A NEW POINT ON EULER LINE

FIRST SYNTHETIC PROOF

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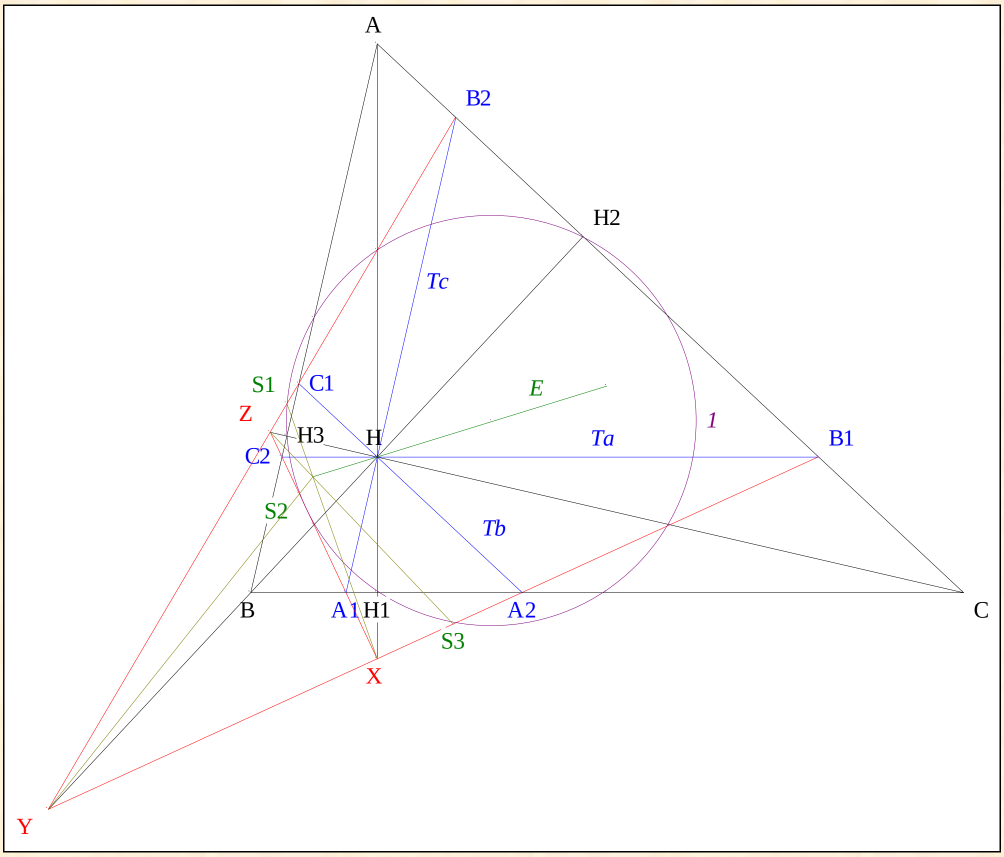
THE YAKUB ALIYEV's RESULT



Hypothesis :	ABC	a triangle,
	H	the orthocenter ABC,
	H1, H2, H3	les feet of the A, B, C-altitudes of ABC,
	Ta, *Tb*, *Tc*	the respective parallels to BC, CA, AB trough H,
	B1, C2	the points of intersection of *Ta* wrt AC, AB,
	C1, A2	the points of intersection of *Tb* wrt BA, BC,
	A1, B2	the points of intersection of *Tc* wrt CB, CA,
	X, Y, Z	the points of intersection of A1C2 and A2B1, of B1A2 and B2C1, of C1B2 and C2A1,
	1	the circumcircle of the triangle H1H2H3,
	S1, S2, S3	the second points of intersection of *1* wrt YZ, ZX, XY
et	*E*	the Euler's line of ABC.

Conclusion : XS1, YS2 and ZS3 concur on *E*.

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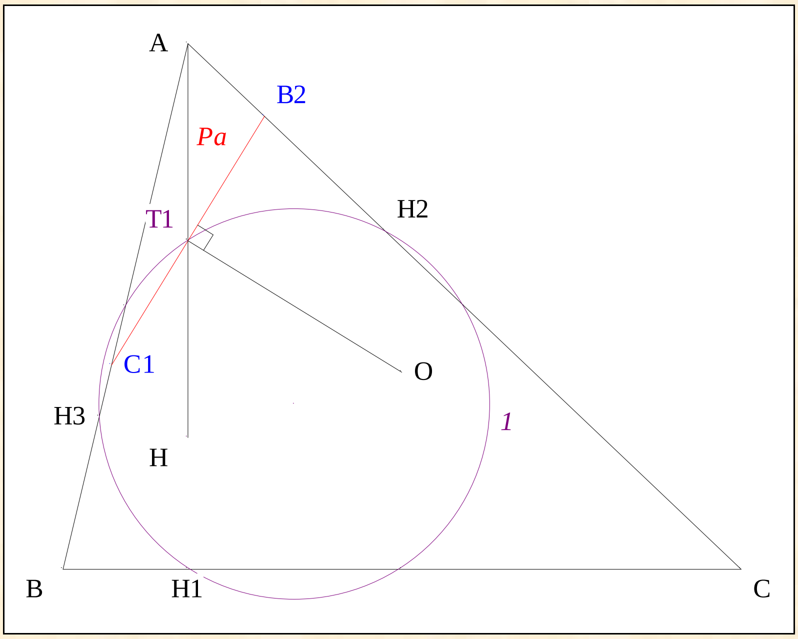
A. TWO LEMMAS

1. Another nature of the Euler's point

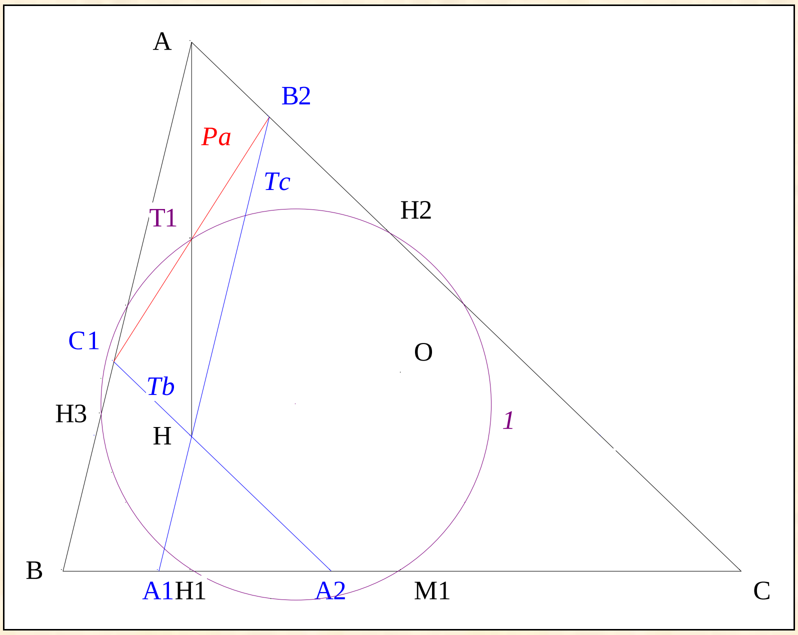


Hypothesis :	ABC	a triangle,
	H	the orthocenter of ABC,
	H_1, H_2, H_3	the feet of the A, B, C-altitudes of ABC,
	1	The Euler's circle of ABC ABC,
	T_1	the midpointof AH,
	O	the center of the circumcircle of ABC,
	Pa	the perpendicular to OT1 trough T_1
and	B_2, C_1	the points of intersection of *Pa* wrt AC, AB.

Conclusion : T_1 is the midpoint of B_2C_1. [1]

Remark : T1 being also the "Euler's A-point of ABC", *1* goes trough T_1. [2]

Scolies : **(1)** another nature of B_2 and C_1

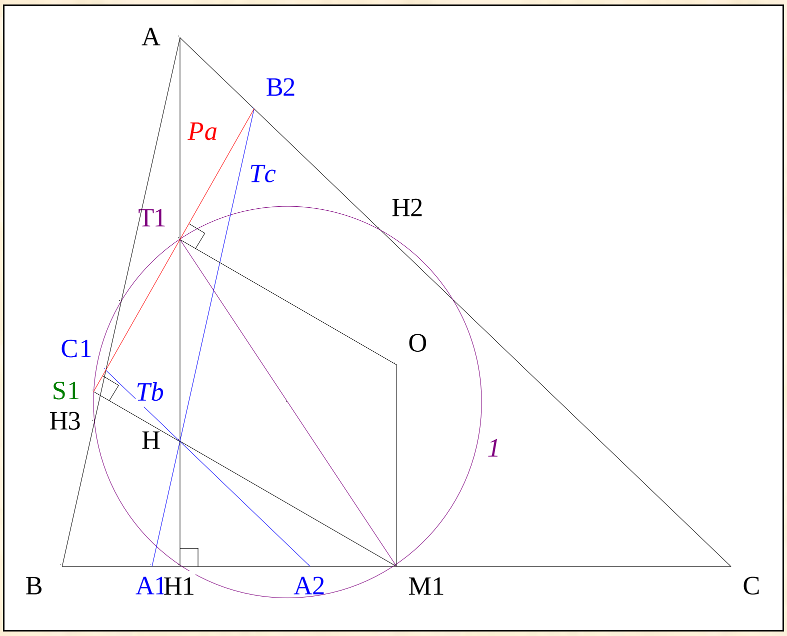


- The quadrilateral AB_2HC_1 is a parallelogram; in consequence, HB_2 // AB and HC_1 // AC.

- **Conclusion:** B_2 and C_1 are the points present in the Aliyev's figure.

(2) $B_2T_1C_1$ = *Pa*.

(3) Nature of the intersection of *Pa* and M_1H

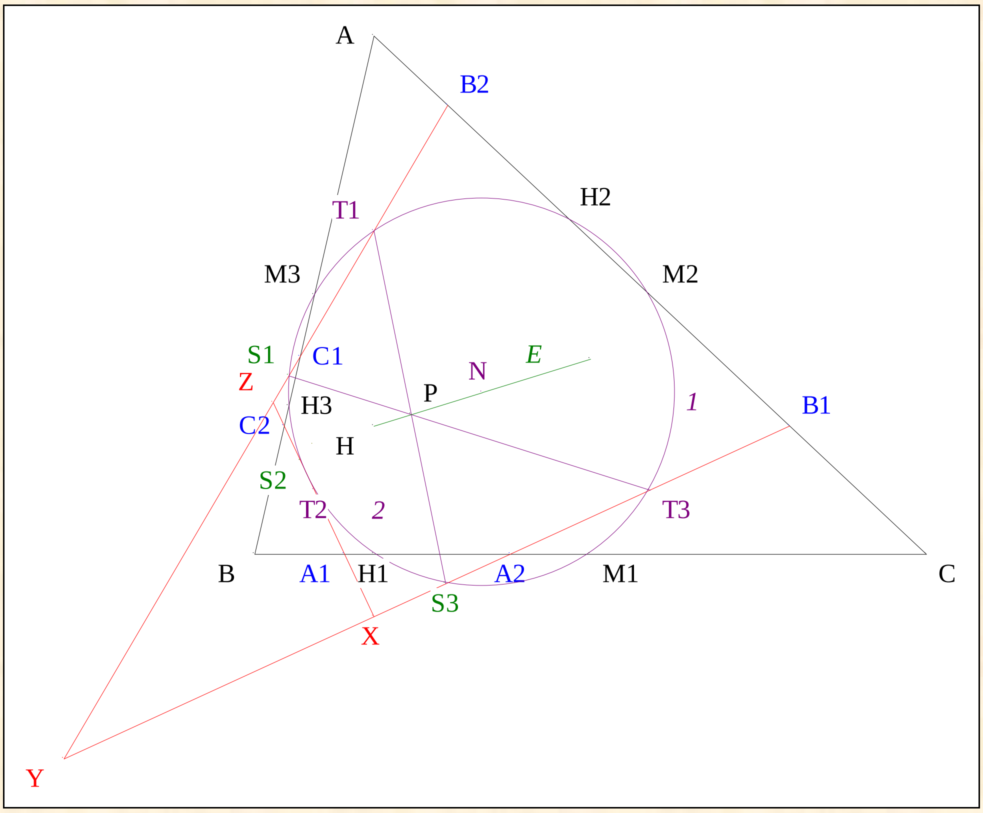


- By hypothesis, *Pa* ⊥ OT_1 ;
  we know that OT_1 // M_1H ;
  in consequence, *Pa* ⊥ M_1H.

- M_1T_1 being a diameter of *1*,
  according Thalès "Triangle inscriptible in a half cercle", *Pa* and M_1H intersect on *1*.

- **Conclusion:** this point of intersection is S_1.

2. A point on the Euler's line



Hypothesis: to the hypothesis and notations of Aliyiev, we add

N the center of *1*,

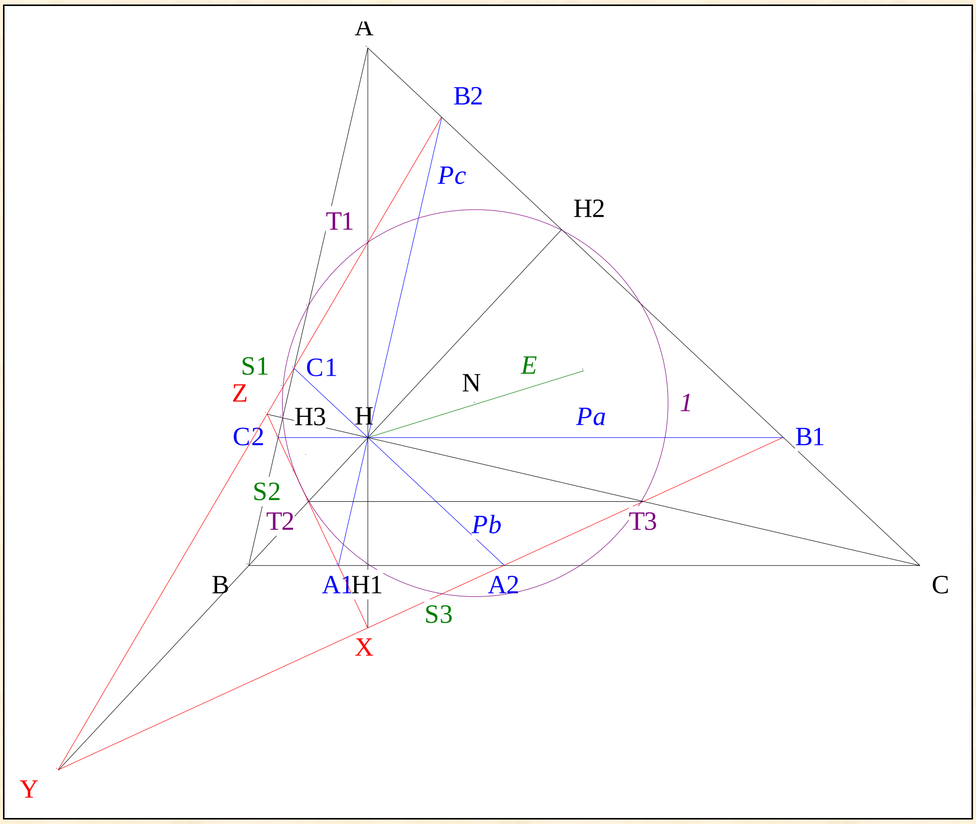
M_1, M_2, M_3 the respective midpoints BC, CA, AB

et P the point of intersection of T_1S_3 and T_3S_1.

Conclusion : P is on *E*.

Proof: according to the Pascal's theorem [3], (PHN) is the Pascal's line of the $T_1S_3M_3T_3S_1M_1T_1$.

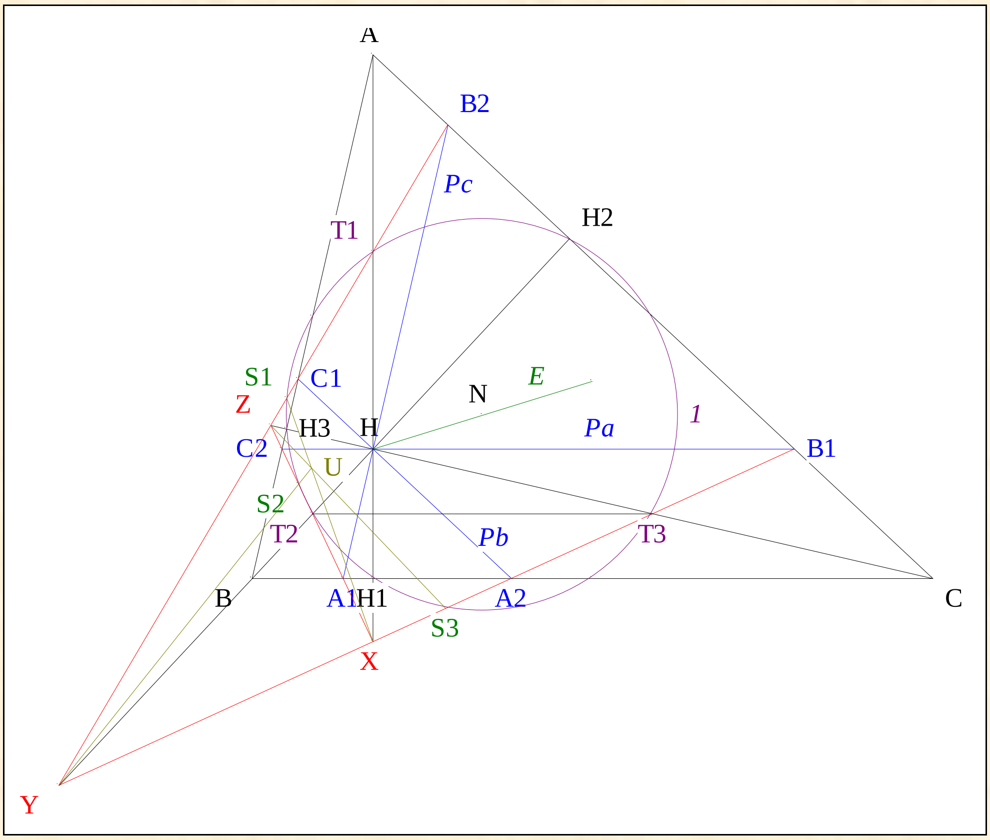
B. THE FIRST SYNTHETIC PROOF



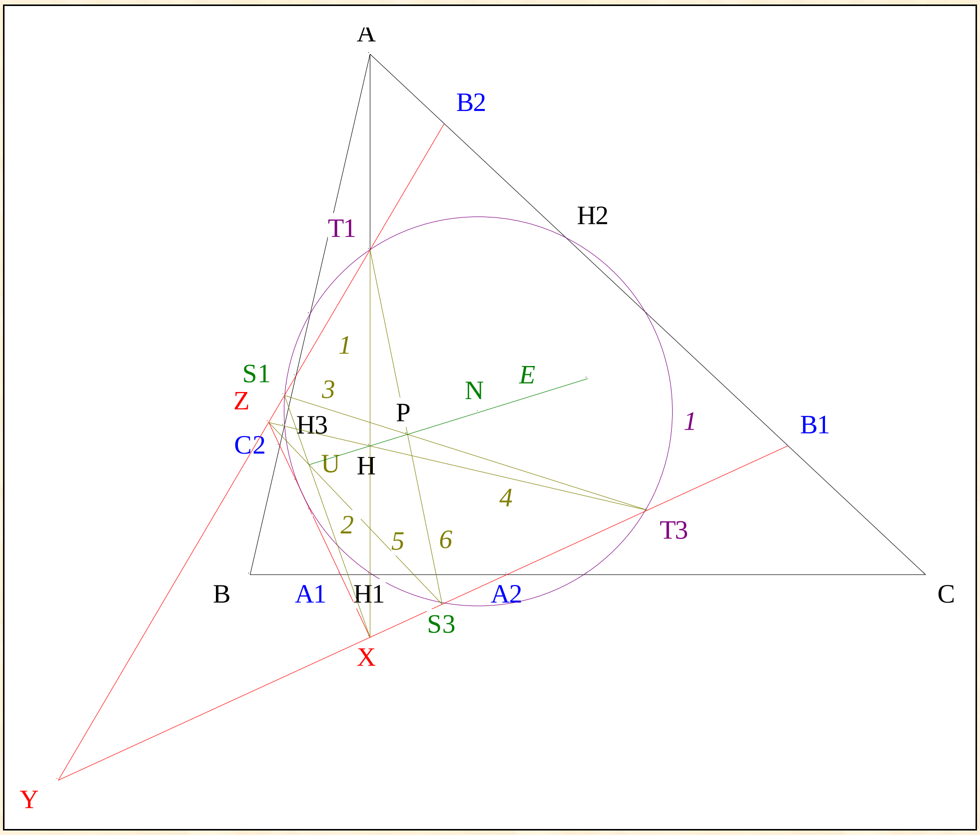
- By hypothesis, C_2B_1 // BC ;
  according to "The middle line theorem" applied to the triangle HBC, BC // T_2T_3 ;
  in consequence, C_2B_1 // T_2T_3.

- According to the Desargues theorem applied to the homothetic triangles AC_2B_1 et HA_1A_2,
  AH goes trough X intersection of C_2A_1 and B_1A_2.

- Mutatis mutandis, we would prove BH goes trough Y intersection of A_2B_1 and C_1B_2
  CH goes trough Z intersection of B_2C_1 and A_1C_2.



- According to the Terquem's theorem [4] applied to XYZ and *1*, XT_1, YT_2 and ZT_3 going trough H, XS_1, YS_2 and ZS_3 are concurrent in U.



- Note P the point of intersection of T_1S_3 and T_3S_1.

- According to Pappus theorem [5], HUP is the Pappus line of the hexagon $T_1XS_1T_3Z\ S_3T_1$.

- According to "A point on the Euler's line", E = PHN ;
  in consequence, U is on E.

- **Conclusion:** XS_1, YS_2 and ZS_3 concur on E.

C. REFERENCES

[1] Ayme J.-L., An Euler point is midpoint; http://www.mathlinks.ro/Forum/viewtopic.php?t=336564.
[2] Ayme J.-L., Les cercles de Morley, Euler,…, G.G.G. vol. **2** p. 3-5 ; http://perso.orange.fr/jl.ayme.
[3] Coxeter, Greitzer, Geometry Revisited, New Mathematical Library, New York (1967) 67.
[4] Terquem O., *Nouvelles Annales* **1** (1842) 403 ; http://www.numdam.org/numdam-bin/feuilleter?j=NAM&sl=0
[5] Coxeter, Greitzer, Geometry Revisited, New Mathematical Library, New York (1967) 67.