6, proof by induction

AM(a1, . . . , an) = (a1 + · · · + an)/n

GM(a1, . . . , an) = n√ a1 · · · an

For n=2,

(a1+a2)/2 ≤ 2√x.y

(a1-a2)2≥0,for all a1,a2 >0

a12-a22-2a1a2≥0

a12+2a1a2+a22-4a1a2≥0

(a1+a2)2≥4a1a2

a1+a2 ≥ 2a1a2, taking the square roots of both sides

(a1+a2)/2 ≥ a1a2

Induction Hypothesis : Assume the statement is true for n-1

Proof : without loss of generality assume that

a1≤a2≤…≤an

Let G be a geometric mean G = n√a1a2…an. Then it follows that

a1≤G≤an.Note that since

a1+an≥a1an/G + G

a1+an-G-a1an/G=a1/G(G-an)+(an-G)=1/G(G-a1)(an-G) ≥0

By the induction hyphothesis

(a2+…+an-1+a1an/G)/n-1≥n-1√Gn/G=G

Hence

a2+…+ an-1+ a1an/G ≥ (n-1)G

and

(a2+…+an-1+a1an/G+G)/n

But since

a1+an≥a1an/G + G

it follows that

(a1+a2+…+an)/n ≥ G