

Complex (!) Combinatorics

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1 Problems

1. How many subsets S of the set $\{1, 2, \dots, 100\}$ are there such that $|S|$ is a multiple of 3?
2. (Important Lemma) Let p be a prime number. $P(z)$ is a polynomial of degree $p - 1$ with rational coefficients. If $e^{\frac{2\pi i}{p}}$ is a root of $P(z)$, then all of the coefficients of $P(z)$ are equal.
3. (Traian Lalescu Contest) How many n -digit numbers, all of whose digits are 1, 3, 4, 6, 7 or 9 have a digit sum which is a multiple of 7?
4. (IMC 1999) We roll a regular six-sided die n times. What is the probability that the sum of the numbers rolled is a multiple of 5?
5. (IMO Shortlist 1987) Three persons A , B and C play the following game: a subset with k elements of the set $\{1, 2, \dots, 1986\}$ is selected uniformly at random. The winner is A , B or C , according to whether the sum of the elements of the selected subset is congruent to 0, 1 or 2 modulo 3. Find all values of k for which this game is a fair game.
6. (Qihong Xie) How many 100 element subsets of the set $\{1, 2, \dots, 2000\}$ have the sum of their elements a multiple of 5?
7. (IMO 1995) Let $p > 2$ be a prime number and let $A = \{1, 2, \dots, 2p\}$. Find the number of subsets of A each having p elements and whose sum is divisible by p .