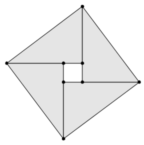
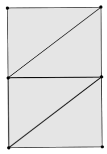
Problems of 2nd Iranian Geometry Olympiad 2015 (Elementary)

1. We have four wooden triangles with sides 3, 4, 5 centimeters. How many convex polygons can we make by all of these triangles?(Just draw the polygons without any proof)
A convex polygon is a polygon which all of it's angles are less than $180°$ and there isn't any hole in it. For example:



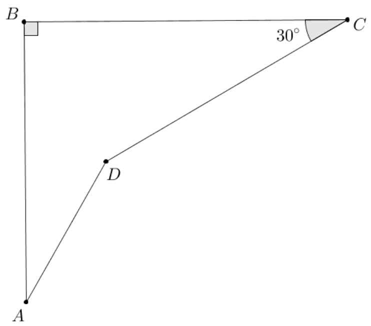
This polygon isn't convex



This polygon is convex

2. Let ABC be a triangle with $\angle A = 60°$. The points M, N, K lie on BC, AC, AB respectively such that $BK = KM = MN = NC$. If $AN = 2AK$, find the values of $\angle B$ and $\angle C$.

3. In the picture below, we know that $AB = CD$ and $BC = 2AD$. Prove that $\angle BAD = 30°$.



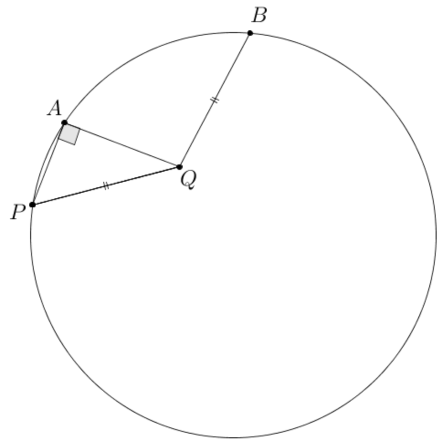
4. In rectangle $ABCD$, the points M, N, P, Q lie on AB, BC, CD, DA respectively such that the area of triangles AQM, BMN, CNP, DPQ are equal. Prove that the quadrilateral $MNPQ$ is parallelogram.

5. Do there exist 6 circles in the plane such that every circle passes through centers of exactly 3 other circles?

Time: 3 hours and 30 minutes
Each problem is worth 8 points

Problems of 2nd Iranian Geometry Olympiad 2015 (Medium)

1. In picture below, the points P, A, B lie on a circle. The point Q lies inside the circle such that $\angle PAQ = 90^\circ$ and $PQ = BQ$. Prove that the value of $\angle AQB - \angle PQA$ is equal to the arc AB.



2. In acute-angled triangle ABC, BH is the altitude of the vertex B. The points D and E are midpoints of AB and AC respectively. Suppose that F be the reflection of H with respect to ED. Prove that the line BF passes through circumcenter of ABC.

3. In triangle ABC, the points M, N, K are the midpoints of BC, CA, AB respectively. Let ω_B and ω_C be two semicircles with diameter AC and AB respectively, outside the triangle. Suppose that MK and MN intersect ω_C and ω_B at X and Y respectively. Let the tangents at X and Y to ω_C and ω_B respectively, intersect at Z. prove that $AZ \perp BC$.

4. Let ABC be an equilateral triangle with circumcircle ω and circumcenter O. Let P be the point on the arc BC. Tangent to ω at P intersects extensions of AB and AC at K and L respectively. Show that $\angle KOL > 90^\circ$.

5. a) Do there exist 5 circles in the plane such that every circle passes through centers of exactly 3 circles?

b) Do there exist 6 circles in the plane such that every circle passes through centers of exactly 3 circles?

Time: 4 hours and 30 minutes
Each problem is worth 8 points

Problems of 2nd Iranian Geometry Olympiad 2015 (Advanced)

1. Two circles ω_1 and ω_2 (with centers O_1 and O_2 respectively) intersect at A and B. The point X lies on ω_2. Let point Y be a point on ω_1 such that $\angle XBY = 90°$. Let X' be the second point of intersection of the line O_1X and ω_2 and K be the second point of intersection of $X'Y$ and ω_2. Prove that X is the midpoint of arc AK.

4. Let ABC be an equilateral triangle with circumcircle ω and circumcenter O. Let P be the point on the arc BC. Tangent to ω at P intersects extensions of AB and AC at K and L respectively. Show that $\angle KOL > 90°$.

3. Let H be the orthocenter of the triangle ABC. Let l_1 and l_2 be two lines passing through H and perpendicular to each other. l_1 intersects BC and extension of AB at D and Z respectively, and l_2 intersects BC and extension of AC at E and X respectively. Let Y be a point such that $YD \parallel AC$ and $YE \parallel AB$. Prove that X, Y, Z are collinear.

4. In triangle ABC, we draw the circle with center A and radius AB. This circle intersects AC at two points. Also we draw the circle with center A and radius AC and this circle intersects AB at two points. Denote these four points by A_1, A_2, A_3, A_4. Find the points B_1, B_2, B_3, B_4 and C_1, C_2, C_3, C_4 similarly. Suppose that these 12 points lie on two circles. Prove that the triangle ABC is isosceles.

5. Rectangles ABA_1B_2, BCB_1C_2, CAC_1A_2 lie otside triangle ABC. Let C' be a point such that $C'A_1 \perp A_1C_2$ and $C'B_2 \perp B_2C_1$. Points A' and B' are defined similarly. Prove that lines AA', BB', CC' concur.

Time: 4 hours and 30 minutes
Each problem is worth 8 points