India
Regional Mathematical Olympiad
2005

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1 Let ABCD be a convex quadrilateral; P,Q, R,S are the midpoints of AB, BC, CD, DA respectively such that triangles AQR, CSP are equilateral. Prove that ABCD is a rhombus. Find its angles.

2 If x, y are integers and 17 divides both $x^2 - 2xy + y^2 - 5x + 7y$ and $x^2 - 3xy + 2y^2 + x - y$, then prove that 17 divides $xy - 12x + 15y$.

3 If a, b, c are positive three real numbers such that $|a - b| \geq c, |b - c| \geq a, |c - a| \geq b$. Prove that one of a, b, c is equal to the sum of the other two.

4 Find the number of 5-digit numbers that each contains the block '15' and is divisible by 15.

5 In a triangle ABC, D is midpoint of BC . If $\angle ADB = 45^\circ$ and $\angle ACD = 30^\circ$, determine $\angle BAD$.

6 Determine all triples of positive integers (a, b, c) such that $a \leq b \leq c$ and $a+b+c+ab+bc+ca = abc + 1$.

7 Let a, b, c be three positive real numbers such that $a + b + c = 1$. Let $\lambda = min\{a^3 + a^2bc, b^3 + b^2ac, c^3 + abc^2\}$ Prove that the roots of $x^2 + x + 4\lambda = 0$ are real.

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