Quick Reply

## High School Olympiads

Concurrent 14 

 Reply



**buratinogiggle**

#1 Sep 16, 2011, 4:06 pm • 3 

Let  $ABC$  be a triangle and a point  $P$ .  $A_1B_1C_1$  is pedal triangle of  $P$ .  $P^*$  is isogonal conjugate of  $P$ .  $A_2B_2C_2$  is pedal triangle of  $P^*$ .  $Q$  is a point on line  $PP^*$ .  $A_2Q, B_2Q, C_2Q$  cut circumcircle  $(A_1B_1C_1)$  again at  $A_3, B_3, C_3$ , respectively.

a) Prove that  $A_1A_3, B_1B_3, C_1C_3$  are concurrent on line  $PP^*$ .

b) Prove that  $AA_3, BB_3, CC_3$  are concurrent.

When  $P, Q \equiv H$  we have problem in the post [concurrent lines formed by incenter and excenters](#).



**Luis González**

#2 Sep 17, 2011, 12:36 am • 2 

a)  $K$  is the center of the pedal circle of  $P, P^*$  (midpoint of  $PP^*$ ) and  $PA_1, PB_1$  cut  $(K)$  again at  $U, V$ . Since  $\angle UA_1A_2$  and  $\angle VB_1B_2$  are right, it follows that  $K \equiv UA_2 \cap VB_2$ . By Pascal theorem for the cyclic hexagon  $A_1UA_2B_1VB_2$ , the intersections  $P \equiv A_1U \cap B_1V, K \equiv UA_2 \cap VB_2$  and  $R \equiv A_2B_1 \cap B_2A_1$  are collinear ( $\star$ ). By Pascal theorem for the cyclic hexagon  $A_1A_3A_2B_1B_3B_2$ , the intersections  $S \equiv A_1A_3 \cap B_1B_3, Q \equiv A_3A_2 \cap B_3B_2$  and  $R \equiv A_2B_1 \cap B_2A_1$  are collinear. Together with ( $\star$ ), we conclude that  $A_1A_3, B_1B_3$  and  $PP^*$  concur at  $S$ . By similar reasoning, we get that  $C_1C_3$  goes through  $S$ .

b) Using the result of the problem [Collinearity and points on the circumcenter](#) (post #2) for  $\triangle A_3B_3C_3$  and the circumcevian triangles  $\triangle A_1B_1C_1, \triangle A_2B_2C_2$  of  $S, Q$  WRT  $\triangle A_3B_3C_3$ , we deduce that the intersections  $A_4 \equiv BC \cap B_3C_3, B_4 \equiv CA \cap C_3A_3$  and  $C_4 \equiv AB \cap A_3B_3$  are collinear, i.e.  $\triangle ABC$  and  $\triangle A_3B_3C_3$  are perspective through  $A_4B_4C_4 \implies AA_3, BB_3, CC_3$  concur.



**buratinogiggle**

#3 Sep 17, 2011, 4:18 pm • 1 

Thank you, very nice proof with Pascal theorem. With your proof, we can prove the following problem

Let  $ABC$  be a triangle, circumcircle  $(O)$ .  $P, Q$  are two arbitrary points.  $PA, PB, PC$  cut  $(O)$  again at  $A_1, B_1, C_1$ , resp.  $QA, QB, QC$  cut  $(O)$  again at  $A_2, B_2, C_3$ , resp.  $R$  is a point on  $PQ$ .  $A_2R, B_2R, C_2R$  cuts  $(O)$  again at  $A_3, B_3, C_3$ . Prove that  $A_1A_3, B_1B_3, C_1C_3$  concur at a point on  $PQ$ .



**drmzjoseph**

#4 Jun 15, 2015, 8:23 am

a) Let  $\gamma$  be the circle passes through  $A_1, B_1, C_1, A_2, B_2, C_2$ , if  $A_1P$  cut again to  $\gamma$  at  $A^*$  then  $A_2A^*$  cut to  $PP^*$  in the center  $L$  of  $\gamma$ . Analogously is define  $B^*$  and  $C^*$ . The composition of involutions that fixed  $\gamma$  with poles  $P, L, Q$ , so changes  $A_1$  with  $A_3$ , further interchanges the antipode points of cut  $PP^*$  with  $\gamma$ . hence is a involution too, so  $A_1A_3, B_1B_3, C_1C_3, PP^*$  are concurrent.

b) **General problem:**

Let  $(A_1, A_2), (B_1, B_2), (C_1, C_2)$  three pairs of points at a conic  $\gamma$ , and on the lines of sides  $BC, CA, AB$  of  $\triangle ABC$ , if  $A_3, B_3, C_3$  lies on  $\gamma$ , suppose that  $\triangle A_1B_1C_1$  is perspective to  $\triangle A_3B_3C_3$  and  $\triangle A_2B_2C_2$  is perspective to  $\triangle A_3B_3C_3$ . Prove that  $\triangle A_3B_3C_3$  is perspective to  $\triangle ABC$

**Proof**

Is sufficient works with  $\gamma$  a circle, because exist a homology between all conics.

Is sufficient prove that  $\prod \frac{\sin \angle BAA_3}{\sin \angle CAA_3} = 1 \iff \prod \frac{\sin \angle C_1B_1A_3}{\sin \angle B_1C_1A_3} \cdot \prod \frac{\sin \angle AC_1A_3}{\sin \angle AB_1A_3} = 1$  this is ceva in the triangles  $ABC$  and  $AB_1C_1$ .

$$\prod \frac{\sin \angle C_1B_1A_3}{\sin \angle B_1C_1A_3} = \prod \frac{\sin \angle C_1A_1A_3}{\sin \angle B_1A_1A_3} = 1$$
$$\prod \frac{\sin \angle AC_1A_3}{\sin \angle AB_1A_3} = \prod \frac{\sin \angle C_2A_2A_3}{\sin \angle B_2A_2A_3} = 1 \text{ is sufficient.}$$

This post has been edited 1 time. Last edited by dmzjoseph, Jun 15, 2015, 9:37 am



Luis González

#5 Jun 15, 2015, 11:32 am

99

1

" dmzjoseph wrote:

b) **General problem:**

Let  $(A_1, A_2), (B_1, B_2), (C_1, C_2)$  three pairs of points at a conic  $\gamma$ , and on the lines of sides  $BC, CA, AB$  of  $\triangle ABC$ , if  $A_3, B_3, C_3$  lies on  $\gamma$ , suppose that  $\triangle A_1B_1C_1$  is perspective to  $\triangle A_3B_3C_3$  and  $\triangle A_2B_2C_2$  is perspective to  $\triangle A_3B_3C_3$ . Prove that  $\triangle A_3B_3C_3$  is perspective to  $\triangle ABC$

The proof I gave to b) in my previous post still works for this general problem. Again using the projective version of [Collinearity and points on the circumcenter](#) (post #2) for  $\triangle A_3B_3C_3$ , we get that  $A_1A_2 \cap B_3C_3, B_1B_2 \cap C_3A_3, C_1C_2 \cap A_3B_3$  are collinear, i.e.  $\triangle ABC$  and  $\triangle A_3B_3C_3$  are perspective.

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## High School Olympiads

Circle is tangent to fixed circle X

↳ Reply



Source: Own, Vietnam IMO team training 2016



buratinogiggle

#1 May 20, 2016, 6:00 pm • 3

Let  $ABC$  be the triangle with  $AB = AC$  and circumcenter  $O$ .  $M$  is midpoint of  $BC$  and  $H$  is projection of  $M$  on  $AB$ .  $P$  lies on perpendicular bisector of  $AH$ .  $Q$  lies on  $PO$  such that  $AQ \perp AC$ .  $E, F$  are projections of  $P, Q$  on  $CA, AB$ , reps. Prove that circle  $(AEF)$  is always tangent to fixed circle when  $P$  moves.



Luis González

#2 May 21, 2016, 12:06 am • 3

Let  $N, L$  be the midpoints of  $AC, AB$  and denote  $\omega$  the circle with center  $O$  tangent to  $AC, AB$  at  $N, L$ . It's known that if a variable circle passing through  $A$  tangent to  $\omega$  cuts  $AC, AB$  again at  $E', F'$ , then  $E' \mapsto F'$  is a projectivity between  $AB$  and  $AC$ . Since  $P \mapsto Q$  is perspectivity through  $O$ , then obviously  $E \mapsto F$  is a projectivity between  $AB$  and  $AC$ . Thus it suffices to show that  $\odot(AEF)$  is tangent to  $\omega$  for 3 particular positions of  $P$ .

Let the perpendicular bisector  $\tau$  of  $AH$  cut  $AC$  at  $J$ . When  $P \equiv \tau \cap ON$  and  $P$  is at infinity, clearly  $\odot(AEF)$  degenerates into  $AC$  and  $AB$ , respectively, trivially tangent to  $\omega$ . When  $P \equiv E \equiv J$ , then  $F \equiv H$ . In this case, notice that  $K \equiv AO \cap \tau$  is the incenter of the J-isosceles  $\triangle AHJ$ , which is clearly midpoint of  $AM \Rightarrow K \in NL \Rightarrow \omega$  is the A-mixtilinear incircle of  $\triangle AHJ$ , i.e.  $\odot(AEF)$  is tangent to  $\omega$ . Consequently,  $\odot(AEF)$  is tangent to  $\omega$  for any  $P$ .



hoangA1K44PBC

#3 May 21, 2016, 9:01 am • 1

Dear Mr. Hung here is my solution.

Because I think it circle is A- Mixtilinear incircle of AEF, so I rewrite the problem as following.

Let triangle ABC with circumcircle (O). J is circumcenter of A- Mixtilinear incircle of ABC. I is its incenter. The line passes through I and perpendicular to AB cut the line passing through C perpendicular to AC at S. SJ cut the line passing through A perpendicular to AC at T. we need to prove that TB is perpendicular to BA.(1)

**Solution**

We need two lemmas : ( I think it has appeared at problem weekly )

Lemma 1( well known): With point we sign in (1) then call X is symmetry of A through O. E, F is the tangents points of (J) at AB, AC. (AEF) cut (O) again at Y then Y, I, X are collinear.

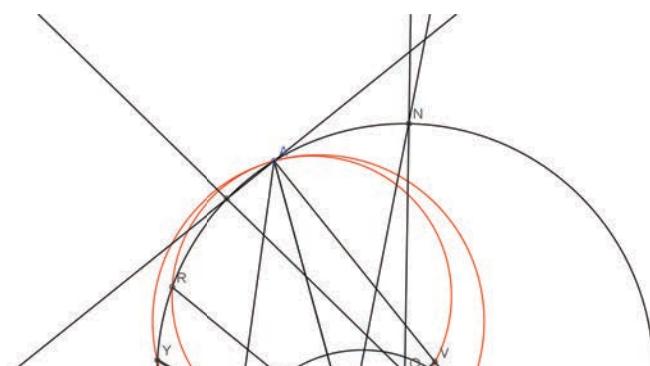
Lemma 2 : With point we sign in (1) then call X is symmetry of A through O. U,V is the tangents point of (I) at AB, AC then (AVU) cut (O) at R then R, I, X are collinear.

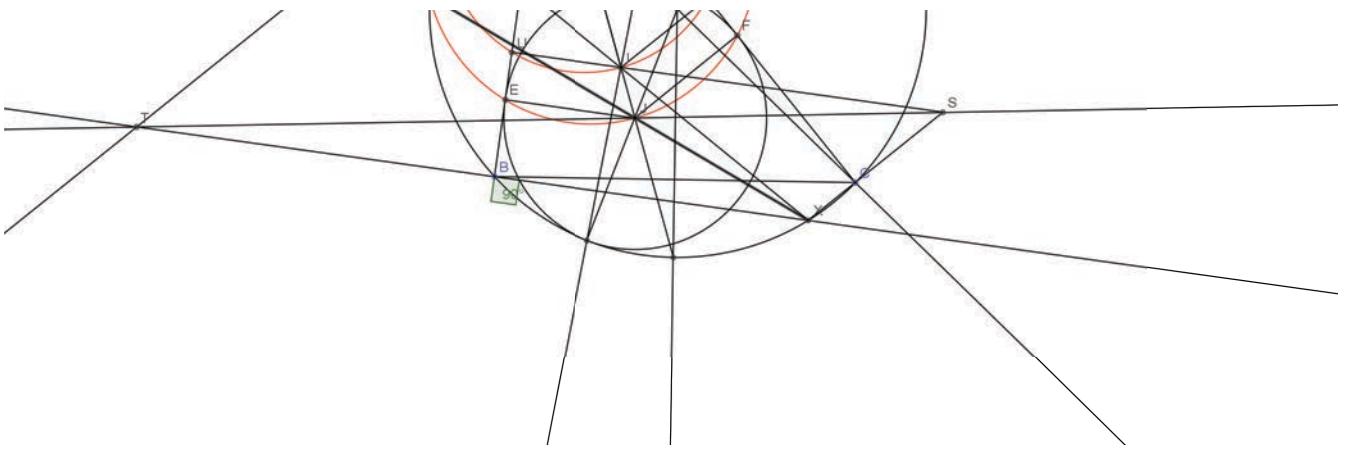
Back to this problem :

call G is symmetry of C through O then we need to prove AG, JS, XB concurrent. So we need to prove X(BJAS) = A(GJXS) .

From IU // AG // CX so A(GJXS) = I(VJXS) but IV is perpendicular to AC, IS is perpendicular to AB. N is midpoint of arc BAC. then I(VJXS) = A(CNRB). projected onto the circle (AVU). Call t is tangent at A of (O). then X(BJAS) = A(BYtC). projected onto the circle (AEF) we have X(BJAS) = A(GJXS). So we have Q.E.D

Attachments:





This post has been edited 1 time. Last edited by hoangA1K44PBC, May 21, 2016, 9:07 am  
Reason: sorry



**buratinogigle**

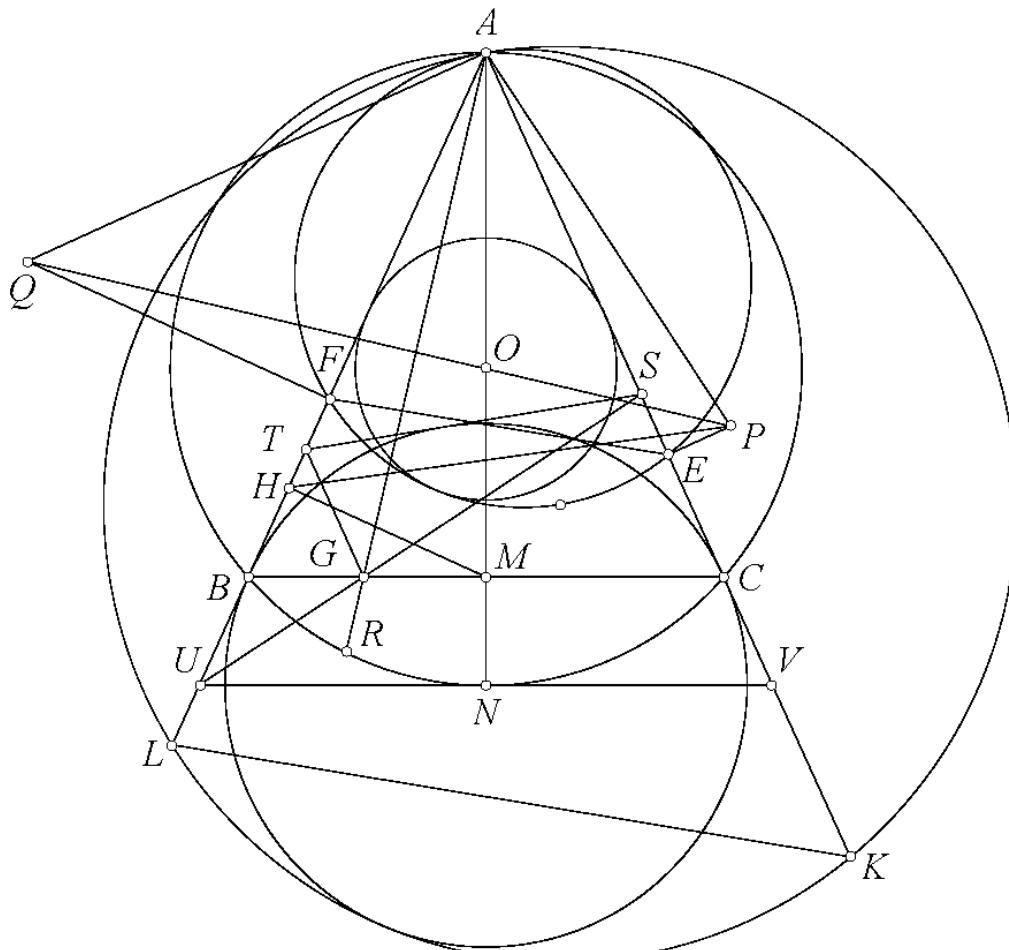
#4 May 21, 2016, 10:32 am • 3

Thank you dear Luis and Hoang for your interest, here is my solution.

Using dilation center  $A$  ratio 2 then  $E, F$  transform to  $K, L$ . Let  $R$  be the reflection of  $A$  through  $PQ$  the  $R$  lies on  $(O)$ . Thus  $K$  is the intersection of circle  $(P)$  circumcircle of triangle  $ARH$  and  $AC$  and  $L$  is the intersection of circle  $(Q)$  which passes through  $A, R$  and is tangent to  $AC$ , with  $AB$ . Now using inversion center  $A$  with power  $AB^2$  then  $R$  transform to  $G$  the intersection of  $AR$  and  $BC$ .  $M$  transform to  $N$  the intersection of  $AM$  and  $(O)$ .  $H$  transform to  $U$  the intersection of tangent at  $N$  of  $(O)$  with  $AB$ . Thus,  $K$  transform to  $S$  the intersection of  $UG$  and  $AC$ .  $L$  transform to  $T$  on  $AB$  such that  $GT \parallel AC$ . We will prove that  $ST$  is tangent to fixed circle. Now  $NU$  cuts  $AC$  at  $V$ . We have,  $\frac{UT}{UA} = \frac{UG}{US} = \frac{VC}{VS}$ , we deduce,  $UT \cdot VS = UA \cdot VC = \frac{UV^2}{4}$ . We easily seen  $ST$  is tangent to circle  $(N)$  which is tangent to  $AB, AC$ . We are done.

See [APMO 2016 P3](#).

Attachments:



Quick Reply



## High School Olympiads

parabola 

 Reply



shmm

#1 May 19, 2016, 3:50 pm

A parabola is drawn on the plane.  
Build its axis of symmetry  
with the help of a  
compass (drawing tool) and a ruler.



Luis González

#2 May 19, 2016, 7:54 pm

In general, given a conic on the plane (parabola, ellipse or hyperbola), its main axes can be drawn with ruler and compass. See the general construction in the topic [how to find the foci of an ellipse](#) (post #5).



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## High School Olympiads

how to find the foci of an ellipse 

 Reply



fmasroor

#1 Oct 4, 2013, 8:02 am

In the plane I have drawn an ellipse. How do I construct its foci?



Dr Sonnhard Graubner

#2 Oct 5, 2013, 12:38 am

hello, see here:

<http://answers.yahoo.com/question/index?qid=20070301184409AAxPGhg>

Sonnhard.



fmasroor

#3 Oct 5, 2013, 3:56 am

I don't have the equation of the ellipse. I only have an ellipse, oriented in any way. I want to find, without any algebra (ie. purely geometrical), the foci.



mavropnevma

#4 Oct 5, 2013, 4:53 am

Draw the long axis, of length  $L$ , and the short axis, of length  $\ell$ . Let  $O$  be their meeting point. The foci will be points on the long axis, symmetrical with respect to  $O$ , at distance  $\frac{1}{2}\sqrt{L^2 - \ell^2}$  from  $O$ .



Luis González

#5 Oct 5, 2013, 5:08 am • 2

mavropnevma, actually drawing the main axes is what the problem is.

Center  $O$  can be found using conjugate diameters. Draw two any parallel chords in the conic, the line connecting their midpoints is the conjugate diameter of their direction, passing through  $O$ . Repeat this for two more parallel chords to get another diameter and then  $O$ . Conjugate diameters  $u, v$  in any conic clearly form an involutive pencil, hence main axes are nothing but the perpendicular rays in the referred involution.

Label  $C$  the given conic with center  $O$ . Draw arbitrary circle  $(K)$  with center  $K$  that goes through  $O$ . If  $u, v$  cuts  $(K)$  again at  $U, V$ , then  $U \mapsto V$  is an involution on  $(K)$ . Construct the fixed pole  $I$  of this involution (two pairs of homologous points are sufficient).  $KI$  cuts  $(K)$  at  $X, Y \Rightarrow OX, OY$  are the axes of  $C$ .

 Quick Reply

## High School Olympiads

### Isogonal conjugate of inversion

[Reply](#)

Source: Goormaghtigh's Generalization of Musselman's Theorem, VMO team training 2016

**buratinogigle**

#1 May 17, 2016, 9:27 pm • 1

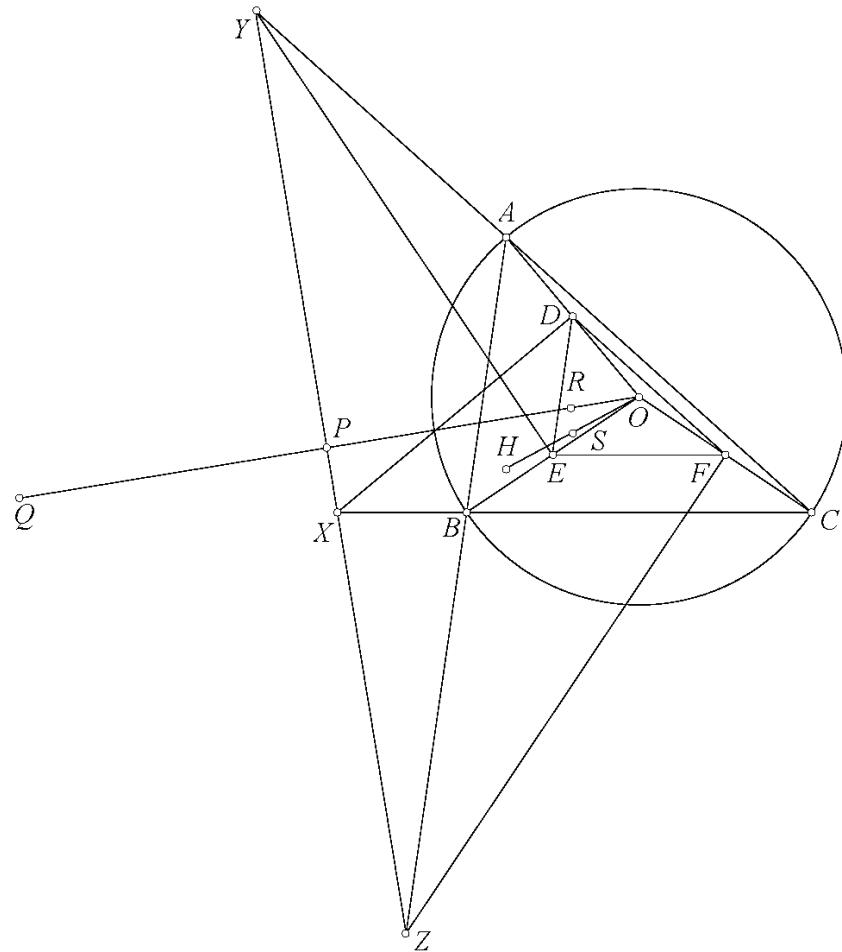
Let  $ABC$  be a triangle inscribed in circle  $(O)$ .  $D, E, F$  devide segment  $OA, OB, OC$ , resp in the same ratio.  $X, Y, Z$  lie on  $BC, CA, AB$ , resp such that  $DX \perp OA, EY \perp OB, FZ \perp OC$ .

a) Prove that  $X, Y, Z$  are collinear on line  $d$ .

b) Let  $P$  be the projection of  $O$  on  $d$ .  $Q$  lie on  $OP$  such that  $AQ \parallel DP$ . Prove that isogonal conjugate with triangle  $ABC$  of inversion of  $Q$  through  $(O)$  lies on Euler of triangle  $ABC$ .

#### Reference

Attachments:

**TelvCohl**

#2 May 18, 2016, 4:19 am • 2

(a) Let  $\triangle A_1B_1C_1$  be the tangential triangle of  $\triangle DEF$ . From Kariya's theorem  $\Rightarrow AA_1, BB_1, CC_1$  are concurrent, so from Desargues' theorem we get  $X, Y, Z$  are collinear at the perspectrix of  $\triangle ABC$  and  $\triangle A_1B_1C_1$ .

(b) Let  $\triangle M_aM_bM_c$  be the medial triangle of  $\triangle ABC$  and let  $V^*$  be the image of an arbitrary point  $V$  under the inversion WRT

$\odot(DEF)$ . Since  $\triangle DEF$ ,  $\triangle M_a^* M_b^* M_c^*$  are bilogic with common orthology center  $O$ , so  $DM_a^*, EM_b^*, FM_c^*$  are concurrent on the circum-rectangular hyperbola of  $\triangle DEF$  passing through  $O$  (isogonal conjugate of Euler line).

Since  $OX$  is the diameter of  $\odot(DOM_a)$ , so we get  $X^*$  is the projection of  $O$  on  $DM_a^*$ . Analogously, we can prove  $Y^*, Z^*$  is the projection of  $O$  on  $EM_b^*, FM_c^*$ , respectively, so we get  $P^*$  is the perspector of  $\triangle DEF$ ,  $\triangle M_a^* M_b^* M_c^*$  and the isogonal conjugate of  $P^*$  WRT  $\triangle DEF$  lies on the Euler line of  $\triangle DEF$  ... ( $\spadesuit$ ).

Finally, it is obvious that  $\triangle ABC \cup Q$  and  $\triangle DEF \cup P$  are homothetic, so from ( $\spadesuit$ ) we conclude that the isogonal conjugate (WRT  $\triangle ABC$ ) of the image of  $Q$  under the inversion WRT  $\odot(ABC)$  lies on the Euler line of  $\triangle ABC$ .



Luis González

#3 May 18, 2016, 7:58 am • 3

a) Animate  $D, E, F$  on  $OA, OB, OC$ . Series  $D, E, F$  are similar and so are  $X, Y, Z$ , because the directions  $XD, YE, ZF$  are fixed, therefore  $\tau \equiv YZ$  envelopes a parabola  $\mathcal{P}$  tangent to  $AB, AC$ . When  $D, E, F$  coincide with  $A, B, C$ , then  $\tau$  becomes the Lemoine axis of  $\triangle ABC$  and when  $D \equiv E \equiv F \equiv O$ , then  $\tau$  is the orthotransversal of  $O$  WRT  $\triangle ABC \implies \mathcal{P}$  is the Kiepert parabola of  $\triangle ABC$ . Similarly  $XY$  touches  $\mathcal{P} \implies X, Y, Z$  are collinear on a tangent  $\tau$  of  $\mathcal{P}$ .

b) Let  $\triangle A'B'C'$  be the tangential triangle of  $\triangle ABC$  and the perpendicular  $\tau'$  from  $Q$  to  $OP$  cuts  $B'C', C'A', A'B'$  at  $X', Y', Z'$ . Since  $OP : OQ = OD : OA = OE : OB = OF : OC$ , then by homothety it follows that  $X', Y', Z'$  lie on  $OX, OY, OZ$ , resp, thus the series  $X', Y', Z'$  are also projective, even similar, as  $X', Y', Z'$  go to infinity when  $\tau$  is the line at infinity  $\implies \tau'$  envelopes a parabola  $\mathcal{P}'$  inscribed in  $\triangle A'B'C'$ . Hence the inverse  $Q^*$  of  $Q$  on  $(O)$  (pole of  $\tau'$  WRT  $(O)$ ) describes the dual  $\mathcal{J}$  of  $\mathcal{P}'$  WRT  $(O)$ . As  $\mathcal{P}'$  is a parabola, then it follows that  $\mathcal{J}$  passes through  $A, B, C$  and  $O$ . When  $D, E, F$  coincide with  $A, B, C$ , then clearly  $\tau \equiv \tau' \implies Q^*$  becomes the symmedian point  $K$  of  $\triangle ABC \implies K \in \mathcal{J} \implies \mathcal{J}$  is the Jerabek hyperbola of  $\triangle ABC$ ; isogonal conjugate of its Euler line  $\implies$  isogonal conjugate of the inverse of  $Q$  is then on the Euler line of  $\triangle ABC$ .

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## High School Olympiads

A-symmedian 

 Reply



Source: Own from some results.



doxuanlong15052000

#1 May 15, 2016, 8:05 pm

Let  $ABC$  be a triangle with  $(N)$  be the Euler-circle.  $(N)$  cuts  $(BOC)$  at  $X$  and  $Y$ .  $AX$  cuts  $BC$  at  $L$ . Let  $T$  be a point on  $BC$  such that  $BL = CT$ . The conjugate equiangular of  $AT$  cuts  $(O)$  at  $R$ .  $AY$  cuts  $(O)$  at  $H$ .  $S$  is the intersection of the tangent of  $(O)$  at  $R$  and  $H$ . Then  $AS$  is  $A$ -symmedian.



Luis González

#2 May 15, 2016, 9:12 pm • 1 

It's well-known that  $AX, AY$  are isogonals WRT  $\angle BAC$  (note that  $\odot(BOC)$  is the image of  $(N)$  under inversion with center  $A$ , power  $\frac{1}{2}AB \cdot AC$  followed by reflection on the bisector of  $\angle BAC$ ). Thus, the problem is a particular case of the following configuration:

Let  $U, V$  be the isotomic points on the side  $BC$  of  $\triangle ABC$ . The isogonals of  $AU, AV$  WRT  $\angle BAC$  cut  $\odot(ABC)$  again at  $M, N$ . The tangents of  $\odot(ABC)$  at  $M, N$  meet at  $S$ . Then  $AS$  is the A-symmedian of  $\triangle ABC$ .

Let the tangent of  $\odot(ABC)$  at  $A$  cut  $BC$  at  $L$  and let the parallel from  $A$  to  $BC$  cut  $\odot(ABC)$  again at  $P$ . Inversion with center  $A$ , power  $AB \cdot AC$ , followed by reflection on the bisector of  $\angle BAC$  swaps  $\odot(ABC)$  and  $BC$ . Consequently,  $U \mapsto M, V \mapsto N$  and  $L \mapsto P$ . By symmetry  $APVU$  is an isosceles trapezoid (cyclic), hence it follows that  $L, M, N$  are collinear  $\implies AS$  is the polar of  $L$  WRT  $\odot(ABC)$ , i.e. the A-symmedian of  $\triangle ABC$ .

 Quick Reply

## High School Olympiads

Conics in triangle X

Reply



Source: maybe well-known



jammy

#1 May 14, 2016, 9:04 pm

Consider a triangle  $ABC$  and three conics, each with focus at one vertex and directrix at the opposite side to that vertex. Prove that these conics intersect the sides of  $\triangle ABC$  at six points which lie on a conic.



Luis González

#2 May 14, 2016, 9:43 pm • 1

Denote  $\epsilon_A, \epsilon_B, \epsilon_C$  the eccentricities of the conics  $\mathcal{C}_A, \mathcal{C}_B, \mathcal{C}_C$  with foci  $A, B, C$  and directrices  $BC, CA, AB$ , respectively.  $\mathcal{C}_B$  and  $\mathcal{C}_C$  cut  $\overline{BC}$  at  $A_1$  and  $A_2$  and we define  $\{B_1, B_2\} \in \overline{CA}$  and  $\{C_1, C_2\} \in \overline{AB}$  similarly. Then

$$\frac{A_1B}{\text{dist}(A_1, AC)} = \epsilon_B \implies \frac{A_1B}{A_1C} = \epsilon_B \cdot \sin C. \text{ Similarly } \frac{A_2C}{A_2B} = \epsilon_C \cdot \sin B.$$
$$\implies \frac{A_1B}{A_1C} \cdot \frac{A_2B}{A_2C} = \frac{\epsilon_B}{\epsilon_C} \cdot \frac{\sin C}{\sin B}.$$

Multiplying the cyclic expressions together, we deduce by Carnot's theorem that  $A_1, A_2, B_1, B_2, C_1, C_2$  lie on a same conic.

Quick Reply

## Concurrent related to three Symmedian points



Reply



Source: Own (Inspired from mineiraojose's problem)

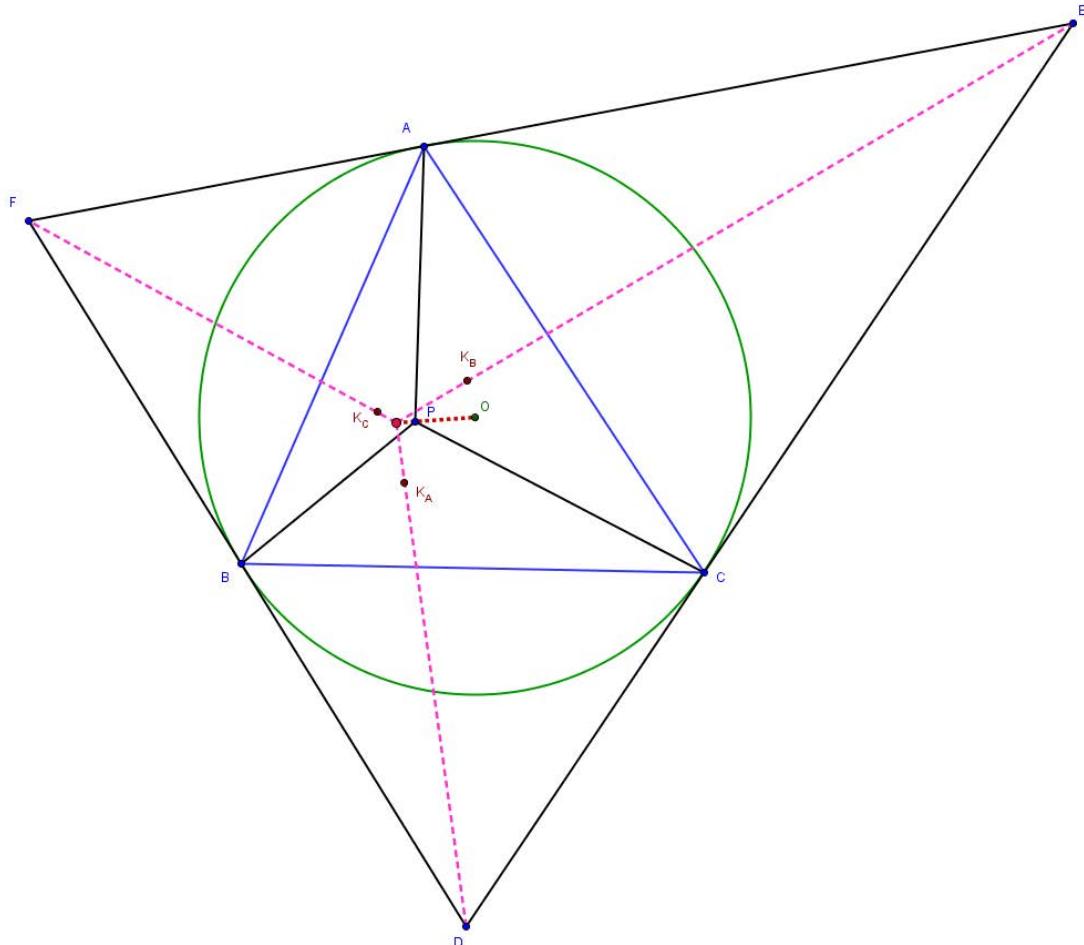


TelvCohl

#1 May 14, 2016, 2:33 am • 3

Given a  $\triangle ABC$  inscribed in  $\odot(O)$  and a point  $P$ . Let  $\triangle DEF$  be the tangential triangle of  $\triangle ABC$  and let  $K_A, K_B, K_C$  be the symmedian point of  $\triangle BPC, \triangle CPA, \triangle APB$ , respectively. Prove that  $OP, DK_A, EK_B, FK_C$  are concurrent.

Attachments:



Luis González

#2 May 14, 2016, 4:41 am • 5

Let  $S_A, S_B, S_C$  be the feet of the P-symmedians of  $\triangle PBC, \triangle PCA, \triangle PAB$ , respectively. Tangents of  $\odot(PBC)$ ,  $\odot(PCA)$ ,  $\odot(PAB)$  at  $P$  cut  $BC, CA, AB$  at  $U_A, U_B, U_C$ , respectively. Since  $U_A P^2 = U_A B \cdot U_A C \implies U_A$  has equal WRT ( $O$ ) and the degenerate circle  $P$ . Likewise,  $U_B$  and  $U_C$  have equal powers WRT ( $O$ ) and  $P \implies U_A, U_B, U_C$  are collinear on the radical axis of ( $O$ ),  $P \implies \overline{U_A U_B U_C} \perp PO$  ( $\star$ ).

On the other hand, since  $(S_A, B, C, U_A) = -1$  and  $DB, DC$  are tangents of ( $O$ ), it follows that  $DS_A$  is the polar of  $U_A$  WRT ( $O$ ) and analogously  $ES_B$  and  $FS_C$  are the polars of  $U_B$  and  $U_C$  WRT ( $O$ )  $\implies DS_A, ES_B, FS_C$  concur at the pole  $Q$  of  $\overline{U_A U_B U_C}$  WRT ( $O$ )  $\implies OQ \perp \overline{U_A U_B U_C}$ . Together with ( $\star$ ), we deduce that  $O, P, Q$  are collinear.

If the perpendicular bisector  $DO$  of  $BC$  cuts the P-symmedian  $PS_A$  of  $\triangle PBC$  at  $D'$ , then  $D'B, D'C$  are tangents of  $\odot(PBC)$   $\implies (K_A, P, S_A, D') = -1$ . Hence, if  $DK_A$  cuts  $OP$  at  $R$ , we have  $(R, P, Q, O) = D(K_A, P, S_A, D') = -1 \implies R$  is fixed on  $\overline{OPQ}$ . Similarly  $EK_B$  and  $FK_C$  hit  $OP$  at the same point  $R$ .

Quick Reply

## High School Olympiads

Concurrent on Simson line 

 Reply



**langkhach11112**

#1 May 13, 2016, 6:39 pm

Given  $\Delta ABC$  and its circumcircle  $(O)$ . Point  $P$  lies on  $(O)$ .  $d$  is an arbitrary line go through  $P$ . Points  $A_1, B_1, C_1$  lies on  $d$  such that  $AA_1, BB_1, CC_1 \perp d$ . Points  $A_2, B_2, C_2$  lies on  $BC, CA, AB$ , respectively, such that  $A_1A_2 \perp BC, B_1B_2 \perp CA, C_1C_2 \perp AB$ . Prove that  $A_1A_2, B_1B_2, C_1C_2$  are concurrent on the Simson line of  $P$  wrt  $\Delta ABC$ .



**Luis González**

#2 May 13, 2016, 7:41 pm

This is well-known;  $A_1A_2, B_1B_2, C_1C_2$  concur at the orthopole of  $d$  WRT  $\Delta ABC$ . Now see the previous lemma at [Six orthopoles lie on a circle](#).



**jam10307**

#3 May 13, 2016, 8:42 pm

This is also pretty nice with complex numbers, when we set the circumcircle (denoted as  $\omega$ ) as the unit circle. Let  $P, P'$  be the points that line  $d$  intersects  $\omega$ . Then we just spam

$$\ell_1 \cap \ell_2 = \frac{ab(c+d) - cd(a+b)}{ab - cd}, c \perp ab = \frac{1}{2}(a + b + c - ab\bar{c}).$$



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## High School Olympiads

Six orthopoles lie on a circle X

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**buratinogiggle**

#1 Sep 3, 2012, 12:00 am

Let  $ABC$  and  $A'B'C'$  be two triangles inscribed in circle  $(O)$ . Prove that orthopoles of  $B'C'$ ,  $C'A'$ ,  $A'B'$  with respect to triangle  $ABC$  and orthopoles of  $BC$ ,  $CA$ ,  $AB$  with respect to triangle  $A'B'C'$  lie on a circle.



**Luis González**

#2 Sep 3, 2012, 2:58 am • 3

**Lemma.** Line  $\tau$  cuts the circumcircle of  $\triangle ABC$  at  $P, Q$ . Then Simson lines of  $P, Q$  with respect to  $\triangle ABC$  meet at the orthopole of  $\tau$  with respect to  $\triangle ABC$ .

**Proof.** Let  $A'$  be the orthogonal projection of  $A$  on  $\tau$ .  $S, T, X$  are the orthogonal projections of  $Q$  on  $CB, BA, AC$  and  $M, N, Y$  the orthogonal projections of  $P$  on  $BC, CA, AB$ . Thus,  $A, Q, X, A'$  and  $A, P, Y, A'$  are concyclic. Using directed angles  $(\text{mod } \pi)$ , we have

$$\angle AA'X = \frac{\pi}{2} + \angle QA'X = \frac{\pi}{2} + \angle QAC, \quad \angle AA'N = \angle APN = \frac{\pi}{2} - \angle PAC$$

$$\angle NAX' = \angle AA'X - \angle AA'N = \frac{\pi}{2} + \angle QAC - \left( \frac{\pi}{2} - \angle PAC \right) = \angle PAQ$$

Let  $K \equiv SX \cap MY$ . Since the angle between the Simson lines of  $P, Q$  is half the measure of the arc  $PQ$  of the circumcircle (\*), it follows that  $\angle NAX' = \angle NKX \Rightarrow N, K, A', X$  are concyclic. Consequently

$$\angle A'KX = \angle A'NX = \angle APQ = \angle ACQ = \angle XSQ \Rightarrow KA' \parallel QS$$

That is,  $KA' \perp BC$ . Similarly, if  $B'$  denotes the orthogonal projection of  $B$  on  $\tau$ , we have  $KB' \perp CA \Rightarrow K$  is the orthopole of  $\tau$  with respect to  $\triangle ABC$ .

Based on the previous lemma, the pairwise Simson lines of  $A', B', C'$  WRT  $\triangle ABC$  meet at the orthopoles  $A_0, B_0, C_0$  of  $B'C', C'A', A'B'$  WRT  $\triangle ABC$ . In addition,  $\triangle A_0B_0C_0 \sim \triangle A'B'C'$ , because of the assertion (\*). Let  $H, H'$  be the orthocenters of  $\triangle ABC, \triangle A'B'C'$ . Then  $B_0C_0, C_0A_0, A_0B_0$  pass through the midpoints  $X_0, Y_0, Z_0$  of  $HA', HB', HC'$ , lying on the 9-point circle  $(N, \frac{R}{2})$  of  $\triangle ABC \Rightarrow \triangle X_0Y_0Z_0 \sim \triangle A'B'C' \sim \triangle A_0B_0C_0$ . If  $U$  is midpoint of  $HH'$ , then  $UY_0$  is H-midline of  $\triangle HH'B' \Rightarrow UY_0 \parallel B'H' \perp A'C'$ . Similarly,  $UZ_0 \perp A'B' \Rightarrow \angle Y_0UZ_0 = \angle B'A'C' = \angle B_0A_0C_0 \text{ (mod } \pi) \Rightarrow U \in \odot(A_0Y_0Z_0)$ . Likewise,  $U \in \odot(B_0Z_0X_0) \Rightarrow U$  is the Miquel point of  $\triangle X_0Y_0Z_0$  WRT  $\triangle A_0B_0C_0 \Rightarrow U$  is orthocenter of  $\triangle X_0Y_0Z_0$  and circumcenter of  $\triangle A_0B_0C_0$  (see the topic [Own problem](#) and elsewhere)  $\Rightarrow \odot(A_0Y_0Z_0) \cong (N, \frac{R}{2})$ .

Now, easy angle chase reveals that the angle between a sideline of  $ABC$  ( $A'B'C'$ ) and the Simson line of its opposite vertex WRT  $A'B'C'$  ( $ABC$ ) is constant. Let  $\lambda$  be the measure of this angle. From  $UY_0 \parallel B'H'$ , we have then  $UA_0 = R \cdot \sin \angle(B'H', A_0C_0) = R \cdot \sin(\frac{\pi}{2} - \lambda) = R \cdot \cos \lambda$ . As a result, orthopoles of  $B'C', C'A', A'B'$  WRT  $\triangle ABC$  lie on a circle  $(U, \varrho)$  centered at the midpoint  $U$  of  $HH'$  with radius  $\varrho = R \cdot \cos \lambda$ . By similar reasoning, the orthopoles of  $BC, CA, AB$  WRT  $\triangle A'B'C'$  lie on the same circle  $(U, \varrho)$ .

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## High School Olympiads



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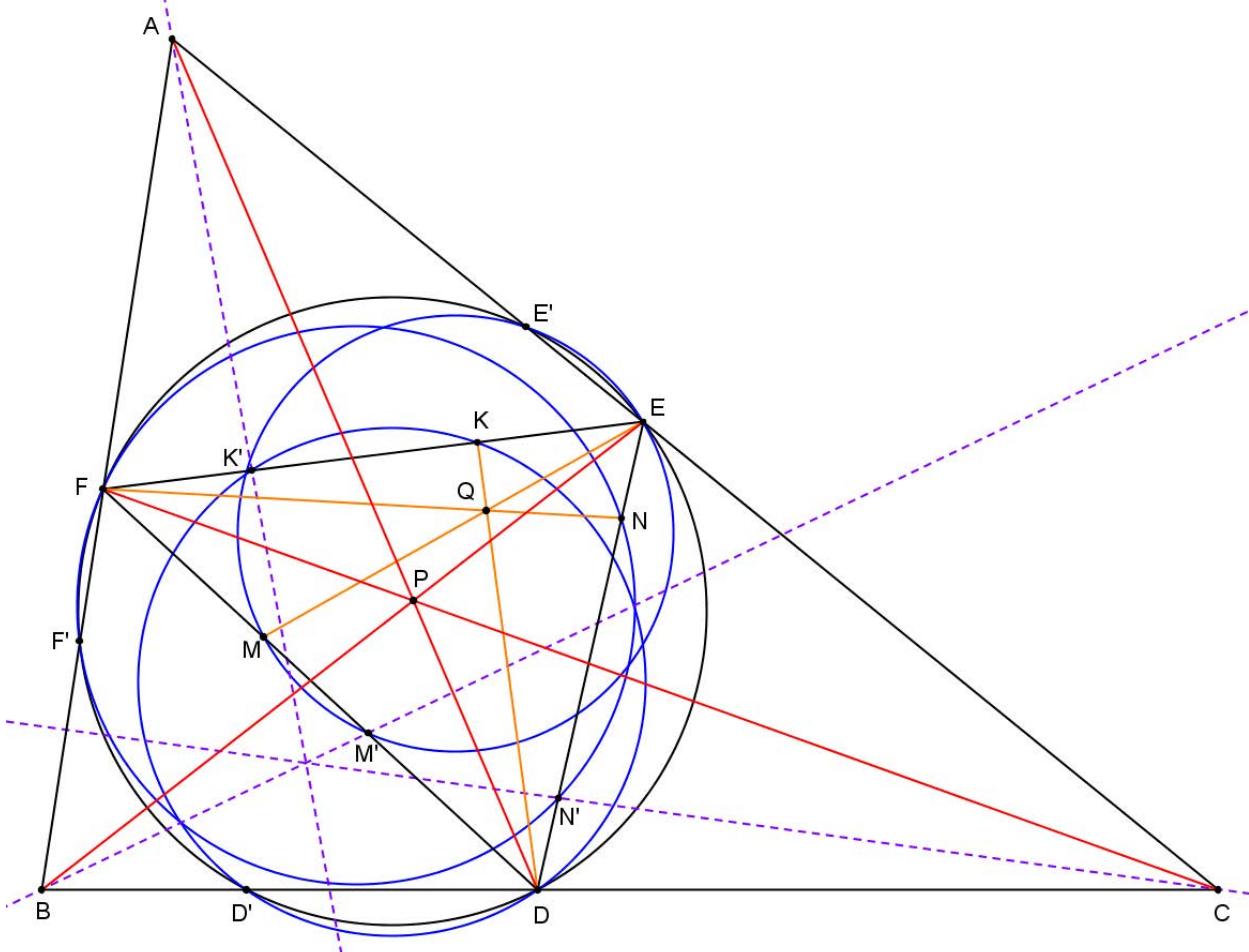
mjuk

#1 May 13, 2016, 12:40 am • 2

Let  $\triangle ABC$  be a triangle, let  $P, Q$  be points. Let  $\triangle DEF$  be cevian triangle of  $P$  wrt.  $\triangle ABC$ . Let  $\odot DEF$  intersect  $BC, CA, AB$  at  $D', E', F'$  respectively. Let  $\triangle KMN$  be cevian triangle of  $Q$  wrt.  $\triangle DEF$ . Let  $K' = EF \cap \odot KDD'$ ,  $M' = DF \cap \odot MEE'$  and  $N' = DE \cap \odot NFF'$ .

Prove that  $AK', BM', CN'$  are concurrent.

Attachments:



This post has been edited 1 time. Last edited by njuk, May 13, 2016, 12:42 am  
Reason: picture edit



Luis González

#2 May 13, 2016, 4:33 am • 2

Let  $EF, FD, DE$  cut  $BC, CA, AB$  at  $U, V, W$  and let  $EF, FD, DE$  cut  $PA, PB, PC$  at  $A', B', C'$ , respectively. Then  $UE \cdot UF = UD \cdot UD' = UK \cdot UK' \implies U$  is the center of the involution that swaps  $E, F$  and  $K, K' \implies$

$$(K, F, E, U) = (K', E, F, \infty) \implies \frac{K'E}{K'F} = \frac{KF}{KE} \cdot \frac{UE}{UF} = \frac{KF}{KE} \cdot \frac{A'E}{A'F}.$$

Hence, multiplying the cyclic expressions together, we deduce by Ceva's theorem that  $DK', EM', FN'$  concur. Thus by Cevian Nest Theorem, it follows that  $AK', BM', CN'$  concur.

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## High School Math

A strange geometry problem X

Reply



finn123

#1 May 11, 2016, 9:24 pm

For each point O on diameter AB of a circle, perform the following construction: Let the perpendicular to AB at O meet the circle at point P. Inscribe circles in the figures bounded by the circle and the lines AB, OP. Let the R and S be the points at which the two incircles to the curvilinear triangles AOP and BOP are tangent to the diameter AB. Show that angle RPS is independent of the position of O.



Luis González

#2 May 12, 2016, 9:58 pm

According to <http://www.artofproblemsolving.com/community/c6h416223>, PR and PS bisect  $\angle OPA$  and  $\angle OPB$ , respectively. Since  $\angle APB = 90^\circ$ , it follows that  $\angle RPS = 45^\circ$ .

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## High School Olympiads

**AC bisects angle PAB** 

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Source: Israel 1995



**sergei93**

#1 Jul 4, 2011, 5:11 am • 2

Let  $PQ$  be the diameter of semicircle  $H$ . The circle  $O$  is internally tangent to  $H$  and is tangent to  $PQ$  at  $C$ . Let  $A$  be a point on  $H$ , and let  $B$  be a point on  $PQ$  such that  $AB$  is perpendicular to  $PQ$  and is also tangent to  $O$ . Prove that  $AC$  bisects  $\angle PAB$ .

*I'm looking particularly for non-inversive solutions. The natural solution uses inversion.*



**Luis González**

#2 Jul 4, 2011, 5:48 am • 2

We assume that  $C$  is between  $P$  and  $B$ . Let  $A'$  and  $(O')$  be the reflections of  $A$  and  $(O)$  about  $PQ$ . Obviously,  $(O')$  is also tangent to  $\odot(PAA')$ . Therefore,  $(O), (O)'$  are the Thebault circles of the angle bisector  $PB$  of the P-isosceles  $\triangle APA'$ . Hence,  $C \equiv PB \cap OO'$  is the incenter of  $\triangle PAA'$ , i.e.  $AC$  bisects  $\angle PAB$ .

See also <http://www.artofproblemsolving.com/Forum/viewtopic.php?f=46&t=146656>



**sunken rock**

#3 Jul 15, 2011, 12:20 am

Take  $A'$  the reflection of  $A$  in  $PQ$ ,  $D$  and  $E$  the common points of circle  $O$  with the semicircle and  $AB$ , respectively. Inversion of pole  $Q$  and power  $\rho = QB \cdot PQ$  sends the circle  $\odot APA'Q$  into the line  $AA'$ , hence the point  $D$  to  $E$ , i.e. it preserves the circle  $O$ .

That means  $\rho = QA^2 = QE \cdot QD = QC^2$  and  $C$  is the incenter of  $\triangle APA'$ .

Note: in the link proposed by Luis we can see something similar to this one; it does not, however, make the connection to inversion (it does show that the circle  $\odot CDE$  remains unchanged through that inversion)!

Best regards,  
sunken rock



**Farenhajt**

#4 Jul 15, 2011, 4:12 am

This is metric solution.

Let  $PB = p$ ,  $QB = q$ ,  $CB = r$  and  $M$  the midpoint of  $PQ$ . Then  $AB = \sqrt{pq}$ ,  $AP = \sqrt{p^2 + pq}$ . By Pythagoras

$$MO^2 - MC^2 = OC^2 \implies \left(\frac{p+q}{2} + r\right)^2 - \left(\frac{q-p}{2} + r\right)^2 = r^2$$

$$r^2 + 2rq - pq = 0 \implies r = \sqrt{pq + q^2} - q \implies PC = p + q - \sqrt{pq + q^2}$$

$$\text{Hence } \frac{PC}{CB} = \frac{\sqrt{p+q}(\sqrt{p+q} - \sqrt{q})}{\sqrt{q}(\sqrt{p+q} - \sqrt{q})} = \frac{\sqrt{p^2 + pq}}{\sqrt{pq}} = \frac{PA}{AB} \text{ and the conclusion follows.}$$

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## High School Olympiads



## Tri polar of Infinity point passes through Fe



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Source: Own

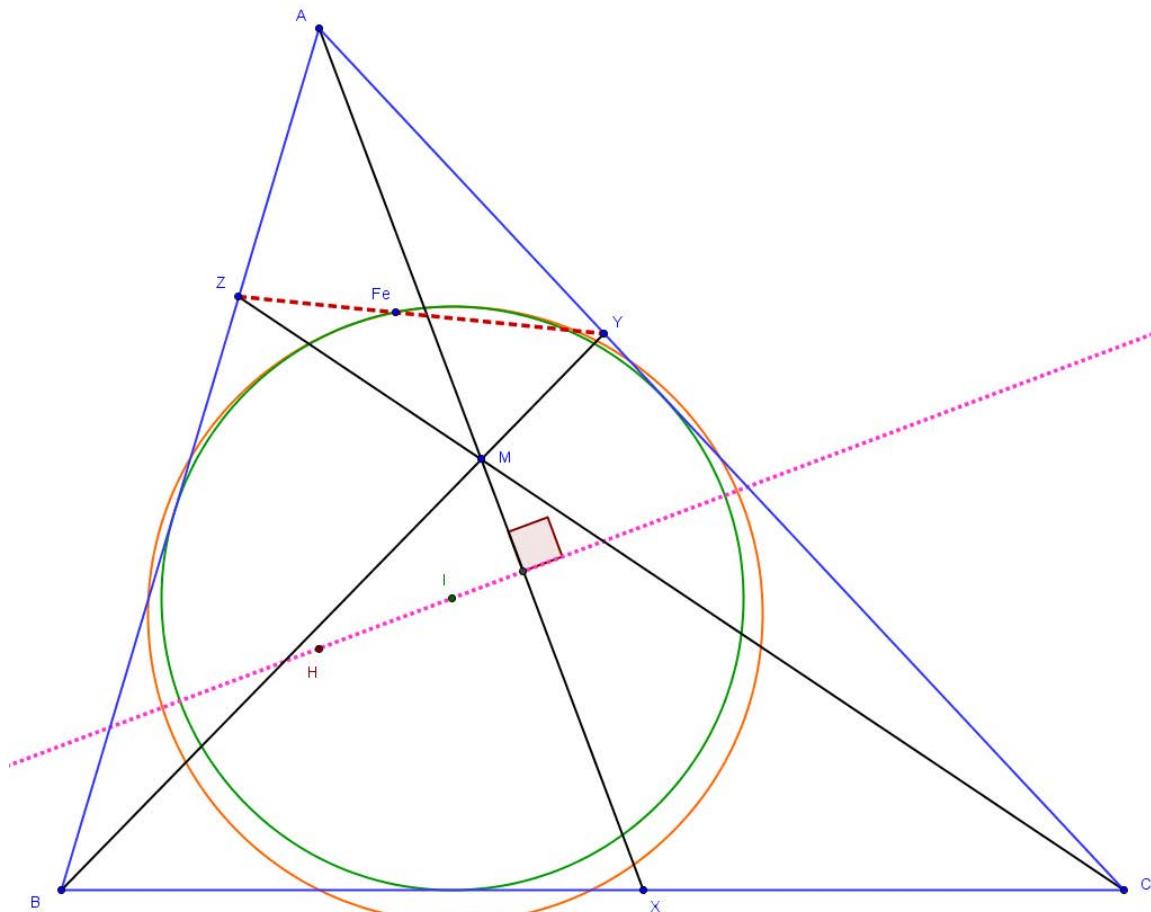


TelvCohl

#1 May 9, 2016, 6:56 pm • 4

Given a  $\triangle ABC$  with incenter  $I$ , orthocenter  $H$  and the Feuerbach point  $F_e$ . Let  $X$  be the point on  $BC$  such that  $AX \perp IH$ . Let  $M$  be the midpoint of  $AX$  and let  $Y \equiv BM \cap CA$ ,  $Z \equiv CM \cap AB$ . Prove that  $F_e, Y, Z$  are collinear.

Attachments:



Luis González

#2 May 10, 2016, 8:17 am • 6

Let  $D, E, F$  be the midpoints of  $BC, CA, AB$  and let the excircles against  $A, B, C$  touch  $BC, CA, AB$  at  $U, V, W$ , respectively.  $EF, FE, DE$  cut  $VW, WU, UV$  at  $A', B', C'$ .  $\triangle UVW$  is pedal triangle of Bevan point  $Be$  of  $\triangle ABC$ , lying on the line connecting  $I$  and the circumcenter  $O$  of  $\triangle ABC$ . Thus, the isogonal conjugate  $Be^*$  of  $Be$  WRT  $\triangle ABC$  is on its Feuerbach hyperbola with center  $F_e \implies F_e$  is the Poncelet point of  $A, B, C, Be^*$ . Consequently, by 1st Fontené theorem, we deduce that  $F_e, U, A'$  are collinear and similarly  $F_e, V, B'$  and  $F_e, W, C'$  are collinear i.e.  $F_e \equiv UA' \cap VB' \cap WC'$ . But if  $\mathcal{M}$  is the inconic tangent to  $BC, CA, AB$  at  $U, V, W$  (Mandart inellipse of  $\triangle ABC$ ), then according to [concurrent at one point in \(I\)](#) (see the generalization at post #4), it follows that  $F_e \equiv UA' \cap VB' \cap WC' \in \mathcal{M}$ .

On the other hand, it's known that, in general  $AA', BB', CC'$  concur at the intersection of the trilinear polars of the Nagel point  $Na \equiv AU \cap BV \cap CW$  and the centroid  $G \equiv AD \cap BE \cap CF \implies AA'$  is parallel to the trilinear polar  $\tau$  of  $Na$ . But from the problem [hard geometry problem](#), we have  $IH \perp \tau \implies AA' \perp IH \implies AA' \equiv AX \implies M \equiv A'$ . Now by the converse of Newton theorem, it follows that  $YZ$  is tangent to  $\mathcal{M}$  at a point collinear with  $U$  and  $A'$ , which is none other than  $F_e \implies F_e \in YZ$ .

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## High School Olympiads

concurrent at one point in  $(I)$  

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**DonaldLove**

#1 Sep 11, 2013, 3:42 pm

consider triangle  $ABC$ , the inscribed circle  $(I)$  is tangent to  $BC, CA, AB$  at  $A_0, B_0, C_0$ .  $AI, BI, CI$  intersect  $BC, CA, AB$  at  $A_1, B_1, C_1$ .  $B_0C_0, C_0A_0, A_0B_0$  intersect  $B_1C_1, C_1A_1, A_1B_1$  at  $A_2, B_2, C_2$ . prove that  $A_0A_2, B_0B_2, C_0C_2$  intersect at one point on  $(I)$



**jayme**

#2 Sep 11, 2013, 9:27 pm • 1 

Dear Mathlinkers,

only to begin,  $A_2B_2C_2$  is the Pelletier triangle of  $ABC$ .

Now look that  $A_0B_0C_0$  and  $A_1B_1C_1$  are two cevian triangle of  $ABC$ ...

Sincerely

Jean-Louis



**DonaldLove**

#3 Sep 11, 2013, 9:49 pm

dear jayme,

can you explain more simple please?



**Luis González**

#4 Sep 12, 2013, 1:13 am • 5 

**Generalization:**  $P_0, P_1$  are two points on the plane of  $\triangle ABC$ .  $\triangle A_0B_0C_0$  and  $\triangle A_1B_1C_1$  are the cevian triangles of  $P_0, P_1$  WRT  $\triangle ABC$ .  $A_2 \equiv B_0C_0 \cap B_1C_1$  and  $B_2, C_2$  are defined similarly. Then  $A_0A_2, B_0B_2, C_0C_2$  concur on the inconic  $\mathcal{K}$  with perspector  $P_0$ .

Let  $\mathcal{C}$  be the conic through  $A, B, C, P_0, P_1$ . Then  $B_0C_0$  and  $B_1C_1$  are the polars of  $A_0$  and  $A_1$  WRT  $\mathcal{C} \implies A_0A_1 \equiv BC$  is the polar of  $A_2 \equiv B_0C_0 \cap B_1C_1$  WRT  $\mathcal{C} \implies A_2B, A_2C$  are tangents of  $\mathcal{C}$ . By similar reasoning  $B_2C, B_2A$  and  $C_2A, C_2B$  are tangents of  $\mathcal{C} \implies A \in B_2C_2, B \in C_2A_2, C \in A_2B_2$ . Further  $\triangle A_2B_2C_2$  is then polar triangle of  $\mathcal{C}$  WRT  $\triangle ABC \implies \triangle A_2B_2C_2$  and  $\triangle ABC$  are perspective  $\implies$  the line through  $Y \equiv BA_2C_2 \cap CA$  and  $Z \equiv CA_2B_2 \cap AB$  is their perspetrix  $\tau$ . Hence if  $P \equiv \tau \cap A_0A_2$ , then by Brianchon's theorem there is a conic tangent to  $BC, CY, YZ, ZB$  at  $A_0, B_0, P, C_0$ , respectively  $\implies$  it coincides with  $\mathcal{K} \implies P \in \mathcal{K}$ . Analogously,  $B_0B_2$  and  $C_0C_2$  go through  $P$ .

P.S. When  $P_0$  and  $P_1$  coincide with the Gergonne point and the incenter, as the proposed problem, then  $\mathcal{K}$  becomes  $(I)$  and  $P$  is then the tangency point of  $(I)$  with the perspectrix of  $\triangle ABC$  and its Pelletier triangle  $\triangle A_2B_2C_2$ , the 6th Stevanovic point of  $\triangle ABC$ .

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## High School Olympiads

hard geometry problem 

 Reply

Source: own



**littlepascal**

#1 Oct 24, 2014, 7:41 am

Given a triangle ABC, let N is the Nagel Point, I is the incenter, and H is the orthocenter. Let AN meets BC on D, BN meets AC on E, CN meets AB on F.

Prove that perspective axis of triangle ABC and DEF is orthogonal to IH.

Sorry for my bad English @@



**Luis González**

#2 Oct 24, 2014, 8:40 am • 1 

$\triangle ABC$  and  $\triangle DEF$  are perspective through the Nagel point  $N$  and orthologic, being the Bevan  $B_e$  of  $\triangle ABC$  one of the orthology center (concurrency point of the normals to BC, CA, AB at D, E, F). By Sondat's theorem  $NB_e$  is perpendicular to their perspectrix. Thus, it suffices to show that  $IH \parallel NB_e$ .

Let  $\triangle A_0B_0C_0$  be the antimedial triangle of  $\triangle ABC$ . Its orthocenter is the reflection  $L$  of  $H$  on  $O$  (De Longchamps point of ABC) and its incenter is  $N$ . Thus, lines  $IH$  and  $NL$  are parallel, as they are homologues under the homothety that takes  $\triangle ABC$  into  $\triangle A_0B_0C_0$ .  $B_e$  is reflection of  $I$  on  $O$ , falling then on the parallel  $NL$  from  $L$  to  $IH$ , i.e.  $NB_e \parallel IH$ .



**littlepascal**

#3 Oct 24, 2014, 8:48 am

What is Sondat's theorem??



**TelvCohl**

#4 Oct 24, 2014, 10:49 am • 1 

**Another approach:**

Let  $D'$ ,  $E'$ ,  $F'$  be the tangent point of the  $\odot(I)$  with  $BC$ ,  $CA$ ,  $AB$ , respectively .

Let  $X = E'F' \cap BC$ ,  $Y = F'D' \cap CA$ ,  $Z = D'E' \cap AB$  and  $l$  be the line pass through  $X$ ,  $Y$ ,  $Z$  .

Let  $D''$ ,  $E''$ ,  $F''$  be the midpoint of  $E'F'$ ,  $F'D'$ ,  $D'E'$  .

Easy to see  $D''$  lie on  $\odot(AX)$ ,  $E''$  lie on  $\odot(BY)$ ,  $F''$  lie on  $\odot(CZ)$  .

Since  $IA \cdot ID'' = IB \cdot IE'' = IC \cdot IF''$ ,

so  $IH$  is the radical axis of  $\odot(AX)$ ,  $\odot(BY)$ ,  $\odot(CZ)$ ,

hence the Newton line of complete quadrilateral form by  $\triangle ABC$  and  $l$  is perpendicular to  $IH$  .

By a well known property of complete quadrilateral (see **Remark**) we get

the perspective axis of  $\triangle ABC$  and  $\triangle DEF$  is perpendicular to  $IH$  .

Q.E.D

**Remark:**

Let  $d'$  be the isotomic line of  $d$  .

Then the Newton line of complete quadrilateral form by  $\triangle ABC$  and  $d$  is parallel to  $d'$ .

Proof :

Let  $D, E, F$  be the intersection of  $d$  and  $BC, CA, AB$ .

Let  $D', E', F'$  be the intersection of  $d'$  and  $BC, CA, AB$ .

Let  $X, Y, Z$  be the midpoint of  $BC, CA, AB$  and  $X' = AD \cap YZ, Y' = BE \cap ZX, Z' = CX \cap XY$ .

Easy to see  $\triangle XYZ \cap X' \cap Y' \cap Z' \sim \triangle ABC \cap D' \cap E' \cap F'$ ,  
so the Newton line  $X'Y'Z'$  of  $\triangle ABC \cap d$  is parallel to  $d' = D'E'F'$ .

This post has been edited 1 time. Last edited by TelvCohl, Mar 31, 2016, 3:39 am

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## High School Olympiads

Nice geometry: EMMO 2016, Jr. 5 

 Reply

Source: Based largely on a problem of Tran Quang Hung (buratinogigle) and its solution by Telv.



**AdithyaBhaskar**

#1 May 7, 2016, 9:40 pm • 2 

Let  $\triangle ABC$  be a triangle with circumcenter  $O$  and circumcircle  $\Gamma$ . The point  $X$  lies on  $\Gamma$  such that  $AX$  is the  $A$ -symmedian of triangle  $\triangle ABC$ . The line through  $X$  perpendicular to  $AX$  intersects  $AB, AC$  in  $F, E$ , respectively. Denote by  $\gamma$  the nine-point circle of triangle  $\triangle AEF$ , and let  $\Gamma$  and  $\gamma$  intersect again in  $P \neq X$ . Further, let the tangent to  $\Gamma$  at  $A$  meet the line  $BC$  in  $Y$ , and let  $Z$  be the antipode of  $A$  with respect to circle  $\Gamma$ . Prove that the points  $Y, P, Z$  are collinear.

Notes: 1. The  $A$ -symmedian of triangle  $\triangle ABC$  is the reflection of the  $A$ -median in the  $A$ -angle bisector.

2. The antipode of a point with respect to a circle is the point on the circle diametrically opposite to it.

*Proposed by Adithya Bhaskar*

This post has been edited 2 times. Last edited by AdithyaBhaskar, May 7, 2016, 10:55 pm



**aopser123**

#2 May 7, 2016, 10:52 pm • 1 

 AdithyaBhaskar wrote:

The point  $X$  lies on  $\Gamma$  such that  $AP$  is the  $A$ -symmedian of triangle  $\triangle ABC$ .

?



**AdithyaBhaskar**

#3 May 7, 2016, 10:54 pm • 1 

Sorry, edited.



**Luis González**

#4 May 8, 2016, 2:25 am • 5 

Obviously  $Z \in EF$  and since  $AX$  is the polar of  $Y$  WRT  $\Gamma$ , then  $AX$  is perpendicular to  $OY$  at  $S$ . Therefore,  $OY$  is the  $A$ -midline of  $\triangle AEF$ , cutting  $AE, AF$  at their midpoints  $U, V \Rightarrow \odot(XUV)$  is 9-point circle of  $\triangle AEF$ .

Let  $P^*$  be the 2nd intersection of  $YZ$  with  $\Gamma$ . Then  $\angle XP^*Z = \angle XAZ = \angle AYO = \angle AP^*S = \angle XYO \Rightarrow \odot(P^*XY)$  is tangent to  $OY$  and  $\angle SP^*X = \angle AP^*Z = 90^\circ$ . Thus if  $M \equiv P^*X \cap OY$ , we have

$MS^2 = MP^* \cdot MX = MY^2 \Rightarrow M$  is midpoint of  $YS$ . Together with  $(S, U, V, Y) = -1$ , we have

$MS^2 = MY^2 = MU \cdot MV \Rightarrow MP^* \cdot MX = MU \cdot MV \Rightarrow P^* \in \odot(XUV) \Rightarrow P \equiv P^* \Rightarrow P, Y, Z$  are collinear.



**anantmudgal09**

#5 May 24, 2016, 1:42 am

Here is a quick sketch: a standard  $\sqrt{bc}$  inversion reduces this to an intermixing of Russian MO 2015 10th grade P7 and IMO Shortlist 2011 G4.



**pi37**

#6 May 24, 2016, 6:18 am • 1 

Let  $M, N$  be the midpoints of  $AE, AF$ . Note that since  $(A, X; B, C)$  is harmonic,  $YX$  is tangent to  $\Gamma$ , so  $MN$ , the perpendicular bisector of  $AX$ , is the line  $OY$ . Now let  $MN$  intersect  $AX$  at  $D$ , and note that

$(D, Y; M, N) = (AD, AY; AC, AB) = -1$ . Consider an inversion about  $Y$  preserving  $\Gamma$ . It fixes  $A, X$  and swaps  $B, C$ , and we wish to show it swaps  $P, Z$ . It suffices to show that the image of  $\gamma$  passes through  $Z$ . Let  $M', N', D'$  be the inverses of  $M, N, D$ . Since  $(Y, D; M, N)$  is harmonic,  $D'$  is the midpoint of  $M'N'$ . But since  $D'$  is the midpoint of  $AX$ ,  $D' = O$ . Thus the center of  $\gamma'$  lies on the perpendicular bisector of  $M'N'$ , which, since  $XZ \parallel MN$ , is the perpendicular bisector of  $XZ$ . This implies  $\gamma'$  passes through  $Z$ , as desired.

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## High School Olympiads



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Source: EMMO 2016, Sr

**Ankoganit**#1 May 7, 2016, 9:45 pm • 1 

In  $\triangle ABC$ , points  $A_1, B_1, C_1$  are feet of altitudes opposite to vertices  $A, B, C$  respectively. Let points  $A_b, A_c$  be on the lines  $AB, AC$  such that  $\angle BA_bB_1 = \angle ABC$  and  $\angle CA_cC_1 = \angle ACB$ . Let  $A_bA_c \cap B_1C_1 = X_A$ . Analogously define  $X_B, X_C$  are defined. Prove that lines  $AX_A, BX_B, CX_C$  concur on circumcircle of  $\triangle ABC$ .

Anant Mudgal, IMOTC-2016, Senior Batch

This post has been edited 1 time. Last edited by Ankoganit, May 8, 2016, 8:33 pm

**Luis González**#2 May 8, 2016, 12:17 am • 3 

Since  $\angle BA_bB_1 = \angle ABC = \angle AB_1C_1$ , then  $\odot(AA_bB_1)$  is tangent to  $B_1C_1$  and similarly  $\odot(AA_cC_1)$  is tangent to  $B_1C_1$ . Thus using the result of the problem [AF perpendicular to Euler line](#) on  $\triangle AB_1C_1$ , we deduce that  $AX_A$  is perpendicular to the Euler line of  $\triangle AB_1C_1$ . But since  $B_1C_1$  is antiparallel to  $BC$  WRT  $AB, AC$ , then it follows that  $AX_A$  is the isogonal (WRT  $\angle BAC$ ) of the perpendicular from  $A$  to the Euler line  $OH$  of  $\triangle ABC \implies AX_A$  cuts  $\odot(ABC)$  again at the anti-Steiner point of  $OH$  WRT  $\triangle ABC$ , i.e. it's Euler reflection point. Similarly  $BX_B$  and  $CX_C$  pass through this point.

**anantmudgal09**

#3 May 8, 2016, 6:07 pm

Proposed by Me. 

P.S.- Did anyone do it in Jr batch?

**Ankoganit**

#4 May 8, 2016, 8:34 pm

 anantmudgal09 wrote:

P.S.- Did anyone do it in Jr batch?

I guess you expect a 'no', and your guess is right AFAIK. 😊

**anantmudgal09**

#5 May 8, 2016, 8:56 pm

 Ankoganit wrote: anantmudgal09 wrote:

P.S.- Did anyone do it in Jr batch?

I guess you expect a 'no', and your guess is right AFAIK. 😊

Hmm, I did expect a No indeed, but expected some reasonable progress since my solution is highly motivated.

**WizardMath**

#6 May 17, 2016, 7:49 pm

No, Arya solved it using Cevian Nest and some angle chasing(as far as he claims).

 Quick Reply



## High School Olympiads

**AF perpendicular to Euler line** 

 Reply

Source: CHKMO 2014



**boblimb**

#1 Feb 8, 2015, 9:43 pm • 1 

Let  $\triangle ABC$  be a scalene triangle, and let  $D$  and  $E$  be points on sides  $AB$  and  $AC$  respectively such that the circumcircles of triangles  $\triangle ACD$  and  $\triangle ABE$  are tangent to  $BC$ . Let  $F$  be the intersection point of  $BC$  and  $DE$ . Prove that  $AF$  is perpendicular to the Euler line of  $\triangle ABC$ .

This post has been edited 1 time. Last edited by boblimb, Nov 10, 2015, 6:47 pm  
Reason: Fix latex



**Luis González**

#2 Feb 8, 2015, 10:26 pm • 4 

Let  $Q, R$  be the projections of  $B, C$  on  $AC, AB$ .  $H \equiv BQ \cap CR$  is the orthocenter of  $\triangle ABC$ . Inversion with center  $A$  and power  $AB \cdot AR = AC \cdot AQ$  takes  $BC$  into  $\omega_A \equiv \odot(AQHR)$  and  $\odot(ABE), \odot(ACD)$  into the tangents of  $\omega_A$  at  $R, Q$ . They pass through the midpoint  $M$  of  $BC$  (well-known) and cut  $AC, AB$  at the inverses  $E', D'$  of  $E, D$ . Thus  $DE$  is taken into  $\odot(AD'E')$  cutting  $\omega_A$  again at the inverse  $F'$  of  $F$ . Now from the problem [Concyclic Quadrilateral](#),  $F'$  is on Euler line  $OH$  of  $\triangle ABC$ , i.e.  $OHF' \perp AF'F$ , as desired.



**TelvCohl**

#3 Jan 24, 2016, 7:57 pm • 3 

Let  $O, H$  be the circumcenter, orthocenter of  $\triangle ABC$ , respectively. Let  $\odot(H, HA)$  cuts  $CA, AB$  again at  $Y, Z$ , respectively. Clearly,  $YZ$  is antiparallel to  $BC$  WRT  $\angle BAC$ , so  $B, C, Y, Z$  lie on a circle  $\Omega$ . From  $\angle DCB = \angle BAC = \angle CZB$  we get  $CD$  is tangent to  $\Omega$  at  $C$ . Similarly, we can prove  $BE$  is tangent to  $\Omega$  at  $B$ , so from Pascal theorem (for  $BBCCYZ$ )  $\Rightarrow F$  lies on  $YZ$ , hence  $F$  is the radical center of  $\Omega, \odot(O), \odot(H, HA) \Rightarrow AF$  is the radical axis of  $\odot(O)$  and  $\odot(H, HA) \Rightarrow AF$  is perpendicular to the Euler line  $OH$  of  $\triangle ABC$ .



**Aiscrim**

#5 Feb 23, 2016, 6:19 pm • 1 

Let  $D, E, F$  be the projections of  $A, B, C$  on  $BC, CA, AB$  and let  $\{X_B\} = DF \cap AC$ ,  $\{X_c\} = DE \cap AB$ . As  $X_B A \cdot X_B C = X_B F \cdot X_B D$ , we get that  $X_B$  is on the radical axis of the nine point circle and the circumcircle, whence  $X_B X_C$  is perpendicular on the Euler line. It is therefore enough to prove that  $AF \parallel X_B X_C$ , which is a relatively simple [bary bash](#).



**Dukejukem**

#6 Feb 24, 2016, 8:18 am

Let  $G, H, N$  be the centroid, orthocenter, nine-point center of  $\triangle ABC$ , respectively. Let  $\triangle M_a M_b M_c$  be the medial triangle and  $\triangle H_a H_b H_c$  be the orthic triangle. Let  $Y, Z$  be the reflections of  $A$  in  $H_b, H_c$  respectively. Denote  $P \equiv M_b M_c \cap H_b H_c$ .

**Lemma:**  $AP$  is perpendicular to the Euler line  $e$ .

**Proof:** Take  $Q \equiv M_b H_c \cap M_c H_b$ . Then  $Q \in e \equiv GH$  by Pappus' Theorem for  $M_b B H_b M_c C H_c$ . Hence,  $e \equiv QN$ , which is perpendicular to  $AP$  by Brokard's Theorem for  $M_b M_c H_b H_c$ . ■

Dilating the above result with center  $A$  and ratio 2, it follows that the line connecting  $A$  and  $F^* \equiv YZ \cap BC$  is perpendicular to  $e$ . We will show that  $F^* \equiv F$ .

Note that  $\triangle ABY \sim \triangle ZCA$ . Moreover,  $\angle BCA = \angle CDB = \angle CDZ$ , which entails  $ABYC \sim ZCAD$ . Therefore,  $CA/CY = DZ/DA$ . Similarly,  $BA/BZ = EY/EA$ . Thus, Menelaus' Theorem for  $\triangle AYZ$  cut by  $\overline{DEF}$  and  $\overline{BCF^*}$

yields  $FY/FZ = F^+Y/F^+Z$ , i.e.  $F^+ \equiv F$ , as desired.  $\square$

This post has been edited 1 time. Last edited by Dukejukem Feb 24, 2016, 8:21 am



**Complex2Liu**

#7 Feb 25, 2016, 9:29 pm

Here is my solution with complex bashing. Let  $x, y, z$  denote the length of  $\overline{AB}, \overline{BC}, \overline{CA}$ , and  $a, b, c$  be the coordinate of  $A, B, C$  respectively.

Applying Menelaus's Theorem from  $DEF$  and  $\triangle ABC$  yields that

$$\frac{BD}{DA} \cdot \frac{AE}{EC} \cdot \frac{CF}{FB} = 1. \quad (1)$$

Since  $\overline{BC}$  is tangent to  $\odot(ADC)$ , so  $BC^2 = BD \cdot BA \implies BD = \frac{y^2}{x}$ ,  $DA = \frac{x^2 - y^2}{x} \implies \frac{BD}{DA} = \frac{y^2}{x^2 - y^2}$ ,

similarly  $\frac{AE}{EC} = \frac{y^2 - z^2}{y^2}$ . Combining (1) we get  $\frac{CF}{FB} = \frac{x^2 - y^2}{y^2 - z^2}$ .

Clearly  $x^2 = (a - b)(\bar{a} - \bar{b}) = 2 - \frac{a^2 + b^2}{ab}$ , similarly  $y^2 = 2 - \frac{b^2 + c^2}{bc}$ ,  $z^2 = 2 - \frac{c^2 + a^2}{ac}$ . Thus we can calculate the coordinate of  $F$

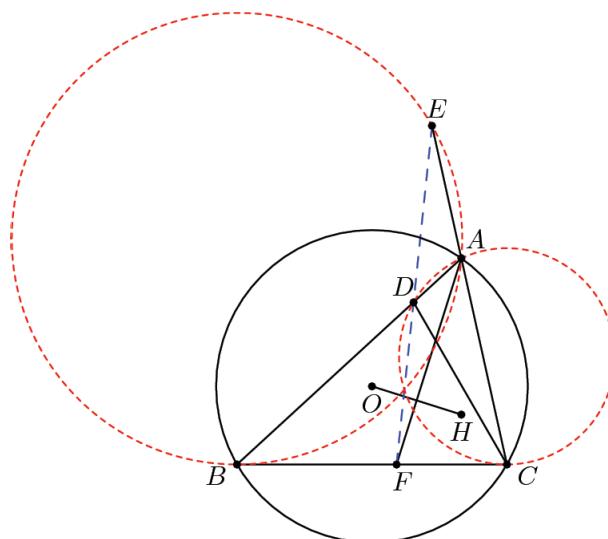
$$f = \frac{x^2 - y^2}{x^2 - z^2}b + \frac{y^2 - z^2}{x^2 - z^2}c = \frac{(a - c)(b^2 - ac)}{(b - c)(a^2 - bc)}b + \left(1 - \frac{(a - c)(b^2 - ac)}{(b - c)(a^2 - bc)}\right)c. \quad (2)$$

Define  $f' = kb + (1 - k)c$  is a point on  $\overline{BC}$  such that  $AF' \perp OH$ , where  $k$  is a constant. It suffices to prove that  $f \equiv f'$ . We have

$$\begin{aligned} AF' \perp OH &\iff 0 = \frac{a - f'}{a + b + c} + \overline{\left(\frac{a - f'}{a + b + c}\right)} \\ &= \frac{a - kb - c + kc}{a + b + c} + \frac{\frac{1}{a} - \frac{k}{b} - \frac{1}{c} + \frac{k}{c}}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} \\ &= \frac{(a - kb - c + kc) \cdot (ab + bc + ca)}{(ab + bc + ca)(a + b + c)} + \frac{(bc - kac - ab + kab) \cdot (a + b + c)}{(ab + bc + ca)(a + b + c)} \\ &= \frac{k(a^2b - a^2c + bc^2 - b^2c) - (ab^2 + ac^2 - b^2c - a^2c)}{(a + b + c)(ab + bc + ca)} \\ &= \frac{k(a^2 - bc)(b - c) - (b^2 - ac)(a - c)}{(a + b + c)(ab + bc + ca)} \end{aligned}$$

Hence  $k = \frac{(b^2 - ac)(a - c)}{(a^2 - bc)(b - c)}$ . Compare with (2) we conclude that  $f \equiv f'$ , as desired. We're done.

A diagram below:



**EulerMacaroni**

#8 May 8, 2016, 1:34 am

Notice that the problem is equivalent to showing that line  $AF$  is the radical axis of the circles with diameters  $\overline{AH}$  and  $\overline{AO}$ . Consider the transformation which is a composition of inversion about  $A$  with power  $r^2 = AB \cdot AC$ , reflection about the  $A$ -angle bisector, and dilation from  $A$  with ratio  $\frac{1}{2}$ . We can now recast the problem as the following:

In triangle  $ABC$ , suppose the tangents to  $\odot(ABC)$  at  $B$  and  $C$  intersect  $AC$  and  $AB$  at  $D$  and  $E$ , respectively. Furthermore, the perpendicular bisectors of  $\overline{AB}$  and  $\overline{AC}$  intersect  $AC$  and  $AB$  at  $P$  and  $Q$ , respectively. If  $\odot(ADE)$  intersects  $\odot(ABC)$  again at  $F$ , show that lines  $AF$ ,  $PQ$ , and  $BC$  are concurrent.

Since  $\angle BPC = 180^\circ - 2\angle BAC = \angle BQC$ , it follows that  $\odot(BQPC)$  is cyclic, and  $X \equiv BD \cap CE$  lies on this circle. By angle chasing, we get that  $PX \parallel BC$  and  $QX \parallel AC$ . Then

$$(D, C; P, P_{AC\infty}) \stackrel{X}{=} (B, E; P_{AB\infty}, Q) \implies \frac{PC}{PD} = \frac{QB}{QE}$$

which implies that  $F$  is the spiral center sending  $DE$  to  $PQ$ , hence  $\odot(AFPQ)$  is cyclic. The conclusion now follows from the radical axis theorem on  $\odot(ABC)$ ,  $\odot(BPQC)$ , and  $\odot(AFPQ)$ .

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## High School Olympiads

### Concurrency from exterior equilateral triangles X

[Reply](#)

Source: Baltic Way 2003

**WakeUp**

#1 Nov 7, 2010, 8:22 pm

Equilateral triangles  $AMB$ ,  $BNC$ ,  $CKA$  are constructed on the exterior of a triangle  $ABC$ . The perpendiculars from the midpoints of  $MN$ ,  $NK$ ,  $KM$  to the respective lines  $CA$ ,  $AB$ ,  $BC$  are constructed. Prove that these three perpendiculars pass through a single point.

**Luis González**

#2 Nov 7, 2010, 11:48 pm

Let  $L$  be the reflection of  $N$  across line  $BC$ . Since  $\angle LBC = \angle MBA = 60^\circ$ , we deduce that  $\angle MBL = \angle ABC$  and because of  $MB = MA$  and  $BL = BC$ , it follows that  $\triangle ABC \cong \triangle MBL$  by SAS criterion  $\implies ML = AC = AK$ . Similar reasoning yields  $KL = AM \implies AMLK$  is a parallelogram, thus  $AL$  and  $MK$  bisect each other at  $U$ . If  $H_a$ ,  $M_a$  denote the foot of the A-altitude and midpoint of  $BC$ , then perpendicular  $\ell_a$  from  $U$  to  $BC$  is the midline of the trapezoid  $ALM_aH_a \implies \ell_a$  passes through the 9-point center of  $\triangle ABC$ . Likewise, perpendiculars from midpoints of  $MN$ ,  $NK$  to  $CA$ ,  $AB$  pass through the 9-point center of  $\triangle ABC$ .

P.S. The result is still true for similar isosceles triangles  $AMB, BNC, CKA$  and the proof is similar.

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## High School Olympiads

Similar Isosceles Triangles Median Concurrence X

[Reply](#)



Source: (China) WenWuGuangHua Mathematics Workshop



XmL

#1 Jan 27, 2013, 6:41 am

See Attachment.

This problem is proposed by PCHP from WenWuGuangHua Mathematics Workshop in China

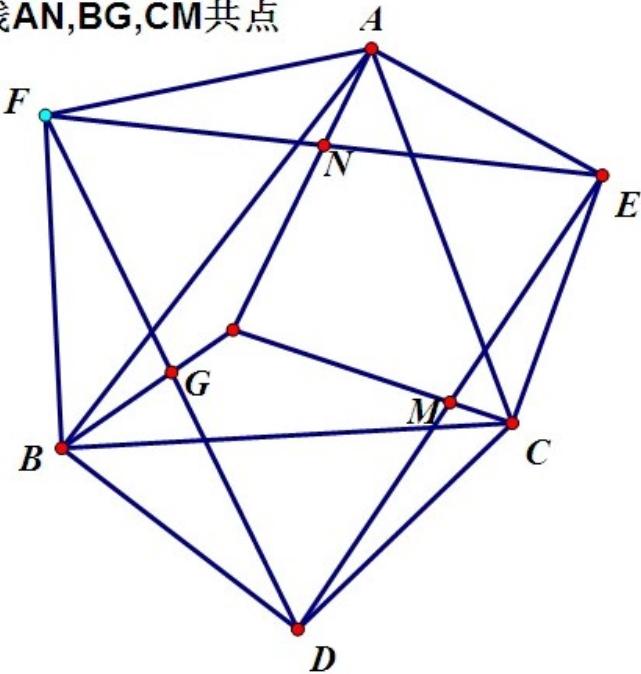
Attachments:

已知 (文武光华数学工作室 南京 潘成华)  $\triangle AFB \sim \triangle AEC$

$\sim \triangle BDC$ ,  $AF = BF$ , 点  $M, N, G$  分别是  $ED, EF, DF$  中点

(2013 1 17 19:46)

求证 直线  $AN, BG, CM$  共点



$\triangle AFB \sim \triangle AEC \sim \triangle BDC$ ,  $AF = BF, M, N, G$  are the midpoints of  $ED, EF, DF$ .  
Prove:  $AN, BG, CM$  are concurrent.



Luis González

#2 Jan 27, 2013, 8:58 am

Let  $D', E', F'$  be the reflections of  $D, E, F$  about  $BC, CA, AB$ .  $\triangle FAB$  and  $\triangle D'CB$  are then spirally similar with center  $B \Rightarrow \triangle FBD'$  and  $\triangle ABC$  are also spirally similar with center  $B \Rightarrow \frac{FD'}{AC} = \frac{FB}{AB} = \frac{AE}{AC} \Rightarrow FD' = AE$ . Similarly,  $ED' = AF \Rightarrow AED'F$  is a parallelogram with diagonal intersection  $N \equiv AD' \cap EF$ . Likewise,  $BG$  and  $CM$  pass through  $E'$  and  $F'$ , thus  $AN \equiv AD', BG \equiv BE'$  and  $CM \equiv CF'$  concur at the inner Kiepert perspector of  $\triangle ABC$  relative to  $\angle DBC = \angle ECA = \angle FAB$ .

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## High School Olympiads



## Kiepert center lies on the Euler line of Kiepert triangle



Reply



Source: Own



TelvCohl

#1 May 4, 2016, 8:53 pm • 2

Given a  $\triangle ABC$  and an acute angle  $\theta$ . Construct isosceles triangles  $\triangle BDC, \triangle CEA$  and  $\triangle AFB$  with base angle  $\theta$  on the sides of  $\triangle ABC$ . Let  $H_D, H_E, H_F$  be the orthocenter of  $\triangle BDC, \triangle CEA, \triangle AFB$ , respectively. Prove that  $AH_D, BH_E, CH_F$  and the Euler line of  $\triangle DEF$  are concurrent.



Scorpion.k48

#2 May 4, 2016, 10:22 pm • 1

**Solution:**

Let  $X, Y, Z$  be the circumcenter of  $\triangle AFB, \triangle AEC, \triangle BH_DC$ . We have

$(XB, XA) \equiv (YA, YC) \equiv (ZB, ZC) \equiv 2\theta (\text{mod } \pi)$  and  $\triangle AXB, \triangle AYC, \triangle BZC$  are isosceles, so  $\triangle XBA \stackrel{\sim}{\rightarrow} \triangle ZBC \stackrel{\sim}{\rightarrow} \triangle YAC$ . Therefore,  $\triangle XBZ \stackrel{\sim}{\rightarrow} \triangle ABC \stackrel{\sim}{\rightarrow} \triangle YZC$ . Because  $ZB = ZC$ , we have  $\triangle XBZ \stackrel{\sim}{\rightarrow} \triangle YZC$ .

Therefore,  $ZY = XB = YA$  and  $XZ = YC = XC$ . We have  $AXZY$  is parallelogram.

We have  $(XF, XZ) \equiv (XF, YA) + (YA, XZ) \equiv (YA, YE) + (YZ, YA) \equiv (YZ, YE) (\text{mod } \pi)$  and  $XF = YZ, XZ = YE$ , so  $\triangle XFZ \stackrel{\sim}{\rightarrow} \triangle YZE$  and we have  $ZF = ZE$ . Let  $O, H$  be circumcenter, orthocenter of  $\triangle DEF$ , we have  $ZO \perp EF, DH \perp EF$

We have a **Lemma**:  $AH_D \perp EF$ . So  $ZO // DH // AH_D$ . Let  $S = AH_D \cap OH$  and  $X', Y'$  be circumcenter of  $\triangle AH_F B, \triangle AH_E C$ . Because  $\triangle BDC \stackrel{\sim}{\rightarrow} \triangle CEA \stackrel{\sim}{\rightarrow} \triangle AFB$ , so  $\frac{ZH_D}{ZD} = \frac{Y'H_E}{YZ} = \frac{X'H_F}{YE} = k$ . Using Thales  $\frac{OS}{OH} = \frac{ZH_D}{ZD} = k$ . Similarly, we have  $BH_E, CH_F$  goes through  $S$  and we have done.

This post has been edited 1 time. Last edited by Scorpion.k48, May 4, 2016, 10:32 pm



Luis González

#3 May 5, 2016, 10:55 am • 2

Let  $X, Y, Z$  be the midpoints of  $BC, CA, AB$ . If  $AH_D$  cuts  $\odot(BCH_D)$  again at  $J$ , we get

$\angle BJH_D = \angle CJH_D = \angle BDX$ . If the parallels from  $Y, Z$  to  $J$ ,  $JB$  meet at  $L$ , then  $\triangle LYZ$  and  $\triangle JCB$  are homothetic with center  $A \implies L \in AH_D$  and  $\angle ZLJ = \angle YLJ = \angle BJH_D = \angle BDX = \angle AEY = \angle AFZ \implies AEYL$  and  $AFZL$  are cyclic  $\implies L$  coincides with the projection of  $A$  on  $EF \implies AH_D \perp EF$ .

On the other hand, it's known that  $\triangle ABC$  and  $\triangle DEF$  have the same centroid  $G$  (see [prove that two triangles share their centroid](#)), thus if  $T$  is the orthocenter of  $\triangle DEF$ , then  $TG$  is Euler line of  $\triangle DEF$  cutting  $AH_D$  at  $P$ . If  $U \equiv AX \cap DT$ , then from  $(DT \parallel H_D A) \perp EF$ , we obtain

$$k = \frac{XD}{DH_D} = \frac{UX}{AU} = \frac{\frac{1}{2}AG + UG}{AU} = \frac{1}{2} \cdot \frac{PG}{PT} + \frac{TG}{PT} = \frac{1}{2} + \frac{3}{2} \cdot \frac{TG}{PT} \implies \frac{TG}{PT} = \frac{2}{3} \left( k - \frac{1}{2} \right).$$

Since  $k$  is clearly a constant, then it follows that  $BH_E$  and  $CH_F$  hit  $TG$  at the same point  $P$ .



Luis González

#4 May 6, 2016, 11:21 pm

Another property of the Kiepert's perspector  $K(90^\circ - \theta) \equiv AH_D \cap BH_E \cap CH_F$ . This point is collinear with the Kiepert's perspector  $K(-\theta)$  and the 9-point center  $N$  of  $\triangle ABC$ .

**Proof:** Let  $A', B', C'$  be the midpoints of  $EF, FD, DE$ , respectively. From [Concurrency from exterior equilateral triangles](#) (see post #2), we get that the perpendiculars from  $A', B', C'$  to  $BC, CA, AB$  concur at the 9-point center  $N$  of  $\triangle ABC$  and from [Similar Isosceles Triangles Median Concurrence](#),  $AA', BB', CC'$  concur at  $K(-\theta)$ . Thus  $\triangle ABC$  and  $\triangle A'B'C'$  are orthologic with orthology centers  $N, K(90^\circ - \theta)$  and perspective with perspector  $K(-\theta)$ . By Sondat's theorem,  $N, K(-\theta)$  and  $K(90^\circ - \theta)$  are collinear.

Quick Reply



## High School Olympiads

Nine-point center lies on Euler line X

Reply



Scorpion.k48

#1 May 6, 2016, 8:17 pm

Let  $\triangle ABC$  with nine-point center  $N$ . Let  $\triangle N_a N_b N_c$  is an antipedal triangle of  $N$  WRT  $\triangle ABC$ . Prove that nine-point center of  $\triangle N_a N_b N_c$  lies on Euler line of  $\triangle ABC$



Luis González

#2 May 6, 2016, 8:36 pm • 1

This was discussed before at [Nine-point Center Lies On OI](#). See lemma 2 at post #7 and the previous lemma in my solution at post #12.

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## High School Olympiads

Nine-point Center Lies On OI X

↶ Reply



Arab

#1 Aug 20, 2013, 6:20 pm

Let  $O, I$  be the circumcenter and incenter of  $\triangle ABC$  and  $D = AI \cap BC, E = BI \cap CA, F = CI \cap AB$ . Then the nine-point center of  $\triangle DEF$  lies on  $OI$ .

Or

Let  $\triangle A'B'C'$  be the pedal triangle of  $\triangle ABC$  where  $A' \in BC, B' \in CA, C' \in AB$ . If  $X = AA' \cap B'C', Y = BB' \cap C'A', Z = CC' \cap A'B'$ , then the nine-point center of  $\triangle XYZ$  lies on the Euler Line of  $\triangle ABC$ .

Is it well-known?



jayme

#2 Aug 20, 2013, 9:09 pm

Dear Mathlinkers,  
I think through my tool that there is a typo... you mind perhaps that DEF is the i-pedal triangle of ABC  
Sincerely  
Jean-Louis



Math-lover123

#3 Aug 20, 2013, 11:30 pm

If  $\triangle DEF$  is intouch triangle of  $\triangle ABC$  then the result is well known and can be proved using homothety or inversion.



XmL

#4 Aug 21, 2013, 5:06 am • 1 thumb up

Arab wrote:

Let  $O, I$  be the circumcenter and incenter of  $\triangle ABC$  and  $D = AI \cap BC, E = BI \cap CA, F = CI \cap AB$ . Then the nine-point center of  $\triangle DEF$  lies on  $OI$ .

Or

Let  $\triangle A'B'C'$  be the pedal triangle of  $\triangle ABC$  where  $A' \in BC, B' \in CA, C' \in AB$ . If  $X = AA' \cap B'C', Y = BB' \cap C'A', Z = CC' \cap A'B'$ , then the nine-point center of  $\triangle XYZ$  lies on the Euler Line of  $\triangle ABC$ .

Is it well-known?

The first one is one of the equivalences I talked about! 😊 I will try these again.



Arab

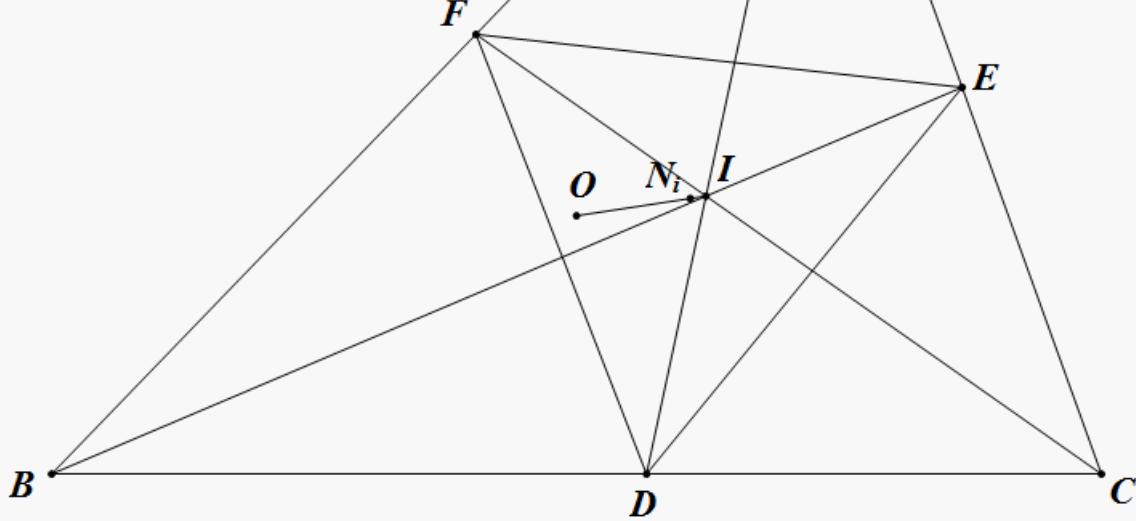
#5 Aug 21, 2013, 6:33 am

@jayme and all mathlinkers  
There are no typos. Please have a check again.

@XmL

Oh, that's really nice. In fact these two are equal.

$$\angle ON_I = 180.00000^\circ$$



**duanby**

#6 Nov 10, 2013, 1:33 pm • 1

Here's my solution:

use the first notion, let  $L, M, N$  be the midpoint of  $EF, DF, DE$   
to prove the nine point center of  $\triangle ABC$  collinear with  $O, I$

Use barycentric coordinate

it's sufficient to prove the radical axis of  $(O), (DEF)$  is parallel to that of  $(O), (I)$

the coordinate  $D(0, b, c) E(a, 0, c) F(a, b, 0)$

$L(a(2a + b + c), b(a + c), c(a + b))$

$M(a(b + c), b(a + 2b + c), c(a + b))$

$N(a(b + c), b(a + c), c(a + b + 2c))$

$$\text{Assume } (LMN) : -\sum a^2yz + (x + y + z)\left(\frac{\alpha}{a}x + \frac{\beta}{b}y + \frac{\gamma}{c}z\right) = 0$$

$$\text{The radical axis of } (O), (I) : \sum (s - a)^2x = 0$$

$$\text{It's sufficient to prove } \sum_{cyc} \alpha(b - c) = 0$$

For  $\alpha, \beta, \gamma$  we have

$$2(a + b)(a + c)[\alpha(2a + b + c) + \beta(a + c) + \gamma(a + b)]$$

$$= abc[a(a + c)(a + b) + b(a + b)(2a + b + c) + c(a + c)(2a + b + c)] \text{ and two other similar equation by cycling the letter}$$

Or in short,

$$\alpha(2a + b + c) + \beta(a + c) + \gamma(a + b) = P$$

$$\alpha(b + c) + \beta(a + 2b + c) + \gamma(a + b) = Q$$

$$\alpha(b + c) + \beta(a + c) + \gamma(a + b + 2c) = R$$

$$\implies 2(a + b)(b + c)(c + a)\alpha = P(b + c)(a + b + c) - Qc(a + c) + Rb(a + b)$$

$$\begin{aligned}
& \text{Therefore } \sum_{cyc} \alpha(b - c) = 0 \\
& \iff \sum_{cyc} (b - c)(b + c)(a + b + c)P - Qc(a + c)(b - c) - Rb(a + b)(b - c) = 0 \\
& \iff \sum_{cyc} P(b + c)(b^2 - c^2) = 0 \\
& \iff \sum_{cyc} (b + c)^2(b^2 - c^2)[a(a + c)(a + b) + b(a + b)(2a + b + c) + c(a + c)(2a + b + c)] = 0 \\
& \iff \sum_{cyc} (a^3b^4 - b^3c^4) + (b^4c^3 - a^3c^4) + (3a^2b^5 - 3b^2c^5) + (3c^2b^5 - 3a^2c^5) + (3b^6c - 3ac^6) + (3ab^6 - 3bc^6) + (b^7 - c^7) + (9a^2b^4c - 9ab^2c^4) + (9ab^4c^2 - 9a^2bc^4) + (2a^3b^3c - 2a^3bc^3) + (6a^2b^3c^2 - 6a^2b^2c^3) + (9ab^5c - 9abc^5) = 0
\end{aligned}$$

which is true.



IDMasterz

#7 Jul 17, 2014, 2:20 am • 10

XML and I have found a synthetic solution to this problem, and will be outlined in an article we hope to be completed in due course.

### Lemma 1:

Let  $ABC$  be a triangle with  $H$  its orthocentre,  $O$  its circumcentre,  $K$  its kosnita point, and  $N$  is its ninepoint centre. Let  $BN \cap AC = N_B$ ,  $CN \cap AB = N_C$ , and  $N_B N_C \cap BC = A_1$ . Let  $K_A$  be the reflection of  $K$  over  $BC$ . Then,  $HK_A \perp AA_1$ .

### Proof:

Since  $BH \perp AC$ , therefore it suffices to prove  $\angle(HK_A, HC) = \angle(AA_1, AB)$ . Let  $H'$  denote the reflection of  $H$  over  $BC$ , which lies on  $(O)$ , define  $H'K \cap (O) = A_2$ ,  $AK \cap (O) = A_3$ , we will prove that  $A_2 C A_3 B$  is a harmonic quadrilateral.

Let  $O_B, O_C$  denote the reflections of  $O$  over  $AC, AB$  respectively. It's well known that  $N$  is the midpoint of  $BO_B, CO_C$ , hence  $BCO_B O_C$  is a parallelogram. Consider the isogonal conjugate of  $K$  wrt  $CBH'$ , denoted by  $K'$ . Since  $\angle CBH' = 90 - \angle C = \angle ABO_C, \angle K'BH' = \angle KBC = \angle ABN$ , therefore  $\angle CBK' = \angle ABN \Rightarrow BK', BO_C$  are isogonals wrt  $\angle NBC$ . Symmetrical result can be obtain on  $\angle NCB$ , and since  $BO_C \parallel CO_B \parallel ND$  where  $D$  is the midpoint of  $BC$ , therefore the isogonal conjugate of  $K'$  lies on  $ND$  and moreover  $K' \in (NBC)$ . Hence  $ND, NK'$  are isogonals wrt  $\angle BNC \Rightarrow BNCK'$  is a harmonic quadrilateral.

Let  $AN \cap (O) = A_4, K'H' \cap (NBC) = K*$ . Note that  $\angle(AA_4, H'A_4) = \angle(AC, CH') = \angle(NC, CK') = \angle(NK*, K*K')$ , therefore  $N, A_4, K*, H'$  are concyclic. By the radical axes theorem among  $(O), (NBC), (NA_4H'K*)$ ,  $NK*, H'A_4$  concur on  $BC$ . This means  $H'(C, B; K', A_4) = K*(C, B; K', N) = -1$  since  $BHCK'$  is harmonic. Now apply a reflection over the angle bisector of  $BH'C$ . We have  $H'(C, B; A_2, A_3) = -1 \Rightarrow A_2 C A_3 B$  is a harmonic quadrilateral.

Now  $A_2 C A_3 B$  is a harmonic quadrilateral  $\Rightarrow (AC, AB; AK, AA_2) = -1$ . When we reflect these pencils over the angle bisector of  $\angle BAC$ ,  $AB \rightarrow AC, AC \rightarrow AB, AK \rightarrow AN$ . Let  $AA_2 \rightarrow j$ . Since the reflection preserves the cross ratio of these pencils, therefore  $(AC, AB; AN, j) = -1$ . Since by properties of complete quadrilaterals  $(AC, AB; AN, AA_1) = -1$ , therefore  $j \equiv AA_1 \Rightarrow AA_2, AA_1$  are isogonals wrt  $\angle BAC$   
 $\Rightarrow \angle(HK_A, HC) = \angle(CH', H'A_2) = \angle(AC, AA_2) = \angle(AA_1, AB)$  and we are done.

### Corollary:

In the same configuration, define  $B_1$  similarly to  $A_1$ , and  $K_B$  similarly to  $K_A$ . If the perpendiculars from  $O$  to  $AA_1, BB_1$  intersect  $NK_A, NK_B$  at  $X, Y$ , then  $XY \parallel K_A K_B$ .

### Lemma 2:

Let  $ABC$  be a triangle and  $N$  its ninepoint centre, and its antipedal triangle be  $A^*B^*C^*$ . The ninepoint centre,  $N'$ , of  $A^*B^*C^*$  lies on the euler line of  $ABC$ .

### Proof:

Let  $AN \cap \odot NBC = T, BN \cap \odot NAC = U, CN \cap NAB = V$ . We have

$\angle ATA^* = \frac{\pi}{2} = \angle(AN, B^*C^*) \Rightarrow TA^* \parallel B^*C^*$ . Let  $M_A M_B M_C$  be the medial triangle  $A^*B^*C^*$ , and notice that  $N'$  is the centre of  $M_A M_B M_C$ . Consider the circles with diameters  $NM_A, NM_B, NM_C$ , and denote them by  $\Gamma_A, \Gamma_B, \Gamma_C$ . Notice the circles contain  $A, B, C$  respectively by definition. Let  $\Gamma_A \cap \odot NBC = P$ . As  $A^*T \parallel AM_A$ , by the converse of Reim's theorem,  $P \in A^*M_A$ .

It follows that  $(A * M, A^*T, A^*B^*, A^*C^*) = -1 \Rightarrow PBTC$  is a harmonic quadrilateral. Thus,

$(NP, NT, NB, NC) = -1 \implies NP$  intersects  $BC$  at the trilinear polar of  $N$  with respect to  $ABC$ . By radical axis theorem on  $\Gamma_A, \odot NBC, ABC$ , it follows that if  $\Gamma_A$  intersects  $\odot ABC$  at  $A_2$ , then  $AA_2 \cap NP \in BC = A_1$  where  $A_1$  is defined as in lemma 1.

Suppose we let  $O_1, O_2, O_3$  be the centres of  $\Gamma_A, \Gamma_B, \Gamma_C$ . The homothety centered at  $N$  with ratio 2 takes  $O_1O_2O_3 \mapsto M_AM_BM_C$ , so if we let the centre of  $O_1O_2O_3$  be  $O_4$ , we have  $O_4 \mapsto O$ . Clearly, it suffice to prove that  $O_4$  lies on the euler line of  $ABC$ . Due to  $AA_1$  being the radical axis of  $ABC$  and  $\Gamma_A$ , it follows if  $O$  is the centre of  $ABC$  that  $OO_1 \perp AA_1$  and similarly,  $OO_2 \perp BB_1, OO_3 \perp CC_1$ .

If  $K_A$  is defined as in lemma 1 and similarly  $K_B, K_C$ , then  $K_AK_BK_C$  is homothetic to  $O_1O_2O_3$ , and it is well known that  $N$  is the circumcentre of  $K_AK_BK_C$  meaning  $O_4O_1O_2$  is therefore homothetic to  $NK_AK_B$ .

Let the centre of homothety be denoted by  $H_O$ . Obviously,  $H_O = O_1K_A \cap O_2K_B, H_O$  lies on the euler line of  $ABC$ . Considering triangles  $OO_1O_2$  and  $NK_BK_C$ , it suffice to show  $NO \cap K_AO_1 \cap K_BO_2$  exists, or that the triangles are perspective from a point. By Desargues' theorem, it suffice to show that these triangles are perspective from a point, or indeed that  $OO_1 \cap NK_A = X', OO_2 \cap NK_B = Y', K_AK_B \cap O_1O_2 = P_\infty$  are collinear, or that  $X'Y' \parallel K_AK_B$ . But as  $OO_1 \perp AA_1, OO_2 \perp BB_1, X' = X, Y' = Y$ , so we are done the corollary.

### Lemma 3:

Let  $\triangle A'B'C'$  be the incentral triangle of the orthic triangle of  $ABC$ . Prove that the center of homothety of  $\triangle A'B'C', \triangle A^*B^*C^*$  ( $A^*$  is defined as in previous lemmas) lies on the Euler line of  $\triangle ABC$ .

### Proof:

It suffices to prove  $C'C^*, B'B^*, HN$  concur. Let  $NB^* \cap BH = E, NC^* \cap CH = F$ . By Desargue's theorem on  $\triangle HB'C', \triangle NB^*C^*$ , the concurrence is equivalent to  $EF, B'C', B^*C^*$  concur or simply  $EF \parallel B^*C^*$ .

Let the lines through  $B^*, C^*$  perpendicular to  $AC, AB$  respectively meet at  $M$ , hence  $N, M$  are isogonal conjugates with respect to  $A^*B^*C^* \Rightarrow$  the circumcenter of  $ABC, O$  lies on  $NM$  (well known property of isogonal conjugates)  $\Rightarrow M$  lies on the Euler line of  $ABC$ .

Since  $H, N, M$  are collinear, therefore  $\frac{NE}{NB^*} = \frac{HN}{MN} = \frac{NF}{NC^*} \Rightarrow EF \parallel B^*C^*$  and we are done.

### Main Proof:

Combining both lemmas 2.5 and 2.4 means the ninepoint centre of  $A'B'C'$  lies on the euler line of  $ABC$ , as a homothety sends ninepoint centre to ninepoint centre. The orthocentre  $H$  of  $ABC$  is the incentre of orthic triangle, and the ninepoint centre of  $ABC$  is the circumcentre of the orthic triangle. Thus, taking the orthic triangle of  $ABC$  as a reference, we have the ninepoint centre of the incentral triangle lies on the line adjoining the circumcentre and incentre, which is the isogonal conjugate of the feuerbach hyperbola.

### The problem is solved.

This post has been edited 4 times. Last edited by IDMasterz, Aug 17, 2014, 5:57 pm



**XmL**

#8 Jul 17, 2014, 2:37 am • 1

IDMasterz is too pro.

This post has been edited 2 times. Last edited by XmL, Jul 17, 2014, 6:23 am

“

▲



**IDMasterz**

#9 Jul 17, 2014, 2:40 am • 1

I love you man, just remember that you're more pro though.

“

▲



**Arab**

#10 Jul 18, 2014, 6:56 pm • 2

I love you two! Thank you for the exquisite work!

“

▲



**XmL**

#11 Aug 16, 2014, 9:01 am • 2

“

▲

I've found a second synthetic proof for this while investigating the nine-point center, it's now posted on my blog



Luis González

#12 Sep 25, 2015, 5:37 am • 2

**Lemma:**  $N$  is the 9-point center of  $\triangle ABC$  and  $\triangle N_a N_b N_c$  is the antipedal triangle of  $N$  WRT  $\triangle ABC$ . Then the 9-point center of  $\triangle N_a N_b N_c$  is on the Euler line of  $\triangle ABC$ .

**Proof:** Let  $O, K$  be the circumcenter and Kosnita point of  $\triangle ABC$  (isogonal conjugate of  $N$  WRT  $\triangle ABC$ ).  $\triangle K_a K_b K_c$  is the pedal triangle of  $K$  WRT  $\triangle ABC$  with circumcenter  $U$  (midpoint of  $NK$ ) and orthocenter  $T$ . Midpoint  $S$  of  $UT$  is then 9-point center of  $\triangle K_a K_b K_c$ . From the thread [HK passes through the circumcenter](#), we get that  $O, K, T$  are collinear such that  $OK = 2 \cdot KT$ , hence if  $X$  is midpoint of  $OK$ , we have  $KS \parallel UX \parallel ON$ .

On the other hand, if  $J$  is the isogonal conjugate of  $N$  WRT  $\triangle N_a N_b N_c$  (reflection of  $N$  on  $O$ ), we have  $(K_b K_c \parallel N_b N_c) \perp AN, (K_a K_c \parallel JN_a) \perp BC$ , etc  $\Rightarrow \triangle K_a K_b K_c \cup K \sim \triangle N_a N_b N_c \cup J$  are homothetic. Thus if  $V$  is the 9-point center of  $\triangle N_a N_b N_c \Rightarrow JV \parallel KS \parallel ON \Rightarrow V \in ON$ , i.e.  $V$  is on Euler line of  $\triangle ABC$ .

Back to the problem, let  $\triangle I_a I_b I_c$  be the excentral triangle of  $\triangle ABC$  with circumcenter  $Be \in OI$ ; the Bevan point of  $\triangle ABC$ .  $\triangle O_a O_b O_c$  is the antipedal triangle of  $O$  WRT  $\triangle I_a I_b I_c$  and  $J$  is the isogonal conjugate of  $O$  WRT  $\triangle O_a O_b O_c$  (reflection of  $O$  on  $Be$ ). Since  $OI_a \perp EF, OI_b \perp FD, OI_c \perp DE$  (well-known), then it follows that  $\triangle O_a O_b O_c$  are  $\triangle DEF$  are homothetic having parallel sides. Moreover  $(AI \parallel JO_a) \perp I_b I_c$ , etc  $\Rightarrow \triangle DEF \cup I \sim \triangle O_a O_b O_c \cup J \Rightarrow OIJ$  is double under the homothety that takes  $\triangle DEF$  into  $\triangle O_a O_b O_c$ , i.e. it contains their homothety center. Using the previous lemma for  $\triangle I_a I_b I_c$  with 9-point center  $O$ , we get that the 9-point center of  $\triangle O_a O_b O_c$  is on its Euler line  $OI \Rightarrow$  9-point center of  $\triangle DEF$  is on  $OI$  as well.

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## High School Olympiads

prove that two triangles share their centroid X

Reply



minhtue0605

#1 Jun 29, 2012, 6:15 pm

Let be given triangle  $ABC$ .  $X, Y, Z$  are points outside triangle  $ABC$  such that  $\widehat{XBC} = \widehat{YCA} = \widehat{ZAB}$  and  $\widehat{XCB} = \widehat{YAC} = \widehat{ZBA}$ . If  $G$  is the centroid of triangle  $ABC$ , prove that  $G$  is also the centroid of triangle  $XYZ$ .



Luis González

#2 Jun 30, 2012, 1:05 am

$M$  is the midpoint of  $BC$  and  $P$  is the reflection of  $X$  on  $M$ .  $BPCX$  is obviously a parallelogram  $\implies \angle XCB = \angle PBC = \angle ZBA \implies \angle ABC = \angle ZBL$ . But  $\triangle ABZ \sim \triangle BCX$  gives  $\frac{BZ}{BP} = \frac{BZ}{CX} = \frac{BA}{BC}$ , thus  $\triangle ABC \sim \triangle ZBL$  by SAS  $\implies \frac{PZ}{CA} = \frac{BZ}{BA} = \frac{AY}{CA} \implies AY = PZ$ . Similarly,  $AZ = PY$ . Thus  $AYPZ$  is a parallelogram  $\implies PA$  cuts  $YZ$  at its midpoint  $N$ . Therefore,  $L \equiv AM \cap XN$  is the centroid of  $\triangle PAX \implies \overline{LM} : \overline{LA} : -1 : 2$  and  $\overline{LN} : \overline{LX} : -1 : 2 \implies L \equiv G$  is the centroid of  $\triangle ABC$  and  $\triangle XYZ$ .



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## High School Olympiads



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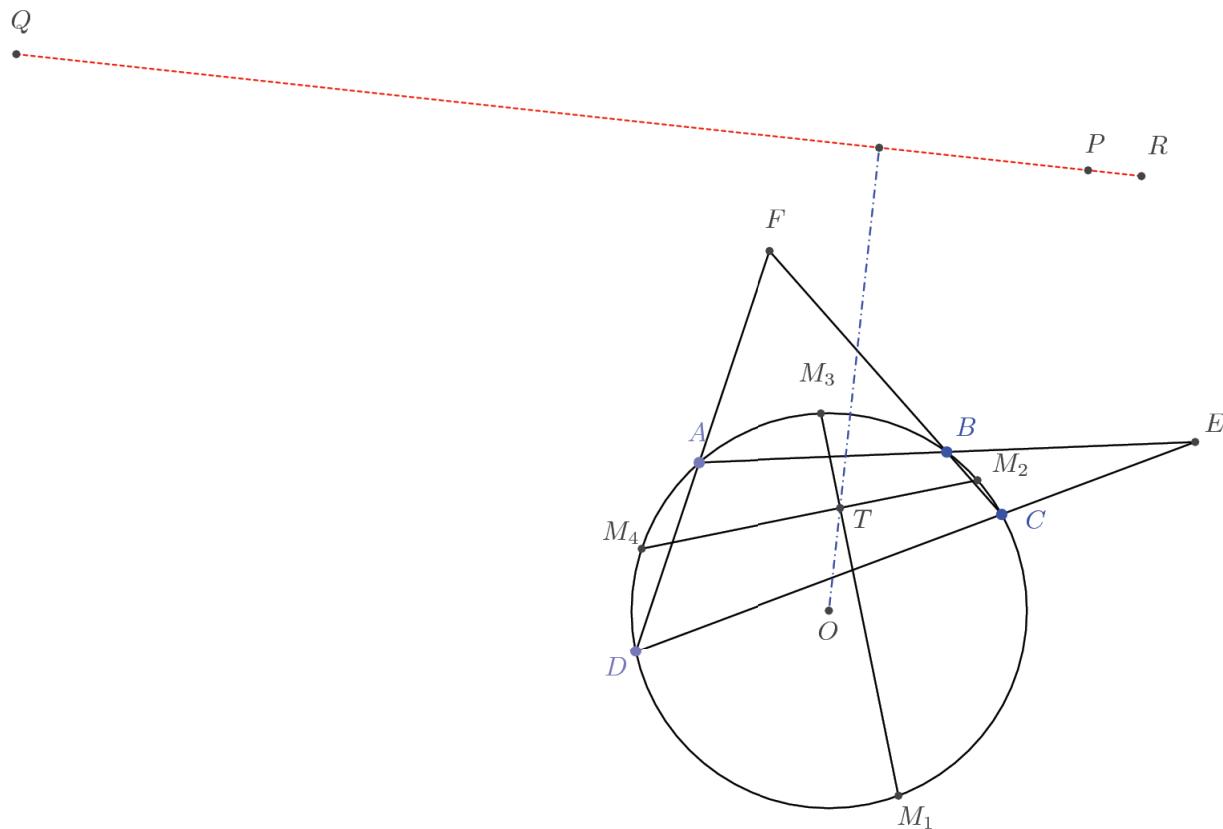


andria

#1 May 2, 2016, 1:59 pm

Let  $ABCD$  be a cyclic quadrilateral with circumcenter  $O$ . Let  $AB \cap CD = E$  and  $AD \cap BC = F$ . External bisectors of  $\angle DAB$  and  $\angle DCB$  intersect at  $P$ , external bisectors of  $\angle ABC$  and  $\angle ADC$  intersect at  $Q$  and external bisectors of  $\angle AED$  and  $\angle AFB$  intersect at  $R$ . Let  $M_1, M_2, M_3, M_4$  be midpoints of inner arcs  $CD, BC, AB, AD$  respectively. Let  $M_1M_3 \cap M_2M_4 = T$ .

Prove that  $P, Q, R$  lie on a line perpendicular to  $OT$ .



Luis González

#2 May 2, 2016, 8:02 pm • 1

Let  $I$  and  $J$  the incenters of  $\triangle FAB$  and  $\triangle EBC$  and let  $K$  and  $L$  be the F-excenter and E-excenter of  $\triangle FCD$  and  $\triangle EDA$ . Let  $S \equiv IK \cap JL$ . For any quadrilateral  $ABCD$  (not necessarily cyclic)  $P, Q, R$  are collinear on the polar of  $S$  WRT  $\odot(IJKL)$  (see [Prove that P, Q, R are collinear](#), post #6). Hence if  $U$  is the center of  $\odot(IJKL)$ , then  $US \perp \overline{PQR}$ .

Particularly, when  $ABCD$  is cyclic, easy angle chase reveals that  $M_1M_3, M_2M_4$  are parallel to the internal bisectors  $IK, JL$  of  $\angle AFB, \angle BEC$  and  $M_1M_2$  is parallel to the external bisector  $JK$  of  $\angle BCD$ , etc. Thus  $M_1M_2M_3M_4 \cup O$  and  $KJIL \cup U$  are homothetic  $\Rightarrow OT \parallel US \Rightarrow OT \perp \overline{PQR}$ .



andria

#3 May 2, 2016, 8:19 pm

My solution:

Since the sum of distances of  $P, Q, R$  from the sides of  $ABCD$  is zero from [Distance sum \(post 8\)](#) we get that they lie on a line perpendicular to  $\frac{\overrightarrow{OM_4}}{|OM_4|} + \frac{\overrightarrow{OM_3}}{|OM_3|} + \frac{\overrightarrow{OM_2}}{|OM_2|} + \frac{\overrightarrow{OM_1}}{|OM_1|} \parallel OT$ .

Q.E.D

This post has been edited 1 time. Last edited by andria, May 2, 2016, 8:22 pm

Quick Reply



## High School Olympiads

Prove that P, Q, R are collinear



Reply



**Amir Hossein**

#1 May 18, 2010, 1:08 am

Let  $ABCD$  be a quadrilateral, and  $E, F$  be intersection points of  $AB, CD$  and  $AD, BC$  respectively.

External bisectors of  $D\hat{A}B$  and  $D\hat{C}B$  intersect at  $P$ , external bisectors of  $A\hat{B}C$  and  $A\hat{D}C$  intersect at  $Q$  and external bisectors of  $A\hat{E}D$  and  $A\hat{F}B$  intersect at  $R$ . Prove that  $P, Q, R$  are collinear.



**yetti**

#2 May 18, 2010, 10:59 am • 1

Let external bisectors  $a, b, c, d$  of the angles  $\widehat{DAB}, \widehat{ABC}, \widehat{BCD}, \widehat{CDA}$  pairwise intersect at  $X \equiv a \cap b, Y \equiv b \cap c, Z \equiv c \cap d, T \equiv d \cap a$ . Likewise, let internal bisectors  $a', b', c', d'$  of these angles pairwise intersect at  $X' \equiv a' \cap b', Y' \equiv b' \cap c', Z' \equiv c' \cap d', T' \equiv d' \cap a'$ . Let  $e, f$  be external and  $e', f'$  internal bisectors of the angles  $\widehat{DEA}, \widehat{AFB}$ . Points  $E, Y, Y', T, T' \in e$  are collinear as incenters/E-excenters of  $\triangle ECB, \triangle EDA$  and similarly, points  $F, Z, Z', X', X$  are collinear as incenters/F-excenters of  $\triangle FDC, \triangle FAB$ . Thus the quadrilaterals  $XYZT$  and  $X'Y'Z'T'$  have common diagonals  $XZ \equiv X'Z'$  and  $YT \equiv Y'T'$  intersecting at  $S$ . Intersections  $I \equiv a' \cap b \equiv T'X' \cap XY, J \equiv a \cap b' \equiv X'Y' \cap TX, K \equiv c' \cap d \equiv Y'Z' \cap ZT, L \equiv c \cap d' \equiv YZ \cap Z'T'$  are all on  $f$  as A-, B-, C-, D-excenters of  $\triangle FAB, \triangle FDC$ .  $P \equiv a \cap c \equiv TX \cap YZ$  and  $Q \equiv b \cap d \equiv XY \cap ZT$  are intersections of the opposite sides of  $XYZT$ . Similarly, let  $P' \equiv a' \cap c' \equiv T'X' \cap Y'Z'$  and  $Q' \equiv b' \cap d' \equiv X'Y' \cap Z'T'$  be intersections of the opposite sides of  $X'Y'Z'T'$ .

$\triangle QTX \equiv \triangle(abd), \triangle P'Y'X' \equiv \triangle(b'a'c')$  are perspective, because intersections  $J \equiv a \cap b', I \equiv b \cap a', K \equiv d \cap c'$  of their corresponding sides are collinear  $\implies$  by Desargues theorem,  $QP', TY', XX'$  are concurrent at  $S$ . Similarly,  $\triangle PXY \equiv \triangle(bca), \triangle Q'X'T' \equiv \triangle(a'd'b')$  are perspective, because intersections  $I \equiv b \cap a', L \equiv c \cap d', J \equiv a \cap b'$  of their corresponding sides are collinear  $\implies PQ', XX', YT'$  are also concurrent at  $S$ .  $XX', PQ', QP'$  are concurrent at  $S \implies \triangle XPQ, \triangle X'Q'P'$  are perspective  $\implies$  intersections  $R \equiv PQ \cap Q'P', J \equiv a \cap b' \equiv XP \cap X'Q', I \equiv b \cap a' \equiv XQ \cap X'P'$  of their corresponding sides are collinear. As a result,  $R \equiv PQ \cap P'Q'$  is on  $f \equiv IJ$ . In exactly the same way,  $R \in e \implies R \equiv e \cap f \cap PQ \cap P'Q'$ .  $\square$



**Amir Hossein**

#3 May 18, 2010, 4:20 pm

" yetti wrote:

Let external bisectors  $a, b, c, d$  of the angles  $\widehat{DAB}, \widehat{ABC}, \widehat{BCD}, \widehat{CDA}$  pairwise intersect at  $X \equiv a \cap b, Y \equiv b \cap c, Z \equiv c \cap d, T \equiv d \cap a$ . Likewise, let internal bisectors  $a', b', c', d'$  of these angles pairwise intersect at  $X' \equiv a' \cap b', Y' \equiv b' \cap c', Z' \equiv c' \cap d', T' \equiv d' \cap a'$ . Let  $e, f$  be external and  $e', f'$  internal bisectors of the angles  $\widehat{DEA}, \widehat{AFB}$ . Points  $E, Y, Y', T, T' \in e$  are collinear as incenters/E-excenters of  $\triangle ECB, \triangle EDA$  and similarly, points  $F, Z, Z', X', X$  are collinear as incenters/F-excenters of  $\triangle FDC, \triangle FAB$ . Thus the quadrilaterals  $XYZT$  and  $X'Y'Z'T'$  have common diagonals  $XZ \equiv X'Z'$  and  $YT \equiv Y'T'$  intersecting at  $S$ . Intersections  $I \equiv a' \cap b \equiv T'X' \cap XY, J \equiv a \cap b' \equiv X'Y' \cap TX, K \equiv c' \cap d \equiv Y'Z' \cap ZT, L \equiv c \cap d' \equiv YZ \cap Z'T'$  are all on  $f$  as A-, B-, C-, D-excenters of  $\triangle FAB, \triangle FDC$ .  $P \equiv a \cap c \equiv TX \cap YZ$  and  $Q \equiv b \cap d \equiv XY \cap ZT$  are intersections of the opposite sides of  $XYZT$ . Similarly, let  $P' \equiv a' \cap c' \equiv T'X' \cap Y'Z'$  and  $Q' \equiv b' \cap d' \equiv X'Y' \cap Z'T'$  be intersections of the opposite sides of  $X'Y'Z'T'$ .

$\triangle QTX \equiv \triangle(abd), \triangle P'Y'X' \equiv \triangle(b'a'c')$  are perspective, because intersections  $J \equiv a \cap b', I \equiv b \cap a', K \equiv d \cap c'$  of their corresponding sides are collinear  $\implies$  by Desargues theorem,  $QP', TY', XX'$  are concurrent at  $S$ . Similarly,  $\triangle PXY \equiv \triangle(bca), \triangle Q'X'T' \equiv \triangle(a'd'b')$  are perspective, because intersections  $I \equiv b \cap a', L \equiv c \cap d', J \equiv a \cap b'$  of their corresponding sides are collinear  $\implies PQ', XX', YT'$  are also concurrent at  $S$ .  $XX', PQ', QP'$  are concurrent at  $S \implies \triangle XPQ, \triangle X'Q'P'$  are perspective  $\implies$  intersections  $R \equiv PQ \cap Q'P', J \equiv a \cap b' \equiv XP \cap X'Q', I \equiv b \cap a' \equiv XQ \cap X'P'$  of their corresponding sides are collinear. As a result,

$R \equiv PQ \cap P'Q'$  is on  $f \equiv IJ$ . In exactly the same way,  $R \in e \implies R \equiv e \cap f \cap PQ \cap P'Q'$ .  $\square$

Nice soln , Thank you!



vittasko

#4 May 19, 2010, 2:13 am

A magic Desarques, by Vladimir. 😊

Kostas Vittas.

Attachments:

[t=349520.pdf \(10kb\)](#)



yetti

#5 May 19, 2010, 2:42 am

Thanks, Kostas. I actually did almost the same problem before at <http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1352360#p1352360>, using central projection. And of course, I remembered your proof of the underlying (more general) lemma at <http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1504248#p1504248> by Desargues theorem. In the special case of 2 common diagonals of the 2 quadrilaterals, some intersections are lost, and I did not see the correct perspective triangles right away. And if I don't try to use Desargues theorem, I would never learn how.



Luis González

#6 Jun 8, 2010, 12:34 pm • 1

Let the external bisectors of  $\angle DAB, \angle ABC, \angle BCD, \angle CDA$  pairwise meet at  $I, J, K, L$ . This is, according to my drawing that  $I, J$  are incenters of  $\triangle AEB, \triangle BFC$  and  $K, L$  are E- and F- excenters of  $\triangle EDC$  and  $\triangle FAD$ . Let  $KC, KD$  cut  $ER$  at  $M, N$ . Then  $S \equiv KE \cap MD \cap NC$  is the incenter of  $\triangle EDC$ . Angle Bisectors  $AI, DS$  of  $\angle(AE, AF)$  and  $\angle(DE, DF)$  meet at the D-excenter  $U$  of  $\triangle ADF \implies U \in FR$  and angle bisectors  $BJ, SC$  of  $\angle(BC, BF)$  and  $\angle(CE, CD)$  meet at the B-excenter  $V$  of  $\triangle BCF \implies V \in FR$ . In other words, the lines  $VN, UM, IK$  concur at  $S \implies \triangle VUI$  and  $\triangle NMK$  are perspective through perspector  $S \implies$  By Desargues theorem, the intersections  $P \equiv MK \cap IU, Q \equiv VI \cap KN$  and  $R \equiv UV \cap MN$  are collinear.  $\square$



- For instance, it is easy to see that the circle  $\omega$  passing through  $E, F, R$  and  $T \equiv IK \cap JL$  is orthogonal to the circumcircle  $(O, R)$  of the cyclic quadrilateral  $IJKL$ . Line  $PQ$  is the polar of  $(O)$  with respect to  $T$ , thus  $OT \perp PQ$  at  $H$ . If  $OT$  cuts  $(O)$  at  $M_0, N_0$ , then  $(M_0, N_0, T, H) = -1 \implies OM_0^2 = ON_0^2 = R^2 = OT \cdot OH \implies (O, R)$  is orthogonal to the pencil of circles passing through  $T, H$ . Particularly  $(O) \perp \omega$ .

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**High School Olympiads****Benelux Mathematical Olympiad 2016, Problem 4**[Reply](#)**socrates**

#1 May 2, 2016, 1:31 am

A circle  $\omega$  passes through the two vertices  $B$  and  $C$  of a triangle  $ABC$ . Furthermore,  $\omega$  intersects segment  $AC$  in  $D \neq C$  and segment  $AB$  in  $E \neq B$ . On the ray from  $B$  through  $D$  lies a point  $K$  such that  $|BK| = |AC|$ , and on the ray from  $C$  through  $E$  lies a point  $L$  such that  $|CL| = |AB|$ . Show that the circumcentre  $O$  of triangle  $AKL$  lies on  $\omega$ .

**Luis González**

#2 May 2, 2016, 2:25 am • 2

Since  $CL = BA$ , then  $\omega \equiv \odot(EBC)$  and  $\odot(AEL)$  meet again at the center  $M$  of the rotation that swaps  $\overline{CL}$  and  $\overline{BA}$ . Thus, it follows that  $MB = MC$  ( $M$  is the midpoint of the arc  $BEDC$ ) and  $MA = ML$ . Similarly  $MK = MA \implies MA = MK = ML \implies M$  is the circumcenter of  $\triangle AKL$ .

**Vietnamisalwaysinmyheart**

#3 May 2, 2016, 9:59 am

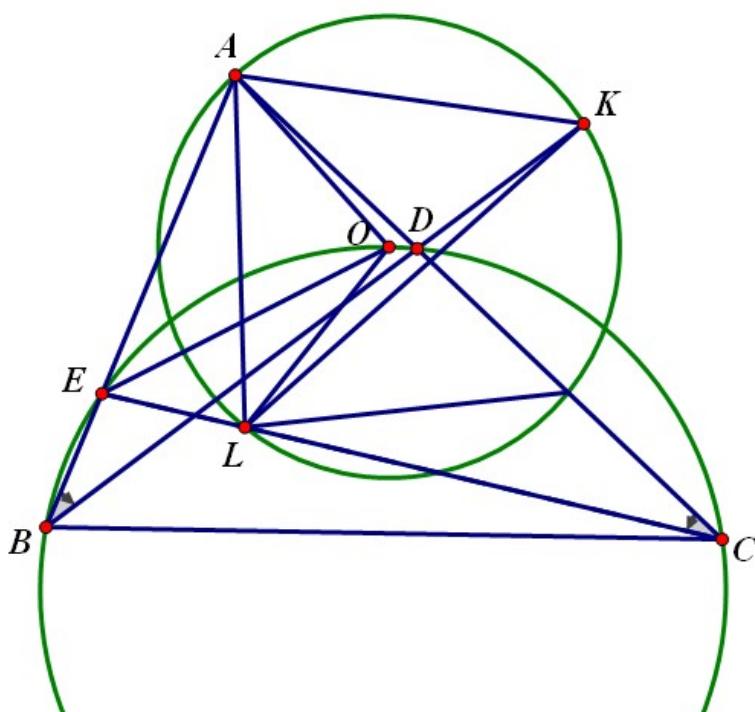
My solution:

Let  $O$  be circumcentre of  $(AKL)$ .We have:  $\Delta BAK = \Delta CLA$ , so:  $AK = AL$  and:  $\widehat{BAK} = \widehat{CLA}$ ,  $\widehat{AKB} = \widehat{ALC}$ Since:  $\widehat{AEL} = 180^\circ - \widehat{BAC} - \widehat{ACE} = \widehat{LAK}$  (easy by angle-chasing) $\widehat{AOL} = 2\widehat{AKL} = 180^\circ - \widehat{LAK}$ Hence,  $A, O, L, E$  are concyclics, likewise,  $A, O, D, K$  are concyclics.

We have that:

 $\widehat{EOD} = 360^\circ - \widehat{AOE} - \widehat{AOD} = 360^\circ - \widehat{ALE} - \widehat{AOD} = \widehat{ALC} + \widehat{AKB} = \widehat{ALC} + \widehat{LAC} = 180^\circ - \widehat{DCE}$ .This implies  $O$  lies on  $\omega$ , as desired.

Attachments:





## Konigsberg

#4 May 19, 2016, 7:23 pm

“ Luis González wrote:

Since  $CL = BA$ , then  $\omega \equiv \odot(EBC)$  and  $\odot(AEL)$  meet again at the center  $M$  of the rotation that swaps  $\overline{CL}$  and  $\overline{BA}$ .

How do we know this?

55



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## High School Olympiads

The sum not depends to x X

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Source: All russian olympiad 2016,Day2,grade 9,P7



MRF2017

#1 May 1, 2016, 4:47 pm

In triangle  $ABC$ ,  $AB < AC$  and  $\omega$  is incircle. The  $A$ -excircle is tangent to  $BC$  at  $A'$ . Point  $X$  lies on  $AA'$  such that segment  $A'X$  doesn't intersect with  $\omega$ . The tangents from  $X$  to  $\omega$  intersect with  $BC$  at  $Y, Z$ . Prove that the sum  $XY + XZ$  not depends to point  $X$ . (Mitrofanov)



Luis González

#2 May 1, 2016, 10:11 pm

Let  $\omega \equiv (I)$  touch  $BC$  at  $D$  and let  $D'$  be the antipode of  $D$  on  $(I)$ . It's well-known that  $D' \in AA'$ . WLOG assume that  $Z$  is between  $D$  and  $Y$  and let the  $Z$ -excircle  $(J)$  of  $\triangle XYZ$  touch  $BC$  at  $A''$ . Since  $X$  is the insimilicenter of  $(I) \sim (J)$  and  $ID' \parallel JA''$ , then  $X, D', A''$  are collinear  $\implies A'' \equiv A'$ . Therefore  $XY + XZ = DA' = AC - AB = \text{const}$ .



FabrizioFelen

#3 May 1, 2016, 10:28 pm

Define  $\Omega$  the  $A$ -excircle of  $\triangle ABC$ ,  $\Gamma$  the  $Y$ -excircle of  $\triangle XYZ$  and let  $D \equiv BC \cap \omega$  and  $D, Y, Z$  lie on  $BC$  in this order.  
 $\implies$  The insimilicenter of  $\omega$  and  $\Gamma$  is  $X$ , the insimilicenter of  $\Omega$  and  $\Gamma$  lie in  $BC$ , the exsimilicenter of  $\Omega$  and  $\omega$  is  $A \implies$  by Monge-D'Alembert theorem we get the insimilicenter of  $\Omega$  and  $\Gamma$  is  $AX \cap BC = A'$  hence the  $\Gamma$  is tangent to  $BC$  in  $A' \implies ZA' = YD \implies$  since  $ZA' = YD$  we get  $XZ + XY = A'D$  which it is fixed.

This post has been edited 1 time. Last edited by FabrizioFelen, May 1, 2016, 10:35 pm



shinichiman

#4 May 2, 2016, 8:11 am • 2

I think the problem is based on the following lemma.

**Lemma.** Triangle  $ABC$  with  $\omega$  is incircle. The  $A$ -excircle is tangent to  $BC$  at  $A'$ .  $X$  is in line  $AA'$ . Tangents from  $X$  to  $\omega$  intersect  $BC$  at  $Y, Z$  then  $BY = CZ$ .

I have a proof for this but it requires to consider the position of point  $X$  on line  $AA'$ , which gives a lot of case work. I hope someone can have a better proof for this.

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## High School Olympiads

Concurrence Point on Euler Line X

Reply



ABCDE

#1 May 1, 2016, 1:54 am • 2

Let  $ABC$  be a triangle with orthocenter  $H$ , nine-point-center  $N$ , and altitudes  $AD$ ,  $BE$ , and  $CF$ . Suppose that  $O_A$ ,  $O_B$ , and  $O_C$  are the circumcenters of  $BNC$ ,  $CNA$ , and  $ANB$  respectively. Let  $AD$  and  $EF$  intersect at  $X$ ,  $BE$  and  $FD$  intersect at  $Y$ , and  $CF$  and  $DE$  intersect at  $Z$ . Prove that  $NH$ ,  $O_AX$ ,  $O_BY$ , and  $O_CZ$  concur.



Luis González

#2 May 1, 2016, 3:09 am • 1

Let  $O$  be the circumcenter of  $\triangle ABC$ . It's well-known that  $YZ$ ,  $ZX$ ,  $XY$  are perpendicular to  $AN$ ,  $BN$ ,  $CN$ , respectively. Thus since  $(O_BO_C \parallel YZ) \perp AN$ , etc, and  $(OO_A \parallel HX) \perp BC$ , etc, then it follows that  $\triangle XYZ \cup H$  and  $\triangle OAOBO_C \cup O$  are homothetic  $\implies OH$ ,  $O_AX$ ,  $O_BY$ ,  $O_CZ$  concur at their homothety center.



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## User Profile

**Luis González****Joined:** February 14, 2009**Status:** Nobody untrained in geometry may enter my house. ~Plato.**Location:** Venezuela**Occupation:** Petroleum engineer

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## High School Olympiads

Collnear  Reply

Source: Own from Y&amp;M



doxuanlong15052000

#1 Apr 19, 2016, 4:18 pm

Let  $ABC$  be a triangle with circumcircle  $(O)$ , nine points circle  $w$ , orthocenter  $H$ .  $P$  is a point on  $OH$ .  $AP, BP, CP$  cut  $(O)$  at  $A_1, B_1, C_1$ .  $HA$  cuts  $(O)$  at  $A_2$ .  $HA_1$  cuts  $w$  at  $A_2$  such that  $A_2$  and  $A_1$  on the other side of  $BC$ . The line pass through  $H$  and parallel to  $XA_1$  cuts  $(AA_2H)$  at  $A_3$ . Similarly, we have  $B_3, C_3$ . Prove that  $A_3, B_3, C_3$  are collinear.



Luis González

#2 Apr 30, 2016, 10:03 pm • 1 

Let  $D, E, F$  be the feet of the altitudes on  $BC, CA, AB$  and let  $DA_1, EB_1, FC_1$  cut  $HA_3, HB_3, HC_3$  at  $A_4, B_4, C_4$ , respectively. Inversion with center  $H$  and power  $HA \cdot HD = HB \cdot HE = HC \cdot HF$  takes  $(O)$  into  $w \implies X, A_1$  are the inverses of  $A, A_2 \implies DA_1$  is the inverse of  $\odot(AA_2H) \implies A_4$  is the inverse of  $A_3$  and likewise  $B_4$  and  $C_4$  are the inverses of  $B_3$  and  $C_3$ .

Since  $D$  is the midpoint of  $HX$  and  $XA_1 \parallel HA_4$ , it follows that  $XA_1PA_4$  is a parallelogram  $\implies A_4$  is the reflection of  $A_1$  on  $D$  and similarly  $B_4, C_4$  are the reflections of  $B_1, C_1$  on  $E, F$ . According to the problem [Similar Hagge circle](#),  $H, A_4, B_4, C_4$  lie on a circle with center on  $OH \implies A_3, B_3, C_3$  are collinear on the inverse of  $\odot(HA_4B_4C_4)$ . Thus in addition  $A_3B_3C_3 \perp OH$ .

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## High School Olympiads

Similar Hagge circle

Reply



**Scorpion.k48**

#1 Sep 21, 2015, 7:56 pm

Let  $\triangle ABC$  with circumcircle  $\odot(O)$  and orthocenter  $H$ .  $P$  is a point lies on Euler line of  $\triangle ABC$ .  $\triangle A_0B_0C_0$  is orthic triangle of  $\triangle ABC$ .  $AP, BP, CP$  cut  $\odot(O)$  at  $A_1, B_1, C_1$ , res. Let  $A_2, B_2, C_2$  be the reflection of  $A_1, B_1, C_1$  to  $A_0, B_0, C_0$ . Prove that  $H, A_2, B_2, C_2$  are cyclic in circle  $\odot(K)$  and  $K$  lies on Euler line of  $\triangle ABC$ .

This post has been edited 1 time. Last edited by Scorpion.k48, Sep 21, 2015, 7:56 pm



**TelvCohl**

#3 Sep 21, 2015, 9:20 pm • 1

Let  $\triangle A_3B_3C_3$  be the circumcevian triangle of  $H$  WRT  $\triangle ABC$ . Let  $A_4, B_4, C_4, A_5, B_5, C_5$  be the antipode of  $A_1, B_1, C_1, A_3, B_3, C_3$  in  $\odot(O)$ , respectively. From Pascal theorem (for  $B_1C_3CC_1B_3B$ ) we get  $B_1C_3 \cap C_1B_3$  lies on the Euler line of  $\triangle ABC$ , so from Pascal theorem (for  $B_1C_3C_4C_1B_3B_4$ )  $\Rightarrow Q \equiv B_3B_4 \cap C_3C_4 \in OH$ . Similarly, we can prove  $Q$  lies on  $A_3A_4 \Rightarrow A_3A_4, B_3B_4, C_3C_4$  and  $OH$  are concurrent at  $Q$ . Analogously, we can prove  $A_1A_5, B_1B_5, C_1C_5$  and  $OH$  are concurrent at  $R$ .



Let  $S$  be the midpoint of  $HQ$ . Since  $A_4$  is the midpoint of  $HA_3$ , so we get  $HA_2 \parallel A_1A_3$  and  $A_0S \parallel A_3Q \parallel A_1R$  ( $\perp A_1A_3$ )  $\Rightarrow$  the perpendicular from  $A_2$  to  $A_2H$  passes through the reflection  $T$  of  $R$  in  $S$ , hence  $A_2$  lie on a circle with diameter  $HT$ . Analogously, we can prove  $B_2$  and  $C_2$  lie on  $\odot(HT) \Rightarrow H, A_2, B_2, C_2$  lie on a circle  $\odot(K)$  with diameter  $HT$  and  $K$  lies on the Euler line of  $\triangle ABC$  due to  $K$  is the midpoint of  $HT$ .



**Luis González**

#4 Sep 22, 2015, 4:21 am • 2

Let  $\triangle A'B'C'$  be the antimedial triangle of  $\triangle ABC$  whose orthocenter is  $H'$ .  $U$  and  $V$  are projections of  $A'$  and  $B'$  on  $B'C'$  and  $C'A'$ . Since  $A_0B_0 \parallel UV$  and  $A_0B_0 = \frac{1}{2}UV$ , then  $Z \equiv UB_0 \cap VA_0$  is common reflection of  $U$  and  $V$  on  $B_0$  and  $A_0$ .  $F \equiv UV \cap AB$  is on radical axis of  $(S) \equiv \odot(H'UC'V)$  and  $(T) \equiv \odot(HBC'A)$   $\Rightarrow C'F$  is their radical axis perpendicular to their center line  $\Rightarrow C'F \perp ST \parallel HOH'$ . Since  $C'F$  is the polar of  $R \equiv AV \cap BU$  WRT  $(O) \Rightarrow RO \perp C'F \Rightarrow R \in OH$ .



Animate  $P$  on  $OH$ .  $A_2, B_2$  move on reflections  $(O_A), (O_B)$  of  $(O)$  on  $A_0, B_0$  that meet at  $H, Z$  and  $P \bar{\wedge} A_1 \bar{\wedge} B_1 \bar{\wedge} A_2 \bar{\wedge} B_2$ . Thus if  $H_A, H_B$  are the antipodes of  $H$  on  $(O_A), (O_B)$ , the pencils  $H_A A_2$  and  $H_B B_2$  are perspective because  $H_A A_2 \equiv H_A Z \equiv H_B B_2 \equiv H_B Z$  when  $P \equiv R$ . Since  $A_2 \equiv B_2 \equiv H$  when  $P \equiv H$ , then  $H_A A_2, H_B B_2, OH$  concur at a point  $M$  and likewise  $H_C C_2$  goes through  $M \Rightarrow A_2, B_2, C_2$  lie on circle with diameter  $HM$  along the Euler line.



**jayme**

#5 Sep 22, 2015, 10:18 am

Dear Mathlinkers,

just a reference in order to put this particular problem in an historical perspective

<http://www.mathlinks.ro/Forum/viewtopic.php?t=319520>



Sincerely  
Jean-Louis



**Scorpion.k48**



“ Luis González wrote:

Animate  $P$  on  $OH$ .  $A_2, B_2$  move on reflections  $(O_A), (O_B)$  of  $(O)$  on  $A_0, B_0$  that meet at  $H, Z$  and  $P \bar{\wedge} A_1 \bar{\wedge} B_1 \bar{\wedge} A_2 \bar{\wedge} B_2$ . Thus if  $H_A, H_B$  are the antipodes of  $H$  on  $(O_A), (O_B)$ , the pencils  $H_AA_2$  and  $H_BB_2$  are perspective because  $H_AA_2 \equiv H_AZ \equiv H_BB_2 \equiv H_BZ$  when  $P \equiv R$ . Since  $A_2 \equiv B_2 \equiv H$  when  $P \equiv H$ , then  $H_AA_2, H_BB_2, OH$  concur at a point  $M$  and likewise  $H_CC_2$  goes through  $M \Rightarrow A_2, B_2, C_2$  lie on circle with diameter  $HM$  along the Euler line.

I don't understand this dear Mr. Luis



livetolove212

#7 Sep 28, 2015, 9:01 am

My old problem

<http://artofproblemsolving.com/community/q3h345850p1857304>

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## High School Olympiads

convex equilateral pentagon X

Reply



Pirkuliyev Rovsen

#1 Apr 5, 2016, 12:42 am

Prove that from any three diagonals of convex equilateral pentagon can make a triangle.



Luis González

#2 Apr 28, 2016, 9:01 pm

Label  $ABCDE$  the vertices of the given pentagon and assume WLOG that  $EC$  is its longest diagonal. If  $P \equiv AC \cap BE$ , then  $P$  is strictly inside  $ABCE$  as the pentagon is convex  $\implies PE < BE$  and  $PC < AC$ . Hence by triangle inequality on  $\triangle PCE$ , we get  $EC < PE + PC < BE + AC \implies AC, BE, CE$  make a triangle.



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## High School Olympiads

Locus of a point X

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ThE-dArK-IOrD

#1 Apr 28, 2016, 6:51 pm

Given  $\triangle ABC$  with altitude  $BE, CF$

Let  $M$  is midpoint of  $BC$ , and  $P$  is arbitrary point on  $BC$

Reflect  $P$  over  $M$  to get point  $Q$

Let  $AP, AQ \cap EF = U, V$  respectively

Let  $(AUV) \cap (APQ) = T \neq A$ , prove that  $T$  lie on a fixed circle when  $P$  varies on  $BC$



Luis González

#2 Apr 28, 2016, 8:38 pm

More general: Let  $E, F$  be arbitrary points on  $AC, AB$ .  $\{P, Q\} \in BC$ , such that  $P \mapsto Q$  is an involution on  $BC$ .  $AP, AQ$  cut  $EF$  at  $U, V$ . Then the 2nd intersection of  $\odot(AUV)$  and  $\odot(APQ)$  moves on a fixed circle.

Inverting with center  $A$ , we get the equivalent problem:  $E, F$  are arbitrary points on  $AC, AB$ .  $\{P, Q\} \in \odot(ABC)$ , such that  $P \mapsto Q$  is an involution on  $\odot(ABC)$ .  $AP, AQ$  cut  $\odot(AEF)$  again at  $U, V$ . Then  $T \equiv UV \cap PQ$  moves on a fixed circle.

Since  $AP \mapsto AQ$  is an involution, then  $U \mapsto V$  is an involution on  $\odot(AEF)$  as well  $\implies PQ$  and  $UV$  go through fixed points  $R$  and  $S$ , respectively (poles of the involutions). If  $K \equiv UF \cap PB$ , then  $\odot(KFB), \odot(ABC), \odot(AEF)$  concur at the Miquel point  $L$  of the complete  $PBFU, BCEF$ . Therefore

$\angle VTQ = \angle AQP + \angle UVQ = \angle KBF + \angle KFB = \angle FLB = \text{const } (\text{mod } 180^\circ)$ . Since  $R, S$  are fixed, then it follows that  $T$  moves on a fixed circle passing through  $R$  and  $S$ .



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## High School Olympiads

Symedian and collinear. 

 Reply

Source: Own.



**doxuanlong15052000**

#1 Apr 3, 2016, 11:16 pm • 1



Let  $ABC$  be a triangle with  $AD$  be  $A$ -symedian ( $D \in (O)$ ),  $t$  be the tangent at  $A$  WRT  $(O)$  and the diameter  $AU$ .  $R$  is a point on  $AC$  and  $L$  is the intersection of  $BR$  and  $AD$  such that  $RA^2 = RL \cdot RB$ .  $T$  is a point on  $AC$  such that  $(TR, AC) = -1$ . The line passes through  $A$  and perpendicular to  $AB$  cuts  $UC$  at  $S$ . Draw  $SM \perp t$  and  $AS$  cuts  $BC$  at  $J$ . Prove that  $T, M, J$  are collinear.



**Luis González**

#2 Apr 26, 2016, 10:04 am • 3



Since  $RA^2 = RL \cdot RB$ , then  $\odot(BAL)$  is tangent to  $AC \Rightarrow L$  is midpoint of  $AD$  (well-known property of the symmedian line), hence  $\odot(CAL)$  is tangent to  $AB$ . Thus  $S' \equiv OL \cap CU$  is the antipode of  $A$  on  $\odot(CAL) \Rightarrow AS' \perp AB \Rightarrow S \equiv S'$ . If  $E \equiv BC \cap AM$  and  $X$  is the 2nd intersection of  $BL$  and  $\odot(CAL)$ , then  $L(X, C, S, A) = L(B, C, E, A) = -1$  and since  $LS \perp LA$ , it follows that  $LS, LA$  bisect  $\angle CLX$ , i.e.  $S$  is midpoint of the arc  $CX$  of  $\odot(CAL) \Rightarrow MS$  is bisector of  $\angle CMX$ .

Let  $J' \equiv MX \cap BC$ . Since  $\angle SCJ' \equiv \angle BCU = \angle BAO = 90^\circ - \angle ACB = \angle SAM = \angle SCM$ , then  $CS$  is bisector of  $\angle MCJ' \Rightarrow S$  is incenter of  $\triangle MCJ'$ , thus by symmetry  $J'S$  is the perpendicular bisector of  $CX$ . But as  $A$  is the midpoint of the arc  $CMX$ , then  $AS$  is the perpendicular bisector of  $CX \Rightarrow J' \in AS \Rightarrow J' \equiv J$ . Now let  $T' \equiv MX \cap AC$ . Since  $S(L, M, C, A) = A(L, E, C, B) = -1 \Rightarrow LMCA$  is harmonic  $\Rightarrow X(L, M, C, A) = X(R, T', C, A) = -1 \Rightarrow T \equiv T' \Rightarrow M, J, T$  are collinear.



**uraharakisuke\_hsgs**

#4 Apr 26, 2016, 8:58 pm



Lemma :  $\triangle ABC$  with symmedian line  $AD$ .  $L$  is the midpoint of  $AD$  so  $AC$  is the tangent of  $(LAB)$

+Prove :

$L$  is the midpoint of  $AD \Rightarrow OL \perp AD$

$D_1$  is the intersection of 2 tangent from  $B$  and  $C$  of  $(O)$

$(OBC)$  cuts  $AB$  at  $G \Rightarrow \angle BGD = \angle BCD = \angle BAC \Rightarrow GD \parallel AC \Rightarrow \angle DAC = \angle ADG = \angle ABL \Rightarrow AC$  is the tangent of  $(LAB)$

Comeback with the problem :

We have :  $RA^2 = RL \cdot RB \Rightarrow AC$  is the tangent of  $(ALB)$

Let  $D_1$  be the intersection of 2 tangents from  $B$  and  $C$  of  $(O)$

$\Rightarrow OLBD_1C$  is cyclic  $\Rightarrow OL \perp AD \Rightarrow L$  is the midpoint of  $AD \Rightarrow AB$  is the tangent of  $(LAC)$

$AS \perp AB \Rightarrow$  the center  $X$  of  $(LAC)$  lies on  $AS$

$CS \perp CA \Rightarrow S$  lies on  $(LAC)$  ( because  $S$  is reflect with  $A$  about  $X$  )

$SM \perp t \Rightarrow M$  lies on  $X$

$N$  is the intersection of  $(BOC)$  and  $AB$

$\Rightarrow \angle NLC = 180^\circ - \angle NBC = 180^\circ - \angle MAC = 180^\circ - \angle MLC$

$\Rightarrow N, L, M$  are collinear

We also have :  $\angle NCL = \angle NBL = \angle LAC \Rightarrow NC$  is the tangent of  $(LAC)$

$\Rightarrow$  tangents at  $A, C$  of  $(ALCM)$  and  $ML$  are concurrent  $\Rightarrow (ACML) = -1$

Let  $BL$  cuts  $(X)$  at  $S_1 \Rightarrow S_1(ACML) = -1$

So, if  $S_1M$  cuts  $AC$  at  $T_1 \Rightarrow (T_1RAC) = -1$

$\Rightarrow T_1 \equiv T \Rightarrow S_1, M, T$  are collinear

We need to prove that  $S_1, M, J$  are collinear

By Pascal

$(S_1 \ S \ A)$

$A \ M \ L)$

=>  $B, J_1$  and  $K_1$  are concurrent ( $K_1$  is the intersection of  $OL$  with  $t$ ,  $J_1$  is the intersection of  $S_1M$  and  $AS$ ) (1)

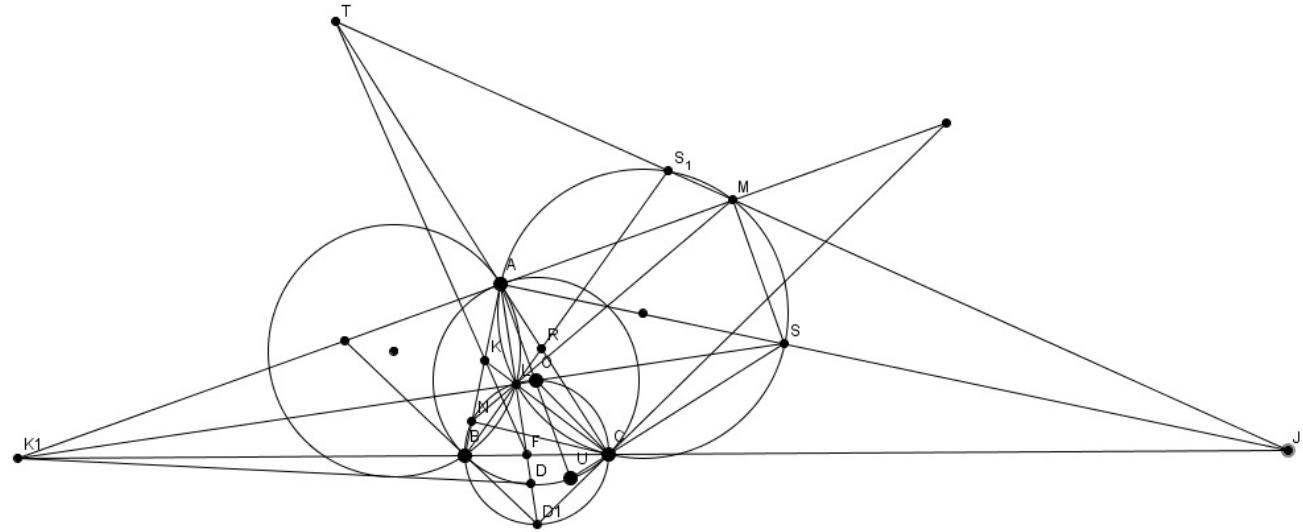
We have :  $K_1A$  is the tangent of  $(O)$  and  $K_1O \perp AD$

=> $K_1D$  is also a tangent of  $(O)$  =>  $K_1, B, C$  are collinear (2)

From (1) and (2) =>  $J_1 \equiv J$

=>  $T, M, J$  are collinear

Attachments:



This post has been edited 2 times. Last edited by uraharakisuke\_hsgs, Apr 26, 2016, 10:19 pm

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## High School Olympiads

Line is parallel to Brocard axis X

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Source: Own



**buratinogigle**

#1 Apr 17, 2016, 10:09 pm • 2

Let  $ABC$  be a triangle inscribed in circle  $(O)$  and orthocenter  $H$ . Let  $X, Y, Z$  lie on  $(O)$  such that  $AX \parallel BC, BY \parallel CA, CZ \parallel AB$ . Let  $XH, YH, ZH$  cut  $BC, CA, AB$  at  $D, E, F$ , reps. Prove that  $AD, BE, CF$  are concurrent at point  $P$  and line  $HP$  is parallel to [Brocard axis](#) of triangle  $ABC$ \$



**buratinogigle**

#2 Apr 18, 2016, 5:46 pm • 2

This problem is true for any point on Euler line.

Let  $ABC$  be a triangle inscribed in circle  $(O)$  with orthocenter  $H$ .  $P$  is any point on Euler of triangle  $ABC$ . Let  $X, Y, Z$  lie on  $(O)$  such that  $AX \parallel BC, BY \parallel CA, CZ \parallel AB$ . Let  $XP, YP, ZP$  cut  $BC, CA, AB$  at  $D, E, F$ , reps. Prove that  $AD, BE, CF$  are concurrent at point  $Q$  and line  $HQ$  is parallel to Brocard axis of triangle  $ABC$ .



**TelvCohl**

#3 Apr 18, 2016, 7:48 pm • 3

**Re:** *buratinogigle wrote:*

Let  $ABC$  be a triangle inscribed in circle  $(O)$  and orthocenter  $H$ . Let  $X, Y, Z$  lie on  $(O)$  such that  $AX \parallel BC, BY \parallel CA, CZ \parallel AB$ . Let  $XH, YH, ZH$  cut  $BC, CA, AB$  at  $D, E, F$ , reps. Prove that  $AD, BE, CF$  are concurrent at point  $P$  and line  $HP$  is parallel to [Brocard axis](#) of triangle  $ABC$

**Lemma :** Given a  $\triangle ABC$  and two points  $P$  and  $Q$ . Let  $\triangle P_a P_b P_c, \triangle Q_a Q_b Q_c$  be the cevian triangle of  $P, Q$  WRT  $\triangle ABC$ , respectively. Let  $T$  be a point on  $PQ$  and  $Q_a T, Q_b T, Q_c T$  cuts  $P_b P_c, P_c P_a, P_a P_b$  at  $D, E, F$ , respectively. Then  $\triangle DEF$  is the cevian triangle of  $S$  WRT  $\triangle P_a P_b P_c$  and  $TS$  is tangent to the circumconic  $\mathcal{C}$  of  $\triangle ABC$  passing through  $P, T$ .

**Proof :** Let  $S \equiv P_b E \cap P_c F$  and  $V \in PQ$  be the intersection of  $P_b Q_c, P_c Q_b$ . From the dual of Desargues' involution theorem (for  $P_b Q_b P_c Q_c$  and  $T$ ) we get there exists an involution which swaps  $(TA, TP), (TP_b, TP_c)$  and  $(TQ_b, TQ_c)$ , but from the dual of Desargues' involution theorem (for  $B P_b C P_c$  and  $T$ ) we get the aforementioned involution also swaps  $TB, TC$ . Note  $P_a$  is the pole of the involution on  $\mathcal{C}$  that swaps  $(B, C), (A, P)$ , so if  $\tau_t$  is the tangent of  $\mathcal{C}$  passing through  $T$ , then we get there is an involution that swaps  $(TB, TC), (TA, TP), (TP_b, TP_c), (TQ_b, TQ_c)$  and  $(\tau_t, TP_a)$ .

From the dual of Desargues' involution theorem (for  $E P_b F P_c$  and  $T$ ) we get there exists an involution that swaps  $(TP_b, TP_c), (TQ_b, TQ_c)$  and  $(TS, TP_a)$ , so  $S$  lies on  $\tau_t$ . Similarly, we can prove the line connecting  $T$  and the intersection of  $P_c F, P_a D$  is tangent to  $\mathcal{C}$  at  $T$ , so we conclude that  $P_a D, P_b E, P_c F$  are concurrent at  $S \in \tau_t$ .

**Remark :** The locus of  $S$  is a line when  $T$  varies on  $PQ$ .

**Proof :** Clearly, pencil  $P_b E \mapsto P_c F$  is a homography, so notice  $E \equiv P_c, F \equiv P_b$  when  $T$  coincide with  $V$  we conclude that the locus of  $S$  (intersection of  $P_b E$  and  $P_c F$ ) is a line.

**Back to the main problem :**

Let  $\triangle A^* B^* C^*$  be the antimedial triangle of  $\triangle ABC$  and let  $G$  be the centroid of  $\triangle ABC$ . From **Lemma**  $\Rightarrow AD, BE, CF$  are concurrent at  $P$  and  $HP$  is tangent to the circumconic of  $\triangle A^* B^* C^*$  passing through  $G$  and  $H$ , so notice the polar of any point WRT the conic through  $A^*, B^*, C^*, G$  passes through its isotomic conjugate WRT  $\triangle ABC$  we get  $HP$  passes through the isotomic conjugate of  $H$  WRT  $\triangle ABC$  which is the anticomplement of the symmedian point of  $\triangle ABC$  WRT  $\triangle ABC$   $\Rightarrow HP$  is parallel to the Brocard axis of  $\triangle ABC$ .

**“** buratinogigle wrote:

This problem is true for any point on Euler line.

Let  $ABC$  be a triangle inscribed in circle  $(O)$  with orthocenter  $H$ .  $P$  is any point on Euler line of triangle  $ABC$ . Let  $X, Y, Z$  lie on  $(O)$  such that  $AX \parallel BC, BY \parallel CA, CZ \parallel AB$ . Let  $XP, YP, ZP$  cut  $BC, CA, AB$  at  $D, E, F$ , reps. Prove that  $AD, BE, CF$  are concurrent at point  $Q$  and line  $HQ$  is parallel to Brocard axis of triangle  $ABC$ .

From the original problem and the remark of the **Lemma** we can get this generalization.



Luis González

#4 Apr 24, 2016, 9:02 am • 3



**“** buratinogigle wrote:

This problem is true for any point on Euler line.

Let  $ABC$  be a triangle inscribed in circle  $(O)$  with orthocenter  $H$ .  $P$  is any point on Euler line of triangle  $ABC$ . Let  $X, Y, Z$  lie on  $(O)$  such that  $AX \parallel BC, BY \parallel CA, CZ \parallel AB$ . Let  $XP, YP, ZP$  cut  $BC, CA, AB$  at  $D, E, F$ , reps. Prove that  $AD, BE, CF$  are concurrent at point  $Q$  and line  $HQ$  is parallel to Brocard axis of triangle  $ABC$ .

Let  $\triangle A'B'C'$  be the antimedial triangle of  $\triangle ABC$ . It's well-known that  $U \equiv BZ \cap CY$  is on the Euler line  $e$  of  $\triangle A'B'C', \triangle ABC$ . When  $P$  runs on  $e$ , clearly  $E \mapsto F$  is a projectivity between  $AC$  and  $AB$ , even a perspectivity, because when  $P \equiv U$ , then  $E \equiv C, F \equiv B \implies Q \equiv BE \cap CF$  moves on a line  $\tau$ .

When  $P$  is the orthocenter of  $\triangle A'B'C'$ , then  $BE, CF$  become Schwatt lines of  $\triangle A'B'C'$  meeting at its symmedian point and when  $P$  is the centroid of  $\triangle A'B'C'$ , then clearly  $BE, CF$  become perpendicular bisectors of  $A'C', A'B'$  meeting at  $H \implies \tau$  is the Brocard axis of  $\triangle A'B'C'$ . Similarly  $AD$  goes through  $\tau \cap BE \cap CF$ . Hence  $AD, BE, CF$  concur at a point  $Q$  on the Brocard axis of  $\triangle A'B'C'$ , thus parallel to the Brocard axis of  $\triangle ABC$ .

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## High School Olympiads

Geometry



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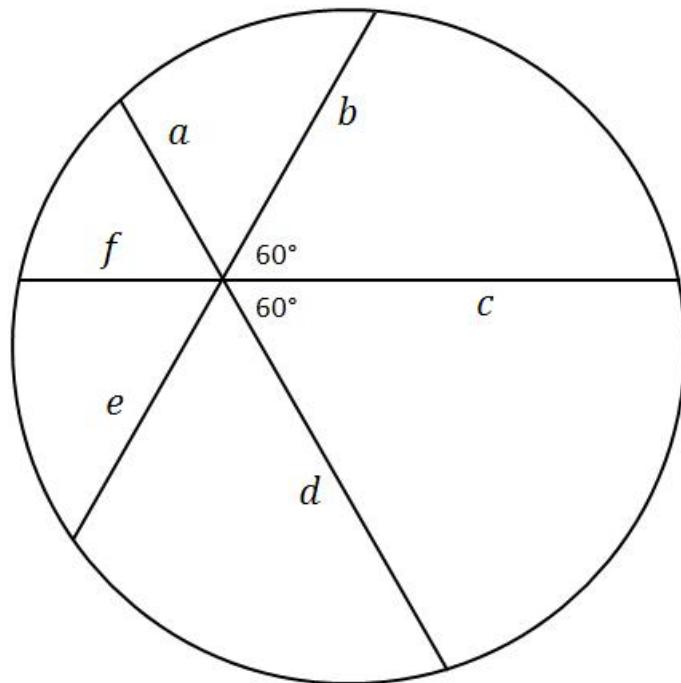


soroush.MG

#1 Apr 23, 2016, 5:42 pm

Prove that:  $a + c + e = b + d + f$

Attachments:



Luis González

#2 Apr 23, 2016, 8:17 pm

See <http://www.artofproblemsolving.com/community/c6h441896>.



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## High School Olympiads



**elegant**

#1 Oct 29, 2011, 6:55 pm

Three chords  $AA_1, BB_1, CC_1$  of a circle (with the order on the circle  $A, B_1, C, A_1, B, C_1$ ) pass through a common point  $K$  and form six angles of  $60^\circ$ . Prove that  $KA + KB + KC = KA_1 + KB_1 + KC_1$ .

**yetti**

#2 Oct 30, 2011, 5:13 am

Let  $(O)$  be the circle with the chords  $AA_1, BB_1, CC_1$  concurrent at  $K$ .

Common internal bisector  $AKA_1$  of  $\angle CKB, \angle C_1KB_1$  meets circles  $\odot(KBC), \odot(KB_1C_1)$  again at  $Z, Z_1$ .  
 $ZB = ZC$  and  $\angle BZC = 180^\circ - \angle CKB = 60^\circ \Rightarrow \triangle BZC$  is equilateral and similarly,  $\triangle B_1Z_1C_1$  is equilateral.  
By Ptolemy for cyclic  $KBZC, KB \cdot ZC + KC \cdot ZB = ZK \cdot BC \Rightarrow ZK = KB + KC$  and similarly,  
 $Z_1K = KB_1 + KC_1$ .

Perpendicular bisectors of  $BC, B_1C_1$  through  $Z, Z_1$  meet at  $O$ . Since  $KBCZ \sim KB_1Z_1C_1$  are similar,

$\angle OZK = \angle OZ_1K \Rightarrow$

$\triangle OZZ_1$  is O-isosceles, just like  $\triangle OAA_1 \Rightarrow$

$$KA + KB + KC = ZK + KA = ZA = Z_1A_1 = Z_1K + KA_1 = KA_1 + KB_1 + KC_1$$

**newsun**

#3 Oct 30, 2011, 7:30 pm

I see this problem from another way..

Assume that this circle centered at  $O$  and which has radius  $R$ .

We have

$$R^2 = OA^2 = (\vec{OK} + \vec{KA})^2 = OK^2 + KA^2 + 2\vec{OK}\vec{KA}$$

$$\text{therefore, } R^2 - OK^2 = KA^2 + 2\vec{OK}\vec{KA}$$

$$\text{hence } KA \cdot KA_1 = KA^2 + 2\vec{OK}\vec{KA}$$

$$\text{so, } KA_1 = KA + 2\vec{OK} \cdot \frac{\vec{KA}}{KA} \quad (1)$$

$$\text{Similarly, } KB_1 = KB + 2\vec{OK} \cdot \frac{\vec{KB}}{KB} \quad (2)$$

$$KC_1 = KC + 2\vec{OK} \cdot \frac{\vec{KC}}{KC} \quad (3)$$

On the other hand,  $\frac{\vec{KA}}{KA}, \frac{\vec{KB}}{KB}, \frac{\vec{KC}}{KC}$  are unit vectors and each pair of them forms angle of  $60^\circ$  then

$$\frac{\vec{KA}}{KA} + \frac{\vec{KB}}{KB} + \frac{\vec{KC}}{KC} = \vec{0}$$

Combining (1)(2)(3)and(4) we have the conclusion.

**Luis González**

#4 Oct 31, 2011, 7:54 am

Antipedal triangles  $\triangle PQR$  and  $\triangle P_1Q_1R_1$  of  $K$  with respect to  $\triangle ABC$  and  $\triangle A_1B_1C_1$  are clearly equilateral with parallel sides. But since  $QR, RP, PQ$  cut the given circle  $(O)$  again at the antipodes of  $A_1, B_1, C_1$ , we deduce that  $\triangle PQR$  and  $\triangle P_1Q_1R_1$  are symmetric about  $O$ . Thus, by Viviani's theorem we get

$$KA + KB + KC = \frac{\sqrt{3}}{2}PQ = \frac{\sqrt{3}}{2}P_1Q_1 = KA_1 + KB_1 + KC_1.$$

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## High School Olympiads

Point on Euler line X

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ThE-dArK-IOrD

#1 Apr 22, 2016, 8:53 pm

Given  $\triangle ABC$  with orthocenter  $H$

Let  $D, E, F$  are midpoints of  $BC, CA, AB$  respectively

Let  $T$  be arbitrary point on Euler line of  $\triangle ABC$

Let  $AT \cap HD = X$ , similarly define  $Y, Z$

Prove that  $\frac{HX}{XD} = \frac{HY}{YE} = \frac{HZ}{ZF}$



Luis González

#2 Apr 22, 2016, 10:03 pm

We use oriented segments throughout the proof. Let  $O$  and  $G$  be the circumcenter and centroid of  $\triangle ABC$  and let  $U \equiv AP \cap OD$ . From  $AH \parallel OU$ , we get

$$\frac{XD}{XH} = \frac{DU}{HA} = \frac{\overline{OU} - \overline{OD}}{\overline{HA}} = \frac{\overline{OU}}{\overline{HA}} - \frac{\overline{GO}}{\overline{GH}} = \frac{\overline{PO}}{\overline{PH}} + \frac{1}{2}.$$

Similarly we get  $\frac{YE}{YH} = \frac{ZF}{ZH} = \frac{\overline{PO}}{\overline{PH}} + \frac{1}{2}$  and the conclusion follows.



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## High School Olympiads

Kosnita points lies on OI line X

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Source: Own



**buratinogigle**

#1 Apr 21, 2016, 11:54 am

Prove that  $OI$  line of a triangle passes through three Kosnita points of three triangles divided by its incenter.



**mineiraojose**

#2 Apr 21, 2016, 12:29 pm • 1

Let  $M$  be the midpoint of arc  $BC$  in  $\odot(ABC)$ . Since  $M$  is the circumcenter of  $\odot(IBC)$  so  $O$  is the circumcenter of  $\odot(MBC)$  so  $IO$  pass through to the Kosnita point of  $\triangle IBC$ . Analogously the others.



**buratinogigle**

#3 Apr 21, 2016, 12:43 pm • 1

Thank you for nice explanation. The following is general problem

Let  $ABC$  be a triangle inscribed in circle  $(O)$  and  $DEF$  is circumcevian of any point  $P$ .  $X, Y, Z$  are reflections of  $D, E, F$  through  $BC, CA, AB$ , reps. Let  $U, V, W, Q$  are isogonal conjugate of  $X, Y, Z, P$  with respect to triangles  $PBC, PCA, PAB, DEF$ . Prove that  $P, Q, U, V, W$  are collinear.



**Luis González**

#4 Apr 21, 2016, 8:06 pm • 3

“ buratinogigle wrote:

Let  $ABC$  be a triangle inscribed in circle  $(O)$  and  $DEF$  is circumcevian of any point  $P$ .  $X, Y, Z$  are reflections of  $D, E, F$  through  $BC, CA, AB$ , reps. Let  $U, V, W, Q$  are isogonal conjugate of  $X, Y, Z, P$  with respect to triangles  $PBC, PCA, PAB, DEF$ . Prove that  $P, Q, U, V, W$  are collinear.

Obviously  $\triangle PBC \sim \triangle PFE$  and  $\angle FEQ = \angle DEP = \angle DCB = \angle BCX$  and likewise  $\angle EFQ = \angle CBX \Rightarrow \triangle QEF \sim \triangle XCB \Rightarrow PQEF \sim PXCB \Rightarrow \angle CPX = \angle QPE \Rightarrow PQ, PX$  are isogonals WRT  $\angle BPC \Rightarrow U \in PQ$  and similarly  $V$  and  $W$  lie on  $PQ$ .

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## High School Olympiads



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**buratinogiggle**

#1 Apr 21, 2016, 12:59 am

Let  $ABC$  be a triangle inscribed in circle  $(O)$ , incenter  $I$  and orthocenter  $H$ .  $A_1, A_2$  lie on  $BC$  such that  $IA_1 \perp IB, IA_2 \perp IC$ .  $AA_1, AA_2$  cut  $(O)$  again at  $A_3, A_4$ .  $A_3A_4$  cuts  $BC$  at  $A_0$ . Similarly, we have  $B_0, C_0$ . Prove that  $A_0, B_0, C_0$  are collinear on a line which is perpendicular to line  $IH$ .

**Luis González**

#2 Apr 21, 2016, 7:43 am • 2

Moreover, we'll prove that  $A_0, B_0, C_0$  are collinear on the trilinear polar of  $X_{85}$  of  $\triangle ABC$  (isotomic conjugate of its Mittenpunkt). But first we introduce a lemma:

**Lemma:**  $P$  is a point inside  $\triangle ABC$  and  $Q$  is the isotomic conjugate of the isomcomplement of  $P$  WRT  $\triangle ABC$ . Then the trilinear polars of  $P$  and  $Q$  WRT  $\triangle ABC$  are parallel.

Consider an affine homology carrying  $\triangle ABC \cup P$  into a  $\triangle ABC$  with orthocenter  $P$ . The trilinear polar of  $P$  is the orthic axis of  $\triangle ABC$  and  $Q$  becomes isotomic conjugate of the symmedian point, whose trilinear polar, the De Longchamps line of  $\triangle ABC$ , is perpendicular to the Euler line, as it is the radical axis of  $\odot(ABC)$  and the circumcircle of the antimedial triangle (well-known). Therefore the trilinear polars of  $P$  and  $Q$  WRT  $\triangle ABC$  are parallel.

**Corollary:** When  $P$  is the Nagel point  $Na$  of  $\triangle ABC$ , then its isomcomplement is the Mittenpunkt  $Mt$  (well-known). Thus from the previous lemma, it follows that the trilinear polar  $\tau$  of the isotomic conjugate of  $Mt$  is parallel to the trilinear polar of  $Na$ . But according to the problem [hard geometry problem](#), the trilinear polar of  $Na$  is perpendicular to  $IH$ , therefore  $IH \perp \tau$ .

Back to the main problem, it's well-known that  $A_1, A_2$  are the tangency points of the B- and C- mixtilinear incircles with  $BC$ , thus we have the ratios  $\frac{A_1C}{A_1B} = \frac{b}{s-b}$  and  $\frac{A_2B}{A_2C} = \frac{c}{s-c} \implies$

$$\frac{A_4B}{A_4C} = \frac{b}{c} \cdot \frac{A_2B}{A_2C} = \frac{b}{c} \cdot \frac{c}{s-c} = \frac{b}{s-c} \text{ and likewise } \frac{A_5C}{A_5B} = \frac{c}{s-b}$$

$$\implies \frac{A_0B}{A_0C} = \frac{A_4B}{A_4C} \cdot \frac{A_5B}{A_5C} = \frac{b(s-b)}{c(s-c)}.$$

It's known that the A-cevian of  $Mt$  divides  $\overline{BC}$  in the ratio  $\frac{c(s-c)}{b(s-b)}$ , thus from the latter expression we deduce that  $A_0$  is on the trilinear polar of the isotomic conjugate of  $Mt$  and analogously  $B_0$  and  $C_0$  fall on this line. Now from the previous corollary, we get  $IH \perp A_0B_0C_0$ .

**TelvCohl**

#3 Apr 21, 2016, 1:28 pm • 2

Let  $\triangle DEF$  be the intouch triangle of  $\triangle ABC$  and let  $AI, AD$  cuts  $\odot(ABC)$  again at  $M, X$ , respectively. Clearly,  $IA$  and  $ID$  are isogonal conjugate WRT  $\angle BIC$ , so there exists an involution that swaps  $(B, C), (A_1, A_2)$  and  $(D, AI \cap BC) \implies$  there is an involution that swaps  $(AB, AC), (AA_3, AA_4)$  and  $(AM, AX)$ , hence  $BC, A_3A_4, MX$  are concurrent at  $A_0$ .

Let  $I_a, I_b, I_c$  be the A-excenter, B-excenter, C-excenter of  $\triangle ABC$ , respectively. Let the isogonal conjugate of  $AD$  WRT  $\angle A$  cuts  $\odot(ABC)$  again at  $X^*$  and  $A_0^* \equiv BC \cap MX^*$ . Note  $X^*$  is the tangency point of  $\odot(ABC)$  and the A-mixtilinear excircle of  $\triangle ABC$ , so  $AI_a \perp I_aA_0^*$ . Similarly, if  $B_0^*, C_0^*$  is the isotomic conjugate of  $B_0, C_0$  WRT  $(C, A), (A, B)$ , respectively, then we get  $BI_b \perp I_bB_0^*$  and  $CI_c \perp I_cC_0^*$ .

Obviously,  $IH$  is the radical axis of  $\odot(AA_0^*I_a), \odot(BB_0^*I_b), \odot(CC_0^*I_c)$ , so notice  $A_0, B_0, C_0$  is the anticomplement (WRT  $\triangle ABC$ ) of the center of these three circles we conclude that  $A_0, B_0, C_0$  are collinear and  $A_0B_0C_0 \perp IH$ .

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## High School Olympiads

Line is tangent to incircle X

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buratinogiggle

#1 Apr 3, 2016, 11:58 pm

Let  $ABC$  be a triangle and incircle  $(I)$ .  $DEF$  is cevian triangle of any point  $P$ . Let  $X, Y, Z$  are poles of  $EF, FD, DE$  with respect to  $(I)$ . Prove that  $AX, BY, CZ$  intersect  $BC, CA, AB$  follow three collinear points on a tangent of  $(I)$ .



Luis González

#2 Apr 4, 2016, 12:39 am • 1

This follows from your previous problem [Poles with cevian triangle](#). If  $(I)$  touches  $BC, CA, AB$  at  $D', E', F'$ , we already know that  $U \equiv E'F' \cap EF \cap BY \cap CZ$ . Thus if  $AX, BY, CZ$  cut  $BC, CA, AB$  at  $X', Y', Z'$ , then by the converse of Newton theorem, it follows that  $Y'Z'$  touches  $(I)$  and similarly  $X'Y'$  touches  $(I) \Rightarrow X', Y', Z'$  are collinear on a tangent of  $(I)$ .

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## High School Olympiads

Poles with cevian triangle X

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buratinogiggle

#1 Oct 8, 2015, 11:02 pm

Let  $ABC$  be a triangle with incircle  $(I)$  and  $DEF$  is cevian triangle of a point  $P$ . Let  $X, Y, Z$  be the poles of lines  $EF, FD, DE$  with respect to  $(I)$ .

a) Prove that  $BY, CZ$  and  $EF$  are concurrent at point  $U$ .

b) Similarly, we have point  $U, V, W$ . Prove that  $AU, BV, CW$  are concurrent.



Luis González

#2 Oct 9, 2015, 12:31 am • 1

a) Let  $(I)$  touch  $BC, CA, AB$  at  $D', E', F'$ . Then  $U \equiv EF \cap E'F', V \equiv FD \cap F'D', W \equiv DE \cap D'E'$  are the poles of  $AX, BY, CZ$  WRT  $(I)$ . If  $\mathcal{C}$  is the circum-conic through  $P$  and  $AD' \cap BE' \cap CF'$ , then  $A, X, V, W$  are collinear on a tangent of  $\mathcal{C}$  (for a proof see [collinear 4](#)), thus their polars  $EF, BY, CZ$  WRT  $(I)$  concur at  $U$ .

b)  $\triangle ABC$  and the polar triangle  $\triangle UVW$  of  $\mathcal{C}$  WRT  $\triangle ABC$  are perspective (well-known), their perspector being  $AU \cap BV \cap CW$ .

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## High School Olympiads

A nice concurrency X

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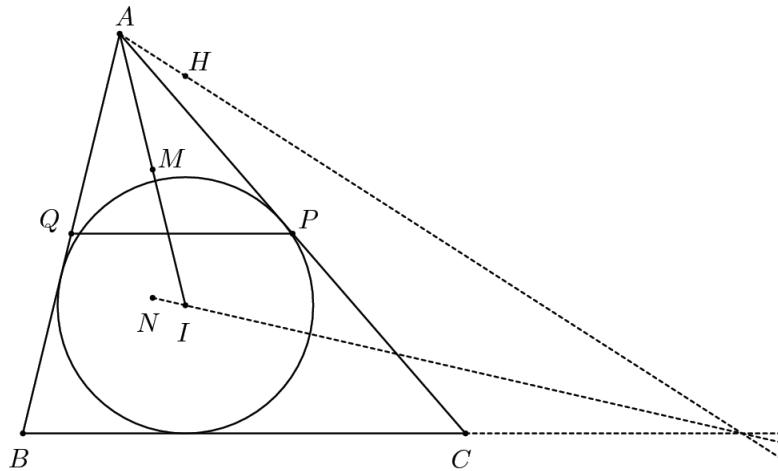
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**mjuk**

#1 Apr 2, 2016, 5:27 am • 1

Let  $\triangle ABC$  be a triangle with incenter  $I$ . Let  $H$  be orthocenter of  $\triangle BIC$ , let  $P, Q, M$  be midpoints of  $AC, AB, AI$ . Let  $N$  be reflection of  $M$  over  $PQ$ . Prove that  $AH, NI, BC$  are concurrent.



This post has been edited 1 time. Last edited by mjuk, Apr 3, 2016, 6:14 pm  
Reason: added latex figure



**Luis González**

#2 Apr 2, 2016, 7:42 am • 1

Let  $R$  be the midpoint of  $BC$  and let  $D, E, F$  be the tangency points of the incircle  $(I, r)$  with  $BC, CA, AB$ .  
 $Fe \equiv \odot(PQR) \cap \odot(DEF)$  is the Feuerbach point of  $\triangle ABC$ .

By 1st Fontené theorem,  $DFe, EF, PQ$  concur at a point  $U$  and if  $L$  is the antipode of  $D$  on  $(I)$ , then  $M, L, Fe$  are collinear (see [Intersect on circle](#) post #4 for a proof of the general case). Since  $M$  is clearly the incenter of  $\triangle APQ$ , it follows that  $MN = r = IL \implies MNIL$  is a parallelogram  $\implies (IN \parallel MLFe) \perp DFe \implies X \equiv BC \cap IN$  is the pole of  $DFe$  WRT  $(I)$ . As  $H$  is the pole of  $PQ$  WRT  $(I)$  (well-known), then we deduce that  $U$  is the pole of  $AH$  WRT  $(I)$   $\implies X \equiv AH \cap NI \cap BC$ . Moreover  $XFe$  is tangent of  $(I)$  at  $Fe$ .



**FabrizioFelen**

#3 Apr 2, 2016, 10:07 am

My solution:

Let  $R$  the point of tangency of incircle and  $BC$ ,  $S = AH \cap BC$ ,  $T = MN \cap BC$  and  $D$  is the foot of perpendicular of  $A$  in  $BC$

Define:  $AB = a + c$ ,  $BC = a + b$ ,  $CA = b + c$ ,  $IR = r$ ,  $AD = h$ ,  $DR = 2y$  and  $RS = z \implies RB = a$ ,  $RC = b$  and  $MN = r$

Since  $H$  is the ortocenter of  $\triangle BIC \implies RI \cdot RH = RB \cdot RC \implies HR = \frac{ab}{r}$

Since  $M$  is the midpoint of  $AI \implies 2MT = AD + IR \implies MT = \frac{h+r}{2} \implies NT = \frac{h-r}{2}$

Since  $AD \parallel RH \implies \frac{AD}{DS} = \frac{RH}{RS} \implies \frac{h}{z+2y} = \frac{ab}{zr} \implies z = \frac{2yab}{hr-ab} \implies \frac{y+z}{z} = \frac{ab+hr}{2ab} \dots (\star)$

But it's well known that  $hr = \frac{2abc}{2abc} = \frac{h}{2(a+b+c)}$  replacing in  $(\star)$  we get  $\frac{h-r}{h-r} = \frac{y+z}{a+b+2c} = \frac{a+b+2c}{2(a+b+c)} \implies$

$\frac{h-r}{2r} = \frac{y+z}{z} \Rightarrow \frac{NT}{IR} = \frac{a+b}{TS} = \frac{r}{RS}$  ... (★). Since  $NT \parallel IR$  and (★) we get  $N, I, S$  are collinear hence  $AH, NI, BC$  are concurrent.



**mjuk**

#4 Apr 2, 2016, 1:50 pm

99

1

**My solution:**

Let  $\triangle DEF$  be intouch triangle, let  $K$  be projection of  $A$  on  $BC$ , let  $L = MN \cap BC$ ,  $R = MN \cap PQ$ ,  $S = NI \cap AK$ ,  $X = DH \cap PQ$ .  $KA \parallel DH$ , so suffices to show  $\frac{KS}{KA} = \frac{DI}{DH}$ . Let  $[KA = h]$ ,  $[DI = r]$ .

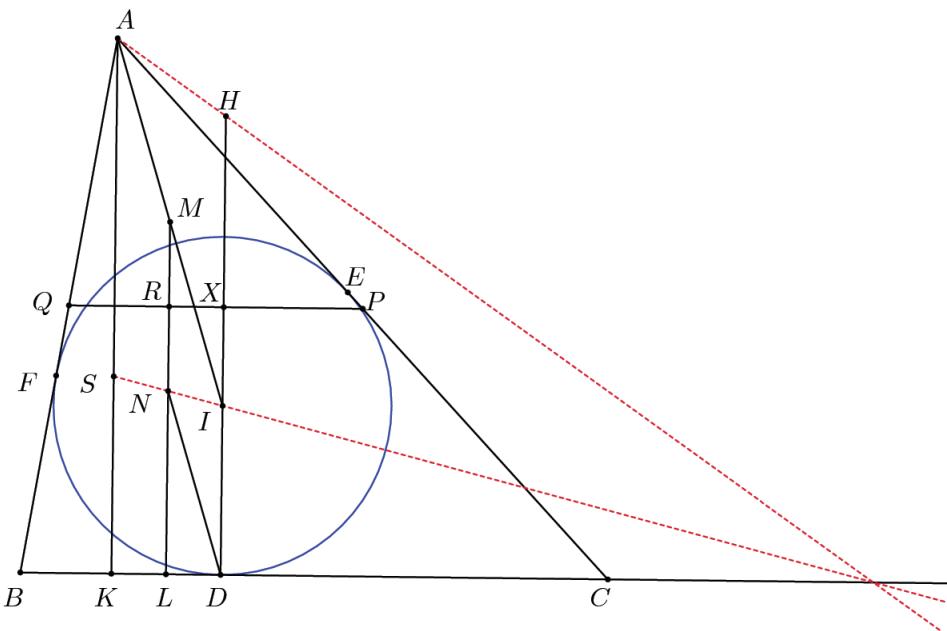
Since  $M$  is midpoint of  $AI$ ,  $M$  is center of incircle of  $\triangle APQ$  with radius  $\frac{r}{2}$ .

$$\rightarrow KS = 2LN - DI = 2(LR - RN) - r = 2\left(\frac{h}{2} - \frac{r}{2}\right) - r \rightarrow [KS = h - 2r].$$

Well-known lemma:  $PQ$  is polar of  $H$  wrt.  $(I)$   $\rightarrow X, H$  are inverses wrt.  $(I)$

$$\rightarrow ID^2 = IX \cdot IH \rightarrow r^2 = \left(\frac{h}{2} - r\right)(DH - r) \rightarrow [DH = \frac{hr}{h - 2r}].$$

And finally  $\frac{KS}{KA} = \frac{h - 2r}{h} = \frac{DI}{DH}$ .



This post has been edited 4 times. Last edited by mjuk, Apr 3, 2016, 1:45 am  
Reason: added figure



**TelvCohl**

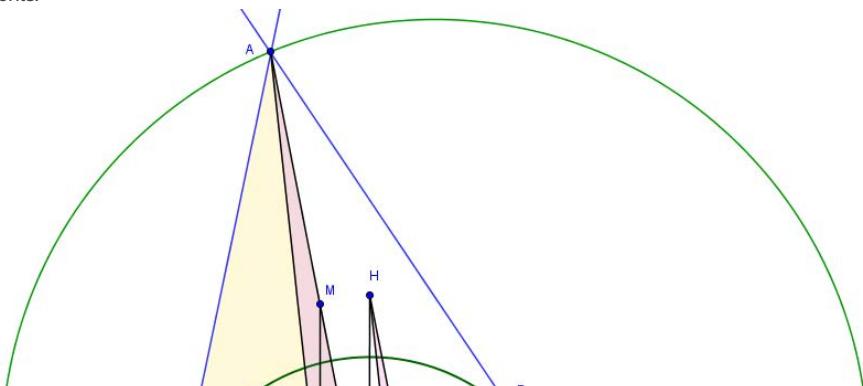
#5 Apr 2, 2016, 2:38 pm • 2

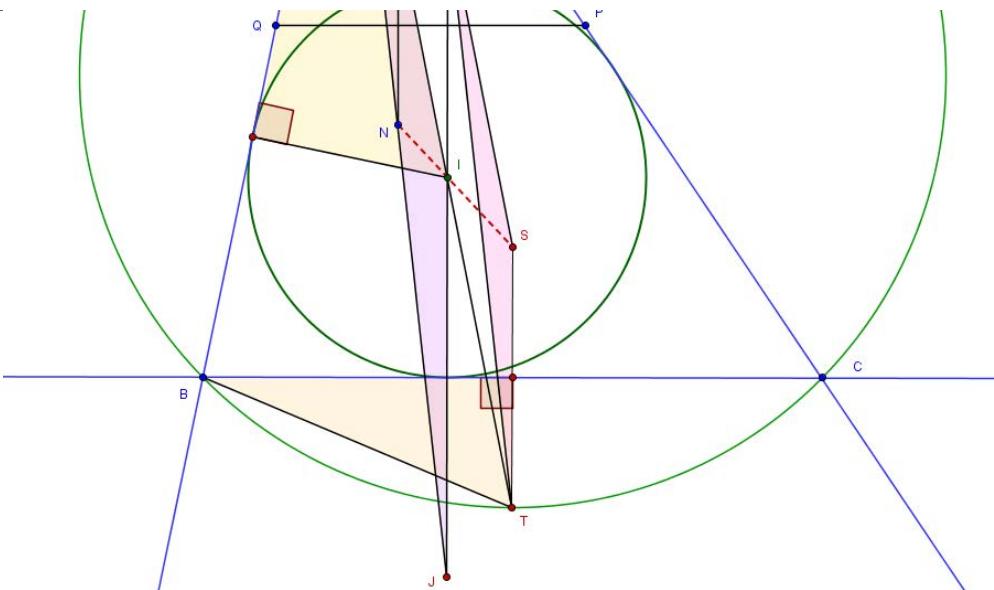
99

1

Let  $T$  be the midpoint of arc  $BC$  in  $\odot(ABC)$  and let  $S$  be the reflection of  $T$  in  $BC$ . Since  $\frac{MI}{MN} = \frac{1}{2} \cdot \frac{AI}{r} = \frac{TI}{2\text{dist}(T, BC)} = \frac{TI}{TS}$ , so  $I, N, S$  are collinear. Let  $J$  be the reflection of  $I$  in  $BC$ . Since  $N$  is the midpoint of  $AJ$  and  $HI \parallel ST$ , so  $HS \parallel AI$  and  $\frac{AI}{IJ} = \frac{MI}{MN} = \frac{TI}{TS} = \frac{HS}{ST} \Rightarrow \triangle AIJ \sim \triangle HST$  are homothetic, hence  $AH, NI, BC$  are concurrent.

Attachments:





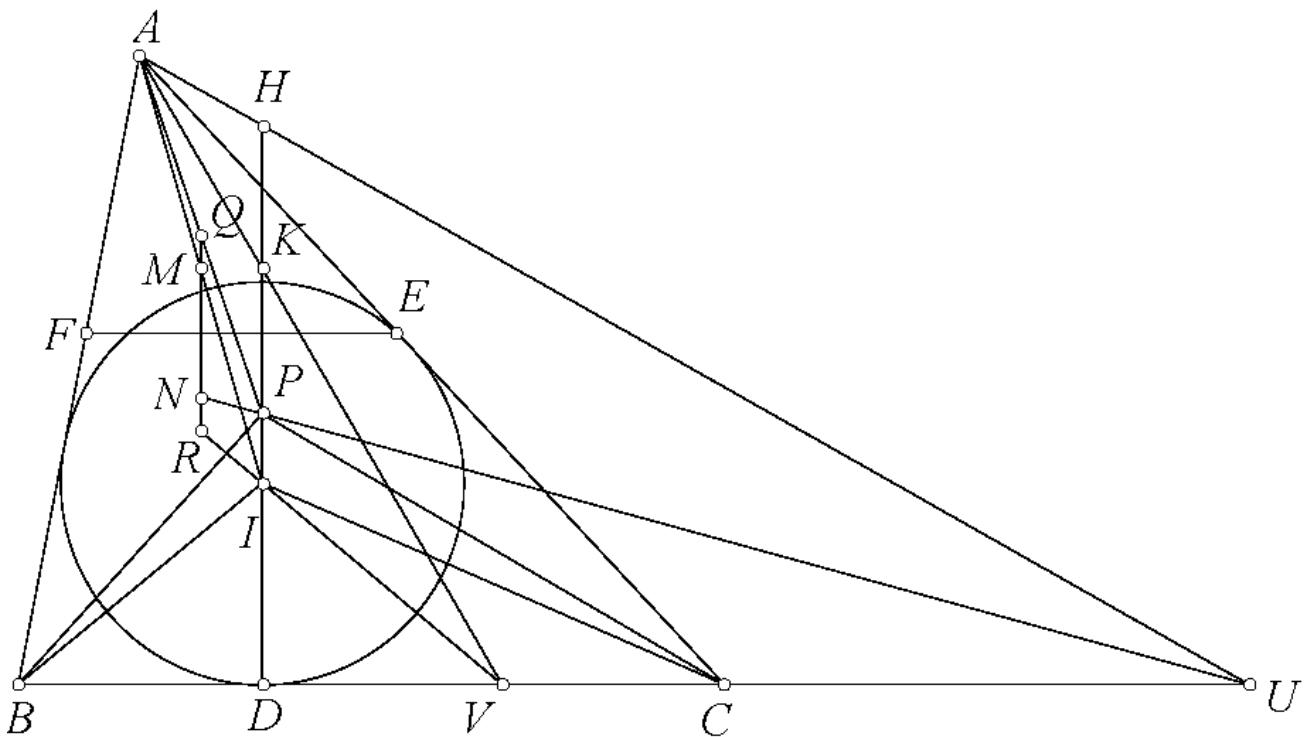
**buratinogigle**

#6 Apr 2, 2016, 11:46 pm

I see a problem as following

Let  $ABC$  be a triangle with incircle  $(I)$ .  $E, F$  lie on  $CA, AB$  such that  $EF \parallel BC$ .  $H$  is orthocenter of triangle  $IBC$ .  $AH$  cuts  $BC$  at  $U$ .  $M$  is midpoint of  $AI$ .  $N$  is reflection of  $M$  through  $EF$ .  $NU$  cuts  $IH$  at  $P$ .  $K$  is orthocenter of triangle  $PBC$ .  $AK$  cuts  $BC$  at  $V$ .  $Q$  is midpoint of  $AP$ .  $R$  is reflection of  $Q$  through  $EF$ . Prove that  $V, I, R$  are collinear.

Attachments:



**Luis González**

#7 Apr 3, 2016, 4:24 am • 1

“ buratinogigle wrote:

I see a problem as following

Let  $ABC$  be a triangle with incircle  $(I)$ .  $E, F$  lie on  $CA, AB$  such that  $EF \parallel BC$ .  $H$  is orthocenter of triangle  $IBC$ .  $AH$  cuts  $BC$  at  $U$ .  $M$  is midpoint of  $AI$ .  $N$  is reflection of  $M$  through  $EF$ .  $NU$  cuts  $IH$  at  $P$ .  $K$  is orthocenter of triangle  $PBC$ .  $AK$  cuts  $BC$  at  $V$ .  $Q$  is midpoint of  $AP$ .  $R$  is reflection of  $Q$  through  $EF$ . Prove that  $V, I, R$  are collinear.

Animate  $E, F$  on  $AC, AB$ . Obviously  $Q, N, R$  move on the A-midline  $\ell$  of  $\triangle AIH$  and the series  $N, P, Q, K, V$  are clearly all projective. By symmetry  $\overrightarrow{MQ} = \overrightarrow{RN}$ , thus as the series  $N, Q$  are similar, then it follows that the series  $N, R$  are similar as well. Consequently  $R \mapsto V$  is a projectivity between  $\ell$  and  $BC$ . So all we need to prove is that this projectivity is a perspectivity through  $I$ , in other words, that  $R, I, V$  are collinear for 3 particular cases.

When  $E, F$  go to infinity,  $Q, N, R, P$  go to the point at infinity of  $\perp BC$ , thus  $V$  coincides then with the projection of  $I$  on  $BC$   $\implies R, I, V$  are collinear. When  $P \equiv H$ , then obviously  $R, I, V$  are collinear on the angle bisector of  $\angle BAC$ . Finally when  $E, F$  coincide with the midpoints of  $AC, AB$ , then from the original problem we get that  $R, I, V$  are collinear and we are done.



**navi\_09220114**

#8 Apr 3, 2016, 8:27 am

I am sorry to ask but why it is sufficient to prove  $V, I, R$  is colinear in 3 cases after showing they are projective? And how the points  $N, P, Q, K, V$  are projective? Thanks in advance!



**wwwrqnojcm**

#9 Apr 3, 2016, 12:21 pm

“ *mjuk* wrote:

**My solution:**

Well-known lemma:  $PQ$  is polar of  $H$  wrt.  $(I) \rightarrow X, H$  are inverses wrt.  $(I)$

$$\rightarrow ID^2 = IX \cdot IH \rightarrow r^2 = \left(\frac{h}{2} - r\right)(DH - r) \rightarrow DH = \frac{hr}{h - 2r}.$$

$$\text{And finally } \frac{KS}{KA} = \frac{h - 2r}{h} = \frac{DI}{DH}.$$

Could someone please specifically explain this lemma? Thanks.



**mjuk**

#10 Apr 3, 2016, 5:47 pm

“ *wwwrqnojcm* wrote:

Could someone please specifically explain this lemma? Thanks.

You can see [post #2 here](#).



**EinsteinXXI**

#11 Apr 3, 2016, 11:07 pm

Here is my solution :

Let  $AH$  meets  $BC$  at  $G$ .

Suppose that  $(I)$  touches  $BC, CA, AB$  at  $D, E, F$  respectively.

$K$  is the intersection point of  $EF$  and  $PQ$ .

We know that  $A$  is the polar point of  $EF$  WRT  $(I)$ ,  $H$  is the polar point of  $PQ$  WRT  $(I)$  so  $K$  is the polar point of  $AH$  WRT  $(I)$ .

But  $D$  is the polar point of  $BC$  WRT  $(I)$  so  $G$  is the polar point of  $DK$  WRT  $(I)$ .

Hence  $IG$  is perpendicular  $DK$ .

Let  $T, S$  is the intersection points of the line passing  $A$  perpendicular  $BC$  with  $BC, PQ$  respectively,  $R$  is the intersection point of  $ID$  and  $PQ$ .

We have  $MN = AS - IR = ST - IR = DR - IR = ID$  so  $MNDI$  is a parallelogram. So  $IM = ND$ .

We have  $IK^2 - ID^2 = IK^2 - r^2 = -KE \cdot KF = MK^2 - MI^2 = NK^2 - ND^2$  so  $IN \perp DK$ .

Hence  $I, N, G$  is collinear so  $AH, NI, BC$  are concurrent.

This post has been edited 3 times. Last edited by EinsteinXXI, Apr 5, 2016, 9:54 am



user1234567890 edited this post on Apr 5, 2016, 9:54 am

**Lemma:**  $\triangle ABC$  with incenter  $I$ .  $H$  is the orthocenter of  $\triangle IBC$ .  $HD$  cuts  $(I)$  at  $K$ . We have  $HK \cdot HI = r^2$

$AR \perp BC$ ,  $D'$  reflect with  $D$  about  $I$ .  $OM$  cuts  $AR$  at  $S$ ,  $D'M$  cuts  $AR$  at  $T$

$\Rightarrow MM'DI$  is a parallelogram

$\Rightarrow D, M, T$  collinear

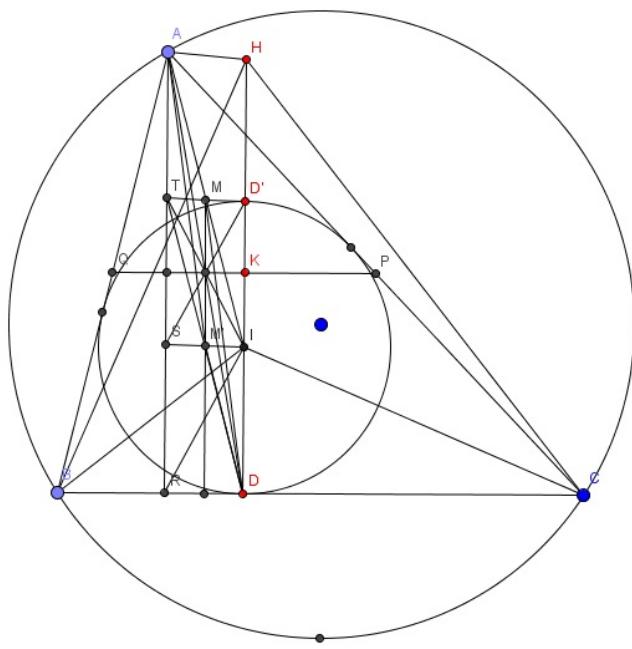
$$ID^2 = IK \cdot IH \Rightarrow r^2 = (DH - r) \left( \frac{h}{2} - r \right)$$

$$\Rightarrow DH = \frac{hr}{(h - 2r)}$$

$$\Rightarrow \frac{DH}{DI} = \frac{RA}{RS}$$

$\Rightarrow AH, SI, DR$  concurrent

Attachments:



This post has been edited 5 times. Last edited by uraharakisuke\_nsgs, Apr 24, 2016, 6:04 pm

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## High School Olympiads

Intersect on circle X

[Reply](#)

**buratinogiggle**

#1 Jan 17, 2010, 10:24 pm

Let  $A'B'C'$  be a pedal triangle of arbitrary point  $P$  with respect to triangle  $ABC$ .  $P^*$  is isogonal conjugate of  $P$  with respect to triangle  $ABC$ .  $R_p$  is circumradius of triangle  $A'B'C'$ . The rays  $[B'P]$ ,  $[C'P]$  intersect circle  $(P^*, 2R_p)$  at  $B''$ ,  $C''$ , respectively. Prove that  $BB''$ ,  $CC''$  intersect on circle  $(P^*, 2R_p)$ .

Note that, this is generalization of the problem on the post [Nice problem about concurrent:D](#)

**livetolove212**

#2 Jan 18, 2010, 9:03 pm • 1

Nice problem dear Mr.Hung 😊

Attachments:

[bai9.PDF \(47kb\)](#)**buratinogiggle**

#3 Jan 19, 2010, 3:13 pm

Great work dear Linh 😊 , all my problems always wait for your nice solution 😊!

**Luis González**

#4 Jul 10, 2011, 8:57 pm • 5

**Proposition:**  $P$  is a point on the plane of  $\triangle ABC$  and  $\triangle P_A P_B P_C$  is the pedal triangle of  $P$  WRT  $\triangle ABC$ .  $A_1, B_1, C_1$  are the midpoints of  $PA, PB, PC$ . Lines  $PP_A, PP_B, PP_C$  cut the pedal circle  $\odot(P_A P_B P_C)$  again at  $P_1, P_2, P_3$ . Then  $P_1 A_1, P_2 B_1$  and  $P_3 C_1$  concur on  $\odot(P_A P_B P_C)$ .

**Proof:** Let  $U$  be the Poncelet point of  $ABCP$ , which lies on the pedal circle of  $P$  WRT  $\triangle ABC$  (for a proof, see post #2 [here](#)).  $\odot(P_A P_B P_C)$  cuts  $BC$  again at  $D$ .  $R$  is the orthogonal projection of  $A$  onto  $PC \Rightarrow \odot(URA_1 C_1)$  is 9-point circle of  $\triangle APC$ .

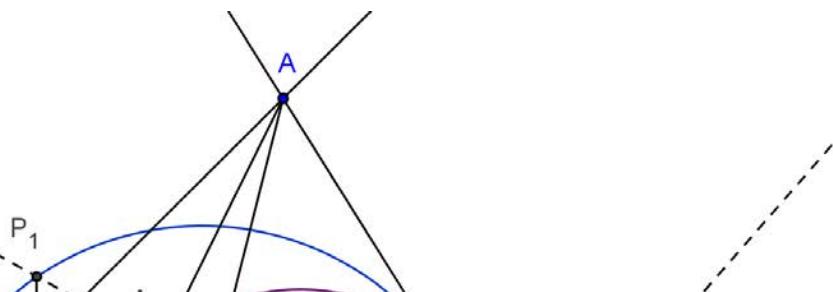
$$\angle DUC_1 = \angle DUP_B - \angle C_1 UP_B = 180^\circ - \angle CPP_B - \angle PRP_B$$

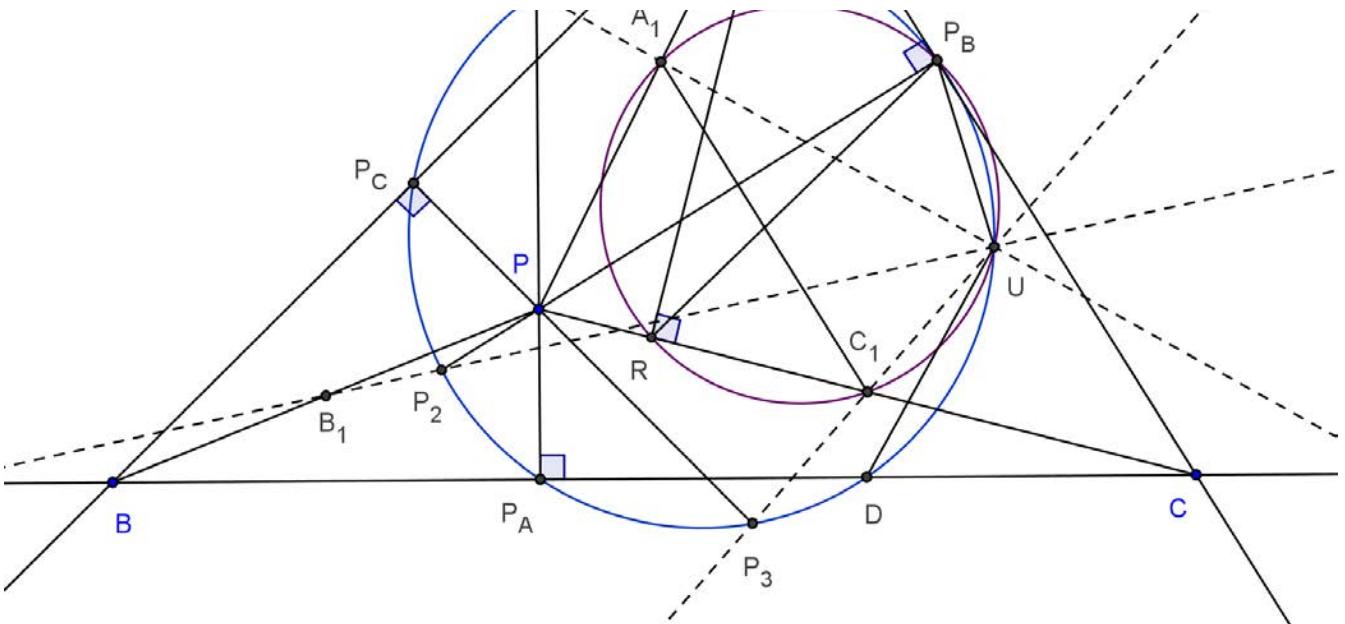
$$\angle DUC_1 = \angle PAC - \angle CPP_B = \angle PAC - \angle RAC = 90^\circ - \angle APC$$

Thus,  $\angle DUA_1 = \angle DUC_1 + \angle C_1 UA_1 = 90^\circ - \angle APC + \angle APC = 90^\circ$ . Since  $\angle DUP_1 = 90^\circ$ , then  $U \in P_1 A_1$ . Likewise,  $P_2 B_1$  and  $P_3 C_1$  pass through  $U$ .

P.S. Note that a dilatation with center  $P$  and factor 2 produces the configuration of the proposed problem. P-Pedal circle  $\odot(P_A P_B P_C)$  with radius  $\rho$  goes to the circle  $\odot(P^*, 2\rho)$  centered at the isogonal conjugate  $P^*$  of  $P$ .

Attachments:





**buratinogiggle**

#5 Jul 11, 2011, 10:52 am

There is a generalization for above



**Problem.** Let  $ABC$  be a triangle and  $A', B', C'$  lies on  $BC, CA, AB$ , resp.  $P$  is Miquel point of  $A'B'C'$  (the common point of circumcircles  $(AB'C')$ ,  $(BC'A')$ ,  $(CA'B')$ ).  $PA', PB', PC'$  cut the circumcircle  $(A'B'C')$  again at  $P_a, P_b, P_c$ . Let  $O_a, O_b, O_c$  be circumcenter of triangles  $AB'C'$ ,  $BC'A'$ ,  $CA'B'$ , resp. Prove that  $O_aP_a, O_bP_b, O_cP_c$  concur on  $(A'B'C')$ .



**skytin**

#6 Jul 11, 2011, 2:24 pm



“ buratinogiggle wrote:

There is a generalization for above

**Problem.** Let  $ABC$  be a triangle and  $A', B', C'$  lies on  $BC, CA, AB$ , resp.  $P$  is Miquel point of  $A'B'C'$  (the common point of circumcircles  $(AB'C')$ ,  $(BC'A')$ ,  $(CA'B')$ ).  $PA', PB', PC'$  cut the circumcircle  $(A'B'C')$  again at  $P_a, P_b, P_c$ . Let  $O_a, O_b, O_c$  be circumcenter of triangles  $AB'C'$ ,  $BC'A'$ ,  $CA'B'$ , resp. Prove that  $O_aP_a, O_bP_b, O_cP_c$  concur on  $(A'B'C')$ .

Use homotety with center at  $P$  and coefficient  $1/2$ , then Luisgeometra's problem for  $O_aO_bO_c$  and  $P$

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## High School Olympiads

geometry  Locked**phantranhuongth**

#1 Apr 1, 2016, 10:48 pm

 $ABC$  circumscribed circle ( $I$ ), inscribed ( $O$ ). ( $I$ ) cut  $BC$  at  $D$ .  $M$  midpoint longbow  $BC$ .A line through  $I$  perpendicular  $IO$  cut  $AO, MD$  at  $X, Y$ Prove  $IX=IY$ **Luis González**

#2 Apr 1, 2016, 11:36 pm

Discussed before at <http://www.artofproblemsolving.com/community/c6h347299>

## High School Olympiads

An application of the butterfly property. 

Reply



Virgil Nicula

#1 Apr 29, 2010, 8:24 pm

Let  $ABC$  be a triangle with incircle  $C(I)$  and circumcircle  $C(O)$ . The incircle touches the side  $[BC]$  in a point  $D$  and the lines  $AI$ ,  $AO$  cut again the circumcircle in the points  $M$ ,  $S$  respectively.

Let  $X \in DM$ ,  $Y \in AO$  be two points so that  $I \in XY$ . Prove that  $IX = IY \iff OI \perp XY$ .



Luis González

#2 May 1, 2010, 5:19 am

Let  $P$  be the midpoint of the arc  $ABC$  and  $T \equiv MD \cap (O)$ , different from  $M$ . Inversion in the circle  $(M)$  with radius  $MB = MC = MI$  swaps  $(O)$  and the sideline  $BC$ ,  $I$  is double and  $D \mapsto T$ , thus  $\odot(IDT)$  is double  $\implies MI$  is tangent to  $\odot(IDT)$ . Hence, it follows that  $\angle ITD = \angle DIM = \angle AMP = \angle ATP$ , but  $\angle MTP$  is right since  $M, P$  are antipodal. Then  $\angle ATI = MTP = 90^\circ$ , which implies that chords  $TS$  and  $AM$  meet at  $I$ . Now, by Butterfly theorem we conclude that  $IX = IY \iff OI \perp XY$ .



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## High School Olympiads

Geometry 

 Reply



Source: It's a well-known result(i forgot the proof)



den\_thewhitelion

#1 Mar 31, 2016, 8:50 pm

Let  $\triangle ABC$  and  $M = \text{midpoint of } (BC), MD \perp AB, ME \perp AC; D \in (AB) \text{ and } E \in (AC)$

Then the  $\odot(BAE)$  and  $\odot(CAD)$  intersect on a point on the A-altitude of the triangle



babu2001

#2 Mar 31, 2016, 9:44 pm • 1 

Let the orthocenter of  $\triangle ABC$  be  $H$ . Let  $BH \cap AC \equiv X$  &  $CH \cap AB \equiv Y$ . Then  $BX \parallel ME$  &  $BC = 2MC$  hence  $CX = 2CE$ . Let the orthocenter of  $\triangle BAE$  be  $H_1$ . Then  $EH_1 \perp AB$  so  $EH_1 \parallel CH$  &  $CX = 2CE$  so  $HX = 2H_1X$  as  $H, H_1, X$  are collinear. Let  $BHH_1 \cap \odot(BAE) \equiv P$ . By a well known result  $P$  is the reflection of  $H_1$  in  $AE$ . Thus

$$HP = HX + XP = \frac{3}{2}HX. \text{ Thus we have } HB \cdot HP = \frac{3}{2}HB \cdot HX. \text{ Similarly let } CH \cap \odot(CAD) \equiv Q, \text{ then}$$

$HC \cdot HQ = \frac{3}{2}HC \cdot HY$ . From the cyclic quadrilateral  $BYXC$  we have  $BH \cdot HX = CH \cdot HY$ . Hence we have  $HB \cdot HP = HC \cdot HQ$ , so  $H$  lies on the radical axis of  $\odot(BAE)$  &  $\odot(CAD)$   $\Rightarrow AH$  is the radical axis of  $\odot(BAE)$  &  $\odot(CAD)$ . Thus  $\odot(BAE) \cap \odot(CAD) \in AH$ , as required.



This post has been edited 2 times. Last edited by babu2001, Apr 1, 2016, 2:07 am

Reason: Improved Latex



den\_thewhitelion

#3 Mar 31, 2016, 10:16 pm

Thanks @babu2001



Luis González

#4 Mar 31, 2016, 11:07 pm

See the topic <http://www.artofproblemsolving.com/community/c6h497171>.



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## High School Olympiads

The line passing through centers is parallel to one side X

Reply



Narcissus

#1 Sep 5, 2012, 10:14 am

Given a triangle  $ABC$  and  $AM$  is one of its medians. Let  $E$  and  $F$  be feet of  $M$  onto sides  $AB$  and  $AC$  respectively. Let  $(O_1)$  be the circumcenter of triangle  $ACE$  and  $(O_2)$  be the circumcenter of triangle  $ABF$ . Prove that  $O_1O_2$  is parallel to  $BC$



Luis González

#2 Sep 5, 2012, 12:18 pm

Let  $H$  be the orthocenter of  $\triangle ABC$ .  $H_A, H_B$  are the feet of the A- and B- altitude.  $MF \parallel BH_B$  is C-midline of  $\triangle BCH_B$ , i.e.  $F$  is midpoint of  $\overline{CH_B}$ . Let  $P$  be the midpoint of  $\overline{HH_A}$ . Then  $\triangle BHH_A$  and  $\triangle BCH_B$  are similar with corresponding B-medians  $BP$  and  $BH$ . Thus if  $Q$  is the reflection of  $P$  about  $BC$ , we have  $\angle BQA = \angle BPH_A = \angle BFA \implies Q \in (O_2)$ . Similarly,  $Q \in (O_1) \implies$  A-altitude  $AQ$  is radical axis of  $(O_1), (O_2) \implies O_1O_2 \perp AQ$ , or  $O_1O_2 \parallel BC$ .

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## High School Olympiads





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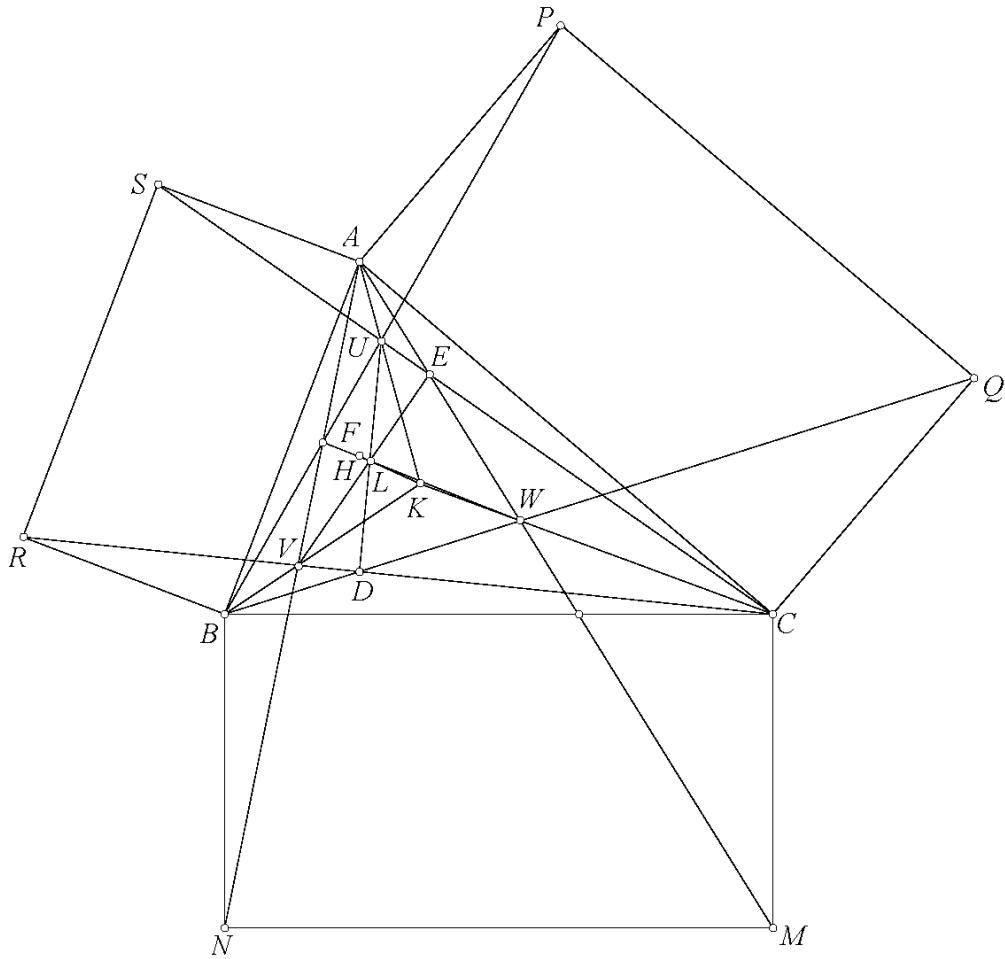
buratinogigle

#1 Mar 31, 2016, 10:05 pm • 1

Let  $ABC$  be a triangle. Construct outside triangle three similar rectangles  $BCMN, CAPQ, ABRS$ . Let  $BQ$  cuts  $CR$  at  $D$ . Similar, we have  $E, F$ . Let  $BP$  cuts  $CS$  at  $U$ . Similarly, we have  $V, W$ .

- Prove that  $AU, BV, CW$  are concurrent at  $K$ .
- Prove that  $DU, EV, FW$  are concurrent at  $L$ .
- Prove that  $KL$  passes through orthocenter  $H$  of triangle  $ABC$ .

Attachments:



Luis González

#2 Mar 31, 2016, 10:40 pm • 2

Denote  $O_1, O_2, O_3$  the centers of the rectangles  $BCMN, CAPQ, ABRS$ , respectively and denote  $\theta$  the base angle of the isosceles  $\triangle O_1 BC, \triangle O_2 CA, \triangle O_3 AB$ . Since  $\angle O_1 BC = \angle O_1 CB = \angle ABS = \angle ACP$  and  $\angle BAS = \angle CAP = 90^\circ$ , then by Jacobi's theorem  $AO_1, BP, CS$  concur, i.e.  $AU \equiv AO_1$ . Similary  $BV \equiv BO_2$  and  $CW \equiv CO_3 \implies AU, BV, CW$  concur at the Kiepert perspector  $K \equiv K(\theta)$ .

Since  $\angle BAR = \angle CAQ$  and  $\angle ABR = \angle ACQ = 90^\circ$ , the by Jacobi's theorem, it follows that  $BQ, CR$  and the A-altitude of  $\triangle ABC$  concur, i.e.  $AD$  is the A-altitude of  $\triangle ABC$  and similarly  $BE$  and  $CF$  are the altitudes issuing from  $B$  and  $C$ . Now see the general configuration at the topic [The Equilateral triangle](#) (post #3 and #4). We conclude that  $DU, EV, FW$  concur at  $L$  and  $K, H, L$  are collinear

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## High School Olympiads

The Equilateral triangle 

 Reply



Source: vankhea



vankhea

#1 Jan 23, 2015, 12:23 pm

Let  $I$  be incenter of equilateral triangle  $\Delta ABC$  and let  $K$  be point lie on circle  $(I)$ . Let  $X, Y, Z$  be midpoints of  $KA, KB, KC$  and let  $D = BZ \cap CY, E = CX \cap AZ, F = AY \cap BX$ . Let  $T$  be concurrent point of  $AD, BE, CF$ .

Prove that  $IT = \frac{1}{4}r$ , with  $r$  be inradii.



Luis González

#2 Jan 23, 2015, 7:38 pm

Let  $(I)$  touch  $BC$  at its midpoint  $M$ . Thus  $KM, BZ$  and  $CY$  are medians of  $\triangle KBC$  concurring at its centroid  $D$ . If  $AD$  cuts  $IK$  at  $T'$ , then by Menelaus' theorem for  $\triangle KIM$  cut by  $\overline{AT'D}$ , we get  $\frac{T'K}{T'I} = \frac{AM}{AT} \cdot \frac{DK}{DM} = -\frac{3}{2} \cdot 2 = -3 \Rightarrow IT' = \frac{1}{4}IK$ . By similar reasoning,  $BE$  and  $CF$  cut  $IK$  at the same point  $T' \Rightarrow T \equiv T' \Rightarrow IT = \frac{1}{4}r$ .

In general we have the following: If  $P$  is an arbitrary point on the plane of  $\triangle ABC$ , the lines joining  $A, B, C$  with the centroids of  $\triangle PBC, \triangle PCA, \triangle PAB$ , resp, concur at the complement of the complement of  $P$  WRT  $\triangle ABC$ .



vankhea

#3 Jan 24, 2015, 10:01 am

Similarly the above problem

Let  $P$  be an arbitrary point on the plan of triangle  $\triangle ABC$ . Let  $X, Y, Z$  be points on  $PA, PB, PC$  respectively.

Let  $D = BZ \cap CY, E = CX \cap AZ, F = AY \cap BX$ .

Prove that  $AD, BE, CF$  concurrent.



Luis González

#4 Jan 28, 2015, 10:49 am

In response to the latter message: The homography  $\{B, Y, Z, C\} \mapsto \{B, F, E, C\}$  is nothing but a homology that fixes  $BC$  and the line pencil through  $A \equiv FY \cap EF$ , thus since  $P \mapsto X$  and  $D \mapsto Q \equiv BE \cap CF \Rightarrow A \in QD$ , in other words  $AD, BE, CF$  concur at  $Q$ , as desired.



Bonus: Since  $YZ \mapsto EF \Rightarrow YZ, EF, BC$  concur at  $U$  and likewise  $ZX, FD, CA$  concur at  $V$  and  $XY, DE, AB$  concur at  $W \Rightarrow \triangle ABC, \triangle DEF$  and  $\triangle XYZ$  are perspective with common perspectrix  $\overline{UVW}$ , thus by the 3 homologies theorem,  $DX, EY, FZ$  concur at  $R$  and  $P, Q, R$  are collinear.

 Quick Reply

## High School Olympiads

Bisect the segment X

↳ Reply



Source: OWN



LeVietAn

#1 Mar 30, 2016, 8:06 am



Dear Mathlinkers,

Let  $\triangle ABC$  with incenter  $I$ . The points  $P$  and  $Q$  lie on the side  $BC$  such that  $AP, AQ$  are isogonal conjugate WRT  $\angle BAC$ . The tangent line at  $B$  of  $\odot(ABP)$  and the tangent line at  $C$  of  $\odot(ACQ)$  intersect at  $D$ . Let  $E = AP \cap IB, F = AQ \cap IC$ . Prove that  $DI$  bisects the segment  $EF$ .



Luis González

#2 Mar 30, 2016, 9:10 am



Let  $M$  and  $N$  be the midpoints of  $BC$  and the arc  $BAC$  of  $\odot(ABC)$ . As  $P$  moves on  $BC$ , clearly  $D$  moves on the perpendicular bisector  $\ell_A$  of  $\overline{BC}$  and since  $\angle DBC = \angle BAE = \angle CAF$ , we deduce that the series  $P, Q, D, E, F, U$  are all projective, where  $U$  is the midpoint of  $EF$ .  $\Rightarrow E \mapsto F$  is a projectivity between  $IB$  and  $IC \Rightarrow U$  describes a hyperbola  $\mathcal{H}$  with asymptotes parallel to  $IB, IC$  and passing through  $I$ , because when  $P \equiv Q \equiv AI \cap BC$ , we have  $E \equiv F \equiv U \equiv I$ .

When  $\{P \equiv B, Q \equiv C\}$ , then  $U \equiv M$  and when  $P \equiv Q$  is identical with the foot of the external bisector of  $\angle BAC$ , then  $E$  and  $F$  become excenters of  $\triangle ABC$  against  $B, C \Rightarrow U \equiv N$ . Therefore  $U \mapsto D$  is a stereographic projection of  $\mathcal{H}$  onto  $\ell_A \Rightarrow I, U, D$  are collinear for any pair  $P, Q$ .



doxuanlong15052000

#3 Mar 30, 2016, 9:45 pm • 2



My solution:

We have  $\angle DBC = \angle BAP = \angle CAQ = \angle DCB \Rightarrow DB = DC \Rightarrow \sin DIB = \sin DBI = \sin IEA = \frac{\sin IEA}{\sin IAF} = \frac{IE}{IF} \Rightarrow DI$  bisects the segment  $EF$ .

This post has been edited 1 time. Last edited by doxuanlong15052000, Mar 30, 2016, 9:47 pm



PROF65

#4 Mar 30, 2016, 10:12 pm



@doxuanlong15

Good job just rectify a typo:

$\frac{IF}{IE}$  instead of  $\frac{IE}{IF}$

↳ Quick Reply

## High School Olympiads

Line joining circumcenters passes through A 

 Reply



Source: Own



livetolove212

#1 Mar 29, 2016, 10:25 am

Given triangle  $ABC$ . Let  $P$  and  $Q$  be arbitrary points on  $BC$  such that  $AP = AQ$ .  $(APQ)$  intersects  $AC, AB$  at  $E, F$ , respectively. Let  $O_1, O_2$  be the circumcenters of triangles  $BPE, CPF$ . Prove that  $O_1O_2$  passes through  $A$ .



Luis González

#2 Mar 29, 2016, 10:43 am

Inversion WRT  $\odot(A, AP)$  swaps  $\odot(APQ)$  and  $PQ$ , therefore it swaps  $B, F$  and  $C, E \implies \odot(BPE)$  goes to  $\odot(CPF) \implies \odot(BPE)$  and  $\odot(CPF)$  are coaxal with their inversion circle  $\odot(A, AP)$  (well-known)  $\implies$  their centers  $A, O_1, O_2$  are collinear.

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**trumpeter's Problem of the Day Forum**

[Reply](#)**trumpeter**

#1 Jan 16, 2016, 12:16 pm

**Problem of the Day: 1/15/16**

In  $\triangle ABC$  with circumcircle  $\Gamma$ , suppose that the tangents to  $\Gamma$  from  $B$  and  $C$  intersect  $AC$  and  $AB$  at  $P$  and  $Q$ , respectively. Similarly, let the perpendicular bisectors of  $\overline{AB}$  and  $\overline{AC}$  intersect  $AC$  and  $AB$  at  $R$  and  $S$ , respectively. Show that  $\Gamma$ , the circumcircle of  $\triangle APQ$  and the circumcircle of  $\triangle ARS$  concur at a point other than  $A$ .

Source: Original

Author: Gunmay Handa

**anantmudgal09**

#2 Jan 17, 2016, 2:17 am

This was a very nice problem, I wonder the motivation behind it. Here is a hint to the solution. I would post my solution which I by far rate to be in the final monthly compilation.

The main idea I had was to use

[Click to reveal hidden text](#)

The rest is some nice trigonometric computation which I would post soon.

Although, this gives technically no information about the point of concurrence of these circles.

**EulerMacaroni**

#3 Jan 17, 2016, 2:38 am

anantmudgal09 wrote:

The main idea I had was to use

[Click to reveal hidden text](#)

This is indeed my solution, but I am curious as to whether there is a better one?

**anantmudgal09**

#4 Jan 17, 2016, 2:58 am

Here is my solution in its full glory

[Solution](#)**Luis González**

#5 Mar 29, 2016, 9:10 am • 1

Let  $O$  be the circumcenter of  $\triangle ABC$  and let  $D \equiv BP \cap CQ$ . From R-isosceles  $\triangle RAB$ , we get

$\angle BRC = 2\angle BAC = \angle BOC \implies R \in \odot(OBDC)$  and likewise  $S \in \odot(OBDC)$ . Thus

$\angle BSD = \angle BCD = \angle BAC \implies DS \parallel AC$  and similarly  $DR \parallel AB$ . Hence if  $B_\infty$  and  $C_\infty$  denote the points at infinity of  $AC$  and  $AB$ , we get  $D(S, B, Q, C_\infty) = D(B_\infty, P, C, R) \implies \frac{SB}{SQ} = \frac{RC}{RP} \implies \overline{SBQ} \sim \overline{RCP} \implies \odot(ABC)$ ,  $\odot(APQ)$  and  $\odot(ARS)$  meet again at the center of the spiral similarity that swaps  $\overline{SBQ}$  and  $\overline{RCP}$

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## trumpeter's Problem of the Day Forum

Problem of the Day: 10/12/15 X

Reply



trumpeter

#1 Oct 12, 2015, 10:26 am

Problem of the Day: 10/12/15

$ABCD$  is a quadrilateral (not necessarily convex) with  $M, N$  midpoints of  $AB, CD$  respectively. Let  $P = AC \cap BD$  and  $Q = (ABP) \cap (CDP) \neq P$ . Prove that  $(AMQ), (CNQ)$ , and  $AC$  are concurrent. Note:  $(XYZ)$  is the circumcircle of  $\triangle XYZ$ .

Source: Original

Author: Daniel Liu



Luis González

#3 Mar 28, 2016, 11:06 am

This holds for all points  $M, N$  on  $AB, CD$ , such that  $\overline{MA} : \overline{MB} = \overline{NC} : \overline{ND}$ .

Let  $X \equiv MN \cap AC$ . Since  $Q$  is the center of the spiral similarity that swaps  $\overline{AB}$  and  $\overline{CD}$ , it follows that  $\triangle QAB \cup M \sim \triangle QCD \cup N \implies \triangle QAM \sim \triangle QCN \implies Q$  is center of the spiral similarity that swaps  $\overline{AM}$  and  $\overline{CN}$   $\implies X \in \odot(AMQ) \cap \odot(CNQ)$  and the conclusion follows.

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## trumpeter's Problem of the Day Forum

**Problem of the Day: 1/30/16** X

[Reply](#)



**trumpeter**

#1 Jan 31, 2016, 11:48 am

### Problem of the Day: 1/30/16

Let  $\omega$  be a circle and let  $A, B, C, D$  be any four points on it such that all points in this problem exist. Let the tangents to  $\omega$  at  $A, B$  meet at  $X$  and those at  $C, D$  meet at  $Y$ . Let  $AB$  and  $CD$  meet at  $P$  and let  $AC, BD$  meet at  $R$  and  $AD, BC$  meet at  $Q$ . Then, if  $QR$  meets  $\omega$  again at  $X_1, X_2$  and  $QR$  meets the circles  $(ABY)$  and  $(CDX)$  again at  $S, T$  respectively then prove that  $(OX_1X_2)$  and  $(OST)$  are tangent to one another at point  $O$  where  $O$  is the centre of  $\omega$ . Here,  $(MNP)$  means the circumcircle of  $\triangle MNP$ .

Source: Original

Author: Anant Mudgal



**Luis González**

#2 Mar 28, 2016, 10:24 am

Clearly  $X, Y, Q, R$  are collinear on the polar of  $P$  WRT  $\omega \equiv (O)$ . Thus the inversion WRT  $(O)$  takes  $QR$  into the circle with  $\Omega$  with diameter  $\overline{OP}$  and takes  $X, Y$  into the midpoints  $M, N$  of  $AB, CD$ , respectively. Hence  $\odot(ABY)$  and  $\odot(CDX)$  go to  $\odot(ABN)$  and  $\odot(CDM)$   $\implies S, T$  go to the second intersections  $U, V$  of  $\Omega \equiv \odot(OMP)$  with  $\odot(ABN)$  and  $\odot(CDM)$ , respectively. If  $\odot(OX_1X_2)$  and  $\odot(OST)$  are tangent, then we need to show that their inverse lines  $\overleftrightarrow{X_1X_2}$  and  $\overleftrightarrow{UV}$  are parallel.

Let  $X_1X_2$  cut  $AB, CD$  at  $L, K$ , respectively. Since  $(L, A, B, P) = (K, D, C, P) = -1$ , it follows that  $PM \cdot PL = PA \cdot PB = PD \cdot PC = PN \cdot PK \implies MNKL$  is cyclic.  $X_1X_2, AB, NU$  are pairwise radical axes of  $(O), \Omega, \odot(ABN)$  concurring at their radical center  $L$ , i.e.  $N, L, U$  are collinear and similarly  $M, K, V$  are collinear  $\implies \angle UVM = \angle LNM = \angle LKM \implies UV \parallel X_1X_2$ , as desired.

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## High School Olympiads

Circumcenter 

 Reply



ThE-dArK-IOrD

#1 Mar 26, 2016, 11:23 pm

For given triangle  $ABC$  with circumcircle  $\omega$  and incenter  $I$ , let incircle touch  $AB, AC$  at  $E, F$  respectively

Let  $AI \cap BC = M$

Extend line  $EF \cap \omega = P, Q$  such that  $P$  lie on different side of  $C$  respect to  $AB$

Let  $(PEM) \cap \omega = K \neq P$  and  $(QFM) \cap \omega = L \neq Q$

Prove that circumcenter of  $\triangle KML$  lie on  $AM$



Luis González

#2 Mar 27, 2016, 8:04 am • 1 

After inverting WRT  $\odot(A, AE)$  we get the following equivalent configuration:

A-mixtilinear excircle of  $\triangle ABC$  touches  $AB, AC$  at  $E, F$  and  $\odot(AEF)$  cuts  $BC$  at  $P, Q$ , such that  $B$  is between  $C, P$ .  $M$  is the midpoint of the arc  $BC$  of  $\odot(ABC)$  and  $\odot(PME), \odot(QFM)$  cut  $BC$  again at  $K, L$ . Then the circumcenter of  $\triangle KML$  lies on  $AM$ .

**Proof:** WLOG assume that  $AC > AB$  and let  $X, Y$  be projections of  $M$  and  $A$  on  $BC$ . Since  $\triangle AEF$  is A-isosceles, its circumcenter is on  $AM \implies AM$  is the A-circumdiameter of  $\triangle APQ \implies \angle XMA = \angle YAM = \angle APQ - \angle AQP = \angle AFQ - \angle AEP$ . But by symmetry  $\angle AEM = \angle AFM \implies \angle XMA = \angle MFQ - \angle MEP = \angle MLK - \angle MKL \implies MA$  is the M-circumdiameter of  $\triangle MKL$ , i.e. the circumcenter of  $\triangle KML$  is on  $AM$ , as desired.



Luis González

#3 Mar 27, 2016, 8:27 am

Notice in the previous proof we have only used  $M$  as any point on the angle bisector of  $\angle BAC$  and  $AE = AF$ . Thus in the original problem,  $E, F$  can be any two points on  $AB, AC$  verifying  $AE = AF$  and  $M$  can be any point on  $AI$ .



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## High School Olympiads

Concurrent lines X

Reply

**mjuk**

#1 Mar 26, 2016, 6:36 pm

Let  $\triangle ABC$  be a triangle with altitudes  $AD, BE, CF$  and orthocenter  $H$ . Let  $X$  be  $H$ -excenter of  $\triangle HEF$ . Define  $Y, Z$  similarly. Prove that  $AX, BY, CZ$  are concurrent.

**Luis González**

#2 Mar 26, 2016, 11:22 pm

This can be proved exactly in the same way as [Baltic way 2009 \(P13\)](#).



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## High School Olympiads

Concurrence 

 Reply

Source: Baltic Way 2009



**Sassha**

#1 Dec 13, 2009, 2:19 am

In a triangle  $ABC$ , draw altitudes  $AD, BE, CF$  and let  $H$  be the orthocenter. Let  $O_1, O_2, O_3$  be the incenters of  $\triangle EHF, \triangle FHD, \triangle DHE$  respectively. Prove that  $AO_1, BO_2, CO_3$  are concurrent.



**Luis González**

#2 Dec 13, 2009, 4:54 am

Construct the triangle  $\triangle A'BC$  outside of  $\triangle ABC$ , such that  $\triangle A'BC \sim \triangle O_1EF$  and define analogously the points  $B', C'$ . Since  $EF$  is antiparallel to  $BC$  with respect to  $AB, AC$ , the rays  $AO_1$  and  $AA'$  are isogonals with respect to  $AB, AC$ . Similarly,  $BB'$  and  $CC'$  are the isogonals of  $BO_2$  and  $CO_3$ . By angle chase, we have then

$$\angle CBA' = \angle ABC' = \frac{\pi - 2\angle B}{4}$$

$$\angle BCA' = \angle ACB' = \frac{\pi - 2\angle C}{4}$$

$$\angle CAB' = \angle BAC' = \frac{\pi - 2\angle A}{4}$$

By Jacobi's theorem, the rays  $AA', BB', CC'$  concur at the Jacobi's perspector  $J$  relative to the triple  $\left(\frac{\pi - 2\angle A}{4}, \frac{\pi - 2\angle B}{4}, \frac{\pi - 2\angle C}{4}\right)$ . Therefore, the rays  $AO_1, BO_2, CO_3$  concur at the isogonal conjugate  $J'$  of  $J$ .



**Sassha**

#3 Dec 29, 2009, 9:46 pm

It could also be proven in the following way making use of another theorem:

Let  $ABC$  be a triangle, and let  $D, E, F$  be points on sides  $BC, CA, AB$ , respectively, such that the cevians  $AD, BE, CF$  are concurrent. If  $M, N, P$  are points on  $EF, FD, DE$ , respectively, then the lines  $AM, BN, CP$  are concurrent if and only if the lines  $DM, EN, FP$  concur. We can use this to solve the problem.

We know from triangle geometry that the orthocenter of a triangle is the incenter of its orthic triangle, therefore  $\angle EFO_1 = \angle DFO_2, \angle FDO_2 = \angle EDO_3, \angle DEO_3 = \angle FEO_1$ .

Then, according to the same Jacobi theorem, the lines  $DO_1, EO_2, FO_3$  are concurrent, and because of the fact that the heights of any triangle are concurrent, the lines  $AO_1, BO_2, CO_3$  meet in one point by the theorem above.

Does anybody know if that theorem has a name? And/or has a proof that does involve barycentric coordinates or complex numbers?

By the way, it was the hardest problem of BW 2009. Only one team solved it.



**Luis González**

#4 Jan 1, 2010, 1:15 am

 Sassha wrote:

It could also be proven in the following way making use of another theorem:

"Let  $\triangle ABC$  be a triangle, and let  $D, E, F$  be points on sides  $BC, CA, AB$ , respectively, such that the cevians  $AD, BE, CF$  are concurrent. If  $M, N, P$  are points on  $EF, FD, DE$ , respectively, then the lines  $AM, BN, CP$  are concurrent if and only if the lines  $DM, EN, FP$  concur."

Does anybody know if that theorem has a name? And/or has a proof that does involve barycentric coordinates or complex numbers?

Dear Sassha the theorem you mentioned is known as Cevian Nest Theorem, you can see a synthetic proof in <http://pagesperso-orange.fr/jl.ayme/> (Volume 3). But I'm afraid that this is not a direct application of the Cevian Nest Theorem as you claimed, since  $O_1, O_2, O_3$  do not lie on the sidelines of the orthic triangle  $\triangle DEF$ .



**Sung-yoon Kim**

#5 Jan 13, 2010, 10:44 pm

I solved this using barycentric coordinates.

Given a triangle  $ABC$ , let  $A_1 = (a_1, a_2, a_3), B_1 = (b_1, b_2, b_3), C_1 = (c_1, c_2, c_3)$ .

$(a_1 + a_2 + a_3 = b_1 + b_2 + b_3 = c_1 + c_2 + c_3 = 1)$  Then if the barycentric coordinate of  $P$  with regard to the triangle  $A_1B_1C_1$  is  $(x, y, z)$ , the barycentric coordinate of  $P$  with regard to the triangle  $ABC$  is

$(xa_1 + yb_1 + zc_1, xa_2 + yb_2 + zc_2, xa_3 + yb_3 + zc_3)$ . Since  $H = \frac{1}{\tan A + \tan B + \tan C}(\tan A, \tan B, \tan C)$ ,

$E = \frac{1}{\tan A + \tan C}(\tan A, 0, \tan C), F = \frac{1}{\tan A + \tan B}(\tan A, \tan B, 0)$ , we can calculate the coordinate of  $O_1$ , and hence the coordinate of  $P_1 = AO_1 \cap BC$ . Apply Ceva's theorem.



**jayme**

#7 Oct 23, 2011, 8:48 pm

For a synthetic proof of the Jacobi's theorem, you can see

<http://perso.orange.fr/jl.ayme> vol. 5 Le theoreme de Jacobi

Sincerely

Jean-Louis

15/8

**MathTwo**

#8 Oct 23, 2011, 9:00 pm

There is also a succinct solution using the trigonometric version of Ceva's theorem. First of all, we see that  $\angle DEF=180-2B$ , and because the altitude from  $B$  bisects this angle, we have that  $\angle FEH=90-B$ . Thus,  $\angle O_1EF=45-B/2$ , and  $\angle O_1FE=45-C/2$ .

Now, we see that  $[FAO_1]/[O_1AE] = AF \cdot \sin \angle FAO_1/AE \cdot \sin \angle EAO_1$ . However, we see that  $[FAO_1]/[O_1AE]$  is also equal to  $AF \cdot FO_1 \cdot \sin \angle AFO_1/AE \cdot EO_1 \cdot \sin \angle AEO_1$ . Thus,  $\sin \angle FAO_1/\sin \angle EAO_1 = FO_1 \cdot \sin \angle AFO_1/EO_1 \cdot \sin \angle AEO_1$ . Now, because triangles  $AFE$  and  $ACB$  are similar  $\angle AFE=\angle ACB$ . Thus  $AFO_1=45-C/2+C=45+C/2$ . Similarly,  $\angle AEO_1=45+B/2$ . Using the law of sines in triangle  $FO_1E$ , we get that  $O_1E/O_1F=\sin \angle 45-C/2 / \sin \angle 45-B/2$ . Putting everything together, we have  $\sin \angle FAO_1/\sin \angle EAO_1 = \sin(45-C/2) \cdot \sin(45+B/2) / \sin(45-B/2) \cdot \sin(45+C/2)$ . Taking the cyclic product of the LHS will then yield the desired result.

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## High School Olympiads

Ratio segments 

 Reply



Source: Own



buratinogiggle

#1 Mar 24, 2016, 10:22 pm • 1 

Let  $ABC$  be a triangle with circumcenter  $O$  and  $P$  is any point.  $PB, PC$  cut  $CA, AB$  at  $E, F$ . The lines passes through  $P$  and is perpendicular to  $OP$  cuts  $BC, EF$  at  $M, N$ .  $XYZ$  symmetric of pedal triangle of  $P$  through  $P$ . Prove that if  $AX, BY, CZ$  are concurrent then  $PM = 2PN$ .



Luis González

#2 Mar 24, 2016, 10:30 pm • 1 

$AX, BY, CZ$  are concurrent  $\iff P$  is on the Thomson cubic of  $\triangle ABC \iff$  polar of  $P$  WRT  $\odot(ABC)$  and trilinear polar of  $P$  WRT  $\triangle ABC$  are parallel. Now use the result of the problem Metric relation  $XP=2YP$ .



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## High School Olympiads

Metric relation  $XP=2YP$  X

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Source: Own



TelvCohl

#1 Dec 14, 2014, 8:50 pm • 3

Let  $O$  be the circumcenter of  $\triangle ABC$ .

Let  $\ell_p$  be the trilinear polar of  $P$  WRT  $\triangle ABC$ .

Let  $\triangle DEF$  be the cevian triangle of  $P$  WRT  $\triangle ABC$ .

Let a line through  $P$  perpendicular to  $OP$  intersect  $BC, EF$  at  $X, Y$ , respectively.

Prove that  $OP \perp \ell_p \iff XP = 2YP$

This post has been edited 1 time. Last edited by TelvCohl, Aug 2, 2015, 12:06 am



shinichiman

#2 Dec 23, 2014, 4:31 pm • 3

It should be  $XP = 2YP \iff \ell_p \parallel XY$ . For convenience, I edited the problem:

Let  $\ell_p$  be the trilinear polar of  $P$  with respect to  $\triangle ABC$ . Let  $\triangle DEF$  be the cevian triangle of  $P$  with respect to  $\triangle ABC$ .

$EF, FD, DE$  cut  $BC, CA, AB$  at  $A_1, B_1, C_1$  respectively. A line  $d$  through  $P$  cuts  $BC, CA, AB, EF, FD, DE$  at  $A_2, B_2, C_2, A', B', C'$  respectively. Prove that  $\ell_p \parallel d \iff PA_2 = 2PA'$ .

( $\implies$ ) If  $\ell_p \parallel d$ . It is easy to prove that  $\frac{\overline{PB_2}}{\overline{PC_2}} = \frac{\overline{PB'}}{\overline{PC'}}$ . Similarly, we can get  $\frac{\overline{PB_2}}{\overline{PB'}} = \frac{\overline{PC_2}}{\overline{PC'}} = \frac{\overline{PA_2}}{\overline{PA'}} = -k$ . We can see that  $k \geq 0$ .

We have  $(BA, FC_1) = -1$  so  $D(BA, FE) = D(A_2P, B'C') = -1$  or  $(A_2P, B'C') = -1$ . By Descartes' identity we have  $\frac{2}{\overline{PA_2}} = \frac{1}{\overline{PB'}} + \frac{1}{\overline{PC'}}$  or  $\frac{1}{\overline{PB'}} + \frac{1}{\overline{PC'}} + \frac{2}{k\overline{PA'}} = 0$ . Similarly, we also obtain  $\frac{1}{\overline{PB'}} + \frac{1}{\overline{PA'}} + \frac{2}{k\overline{PC'}} = 0$  and  $\frac{1}{\overline{PA'}} + \frac{1}{\overline{PC'}} + \frac{2}{k\overline{PB'}} = 0$ . Therefore

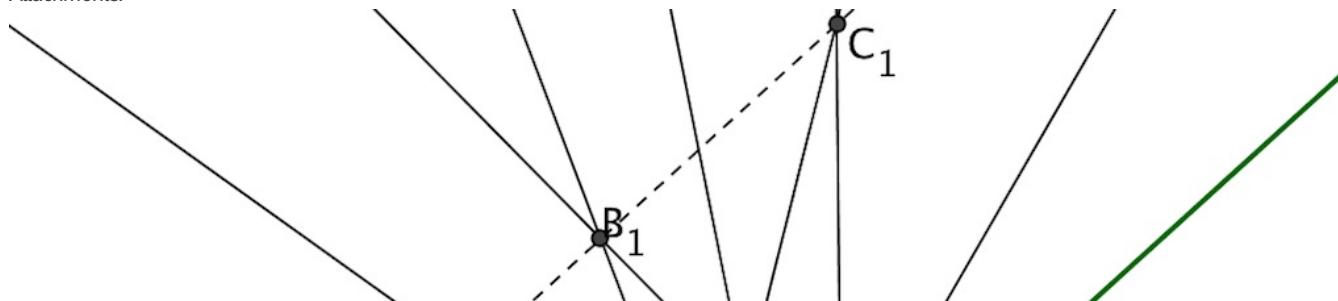
$$\left(\frac{2}{k} + 2\right) \left(\frac{1}{\overline{PB'}} + \frac{1}{\overline{PC'}} + \frac{1}{\overline{PA'}}\right) = 0.$$

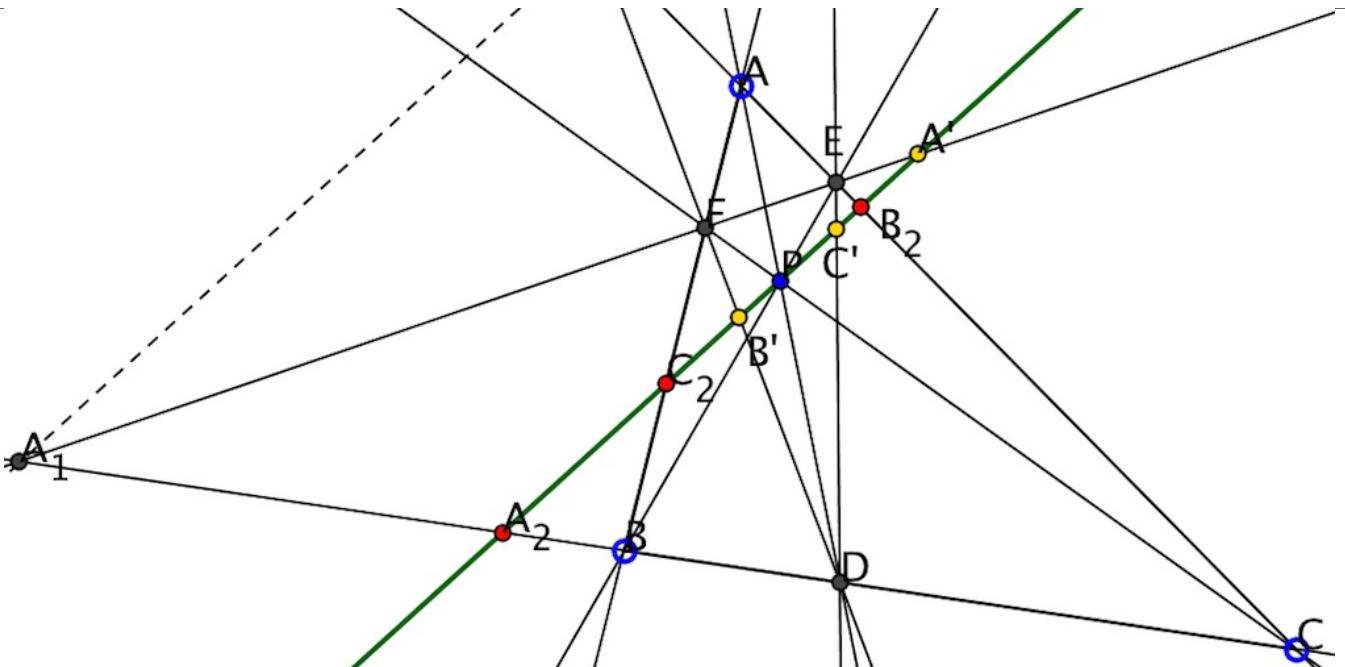
Hence,  $\frac{1}{\overline{PB'}} + \frac{1}{\overline{PC'}} + \frac{1}{\overline{PA'}} = 0$ . We also get that  $\frac{2}{\overline{PA_2}} = \frac{1}{\overline{PB'}} + \frac{1}{\overline{PC'}}$  so  $\frac{2}{\overline{PA_2}} = \frac{-1}{\overline{PA'}}$  or  $\overline{PA_2} = -2\overline{PA'}$ . Thus,  $PA_2 = 2PA'$ .

( $\iff$ ). From  $PA_2 = 2PA'$  we get  $\frac{1}{\overline{PB'}} + \frac{1}{\overline{PC'}} + \frac{1}{\overline{PA'}} = 0$ . And from here, we can easily obtain that

$PB_2 = PB', PC_2 = PC'$  or  $\frac{\overline{PB_2}}{\overline{PB'}} = \frac{\overline{PC_2}}{\overline{PC'}} = \frac{\overline{PA_2}}{\overline{PA'}} = -2$ . We get  $\frac{\overline{PB_2}}{\overline{PC_2}} = \frac{\overline{PB'}}{\overline{PC'}}$ . From here we can find out that  $\ell_p \parallel d$  by drawing a line  $d' \parallel d$  through  $B_1$  and prove that line also passes through  $C_1$ .

Attachments:





**Luis González**

#3 Dec 24, 2014, 10:58 am • 3

We work with the same initial notations of shinichiman's. The image of  $BC$  under homothety with center  $P$  and coefficient  $-\frac{1}{2}$  will cut  $EF$  at only one point, thus it suffices to prove that  $\ell_P \parallel d \implies \overline{PA}_2 : \overline{PA}' = -2$ . The converse is guaranteed by the uniqueness of  $A'$ .

Let  $B_0, C_0$  be the midpoints of  $AC, AB$ .  $B_0C_0$  cuts  $EF$  at  $U$  and  $AU$  cuts  $BC$  at  $V$ . Since  $(A, B, F, C_1) = -1 \implies \overline{FB} : \overline{FC_0} = \overline{FC_1} : \overline{FA}$ , but from  $B_0UC_0 \parallel BC$ , we get  $\overline{FB} : \overline{FC_0} = \overline{FA}_1 : \overline{FU} \implies \overline{FC_1} : \overline{FA} = \overline{FA}_1 : \overline{FU} \implies AU \parallel \ell_P \parallel d$ . Now if  $AA_1$  cuts  $\ell$  at  $Q$ , we get  $\overline{QA}_2 : \overline{QA}' = \overline{AV} : \overline{AU} = 2$ , but since  $A_1(E, C, P, A) = (A', A_2, P, Q) = -1$ , then  $\overline{PA}_2 : \overline{PA}' = -\overline{QA}_2 : \overline{QA}' = -2$ .



**junior2001**

#4 Dec 24, 2014, 11:39 am

wath is trilinear polar ?



**TelvCohl**

#5 Feb 20, 2015, 7:33 pm

“ shinichiman wrote:

It should be  $XP = 2YP \iff \ell_P \parallel XY$ .

Thank you **shinichiman** for point out this and thanks for **shinichiman** and **Luis González** for your nice solution 😊

After thinking this problem again today, I found this problem is quite easy . I'll prove  $\ell_P \parallel d \implies \overline{PA}_2 : \overline{PA}' = -2$ :

Let  $X = AA_1 \cap d$ .

From  $(A_1A', \ell_P; A_1X, A_1A_2) = -1$  and  $\ell_P \parallel d \implies A'$  is the midpoint of  $A_2X$ , so combine with  $(X, P; A', A_2) = A_1(X, P; A', A_2) = -1$  we get  $PA_2 = 2A'P$ .

Done

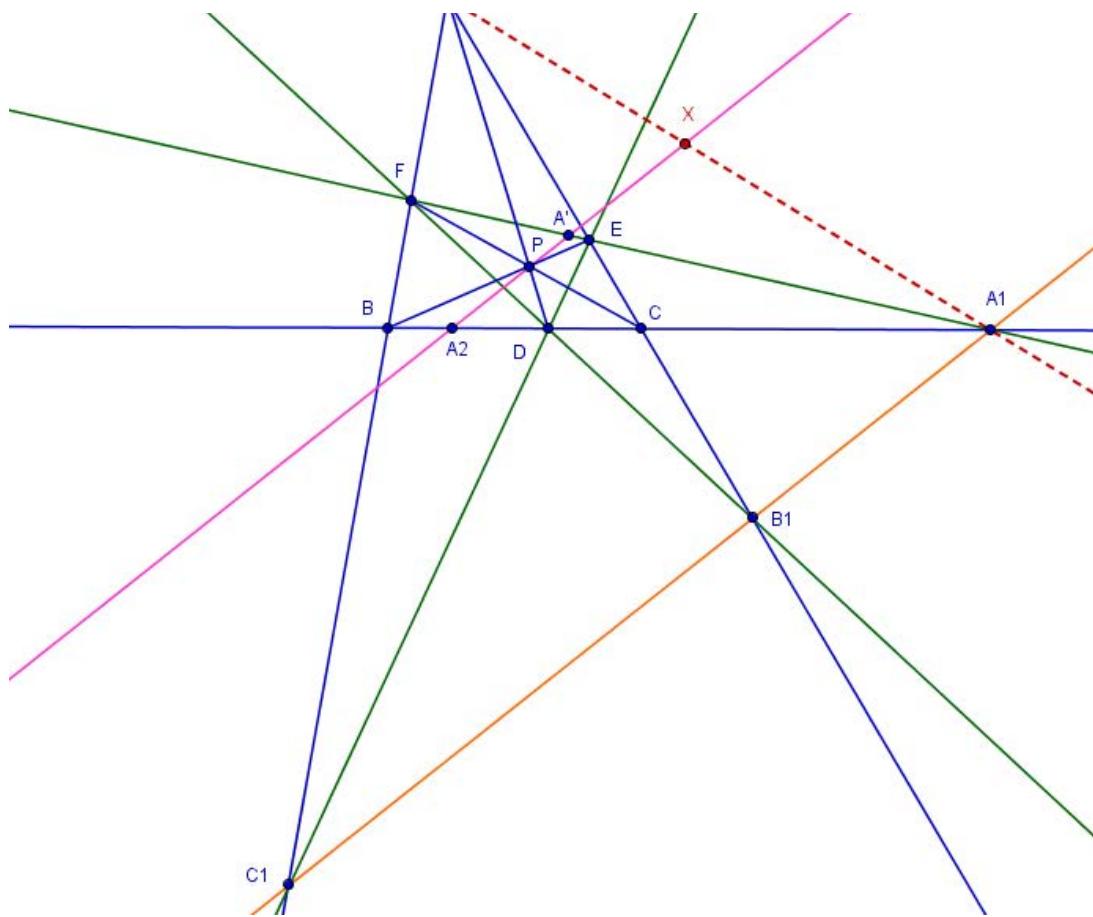
“ junior2001 wrote:

wath is trilinear polar ?

See <http://mathworld.wolfram.com/TrilinearPolar.html> 😊

Attachments:





**toto1234567890**

#6 Feb 21, 2015, 7:41 pm

Yes. It's quite easy but it is very interesting. 😊

99

1



**TelvCohl**

#7 Feb 21, 2015, 7:59 pm • 1

Thank you toto1234567890 😊

I found this problem when I tried to solve the problem [A perspectrix perpendicular to OI \(own\)](#)

This post has been edited 1 time. Last edited by TelvCohl, Aug 2, 2015, 12:07 am

99

1



**toto1234567890**

#8 Feb 22, 2015, 4:41 pm

At first I tried to use butterfly theorem and Pascal's Theorem and some harmonic Divisions but the first two wasn't needed. 😊

It's quite funny that the intersection points of the lines and the line parallel to the trilinear polar and going through  $P$  are almost Homothetilike(?) .

99

1

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## High School Olympiads

3 circles with common tangency point 

 Reply

Source: me



yetti

#1 Apr 10, 2012, 3:19 am

Let  $ABCD$  be cyclic quadrilateral with circumcircle  $(O)$  and diagonal intersection  $E \equiv AC \cap BD$ .

Let the quadrilateral opposite sides meet at  $F \equiv BC \cap AD$ . WLOG, assume that points  $C, D, F$  are on the same side of the line  $AB$ .

Let  $(J), (K)$  be incircles of curvilinear  $\triangle ABE, \triangle ABF$ , internally tangent to the arc  $\widehat{AB}$  of circle  $(O)$ .

Show that circles  $(J), (K), (O)$  have common tangency point  $T$ .

[Similarly...](#)



Luis González

#2 Apr 10, 2012, 8:43 am • 1

Let  $U, V, W$  be the incenters of  $\triangle ABC, \triangle ABD, \triangle ABE$ .  $UV$  cuts  $BC, AD$  at  $P, Q$ .  $(J)$  touches  $EA, EB$  at  $M, N$  and  $(O)$  internally at  $T$ . According to the topic [incenter of triangle](#),  $W, T, B, U, N$  and  $W, T, A, V, M$  are concyclic. Hence  $\angle TVP = \angle TAC = \angle TBP \pmod{\pi} \implies T, B, P, V$  are concyclic. Likewise,  $T, A, Q, U$  are concyclic  $\implies \angle TPB = \angle TVW = \angle TAU = \angle TQP \implies BC$  is tangent to  $\odot(TPQ)$  at  $P$ . Similarly,  $AD$  is tangent to  $\odot(TPQ)$  at  $Q$ . Furthermore,  $\angle BTP = \angle WVM = \angle WAC = \frac{1}{2}\angle BTC \implies TP$  bisects  $\angle BTC$ , thus  $\odot(TPQ)$  is tangent to  $(O)$  at  $T \implies (K) \equiv \odot(TPQ)$ .



yetti

#3 Apr 10, 2012, 12:21 pm

$I_C, I_D$  are incenters of  $\triangle ABC, \triangle ABD$ .  $AI_C, BI_C$  cut  $(O)$  again at  $X_C, Y_C$  and  $AI_D, BI_D$  cut  $(O)$  again at  $X_D, Y_D$ .  $CI_C, DI_D$  meet at  $Z \in (O)$ .

$I_CI_D$  cuts  $AB$  at  $S, AC, BD$  at  $P_J, Q_J$  and  $AD, BC$  at  $P_K, Q_K$ , respectively.  $SZ$  cuts  $(O)$  again at  $T$ .

Intersections  $S \equiv AB \cap TZ, I_D \equiv DZ \cap BY_D$  and point  $P_K \in AD$  are collinear  $\implies P_K \in ZY_D$  by Pascal theorem for  $AZTBY_DZ$ .

Intersections  $S \equiv AB \cap TZ, I_C \equiv CZ \cap AX_C$  and point  $Q_K \in BC$  are collinear  $\implies Q_K \in ZX_C$  by Pascal theorem for  $AZTBX_CC$ .

Similarly,  $P_J \in TY_C$  and  $Q_J \in TX_D$ .

$ABI_CI_D$  is cyclic with circumcircle  $(Z)$  and  $I_C \in AX_C, I_D \in BY_D \implies \angle AI_CI_D = \angle ABI_D \equiv \angle ABy_D = \angle AX_CY_D \implies X_CY_D \parallel I_CI_D$ .  
Similarly,  $X_DY_C \parallel I_CI_D$ .

$\triangle TP_JQ_J \sim \triangle TY_CX_D$  are centrally similar with similarity center  $T \in (O)$  and so are their circumcircles  $(J) \equiv \odot(TP_JQ_J), (O) \equiv \odot(TY_CX_D) \implies$  they are internally tangent at  $T$ .

Tangents of  $(O)$  at  $Y_C, X_D$  are parallel to  $AC, BD \implies (J)$  is tangent to  $AEC, BED$  at  $P_J, Q_J$ .

$\triangle TP_KQ_K \sim \triangle TY_AX_C$  are centrally similar with similarity center  $T \in (O)$  and so are their circumcircles  $(K) \equiv \odot(TP_KQ_K), (O) \equiv \odot(TY_AX_C) \implies$  they are internally tangent at  $T$ .

Tangents of  $(O)$  at  $Y_D, X_C$  are parallel to  $AD, BC \implies (K)$  is tangent to  $ADF, BCF$  at  $P_K, Q_K$ .

Combined,  $(J), (K), (O)$  are internally tangent at  $T$ .

The proposed problem was intended to provide an alternative solution of the problem at  
<http://www.artofproblemsolving.com/Forum/viewtopic.php?f=47&t=463508>.



**armpist**

#4 Apr 10, 2012, 6:49 pm

Dear Yetti, Luis and MLs,

Proposed problem is Poncelet theorem for a pencil of circles. Amazing what a tender Frenchman can discover when in a POW camp in Russian winter!

On top of what posted here, it also mentions another tangent circle (for it would be frivolous and lighthearted to think that the last pair of quadrilateral sides does not have a circle with the same point touching qualities ).

It also states that all 6 tangent points are collinear.

Now the question is : What are the incenters that this line speaks through ( a la Sawayama ) ?

Friendly,

M.T.



**yetti**

#5 Apr 11, 2012, 4:24 am

Assuming cyclic  $ABCD$ , with points  $C, D, F \equiv BC \cap AD$  on the same side of the line  $AB$ .

Incenters  $I_C, I_D$  of  $\triangle ABC, \triangle ABD$ , C-excenter  $L_C$  of  $\triangle BCD$  and D-excenter  $L_D$  of  $\triangle ADC$  are collinear.

Suppose  $(J), (K), (O)$  are 3 coaxal circles, forming intersecting, tangent, or non-intersecting pencil.  $A \in (O)$  is arbitrary point.

Tangent to  $(J)$  from  $A$  cuts  $(O)$  again at  $C$ . Tangent to  $(K)$  from  $C$  cuts  $(O)$  again at  $B$ . Tangent to  $(J)$  from  $B$  cuts  $(O)$  again at  $D$ .

Then tangent to  $(K)$  from  $D$  always cuts  $(O)$  again at  $A \implies$  infinite number of solutions always exist for any 3 coaxal circles  $(J), (K), (O)$ , one for any  $A \in (O)$ .

I have never heard about Poncelet porism applied to a pencil of 3 circles. Perhaps you could give a reference for your claim. For now, I would say that Poncelet had something different in mind.



**drmzjoseph**

#6 Mar 18, 2015, 6:51 pm

### General Problem

Let  $ABCD$  be cyclic quadrilateral with circumcircle  $(O)$  and diagonal intersection  $E \equiv AC \cap BD$ .

Let  $(J)$  a circle tangent  $\overrightarrow{EB}$  and  $\overrightarrow{EA}$  in  $M$  and  $N$  respectively.  $X \equiv BC \cap MN; Y \equiv AD \cap MN$

Let  $(K)$  the circle tangent to  $BC$  and  $AD$  in  $X$  and  $Y$  respectively.

Show that  $(J), (K)$  and  $(O)$  are coaxial circles.



**TelvCohl**

#8 Mar 18, 2015, 7:25 pm • 1

**drmzjoseph wrote:**

### General Problem

Let  $ABCD$  be cyclic quadrilateral with circumcircle  $(O)$  and diagonal intersection  $E \equiv AC \cap BD$ .

Let  $(J)$  a circle tangent  $\overrightarrow{EB}$  and  $\overrightarrow{EA}$  in  $M$  and  $N$  respectively.  $X \equiv BC \cap MN; Y \equiv AD \cap MN$

Let  $(K)$  the circle tangent to  $BC$  and  $AD$  in  $X$  and  $Y$  respectively.

Show that  $(J), (K)$  and  $(O)$  are coaxial circles.

My solution:

Let  $\mathcal{P}(P, \odot)$  be the power of a point  $P$  WRT circle  $\odot$ .

From Menelaus theorem we get  $\frac{AY}{AN} = \frac{DY}{DM}$  and  $\frac{BX}{BM} = \frac{CX}{CN} \dots (\star)$

From  $\angle YNA = \angle XMB, \angle NAY = \angle MBX \Rightarrow \triangle AYN \sim \triangle BXM$ ,

so we get  $\frac{\mathcal{P}(A, \odot(K))}{\mathcal{P}(A, \odot(J))} = \frac{AY^2}{AN^2} = \frac{BX^2}{BM^2} = \frac{\mathcal{P}(B, \odot(K))}{\mathcal{P}(B, \odot(J))}$ ,

hence combine with  $(\star) \Rightarrow \frac{\mathcal{P}(D, \odot(K))}{\mathcal{P}(D, \odot(J))} = \frac{\mathcal{P}(A, \odot(K))}{\mathcal{P}(A, \odot(J))} = \frac{\mathcal{P}(B, \odot(K))}{\mathcal{P}(B, \odot(J))} = \frac{\mathcal{P}(C, \odot(K))}{\mathcal{P}(C, \odot(J))}$ .

i.e.  $\odot(J), \odot(K), \odot(ABCD) \equiv \odot(O)$  are coaxial

Done 😊



**buratinogigle**

#9 May 9, 2015, 6:54 pm • 1 ↗

Here is an extension of Poncelet theorem <http://www.artofproblemsolving.com/community/c6h1076831p4710069>

Acutally, I found this extension two years ago, Tran Minh Ngoc wrote an aticle about it on Mathematics and Young of Vietnam, this is link of his article

<https://tranminhngocctlhp.wordpress.com/2015/03/14/bo-de-poncelet-mo-rong-va-ung-dung/>

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**High School Olympiads****Mixtilinear Geometry Problem** X[Reply](#)**comboisreallyhard**

#1 Mar 23, 2016, 5:49 am • 2

In  $\triangle ABC$ , let  $M$  be the midpoint of the feet of the  $A$ -angle bisectors. Suppose  $T$  is the touchpoint of the  $A$ -mixtilinear incircle on  $\odot(ABC)$ ; put  $M_a$  as the midpoint of major arc  $BAC$  and  $R$  as the midpoint of the midpoints of major arcs  $ABC$  and  $ACB$ . Show that lines  $MT$  and  $M_aR$  intersect on  $\odot(ABC)$ .

**Luis González**

#2 Mar 23, 2016, 6:26 am • 1

It's well-known that  $MA$  is tangent to  $(O) \equiv \odot(ABC)$  and  $T, I, M_a$  are collinear. If  $I_a, I_b, I_c$  are the excenters of  $\triangle ABC$  against  $A, B, C$ , then clearly  $I_aR \equiv I_aM_a$  is the  $I_a$ -median of  $\triangle I_aI_bI_c \implies I_aM_a$  cuts  $(O)$  again at the tangency point  $S$  of the  $A$ -mixtilinear excircle of  $\triangle ABC$  with  $(O)$  (extraversion of the mixtilinear incircle case).

Let  $X$  and  $Y$  be the tangency points of the incircle ( $I$ ) and  $A$ -excircle ( $I_a$ ) with  $BC$ . The inversion with center  $A$ , power  $AB \cdot AC$  followed by reflection on  $AI$  swaps  $(O)$  and  $BC$ , thus it swaps  $(I)$  and the  $A$ -mixtilinear excircle and  $(I_a)$  and the  $A$ -mixtilinear incircle. This inversion then swaps  $(X, S), (Y, T)$  and  $(M, U)$ , being  $U$  the 2nd intersection of  $(O)$  with the parallel from  $A$  to  $BC$ . By symmetry  $AUYX$  is an isosceles trapezoid (cyclic), thus  $S, T, M$  are collinear and the conclusion follows.

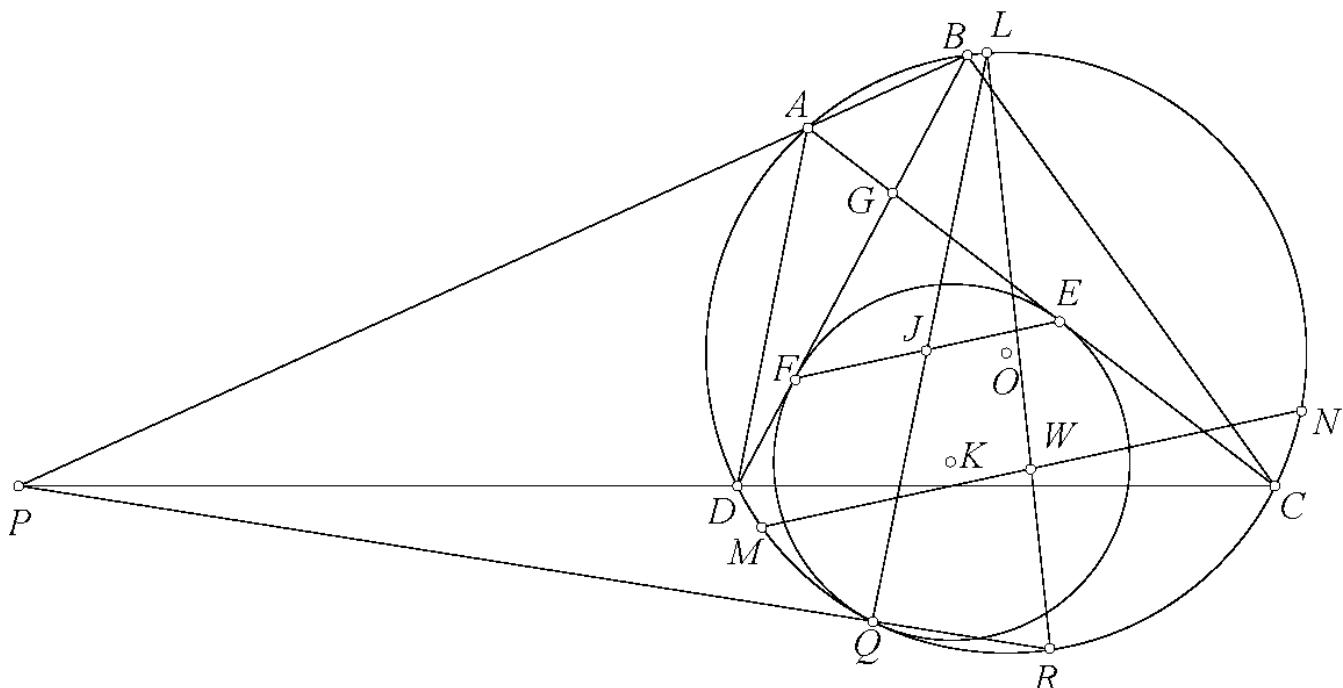
**buratinogigle**

#3 Mar 23, 2016, 10:51 pm • 1

Nice problem, this is general problem.

Let  $ABCD$  be cyclic quadrilateral inscribed in circle  $(O)$  with  $AC$  cuts  $BD$  at  $G$ ,  $AB$  cuts  $CD$  at  $P$ . Circle  $(K)$  is tangent to segment  $GC, GD$  at  $E, F$  and  $(O)$  at  $Q$ .  $M, N$  are midpoints of major arcs  $BC, AD$ .  $J$  is midpoint of  $EF$ .  $QJ$  cuts  $(O)$  again at  $L$ .  $PQ$  cuts  $(O)$  again at  $R$ . Prove that  $LR$  bisects segment  $MN$ .

Attachments:





**“ buratinogigle wrote:**

Nice problem, this is general problem.

Let  $ABCD$  be cyclic quadrilateral inscribed in circle  $(O)$  with  $AC$  cuts  $BD$  at  $G$ ,  $AB$  cuts  $CD$  at  $P$ . Circle  $(K)$  is tangent to segment  $GC$ ,  $GD$  at  $E, F$  and  $(O)$  at  $Q$ .  $M, N$  are midpoints of major arcs  $BC, AD$ .  $J$  is midpoint of  $EF$ .  $QJ$  cuts  $(O)$  again at  $L$ .  $PQ$  cuts  $(O)$  again at  $R$ . Prove that  $LR$  bisects segment  $MN$ .

Let  $H \equiv AD \cap BC$ .  $\omega_1$  is the circle tangent to  $\overline{HA}, \overline{HB}$  and externally tangent to  $(O)$  at  $S$ .  $\omega_2$  is the circle tangent to the extensions of  $\overline{AD}, \overline{BC}$  and externally tangent to  $(O)$  at  $R^*$ . Using the result of the problem [3 circles with common tangency point](#), we get that there are circles  $\lambda_1, \lambda_2$  tangent to  $AC, BD$  and externally tangent to  $(O)$  at  $S, R^*$ , resp. Similarly there are circles  $\epsilon_1, \epsilon_2$  tangent to  $AB, CD$  and externally tangent to  $(O)$  at  $Q, R^*$ . Since  $P$  is the exsimilicenter of  $\epsilon_1 \sim \epsilon_2$  and  $Q, R^*$  are the insimilicenters of  $(O) \sim \epsilon_1$  and  $(O) \sim \epsilon_2$ , then  $P, Q, R^*$  are collinear  $\implies R \equiv R^*$ . Since  $G, S$  are the insimilicenters of  $(K) \sim \lambda_1, (O) \sim \lambda_1$  and  $Q$  is the exsimilicenter of  $(K) \sim (O)$ , then  $Q, G, S$  are collinear. Since  $H$  is the exsimilicenter of  $\omega_1 \sim \omega_2$  and  $S, R$  are the insimilicenters of  $(O) \sim \omega_1$  and  $(O) \sim \omega_2$ , then  $H, S, R$  are collinear.

Let  $U$  and  $V$  be the tangency points of  $\omega_1$  and  $\omega_2$  with  $AD$ . From the external tangency of  $(O)$  and  $\omega_1$ , we deduce that  $SU$  is external bisector of  $\angle ASD$ , i.e.  $N \in SU$  and likewise  $N \in RV$ . As the exsimilicenter  $H$  of  $\omega_1 \sim \omega_2$  is also center of their direct inversion, it follows that  $HU \cdot HV = HS \cdot HR \implies UVRH$  is cyclic  $\implies NU \cdot NS = NR \cdot NV \implies N$  has equal power WRT  $\omega_1, \omega_2$ . Similarly  $M$  has equal power WRT  $\omega_1, \omega_2 \implies MN$  is the radical axis of  $\omega_1, \omega_2 \implies MN$  and the internal common tangents of  $(O), \omega_1$  and  $(O), \omega_2$  concur at the radical center of  $(O), \omega_1, \omega_2 \implies SMRN$  is harmonic  $(\star)$ .

Let  $QE, QF$  cut  $(O)$  again at  $X, Y$ . Since  $Q$  is the exsimilicenter of  $(K) \sim (O)$ , then  $XY \parallel EF \parallel MN$ .  $QG$ , being the Q-symmedian of  $\triangle QEF$ , is the isogonal of its Q-median  $QJ$  WRT  $\angle EQF \equiv \angle XQF \implies LS \parallel XY \parallel MN$ . Together with  $(\star)$ , we get  $L(R, M, N, S) = -1 \implies LR$  passes through the midpoint of  $MN$ , as desired.

Quick Reply

## High School Olympiads

Line passes through centroid



Reply



Source: Own



**buratinogigle**

#1 Mar 22, 2016, 8:27 pm • 1

Let  $ABC$  be a triangle inscribed in circle  $(O)$  and incenter  $I$ . Circle  $(\omega_a)$  touches  $(O)$  at  $A$  and touches  $BC$  at  $D$ . Similarly, we have circles  $(\omega_b)$ ,  $(\omega_c)$  and  $E, F$ . Let  $K$  be radical center of  $(\omega_a)$ ,  $(\omega_b)$ ,  $(\omega_c)$ . Prove that line  $IK$  passes through centroid of triangle  $DEF$ .



**Luis González**

#2 Mar 23, 2016, 5:23 am • 2

You rediscovered the 2nd Hatzipolakis-Yiu point;  $X_{595}$  on ETC, which was exactly defined as the radical center of those 3 circles. Circle  $\omega_a$  passing through  $A \equiv (1 : 0 : 0)$  and tangent to  $BC$  at  $(0 : b : c)$  has barycentric equation

$$\omega_a \equiv a^2yz + b^2zx + c^2xy - \frac{a^2(x+y+z)}{(b+c)^2}(c^2y + b^2z) = 0.$$

Similarly, the equations of  $\omega_b$  and  $\omega_c$  are given by

$$\omega_b \equiv a^2yz + b^2zx + c^2xy - \frac{b^2(x+y+z)}{(c+a)^2}(a^2z + c^2x) = 0$$

$$\omega_c \equiv a^2yz + b^2zx + c^2xy - \frac{c^2(x+y+z)}{(a+b)^2}(b^2x + a^2y) = 0.$$

Their radical center is then  $K \equiv X_{595} \equiv (a^2(a^2 + ab + ac - bc))$ . Thus easy algebraic manipulation shows that  $K$  is indeed on the line through  $I \equiv (a : b : c)$  and the centroid  $(a(b+c)(2a+b+c) : b(c+a)(2b+a+c) : c(a+b)(2c+a+b))$  of  $\triangle DEF$ . Furthermore this line also passes through the Schiffler point  $X_{21}$  of  $\triangle ABC$ .

Quick Reply

## High School Olympiads

Radical center and collinear ! 

 Locked



Source: own



TacH

#1 Mar 22, 2016, 11:07 pm

Let  $ABC$  be a scalene triangle  $I, O$  be Incenter and circumcenter

Let the Incenter tangent to  $BC, CA, AB$  at  $D, E, F$  respectively

Line  $EF$  cut the circumcircle of  $\triangle ABC$  at  $D_1, D_2$

Line  $ED$  cut the circumcircle of  $\triangle ABC$  at  $F_1, F_2$

Line  $DF$  cut the circumcircle of  $\triangle ABC$  at  $E_1, E_2$

Let  $S$  be the radical center of circumcircle of  $\triangle DD_1D_2, \triangle EE_1E_2$  and  $\triangle FF_1F_2$

Prove that  $S, I, O$  are collinear



Luis González

#2 Mar 23, 2016, 12:25 am

It's an old problem. See [Radical center of DEF](#) and [Radical center and line OI](#) (second problem).



## High School Olympiads

Radical center of DEF X

Reply



mathVNpro

#1 Jan 26, 2009, 8:04 am

Let triangle ABC inscribed in circumcircle  $(O)$ ,  $(I)$  is its incenter. Let D, E, F perspectively be the tangency point of  $(I)$  with BC, CA, AB. EF intersects  $(O)$  at  $X_1, X_2$ . Similar for  $Y_1, Y_2; Z_1, Z_2$ . Prove that the radical center of  $(DX_1X_2), (EY_1Y_2), (FZ_1Z_2)$  lies on the Euler line of triangle DEF.

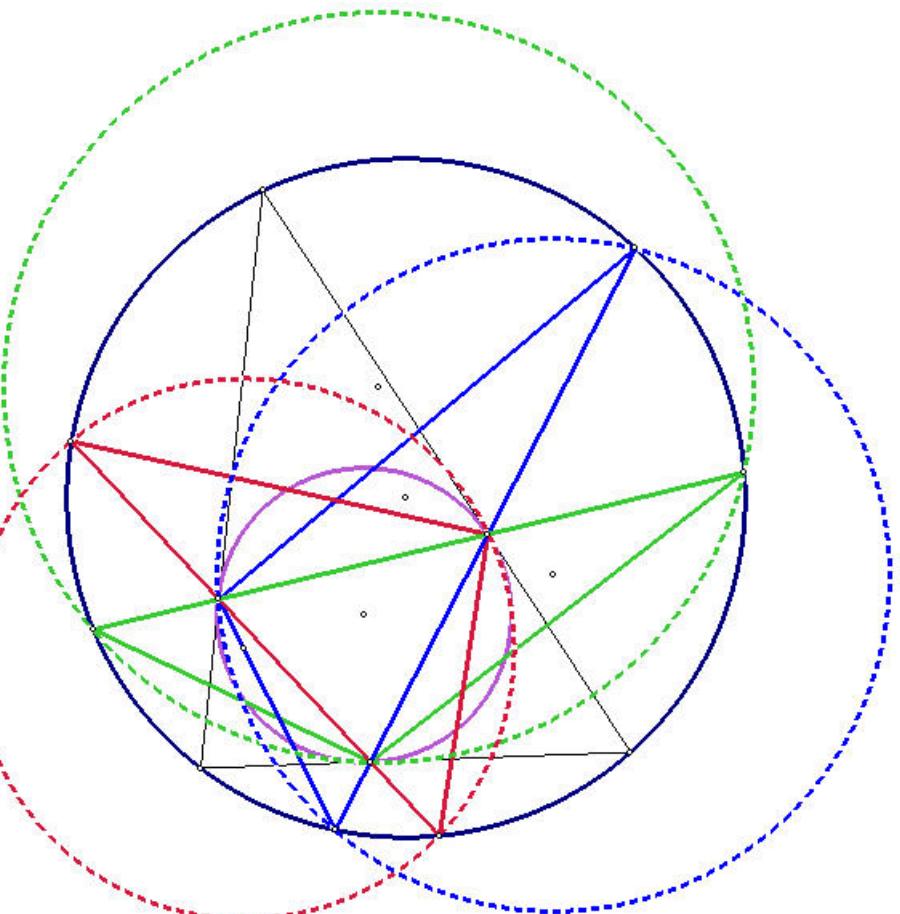


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#2 Jan 26, 2009, 8:22 am

Here is the figure

Attachments:



mathVNpro

#3 Jan 26, 2009, 11:07 pm

Mashimaru has had the solution, , he will posted it later



The QuattoMaster 6000

#4 Jan 28, 2009, 12:21 pm • 1



## Solution



yetti

#5 Jan 28, 2009, 12:49 pm

Let the angle bisectors  $AI, BI, CI$  cut the circumcircle  $(O)$  again at  $X, Y, Z$ . Let  $XX'$  be a diameter of  $(O)$ .  $I$  is orthocenter of  $\triangle XYZ \implies IYX'Z$  is a parallelogram, its diagonals  $IX', YZ$  cutting each other in half at  $K$ . Let  $(K)$  be a circle with center  $K$  and radius  $KD$ .  $IX'$  cuts  $(O)$  again at  $T$ , which is tangency point of the mixtilinear A-incircle  $(J_a)$  of the  $\triangle ABC$  with its circumcircle  $(O)$ .

Parallel to  $XT$  through  $D$  cuts  $(I)$  again at  $P$ . The angle  $\angle X'TX$  is right,  $IK \perp XT$ . Therefore,  $DP \perp IK$  is radical axis of  $(I), (K)$ . Perpendicular to  $AI$  through  $I$  cuts  $CA, AB$  at  $E_a, F_a$ ; these are tangency points of  $(J_a)$  with  $CA, AB$ . The parallels  $EF \parallel E_aF_a$  are similar with center  $A$  and coefficient  $k = \frac{AI}{AJ_a}$ .  $XT$  cuts  $(J_a)$  again at  $D_a$ .  $T$  is the external similarity center of  $(O), (J_a) \implies$  tangent of  $(J_a)$  at  $D_a$  is parallel to tangent of  $(O)$  at  $X$ , which is parallel to tangent of  $(I)$  at  $D$ . Since  $A$  is the external similarity center of  $(I), (J_a)$ , it follows that  $A, D, D_a$  are collinear. But then the parallels  $DP \parallel XT$  are similar with the same center  $A$  and coefficient  $k = \frac{AI}{AJ_a}$ .

$(U), (V)$  are circles with diameters  $IA, IX$  and common tangent  $E_aF_a$  at  $I$ .  $(U)$  goes through  $E, F$  and  $(V)$  goes through  $T$  on account of the right angles at these points.  $(U)$  cuts  $(O)$  again at  $S$ . Pairwise radical axes  $AS, E_aF_a, XT$  of  $(O), (U), (V)$  meet at their radical center  $Q$ . Pairwise radical axes  $AS, EF$  of  $(O), (U)$  and  $(U), (I)$  meet at the radical center  $R$  of  $(O), (U), (I)$ . Radical axis of  $(O), (I)$  goes through  $R$ . From similarity of the parallels  $EF \parallel E_aF_a$  and  $DP \parallel XT$  with the same center  $A$  and coefficient  $k$ , it follows that the radical axis  $DP$  of  $(I), (K)$  also goes through  $R$ . But then  $R$  is also radical center of  $(O), (I), (K)$  and the radical axis of  $(O), (K)$  also goes through  $R$ . But this radical axis is perpendicular to the center line  $OK$ , i.e., parallel to  $EF$ , hence it is identical with  $EF$ . As a result,  $(K)$  is the circle  $\odot(DX_1X_2)$ . In exactly the same way, circles  $(L), (M)$  centered at the midpoints  $L, M$  of  $ZX, XY$  and with radii  $LE, MF$  are the circles  $\odot(EY_1Y_2), \odot(FZ_1Z_2)$ .

$D$  is radical center of the circles  $(O), (L), (M)$ . Radical axis of  $(L), (M)$  goes through  $D$  and is perpendicular to their center line  $LM \parallel YX \parallel EF$ , it is the D-altitude of the  $\triangle DEF$ . Similarly, radical axes of  $(M), (K)$  and  $(K), (L)$  are the E- and F-altitudes, they meet at the orthocenter of  $\triangle DEF$  on its Euler line.



mathVNpro

#6 Jan 29, 2009, 10:17 pm

This beautiful geometry problem can be stated in another way, this statement is a little more complicated than the one I had posted: "Prove that the radical center of  $(DX_1X_2), (EY_1Y_2), (FZ_1Z_2)$  lies on  $OI$ ." Actually, to prove this, it is not really hard

. We just have to prove that  $O$  lies on the Euler line of triangle  $DEF$  by using the inversion with Pole and Polar. You can try



April

#7 Jan 30, 2009, 5:56 am • 1

This problem was proposed by **Cosmin Pohoata** and **Darij Grinberg** on *Mathematical Reflections*. And here is my solution:

Let  $S$  be the intersection of  $EF$  with the sideline  $BC$ , and  $M$  is the mid-point of  $BC$ . We have the division  $(BDCS)$  is a harmonic, so that  $SD \cdot SM = SB \cdot SC = SX_1 \cdot SX_2$ . Thus,  $M$  lies on the circumcircle of triangle  $DX_1X_2$ .

Let us call  $P, Q$  the mid-points of arcs  $BC$  and  $AB$  of the circle  $\rho(O)$  (not containing  $A, C$ ), respectively. Let  $N$  the mid-point of the arc  $BC$  containing  $A$  of  $\rho(O)$ ,  $O_1$  the mid-point of the segment  $IN$ . We have  $OO_1$  is parallel to  $IP$ , it means  $OO_1$  is perpendicular to  $X_1X_2$ , and so it's also the perpendicular bisector of  $X_1X_2$ . On the other hand, we have the quadrilateral  $IDMN$  is a trapezoid, so  $O_1$  lies on the perpendicular bisector of  $DM$ . Now, we have the quadrilateral  $DMX_2X_1$  is cyclic and  $O_1$  is the intersection of the perpendicular bisectors of the segments  $DM$  and  $X_1X_2$ , so that  $O_1$  is the center of the circumcircle of triangle  $DX_1X_2$ .

We can easily see that  $OO_1$  is parallel to  $IP$  and  $OO_1 = \frac{1}{2} \cdot IP$ . Similarly,  $OO_3$  is parallel to  $IQ$  and  $OO_3 = \frac{1}{2} \cdot IQ$ ,

where  $O_3$  is the center of the circumcircle of triangle  $FZ_1Z_2$ . Hence, we obtain  $O_1O_3$  is parallel to  $PQ$ . Now, let's take a look at the circumcircles of triangles  $DX_1X_2$  and  $FZ_1Z_2$ , we have the radical exits of the circles is the line  $d_2$ , which passes through  $E$  and perpendicular to  $O_1O_3$ , i.e.  $d_2 \perp PQ$ . It follows  $d_2$  is the E-altitude of triangle  $DEF$ . Similarly, we have the radical axis  $d_3$  of the circumcircles of triangles  $DX_1X_2$  and  $EY_1Y_2$  is the F-altitude of triangle  $DEF$ . Hence, we deduce the radical center of the circles  $DX_1X_2, EY_1Y_2$ , and  $FZ_1Z_2$  is the orthocentre of triangle  $DEF$ , which has known that lies on  $OI$ . The solution is completed.

Hope that I didn't make any mistake! 😊



mathVNpro

#8 Jan 30, 2009, 7:20 am

When I proved this problem, I didn't use any midpoints of AB, BC, CA. I noticed that if we let C1, B1 respectively be the midpoint of arc AB, AC then the midpoint of B1C1 are the center of the circumcircle of triangle DX1X2. To prove this I use the similarity that maps (I) to (O), and directed angle 😊 😎

Then I also noticed that the radical center of (DX<sub>1</sub>X<sub>2</sub>), (EY<sub>1</sub>Y<sub>2</sub>), (FZ<sub>1</sub>Z<sub>2</sub>) are the orthocenter of triangle DEF, hence it lies on the Euler line of triangle DEF.

Proof is done then!!! 🎉

To April, Yetti, and The QuattoMaster 6000: Thanks for your all nice solution, you have widened my eyes 😊

To April: May I ask your in Vietnamese???? 😊 I am just curious 😊

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## High School Olympiads

Radical center and line OI



Reply



Source: Own



livetolove212

#1 Aug 22, 2009, 4:32 pm

Given triangle  $ABC$  with incircle  $(I)$ , circumcircle  $(O)$ .  $(I)$  touches  $BC, AC, AB$  at  $A', B', C'$ . Prove that the radical center  $L$  of  $(A, AA')$ ,  $(B, BB')$ ,  $(C, CC')$  lies on the line  $OI$ . Moreover, if  $B'C'$  cuts  $(O)$  at  $A_1, A_2$ , similar for  $B_1, B_2, C_1, C_2$  then  $L$  is the radical center of  $(A'A_1A_2)$ ,  $(B'B_1B_2)$  and  $(C'C_1C_2)$



Luis González

#2 Sep 3, 2009, 1:00 pm • 1

Let  $L$  be the orthocenter of the intouch triangle  $\triangle A'B'C'$  lying on its Euler line  $IO$ .  $H$  is the foot of the perpendicular from  $A'$  to  $B'C'$  and  $S$  is the intersection of  $A'H$  with the external bisector of  $\angle BAC$ . Let  $\mathcal{P}_a$  denote the power of  $L$  WRT  $(A, AA')$ . Then we have  $\mathcal{P}_a = (AA')^2 - AL^2$ , but since  $AS \perp SA' \implies \mathcal{P}_a = (SA')^2 - LS^2$

$$\mathcal{P}_a = (LS + LA')^2 - LS^2 = (LA')^2 + 2LS \cdot LA'$$

$$\mathcal{P}_a = (LA')^2 + 2LA'(SH + LH) = (LA')^2 + 2LA' \cdot SH + 2LA' \cdot LH$$

Denote  $J \equiv AI \cap B'C'$ . We have then

$$\mathcal{P}_a = (2IJ)^2 + 4IJ \cdot SH + 2LA' \cdot LH = 4IJ(IJ + SH) + 2LA' \cdot LH$$

$$\mathcal{P}_a = 4IJ \cdot IA + 2LA' \cdot LH = 4r^2 + 2LA' \cdot LH$$

$LA' \cdot LH$  equals the power of the inversion with center  $L$  taking  $(I)$  into the nine-point circle of  $\triangle A'B'C'$ . Hence,  $\mathcal{P}_a$  is symmetric in terms of  $a, b, c \implies L$  has equal power WRT the circles  $(A, AA')$ ,  $(B, BB')$ ,  $(C, CC')$ . ■

Let  $D \equiv B'C' \cap BC$  and  $M_a$  be the midpoint of  $BC$ . From the harmonic cross ratio  $(B, C, A', D)$ , it follows that  $DB \cdot DC = DM_a \cdot DA'$ , but  $D$  has equal power WRT  $(O)$  and  $\odot(A'A_1A_2) \implies DB \cdot DC = DA_1 \cdot DA_2 \implies DA_1 \cdot DA_2 = DM_a \cdot DA \implies M_a \in \odot(A'A_1A_2)$ . Analogously  $M_b, M_c$  lie on  $\odot(B'B_1B_2), \odot(C'C_1C_2)$ , respectively. Center  $O_a$  of  $\odot(A'A_1A_2)$  is the intersection of the perpendicular bisector of  $M_aA'$  with the perpendicular to  $A_1A_2$  through  $O$ . In other words, if  $A''$  is the intersection of  $IA'$  with the perpendicular to  $A_1A_2$  from  $O$ , then  $O_a$  is the midpoint of  $OA''$ . Similarly,  $O_b, O_c$  are midpoints of  $OB'', OC''$ . Note that  $\triangle OA''B''$  and  $\triangle LA'B'$  are homothetic with homothety center  $I$ , since  $OA'' \parallel LA', OB'' \parallel LB'$  and  $OL, A''A'', B''B'$  concur at  $I$ . Thereby,  $A''B'' \parallel A'B' \parallel O_aO_b$ . Hence, radical axis  $\mathcal{L}_c$  of  $(O_a), (O_b)$  goes through  $C'$ , due to  $C'A_1 \cdot C'A_2 = C'B_1 \cdot C'B_2$  and it is orthogonal to  $O_aO_b \parallel A'B' \implies \mathcal{L}_c$  is identical with the C'-altitude of  $\triangle A'B'C'$ . Analogously,  $\mathcal{L}_a, \mathcal{L}_b$  are identical with the A'- and B'- altitudes of  $\triangle A'B'C' \implies L$  is the radical center of  $(O_a), (O_b), (O_c)$ . ■



livetolove212

#3 Sep 3, 2009, 2:13 pm

Dear Luis,

Thanks you very much for your nice solution! I have learned so much from you ☺

The orthocenter of triangle  $A'B'C'$  is the radical center of three circles  $(A'A_1A_2), (B'B_1B_2), (C'C_1C_2)$  was mentioned in a nice problem which was proposed by **Cosmin Pohoata** and **Darij Grinberg**.

My solution is base on this lemma: "Given triangle  $ABC$  and circumcenter  $O$ , incenter  $I$ .  $M$  is inside triangle  $ABC$ .  $MA' \perp BC, MB' \perp AC, MC' \perp AB$  then  $M$  lies on  $OI$  iff  $AC' + BA' + CB' = p$ " ☺

Quick Reply



## High School Olympiads

Major - ization 

 Reply

Source: Euler - SIN



armpist

#1 Jun 27, 2009, 6:56 am

It seems that the Euler triangle identity, although has been very very good, still does not go far enough. The following is not full generalization, but maybe can claim a majorization status.

The product of distances from circumcenter to two isogonal conjugate points equals the product of distance from their midpoint to nine-point center and circumdiameter.

With the diameter being part of the equality, it could be viewed as some expansion of the SIN theorem as well.

M.T.

Attachments:

[Euler-SIN.doc \(25kb\)](#)



Luis González

#2 Mar 22, 2016, 1:10 pm • 1 

**Problem:** Let  $\odot(O, R)$  and  $N$  be the circumcircle and 9-point center of  $\triangle ABC$ .  $P, Q$  are isogonal conjugates WRT  $\triangle ABC$  and  $T$  is the midpoint of  $\overline{PQ}$  (center of the pedal circle of  $P, Q$ ), then  $NT = \frac{OP \cdot OQ}{2R}$ .

**Proof:** Let  $\triangle DEF$  be the antipedal triangle of  $P$  WRT  $\triangle ABC$  with circumcircle  $\odot(U, \rho)$  and let  $\triangle Q_A Q_B Q_C$  be the pedal triangle of  $Q$  WRT  $\triangle ABC$ .  $P^*$  is the isogonal conjugate of  $P$  WRT  $\triangle DEF$  and  $Q^*$  is the isogonal conjugate of  $Q$  WRT  $\triangle Q_A Q_B Q_C$ .  $X, Y, Z$  are the circumcenters of  $\triangle PBC, \triangle PCA, \triangle PAB$  and  $X', Y', Z'$  are the reflections of  $X, Y, Z$  on  $BC, CA, AB$ .

Clearly the 9-point centers  $A_P, B_P, C_P$  of  $\triangle PBC, \triangle PCA, \triangle PAB$  are the midpoints of  $PX', PY', PZ' \Rightarrow \triangle A_P B_P C_P$  is the image of  $\triangle X'Y'Z'$  under homothety  $\mathcal{H}(P, \frac{1}{2}) \Rightarrow$  radius of  $\odot(A_P B_P C_P)$  is half the radius  $\rho$  of  $\odot(A' B' C')$ , but  $\rho = \frac{1}{2}PU$  (see [area of triangle ABC](#); remark at post #2). Moreover, we have  $\triangle A_P B_P C_P \cup N \cup T \sim \triangle Q_A Q_B Q_C \cup Q \cup Q^* \sim \triangle DEF \cup P^* \cup P$  (see [Similar triangle related to 9-point center](#)). Consequently

$$\frac{NT}{PP^*} = \frac{\frac{1}{2}\rho}{\rho} \Rightarrow NT = \frac{2 \cdot OP \cdot \frac{1}{4}PU}{\rho} = \frac{OP \cdot PU}{2\rho} \quad (1).$$

On the other hand, since  $\triangle DEF$  and  $\triangle Q_A Q_B Q_C$  are homothetic, then by [Gergonne-Arn theorem](#), we get  $[ABC]^2 = [DEF] \cdot [Q_A Q_B Q_C]$ . Using Euler's pedal triangle theorem this expression is equivalent to

$$\frac{[ABC]}{[DEF]} = \frac{[Q_A Q_B Q_C]}{[ABC]} \Rightarrow \frac{\rho^2 - PU^2}{4\rho^2} = \frac{R^2 - OQ^2}{4R^2} \Rightarrow \frac{PU}{\rho} = \frac{OQ}{R} \quad (2).$$

Combining (1) and (2) yields  $NT = \frac{OP \cdot OQ}{2R}$ .

**Luis González wrote:**

**Problem:** Let  $\odot(O, R)$  and  $N$  be the circumcircle and 9-point center of  $\triangle ABC$ .  $P, Q$  are isogonal conjugates WRT  $\triangle ABC$  and  $T$  is the midpoint of  $\overline{PQ}$  (center of the pedal circle of  $P, Q$ ), then  $NT = \frac{OP \cdot OQ}{2R}$ .

Let  $\triangle A_1B_1C_1$  be the circumcevian triangle of  $P$  WRT  $\triangle ABC$  and  $A_2, B_2, C_2$  be the reflection of  $A_1, B_1, C_1$  in  $BC, CA, AB$ , respectively. From [Some properties of Hagge circle](#) we get  $A_2, B_2, C_2$  and the orthocenter  $H$  of  $\triangle ABC$  are concyclic. Furthermore, if  $Q^*$  is the reflection of  $Q$  in  $N$ , then  $\triangle A_1B_1C_1 \cup O \cup P \sim \triangle A_2B_2C_2 \cup Q^* \cup T$ , so

$$NT = \frac{Q^*P}{2} = \frac{OP}{2} \cdot \frac{\text{radius of } \odot(A_2B_2C_2)}{\text{radius of } \odot(A_1B_1C_1)} = \frac{OP}{2} \cdot \frac{HQ^*}{R} = \frac{OP \cdot OQ}{2R}.$$

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## High School Olympiads

area of triangle ABC X

↳ Reply



Source: vankhea



vankhea

#1 Dec 26, 2013, 7:46 am

Let  $P$  be any point in the plane of triangle  $\Delta ABC$ . Let  $X, Y, Z$  be circumcenter of triangles  $\Delta APB, \Delta BPC, \Delta CPA$  respectively.  $X', Y', Z'$  be points of reflection  $X, Y, Z$  with mirror  $AB, BC, CA$  respectively.

Prove that  $|[XYZ] - [X'Y'Z']| = [ABC]$



Luis González

#2 Mar 22, 2016, 1:09 pm



Relabel  $X, Y, Z$  the circumcenters of  $\Delta PBC, \Delta PCA, \Delta PAB$ , thus  $X', Y', Z'$  are the reflections of  $X, Y, Z$  on  $BC, CA, AB$ . Let  $\Delta DEF$  be the antipedal triangle of  $P$  WRT  $\Delta ABC$  ( $D, E, F$  againts  $A, B, C$ ). Denote  $\widehat{A}, \widehat{B}, \widehat{C}$  and  $\widehat{D}, \widehat{E}, \widehat{F}$  the angles of  $\Delta ABC$  and  $\Delta DEF$ . Easy angle chase reveals that  $\widehat{Y'AZ'} = |\widehat{A} - \widehat{D}|$  and  $\widehat{EPF} = \widehat{A} + \widehat{D}$ , thus by cosine theorem, we obtain:

$$\begin{aligned} Y'Z'^2 &= AY'^2 + AZ'^2 - 2 \cdot AY' \cdot AZ' \cdot \cos(\widehat{A} - \widehat{D}) = AY^2 + AZ^2 - 2 \cdot AY \cdot AZ \cdot \cos(\widehat{A} - \widehat{D}) = \\ &= \frac{1}{4}(PE^2 + PF^2) - \frac{1}{2}PE \cdot PF \cos(\widehat{A} - \widehat{D}) = \frac{1}{4}(EF^2 + 2 \cdot PE \cdot PF \cdot \cos \widehat{EPF}) - \frac{1}{2}PE \cdot PF \cos(\widehat{A} - \widehat{D}) = \\ &= \frac{1}{4}EF^2 - \frac{1}{2}PE \cdot PF[\cos(\widehat{A} - \widehat{D}) - \cos(\widehat{A} + \widehat{D})] = \frac{1}{4}EF^2 - PE \cdot PF \cdot \sin \widehat{A} \cdot \sin \widehat{D} = \\ &= \frac{1}{4}EF^2 - AB \cdot AC \cdot \frac{\sin \widehat{A} \cdot \sin \widehat{D}}{\sin \widehat{E} \cdot \sin \widehat{F}} = \frac{1}{4}EF^2 - \frac{[ABC]}{[DEF]} \cdot EF^2 \implies \frac{Y'Z'^2}{EF^2} = \frac{1}{4} - \frac{[ABC]}{[DEF]} \quad (\star). \end{aligned}$$

Analogously we get the same expression for the ratios  $\frac{Z'X'^2}{FD^2}$  and  $\frac{X'Y'^2}{DE^2}$ , which means that  $\Delta X'Y'Z' \sim \Delta DEF$  and since  $\Delta XYZ$  is the image of  $\Delta DEF$  under homothety  $\mathcal{H}(P, \frac{1}{2})$ , then  $[XYZ] = \frac{1}{4}[DEF]$ . Therefore the expression  $(\star)$  becomes

$$\frac{[X'Y'Z']}{[DEF]} = \frac{[XYZ]}{[DEF]} - \frac{[ABC]}{[DEF]} \implies [ABC] = [XYZ] - [X'Y'Z'].$$

**Remark:** Let  $\rho, \varrho$  denote the radii of the circumcircles of  $\Delta DEF, \Delta X'Y'Z'$ , resp and let  $U$  denote the circumcenter of  $\Delta DEF$ . Using Euler's pedal triangle theorem, the latter area relation is equivalent to

$$\frac{\varrho^2}{\rho^2} = \frac{1}{4} - \frac{\rho^2 - PU^2}{4\rho^2} \implies PU = 2\rho.$$



TelvCohl

#3 Mar 22, 2016, 8:37 pm

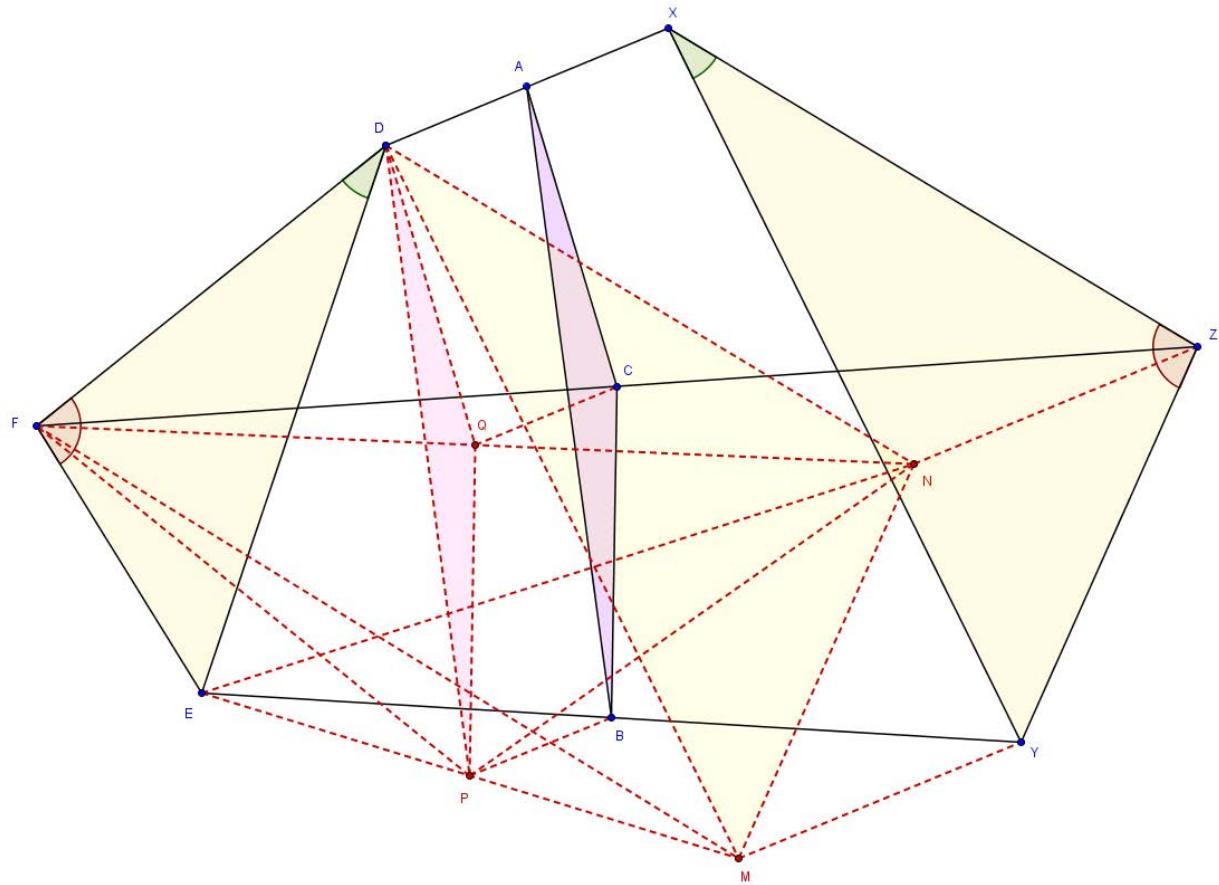


**Generalization :** Given two triangles  $\Delta DEF, \Delta XYZ$  such that  $\Delta DEF \sim \Delta XYZ$ . Let  $A, B, C$  be the midpoint of  $DX, EY, FZ$ , respectively. Then  $|[DEF] - [XYZ]| = 4[ABC]$ .

**Proof :** Let  $M, N$  be the image of  $Y, Z$  under translation  $\mathbf{T}(\overrightarrow{XD})$ , resp. and  $P, Q$  be the midpoint of  $EM, FN$ , respectively. Obviously,  $\Delta DPQ$  is the image of  $\Delta ABC$  under the translation  $\mathbf{T}(\frac{1}{2}\overrightarrow{XD})$ . From  $DE \cdot DN = DF \cdot DM$  and  $\angle EDN = \angle FDM$  we get  $[DEN] = [DFM]$ , so we conclude that

$$[DMN] - [DEF] = [DMN] + [DEN] - [DEF] - [DFM] = 2[DPN] - 2[DPF] = 4[DPQ].$$

Attachments:



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## High School Olympiads

Similar triangle related to 9-point center X

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Source: Own



TelvCohl

#1 Jan 29, 2015, 8:04 pm

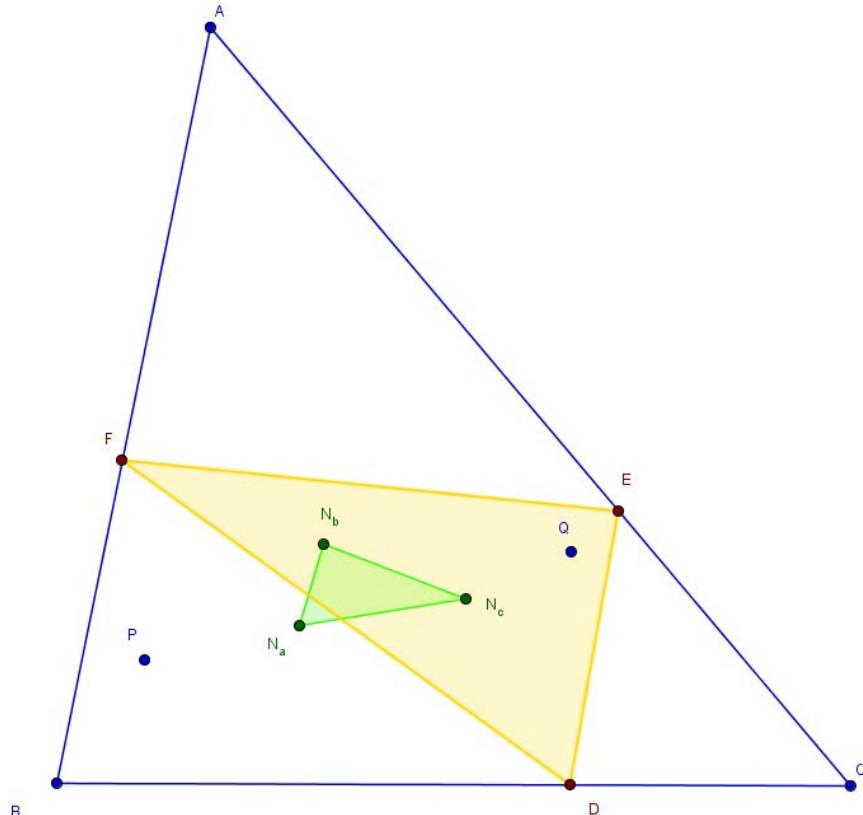
Let  $P, Q$  be the isogonal conjugate of  $\triangle ABC$ .

Let  $D, E, F$  be the projection of  $Q$  on  $BC, CA, AB$ , respectively.

Let  $N_a, N_b, N_c$  be the 9-point center of  $\triangle BPC, \triangle CPA, \triangle APB$ , respectively.

Prove that  $\triangle DEF \sim \triangle N_a N_b N_c$

Attachments:



Luis González

#2 Jan 30, 2015, 1:48 am • 1 ↳

Let  $M, N, L$  and  $X, Y, Z$  denote the midpoints of  $BC, CA, AB$  and  $PA, PB, PC$ . 9-point circles  $(N_a), (N_b), (N_c)$  of  $\triangle PBC, \triangle PCA, \triangle PAB$  concur at Poncelet point  $T$  of  $ABCP$ , thus  $N_b N_c \perp TX, N_c N_a \perp TY$  and  $N_a N_b \perp TZ$   $\Rightarrow \angle(N_a N_c, N_a N_b) = \angle(TY, TZ) = \angle(MY, MZ) = \angle(PC, PB) = \angle(DE, DF)$  and similarly we'll have  $\angle(N_b N_a, N_b N_c) = \angle(EF, ED) \Rightarrow \triangle N_a N_b N_c \sim \triangle DEF$ .



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## High School Olympiads

Perpendicular following tangent circles X

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Source: China Team Selection Test 2016 Test 2 Day 2 Q6

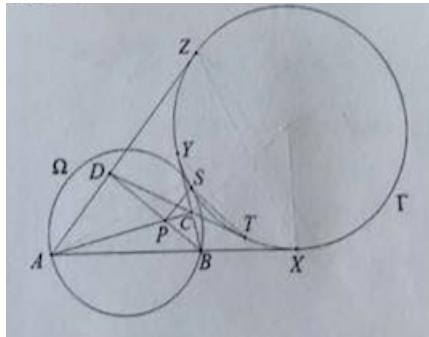


leeky

#1 Mar 21, 2016, 11:53 am • 2

The diagonals of a cyclic quadrilateral  $ABCD$  intersect at  $P$ , and there exist a circle  $\Gamma$  tangent to the extensions of  $AB, BC, AD, DC$  at  $X, Y, Z, T$  respectively. Circle  $\Omega$  passes through points  $A, B$ , and is externally tangent to circle  $\Gamma$  at  $S$ . Prove that  $SP \perp ST$ .

Attachments:



ABCDE

#2 Mar 21, 2016, 12:42 pm • 3

Let  $Q$  be the intersection of  $XY$  and  $ZT$  and  $P'$  be the intersection of  $YZ$  and  $XT$ . By Pascal on  $XYZTT$ ,  $Q, C$ , and  $P'$  are collinear. By Pascal on  $XXYZT$ ,  $A, Q$ , and  $P'$  are collinear. Hence,  $P'$  is on  $AC$ . Similarly, we can show that  $P'$  is on  $BD$ , so  $P' = P$ .

Now, invert about  $P$  fixing  $\Gamma$ , and denote by  $K'$  the image of point  $K$  under this inversion. Since line  $ABX$  is tangent to  $\Gamma$ , the circumcircle of  $PA'B'T$  is tangent to  $\Gamma$ . Because  $ABCD$  is cyclic,  $A'B'$  is parallel to line  $CD$ , which is the tangent to  $\Gamma$  at  $T$ . This implies that  $TA' = TB'$ . If  $T^*$  is the antipode of  $T$  with respect to  $\Gamma$ , then clearly the circumcircle of  $T^*A'B'$  is also tangent to  $\Gamma$  at  $T^*$ , so  $T^* = S'$ . But this means that  $\angle PST = \angle PXS' = \angle XT^* = 90^\circ$ , as desired.



TelvCohl

#3 Mar 21, 2016, 12:54 pm • 2

Let  $J$  be the pole of  $SX$  WRT  $\Gamma$  and let  $V \equiv AB \cap YZ$ . It's well-known that  $P$  lies on  $XT, YZ$  and  $(A, B; V, X) = -1$ , so from  $JX^2 = JA \cdot JB$  we get  $J$  is the midpoint of  $VX$ , hence  $J$  is the circumcenter of  $\triangle SVX$ . Since  $TX, YZ$  is parallel to the bisector of  $\angle(AB, CD), \angle(BC, DA)$ , respectively, so  $\angle XPV = 90^\circ \Rightarrow P, S, V, X$  lie on a circle with diameter  $VX$ , hence we conclude that  $\angle TSP = \angle XSP - \angle XST = \angle XVP - \angle VXP = 90^\circ$ .



Luis González

#4 Mar 21, 2016, 1:10 pm • 2

Let  $O$  and  $K$  be the centers of  $\odot(ABCD)$  and  $\Gamma$ .  $E \equiv AD \cap BC$  and  $F \equiv AB \cap CD$ . Since  $E(F, Y, Z, P) = -1$ , then the pole of  $EF$  WRT  $\Gamma$  is on  $EP$  and similarly it must be on  $FP \Rightarrow P$  is the pole of  $EF$  WRT  $\Gamma \Rightarrow X, T, P$  are collinear and  $EF \perp PK$  at  $Q$ , i.e.  $K \in OP$ . Since the polar of  $Q$  WRT  $(O)$  passes through  $P$ , then it follows that  $P, Q$  are the limiting points of  $\Gamma, (O)$ . Thus if the common tangent of  $\Gamma, \Omega$  cuts  $AB$  at  $M$ , we get  $MS^2 = MX^2 = MA \cdot MB \Rightarrow \odot(M, MS)$  is orthogonal to  $(O)$  and  $\Gamma \Rightarrow P \in \odot(M, MS) \Rightarrow \angle PST = \angle MSP + \angle MST = 90^\circ - \angle SXT + \angle SXT = 90^\circ$ .



leeky

#5 Mar 21, 2016, 4:15 pm

Let  $AD \cap BC = E$ ,  $AB \cap CD = F$ ,  $XY \cap TZ = G$ ,  $XZ \cap TY = H$ ,  $YZ \cap AB = Q$ . Note that polar of  $H$  wrt  $\Gamma$  is  $AC$ , polar of  $G$  is  $BD$ , hence polar of  $GH$  must be  $P = XT \cap YZ$ .  $YZ, XT$  are parallel to the internal angle bisectors of  $\angle AEB, \angle AFB$ , which are also parallel to the internal bisectors of  $\angle APB, \angle APD$  since  $ABCD$  is cyclic  $\Rightarrow PZ \perp PX$ , and  $PQ$  is the angle bisector of  $\angle APB \Rightarrow (A, B; Q, X) = -1$ .

Let the tangent at  $S$  to the two circles meet  $AB$  at  $M$ .  $MX^2 = MS^2 = MA \times MB$ , so  $M$  is the midpoint of  $QX$ , combining with  $MS = MX \Rightarrow QS \perp SX$ . Hence  $Q, P, S, X$  lie on a circle with diameter  $QX \Rightarrow \angle PSQ = \angle PXQ = \angle TSX \Rightarrow \angle PST = \angle QSX = 90^\circ$ .



Dukejukem

#6 Mar 22, 2016, 5:19 am

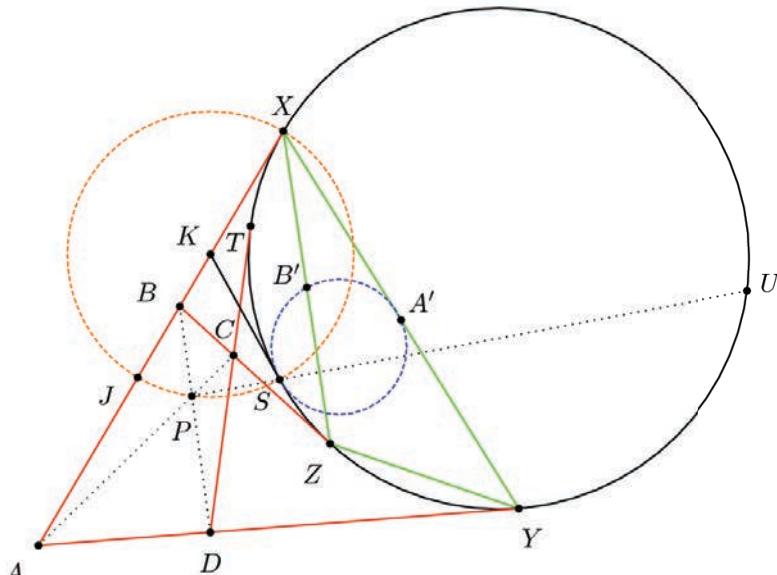
Let  $O$  be the center of  $\Gamma$  and let  $U$  be the antipode of  $T$  wrt  $\Gamma$ . Set  $A', B'$  as the midpoints of  $\overline{XY}, \overline{XZ}$  respectively. It's enough to show that  $U, P, S$  are collinear.

Since  $A, C, P$  are collinear, their polars wrt  $\Gamma$  are concurrent. Hence, the polar of  $P$  passes through  $XZ \cap YT$ . Similarly, the polar of  $P$  passes through  $XY \cap ZT$ . Thus, by Brokard's Theorem,  $P \equiv XT \cap YZ$ .

On the other hand, since  $ABCD, AXOY, CZOT$  are cyclic, we have

$$\angle XYO = 180^\circ - \angle XAY = 180^\circ - \angle BAD = \angle BCD = \angle ZCT = 180^\circ - \angle ZOT = \angle ZOU.$$

It follows that  $XYZU$  is an isosceles trapezoid. In particular,  $XU \parallel YZ$  and consequently  $XT \perp YZ$ .



Meanwhile, inversion in  $\Gamma$  sends  $\odot(ABS) \mapsto \odot(A'B'S)$ . Therefore,  $\odot(A'B'S)$  is tangent to  $\Gamma$  as well. Hence,  $K \equiv XX \cap SS \cap A'B'$  is the radical center of  $\Gamma, \odot(A'B'S), \odot(A'B'X)$ . Clearly  $KS = KX$  by equal tangents; also  $KX = KP$  because  $A'B'$  is the perpendicular bisector of  $XP$ . Therefore,  $K$  is the circumcenter of  $\triangle SXZ$ . Thus, if  $J$  is the reflection of  $X$  in  $K$ , we have  $(JP \parallel XU) \perp XP$ . By the converse of Reim's Theorem for  $XXJ$  and  $UPS$ , it follows that  $U, P, S$  are collinear, as desired.

**Remark:** This problem is essentially [2011 ISL G4](#), with  $\triangle XYZ$  as the reference triangle.

This post has been edited 3 times. Last edited by Dukejukem Mar 22, 2016, 8:51 pm



suli

#7 Mar 22, 2016, 11:16 am • 1

Projective KO:

1. Lemma:  $P$  is  $YZ$  intersect  $TX$ .

Proof.

1a. With respect to  $\Gamma$ , the pole of  $P$  is the line through the polars of  $BD$  and  $AC$ .

1b. But the polar of  $BD$  is the intersection of the poles of  $B$  and  $D$ , or  $XY$  and  $TZ$ .

1c. Similarly  $AC$  polar is intersection of  $XZ$  and  $YT$ .

1d. Thus pole of  $P$  is line through aforementioned intersections. But by Brokard's theorem the polar of this line is also the intersection of  $YZ$  and  $TX$ .

2.  $YZ$  intersect  $AB$  at  $M$  such that  $A, B; M, X$  harmonic.

Proof.

2a.  $BY$  and  $BX$  are tangents, so  $Z, A, BY, ZB, ZX$  form harmonic bundle.

2b. Thus  $Z, A, ZM, ZB, ZX$  are harmonic, so  $A, B; M, X$  harmonic.

3.  $\angle ZPX = 90^\circ$  by easy angle chasing because  $ABCD$  is cyclic, so  $\angle A + \angle D = 180^\circ$ .

4. Because the two circles are tangent at  $S$ , easy angle chasing show that  $SX$  is external angle bisector of  $ABS$ .

5. Thus by famous lemma  $\angle MSX = 90^\circ$ .

6.  $MPSX$  is cyclic.

7.  $PST$  and  $MSX$  are spirally similar, so  $\angle PST = \angle MSX = 90^\circ$ .



v\_Enhance

#8 Apr 2, 2016, 6:03 am • 2

Solution with Danielle Wang: Ignore  $ABCD$  cyclic for now, and focus entirely on  $\Gamma$ .

Let  $Q$  be the inverse of  $P$  with respect to  $\Gamma$ . Since  $P = AC \cap BD$ , it follows  $P$  lies on the polars of  $\overline{TY} \cap \overline{XZ}$  and  $\overline{YZ} \cap \overline{TX}$ . By Brokard's Theorem, this implies  $P = \overline{YZ} \cap \overline{XT}$ . Therefore  $Q$  is the Miquel point of cyclic quadrilateral  $YTXZ$ . Next let  $\gamma$  be the circumcircle of  $\triangle PQX$ , and  $M$  its center. Thus  $\gamma$  is orthogonal to  $\Gamma$ . So if  $W$  is the second intersection of  $\Gamma$  and  $\gamma$ , then  $\overline{OW}$  and  $\overline{OX}$  are tangents to  $\gamma$ . Angle chasing,

$$\angle WPT = \frac{1}{2}\angle WMX = 90^\circ - \angle WOX = \angle WTX - 90^\circ \implies \angle PWT = 90^\circ.$$

Thus we have shown that a circle centered at  $M \in AB$  passes through  $P, W, Q, X$  with  $\angle PWT = 90^\circ$ .

Now suppose  $ABCD$  is cyclic, centered at  $N$  with circumcircle  $\omega$ . If  $E = AB \cap CD, F = BC \cap DA$ , then  $Q \in EF$ ,  $PQ \perp EF$ , so  $Q$  is the Miquel point of cyclic quadrilateral  $ABCD$ . Consequently,  $N, P, Q, O$  collinear, and  $P$  and  $Q$  are inverses with respect to both  $\omega$  and  $\Gamma$ . Now the circle with diameter  $PQ$  is orthogonal to both  $\omega$  and  $\Gamma$ , thus the midpoint  $H$  of  $PQ$  is on the radical axis of  $\omega$  and  $\Gamma$ . Thus from  $HM \perp PQ, M$  lies on this radical axis as well. Then  $MA \cdot MB = MW^2$ , so  $W = S$  and we're done.

