India
Regional Mathematical Olympiad
1993

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1 Let ABC be an acute angled triangle and CD be the altitude through C. If $AB = 8$ and $CD = 6$, find the distance between the midpoints of AD and BC.

2 Prove that the ten's digit of any power of 3 is even.

3 Suppose $A_1, A_2, A_3, \ldots, A_{20}$is a 20 sides regular polygon. How many non-isosceles (scalene) triangles can be formed whose vertices are among the vertices of the polygon but the sides are not the sides of the polygon?

4 Let $ABCD$ be a rectangle with $AB = a$ and $BC = b$. Suppose r_1 is the radius of the circle passing through A and B touching CD; and similarly r_2 is the radius of the circle passing through B and C and touching AD. Show that

$$r_1 + r_2 \geq \frac{5}{8}(a + b).$$

5 Show that $19^{93} - 13^{99}$ is a positive integer divisible by 162.

6 If a, b, c, d are four positive reals such that $abcd = 1$, prove that $(1+a)(1+b)(1+c)(1+d) \geq 16$.

7 In the group of ten persons, each person is asked to write the sum of the ages of all the other nine persons. Of all ten sums form the nine-element set $\{82, 83, 84, 85, 87, 89, 90, 91, 92\}$, find the individual ages of the persons, assuming them to be whole numbers.