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Problem 1. A 10-digit number is called *cute* if each of its digits belong to the set $\{1, 2, 3\}$ and the difference between each pair of consecutive digits is 1.

(a) Find the total number of cute numbers.
(b) Prove that 1408 divides the sum of all cute numbers.

Problem 2. For $n \geq 1$, the set $\{1, 2, ..., 2n\}$ can be partitioned into n pairs

$$\{a_1, b_1\}, ..., \{a_n, b_n\}$$

such that for each $1 \leq i \leq n$, $a_i + b_i$ is a prime.

Problem 3. In each square of a 4×4 board, a lightbulb is placed. Each lightbulb is either on or off. A ***move*** consists of selecting a lightbulb, and switching the states (from on to off or from off to on) **of that lightbulb *and* all lightbulbs sharing an edge with it**. Is it possible to find a sequence of moves to switch all lightbulbs on, for all starting configurations?

Problem 4. Find all positive integer solutions for (a_1, a_2, a_3) which satisfy the following conditions:

1. $a_1 > a_2 > a_3$
2. $gcd(a_1, a_2, a_3) = 1$
3. $$a_1 = \sum_{i=1}^{3} gcd(a_i, a_{i+1})$$

   $(a_4 = a_1)$