AM=CN in Russia

Source: AllRussian-2014, Grade 11, day1, P4

mathuz
1229 posts

Apr 30, 2014, 12:51 am • 1 PM #1

Given a triangle ABC with $AB > BC$, Ω is circumcircle. Let M, N are lie on the sides AB, BC respectively, such that $AM = CN$. $K(.) = MN \cap AC$ and P is incenter of the triangle AMK, Q is K-excenter of the triangle CNK (opposite to K and tangents to CN). If R is midpoint of the arc ABC of Ω then prove that $RP = RQ$.

M. Kungodjin

This post has been edited 1 time. Last edited by Amir Hossein, May 18, 2014, 8:45 pm
Reason: Fixed. Changed "Let M, N are lie on the sides AB, AC respectively," to "Let M, N are lie on the sides AB, BC respectively."

duanby
61 posts

Apr 30, 2014, 11:26 am • 1 PM #2

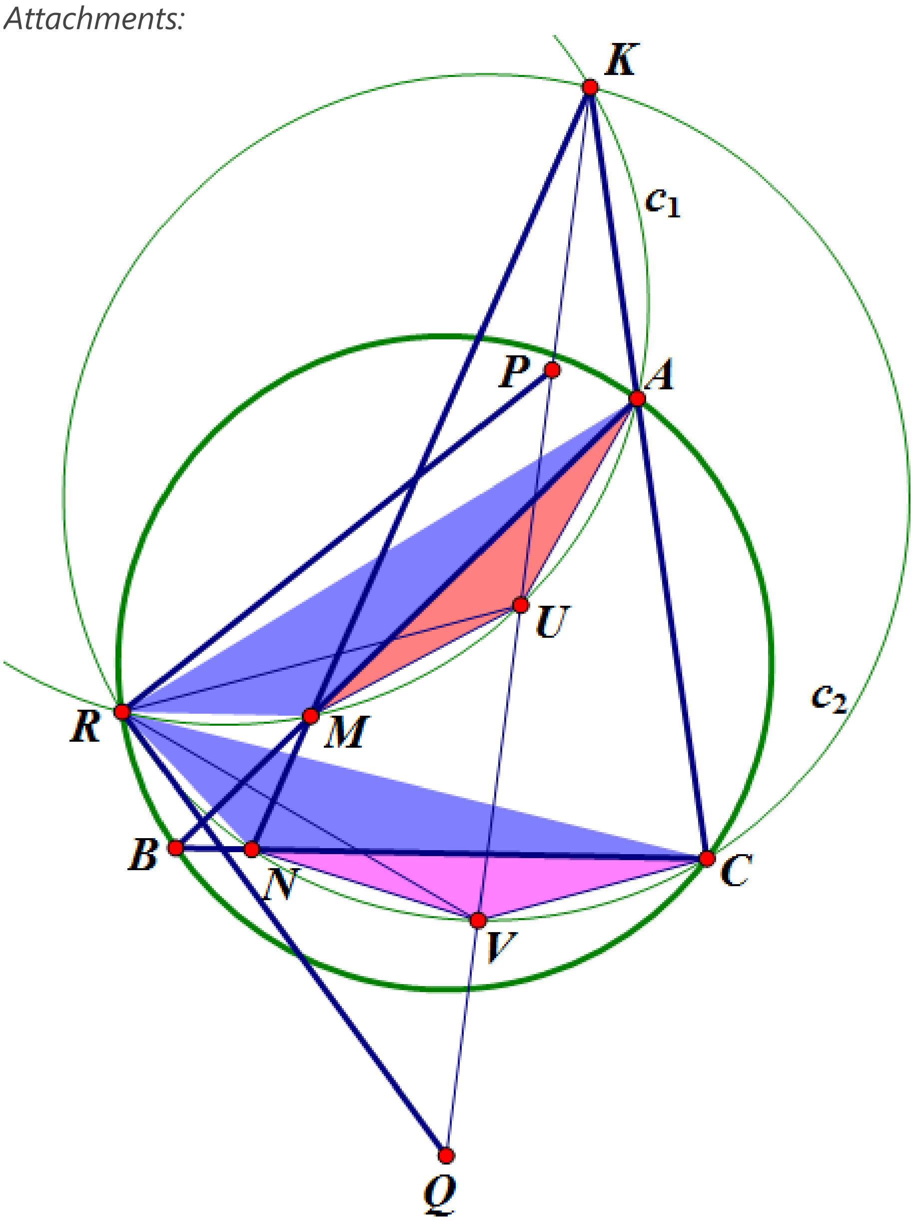
There's a typo:N lie on BC
my solution:
Let U,V,W be the midpoint of MN,AC,PQ
It's sufficient to prove U,V,W,C are concyclic
which is equivaient to
KV+KU=$\frac{1}{2}(KM + KA - MA + KN + KC + NC) = \frac{1}{2}(KM + KN + KA + I$

yunxiu
565 posts

May 6, 2014, 3:29 pm • 3 PM #3

Denote $(KAM) \cap KP = U, (KCN) \cap KQ = V$.
Then $\angle UAM = \angle UMA = \frac{1}{2}\angle K = \angle VCN = \angle VNC$, so
$\Delta UAM \cong \Delta VCN$, and we have $UP = UA = VC = VQ$, so
$RP = RQ \Leftrightarrow RU = RV$.
But from $\Delta RAM \cong \Delta RCN$, we have $\Delta RUM \cong \Delta RVN$, so we done.

Attachments:



Lawasu
208 posts

May 27, 2014, 6:52 pm • 2 PM #4

R is the Miquel point of the quadrilateral $AMNC$, ie there is a rotation centered in R which sends $\odot(DNC)$ to $\odot(DMA)$, therefore the image of the midpoint of the small arc (NC) will be the midpoint of the small arc (MA), and this immediately implies the problem.

See here: http://www.cut-the-knot.org/Curriculum/Geometry/SpiralSim.shtml

pi37
2079 posts

Aug 18, 2014, 6:19 am PM #5

Note that $AR=CR, AM=CN, \angle RAM=\angle RCN$, so $RAM\cong RCN$. So R is the miquel point of complete quadrilateral $BMACNK$. Then R lies on the perpendicular bisectors of MN, AC. Let D, E be the midpoints of MN, AC. Suppose $(KDER)$ meets KPQ at X; it suffices to show X is the midpoint of PQ. Let $(KAM), (KNC)$ meet KPQ at F, G respectively. Note that X is the midpoint of FG. Now F is the arc midpoint of AM in (KAM), so it is the center of circle PAM. Thus

$$FP=FM=\frac{AM}{2\cos\angle AKM/2}=\frac{CN}{2\cos\angle CKN/2}=GN=GQ$$

so X is the midpoint of PQ.

junioragd
277 posts

Sep 27, 2014, 6:55 pm PM #6

Let I be the intersection of KP with the circimcircle of KAM and J be the intersection point of KQ with the circumcircle of KCN.Now,because AMI is congruent to CNJ we have $IP=JQ$ so it is enough to prove $RI=RJ$.It is easy to show that RAM and RNC are congruent so from $MI=NJ$ and the previous congruence we have RMI is congruent to RNJ so we are finished.

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