FIRST AND SECOND

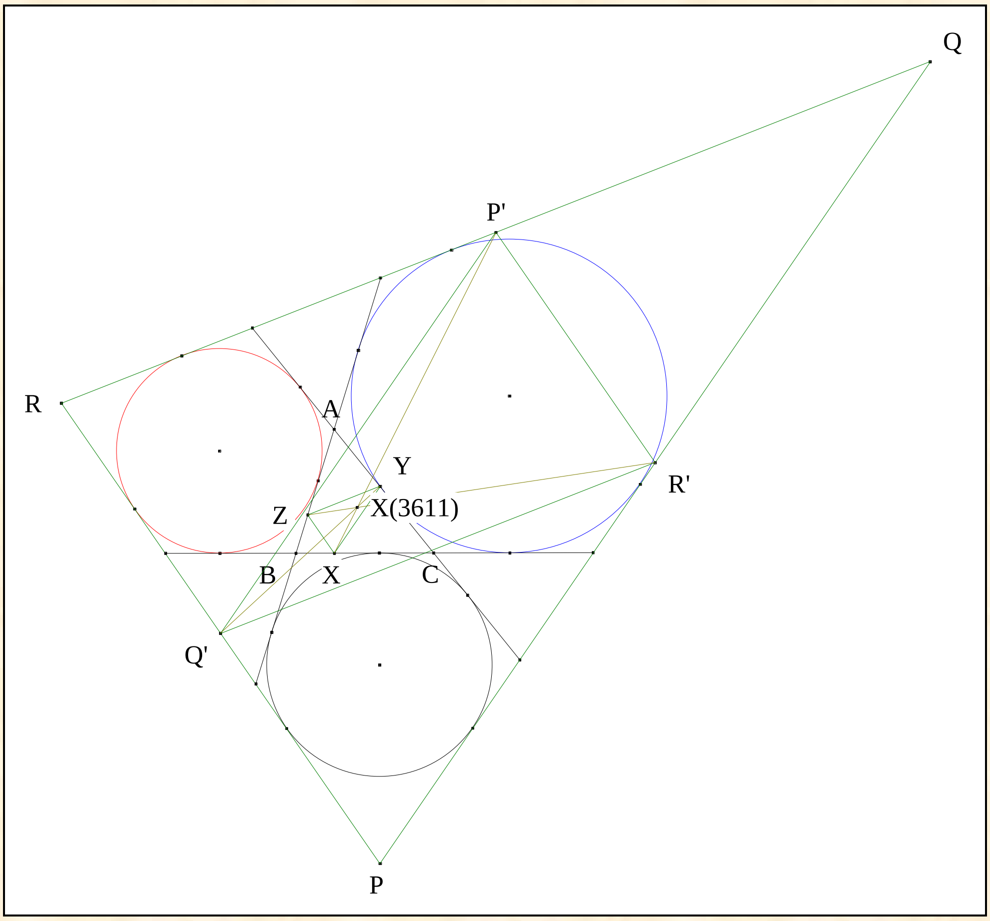
AYME - MOSES PERSPECTORS

OR

X(3610) and X(3611)

†

Jean - Louis AYME [1]



Abstract. The paper presents a triangle and two remarkable centers which bear the name of the author, and a short reflection on the allocation of names in Geometry. Two results of the author are exposed.
The figures are all in general position and all cited theorems can all be demonstrated synthetically.

Résumé. L'article présente un triangle et deux centres remarquables qui portent le nom de l'auteur ainsi qu'une courte réflexion sur l'attribution des noms en géométrie. Deux résultats de l'auteur sont exposés.

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1 St.-Denis, Île de la Réunion (France).

Toutes les figures sont en position générale et tous les théorèmes cités peuvent tous être démontrés synthétiquement.

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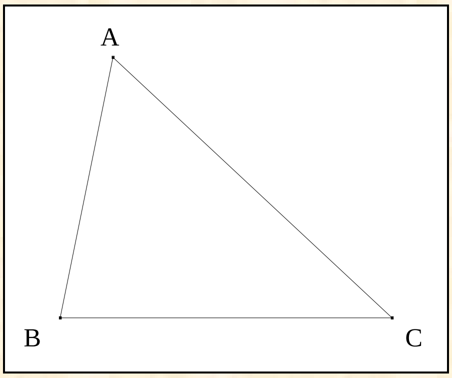
A. SOME GEOMETRIC ELEMENTS

1. Triangle

VISION

Figure :



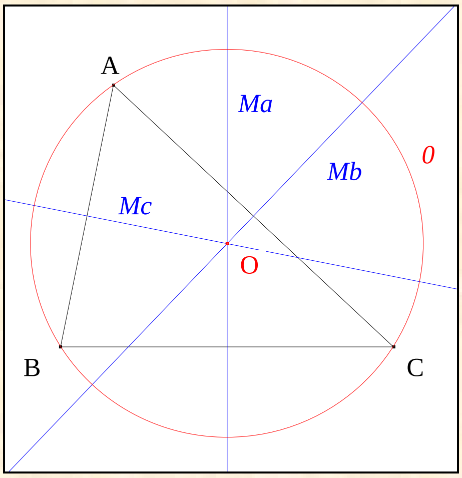
Finition : A, B, C three distinct and non-aligned points.

Definitions :
(1) (A, B, C) is "a triangle" more simply noted ABC
(2) A, B, C are "the vertice of ABC".

2. Circumcircle

VISION

Figure :



Finition : ABC a triangle,
Ma, *Mb*, *Mc* the A, B, C- perpendicular bisectors of ABC,
O the point of concurs of *Ma*, *Mb*, *Mc*
and *0* the circle centered at O and passing through A, B, C.

Definition : *1* is "the circumcircle of ABC".

Historic note : this circle is presented in the proposal 5 of Book **IV** of the Elements of Euclid of Alexandria.

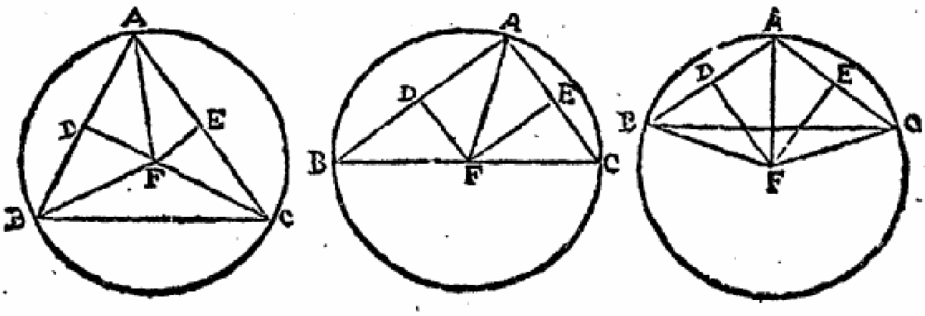
PROB. 5. PROP. V.

A l'entour d'vn triangle donné, deſcrire vn cercle.

Soit le triangle donné ABC, à l'entour duquel il faut deſcrire vn cercle.

ELEMENT. 157

Soient couppez en deux egalement les deux coſtez AB & AC aux poincts D & E par la 10, p. 1. & par la 11. pr. 1. d'iceux poincts D & E, ſoient leuees les perpendiculaires DF, EF, ſe rencontrans au poinct F, lequel ſera ou dans le triangle, ou au coſté BC, ou hors le triangle: & apres auoir mené les trois



lignes FA, FB, FC, les deux triangles ADF, BDF, auront les coſtez AD, BD egaux, & DF commun, & les deux angles au poinct D egaux pour eſtre droicts: donc les baſes AF, BF, ſeront egales par la 4. prop. 1. Par meſme diſcours AF, CF, ſeront auſſi egales: & par la 1. com. ſent. les trois lignes FA, FB, FC, ſeront egales entr'elles: & partant le cercle deſcrit de F, & de l'interuale FA, paſſera auſſi par les poincts B & C. Nous auons donc deſcrit vn cercle à l'entour du triangle donné ABC: Ce qu'il falloit faire.

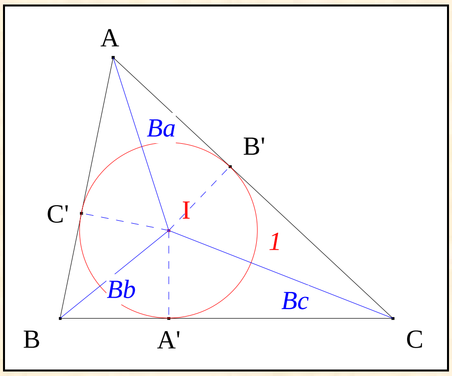
This excerpt comes from "The fifteen books of the geometric elements" of Euclid translated by Henrion[2] and printed in his lifetime in 1632.

2 http://gallica.bnf.fr/.

3. Incircle

VISION

Figure :



Finition :	ABC	a triangle,
	Ba, *Bb*, *Bc*	the A, B, C-internal bisectors of ABC,
	I	the point of concurs of *Ba*, *Bb*, *Bc*,
	A', B', C'	the feet of the perpendicular through I wrt BC, CA, AB
and	*1*	the circle centered at I and passing through A', B', C'.

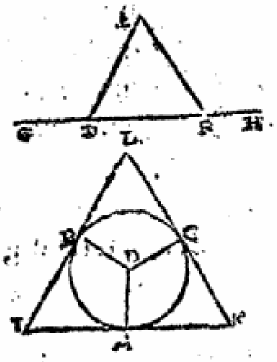
Definition : *1* is "the incircle of ABC".

Historic note : this circle is presented in the proposition 4 of Book **IV** of the Elements of Euclid of Alexandria.

PROB. 3. PROP. III.

A l'entour d'vn cercle donné, descrire vn triangle equiangle à vn triangle donné.

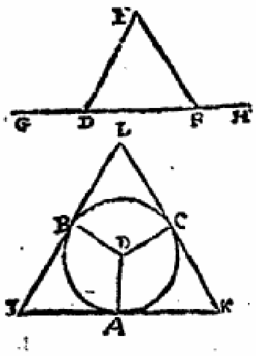
Soit le cercle donné ABC, à l'entour duquel il faut descrire vn triangle equiangle au triangle donné DEF.



Soit prolongé le costé DF de part & d'autre iusques en G & H, & du centre D soit menee, comme on voudra la ligne DA, sur laquelle & au poinct D, soient construits les deux angles ADB egal à EDG, & ADC egal à l'angle HFE par la 23. pr. 1. & aux trois lignes DA, DB, DC, soient menees les trois lignes perpendiculaires IK, IL, KL, lesquelles toucheront le cercle és poincts A, B, C, par le Corol. de la 16. p. 3. & icelles se rencontrans aux trois poincts I, K, L, feront le triangle IKL, lequel ie dis estre le triangle demandé.

Car il appert desia qu'il est circonscrit au cercle; puis que tous les costez

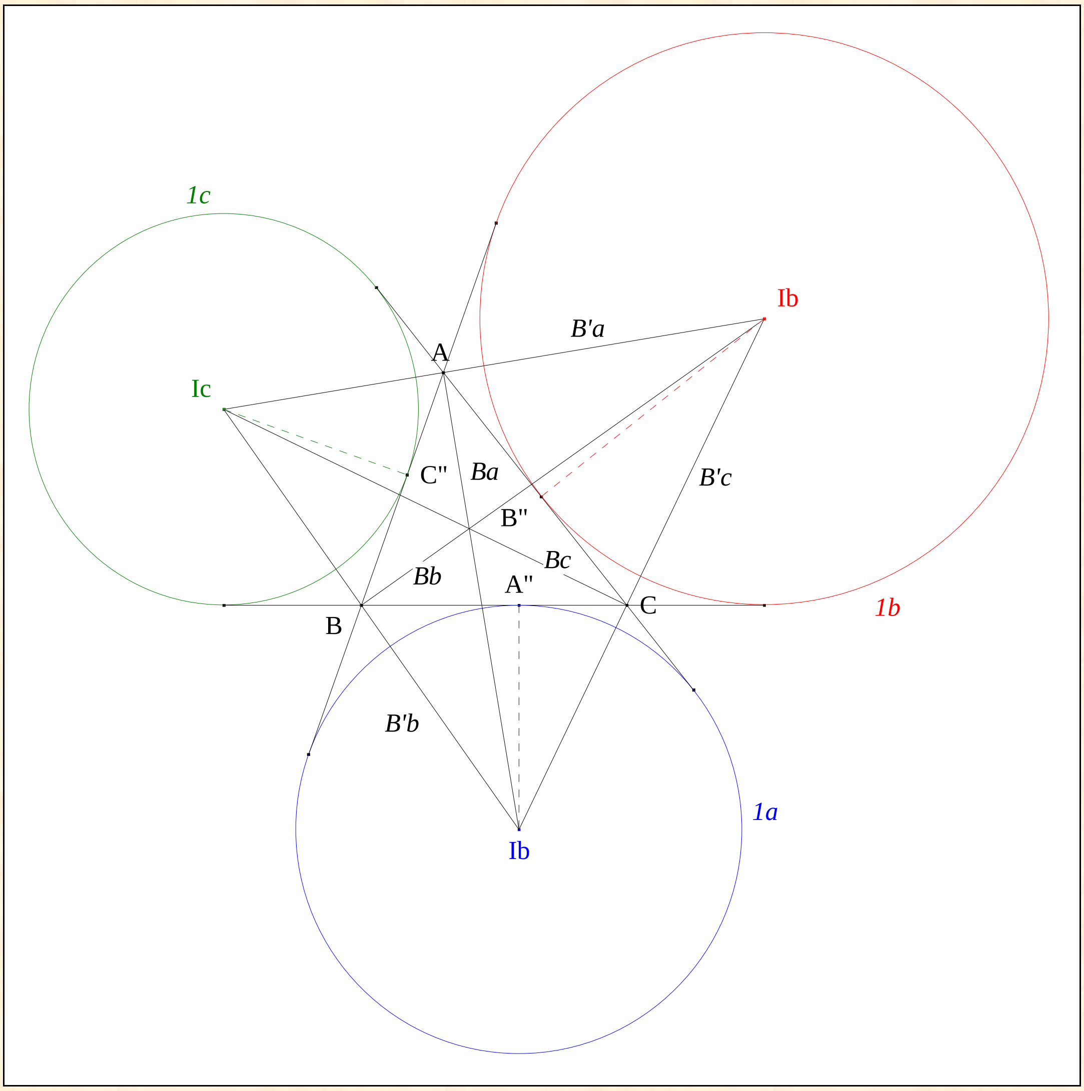
d'iceluy le touchent és poincts A, B, C. Et d'autant que toute figure de quatre costez a les quatre angles egaux à quatre angles droicts comme nous auons demonstré à la 32. p. 1.) le trapeze ADBI aura les quatre angles egaux à quatre droicts. Mais les deux A & B estans droicts par la construction, les deux autres D & I, feront egaux à deux droicts, c'est à dire egaux aux deux GDE & FDE, qui sont egaux à deux angles droicts par la 13. pr. 1. & par la construction ADB est egal à GDE : donc l'angle I sera egal à l'angle EDF. Par mesme discours l'angle K se trouuera egal à l'angle DFE. Et par la 32. p. 1. le troisiesme L sera egal au troisiesme E : ainsi le triangle circonscrit IKL sera equiangle au triangle donné DEF: Parquoy nous auons fait ce qui estoit requis.



4. Excircles

VISION

Figure :



Finition : ABC a triangle,
Ba, *Bb*, *Bc* the A, B, C-internal bisectors of ABC,
B'a, *B'b*, *B'c* the A, B, C-external bisectors of ABC,
Ia, Ib, Ic the points of concurs of *Ba*, *B'b* and *B'c*,
B'a, *Bb* and *B'c*,
B'a, *B'b* and *Bc*,
A", B", C" the feet of the perpendicular through Ia, Ib, Ic wrt BC, CA, AB
and *1a*, *1b*, *1c* the circle centered at Ia, Ib, Ic and passing resp. through A", B", C".

Definition : *1a*, *1b*, *1c* are "the A, B, C-excircles of ABC".

Historic note : the name of "excircle" was introduced in 1812 by Simon-Antoine L'Huillier, professor of the Academy of Geneva (Switzerland).

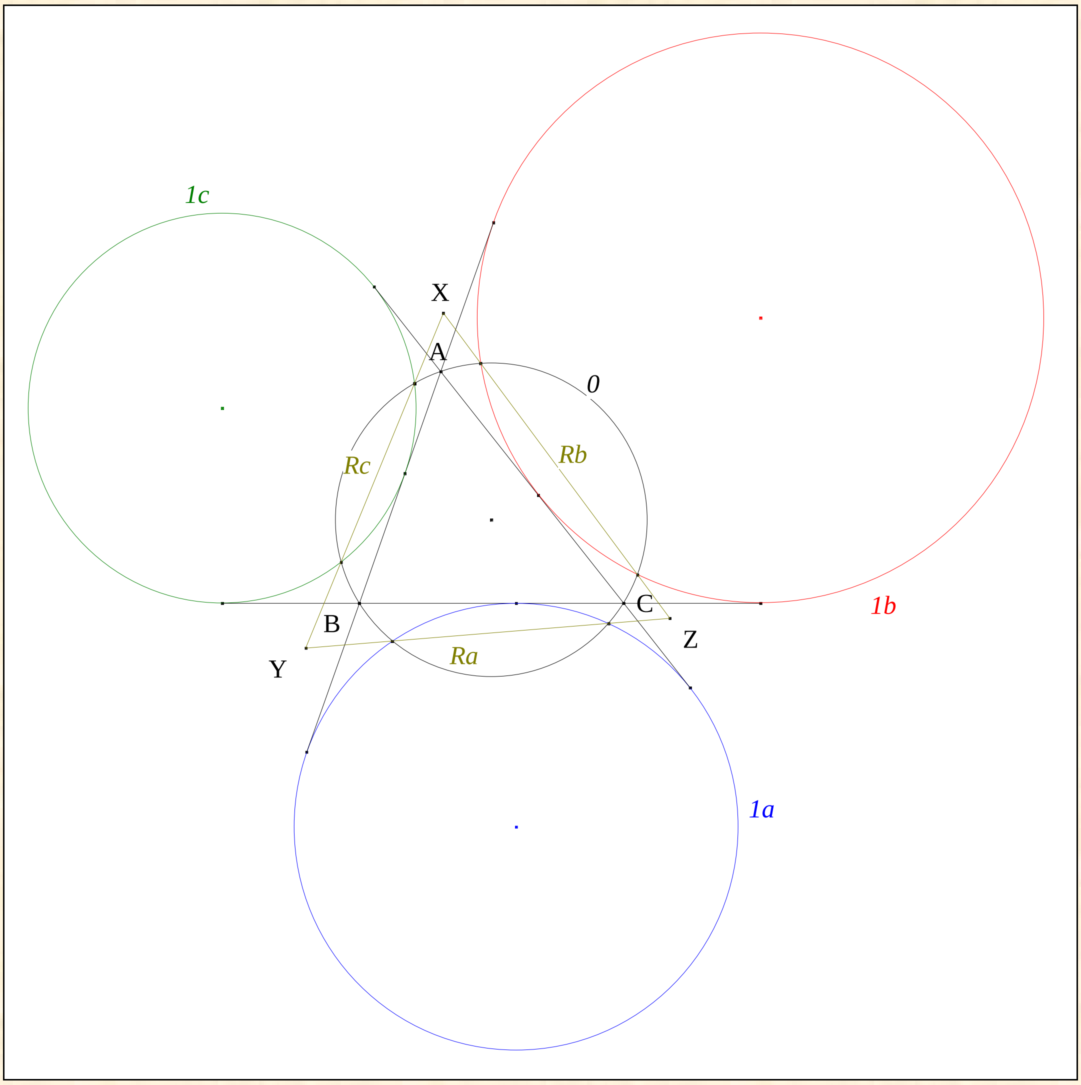
§. VII.

Ce qui vient d'être développé, sur le cercle circonscrit et sur le cercle *inscrit* à un triangle, peut être appliqué, avec de légers changemens, au cercle circonscrit et à l'un des trois cercles *exinscrits* à ce même triangle, savoir : à un cercle qui touche un des côtés du triangle extérieurement, et les prolongemens des deux autres côtés (Voyez mes *Élémens d'analyse*, etc, §. 131.).[3]

5. Ayme's triangle

VISION

Figure :



Finition : ABC a triangle,

[3] L'Huillier S.A., *Annales de Gergonne*, tome **1** p. 156 (1812) ; http://www.numdam.org/numdam-bin/feuilleter?j=AMPA.

	0	the circumcircle of ABC,
	1a, *1b*, *1c*	the A, B, C-excircles of ABC,
	Ra, *Rb*, *Rc*	the radical axis of *0* wrt *1a*, *1b*, *1c*
and	X, Y, Z	the points of intersection of *Rb* and *Rc*, *Rc* and *Ra*, *Ra* and *Rb*.

Definition : XYZ is "the Ayme's triangle of ABC".

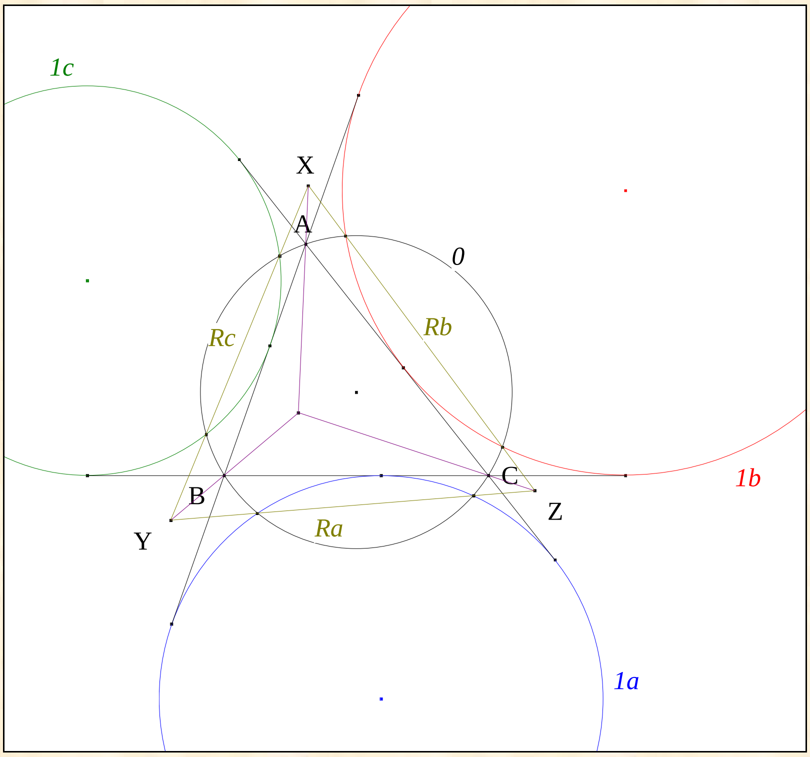
Historic note : this name has been given by Clark Kimberling in ETC [4].
Peter Moses found coordinates of the A-vertex of the Ayme's triangle.

B. THE FIRST AYME-MOSES PERSPECTOR

1. Paul Yiu

VISION

Figure :

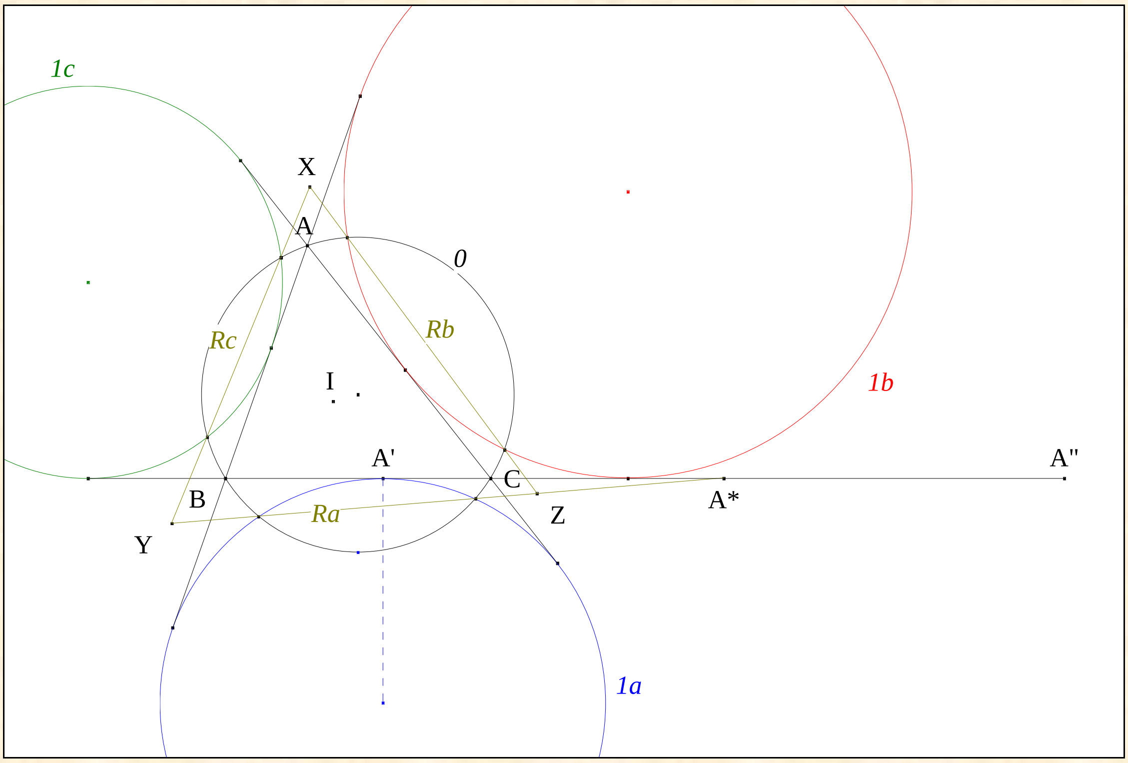


Features : the hypothesis and notations are the same as above.

Given : XYZ is perspective with ABC. [5]

VISUALISATION [6]

4 Kimberling C., Encyclopedia of Triangle Centers ; http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.
5 Yiu P., *The Clawson point and excircles*, Theorem **1**.
6 Of the author.

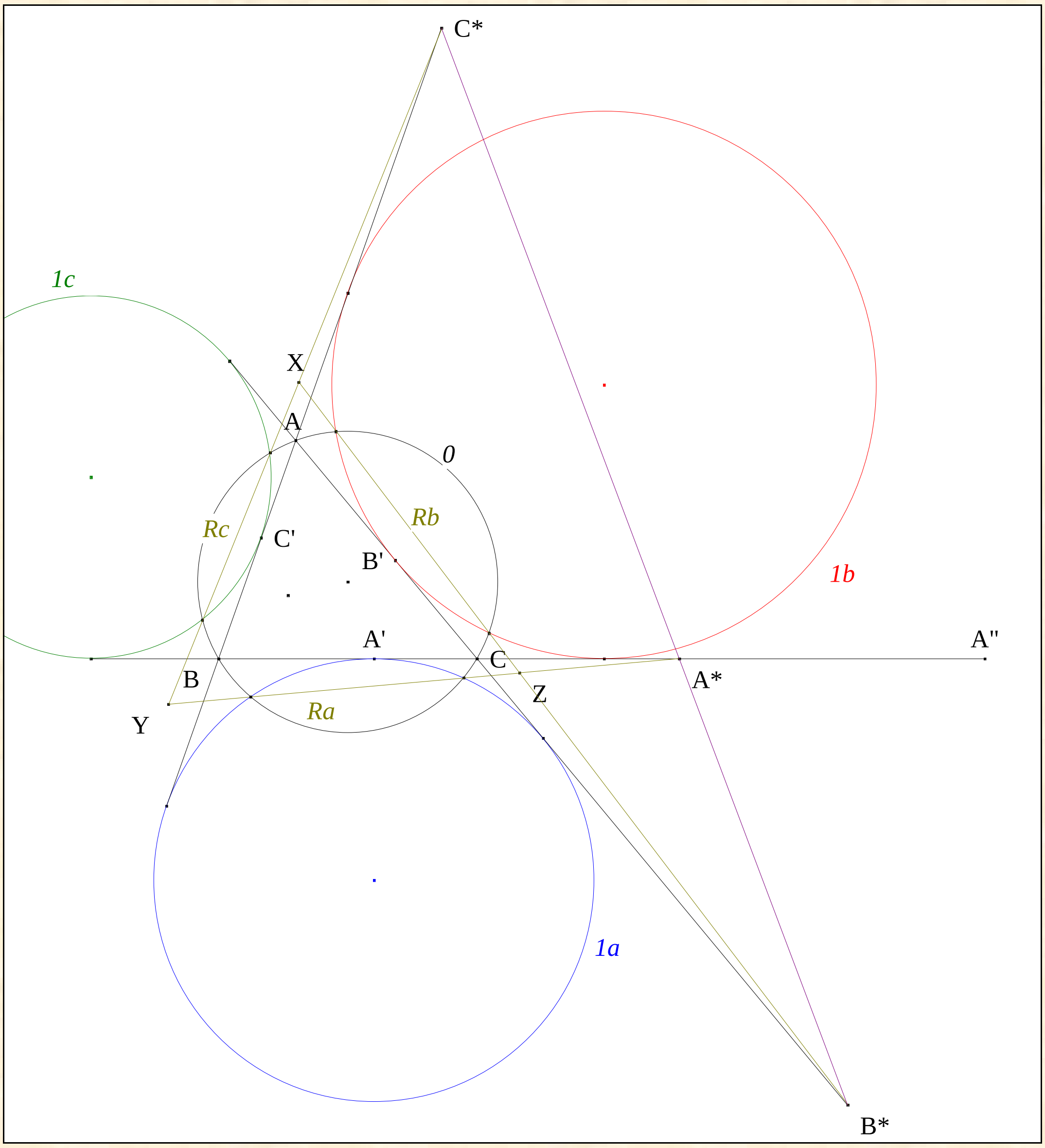


- Note A* the point of intersection of *Ra* and BC.
  A' the point of contact de *1a* with BC
  and A" the symmetric of A' wrt A*.

- **Remarks :** **(1)** A* is the midpoint of the segment A'A"
  (2) A* is situated on the radical axis *Ra* of *0* and *1a*.

- According Gaultier "Radical axis of two intersecting circles", A*A'^{2} = A*B . A*C.

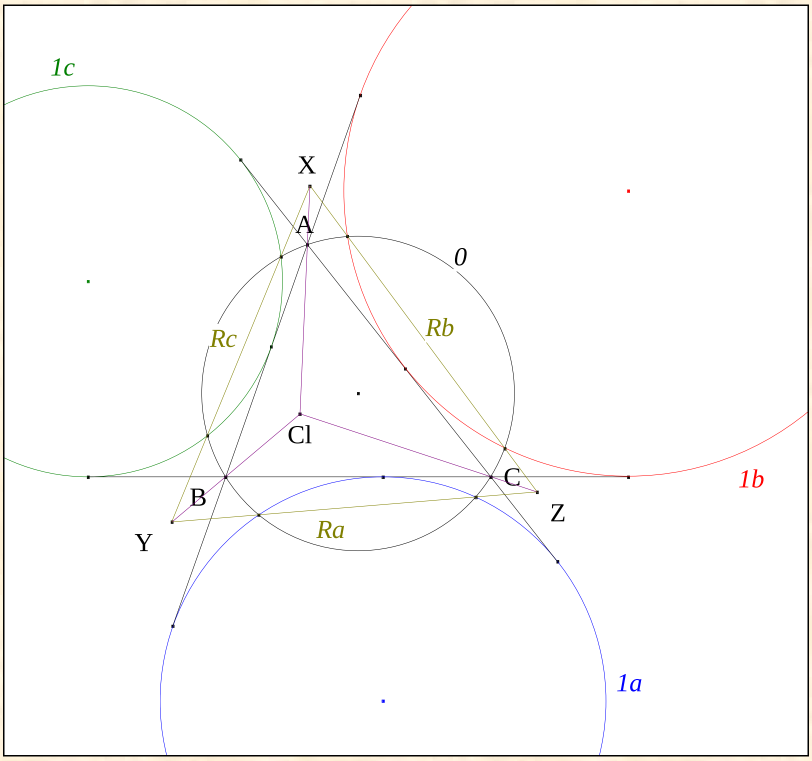
- **Partial conclusion :** according to "MacLaurin's relation" (Cf. Annex **1**),
  the quaterne (B, C, A', A") is harmonic.



- Note B', C' the points of contact resp. of *1b* with CA, *1c* with AB,
  B*, C* the points of intersection resp. of *Rb* and CA, *Rc* and AB,
  and B", C" the symmetrics of B' wrt B*, C' wrt C*.

- Mutatis mutandis, we would prove that the quaterne (C, A, B', B") is harmonic
  the quaterne (A, B, C', C") is harmonic.

- According to "Three collinear midpoints" (Cf. Appendix **1**), A*, B* and C* are collinear.



- **Conclusion :** by definition, XYZ is perspective with ABC.

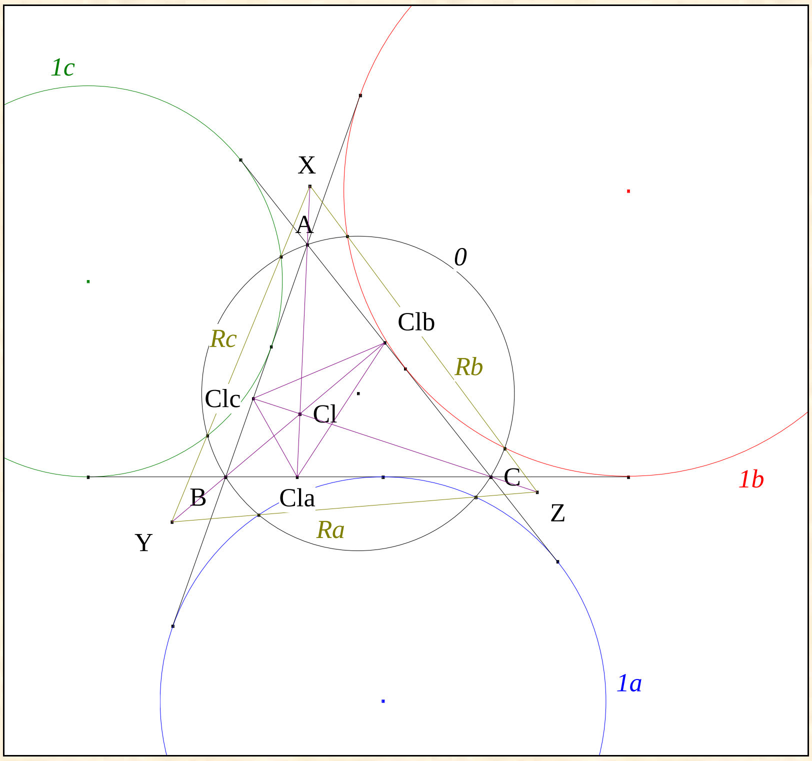
- Note Cl the center of this perspective.

Remarks : **(1)** according to Desargues "The theorem of the two triangles" [7], AX, BY and CZ are concurrent at Cl.

(2) Cl is the Clawson's point of ABC and is indexed under X_{19} by ETC. [8]

(3) The Cl-cevian triangle

[7] Ayme J.L., Une rêverie de Pappus, G.G.G. vol. **6** ; http://perso.orange.fr/jl.ayme.
[8] Kimberling C., Encyclopedia of Triangle Centers ; http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.



- Note Cla Clb Clc the Cl-cevian triangle wrt ABC.

- **Conclusion :** the Cl-cevian triangle is perspective at the Clawson's point with the Ayme's triangle.

Historic note : in March 2000, Paul Yiu[9] wrote

Last December, I have written a short note "The Clawson point and excircles", where I found another simple characterization of the Clawson point :

The radical axes of the circumcircle with the three excircles of a triangle bound a triangle perspective with ABC*, the perspector being the Clawson point* X(19).

Comment : this result has been obtained by using trilinear coordinate.

2. Peter J. C. Moses

- He is asked the following question :

what are the points P for which the cevian triangle is perspective with the Ayme's triangle.

- By computer calculation, he found a cubic named the "Ayme-Moses cubic" which passes through the points X(i) for I = 1, 2, 19, 75, 279, 304, 346, 2184.

9 Yiu P., Clawson point (was Naturality), *Hyacinhos* message # 564 (03/21/2000) ; http://tech.groups.yahoo.com/group/Hyacinthos/ ;
Yiu P., An Introduction to the Geometry of the Triangle (2001) p. 68 ; http://math.fau.edu/Yiu/Geometry.html.

3. X(3610)

- The point X(346) is the isotomic of X(279).
  The point X(279) is the isogonal of X(220).
  The point X(220) is the X(9)-ceva conjugate of X(55)
  where
  (1) X(9) is the Mittenpunkt
  (2) X(55) is the internal center of the homothety between the incircle and the circumcircle.

- The point X(9)-ceva conjugate of X(55) is
  the center of perspective of the X(9) cevian triangle and the X(55) anticevian triangle.

- **Concluion :** the center of perspective of the X(346)-cevian triangle with the Ayme's triangle wrt ABC
  is named
  "the first Ayme-Moses perspector of ABC".
  Peter Moses found coordinates for this perspector.

4. Archive

X(3610) = 1st AYME-MOSES PERSPECTOR

Trilinears f(a,b,c) : f(b,c,a) : f(c,a,b), where $f(a,b,c) = bc(b + c)(a^2 - b^2 - c^2)(a^2 + b^2 + c^2 + 2bc)$
Barycentrics af(a,b,c) : bf(b,c,a) : cf(c,a,b)

In a Hyacinthos message dated January 10, 2011, Jean-Louis Ayme introduced a triangle as follows. Let R_a be the A-radical axis of the circumcircle and let O_a be the A-excircle. Let $T_a = R_a \cap O_a$, and define T_b and T_c cyclically. The Ayme triangle $T_aT_bT_c$ is perspective to triangle ABC and also perspective to many other triangles. Peter Moses found that its perspector with the cevian triangle of X(346) is X(3610). He also found that the A-vertex of the Ayme triangle has first barycentric as follows:

$- (b + c)(a^2 + b^2 + c^2 + 2bc) : b(a^2 + b^2 - c^2) : c(a^2 + b^2 + c^2)$,

from which the other two vertices are easily obtained. The Ayme triangle is perspective to ABC with perspector X(19).

Moses found that the locus of X such that the cevian triangle of X is perspective to the Ayme triangle is a cubic which passes through the points X(i) for i=1, 2, 19, 75, 279, 304, 346, 2184. A barycentric equation for this Ayme-Moses cubic follows:

(Cyclic sum of $ayz[by(a^2 + b^2 - c^2) - cz(a^2 - b^2 + c^2)]$) = 0.

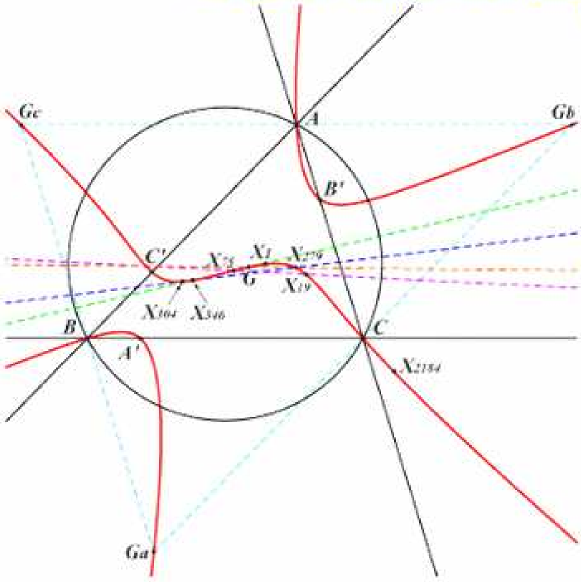
X(3610) lies on these lines:

[10]

[10] Kimberling C., Encyclopedia of Triangle Centers ; http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.

K605 Ayme-Moses cubic, pK(X2, X304)

Home page | Catalogue | Classes | Tables | Glossary | Notations | Links | Bibliography | Thanks | Downloads | Related Curves



Barycentric equation of the curve :

$$\sum_{\text{cyclic}} x^2 \left(\frac{S_B}{b}\, y - \frac{S_C}{c}\, z \right) = 0$$

$$\iff \sum_{\text{cyclic}} \left(\frac{S_B}{b}\, z - \frac{S_C}{c}\, y \right) yz = 0$$

Points on the curve :

X(1), X(2), X(19), X(75), X(279), X(304), X(346), X(2184)

vertices of the antimedial triangle

vertices of the cevian triangle of X(304)

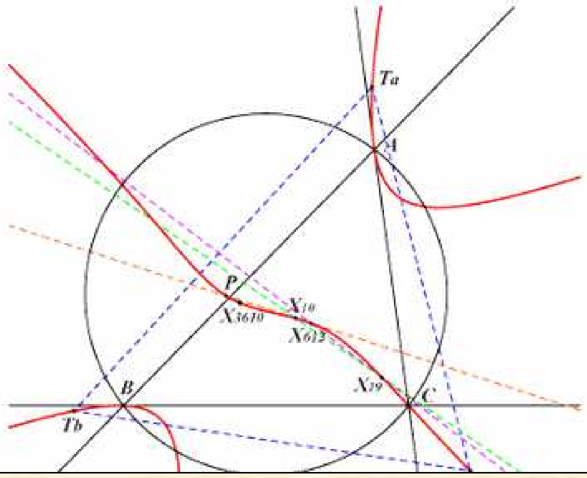
Geometric properties :

The radical triangle is the triangle TaTbTc formed by the radical axes of the circumcircle and the three excircles.

It is perspective to ABC at X(19) and to the cevian triangle of any point P that lies on the Ayme-Moses cubic K605 (Hyacinthos #19710).

K605 is the isotomic pivotal cubic with pivot X(304), the isotomic conjugate of the Clawson point X(19).

The locus of the perspectors is another pivotal cubic with pole X(10) x X(612), pivot the intersection P of the lines X(1)X(2) and X(19)X(346), passing through X(10), X(19), X(612), X(3610).



[11]

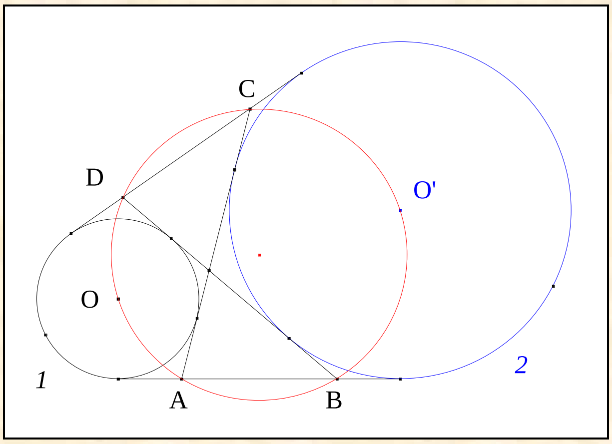
[11] http://bernard.gibert.pagesperso-orange.fr/Exemples/k605.html

C. THE SECOND AYME-MOSES PERSPECTOR

1. A lemma or the result of Jean-Ch. Dupain

VISION

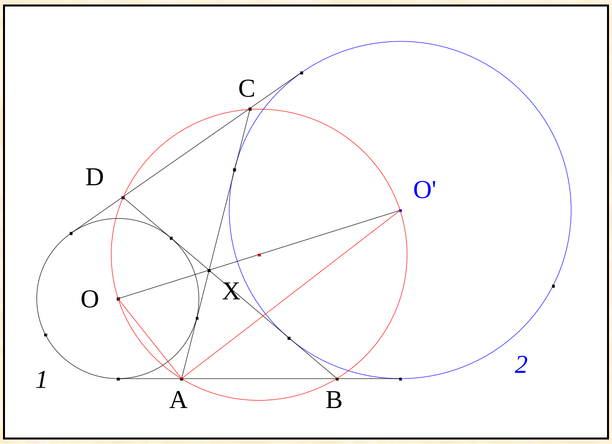
Figure :



Features : *1, 2*	two external circles from the other,
	O, O'	the centers of *1*, *2*,
et	A, B, C, D	the four points of intersection of the internal and external tangents as shown in figure.

Given : A, B, C, D, O and O' are on the circle with diameter OO'. [12]

VISUALIZATION

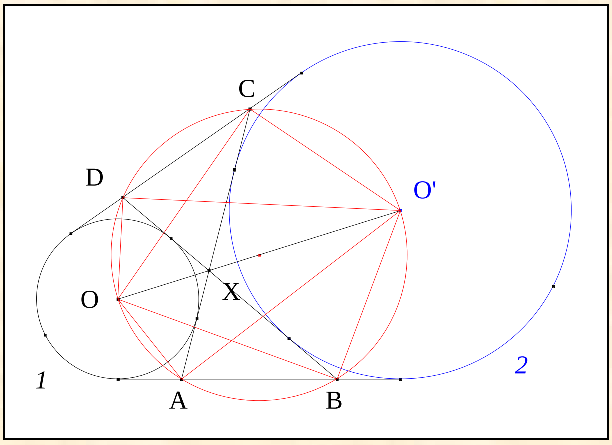


- Note X the point of concurs of AC, BD and OO'.

- **Remark :** wrt the triangle XAB, **(1)** AO' is the A-external bisector of XAB
  (2) AO is the A-internal bisector of XAB.

[12] Dupain J. Ch., Note sur les tangentes communes à deux cercles, *Nouvelles annales de mathématiques, journal des candidats aux écoles polytechnique et normale*, Sér. 2, **8** (1869) 458-459 ; http://www.numdam.org/numdam-bin/feuilleter?j=NAM&sl=0.

- **Partial conclusion :** the triangle AOO' is A-right angle.



- Mutatis mutandis, we would prove that the triangle BOO' is B-right angle
the triangle COO' is C-right angle
the triangle DOO' is D-right angle.

- **Conclusion :** according to Thalès "Right triangle inscriptible in a half circle",
A, B, C and D are on the circle with diameter OO'.

Theorem : two circles have for centers O and O' ;
the circle with diameter OO' passes through the four points of intersection of the internal and external tangents.

2. Archive

NOTE SUR LES TANGENTES COMMUNES A DEUX CERCLES;

PAR M. J.-CH. DUPAIN.

Les Traités de Géométrie qui donnent la construction des tangentes communes à deux cercles négligent ordinairement une vérification très-simple.

(*) *Voir*, par exemple, le *Cours d'Analyse* de STURM.

(459)

Soient O, C les centres des deux cercles, A l'intersection d'une tangente intérieure et d'une tangente extérieure; il est visible que AO et AC sont bissectrices des angles de ces tangentes; d'où il résulte que l'angle OAC est droit, et que la circonférence décrite sur OC comme diamètre passe par les quatre points où les tangentes intérieures rencontrent les tangentes extérieures.

3. Historic note

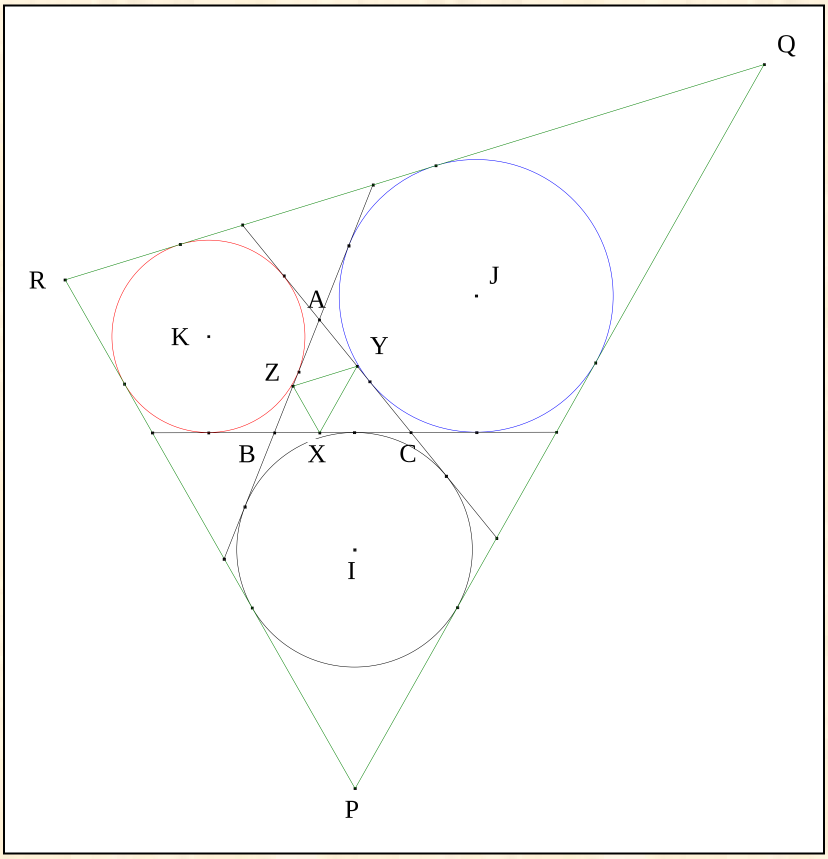
this result of the "agrégé" professor in lycée and former student of the "École normale" (promotion 1848 S) Jean Ch. Dupain, will be recovered by C. Reinhardt[13] in 1887.

4. The Clawson's point

VISION

Figure :

13 Reinhardt C., *Schlömilch* **32** (1887) 183.

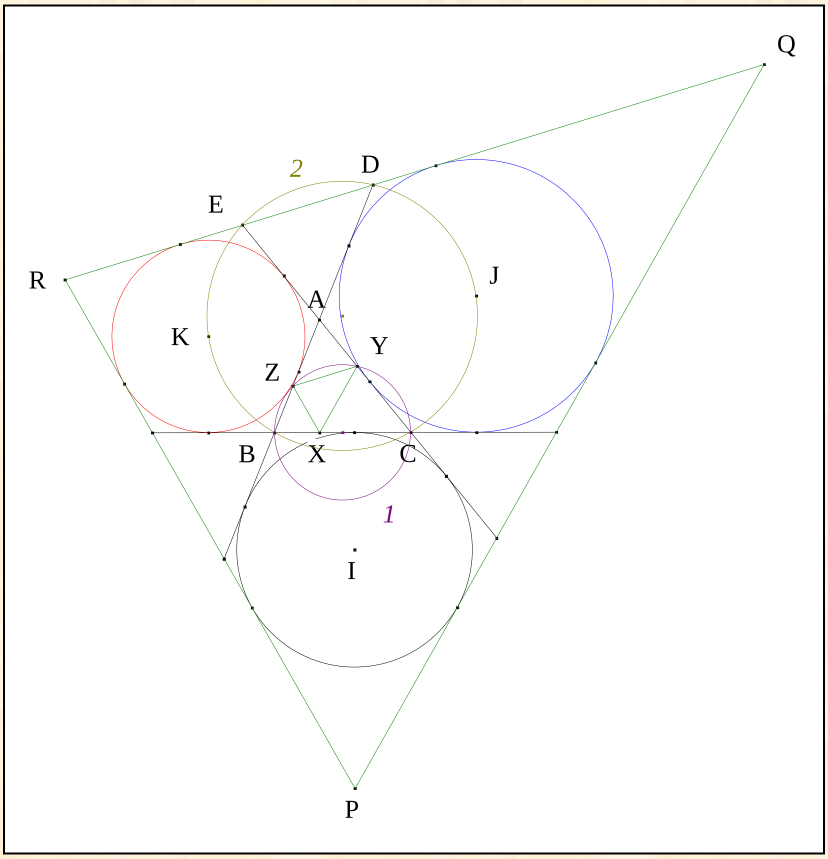


Features : ABC an acute triangle,
XYZ the orthic triangle of ABC,
and PQR the extangent triangle of ABC.

Given : PX, QY and RZ are concurrent.[14]

VISUALIZATION

[14] Clawson J. W., Points on the circumcircle, *American Mathematical Monthly* **32** (1925) 169-174
Clawson J. W. and Goldberg M., Problem **3132**, *American Mathematical Monthly* **32** (1926) 285 [Proposed 1925].



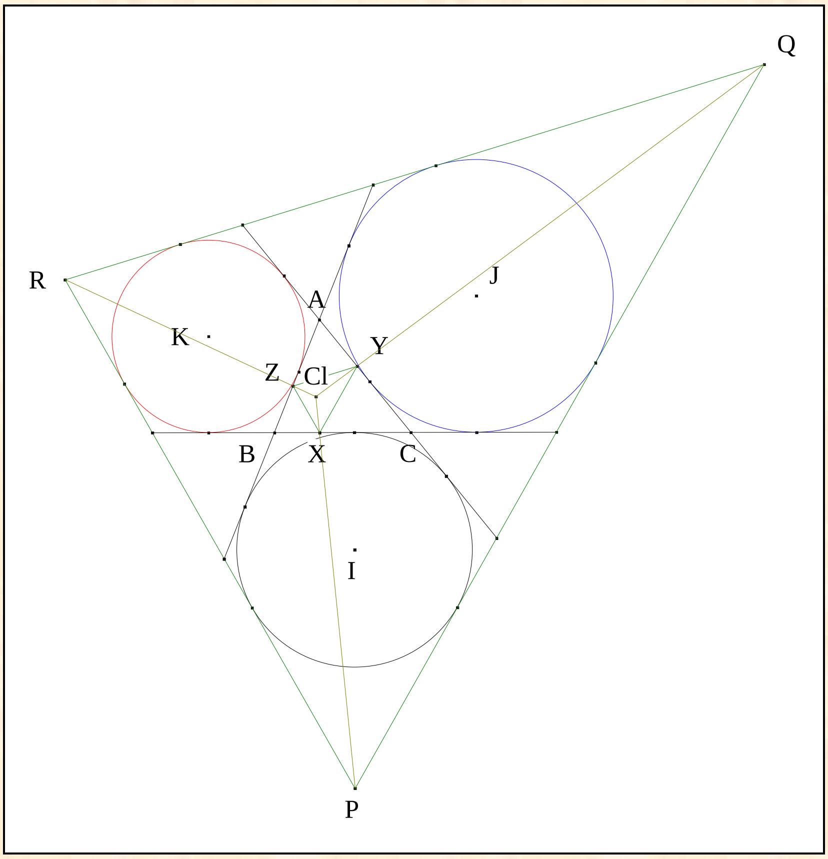
- Note *1* the circle with diameter BC ; it goes through Y and Z ;
  D, E the points of intersection of AB, AC with QR
  and *2* the circle with diameter JK.

- According to **C. 1.** A lemma, B, C, D and E are on *2*.

- The circles *1* and *2*, the basic points B and C, the monians YCE and ZBD,
  lead to the Reim's theorem **0** ;
  consequently YZ // ED i.e. YZ // QR.

- Mutatis mutandis, we would prove that ZX // RP and XY // PQ.

- **Partial conclusion :** PQR and XYZ are homothetic.



- **Conclusion :** XYZ and PQR being not equal,
  according to Desargues "The weak theorem" (Cf. Annexe 2)
  applied to XYZ and PQR, PX, QY and RZ are concurrent.

- Note Cl this point of concurs.

Remark : Cl is "the Clawson's point of ABC" [15] and is indexed under X_{19} by ETC.[16]

Comment : the author states that he has no synthetic proof to show that it is the same point from the Ayme triangle or from the extangent triangle. The problem remains open.

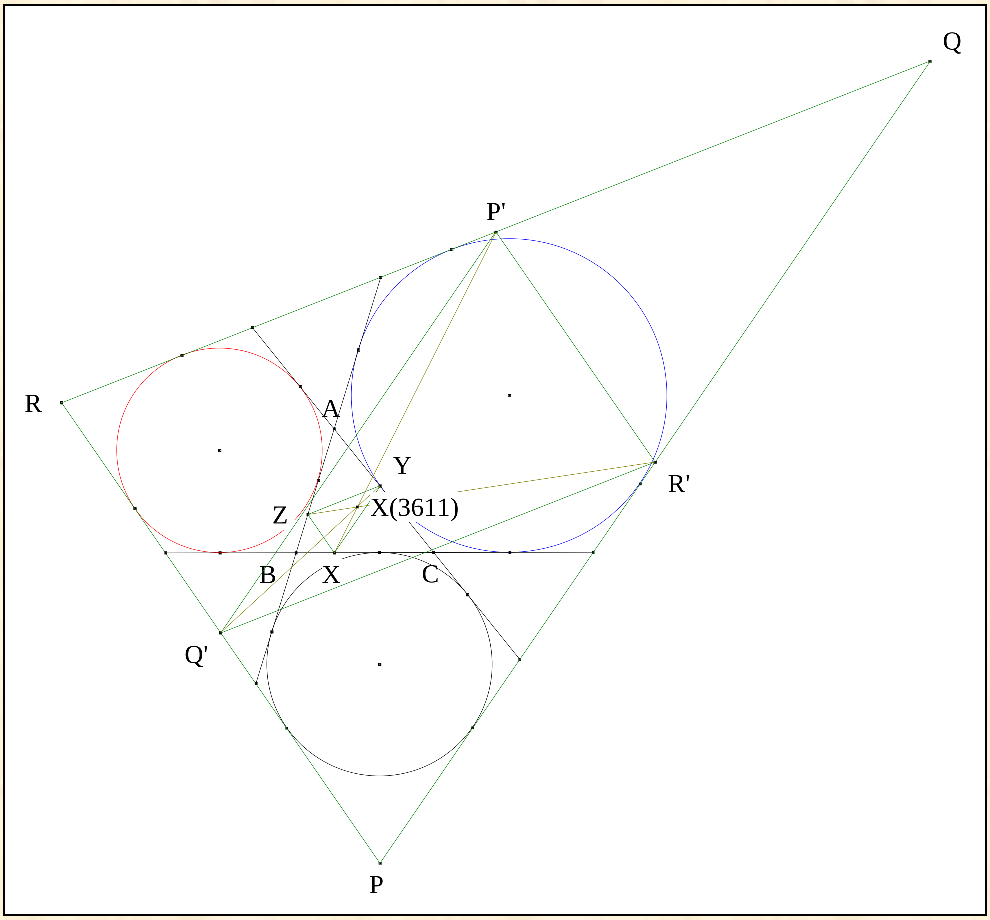
5. X(3611)

VISION

Figure :

[15] Clawson J. W., Points on the circumcircle, *American Mathematical Monthly* **32** (1925) 169-174
Clawson J. W. and Goldberg M., Problem 3132, *American Mathematical Monthly* **32** (1926) 285 [Proposed 1925].

[16] Kimberling C., Encyclopedia of Triangle Center ; http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.



Features : ABC an acute triangle,
XYZ the orthic triangle of ABC,
PQR the extangent triangle of ABC
and P'Q'R' the medial triangle of PQR

Given : P'X, Q'Y and R'Z are concurrent.[17]

VISUALIZATION

- **Conclusion :** P'Q'R' and XYZ being homothetic,
according to Desargues "The weak theorem" (Cf. Annexe **2**)
applied to P'Q'R' and XYZ, P'X, Q'Y and R'Z are concurrent.

Remark : this point of concurs is "the second Ayme-Moses perspector"
and is indexed under X_{3611} by ETC.[18]
Peter Moses found coordinates for this perspector.

17

18 Kimberling C., Encyclopedia of Triangle Center ; http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.

6. Archive

X(3611) = 2nd AYME-MOSES PERSPECTOR

Trilinears f(a,b,c) : f(b,c,a) : f(c,a,b), where $f(a,b,c) = a(b + c) (b^2 + c^2 - a^2)[a^3(b + c) + (b - c)^2(b^2 + c^2 - a^2 - ab - ac]$
Barycentrics af(a,b,c) : bf(b,c,a) : cf(c,a,b)

In a Hyacinthos message dated January 7, 2011, Jean-Louis Ayme noted that the orthic triangle of ABC is perspective to the medial triangle of the extangents triangle of ABC. Peter Moses found coordinates for the perspector, X(3612).

X(3611) lies on these lines:
19,51 25,3197 40,185 42,1409 55,184 65,225 72,228 209,3198 511,3101 1899,2550

[19]

D. ABOUT THE ALLOCATION OF NAMES

Without risk to mislead us seriously, we can say that to early XIX century, around two thousand geometers have contributed to the development of Geometry. If some of them remained still present in our manuals with a remarkable result, many have fallen into oblivion.

Geometry under the aspect of an admirable construction made "man by man", contains so many results without reference to a geometer, that a method becomes necessary to retain the greatest number.
Today, an approach consists to find the name of their author if the archive, often scattered, permit it.
Also, the geometer to be grace again to pronounce the name of a forgotten geometer, will be not only revived in the mind of the listener, the figure which haunts him, but also enable it to act on the figure in developing the creative act in a certain way. In the contrary case, he will give a secular name what is not always ideal.

Start with the example of the "orthocenter" of a triangle.
The first to regard this is Archimedes of Syracuse in its **5** and **12** of his book titled *Scholia lemmas*.
Formerly, this remarkable point was with the name of its author and, until in 1865, where Besant and Ferrers called it "orthocenter".
Include the theorem of the "Six segments" that are attributed to Ptolemy to the XIX century before restore the name of its author, Menelaus.
Talk about the so-called "Simson's line". Although the Edinburgh historian, John Sturgeon Mackay[20] did found no trace in his works, he showed that this error paternity came from the French geometer François Joseph Servois in 1814 wrote

the following theorem, which is, I believe, of Simson...[21].

This error would be resumed by Jean-Victor Poncelet[22], omitting the note of F. J. Servois, which would definitely perpetuate this fact. It is in 1799 that William Wallace[23] had discovered "this line", well after the death of Robert Simson in 1768, in Glasgow.
Finally, discussing the geometric centre X_{19} which first bore the name of "focal point" in 1925, "crucial" in 1982, "Lyness" (1983), and then "Clawson" to see, finally, that Émile Lemoine[24] had already studied it in 1886.

A proposed name today may tomorrow be replaced by another.

Issue an idea, it is also awaken a latent energy, see creating a being, or rather to call it to a degree of reality. Issued by a person, an idea becomes present and can be captured by all being sensitive to the vibrations of the universal unconscious...

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19 Kimberling C., Encyclopedia of Triangle Centers ; http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.
20 MacKay J. S., *Proceedings Edinburgh Math Soc.*, (1890-1891) 83.
21 Servois F. J., *Annales de Gergonne* **4** (1813-14) 250-251.
22 Poncelet J. V., *Traité des propriétés projectives des figures* (1822).
23 Wallace W. (1768-1843), *Leybourne's* mathem. repository (old series) **2** (1798) 111.
24 Lemoine E., Quelques questions se rapportant à l'étude des antiparallèles des côtés d'un triangle, Bulletin de la S. M. F., tome **14** (1886) 107-128 et plus précisément à la page 114.

Under this point of view, we can better understand that most of the discoveries can be claimed by at least two researchers.
This was the case of Japanese Kariya who find again in 1904, a point already considered by Lemoine in 1896 and spare no effort to attribute to him this point.
Recall of brother Gabriel-Marie reflection in his book, F. G-M... :

être devancé dans la découverte d'un théorème particulier,
que les premiers auteurs ont rencontré et présenté comme question isolée,
n'ôte point le mérite de ceux qui rencontrent ultérieurement la même question,
qui la creusent, qui la développent, la complètent,
et en font le simple point de départ d'une étude importante et bien originale.

In refusing this research paternity of geometers as Adrian Oldknow awarded without complex in 1995 his own name at a point and in points known or little-known, the names of his knowledge as Fletcher, Nobbs, Griffiths, Rigby recognizing it as follows[25]

We know have the small matter of the 10 *points* O, O', K, K', M, M', D', E', F' *and* T.
If they haven't *already been claimed I would like to offer T as the Fletcher point,* D', E', F' *as the Nobbs points,*
M, M' *as the Griffiths points and* K, K' *as the inner and outer Rigby points.*
Which leaves O *and* O'*-modesty forbids- or does it* ?

This approach was taken over in 2006 by Deko Dekov, which gives the names of three pioneers of Bulgarian mathematics to three centers.
Some geometers gave the name of their University such as Exeter or their high school as of Steinbart at one point.
To remedy all these excesses, the geometers of the ex-USSR called not the results by the name of their discoverer except that of Euler, because they were regarded as simple results and that their discoverer were not Soviet.
Beyond all these aspects, Clark Kimberling and Edward Brisse developed the foundations of two classifications by assigning to each centre a rating and ranking.

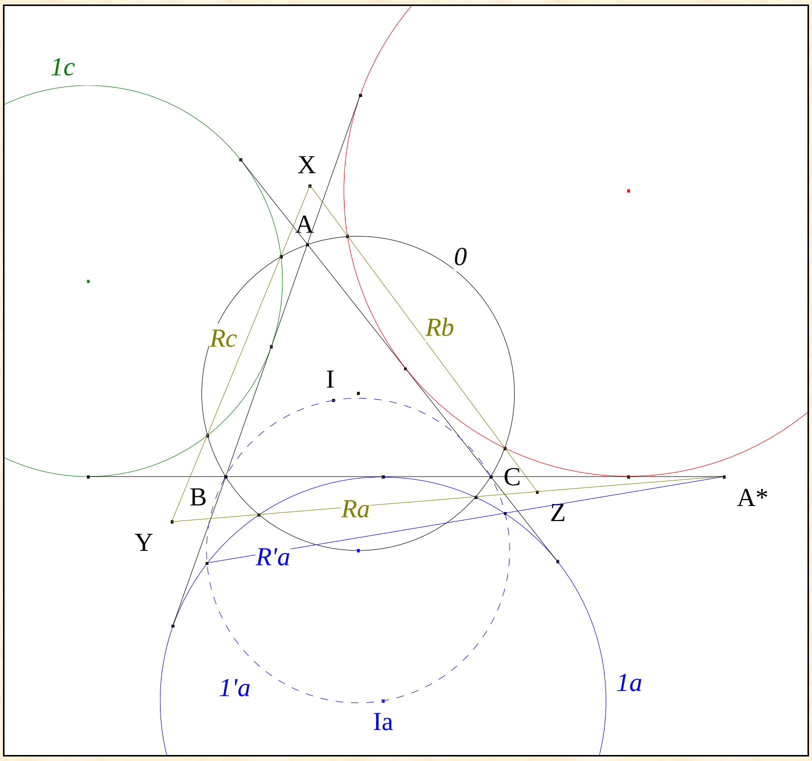
[25] Oldknox A., Computer Aided research into Triangle Geometry, *Mathematical Gazette* (1995 ou 1996) 263-273, 272.

E. AYME's RESULTS

1. With a Mention's circle

VISION

Figure :

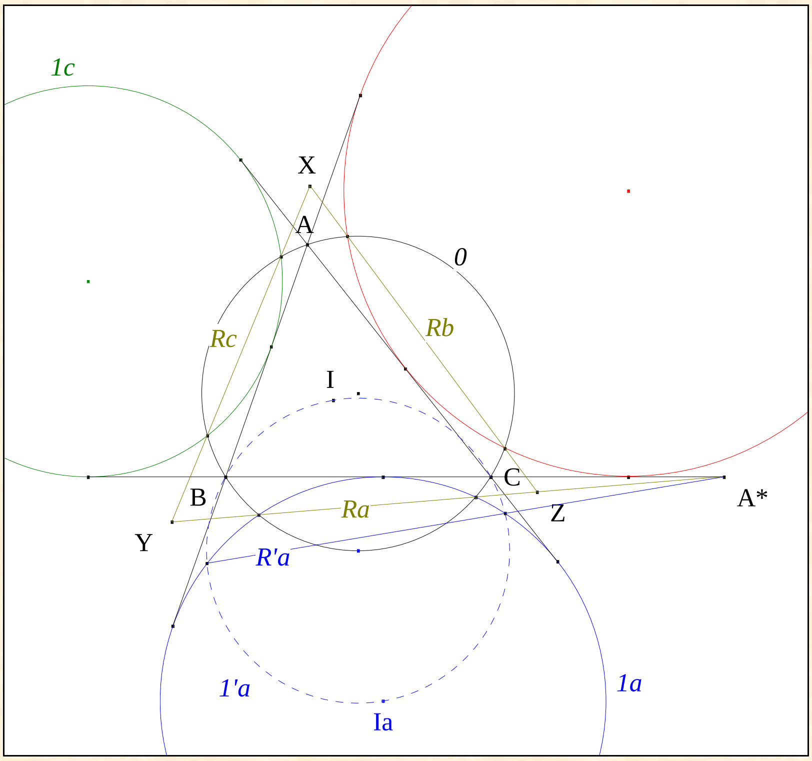


Features : to the hypothesis and notations, we add
1'a the A-Mention's circle of ABC (Cf. Annex **3**)
and *R'a* the radical axis of *1a* and *1'a*.

Given : *R'a* goes through A*.

VISUALISATION [26]

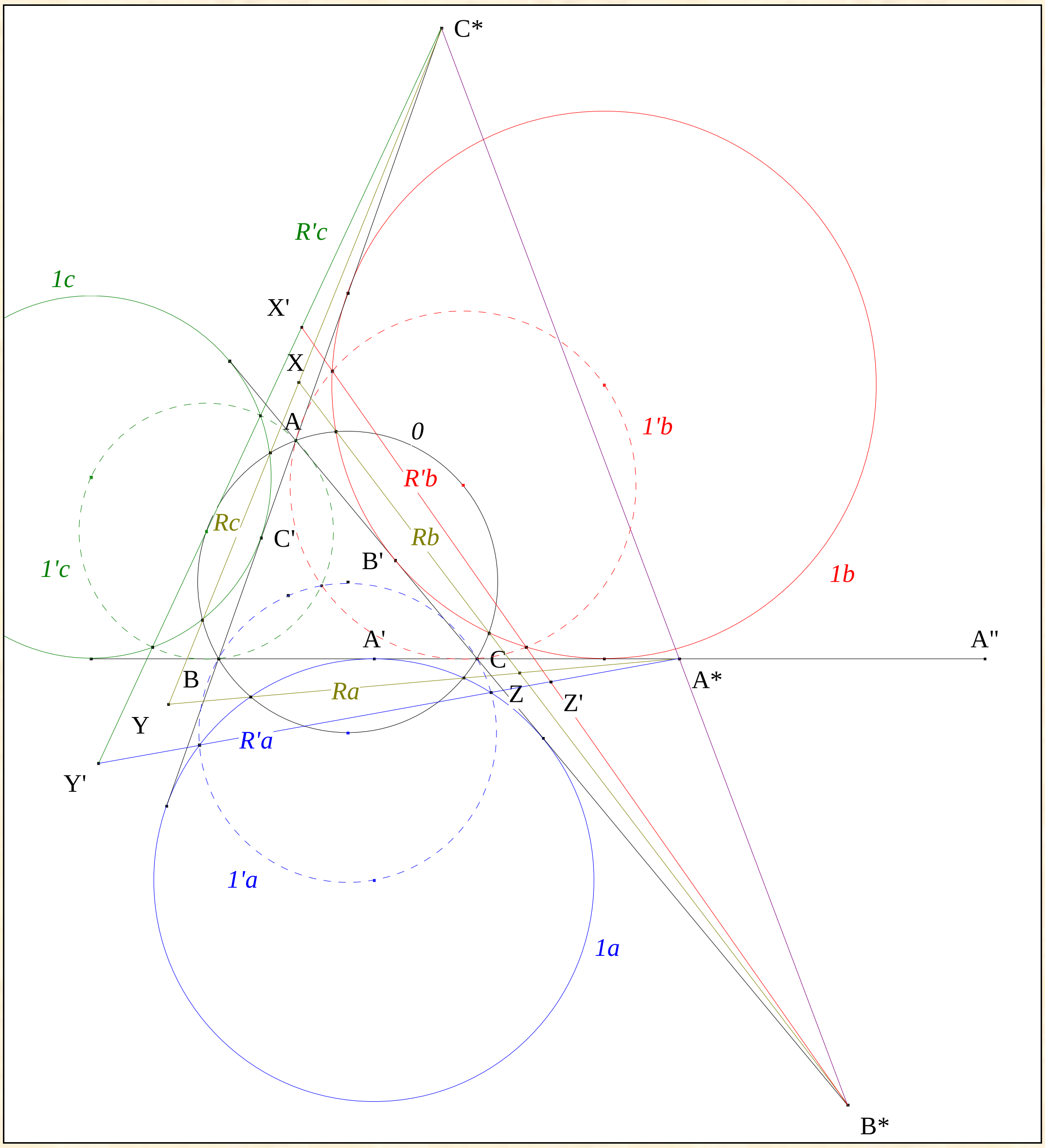
[26] Of the author.



- **Conclusion :** according to Monge "The theorem of the three chords" [27] applied to *0*, *1a* and *1'a*, *R'a* goes through A*.

Remark : another triangle perspective with the Ayme's triangle XYZ

[27] Ayme J.-L., Le théorème des trois cordes, G.G.G. vol. **6** ; http://perso.orange.fr/jl.ayme.



- Note $\quad$ *1'b*, *1'c* $\quad$ the B, C-Mention's circles of ABC
  R'b, *R'c* $\quad$ the radical axis of *1b* and *1'b*, *1c* and *1'c*,
  and $\quad$ X', Y', Z' $\quad$ the points of intersection of *R'b* and *R'c*, *R'c* and *R'a*, *R'a* and *R'b*.

- A*B*C* is the common perspectrix of the triangles ABC, XYZ and X'Y'Z'.

- **Conclusion :** according to Desargues "The theorem of the two triangles" [28],
  ABC, XYZ and X'Y'Z' are perspective. [29]

- Note $\quad$ U_{xa}, $U_{ax'}$, $U_{x'x}$ $\quad$ the perspectors of XYZ and ABC, ABC and X'Y'Z', X'Y'Z' and XYZ.

Remarks : $\quad$ **(1)** $\quad$ according to Casey "Three perpective triangles in pair" (Cf. Annex **4**),

U_{xa}, $U_{ax'}$ and $U_{x'x}$ are collinear.

(2) $\quad$ We know that $\quad$ U_{xa} is the Clawson's point X_{19} of ABC.

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[28] Ayme J.L., Une rêverie de Pappus, G.G.G. vol. **6** ; http://perso.orange.fr/jl.ayme.

[29] Ayme J.-L., Clawson 4, *Hyacinthos* message # 19658 (01/04/2011) ; http://tech.groups.yahoo.com/group/Hyacinthos/.

(3) $U_{ax'}$ is indexed under X_{3062} by ETC. [30]

(4) According to Francisco Javier Garcia Capitan[31], $U_{x'x}$ is not in ETC. [32]

(5) According to Peter J. C. Moses[33] $U_{x'x}$ is on the central lines

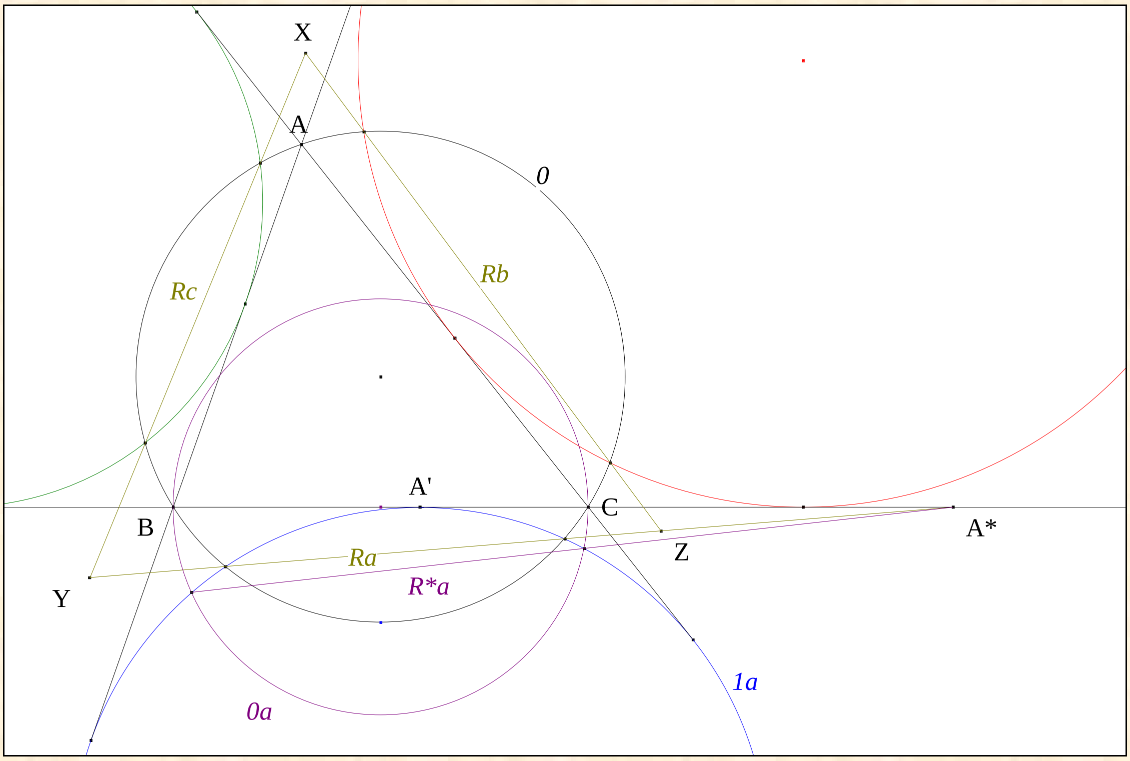
{8, 2893, 2897} , {10, 971} , {19, 3062} , {374, 1903} , {612, 1419}.

Historic note : the perspector X_{3062} (which is the isogonal of X_{165}) has been identified by Francisco Javier Garcia Capitan.

2. With a Thales circle

VISION

Figure :



Features : to the hypothesis and notations previous, we add

0a the A-Thales circle [34]

and *R*a* the radical axis of *0* and *0a*

Given : *R*a* goes through A*.

VISUALIZATION

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30 Ayme J.-L., Clawson 4, *Hyacinthos* message # 19658 (01/04/2011) ; http://tech.groups.yahoo.com/group/Hyacinthos/.

31 Garcia Capitan F. J ., Clawson 5, *Hyacinthos* message # 19662 (01/04/2011) ; http://tech.groups.yahoo.com/group/Hyacinthos/.

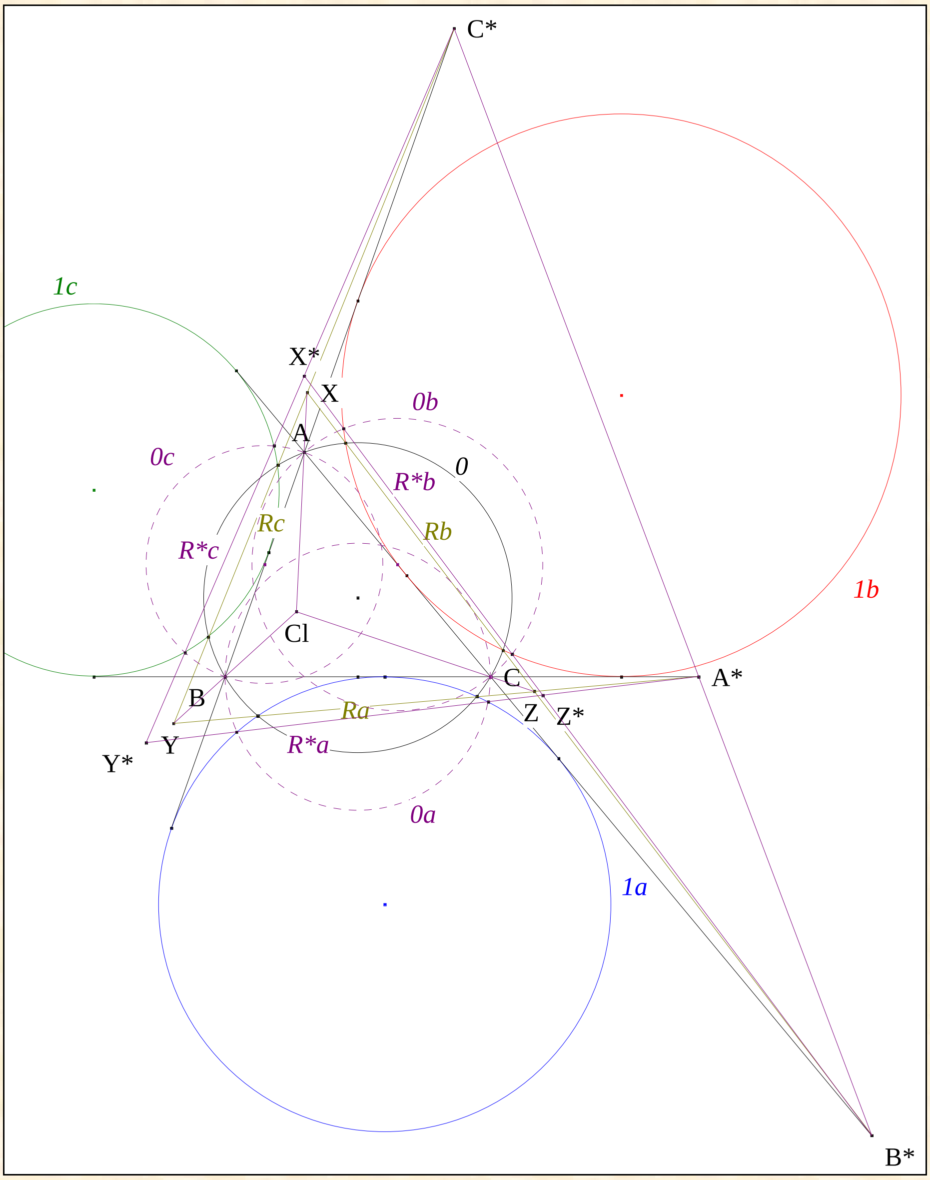
32 Ayme J.-L., Clawson 5, *Hyacinthos* message # 19661 and 19663 (01/04/2011) ; http://tech.groups.yahoo.com/group/Hyacinthos/.

33 Moses P. J. C., Clawson 5, *Hyacinthos* message # 19664 (01/04/2011) ; http://tech.groups.yahoo.com/group/Hyacinthos/.

34 The circle with diameter BC.

- **Conclusion :** according to Monge "The theorem of the three chords" [35] applied to *0*, *1a* and *0a*, *R*a* goes through A*.

Remarks : **(1)** another triangle perspective with the Ayme's triangle XYZ



- Note *0b*, *0c* the B, C-Thales circles of ABC
  *R*b*, *R*c* the radical axis of *0* and *0b*, *0* and *0c*,
  and X*, Y*, Z* the points of intersection of *R*b* and *R*c*, *R*c* and *R*a*, *R*a* and *R*b*.

- A*B*C* is the common perspectrix of the triangles ABC, XYZ and X*Y*Z*.

- **Conclusion :** according to Desargues "The theorem of the two triangles" [36], ABC, XYZ and X*Y*Z* are perspective. [37]

[35] Ayme J.-L., Le théorème des trois cordes, G.G.G. vol. **6** ; http://perso.orange.fr/jl.ayme.
[36] Ayme J.L., Une rêverie de Pappus, G.G.G. vol. **6** ; http://perso.orange.fr/jl.ayme.
[37] Ayme J.-L., Clawson 6, *Hyacinthos* message # 19670 (01/05/2011) ; http://tech.groups.yahoo.com/group/Hyacinthos/.

(2) According to Casey "Three perpective triangles in pair" (Cf. Annex **4**), the two new perspectors are collinear with the Clawson's point.

3. A generalization

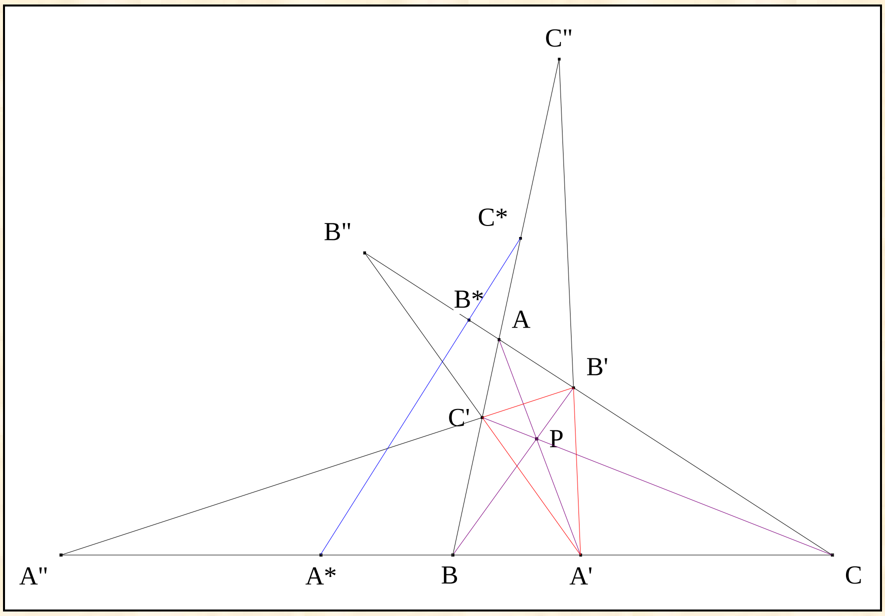
We can consider a triad of circles passing resp. through B and C, C and A, A and B which radical axis with the resp. A, B, C-excircles lead to a triangle in perspective with the Ayme's triangle.
Again, the two new perspectors are collinear with the Clawson's point.

F. APPENDIX

1. Three collinear midpoints

VISION

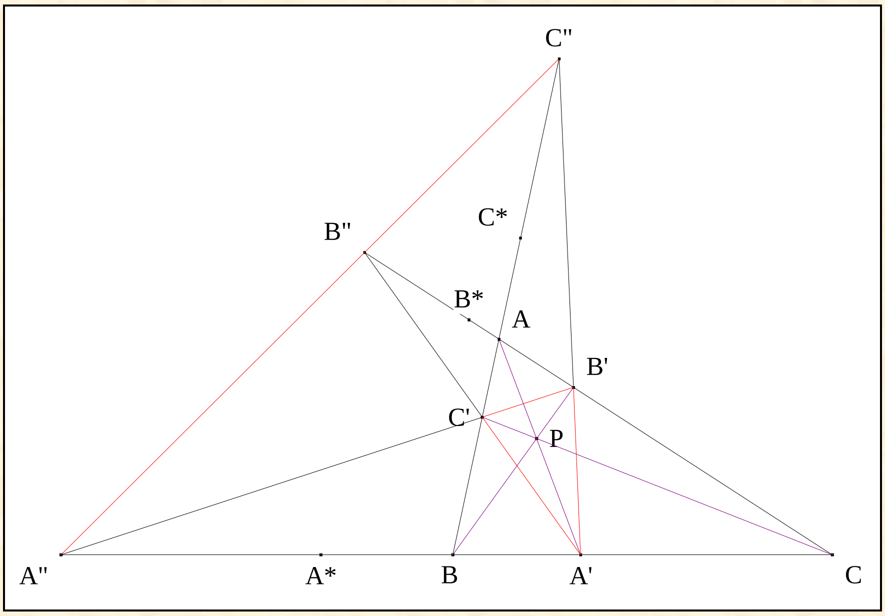
Figure :



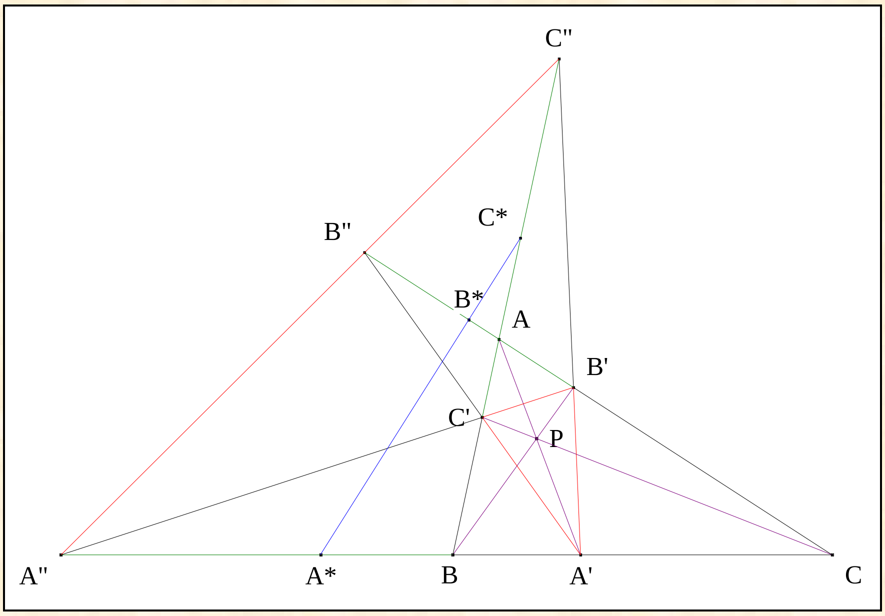
Features : ABC a triangle,
P a point,
A'B'C' the P-cevian triangle of ABC
A", B", C" the points of intersection of B'C' and BC, C'A' and CA, A'B' and AB
and A*, B*, C* the midpoints of the segments A'A", B'B", C'C".

Given : A*, B* et C* are collinear.

VISUALIZATION



- **Remark :** A"B"C" is the trilinear polar of P
  or
  the arguesian of the perspective triangle ABC and A'B'C'.



- **Conclusion :** according to "The Gauss line" [38]
  applied to the complete quadrilateral A’’B'A'B’’, A*, B* and C* are collinear.

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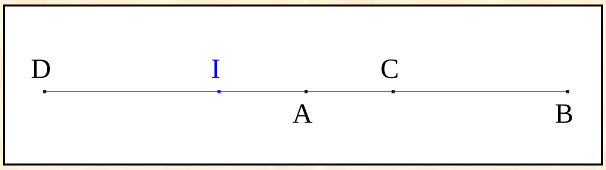
[38] Ayme J.-L., La droite de Newton, G.G.G. vol. **1** ; http://perso.orange.fr/jl.ayme.

G. ANNEX

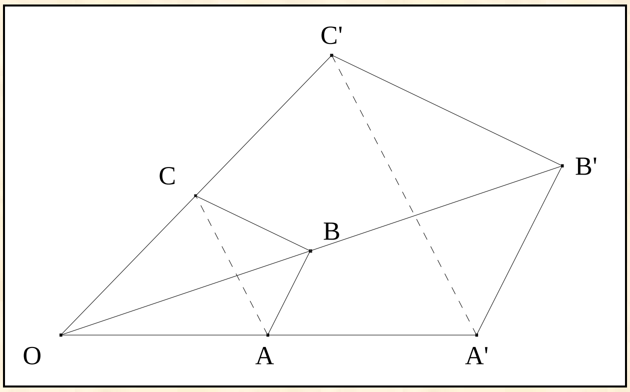
1. MacLaurin's relation [39]



Features : (A, B, C, D) a harmonic quaterne
and I the midpoint of the segment CD.

Given : $IC^2 = IA \cdot IB$

2. The Desargues weak theorem

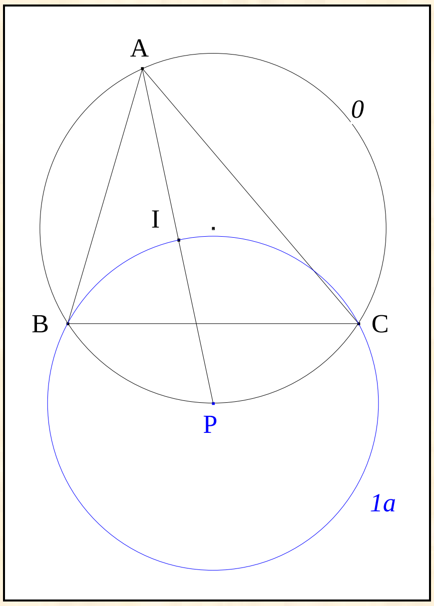


Features : ABC a triangle,
and A'B'C' a triangle so that **(1)** AA' and BB' concur at O
(2) AB parallel to A'B'
(3) BC parallel to B'C'.

Given : CC' goes through O *if, and only if,* AC is parallel to A'C'.

3. A Mention's circle

[39] MacLaurin C..

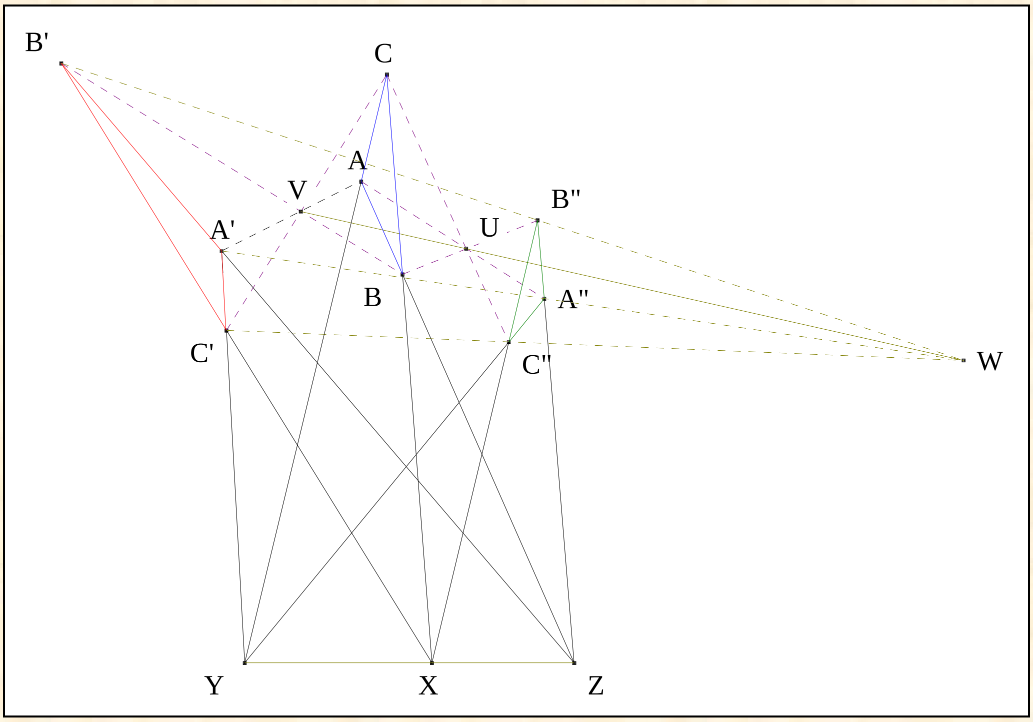


Features :	ABC	a triangle,
	0	the circumcircle of ABC,
	I	the incenter of ABC,
	P	the second point of intersection of AI with *0*
and	*1a*	the circle centered at P through B and C.

Given : *1a* goes through I.

Definition : *1a* is "the A-Mention's circle of ABC".

4. Three perpective triangles in pair [40]



[40] Casey J., A sequel to Euclid, propositions **13** (1881) 77.

Features :	ABC, A'B'C', A"B"C"	three perspective triangles in pair,	
	U, V, W	the perspectors of ABC and A"B"C", ABC and A'B'C', A'B'C' and A"B"C",	
and	X, Y, Z	the points of concurs of	BC, B'C' and B"C", CA, C'A' and C"A", AB, A'B' and A"B".

Given : *if*, X, Y and Z are collinear *then*, U, V and W are collinear.