2000 Iran MO (2nd round)

National Math Olympiad (Second Round) 2000

Day 1

1 21 distinct numbers are chosen from the set $\{1, 2, 3, \ldots, 2046\}$. Prove that we can choose three distinct numbers a, b, c among those 21 numbers such that

$$bc < 2a^2 < 4bc$$

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2 The points D, E and F are chosen on the sides BC, AC and AB of triangle ABC, respectively. Prove that triangles ABC and DEF have the same centroid if and only if

$$\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB}$$

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3 Let $M = \{1, 2, 3, \ldots, 10000\}$. Prove that there are 16 subsets of M such that for every $a \in M$, there exist 8 of those subsets that intersection of the sets is exactly $\{a\}$.

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Day 2

1 Find all positive integers n such that we can divide the set $\{1, 2, 3, \ldots, n\}$ into three sets with the same sum of members.

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2 In a tetrahedron we know that sum of angles of all vertices is 180°. (e.g. for vertex A, we have $\angle BAC + \angle CAD + \angle DAB = 180^\circ$.)
Prove that faces of this tetrahedron are four congruent triangles.

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3 *Super number* is a sequence of numbers $0, 1, 2, \ldots, 9$ such that it has infinitely many digits at left. For example $\ldots 3030304$ is a *super number*. Note that all of positive integers are *super numbers*, which have zeros before they're original digits (for example we can represent the number 4 as $\ldots, 00004$). Like positive integers, we can add up and multiply *super numbers*. For example:

\[\begin{array}{cc}& \ \ \ \ldots 3030304 \ &+ \ldots4571378\ &\overline{\qquad \qquad \qquad }\ & \ \ \ \ldots 7601682 \end{array} \]

And

\[\begin{array}{cc}& \ \ \ \ldots 3030304 \ &\times \ldots4571378\ &\overline{\qquad \qquad \qquad }\ & \ \ \ \ldots 4242432 \ & \ \ \ \ldots 212128 \ & \ \ \ \ldots 90912 \ & \ \ \ \ldots 0304 \ & \ \ \ \ldots 128 \ & \ \ \ \ldots 20 \ & \ \ \ \ldots 6 \ &\overline{\qquad \qquad \qquad } \ & \ \ \   \ldots 5038912 \end{array}\]

a) Suppose that A is a *super number*. Prove that there exists a *super number* B such that $A + B = \overleftarrow{0}$ (Note: $\overleftarrow{0}$ means a super number that all of its digits are zero).

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b) Find all *super numbers* A for which there exists a *super number* B such that $A \times B = \overleftarrow{0}\ 1$ (Note: $\overleftarrow{0}\ 1$ means the super number $\ldots 00001$).

c) Is this true that if $A \times B = \overleftarrow{0}$, then $A = \overleftarrow{0}$ or $B = \overleftarrow{0}$? Justify your answer.