L'AUTEUR CHEZ E.T.C.

LINKS

ENCYCLOPEDIA OF TRIANGLE CENTERS

Paul Yiu's Introduction to the Geometry of the Triangle and other works

Jean-Louis Ayme's Geometry * Géométrie * Geometria

1

1. Une courte présentation de l'auteur du site E.T.C.



Clark Kimberling est né le 7 novembre 1942 à Hinsdale (Illinois, États-Unis).

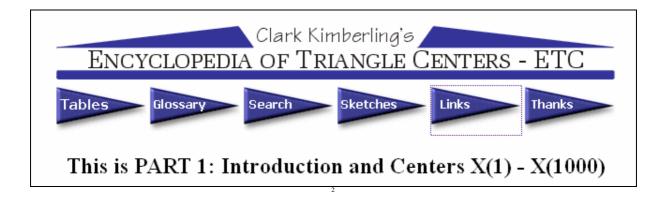
En 1970, il obtient un doctorat en mathématiques à l'Institut de technologie de l'Illinois et est, depuis, professeur de mathématiques à l'université privée méthodiste d'Evansville (Indiana).

Il est connu mondialement pour le site *Encyclopedia of Centers Triangle* qu'il a créé dans lequel il s'intéresse aux éléments remarquables du triangle.

Pour ses loisirs, il est membre du chœur de St. Paul l'Église épiscopale St-Paul d'Evansville, ainsi que compositeur en résidence. Ce musicien chevronné compose de nombreux solos et écrit de des articles pour *The Hymn*, un journal de la société de l'hymne édité aux États-Unis et au Canada.

http://faculty.evansville.edu/ck6/encyclopedia/links.html

2. La présence de l'auteur



X(346) = ISOTOMIC CONJUGATE OF X(279)

Trilinears $bc(b+c-a)^2: ca(c+a-b)^2: ab(a+b-c)^2$

 $= \cos(A/2) \csc^{3}(A/2) : \cos(B/2) \csc^{3}(B/2) : \cos(C/2) \csc^{3}(C/2)$

Barycentrics $(b + c - a)^2 : (c + a - b)^2 : (a + b - c)^2$

The cevian triangle of X(346) is perspective to the Ayme triangle; see X(3610).

X(346) lies on these lines:

2,37 6,145 8,9 45,594 69,144 78,280 100,198 219,644 220,1043 253,306 279,304 281,318 573,1018

X(346) = isogonal conjugate of X(1407)

X(346) = isotomic conjugate of X(279)

X(346) = X(312)-Ceva conjugate of X(8)

X(346) = X(200)-cross conjugate of X(8)

X(346) = crosspoint of X(312) and X(341)

X(346) = crosssum of X(604) and X(1106)

http://faculty.evansville.edu/ck6/encyclopedia/ETC.html

2

X(612) = INTERSECTION OF LINES X(1)X(2) AND X(9)X(31)

 $f(a,b,c): f(b,c,a): f(c,a,b), \text{ where } f(a,b,c) = a^2 + b^2 + c^2 + 2bc$ (M. Iliev, 5/13/2007)

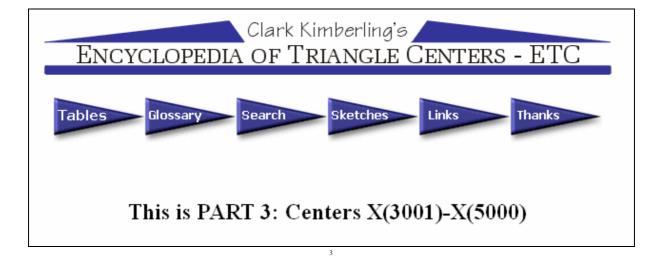
 $bc + S_{\omega} : ca + S_{\omega} : ab + S_{\omega}$ (C. Lozada, 9/07/2013)

Barycentrics af(a,b,c): bf(b,c,a): cf(c,a,b)

X(612) is the homothetic center of the incentral triangle and the Ayme triangle; see X(3610).

X(612) lies on these lines: 1,2 6,210 9,31 12,34 19,25 21,989 22,35 38,57 63,171 165,990 394,611 404,988 495,1060 518,940

X(612) = crossdifference of every pair of points on line X(649)X(905)



X(3610) = 1st AYME-MOSES PERSPECTOR

Trilinears $f(a,b,c): f(b,c,a): f(c,a,b), \text{ where } f(a,b,c) = bc(b+c)(a^2-b^2-c^2)(a^2+b^2+c^2+2bc)$

 $Barycentrics \quad af(a,b,c):bf(b,c,a):cf(c,a,b)$

In a Hyacinthos message dated January 10, 2011, Jean-Louis Ayme introduced a triangle as follows. Let RA be the radical axis of the circumcircle and the A-excircle, and define RB and RC cyclically. Let TA = RB∩RC, and define T_B and T_C cyclically. (T_A is also the radical center of the circumcircle and the B- and C- excircles.) The Ayme triangle $T_AT_BT_C$ is perspective to triangle ABC and also perspective to many other triangles. Peter Moses found that its perspector with the cevian triangle of X(346) is X(3610). He also found that the A-vertex of the Ayme triangle has first barycentric as follows:

 $-(b+c)(a^2+b^2+c^2+2bc)$; $b(a^2+b^2-c^2)$; $c(a^2+b^2+c^2)$.

from which the other two vertices are easily obtained. The Ayme triangle is perspective to ABC with perspector X(19).

Moses found that the locus of X such that the cevian triangle of X is perspective to the Ayme triangle is a cubic which passes through the points X() for i = 1, 2, 19, 75, 279, 304, 346, 2184. A barycentric equation for this Ayme-Moses cubic follows

(Cyclic sum of ayz[by($a^2 + b^2 - c^2$) - cz($a^2 - b^2 + c^2$]) = 0.

X(3610) lies on these lines: 10,37 19,346 612,2345

X(3611) = 2nd AYME-MOSES PERSPECTOR

Trilinears $f(a,b,c): f(b,c,a): f(c,a,b), \text{ where } f(a,b,c) = a(b+c) \ (b^2+c^2-a^2)[a^3(b+c)+(b-c)^2(b^2+c^2-a^2-ab-ac+2bc)]$

Barycentrics af(a,b,c):bf(b,c,a):cf(c,a,b)

In a Hyacinthos message dated January 7, 2011, Jean-Louis Ayme noted that the orthic triangle of ABC is perspective to the medial triangle of the extangents triangle of ABC. Peter Moses found coordinates for the perspector,

http://faculty.evansville.edu/ck6/encyclopedia/ETCPart3.html

X(3657) = XYME PERSPECTOR

 $Trilinears \qquad f(a,b,c): f(b,c,a): f(c,a,b), \ where \ f(a,b,c) = (b^2-c^2)/[2a(b+c)s_Bs_C-S^2-S_BS_C]$

 $Barycentrics \quad af(a,b,c):bf(b,c,a):cf(c,a,b)$

Let AB'C' be the medial triangle of the reference triangle ABC. Let P be the point of intersection of the lines X(1)X(3) and BC, and let P' be the point where the line through P perpendicular to line AX(3) meets that line. Let L_A be the line PP', and define L_B and L_C cyclically. Let $A'=L_B\cap L_C$, and define P' and define PP' and define PP' and define PP' and define PP' and PP

The Ayme triangle A"B"C" is also perspective to these triangles: tangential, orthic, intangents, extangents, and the circumorthic. (Peter Moses, November 7, 2011)

X(3657) lies on these lines

3,513 65,924 68,521 69,693 71,661 72,523 74,915 895,2990

X(3658) = ISOGONAL CONJUGATE OF AYME PERSPECTOR

filinears f(a,b,c): f(b,c,a): f(c,a,b), where $f(a,b,c) = [2a(b+c)s_Bs_C - S^2 - S_BS_C]/(b^2 - c^2)$

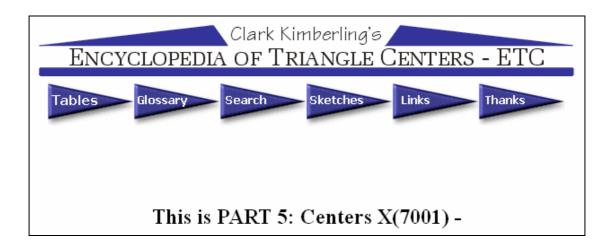
Barycentrics af(a,b,c): bf(b,c,a): cf(c,a,b)

As a point on the Euler line, X(3658) has Shinagawa coefficients ($\$aS_A^3\$(E+F) - \$aS_A^2\$(E+F)^2 - 2S^2 + 3\$aS_A\$FS^2 - \$a\$(E+F)FS^2$, $2\$aS_BS_C\$S^2 - 2\$aS_A^2\S^2).

X(3658) lies on the Euler line.

X(3658) lies on these lines:

2,3 100,110 108,925 109,2617 476,1290 1292,1302



X(9895) = ORTHOLOGIC CENTER OF THESE TRIANGLES: AYME TO APUS

 $Triline ars \quad (b+c)*a^5 + (b^2 + c^2)*a^4 + 2*b*c*(b+c)^2 *a^2 - (b+c)*((b^2 - c^2)^2 - 4*b^2 *c^2)*a - (b^2 - c^2)^2 *(b+c)^2 : : \\ Triline ars \quad (b+c)*a^5 + (b^2 + c^2)*a^4 + 2*b*c*(b+c)^2 *a^2 - (b+c)*((b^2 - c^2)^2 - 4*b^2 *c^2)*a - (b^2 - c^2)^2 *a^4 + 2*b*c*(b+c)^2 : : \\ Triline ars \quad (b+c)*a^5 + (b^2 + c^2)*a^4 + 2*b*c*(b+c)^2 *a^2 - (b+c)*((b^2 - c^2)^2 - 4*b^2 *c^2)*a - (b^2 - c^2)^2 *a^4 + 2*b*c*(b+c)^2 *a^2 - (b+c)*((b^2 - c^2)^2 - 4*b^2 *c^2)*a - (b^2 - c^2)^2 *a^2 + (b^2 - c^2)^2 + (b^2 - c^2)^2 *a^2 + (b^2 - c^2)$

 $X(9895) = (\mathbf{r}^{\wedge}2 - \mathbf{s}^{\wedge}2 + 2 * \mathbf{R} * \mathbf{r}) * X(5) - (\mathbf{r} + 2 * \mathbf{R} + \mathbf{s}) * (\mathbf{r} + 2 * \mathbf{R} - \mathbf{s}) * X(10)$

The reciprocal orthologic center of these triangles is X(3).

X(9895) lies on these lines

(17535), (3,19), (5,10), (58,1731), (65,1714), (72,5271), (197,6642), (389,916), (392,7532), (405,1824), (442,1829), (612,3295), (942,6678), (1697,9817), (1872,6913), (1902,8226) (1953,3682), (2262,5752), (2355,2915), (2475,5146), (3101,7523), (3702,5082), (9052,9822)

 $X(9895) = midpoint of X(i), X(j) for these {i,j}: {3,1871}$

 $X(9895) = \{X(i),X(j)\}$ -harmonic conjugate of X(k) for these (i,j,k): (1,9816,7535)

X(9940) = ORTHOLOGIC CENTER OF THESE TRIANGLES: ASCELLA TO AYME

 $Th linears \quad (b+c)*a^5 - (b^2 - 4*b*c+c^2)*a^4 - 2*(b^3 + c^3)*a^3 + 2*(b^4 + c^4 - 3*(b^2 + c^2)*b*c)*a^2 + (b+c)*(b-c)^2 *(a*(b^2 + c^2) - (b^2 - c^2)*(b-c)) : 1 + (b+c)*$

X(9940) = (2 *R+r) *X(1)+(2 *R-r) *X(3)

The reciprocal orthologic center of these triangles is X(10).

X(9940) lies on these lines

(1,3), (4,5439), (5,142), (7,5812), (30,5806), (72,631), (84,6913), (140,912), (226,6922), (244,4300), (355,443), (452,2096), (515,3812), (518,5771), (581,3752), (938,6916), (944,3753), (938,6916)(946,3742), (960,5884), (1001,1158), (1006,3916), (1125,6001), (1210,6907), (1439,5909), (1465,4303), (1490,5437), (1768,5259), (1829,7501), (1858,5433), (1871,7490), (1876,7412), (2771,5972), (2801,3634), (3090,5927), (3149,3306), (3218,6986), (3419,6897), (3487,6926), (3523,3868), (3555,5657), (3616,6935), (3698,5881), (4297,5883), (4675,5713), (5249,6831), (5435,6988), (5440,6940), (5534,9709), (5658,6964), (5705,5784), (5715,6173), (5722,6850), (5722,68908), (5770,5791), (5787,6826), (5805,6851), (5806,6847), (5887,6857), (6864,9799), (8099,8733), (8582,8728)

X(9940) = complement of X(5777)

X(9940) = midpoint of X(i), X(j) for these (i,j): (3,942), (960,5884), (4297,7686), (5836,5882)

X(9940) = reflection of X(i) in X(j) for these (i,j): (5044,140)

X(9940) = (X(i),X(j)) -harmonic conjugate of X(k) for these (i,j,k): (3,1482,6282), (3,2095,40), (3,5708,5709), (7,5865,5812), (57,8726,3), (443,5768,355), (1490,5437,6918), (4297,5883,7686), (443,5768,355), (443,5768 (5770.6989.5791)

X(9947) = ORTHOLOGIC CENTER OF THESE TRIANGLES: ATIK TO AYME

Trilinears $(3*\sin(A/2)+\sin(3*A/2))*\cos((B-C)/2)-(\cos(A)+3)*\cos(B-C)+2*\cos(A)-2$:

X(9947) = (4*R-r)*X(4)+(4*R+r)*X(8)

The reciprocal orthologic center of these triangles is X(10)

(3,5234), (4,8), (5,5947), (10,971), (20,3697), (40,3062), (65,9656), (84,9709), (165,3983), (210,5691), (354,7989), (515,5044), (516,4662), (518,5806), (942,5290), (944,5129), (1385,5720), (1490,9708), (1698,5789), (1864,9578), (2551,5787), (2801,3812), (3091,3555), (3579,7330), (3740,4297), (3889,5068), (5049,8227), (5261,5728), (5534,6913), (5817,9844), (5918,9588), (6918,7091), (7956,9842), (8582,8728)

 $X(9947) = midpoint of X(i), X(j) for these {i,j}: {8,9856}, {355,5777}$

 $\begin{array}{l} X(9947) = reflection \ of \ X(i) \ in \ X(j) \ for these \ (i,j): (5045,5) \\ X(9947) = (X(i),X(j)) - harmonic \ conjugate \ of \ X(k) \ for these \ (i,j,k): (8,5927,9856) \end{array}$

X(9955) = ORTHOLOGIC CENTER OF THESE TRIANGLES: 3RD EULER TO AYME

 $Barycentrics \quad (b+c)*a^3+2*(b^2-b*c+c^2)*a^2-(b^2-c^2)*(b-c)*a-2*(b^2-c^2)^2 :: \\$

X(9955) = 3*X(5)-X(10)

The reciprocal orthologic center of these triangles is X(10).

X(9955) lies on these lines:

(1,381), (2,3579), (3,1699), (4,1385), (5,10), (8,3545), (11,113), (30,1125), (40,1656), (65,7741), (72,6990), (79,3582), (125,5950), (140,516), (145,355), (165,3526), (226,496), (376,5550), (382,3576), (392,2476), (403,1829), (497,6849), (499,1836), (500,3720), (515,546), (519,5066), (547,3634), (551,3845), (582,748), (631,9812), (912,5448), (944,3832), (952,3635), (962,3090) (999,9612), (1001,6985), (1058,5226), (1155,5442), (1319,3585), (1386,3818), (1420,9655), (1482,3632), (1483,3857), (1519,6831), (1594,1902), (1657,7987), (1698,5055), (1702,8976), (1737,7173), (1770,5122), (2646,3583), (2807,5462), (3057,7951), (3218,3652), (3295,5219), (3337,7701), (3434,6896), (3485,5722), (3487,5274), (3525,9778), (3543,3653), (3574,5777), (3587,3646), (3601,9668), (3622,3655), (3627,4297), (3628,6684), (3648,3916), (3651,5284), (3654,5071), (3666,8143), (3670,5492), (3679,8148), (3753,4193), (3812,3825), (3834,5482), (3843,5691), (3858,5882), (3869,6873), (3877,5141), (3922,6971), (3927,5231), (3962,5694), (4004,6830), (4018,5887), (4324,5444), (4663,5476), (4668,5072), (4678,5068), (4701,5844), (5056,5657), (5070,9589), (5074,6706), (5079,7991), (5082,5748), (5126,7354), (5183,5445), (5290,7373), (5439,6845), (5440,6900), (5586,5708), (5698,5805), (5715,6913), (5728,7678), (5812,6846), (5885,6001), (6564,7968), (6565,7969), (6855,8166), (6881,7958), (7393,9911), (8085,8099), (8086,8100), (8164,9785), (8727,9940)

X(9955) = complement of X(3579)

X(9955) = midpoint of X(j), X(j) for these (i,j): (4,1385), (5,946), (546,5901), (551,3845), (1386,3818), (3627,4297), (4301,5690) X(9955) = reflection of X(j) in X(j) for these (i,j): (6684,3628)

X(9955) = (X(0),X(1)) -harmonic conjugate of X(k) for these (i,j,k): (4,5886,1385), (40,7988,1656), (226,496,5045), (946,3817,5), (946,7681,5806), (1125,3838,3824), (1482,3851,5587), (1519,6831,9856), (1699,8227,3), (1770,5433,5122), (3091,5603,355), (3583,5443,2646), (5072,5790,7989), (5219,9614,3295), (7982,7989,5790)

X(9956) = ORTHOLOGIC CENTER OF THESE TRIANGLES: 4TH EULER TO AYME

Barycentrics (b+c)*a^3-2*(b^2+b*c+c^2)*a^2-(b^2-c^2)*(b-c)*a+2*(b^2-c^2)^2::

The reciprocal orthologic center of these triangles is X(10).

(1,1656), (2,355), (3,1698), (4,2355), (5,10), (8,3090), (12,942), (30,3828), (35,7489), (40,381), (57,9654), (65,5694), (72,6829), (80,2646), (100,6920), (113,5952), (116,6706), (119,125), (140,515), (145,7486), (165,382), (392,4193), (403,1902), (495,1210), (498,1837), (499,5252), (511,3844), (516,546), (518,6583), (519,547), (549,4297), (551,1483), (912,3812), (938,8164), (912,3812), (938,8164), (938(952,1125), (958,6911), (962,3545), (971,3826), (993,6924), (999,9578), (1056,5704), (1155,3585), (1376,3560), (1482,3679), (1512,6831), (1532,9856), (1538,4002), (1594,1829), (1697,9669), (1699,3851), (1871,5142), (2476,3753), (2550,6893), (2551,5791), (2800,3918), (2975,6946), (3057,7741), (3085,5722), (3091,5657), (3167,9896), (3295,9581), (3333,5726), (3419,5552), (3434,6898), (3436,6854), (3525,5731), (3526,3576), (3542,5090), (3544,9779), (3616,5067), (3617,5056), (3624,5070), (3626,5844), (3632,9624), (3652,6175), (3656,5071), (3697,6991), (3698,6980), (3754,3838), (3832,6361), (3841,6001), (3843,9588), (3855,9812), (3858,5493), (3869,6874), (3877,5154), (3916,5080), (3925,6842), (3947,6147), (4325,5442), (5054,7987), (5072,7991), (5079,7982), (5086,5440), (5122,7354), (5126,5433), (5174,7551), (5219,5780), (5260,6905), (5285,7562), (5290,5708), (5499,9943), (5550,7967), (5554,6933), (5719,6738), (5728,7679), (5787,6989), (5794,6862), (5812,6843), (5927,6937), (6197,7559), (6797,8068), (6913,9709), (6940,9342), (7393,9798), (7529,8193), (8087,8099), (8088,8100), (8582.8728)

X(9956) = complement of X(1385)

X(9956) = midpoint of X(i), X(j) for these (i,j): (4,3579), (5,10), (65,5694), (355,1385), (946,5690), (9940,9947)

 $X(9956) = \text{reflection of } X(\hat{0} \text{ in } X(\hat{0}) \text{ for these (i,j)}: (140,3634), (1125,3628), (5885,3812), (9955,5)$ $X(9956) = (X(\hat{0},X(\hat{0})) - \text{harmonic conjugate of } X(k) \text{ for these (i,j,k}): (2,355,1385), (2,5818,355), (5,5690,946), (8,3090,5886), (10,946,5690), (10,1329,5044), (10,3814,960), (12,1737,942), (40,7989,381), (10,946,5690), (10,1329,5044), (10,3814,960), (10,3814,9$ (495,1210,5045), (1482,5055,8227), (1656,5790,1), (1698,5587,3), (3057,7741,7743), (3585,5445,1155), (3617,5056,5603), (3679,8227,1482), (3812,3822,3824)

X(9957) = ORTHOLOGIC CENTER OF THESE TRIANGLES: HUTSON INTOUCH TO AYME

Trilinears (b+c)*(a^2-(b-c)^2)-6*a*b*c:

X(9957) = (2 *R-r) *X(1)+r*X(3)

The reciprocal orthologic center of these triangles is X(10).

X(9957) lies on these lines

(1,3), (2,3885), (4,7320), (5,7743), (8,392), (10,496), (11,9956), (12,9955), (37,4266), (72,145), (78,4917), (210,3632), (221,1480), (355,497), (376,4308), (381,9578), (382,9580), (390,944), (390(495,946), (515,9856), (518,3244), (519,960), (550,4311), (551,3812), (758,3635), (912,1483), (936,2136), (943,1320), (950,952), (956,5250), (962,1056), (995,4646), (997,3913), (1005,3957), (1004,5399), (1100,5053), (1125,1387), (1149,4642), (1210,5690), (1317,2771), (1365,6018), (1389,2346), (1479,5252), (1490,7966), (1497,5398), (1621,4861), (1699,9654), (1770,5434), (1788,3654), (1829,4222), (1870,1902), (2262,3247), (2292,2611), (3061,3991), (3085,5886), (3241,3555), (3294,4875), (3476,4294), (3485,3656), (3486,5887), (3600,6361), (3616,3753), (3621,3876), (3622,5439), (3623,3868), (3624,3698), (3624,3698), (3625,4662), (3626,3740), (3633,5692), (3636,3742), (3655,4305), (3679,3893), (3680,7160), (3683,5258), (3811,5289), (3820,6736), (3621,3876), (3622,5439), (3622,5439), (3623,3868), (3622,3868), (36(3825,5123), (3870,5730), (3871,5440), (3873,4018), (3892,4084), (3895,5687), (3899,3962), (3921,4678), (3927,6762), (3940,6765), (3954,4746), (3953,4668), (4015,4701), (4059,7278), (4187,6735), (4251,6603), (4314,5882), (4345,5703), (4640,8666), (4673,5295), (4719,4868), (4853,9708), (5011,9327), (5195,7247), (5274,5818), (5587,9669), (5603,5806), (5691,9668), (5728,5766), (5790,9581), (5881,9947), (7992,9845), (8099,8241), (8100,8242), (8583,9709)

$$\begin{split} X(9957) &= \text{midpoint of } X(i), X(j) \text{ for these } (i,j) : \{1,3057\}, \ (65,5697), \ (72,145), \ (3244,3878), \ (3555,3869) \\ X(9957) &= \text{reflection of } X(j) \text{ in } X(j) \text{ for these } (i,j) : \{8,5044\}, \ (65,5045), \ (942,1), \ (960,3884), \ (3625,4662), \ (3754,3636), \ (4701,4015), \ (5836,1125), \ (5881,9947), \ (6797,1387) \\ &= (36,5042), \ (3625,4662), \ (3754,3636), \ (4701,4015), \ (5836,1125), \ (5881,9947), \ (6797,1387) \\ &= (36,5042), \ (3625,4662), \ (3754,3636), \ (4701,4015), \ (5836,1125), \ (5881,9947), \ (6797,1387) \\ &= (36,5042), \ (3625,4662), \ (3754,3636), \ (4701,4015), \ (5881,9947), \ (6797,138$$

X(9957) = (X(i),X(j)) -harmonic conjugate of X(k) for these (i,j,k): (1,35,1319), (1,40,999), (1,46,3304), (1,55,1385), (1,57,7373), (1,65,5045), (1,942,5049), (1,1697,3), (1,3612,1388), (1,3746,2646), (1,5119,56), (1,5697,65), (1,7962,1482), (1,7991,3333), (1,9819,40), (8,392,5044), (8,1058,5722), (8,3890,392), (55,1388,3612), (56,5579,5122), (56,5119,3579), (65,3057,5697), (65,5045,942), (145,3877,72), (1000,1058,8), (1388,3612,1385), (1482,6767,1), (3241,3869,3555), (3636,3754,3742), (4015,4701,4711), (9578,9614,381), (9580,9613,382)

X(9958) = ORTHOLOGIC CENTER OF THESE TRIANGLES: AYME TO INCENTRAL

 $Barycentrics \\ (a+b+c)*(2*a^3-(b^2+c^2)*(a-2*b-2*c))*a^3-2*(b+c)*(b^4+c^4-(b+c)^2*b^*c)*a^4\\ 2-(b^4-c^2)^2*(a^2+b^2+c^2)^2*($

The reciprocal orthologic center of these triangles is X(500).

X(9958) lies on these lines

{4,5278}, {5,182}, {10,30}, (381,1714), (500,612), (6000,9895)

X(9959) = ORTHOLOGIC CENTER OF THESE TRIANGLES: 1ST SHARYGIN TO AYME

 $\begin{array}{ll} \text{Trilinears} & (b+c)^* a'^5 + 2^* (b'^2 + b^* c + c'^2)^* a'^4 - (b+c)^* (b'^2 + b^* c + c'^2)^* a'^4 - (b+c)^* (b'^2 + b^* c + c'^2)^* a'^4 - (b'^2 + b'^2 + b'^2$

The reciprocal orthologic center of these triangles is X(10).

X(9959) lies on these lines

(3,846), (4,9791), (5,4425), (10,2783), (21,104), (40,8245), (256,3931), (511,3743), (517,2292), (942,1284), (1281,6998), (3579,4220), (4199,5777), (5051,9956), (5728,8238), (5884,6176), (41,9791(8099,8249), (8100,8250), (8229,9955), (8240,9957), (8731,9940)

 $X(9959) = midpoint of X(i), X(j) for these {i,j}: {3,5492}, {2292,9840}$

 $X(9959) = \{X(i), X(j)\}\$ -harmonic conjugate of X(k) for these (i, j, k): (846, 8235, 3)