AoPS Quarantine Geometry Olympiad 2020

Day 1

P1 Let $\triangle ABC$ be a triangle with altitude AD. Let V be a variable point on $\odot(ABC)$ and $\overline{VD}$ intersect $\odot(ABC)$ at U. Let $\overline{UM}$ intersect $\odot(ABC)$ at X where M is the midpoint of $\overline{BC}$. Let $\overline{AX} \cap \overline{BC} = Y$. Let $\odot(YVM)$ intersect $\odot(ABC)$ at a point Z. Let Z' be the reflection of Z over $\overline{OH}$. Finally let $\odot(Z'OH)$ intersect $\odot(ABC)$ at $T \neq Z'$. Then prove that $\overline{TZ}$ passes through M.

P2 Let $\triangle ABC$ be a triangle with D as a random point on $\overline{BC}$ such that $\overline{BD} < \overline{BM}$ where M is the midpoint of $\overline{BC}$ and let E, F be on sides $\overline{AB}, \overline{AC}$ such that $\overline{EF} \parallel \overline{BC}$. Let D' be the reflection of D across the midpoint of $\overline{BC}$ and define X, Y as the intersections of $\odot(BD'E)$ and $\odot(CDF)$. Then prove that $\odot(AXY)$ passes through a fixed point as D varies other than A.

P3 Let $\triangle ABC$ be a triangle with circumcenter O. Let M, N be the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and let T be the projection of O on $\overline{MN}$. Let D be the projection of A on $\overline{BC}$. Let $\overline{TD}$ intersect $\odot(BOC)$ at points U and V. Let $\odot(AUV)$ intersct $\overline{MN}$ at points X, Y. Let $\overline{AY}$ intersect $\odot(AMN)$ at R and $\overline{AX}$ intersect $\odot(AMN)$ at S. Then show that $\overline{AO}, \overline{RS}, \overline{MN}$ are concurrent.

Time: 4 hours and 30 minutes.
Each problem is worth 7 points.