India
Regional Mathematical Olympiad
2007

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1 Let ABC be an acute-angled triangle; AD be the bisector of $\angle BAC$ with D on BC; and BE be the altitude from B on AC. Show that $\angle CED > 45^\circ$. **[weightage 17/100]**

2 Let a, b, c be three natural numbers such that $a < b < c$ and $gcd(c-a, c-b) = 1$. Suppose there exists an integer d such that $a+d, b+d, c+d$ form the sides of a right-angled triangle. Prove that there exist integers, l, m such that $c + d = l^2 + m^2$. **[Weightage 17/100]**

3 Find all pairs (a, b) of real numbers such that whenever α is a root of $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of the equation. **[Weightage 17/100]**

4 How many $6-digit$ numbers are there such that-: a)The digits of each number are all from the set $\{1, 2, 3, 4, 5\}$ b)any digit that appears in the number appears at least twice ? (Example: 225252 is valid while 222133 is not) **[weightage 17/100]**

5 A trapezium $ABCD$, in which AB is parallel to CD, is inscribed in a circle with centre O. Suppose the diagonals AC and BD of the trapezium intersect at M, and $OM = 2$. **(a)** If $\angle AMB$ is 60°, find, with proof, the difference between the lengths of the parallel sides. **(b)** If $\angle AMD$ is 60°, find, with proof, the difference between the lengths of the parallel sides. **[Weightage 17/100]**

6 Prove that: **(a)** $5 < \sqrt{5} + \sqrt[3]{5} + \sqrt[4]{5}$ **(b)** $8 > \sqrt{8} + \sqrt[3]{8} + \sqrt[4]{8}$ **(c)** $n > \sqrt{n} + \sqrt[3]{n} + \sqrt[4]{n}$ for all integers $n \geq 9$. **[Weightage 16/100]**