Geometry Camp 2009

Problem set

Easy [1]

Problem 0. In $\triangle ABC$ $AB = AC$ iff[2] $\angle B = \angle C$ (Kidding!)

Problem 1. Prove that a quadrilateral in which both pairs of opposite angles are equal has to be a parallelogram.

$\star$ **Problem 2.** [3](a) P, Q, R are points on the sides BC, CA, AB of $\triangle ABC$. Prove that the perpendiculars to the sides at these points meet in a common point iff

$$BP^2 + CQ^2 + AR^2 = PC^2 + QA^2 + RB^2$$

(b) Given a triangle ABC, Let L, M, N be the feet of the perpendiculars from the point K to the sides BC, CA, AB respectively. Prove that the perpendiculars from A, B, C to MN, NL, LM respectively are concurrent.

Problem 3. [4] (a) Let two segments AB and CD intersect at P. Prove that $AP \cdot PB = CP \cdot PD$ iff A,B,C,D are concyclic.
(b) Let A, B, T be three distinct points on ω, and P be a point on the extension of BA. Prove that PT is tangent to ω iff $PT^2 = PA \cdot PB$.

Problem 4. The convex hexagon $ABCDEF$ is inscribed in a circle. Prove that the diagonals AD, BE, CF are concurrent iff

$$AB \cdot CD \cdot EF = BC \cdot DE \cdot FA$$

Problem 5. The circles S_1 and S_2 intersects A. Through A any pair of straight lines BAC and B_0AC_0 are drawn with B, B_0 on S_1 and C, C_0 on S_2. Prove that the chord BB_0 of S_1 and C_0C of S_2 are inclined at a constant angle.

$\star$ **Problem 6.** The centers of two circles of unequal radii r_1, r_2 are a distance r_{12} apart. Calculate the distance between the two centers of similitude.[5]

Problem 7. In $\triangle ABC$, P lies on BC (possibly on the extensions). Prove that,

$$\frac{BP}{PC} = \frac{AB \times \sin \angle BAP}{AC \times \sin \angle PAC}$$

$\star\star$ **Problem 8.** A, B are two fixed points. P moves so that $\frac{PA}{PB}$ is constant. Prove that the locus of P is a circle. [6]

Problem 9. Let H is the orthocenter of $\triangle ABC$ and D be the foot of perpendicular from A on BC. Prove that $AH = a \cot A = 2R \cos A$ (R circumradius), and $HD = 2R \cos B \cos C$.

$\star$ **Problem 10.** Let the extension of BO meet the circumcircle of $\triangle ABC$ at Q. Prove that $AQCH$ is the parallelogram. (O circumcenter, H orthocenter)

Problem 11. $area(ABC) = \sqrt{rr_1r_2r_3}$.

$\star\star$ **Problem 12.** $r_1 + r_2 + r_3 = 4R + r$. (R, r, r_1, r_2, r_3 are the radius of circumcircle, incircle, and

[1] This does not mean that they are not "problems". You will not be able to solve them unless you pay enough attention, and try hard enough!

[2] *iff* means if and only if. When some problem includes this, it asks to prove "both ways". *This is important!*

[3] Asterisk ($\star$) indicates the difficulty level of the problems. Note that: "difficulty" is relative, and the difficult level of double asterisk problem of "Easy" set is not the same as that of "Not so easy!!!" set.

[4] Yes...the opposite statement of "the power of point" theorem is true, and undoubtedly very important.

[5] Plane Euclidean page 115.

[6] The circle of Appolonius.

the excircles respectively.)

Problem 13.[7] Let $\triangle ABC$ be a triangle, and let P, Q, R be any points in the plane distinct from A, B, C, respectively. Then AP, BQ, CR are concurrent if and only if

$$\frac{\sin\angle CAP}{\sin\angle PAB}\cdot\frac{\sin\angle ABQ}{\sin\angle QBC}\cdot\frac{\sin\angle BCR}{\sin\angle RCA}=1.$$

Problem 14.[8](a) Suppose the angle bisector of $\angle BAC$ intersect the circumcircle of $\triangle ABC$ at $X \neq A$. Let I be a point on the line segment AX. Then I is the incenter of $\triangle ABC$ if and only if $XI = XB = XC = XI_a$.
(b) Find the length AX in terms of the side lengths and angles, using trigonometry.

★★ **Problem 15.** Two circles intersect at the points A and B. Tangent lines drawn to both of the circles at the point A intersect the circles at the points M and N. The lines BM and BN intersect the circles once more at the points P and Q respectively. Prove that the segments MP and NQ are equal.

Problem 16. (Perpendicular Lemma)[9] On a plane, for distinct points R, S, X, Y we have $RX^2 - SX^2 = RY^2 - SY^2$ if and only if $RS \perp XY$.

★ **Problem 17.** Two circles meet at P and Q. A line intersects segment PQ and meets the circles at the points A, B, C, D in that order. Prove that $\angle APB = \angle CQD$.

★★ **Problem 18.** A convex quadrilateral $ABCD$ is given for which $\angle ABC + \angle BCD < \pi$. The comon point of the lines AB and CD is E. Prove that $\angle ABC = \angle ADC$ iff

$$AC^2 = CD \cdot CE - AB \cdot AE$$

★ **Problem 19.** The quadrilateral $ABCD$ is inscribed in a circle. The lines AB and CD meet at E, while the diagonals AC and BD meet at F. The circumcircle of the triangles AFD and BFC meet again at H. Prove that $\angle EHF = \frac{\pi}{2}$.

Problem 20. Prove that the radical axis of two intersecting circles passes through the intersection points.

Problem 21. Given three circles in plane whose centers do not lie on one line. Let us draw radical axes for each pair of these circles. Prove that all the three radical axes meet at one point.[10]

★★ **Problem 22.** Let M be the midpoint of the altitude BE in $\triangle ABC$ and suppose that the excircle opposite B touches AC at Y. Then the line MY goes through the incenter I.

★ **Problem 23.** [11] Let I be the incenter of $\triangle ABC$. Then AI is the bisector of angle A. If X and Y are the points of contact of the incircle on BC and AC then prove that the lines AI, XY and the perpendicular from B to AI are concurrent at a point P.

★ **Problem 24.** Let r, R be the inradius and circumradius of $\triangle ABC$. Prove that

$$\cos A + \cos B + \cos C = 1 + \frac{r}{R}$$

★ **Problem 25.** If h_a, h_b, h_c are the lengths of the altitudes of $\triangle ABC$, whose incircle has center I and radius r. Prove that
(a)

$$\frac{r}{h_a} + \frac{r}{h_b} + \frac{r}{h_c} = 1$$

[7] Trigonometric form of Ceva's Theorem, both of the forms are handy proving concurrency.
[8] This is a very important lemma. Don't forget!
[9] More applications: `http://www.math.ust.hk/excalibur/v12_n3.pdf`
[10] This point is called the radical center of the three circles.
[11] If the incircle touches AB at Z, then we can also deduce that B, Z, I, P, X are concyclic! This is indeed a very useful and surprizing result.

(b)

$$h_a + h_b + h_c \geq 9r.$$

Problem 26. (Leibniz's[12] Inequality) In a triangle ABC with circumradius R prove that $9R^2 \geq a^2 + b^2 + c^2$.

⋆⋆ **Problem 26.** Let ABC be an acute triangle. Let D be on side BC such that $AD \perp BC$. Let H be a point on the segment AD different from A and D. Let line BH intersect side AC at E and line CH intersect side AB at F. Prove that $\angle EDA = \angle FDA$.

⋆⋆ **Problem 27.** The circles S_1 and S_2 intersect at M and N. Show that if vertices A and C of a rectangle $ABCD$ lie on S_1 while vertices B and D lie on S_2, then the intersection of the diagonals of the rectangle lies on the line MN.

Problem 28. The convex quadrilateral $ABCD$ is inscribed in the circle S_1. Let O be the intersection of AC and BD. Circle S_2 passes through D and O, intersecting AD and CD at M and N, respectively. Lines OM and AB intersect at R, lines ON and BC intersect at T; and R and T lie on the same side of line BD as A. Prove that O, R, T, and B are concyclic.

Medium

Problem 1. Two circles with centers A and B intersect at right angles. Their common chord meets AB at F. DE is a chord of the first circle passing through B. Prove that A, D, E, F are concyclic.

⋆ **Problem 2.** L is the midpoint of the side BC of $\triangle ABC$. The circle through L which touches AB at B and the circle through L which touches AC at C meet at D. Prove thar $LA \cdot LD = LB^2$

Problem 3. The centers of two circles S_1, S_2 and their common tangents intersect at T. AP and AQ are the tangents at A to the two circles. Prove that AT bisects $\angle PAQ$.

Problem 4. Two circles AP_1Q_1 and AP_2Q_2 cut at A. The lines P_1P_2, Q_1Q_2 are their common tangents. Prove that the circles AP_1P_2 and AQ_1Q_2 touch each other.

⋆⋆ **Problem 5.**[13] Prove that there exists a point P inside $\triangle ABC$ such that $\angle BA\Omega = \angle CA\Omega = \angle AC\Omega = \omega$. Prove that

$$\cot\omega = \cot A + \cot B + \cot B$$

⋆ **Problem 6.** Let the extension of DG meet the circumcircle at D_2. Prove that $GD = \frac{1}{2}D_2G$. (G centroid,D feet of perpendicular from A.)

⋆ **Problem 7.** If O_A, O_B, O_C are the reflections of O in the sides BC, CA, AB respectively, prove that AO_A, BO_B, CO_C are concurrent. (O cirumcenter).

Problem 8. On sides AB and AC of triangle ABC there are given points D, E such that DE is tangent of circle inscribed in triangle ABC and $DE \parallel BC$. Prove

$$AB + BC + CA \geq 8DE$$

Problem 9. For every triangle ABC, let D, E, F be a point located on segment BC, CA, AB, respectively. Let P be the intersection of AD and EF. Prove that:

$$\frac{AB}{AF} \times DC + \frac{AC}{AE} \times DB = \frac{AD}{AP} \times BC$$

⋆ **Problem 10.** Prove that in any triangle ABC ,

[12] Have you heard of Leibniz's identity? If you haven't, ask someone!
[13] Brocard angle

$$\sqrt{r_a} + \sqrt{r_b} + \sqrt{r_c} \geq 3\sqrt{3r}$$

Where r, r_a, r_b, r_c are the radius of the incircle, $A-, B-, C-$ excircles of $\triangle ABC$.

★★ **Problem 11.** Let ABC be a triangle and let P be a point on the angle bisector AD, with D on BC. Let E, F and G be the intersections of AP, BP and CP with the circumcircle of the triangle, respectively. Let H be the intersection of EF and AC, and let I be the intersection of EG and AB. Determine the geometric place of the intersection of BH and CI when P varies.

★ **Problem 12.** A circle has center on the side AB of the cyclic quadrilateral $ABCD$. The other three sides are tangent to the circle. Prove that $AD + BC = AB$.

★ **Problem 13.** A circumference α intersects with circumference β in points A and B. There is a tangent line to both circumferences α and β which intersects them in points C and D respectively. Points C, D, B (B is closer to the tangent line) lie on the circumference γ. Prove, that the radius of circumference γ is the geometric mean of the radiuses of the circumferences α and β.

Problem 14.[14] Let ABC be a triangle, and draw isosceles triangles $\triangle DBC$, $\triangle AEC$, $\triangle ABF$ external to $\triangle ABC$ (with BC, CA, AB as their respective bases). Prove that the lines through A, B, C perpendicular to EF, FD, DE, respectively, are concurrent.

★★ **Problem 15.** ABC is a triangle, with inradius r and circumradius R. Show that:

$$\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)+\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)+\sin\left(\frac{C}{2}\right)\cdot\sin\left(\frac{A}{2}\right)\leq\frac{5}{8}+\frac{r}{4\cdot R}.$$

★ **Problem 16.** Let $ABCD$ be a cyclic quadrilateral. Prove that the incenters of triangles $\triangle ABC, \triangle BCD, \triangle CDA, \triangle DAB$ form a rectangle.

★ **Problem 17.** Let $ABCD$ be a cyclic quadrilateral. Prove that the sum of the inradii of $\triangle ABC$ and $\triangle CDA$ equals the sum of the inradii of $\triangle BCD$ and $\triangle DAB$.

★★ **Problem 18.** [15] In a triangle ABC we have $AB = AC$. A circle which is internally tangent with the circumscribed circle of the triangle is also tangent to the sides AB, AC in the points P, respectively Q. Prove that the midpoint of PQ is the center of the inscribed circle of the triangle ABC.

★ **Problem 19.** Let O be the point of intersection of the diagonals AC and BD of the quadrilateral $ABCD$ with $AB = BC$ and $CD = DA$. Again, let N and K be the feet of perpendiculars from D and B to AB and CD, respectively. Prove that the points N, O, and K are collinear.

★ **Problem 20.** In triangle ABC, let AK, BL, CM be the altitudes and H the orthocenter. Let P be the midpoint of AH. If BH and MK meet at S, and LP and AM meet at T, show that TS is perpendicular to BC.

★★ **Problem 21.** Let D, E, F be the points on the sides BC, CA, AB respectively, of $\triangle ABC$. Let P, Q, R be the second intersection of AD, BE, CF respectively, with the cricumcircle of $\triangle ABC$. Show that

$$\frac{AD}{PD}+\frac{BE}{QE}+\frac{CF}{RF}\geq 9$$

★ **Problem 22.** Let ABC be an acute triangle, AD, BE, CZ be its altitudes and H its orthocenter. Let AI, AI' be the internal and external bisectors of angles A. Let M, N be the midpoints of BC, AH, respectively. Prove that

- MN is perpendicular to EZ.

[14]This problem is a real gem. It has multiple solutions with unique and very,very beautiful ideas. This is USAMO 97.

[15]This is IMO 1978/4. A great problem if you want to learn some homothety. Recommended reading for Homothety: `http://www.math.ust.hk/excalibur/v9_n4.pdf`

- If MN cuts the segments AI, AI' at the points K, L, then $KL = AH$.

⋆⋆ **Problem 23.** Points D and E lie on sides AB and AC of triangle ABC such that $DE \parallel BC$. Let P be an arbitrary point inside ABC. The lines PB and PC intersect DE at F and G, respectively. If O_1 is the circumcenter of PDG and O_2 is the circumcenter of PFE, show that $AP \parallel O_1O_2$.

⋆⋆ **Problem 24.** Let $ABCDEF$ be a convex hexagon such that $AB = BC, CD = DE, EF = FA$. Prove that

$$\frac{BC}{BE} + \frac{DE}{DA} + \frac{FA}{FC} \geq \frac{3}{2}.$$

⋆⋆ **Problem 25.** *Nagel Point N: If the Excircles of ABC touch BC, CA, AB at D, E, F, then the intersection point of AD, BE, CF is called the Nagel Point N.* Prove that

- I, G, N are collinear. (G centroid, I incenter.)
- $GN = 2 \cdot IG$.
- *Speiker center S: The incircle of the medial triangle is called the Speiker circle, and it's center is Speiker center S.* Prove that S is the midpoint of IN.

⋆ **Problem 26.** (a) (*Archemides' Theorem*)Let M be the midpoint of the arc ACB on the circumcircle of $\triangle ABC$ and let MD be the perpendicular to the longer of AC and BC (say AC). Then D bisects the polygonal path ACB that is $AD = DC + CB$.
(b) Let C' be the midpoint of side AB. Prove that CD is parallel to the angle bisector of $\angle C$.
(c) In the same way define $B'E$, $A'F$, and prove that $C'E, B'E, A'F$ are concurrent at the incenter of $\triangle ABC$.

⋆ **Problem 27.** If three cevians AD, BE, CF of $\triangle ABC$ are concurrent at P. Prove that

$$\frac{AD}{AP} + \frac{BE}{BP} + \frac{CF}{CP} \geq \frac{9}{2}.$$

⋆ **Problem 28.** Let $ABCD$ be a convex quadrilateral such that $\angle DAB = \angle ABC = \angle BCD$. Let H and O denote the orthocenter and circumcenter of ABC. Prove that D, O, H are collinear.

NOT SO EASY!!! [16]

⋆ **Problem 1.** Let I and G be the incenter and the centroid of the given triangle ABC. Let M, N, P be the midpoint of BC, CA, AB, respectively and let J be the incenter of triangle MNP. Then we have: I, G, J are collinear and $GI = 2 \cdot GJ$

⋆⋆ **Problem 2.** Let $D.E, F$ be the feet of the angle bisectors of angles A, B, C, respectively, of triangle ABC, and let K_a, K_b, K_c be the points of contact of the tangents to the incircle of ABC through D, E, F (that is, the tangent lines not containing sides of the triangle).

Prove that the lines joining K_a, K_b, K_c to the midpoints of $BC, CA.AB$ respectively, pass through a single point on the incircle of ABC.

Problem 3. In triangle ABC, with $AB > BC$, BM is a median and BL an angle bisector. The line through M parallel to AB meets BL at D and the line through L parallel to BC meets BP at E. Prove that $ED \perp BL$.

⋆⋆ **Problem 4.** Let O be the circumcircle of a ΔABC and let I be its incenter, for a point P of the plane let $f(P)$ be the point obtained by reflecting P' by the midpoint of OI, with P' the homothety of P with center O and ratio $\frac{R}{r}$ with r the inradii and R the circumradii,(understand

[16] Hey...Don't panic!!! These are simply some of the coolest Olympiad problems. They are worth trying even if you can not solve them.

it by $\frac{OP}{OP'} = \frac{R}{r}$). Let A_1, B_1 and C_1 the midpoints of BC, AC and AB, respectively. Show that the rays $A_1f(A)$, $B_1f(B)$ and $C_1f(C)$ concur on the incircle.

$\star\star$ **Problem 5.** Let ABC be an acute-angled triangle, and let P and Q be two points on its side BC. Construct a point C_1 in such a way that the convex quadrilateral $APBC_1$ is cyclic, $QC_1 \parallel CA$, and the points C_1 and Q lie on opposite sides of the line AB. Construct a point B_1 in such a way that the convex quadrilateral $APCB_1$ is cyclic, $QB_1 \parallel BA$, and the points B_1 and Q lie on opposite sides of the line AC.

Prove that the points B_1, C_1, P, and Q lie on a circle.

$\star\star\star$ **Problem 6.** Let $ABCD$ be a quadrilateral, and let E and F be points on sides AD and BC, respectively, such that $\frac{AE}{ED} = \frac{BF}{FC}$. Ray FE meets rays BA and CD at S and T, respectively. Prove that the circumcircles of triangles SAE, SBF, TCF, and TDE pass through a common point.

$\star\star\star$ **Problem 7.** Let ABC be an acute triangle with ω, S, and R being its incircle, circumcircle, and circumradius, respectively. Circle ω_A is tangent internally to S at A and tangent externally to ω. Circle S_A is tangent internally to S at A and tangent internally to ω. Let P_A and Q_A denote the centers of ω_A and S_A, respectively. Define points P_B, Q_B, P_C, Q_C analogously. Prove that

$$8P_AQ_A \cdot P_BQ_B \cdot P_CQ_C \le R^3 \ ,$$

with equality if and only if triangle ABC is equilateral.

$\star$ **Problem 8.** Let ABC be a triangle. A circle passing through A and B intersects segments AC and BC at D and E, respectively. Lines AB and DE intersect at F, while lines BD and CF intersect at M. Prove that $MF = MC$ if and only if $MB \cdot MD = MC^2$.

$\star$ **Problem 9.** Let ABC be an acute, scalene triangle, and let M, N, and P be the midpoints of $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Let the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$ intersect ray AM in points D and E respectively, and let lines BD and CE intersect in point F, inside of triangle ABC. Prove that points A, N, F, and P all lie on one circle.

$\star\star$ **Problem 10.** $ABCDEF$ is a convex hexagon with $AB = BC$, $CD = DE$,$EF = FA$. Prove that

$$\frac{BC}{BE} + \frac{DE}{DA} + \frac{FA}{FC} \geqslant \frac{3}{2}$$

$\star\star$ **Problem 11.** An acute triangle ABC is given. Points A_1 and A_2 are taken on the side BC (with A_2 between A_1 and C), B_1 and B_2 on the side AC (with B_2 between B_1 and A), and C_1 and C_2 on the side AB (with C_2 between C_1 and B) so that

$$\angle AA_1A_2 = \angle AA_2A_1 = \angle BB_1B_2 = \angle BB_2B_1 = \angle CC_1C_2 = \angle CC_2C_1.$$

The lines AA_1, BB_1, and CC_1 bound a triangle, and the lines AA_2, BB_2, and CC_2 bound a second triangle. Prove that all six vertices of these two triangles lie on a single circle.

$\star$ **Problem 12.** The incircle k of a non-isosceles triangle ABC touches the sides AB, BC, CA at C_1, A_1, B_1 , respectively. Lines AA_1, BB_1, CC_1 intersect k again at A_2, B_2, C_2, respectively. Let A_1A_3 and B_1, B_3 be the bisectors of the angles in $A_1B_1C_1$ ($A_3 \quad B_1C_1, B_3 \quad A_1C_1$). Prove that:

(a) A_2A_3 is a bisector of $\angle B_1A_2C_1$;

(b) If the circumcircles of $A_1A_2A_3$ and $B_1B_2B_3$ intersect at P and Q, then the incenter I of ABC lies on the line PQ.

$\star$ **Problem 13.** Let ABC be a triangle, O its circumcenter, S its centroid, and H its orthocenter. Denote by A_1, B_1 and C_1 the centers of the circles circumscribed about the triangles CHB, CHA and AHB respectively.

Prove that the triangle ABC is congruent to the triangle $A_1B_1C_1$ and that the nine-point circle of ABC is also the nine-point circle of $A_1B_1C_1$.

$\star$ **Problem 14.** Let ABC be a triangle and K and L be two points on (AB), (AC) such that

$BK = CL$ and let $P = CK \cap BL$. Let the parallel through P to the interior angle bisector of $\angle BAC$ intersect AC in M. Prove that $CM = AB$.

Problem 15. Let ABC be an acute-angled triangle, and let H be its orthocenter. Let D be the foot of the altitude from B to AC, and let E be the reflection of A on D. The circumcircle of triangle BCE intersects the median from A in an interior point F. Prove that A, D, H and F are concyclic.

⋆ **Problem 16.** Let ABC be triangle with $AB \neq AC$. Point E is such that $AE = BE$ and $BE \perp BC$. Point F is such that $AF = CF$ and $CD \perp BC$. Let D be the point on the line BC such that AD is tangent to the circumcircle of triangle ABC. Prove that D, E, F are collinear.

⋆ **Problem 17.** Let ABC be an isosceles triangle with $AB = AC$. M is the midpoint of BC and O is the point on the line AM such that OB is perpendicular to AB. Q is an arbitrary point on BC different from B and C. E lies on the line AB and F lies on the line AC such that E, Q, F are distinct and collinear. Prove that OQ is perpendicular to EF if and only if $QE = QF$.

⋆ **Problem 18.** An acute-angled triangle ABC is given in the plane. The circle with diameter AB intersects altitude CC' and its extension at points M and N, and the circle with diameter AC intersects altitude BB' and its extensions at P and Q. Prove that the points M, N, P, Q lie on a common circle.

⋆⋆ **Problem 19.** Let O and I be the circumcenter and incenter of triangle $\triangle ABC$, respectively. Let ω_A be the excircle of triangle $\triangle ABC$ opposite to A; let it be tangent to AB, AC, BC at K, M, N, respectively. Assume that the midpoint of segment KM lies on the circumcircle of triangle $\triangle ABC$. Prove that O, N, I are collinear.

⋆ **Problem 20.** Let O be the center of circle ω. Two equal chords of AB and CD of ω intersects at L such that $AL > LB$, and $DL > LC$. Let M and N be points on AL and DL respectively such that $\angle ALC = 2\angle MON$. Prove that the chord of ω passing through M and N is congruent to AB and CD.

⋆⋆ **Problem 21.** Let O be the center of the excircle of $\triangle ABC$ opposite to A. Let M be the midpoint of AC, and let P be the intersection of lines MO and BC. Prove that if $\angle BAC = 2\angle ACB$, then $AB = BP$.

⋆ **Problem 22.** [17] Let $ABCD$ be a cyclic quadrilateral. Let $AB \cap CD = P$[18] and $AD \cap BC = Q$. Let the tangents from Q meet the circumcircle of $ABCD$ at E and F. Prove that P, E, F are collinear.

IMO 2009: *Why don't we try them?*

Problem 2. (*Day 1*) Let ABC be a triangle with circumcentre O. The points P and Q are interior points of the sides CA and AB respectively. Let K, L and M be the midpoints of the segments BP, CQ and PQ. respectively, and let Γ be the circle passing through K, L and M. Suppose that the line PQ is tangent to the circle Γ. Prove that $OP = OQ$.

Problem 4. (*Day 2*) Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E , respectively. Let K be the incentre of triangle ADC. Suppose that $\angle BEK = 45°$. Find all possible values of $\angle CAB$. (Don't forget to check the cases, anyway.)

[17] There is a very ingenious solution using Pole-Polar. For further study: `http://www.math.ust.hk/excalibur/v11_n3.pdf`

[18] $AB \cap CD = X$ means that the intersection point of AB and CD is X. This sign is very common in problem literature.