Mock Inequalities USAMO 2012

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1. Let a, b, c, and d be non-negative real numbers such that $a+b+c+d=4$. Prove that $3(a^2+b^2+c^2+d^2)+4abcd \geq 16$.

2. Let a, b, and c be the sides of a triangle with perimeter 3. Prove that

$$\sum_{cyc} \frac{a^2}{a+2\sqrt{b}-1} \geq \frac{ab^3+bc^3+ca^3+9abc}{3(ab+bc+ca)-abc}.$$

3. Given positive real numbers a, b, and c, show that

$$\sum_{cyc} \sqrt[12]{\frac{a^6+b^4c^2}{b^3c^3}} > \frac{\sqrt[4]{4a}+\sqrt[4]{4b}+\sqrt[4]{4c}}{\sum_{cyc} \sqrt[12]{b^2+c^4}}.$$

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4. For positive reals a, b, and c, prove that

$$\sum_{cyc} \sqrt[3]{\frac{a^2+bc}{b^2+c^2}} \geq 9 \cdot \frac{\sqrt[3]{abc}}{a+b+c}.$$

5. Let a, b, and c be positive reals such that $a^2b+b^2c+c^2a+abc = a+b+c+1$. Prove that

$$\sum_{cyc} \frac{3ab-ab^2}{b^2+bc+1} + \frac{\sum_{cyc} a^3b^2c}{\sum_{cyc} ab} \leq \sum_{cyc} \sqrt{\frac{ab(a^2+b^2)+c(a^3+b^3)}{2ab+c(a+b)}}.$$

6. For $a, b, c \geq 0$, prove that

$$\frac{a^6+b^6+c^6+15}{12} - \frac{3}{a^6+b^6+c^6+3} \geq abc$$