# India
## Regional Mathematical Olympiad
2006

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1 Let $ABC$ be an acute-angled triangle and let $D, E, F$ be the feet of perpendiculars from $A, B, C$ respectively to $BC, CA, AB$. Let the perpendiculars from $F$ to $CB, CA, AD, BE$ meet them in $P, Q, M, N$ respectively. Prove that the points $P, Q, M, N$ are collinear.

2 If $a$ and $b$ are natural numbers such that $a + 13b$ is divisible by 11 and $a + 11b$ is divisible by 13, then find the least possible value of $a + b$.

3 If $a, b, c$ are three positive real numbers, prove that $\frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b} \geq 3$

4 A $6 \times 6$ square is dissected in to 9 rectangles by lines parallel to its sides such that all these rectangles have integer sides. Prove that there are always **two** congruent rectangles.

5 Let $ABCD$ be a quadrilateral in which $AB$ is parallel to $CD$ and perpendicular to $AD$; $AB = 3CD$; and the area of the quadrilateral is 4. if a circle can be drawn touching all the four sides of the quadrilateral, find its radius.

6 Prove that there are infinitely many positive integers $n$ such that $n(n+1)$ can be represented as a sum of two positive squares in at least two different ways. (Here $a^2 + b^2$ and $b^2 + a^2$ are considered as the same representation.)

7 Let $X$ be the set of all positive integers greater than or equal to 8 and let $f : X \to X$ be a function such that $f(x + y) = f(xy)$ for all $x \geq 4, y \geq 4$. if $f(8) = 9$, determine $f(9)$.