Two parallel lines

geometry circumcircle

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Source: Iranian third round 2015 geometry problem 2

andria
714 posts

Sep 10, 2015, 5:41 pm • 1 #1

Let ABC be a triangle with orthocenter H and circumcenter O. Let K be the midpoint of AH. point P lies on AC such that $\angle BKP = 90^{\circ}$. Prove that $OP \parallel BC$.

TelvCohl
1980 posts

Sep 10, 2015, 5:56 pm #2

This problem is a particular case of the problem Perpendicular line :).

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jayme
5161 posts

Sep 11, 2015, 3:58 pm #3

Dear Mathlinkers,

1. E the foot of the B-altitude of ABC
B' the circumtrace of BE,
A'', B'' the antipoles of A, B wrt (O),
(1) the circle with diameter BP
A' the second point of intersection of (1) with (O).
2. by considering two time a converse of the Reim theorem, B''P goes through A', then AH goes through A'
3. By the Pascal theorem, we are done...

Sincerely
Jean-Louis

ATimo
226 posts

Sep 14, 2015, 1:17 am • 2 #4

Let P be the point on AC such that $OP \parallel BC$, we will prove that $\angle PKB = 90$. Let M be the foot of the perpendicular line from P to BC. Suppose that N is the midpoint of BC. Then we have $PM = ON = AK = KH$. $AH \parallel PM$ so $APMK$ and $KPMH$ are parallelograms. So $MK \parallel AC$ and $MH \parallel PK$. So we have to say that MH is perpendicular to BK. $MK \parallel AC$, so BH is perpendicular to MK. KH is also perpendicular to BM, So H is the orthocenter of the triangle $\triangle BKM$. And we are done.

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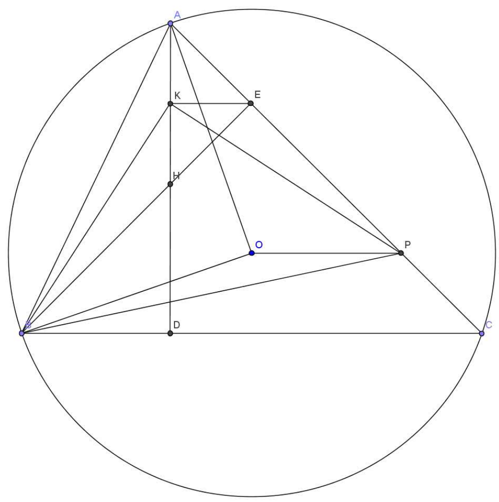
trunglqd91
42 posts

Sep 14, 2015, 4:45 pm #5

My solution:

Let AD, BE are the altitudes of $\triangle ABC$.
Easy to see that $BKEP$ is cyclic.
$\Longrightarrow \angle KBP = \angle AEK = \angle KAE = \angle EBC \Longrightarrow \angle KBE = \angle PBC.$
We have
$\angle OBP = \angle OBC - \angle PBC = 90^o - \angle BAC - \angle KBE = \angle ABE - \angle KBE = \angle ABK (1).$
In the other hand, $\angle BAK = \angle PAO.(2)$ (well-known).
From $(1), (2)$ we deduce O, K is isogonal conjugate WRT $\triangle ABP$.
$\Longrightarrow \angle BPO = \angle APK = \angle KBE = \angle PBC \Longrightarrow OP \parallel BC$. Done

Attachments:



Sep 14, 2015, 11:54 pm #6

huricane
610 posts

<u>*Solution:*</u>Let B' be the symmetric point of B wrt point K.We shall start with the following lemma:

Lemma: $m(\measuredangle KBP) = 90^0 - m(\measuredangle ACB)$

<u>Proof of the lemma:</u>Quadrilateral $ABHB'$ is parallelogram,so $m(\measuredangle HAB') = m(\measuredangle BHA) = 180^0 - m(\measuredangle ACB)$ and since $m(\measuredangle HAC) = 90^0 - m(\measuredangle ACB)$,it results that $B'A \perp AC$.Hence,$m(\measuredangle B'AP) = 90^0 = m(\measuredangle B'KP)$,resulting that quadrilateral $AKPB'$ is concyclic.
Therefore $m(\measuredangle KBP) = m(\measuredangle KB'P) = m(\measuredangle KAP) = 90^0 - m(\measuredangle ACB)$,which ends the proof.$\blacksquare$

Back to the main problem,using the above lemma we obtain $m(\measuredangle KBP) = 90^0 - m(\measuredangle ACB) = m(\measuredangle ABO)$,which implies $\measuredangle ABB' \equiv \measuredangle OBP(\bigstar)$.
But,let's observe that $\triangle AOB \sim \triangle B'PB(\text{case A.A.})$.This helps us see that $\frac{AB}{BO} = \frac{BB'}{BP}(\bigstar\bigstar)$.
Finally,from $(\bigstar)$ and $(\bigstar\bigstar)$ we get $\triangle BAB' \sim \triangle BOP$.So,$m(\measuredangle BOP) = m(\measuredangle BAB') = 90^0 + m(\measuredangle BAC$
that $OP \parallel BC$,which is what we wanted to prove.

This post has been edited 1 time. Last edited by huricane, Sep 15, 2015, 12:14 am
Reason: no need for point L...

Mar 27, 2016, 11:24 pm #7

Garfield
111 posts

This is nice problem but not too hard to complex bash:$k = a + \frac{b+c}{2}$,because P is on chord AB ,$\overline{p} = \frac{a+c-p}{ac}$, now because BK perpendicular to KP so $\frac{b-k}{\overline{b}-\overline{k}} = -\frac{p-k}{\overline{p}-\overline{k}}$ so after some calculations (about 15 minutes) we get $p = \frac{b^2c + abc + b^2a - c^2b}{b^2 + bc - ac - ab}$ and