India

Regional Mathematical Olympiad

1996

$\boxed{1}$ The sides of a triangle are three consecutive integers and its inradius is 4. Find the circumradius.

$\boxed{2}$ Find all triples a, b, c of positive integers such that

$$(1+\frac{1}{a})(1+\frac{1}{b})(1+\frac{1}{c}) = 3.$$

$\boxed{3}$ Solve for real numbers x and y, *to*
xy$^2 = 15x^2 + 17xy + 15y^2$;

$x^2y = 20x^2 + 3y^2 . Suppose$N$isanndigitpositiveintegersuchthat(a)allitsdigitsaredistinct;(b)thesumofanythreecon$
≤ 6. Further, show that starting with any digit, one can find a six digit number with these properties.

Let ABC be a triangle and h_a be the altitude through A. Prove that

$$(b+c)^2 \geq a^2 + h_a^2.$$

Given any positive integer n , show that there are two positive rational numbers a and b , $a \neq b$, which are not integers and which are such that $a-b, a^2-b^2, \ldots a^n-b^n$ are all integers.

If A is a fifty element subset of the set $1, 2, \ldots 100$ such that no two numbers from A add up to 100, show that A contains a square.