Geometry

geometry geometric transformation reflection

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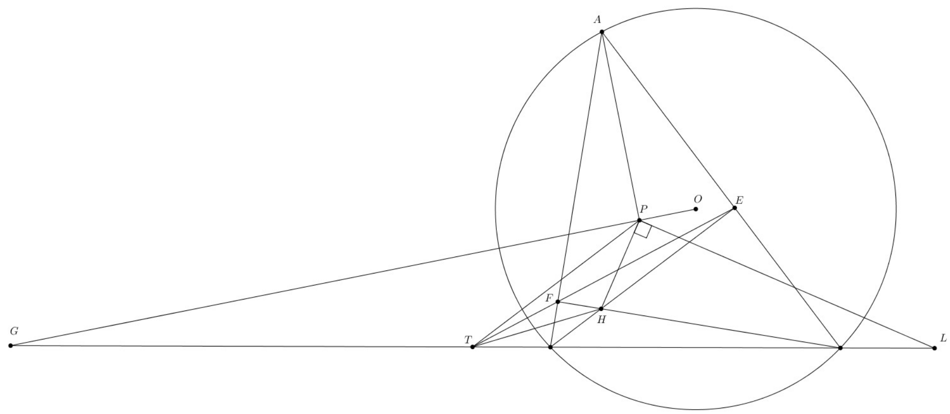
Source: Own

uraharakisuk...
230 posts

Yesterday at 11:21 PM • 7 #1

$\triangle ABC$ with altitudes BE, CF intersect at H. EF cuts BC at T. P is the projection of O on the $A-$ symmedian line. OP cuts BC at G. L reflects with G about T. Prove that $PL \perp PH$

Attachments:



tastymath750...
2930 posts

Today at 2:06 AM • 6 #3

very nice :)

Let D be the foot of the A-altitude, let B_1, C_1 be the midpoints of AC, AB, and let (BHC) meet AC, AB at B_2, C_2. It's well-known (and provable by angle-chasing) that E, F are the mipdoints of AB_2, AC_2. Let X be the projection of A onto OH and let M be the midpoint of BC.

First, we know that $G = AA \cap BC$, and $AA||EF$. Therefore, since $d(AA, EF) = d(EF, B_2C_2)$, we know that if B_2C_2 meets BC at a point L', then $T = EF \cap BC$ is the midpoint of $GL' \implies L' = L$.

By radical axis on $(AH), (AO)$, and the nine-point circle of ABC, we know that $L_1 = EF \cap B_1C_1$ lies on the radical axis of $(AH), (AO)$, which is obviously line AX. But a homothety centered at A with ratio 2 sends EF, B_1C_1 to B_2C_2, BC, so L_1 goes to L, implying that $AL \perp OH$. Remark that $HDXL, AOPX$ are cyclic with diameters HL, OA.

Now take an inversion with power $0.5bc$ centered at A followed by a reflection across the A-angle bisector, so that $D \iff O, P \iff M$. Then clearly $\angle HDP = \angle ADP = \angle AMO = \angle DAM = \angle OAP = \angle OXP$, so that $DHXP$ is cyclic, so $P \in (HL) \implies HP \perp PL$ as desired.

Today at 3:43 AM • 1 #4

rezareza14

98 posts

HARD AND NICE PROBLEM :)
Complex number can solve it but it is Long way. :D
very long! :|
BUT CAN KILL IT. ;)
I solved it by Complex number.
GOOD LUCK 8)

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zadaops
23 posts

Today at 4:00 AM #5

> *rezareza14 wrote:*
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> very long! :|
> BUT CAN KILL IT. ;)
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> GOOD LUCK 8)

Nice solution! :D

EulerMacaroni
666 posts

Today at 4:47 AM • 2 #6

666th post :)

First note that $G \equiv AA \cap BC$; I now claim that AL is perpendicular to the Euler line of $\triangle ABC$. To prove this, define X, Y as the midpoints of $\overline{AB}$, $\overline{AC}$, respectively. Note that the line between $FX \cap EY$ and $FE \cap XY$ is the polar of $FY \cap XE$ with respect to the nine-point circle of $\triangle ABC$ by Brokard's theorem, but by Pappus' theorem on $\overline{FXB}$ and $\overline{YEC}$, we have that $FY \cap XE$ lies on the Euler line of $\triangle ABC$, so the line between A and $FE \cap XY$ is perpendicular to OH. Let this line intersect BC again at L'; it is easy to see that the tangent to $\odot(ABC)$ at A is parallel to EF, so a dilation centered at L' with ratio $\frac{1}{2}$ maps G to T, implying $L' \equiv L$ and proving the claim.

To finish, let $Z \equiv AL \cap OH$; note that quadrilateral $AZPO$ is cyclic. Since A is the spiral center sending segment $\overline{OP}$ to $\overline{MD}$, where D is the foot of A on $\overline{BC}$, we have $\angle HDP = \angle ADP = \angle AMO = \angle OAP = \angle HZP$, so $P \in \odot(HZL)$, as desired.

This post has been edited 1 time. Last edited by EulerMacaroni, Today at 4:49 AM

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livetolove212
810 posts

Today at 3:25 PM • 1 #7

Nice observation from my problem :)
Let M, N be the midpoints of AH, BC. Altitude AD intersects OG at Q. AP intersects BC at K.
We have $\angle PQK = \angle PDK = \angle GAP = \angle AOG$ then $QK \parallel AO \parallel MN$.
Let MN cut EF at J then J is the midpoint of EF or $J \in AK$. We have $\angle DMJ = \angle DQK = \angle DPK$ then $MJPD$ is cyclic. This means D, P, J are belong to circle with diameter MT. We obtain $\angle MPT = 90^\circ$ then