Sharygin Geometry Olympiad 2011

1 Does a convex heptagon exist which can be divided into 2011 equal triangles?

2 Let ABC be a triangle with sides $AB = 4$ and $AC = 6$. Point H is the projection of vertex B to the bisector of angle A. Find MH, where M is the midpoint of BC.

3 Let ABC be a triangle with $\angle A = 60^\circ$. The midperpendicular of segment AB meets line AC at point C_1. The midperpendicular of segment AC meets line AB at point B_1. Prove that line B_1C_1 touches the incircle of triangle ABC.

4 Segments AA', BB', and CC' are the bisectrices of triangle ABC. It is known that these lines are also the bisectrices of triangle $A'B'C'$. Is it true that triangle ABC is regular?

5 Given triangle ABC. The midperpendicular of side AB meets one of the remaining sides at point C'. Points A' and B' are defined similarly. Find all triangles ABC such that triangle $A'B'C'$ is regular.

6 Two unit circles ω_1 and ω_2 intersect at points A and B. M is an arbitrary point of ω_1, N is an arbitrary point of ω_2. Two unit circles ω_3 and ω_4 pass through both points M and N. Let C be the second common point of ω_1 and ω_3, and D be the second common point of ω_2 and ω_4. Prove that $ACBD$ is a parallelogram.