AoPS Quarantine Geometry Olympiad 2020

Day 2

P4 Line ℓ intersects sides $\overline{AB}, \overline{AC}$ of $\triangle ABC$ at D, E. P, Q are the midpoints of $\overline{CD}$ and $\overline{BE}$ respectively. The lines through P, Q perpendicular to ℓ meet the perpendicular bisectors of $\overline{AC}$ and $\overline{AB}$ at M, N respectively. Prove that $\overline{MN} \parallel \overline{PQ}$.

P5 Let $\triangle ABC$ be a triangle and let the incircle meet $\overline{BC}, \overline{CA}, \overline{AB}$ at D, E, F respectively . Let $\overline{DI}$ intersect $\odot(ABC)$ at points X, Y such that X, I, D, Y are in this order. Let $\odot(XFE)$ intersect $\odot(ABC)$ at T and $\odot(DXT)$ intersect the incircle at K. Let $\overline{AX}$ intersect $\overline{BC}$ at M and $\overline{AY}$ intersect $\overline{BC}$ at N and let $\odot(AMN)$ intersect $\odot(ABC)$ at R. Then prove that A, K, R are collinear.

P6 Let $\triangle ABC$ be a triangle with orthocenter H and $\overline{BH}$ meet $\overline{AC}$ at E and $\overline{CH}$ meet $\overline{AB}$ at F. Let $\overline{EF}$ intersect the line through A parallel to $\overline{BC}$ at X and the tangent to $\odot(ABC)$ at A intersect $\overline{BC}$ at Y. Let $\overline{XY}$ intersect $\overline{AB}$ at P and let $\overline{XY}$ meet $\overline{AC}$ at Q. Let O be the circumcenter of $\triangle APQ$ and $\overline{AO}$ meet $\overline{BC}$ at T. Let V be the projection of H on $\overline{AT}$ and M be the midpoint of $\overline{BC}$. Then prove that $\odot(BHC)$ and $\odot(TVM)$ are tangent to each other.

Time: 4 hours and 30 minutes.
Each problem is worth 7 points.