A Multi-Objective Optimization Model for Balancing Efficiency and Equity in Urban Transport Resource Allocation

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Abstract

Urban transport planning often faces a challenge: how to balance efficiency, minimizing travel times, and equity, ensuring fair access to transport resources. This report presents a general framework for a multi-objective optimization model that allocates limited transport resources (vehicles) across city zones to achieve a balance between efficiency and equity. The report focuses on methodology, model formulation, and potential applications, providing a foundation for future research or practical implementation.

1 Introduction

Urban transportation is essential for social and economic development. Efficient transport systems reduce travel time, fuel consumption, and congestion, but focusing solely on efficiency can create inequities, where some zones or populations receive fewer resources.

Equity in transport ensures fair access for all populations, including underserved areas. Balancing efficiency and equity is a critical challenge for urban planners.

This project proposes a multi-objective optimization model to allocate transport resources efficiently while maintaining fairness. The framework is general and can be applied to any city using synthetic or real data.

2 Objectives

The main objectives are:

1. Develop a mathematical model for allocating transport resources across zones.
2. Balance two objectives:
   - Efficiency: minimize total travel time
   - Equity: reduce disparities in travel times between zones
3. Provide a flexible framework applicable to different cities or datasets.
4. Demonstrate trade-offs between efficiency and equity using the model.

3 Literature Review

Research shows that equity is an important consideration in urban transport planning:

- Multi-objective optimization is used to balance competing objectives like efficiency and equity.
- Equity measures include travel time variance, accessibility indices, and social weights for underserved populations.
- Transport modeling often relies on Origin-Destination matrices, population data, and vehicle capacities to simulate allocation scenarios.
- Studies show prioritizing underserved zones improves social outcomes but may slightly increase total travel time.

4 Methodology

4.1 Problem Definition

The city is divided into **zones**, and a limited number of vehicles are available for allocation. Each zone has:

- Population or commuter demand
- Baseline travel times
- Existing resources
- Socioeconomic weight (for equity)

Decision Variable:

$$x_i = \text{number of vehicles allocated to zone } i$$

Constraint:

$$\sum_{i=1}^{n} x_i = \text{Total Vehicles Available}$$

4.2 Model Parameters

Table 1: Model Parameters and Definitions

Parameter	**Description**
T_i^{base}	Base travel time for zone i (minutes)
x_i	Number of vehicles allocated to zone i
α	Effectiveness coefficient (0.05)
w	Weight balancing efficiency and equity ($0 \leq w \leq 1$)
E	Efficiency objective (total weighted travel time)
Q	Equity objective (variance of travel times)

4.3 Objective Functions

Efficiency: Minimize total travel time:

$$E = \sum_{i=1}^{n} T_i(x_i) \cdot D_i$$

Equity: Minimize disparities in travel times:

$$Q = \text{Var}(T_1(x_1), T_2(x_2), ..., T_n(x_n))$$

Travel Time Function:

$$T_i(x_i) = T_i^{base} \cdot \frac{1}{1 + \alpha x_i}, \quad \alpha = 0.05$$

4.4 Combined Objective Function

The overall objective minimizes a weighted sum of efficiency and equity:

$$\min_x \; F(x) = w \cdot E(x) + (1 - w) \cdot Q(x)$$

Subject to:

$$\sum_{i=1}^{n} x_i = V_{\text{total}}, \quad x_i \geq 0$$

4.5 Synthetic Example

A hypothetical city with six zones:

Table 2: Hypothetical City Zones

Zone	Population	Commuters	Base Travel Time (min)	Equity Weight
1	35,000	21,000	25	1.0
2	25,000	13,750	30	0.8
3	30,000	15,000	35	0.9
4	20,000	10,000	40	0.7
5	15,000	6,750	45	0.6
6	10,000	4,000	50	0.5

4.6 Optimization Approach

1. Formulate the multi-objective problem with efficiency and equity objectives.
2. Optimization techniques:
   - Weighted sum method: combine objectives using weights
   - Pareto optimization: generate trade-offs between efficiency and equity
3. Simulate travel times for different allocations.
4. Visualize results using bar charts and Pareto plots.

4.7 Tools

- Python: NumPy, Pandas, Matplotlib
- Optimization libraries: scipy.optimize, pymoo for multi-objective optimization
- Optional GIS: for mapping zones and visualization

5 Results and Discussion

5.1 Synthetic Example Results

The optimization model was applied to the hypothetical six-zone city described in Table **??**. A total of 100 vehicles were allocated among the zones to minimize a weighted combination of efficiency and equity, using $\alpha = 0.05$ and $w = 0.5$.

Table 3: Optimal Allocation and Resulting Travel Times

Zone	**Population**	**Commuters**	**Base Time (min)**	**Optimized Vehicles**	**Travel Time (**
1	35,000	21,000	25	22.82	11.68
2	25,000	13,750	30	17.96	15.81
3	30,000	15,000	35	22.82	16.35
4	20,000	10,000	40	17.38	21.40
5	15,000	6,750	45	12.58	27.63
6	10,000	4,000	50	6.45	37.81

Key Metrics:

- Total weighted travel time: $\approx 1,286,618$ minutes
- Equity (variance of travel times): ≈ 24.02

Key Findings

- Zones with higher commuter populations (1 and 3) receive the most vehicles.
- Trade-off achieved between minimizing total travel time and ensuring fairness.
- Less populated zones have higher travel times, but no extreme inequity.

5.2 Pareto Frontier Analysis

5.3 Additional Visualizations

5.4 Sensitivity and Scenario Analysis

(a) Total Vehicle Variation: When total vehicles were changed to 80 and 120, allocations scaled proportionally. More vehicles reduced overall travel times while maintaining similar distribution patterns.

(b) Rush Hour Scenario: When commuter numbers increased by 20%, the optimal allocation remained nearly identical, showing robustness under demand growth.

5.5 Policy Insights

- Adjusting the weighting parameter w allows policymakers to emphasize either fairness or efficiency.
- High-demand zones (e.g., Zones 1 and 3) yield the greatest travel-time reductions.
- Ensuring a minimum allocation to peripheral zones supports social inclusion.

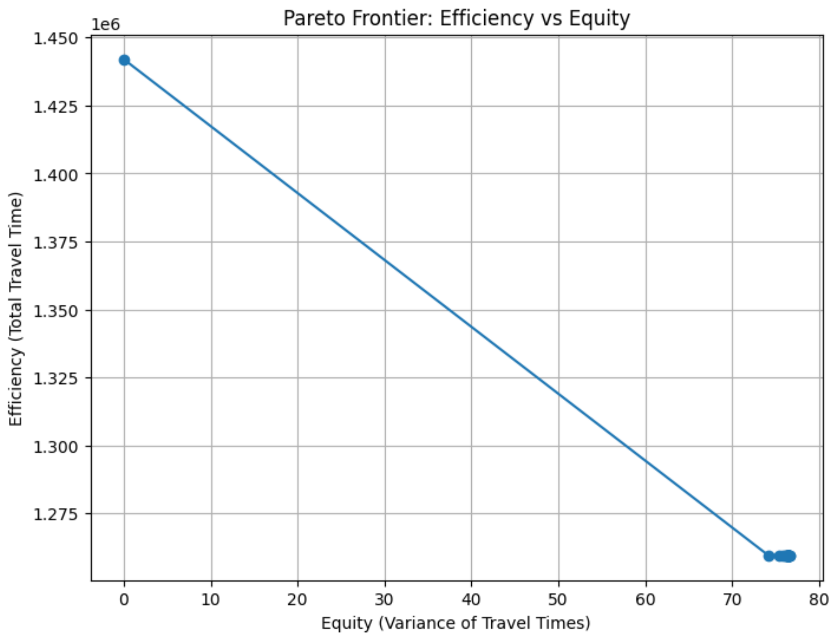

Figure 1: Pareto Frontier showing trade-offs between efficiency and equity. Each point represents an optimized solution for a different weighting w.

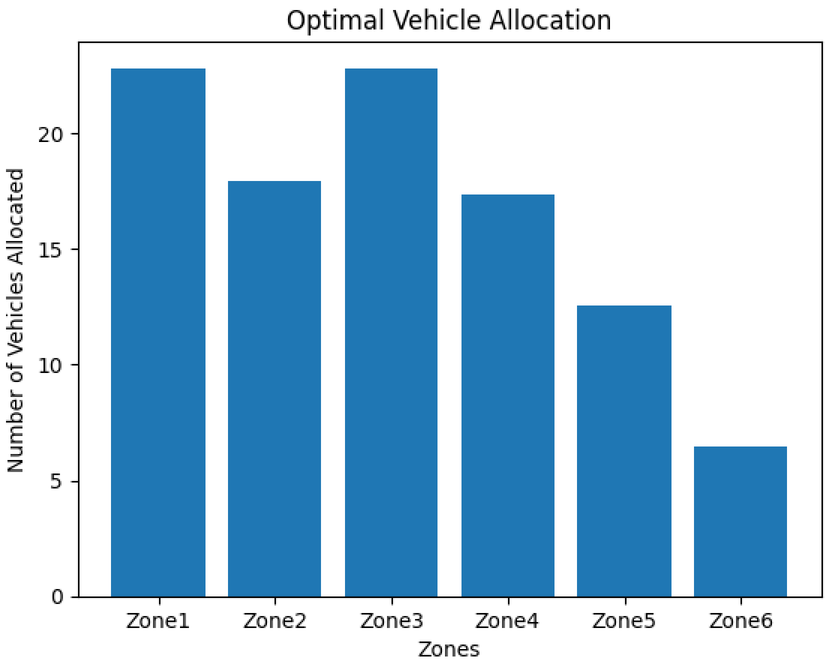

Figure 2: Optimal vehicle allocation across six zones for $w = 0.5$.

6 Discussion

The model demonstrates how multi-objective optimization can support urban transport planning by quantifying trade-offs between efficiency and equity. It shows that modest equity considerations can improve social outcomes without large efficiency losses.

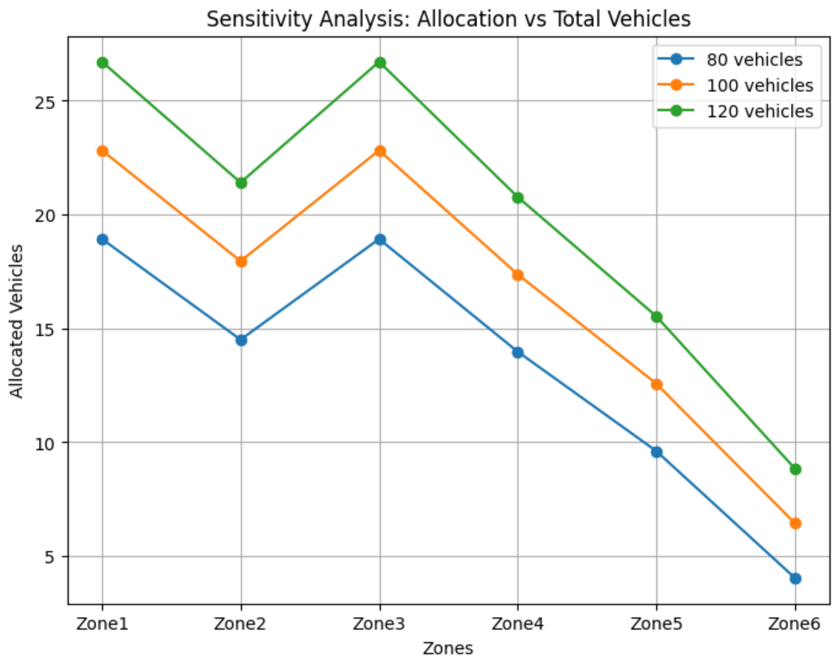

Figure 3: Sensitivity analysis for different total vehicle levels (80, 100, 120).

7 Conclusion

This report presents a framework for balancing efficiency and equity in urban transport planning. Using synthetic data, it:

- Demonstrates systematic vehicle allocation;
- Shows trade-offs between efficiency and equity;
- Provides a foundation for applying similar models to real urban systems.

8 Future Work

- Apply the model to real cities with actual travel data.
- Extend to dynamic demand patterns (peak/off-peak hours).
- Include budget, capacity, or environmental constraints.
- Integrate with GIS and real-time traffic data for smart city management.

A Python Implementation Code

The following Python code was used for simulation and optimization:

```
from scipy.optimize import minimize
import numpy as np

def travel_time(x, base_time, alpha):
```

```
    return base_time / (1 + alpha * x)

def objective(x, base_time, commuters, alpha, w=0.5):
    tt = travel_time(x, base_time, alpha)
    efficiency = np.sum(tt * commuters)
    equity = np.var(tt)
    return w * efficiency + (1 - w) * equity
```

B References

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