

# Junior Balkan MO 2007

Shumen, Bulgaria

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- 1] Let  $a$  be positive real number such that  $a^3 = 6(a+1)$ . Prove that the equation  $x^2+ax+a^2-6=0$  has no real solution.
- 2] Let  $ABCD$  be a convex quadrilateral with  $\angle DAC = \angle BDC = 36^\circ$ ,  $\angle CBD = 18^\circ$  and  $\angle BAC = 72^\circ$ . The diagonals intersect at point  $P$ . Determine the measure of  $\angle APD$ .
- 3] Given are 50 points in the plane, no three of them belonging to a same line. Each of these points is colored using one of four given colors. Prove that there is a color and at least 130 scalene triangles with vertices of that color.
- 4] Prove that if  $p$  is a prime number, then  $7p + 3^p - 4$  is not a perfect square.