A VARIANT

OF

31st I.M.O BEIJING CHINA 1990

AN AESTHETIC PROOF [1]

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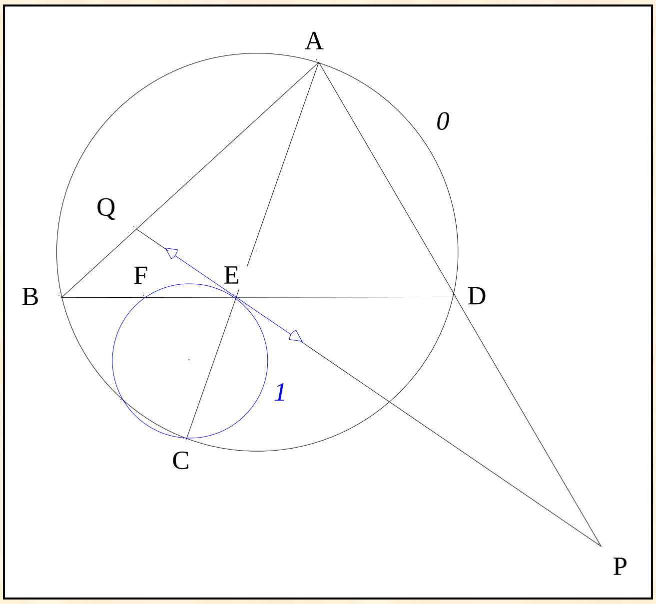
Jean - Louis AYME



Abstract. The author presents an aesthetic proof of a variant of an I.M.O. problem.
The figures are all in general position.

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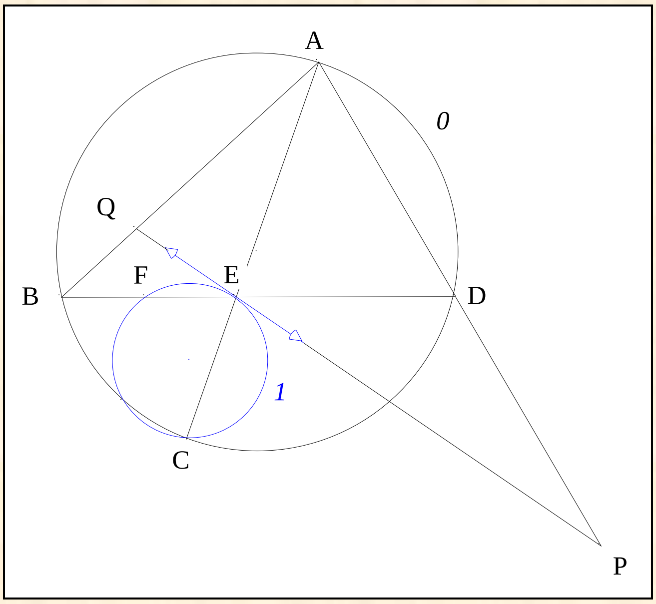
[1] St.-Denis, Île de la Réunion (France), le 29/09/2010

DAY 1 JULY 12, 1990

PROBLEM 1

VISION

Figure :

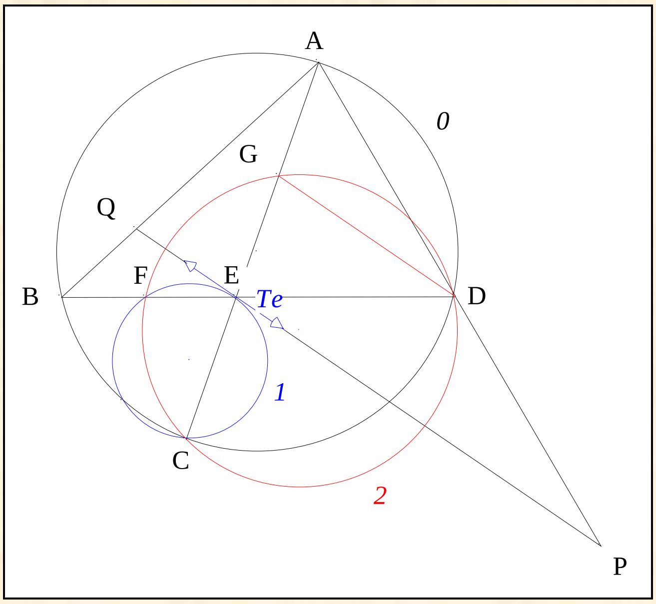


Features :	*0*	a circle,
	A, B, C, D	four points in this order on *1*,
	E	the point of intersection of AC and BD,
	F	a point on the segment BE,
	1	the circle passing through C, E, F,
	Te	the tangent to *1* at E
and	P, Q	the points of intersection of *Te* wrt AD and AB.

Given : $\frac{EQ}{EP} = \frac{FB}{FD}$.[2]

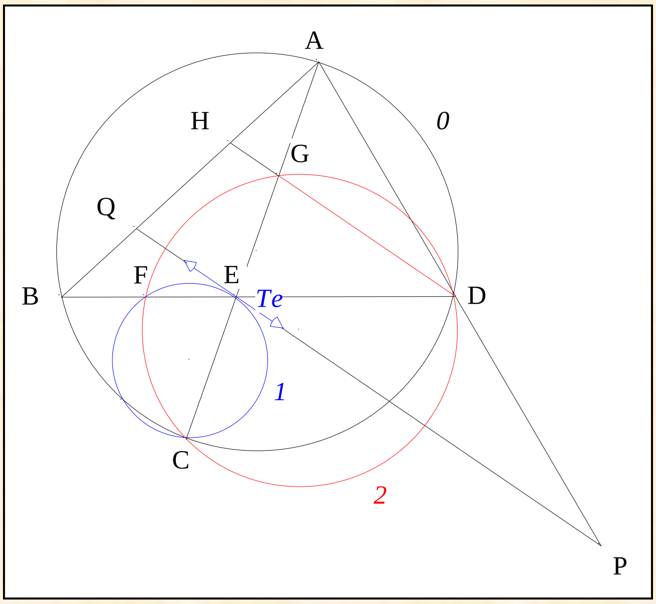
VISUALIZATION

[2] Chords AB and CD of a circle intersect at a point E, Art of problem solving (11/11/2005) ; http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366460.



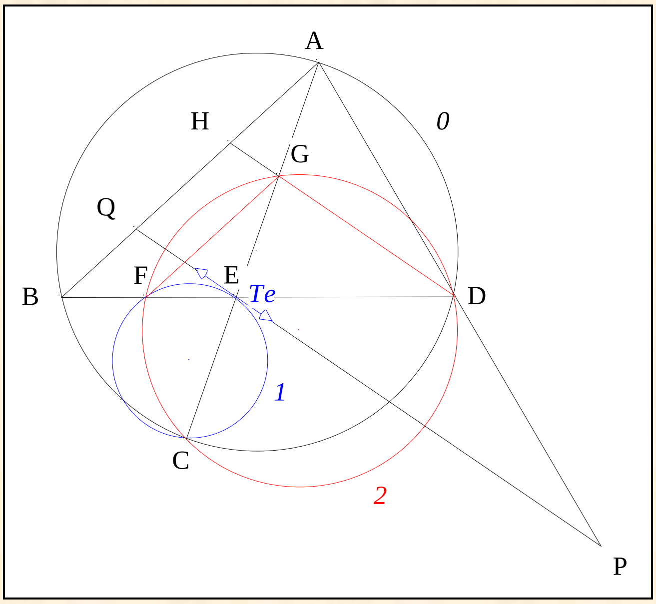
- Note $\quad$ *2* $\quad$ the circle passing through C, D, F
  and $\quad$ G $\quad$ the second point of intersection of AC with *2*.

- The circles *1* and *2*, the basic points C and F, the monians ECG and EFD, lead to the Reim's theorem **1** ; consequently, $\quad$ *Te* // GD.



- Note $\quad$ H $\quad$ the point of intersection of DG and AB.

- **Partial conclusion :** $\quad$ according to Thalès, $\quad \dfrac{EQ}{EP} = \dfrac{GH}{GD}$.



- The circles *2* et *0*, the basic points C and D, the monians GCA and FDB, lead to the Reim's theorem **0** ; consequently, GF // AB.

- According to Thalès, $\frac{GH}{GD} = \frac{FB}{FD}$;

- **Conclusion :** by transitivity of the relation =, $\frac{EQ}{EP} = \frac{FB}{FD}$.

Comments : this problem is considered to be difficult by Jean-Marie Monier[3], professor of Mathématiques spéciales at La Martinière-Montplaisir Highschool in Lyon (France) ; the heavy proof which he presents in his book uses the area technique.
The proof of the professor Theo Koupelis[4] of the University of Wisconsin (Marathon, United States), is more simple : it uses similar triangles and ratios.

The Professor Francisco Rosado Bellot who was the head of the Spanish delegation at this Olympic Games, told that this problem was also solved by the angular, vector technique and analytically. He also told that beside the official solution, he knew of a Colombian and Becheanu.

Historical note : the IMO 1990 which begins the 8 of July until the 19 of that month in the Popular Republic of China assembles 54 states and 308 competitors.

[3] Monier J.-M., *Géométrie*, Tome 7, Dunod (1997) 101.
[4] http://mks.mff.cuni.cz/kalva/imo/isoln/isoln901.html.

Original problem :

Day 1

$\boxed{1}$ Chords AB and CD of a circle intersect at a point E inside the circle. Let M be an interior point of the segment EB. The tangent line at E to the circle through D, E, and M intersects the lines BC and AC at F and G, respectively. If

$$\frac{AM}{AB} = t,$$

find $\frac{EG}{EF}$ in terms of t.