2009 Iran Team Selection Tes.

Iran Team Selection Test 2009

Day 1

1	Let ABC be a triangle and A' , B' and C' lie on BC , CA and AB respectively such that the incenter of $A'B'C'$ and ABC are coincide and the inradius of $A'B'C'$ is half of inradius of ABC . Prove that ABC is equilateral .	khashi70 view topic
2	Let a be a fix natural number . Prove that the set of prime divisors of $2^{2^n}+a$ for $n=1,2,\cdots$ is infinite	khashi70 view topic
3	Suppose that a,b,c be three positive real numbers such that $a+b+c=3$. Prove that : $$\frac{1}{2+a^2+b^2}+\frac{1}{2+b^2+c^2}+\frac{1}{2+c^2+a^2}\leq\frac{3}{4}$$	khashi70 view topic

Day 2

4	Find all polynomials f with integer coefficient such that, for every prime p and natural numbers u and v with the condition: $$p\mid uv-1$$ we always have $p\mid f(u)f(v)-1$.	khashi70 view topic
5	ABC is a triangle and AA' , BB' and CC' are three altitudes of this triangle . Let P be the feet of perpendicular from C' to $A'B'$, and Q is a point on $A'B'$ such that $QA=QB$. Prove that : $\angle PBQ=\angle PAQ=\angle PC'C$	khashi70 view topic
6	We have a closed path on a vertices of a $n\times n$ square which pass from each vertice exactly once . prove that we have two adjacent vertices such that if we cut the path from these points then length of each pieces is not less than quarter of total path .	khashi70 view topic

Day 3

7	Suppose three direction on the plane . We draw 11 lines in each direction . Find maximum number of the points on the plane which are on three lines .	khashi70 view topic
8	Find All Polynomials $P(x,y)$ such that for all reals x,y we have $$P(x^2,y^2)=P\left(\frac{(x+y)^2}{2},\frac{(x-y)^2}{2}\right).$$	khashi70 view topic
9	In triangle ABC, D, E and F are the points of tangency of incircle with the center of I to BC, CA and AB respectively. Let M be the foot of the perpendicular from D to EF. P is on DM such that $DP=MP$. If H is the orthocenter of BIC, prove that PH bisects EF.	khashi70 view topic

Day 4

10	Let ABC be a triangle and $AB\neq AC$. D is a point on BC such that $BA=BD$ and B is between C and D . Let I_c be center of the circle which touches AB and the extensions of AC and BC . CI_c intersect the circumcircle of ABC again at T . If $\angle TDI_c=\frac{\angle B+\angle C}{4}$ then find $\angle A$	khashi70 view topic
11	Let n be a positive integer. Prove that	

	$$3^{\frac{5^{2^n}-1}{2^{n+2}}} \equiv (-5)^{\frac{3^{2^n}-1}{2^{n+2}}} \pmod{2^{n+4}}.$$	khashi70 view topic
12	T is a subset of $1, 2, ..., n$ which has this property : for all distinct $i, j \in T$, $2j$ is not divisible by i . Prove that : $ |T| \leq \frac {4}{9}n + {\mathrm{log}_2n_} + 2$	 khashi70 view topic