Geometric inequalities -- Math Olympiad Training

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5:06 AM

Problem 1: Consider a triangle inscribed a in a circle. Tangents to the circumcircle passing through the vertices of the triangle are constructed. The distances of an arbitrary point on the circle to the straight lines containing the sides of the triangle are equal to *a, b* and *c,* and to the tangents are equal to *x,y* and *z*. Prove that $a^2 + b^2 + c^2 = xy + xz + yz$.

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Problem 2: Let the inscribed circle of the triangle *ABC,* with *A, B, C < 120,* and center *O,* be tangent to the sides of the triangle *AB, BC,* and *CA* at the points C_1, A_1, B_1 respectively. Denote the midpoints of the segments AO, BO and CO by $A_2, B_2,$ and C_2 respectively. Prove that the lines $A_1A_2,\ B_1B_2,$ and C_1C_2 intersect at the same point.

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Problem 3: Suppose that $ABCDEF$ is a convex hexagon with $AB = BC = CD = DE$ and $EF = FA$. Prove that

$$\frac{BC}{BE} + \frac{DE}{DA} + \frac{FA}{FC} \geq \frac{3}{2}$$

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