



Intensity based Image Registration

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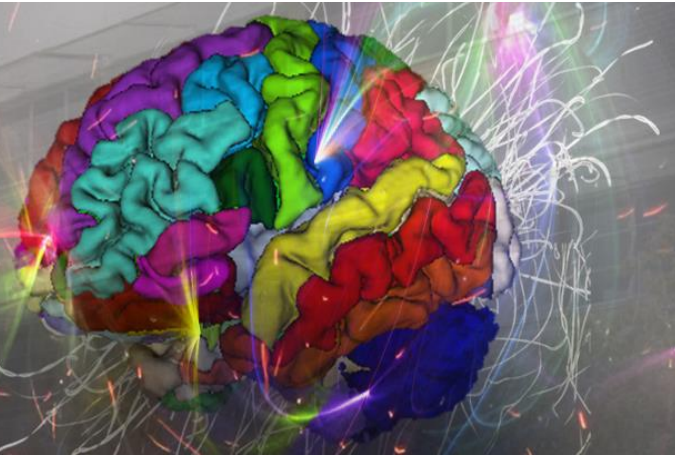


Image Registration . Overview

- Find the transformation or mapping function f

$$T(x', y') = R(f(x, y))$$

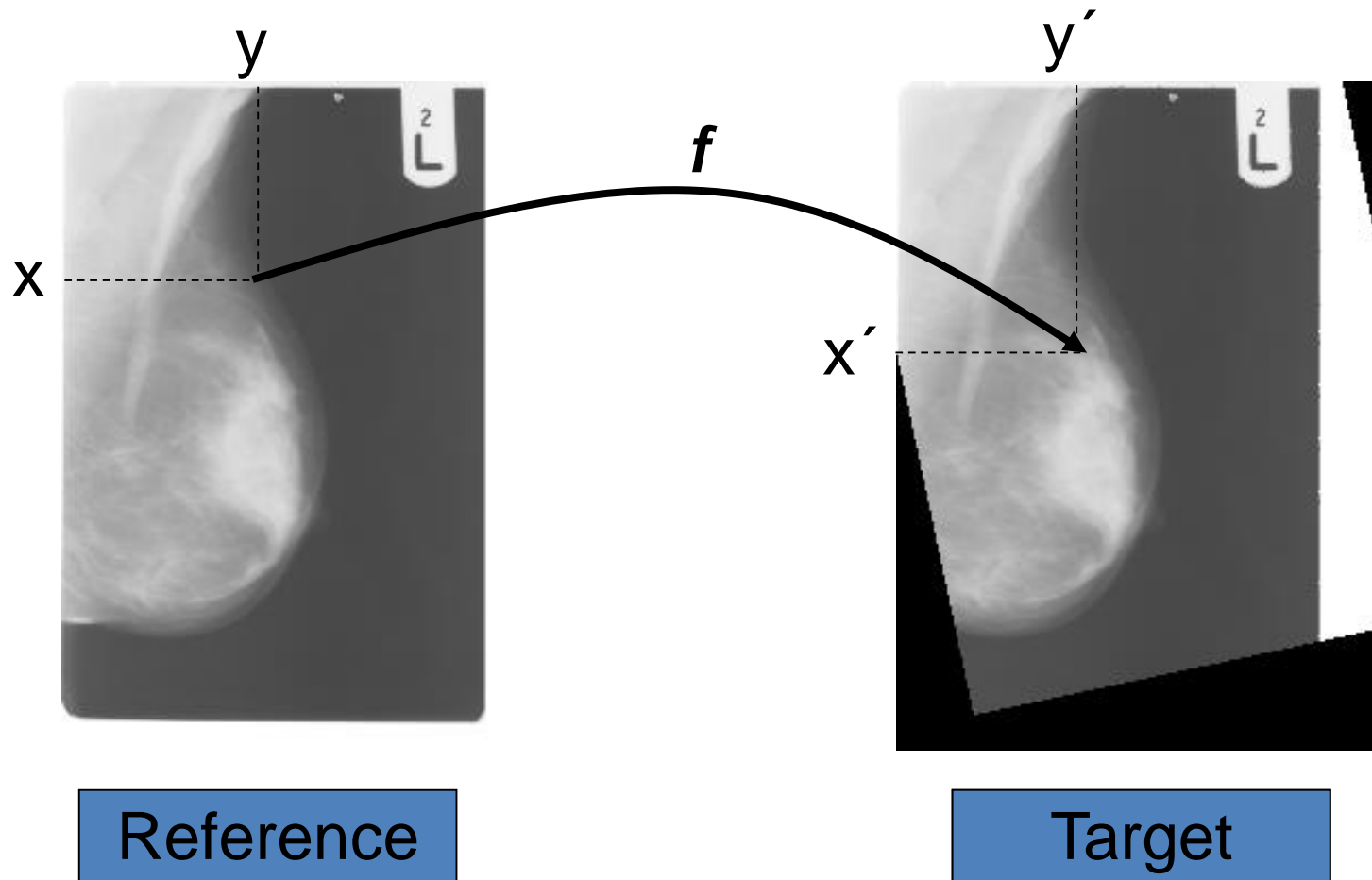


Image Registration. Components

- Classification:
 - Intensity based vs feature based (aka dense vs sparse)

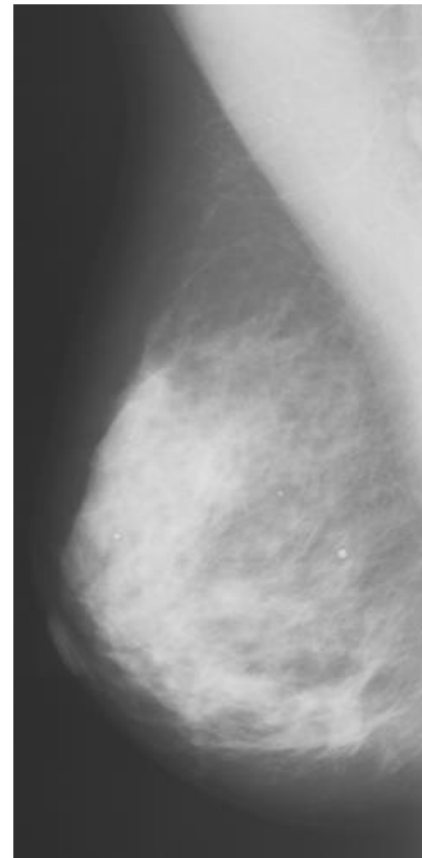
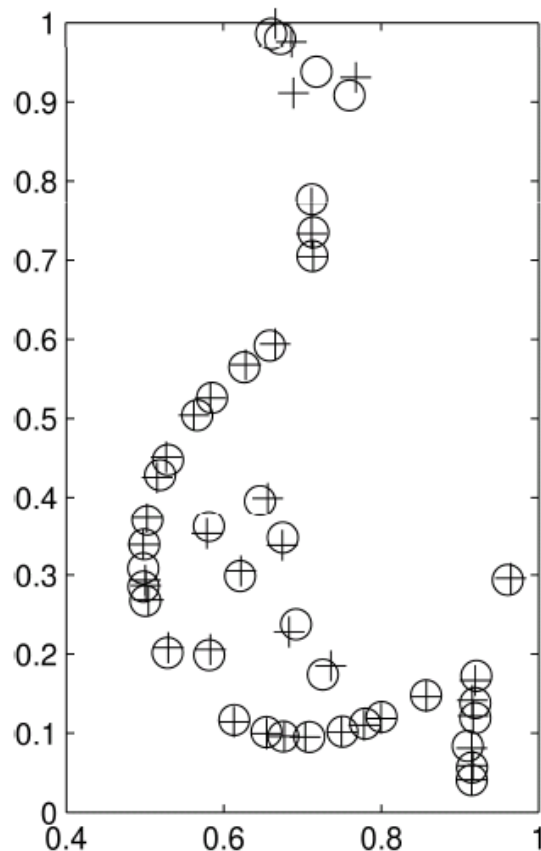
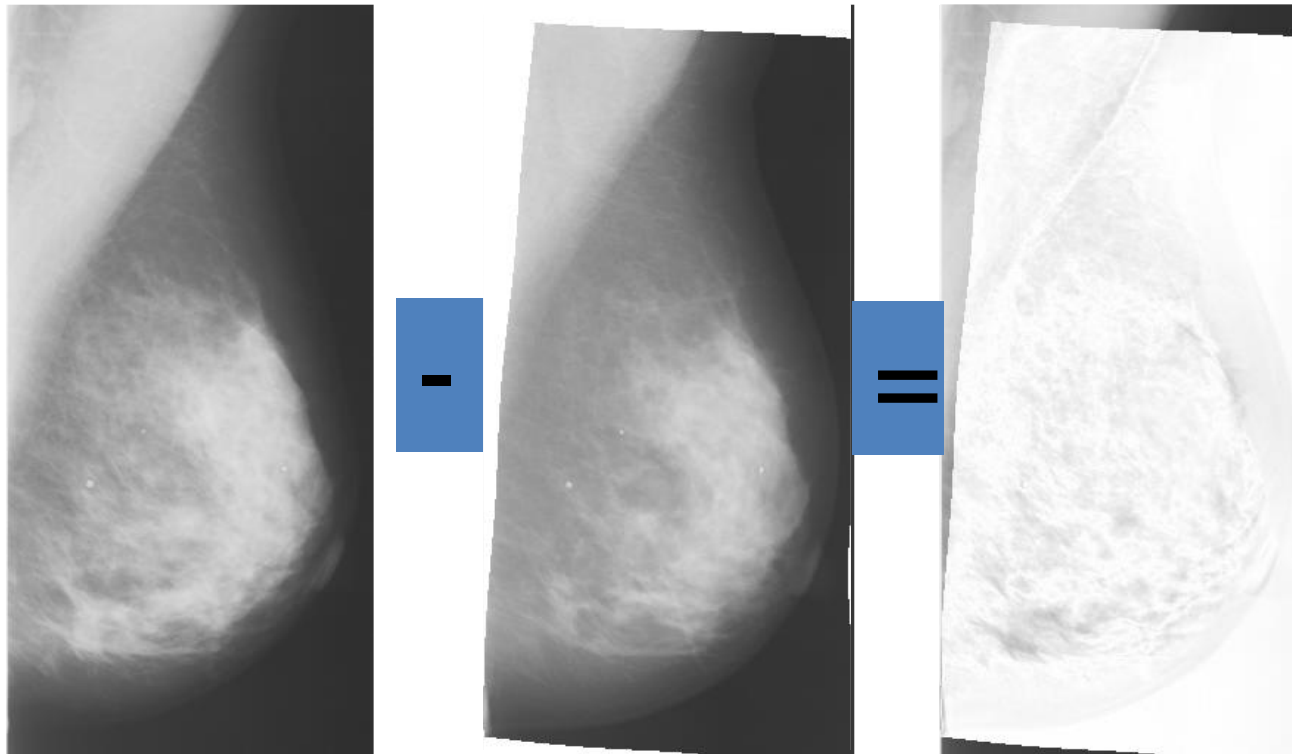


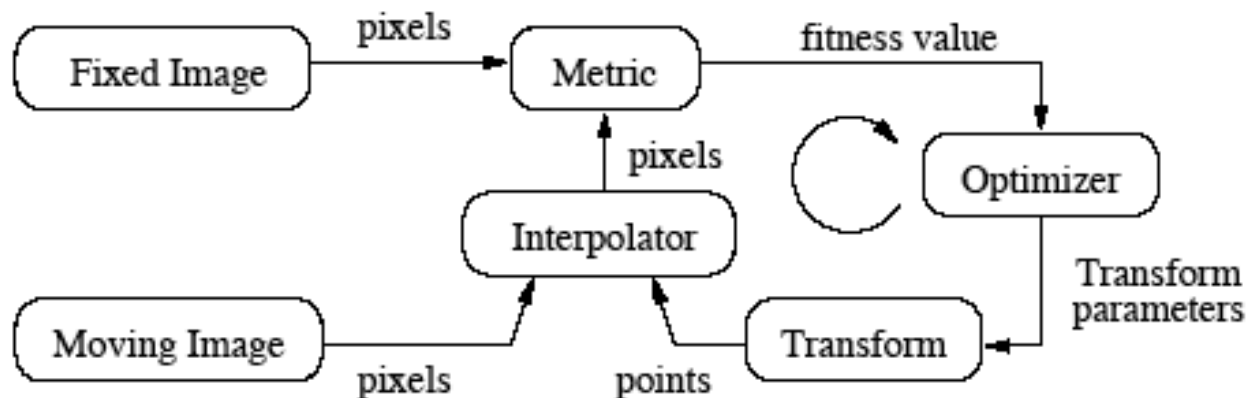
Image Registration. Classification

- Classification:
 - Rigid vs non-rigid (aka deformable, local)



Components

- **Transform:** allowed movement of the moving image, depends on some parameters.
- **Metric:** Measure of similarity between fixed and moving.
- **Interpolator:** Determines the pixel intensity given the position and neighbour pixels.
- **Optimizer:** finds the best metric value with respect to the transformation parameters.



Transform

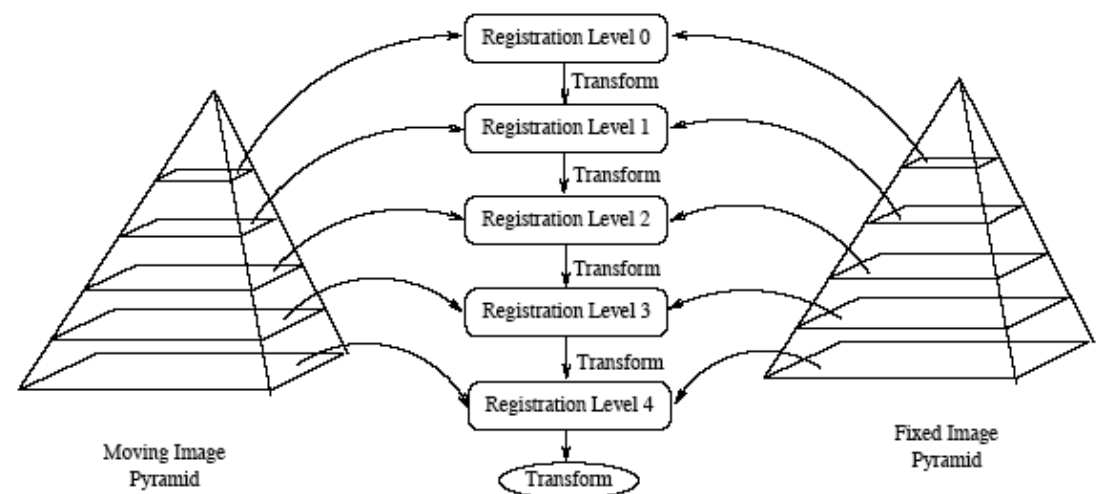
- **Many different classifications**
 - Global vs Local
 - Parametric vs Non-parametric
- Global: all pixels suffer the “same” transformation.
 - Rigid: Rotation and translation
 - Similarity: Rigid + Scaling
 - Affine: Similarity + Shearing and tearing (keeps parallelism)
 - Perspective



¹ Zitová et al., *Image registration methods: A survey*, IVC, 2003

Transform

- **Local:** pixels can move independently at a local level.
 - Thin Plate Splines. Based on point correspondence and spline interpolation
 - B-Splines (Free-Form deformations). Deform grids of points based on B-spline interpolation.
 - Demons, Finite Element Modelling, Poly Rigid, Local Affine, ...
- **Variations**
 - Global + Local
 - Multi-resolution



Global Transform

- Rigid**

$$f_x(x, y) = x \cos \phi + y \sin \phi + t_x$$

$$f_y(x, y) = -x \sin \phi + y \cos \phi + t_y$$

- Affine**

$$f_x(x, y) = a_x x + a_y y + t_x$$

$$f_y(x, y) = b_x x + b_y y + t_y$$

- Projective**

$$f_x(x, y) = \frac{a_x x + a_y y + t_x}{c_x x + c_y y + 1}$$

$$f_y(x, y) = \frac{b_x x + b_y y + t_y}{c_x x + c_y y + 1}$$



Local (non-rigid) Transformations

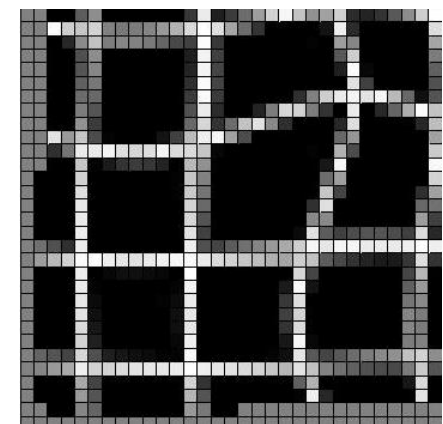
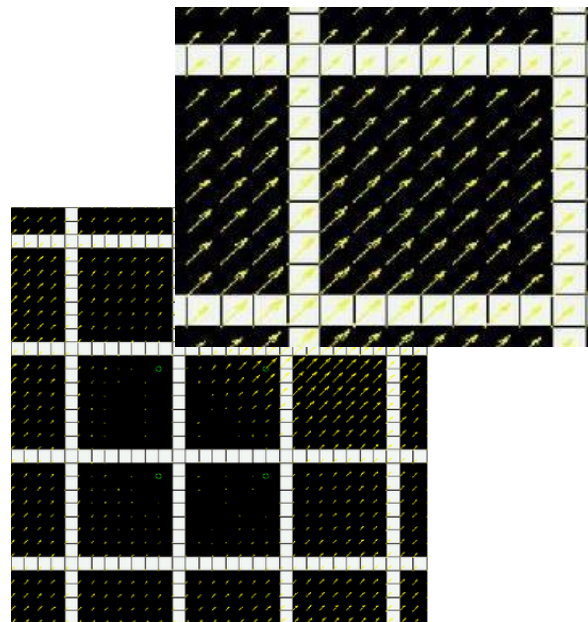
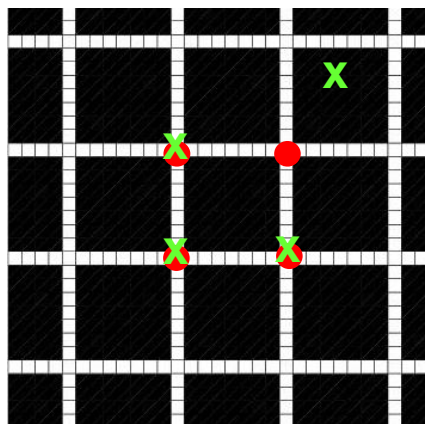
- Thin Plate Splines

$$\iint_{R^2} \left(\left(\frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \right) dx dy$$

$$U(r) = r^2 \log r^2$$

$$f_x(x, y) = a_x x + a_y y + t_x + \sum_{i=1}^N w_i U(|\vec{p}_i - (x, y)|)$$

$$f_y(x, y) = b_x x + b_y y + t_y + \sum_{i=1}^N w_i U(|\vec{p}_i - (x, y)|)$$



Local (non-rigid) Transformations

- B-Splines grid (Free-form deformations)**

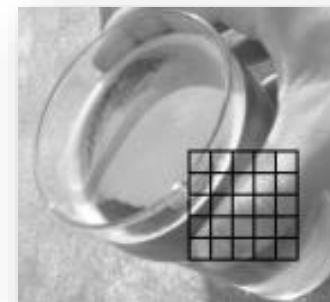
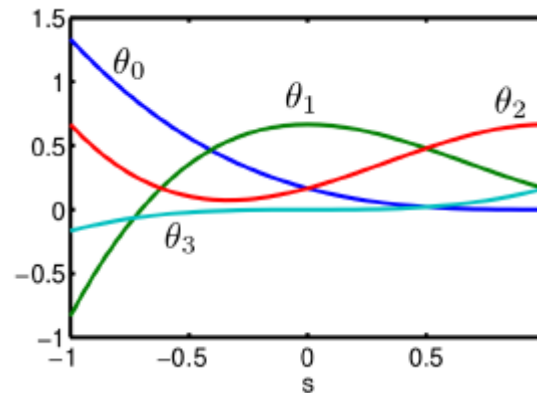
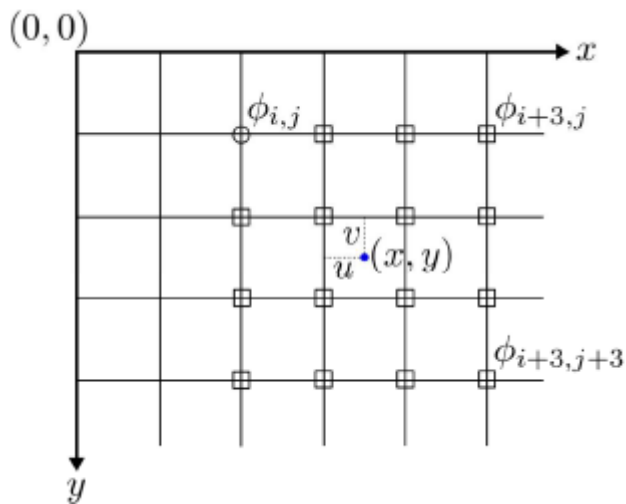
$$\mathbf{T}_{\text{local}}(x, y, z) = \sum_{l=0}^3 \sum_{m=0}^3 \sum_{n=0}^3 B_l(u) B_m(v) B_n(w) \phi_{i+l, j+m, k+n}$$

$$B_0(u) = (1 - u)^3 / 6$$

$$B_1(u) = (3u^3 - 6u^2 + 4) / 6$$

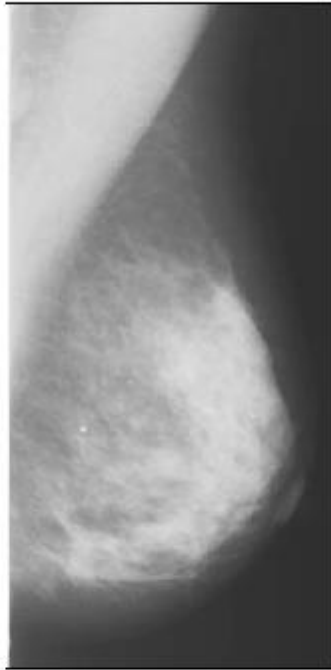
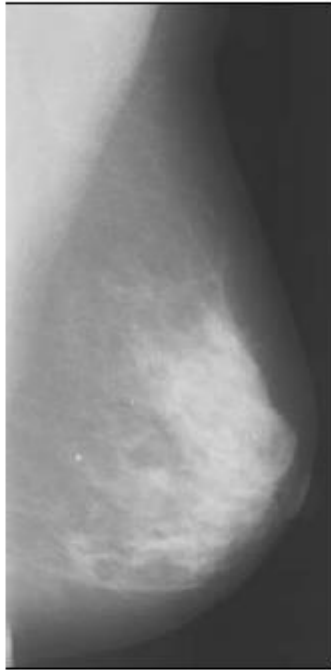

$$B_2(u) = (-3u^3 + 3u^2 + 3u + 1) / 6$$

$$B_3(u) = u^3 / 6.$$



Similarity Metrics

How similar two images are?

Image A	Image B	Image C
		
Measure	0.38306	0.518002

Similarity Metrics

- Measure the similarity between images
- Definition of a metric between image A and B (C).

$$R1) \quad d(A, B) > 0$$

$$R2) \quad d(A, B) = 0 \text{ iff } A = B$$

$$R3) \quad d(A, B) = d(B, A)$$

$$R4) \quad d(A, B) \leq d(A, C) + d(C, B)$$

Similarity Metrics

- Simple Intensity difference (Root mean squared)

$$RMS(A, B) = \frac{1}{N} \sqrt{\sum_{x=0}^X \sum_{y=0}^Y (A(x, y) - B(x, y))^2}$$

- Normalised cross correlation
 - Correlation between pixels in both images.
 - High values denote high similarity
 - Intensity, gradient, etc.

$$R(A, B) = \frac{\sum_{x=0}^X \sum_{y=0}^Y (A(x, y) - \overline{I_A})(B(x, y) - \overline{I_B})}{\sqrt{\sum_{x=0}^X \sum_{y=0}^Y (A(x, y) - \overline{I_A})^2 \sum_{x=0}^X \sum_{y=0}^Y (B(x, y) - \overline{I_B})^2}}$$

Similarity Metrics

- Entropy related measures
 - Mutual information
 - Entropy of the difference image
 - Low values, low entropy, hence similar images?

$$S_{AB} = \sum_{x=0}^X \sum_{y=0}^Y A(x, y) - sB(x, y)$$

$$H(S_{AB}) = - \sum_{i=0}^I p(i) \log(p(i))$$

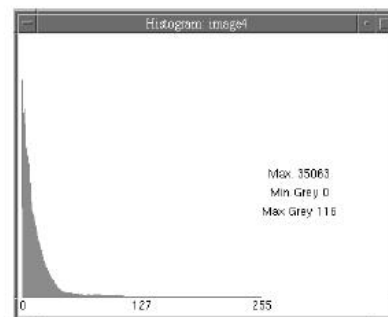
Similarity Metrics

- Entropy of the difference image

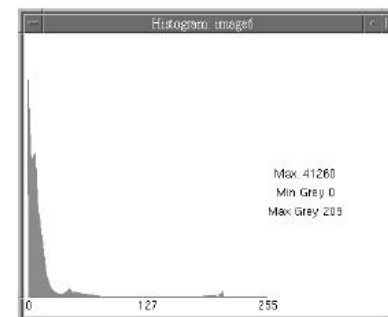
A-B



A-C



1.59448



1.52891

Similarity Metrics

- Mutual Information

$$C(A, B) = \sum_{i=0}^I \sum_{j=0}^J p_{AB}(i, j) \log \frac{p_{AB}(i, j)}{p_A(i)p_B(j)}$$

$$= H(A) + H(B) - H(A, B)$$

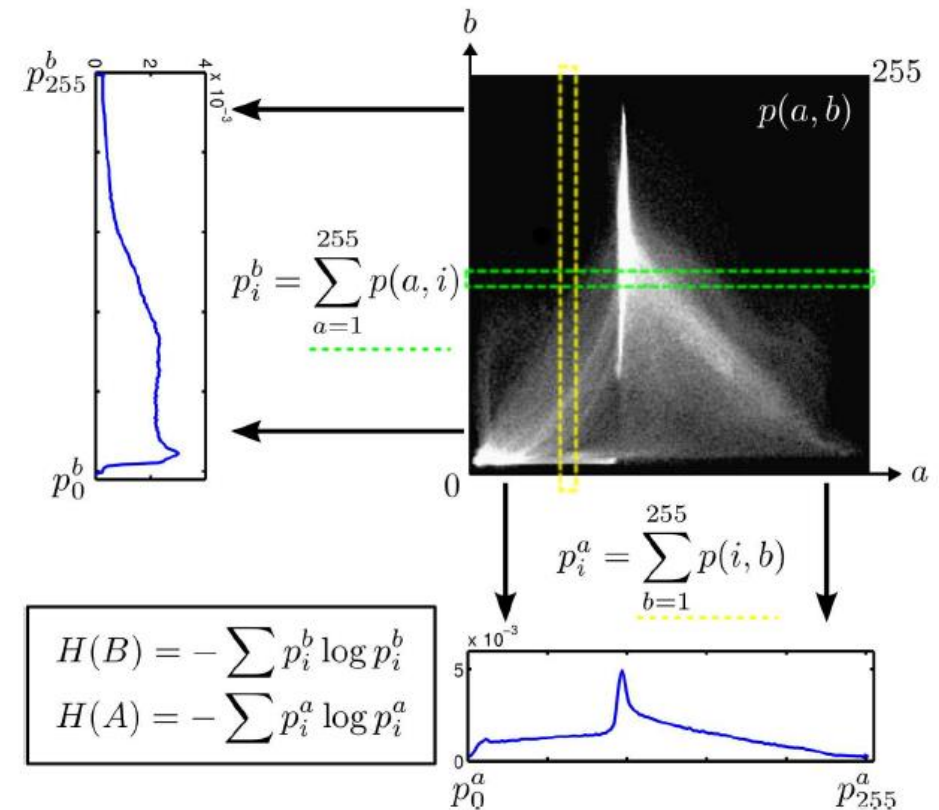
Joint histogram



Before
0.38306

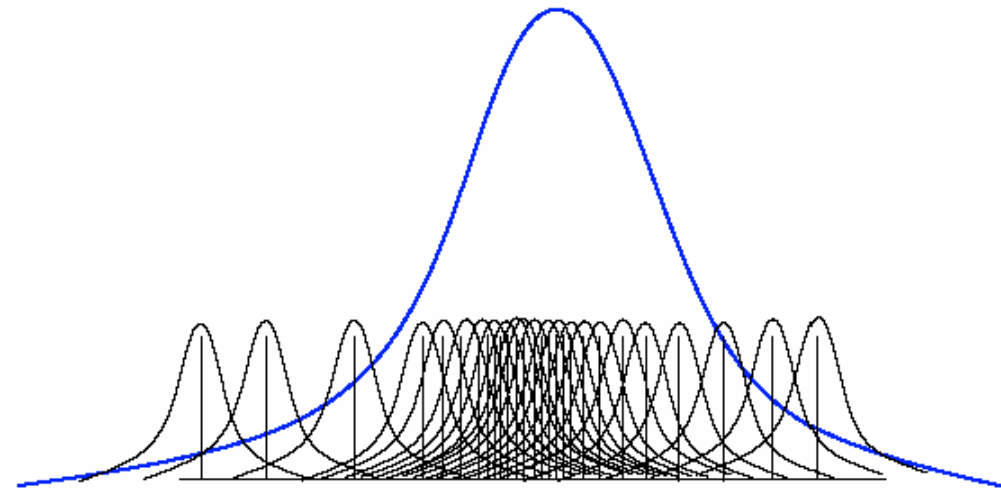
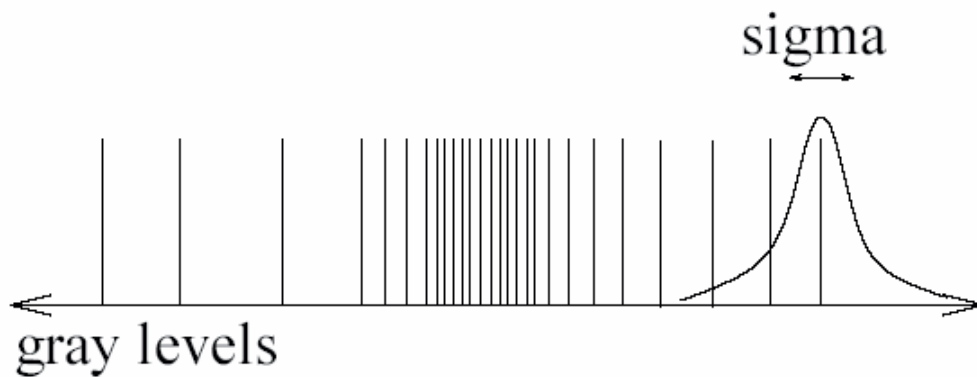


After
0.518002



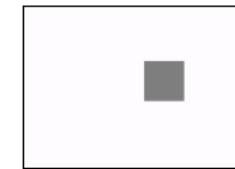
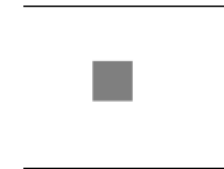
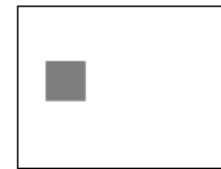
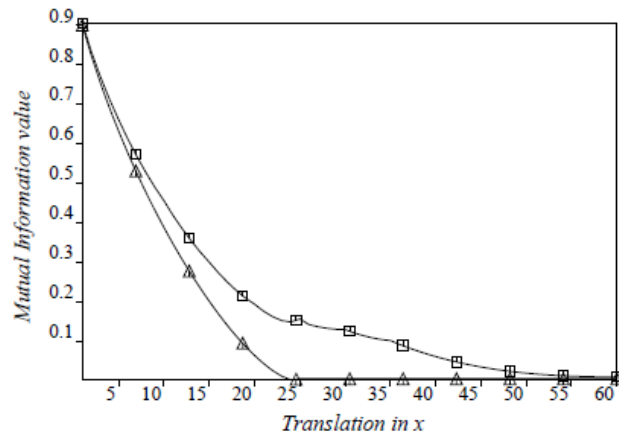
Similarity Metrics

- How to estimate the histogram?
- Has to be a continuous distribution!
- Parzen Window estimation



Similarity Metrics

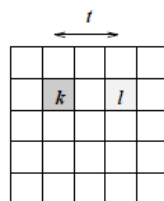
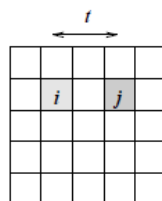
- Variations: Non-overlapping MI.



MI	0.012189	0.012189
Novel	0.183060	0.037711

- Incorporate spatial information into MI
- Pixel pairs instead of single pixels (GLCM)

Image A Image B



A-B	A-C
0.335626	0.441571

Similarity Metrics

- Mathematically express the human perception of similarity
- Does not always work!

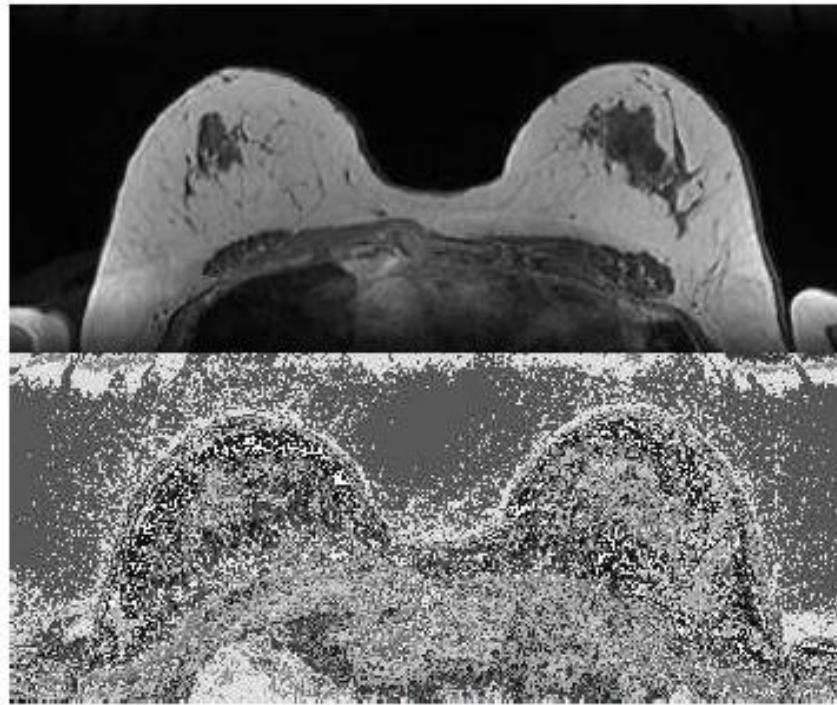


Figure 1. Example of image intensity swapping for an axial slice through an MRI breast volume. Pixel intensity values in the original image **a** are binned and arbitrarily rearranged to give image **b**. The mutual information (MI) between these two images is maximal (here, for 64 bins, $MI=6$) and the images may be regarded as being identical.

Similarity Metrics

- Mathematically express the human perception of similarity
- Does not always work!

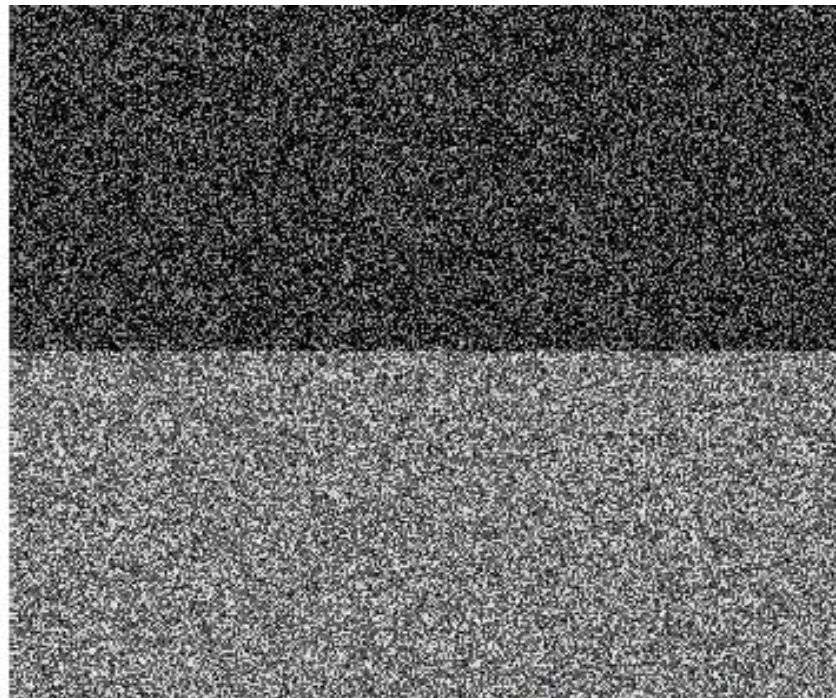
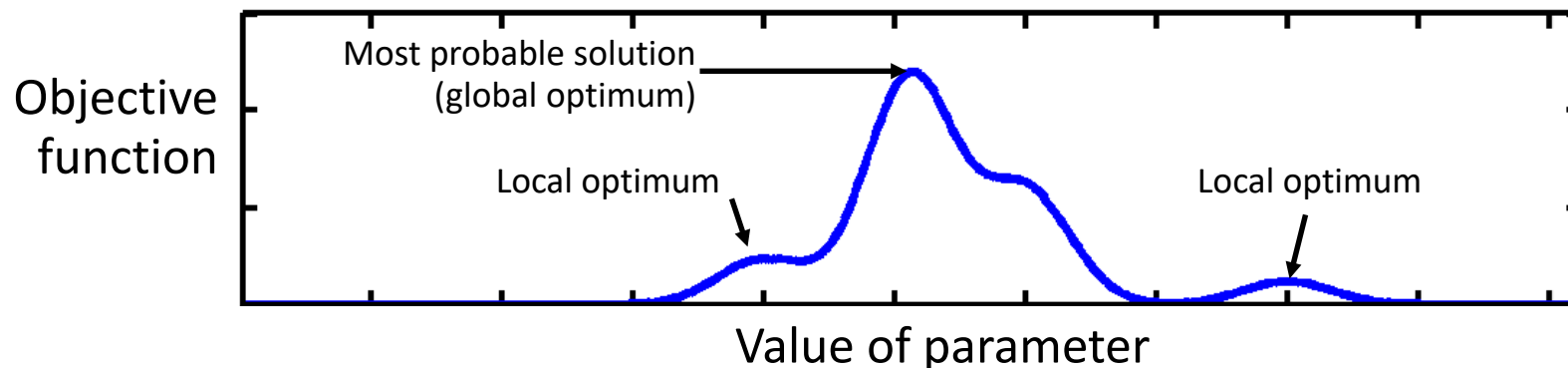


Figure 2. Example of intensity position swapping for an axial slice through an MRI breast volume (1a). The pixel positions are arbitrarily rearranged in space. These images have the same marginal entropy as Figure 1a. Figure 2b has had the intensity values swapped as in Figure 1b. Again, the mutual information (MI) between these two images is maximal (here, for 64 bins, $MI=6$) and the 'images' may be regarded as being identical

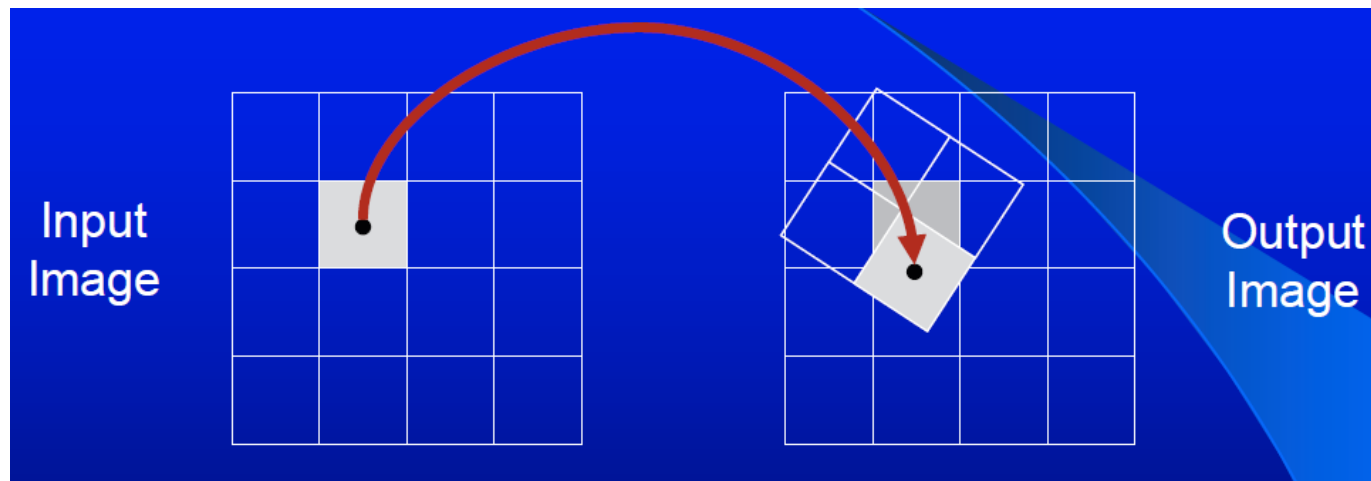
Optimizer

- Finds the transformation parameters that maximises a similarity measure
- Gradient descent algorithms
- Usually need to specify the gradient of the metric with respect to the transformation parameters



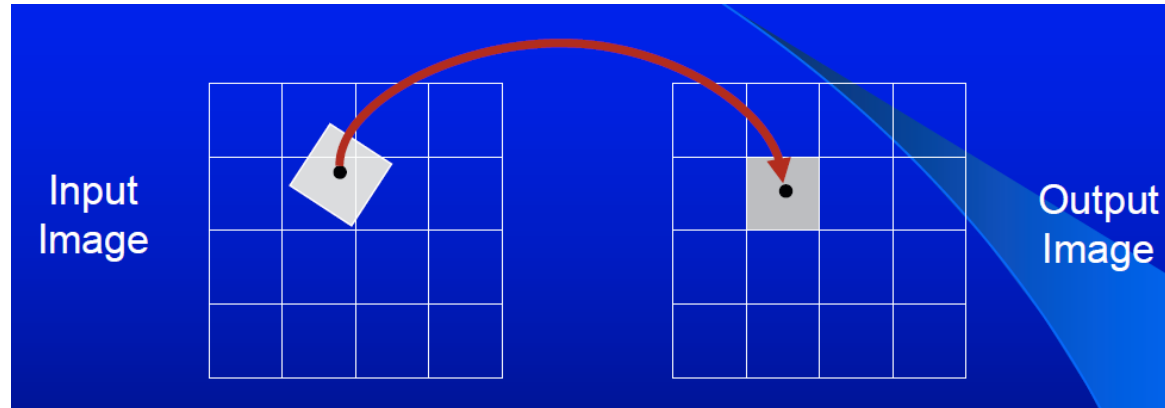
Mapping

- Forward mapping
 - Input image pixel is mapped onto the output image
 - Output pixels with more than one hit: overlap
 - Value must be accumulated from overlapping pixels
 - Output pixels with no hits: hole



Mapping

- Inverse mapping
 - Output pixels are mapped back onto the input image
 - Output pixel value must be interpolated from a neighborhood in the input image
 - Scheme avoids any holes and overlaps in the output image because all pixels are scanned sequentially



Interpolator

- Transforming the image

```
for y=1..ny % loop over rows
    for x=1..nx % loop over columns
        x' = fx(x,y) % transform
        y' = fy(x,y)
        if 1 ≤ x' ≤ nx, & 1 ≤ y' ≤ ny, then % voxel in range
            newB(x,y) = B(x',y') % assign re-sampled value
        end % voxel in range
    end
end
```

- What if x', y' are not integers?
- Inverse transform

Interpolator

- Transformed coordinates are usually non-integers, $m = (x', y')$
- Assign the intensity of
 - Nearest Neighbour

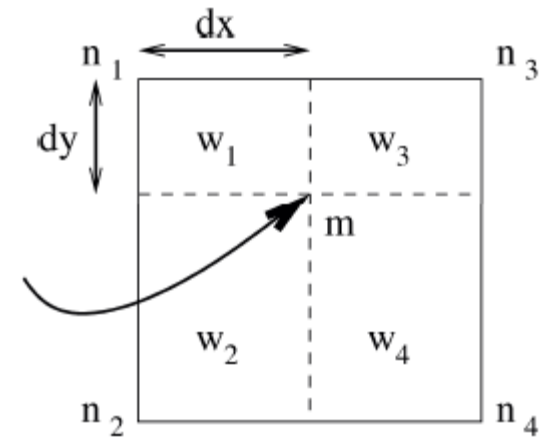
$$B(m) = B(\min_{\forall i}(d_E[m, n_i]))$$

- Linear interpolation

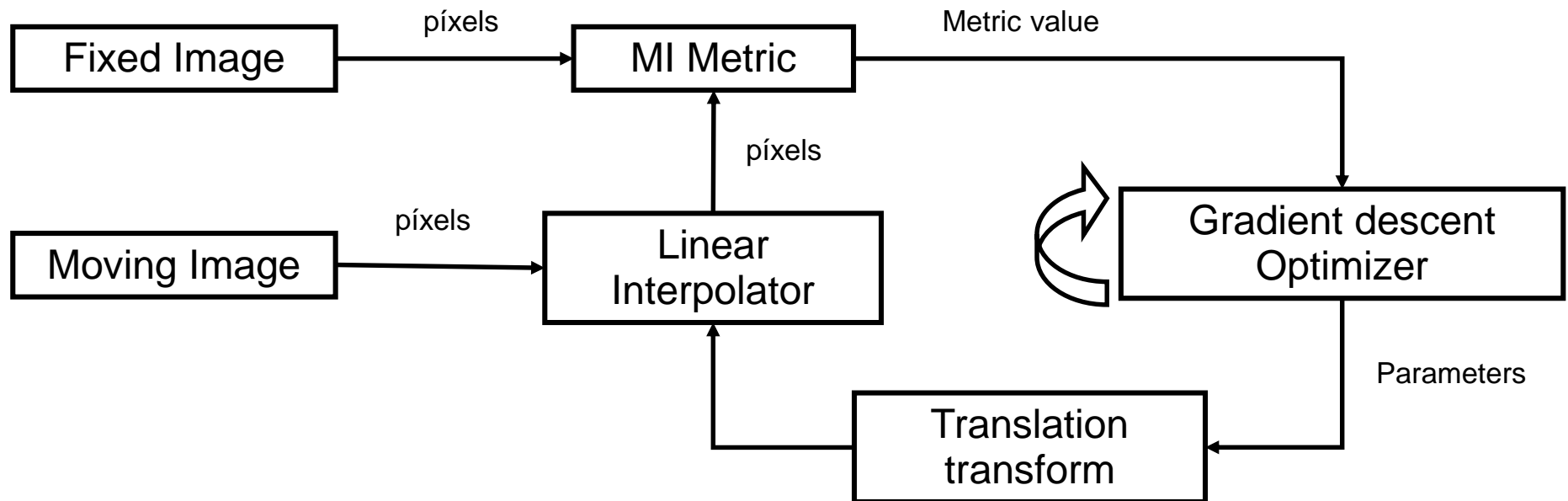
$$B(m) = \sum_i w_i B(n_i)$$

$$w_1 = (1 - dx)(1 - dy) \quad w_2 = (1 - dx)dy \quad w_3 = dx(1 - dy) \quad w_4 = dxdy$$

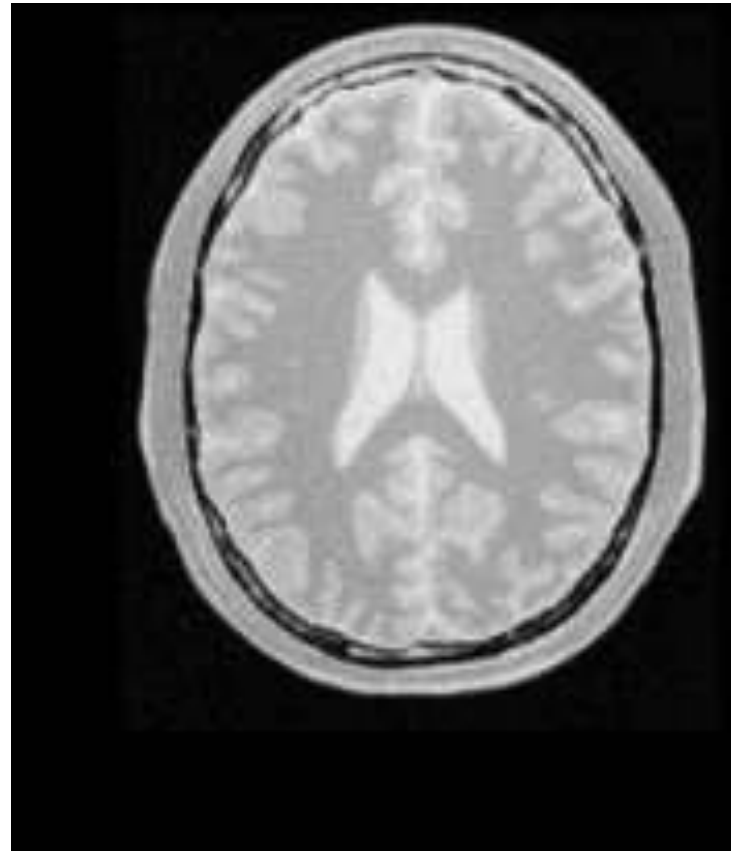
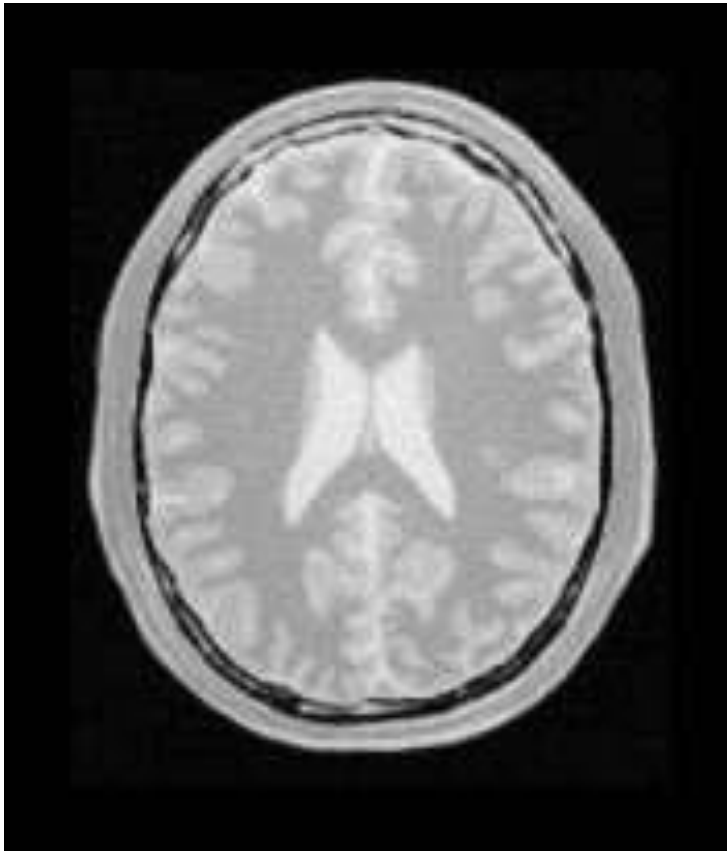
- Cubic Interpolation



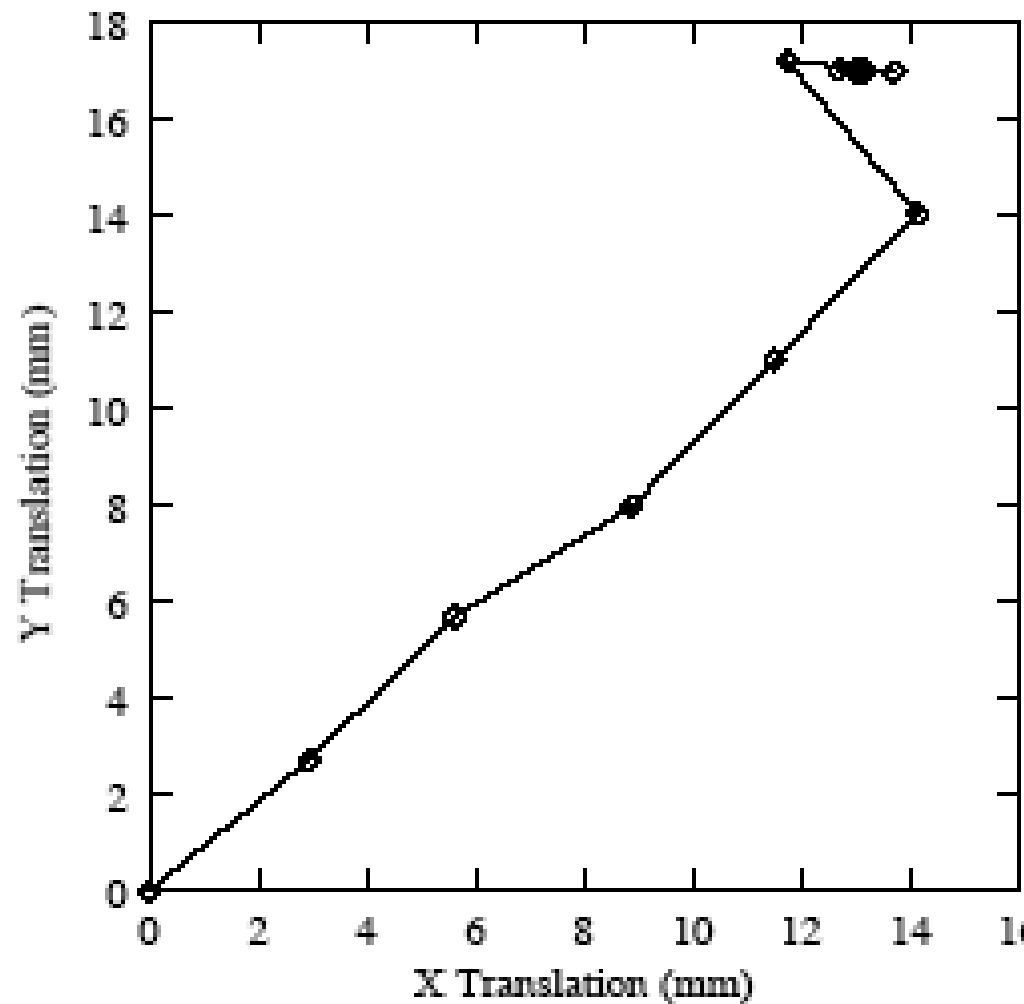
Hello world Registration example



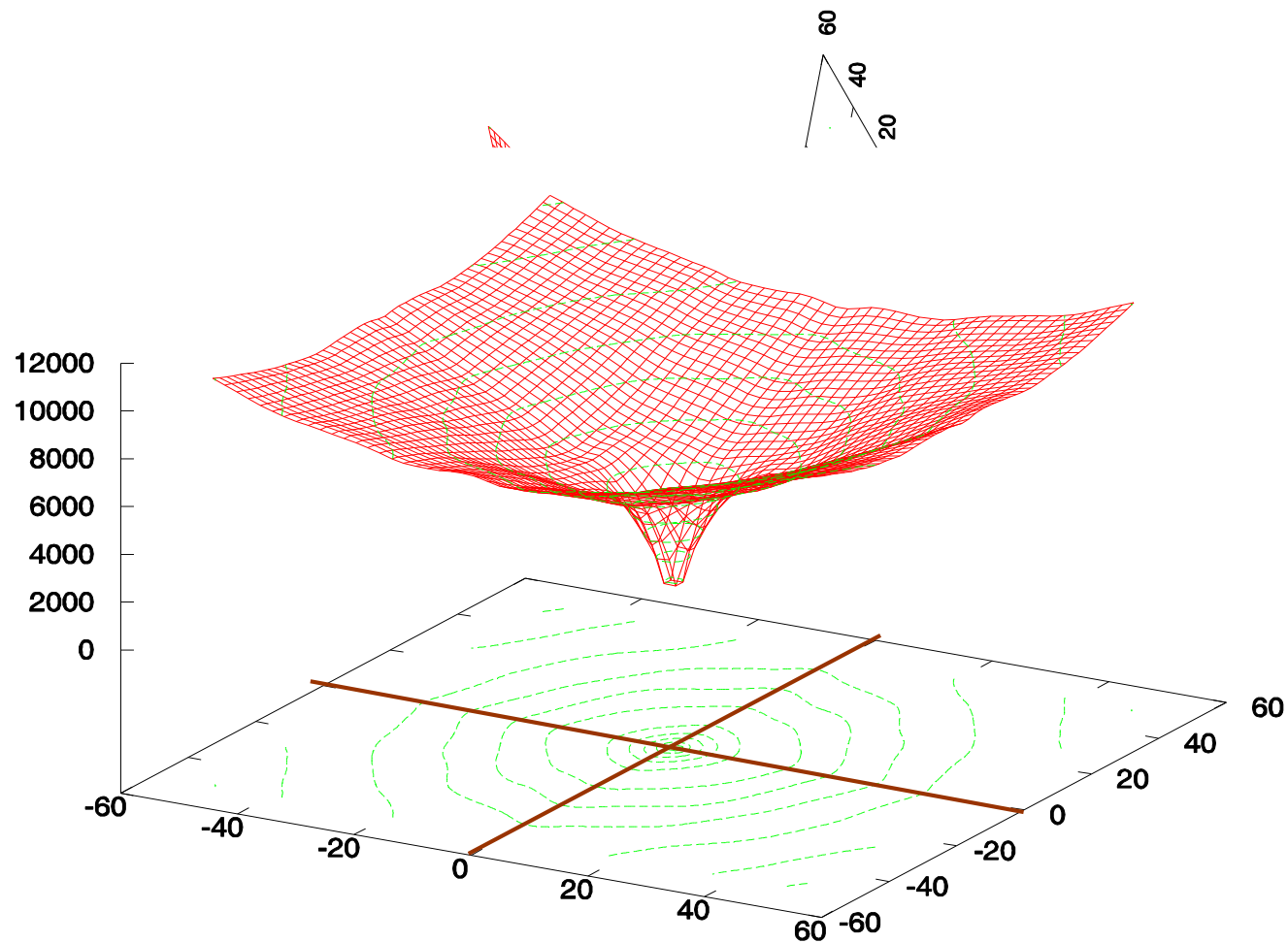
Hello world input



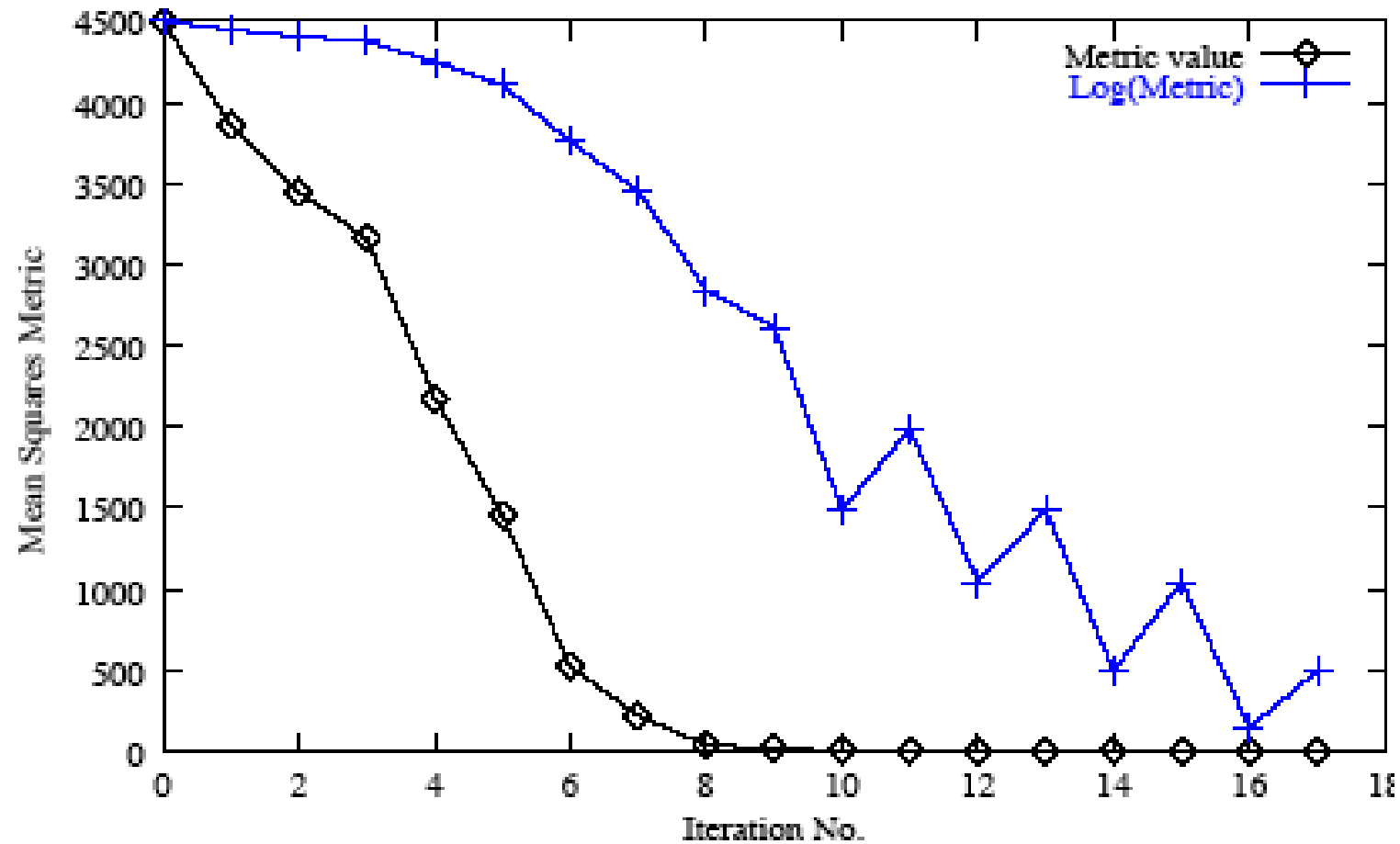
X & Y translation vs. time



X & Y translation vs. time

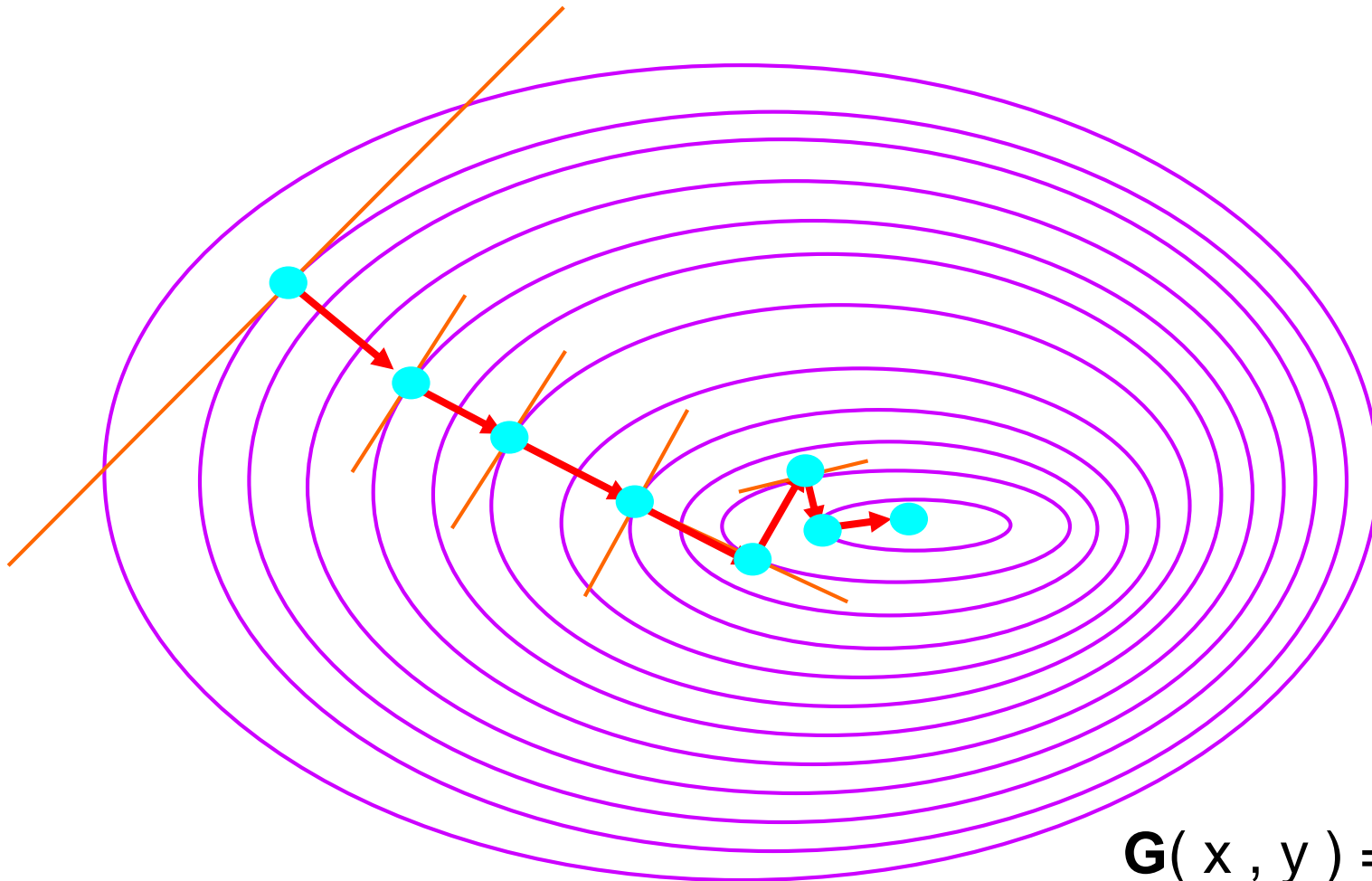


Metric vs. time



Gradient Descent

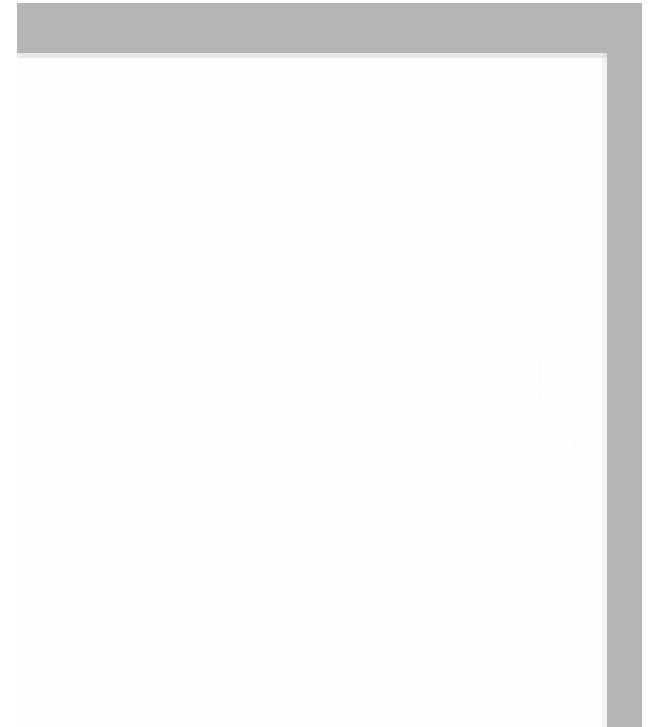
Gradient Descent Optimizer



$$f(x, y)$$

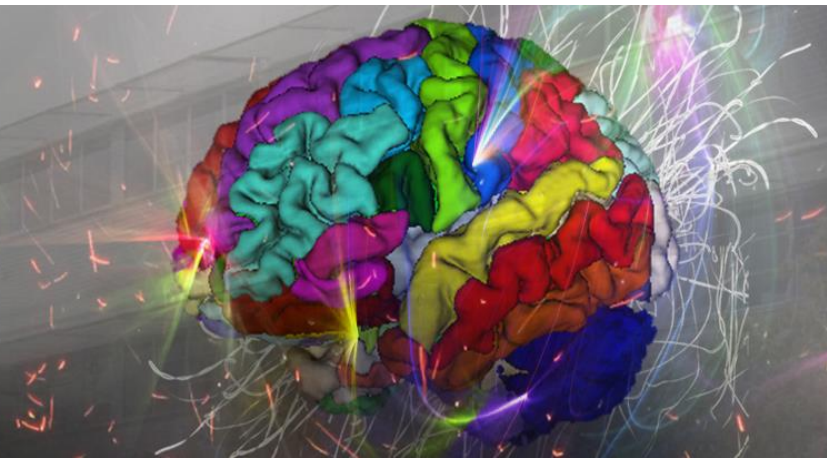
$$\mathbf{G}(x, y) = -\nabla f(x, y)$$

Image comparison



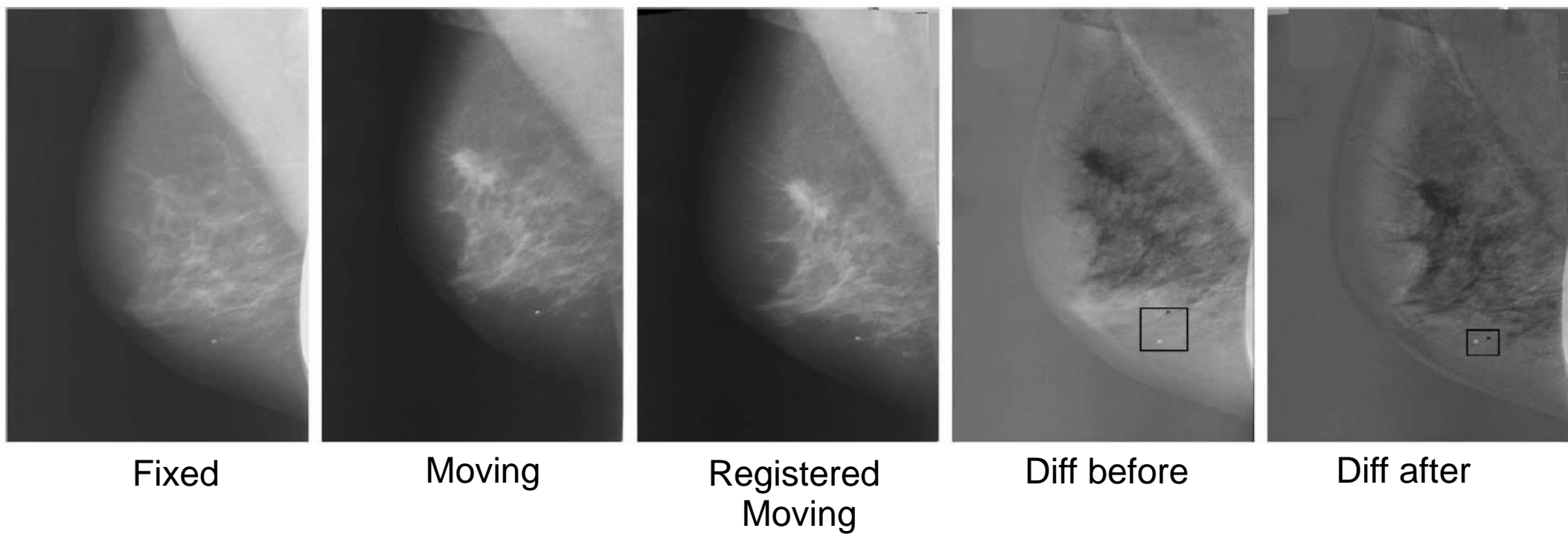


Evaluating your registration



How to evaluate?

You have developed your image registration algorithm... now what?



Validation of algorithms

- Technical validation
 - Accuracy, Robustness, speed, reliability
- Clinical Validation
 - Is it useful for the doctors?
 - Does it help to improve diagnosis/treatment?
- Comercial System
 - FDA Approval

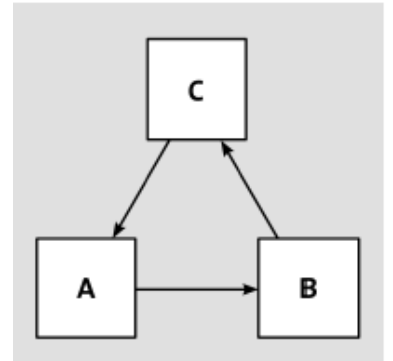
Evaluation criteria

- Robustness
 - Variability for starting conditions of the registration
 - Noise, allowed misregistration, landmark error, etc.
- Consistency
- Visual assessment
 - Subtraction images
 - Checkerboard, overlay
- Quantitative analysis
 - Landmark error
 - Simulated deformations
 - Similarity metrics.
- Ground Truth!?

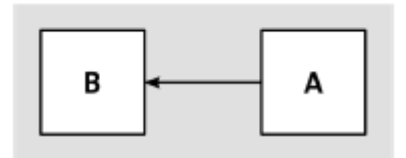
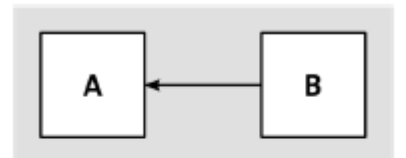
Consistency

- Apply various registration in order to evaluate the consistency.

$$T_{AB}(T_{BC}(T_{CA}(x))) = I(x) = x$$



- Forward /inverse consistency
 - Bijectivity/diffeomorphic registration



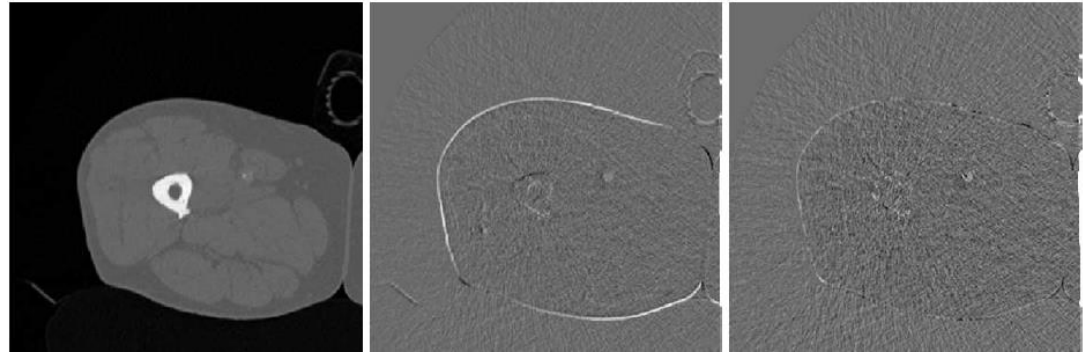
- Does not measure accuracy!!

Visual Assessment

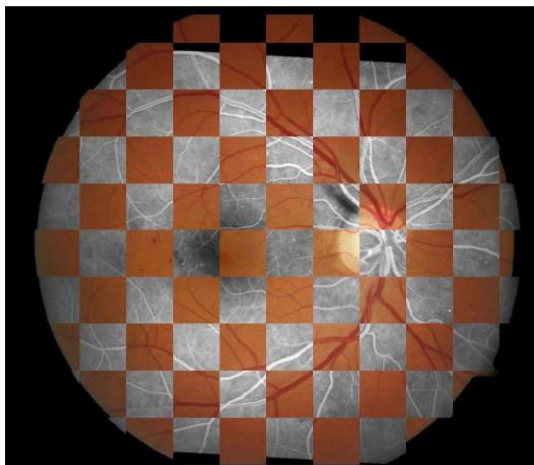
- Qualitative assessment. Observer opinion.
- Intra – inter observer variability!
- Subtraction images
- Overlay
 - Checkboard
 - Contours
 - Colors

Visual Assessment

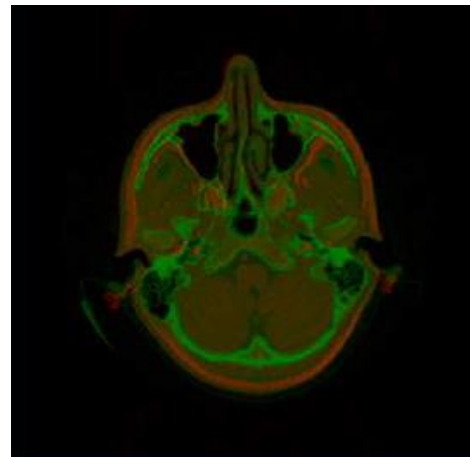
- Subtraction images
 - Absolute?



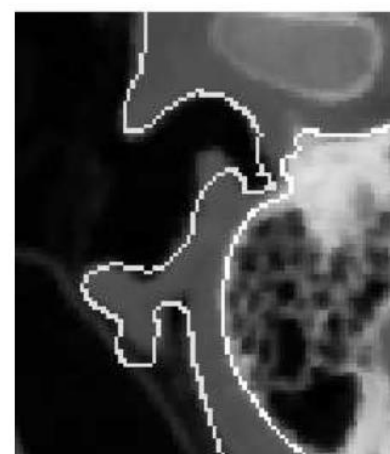
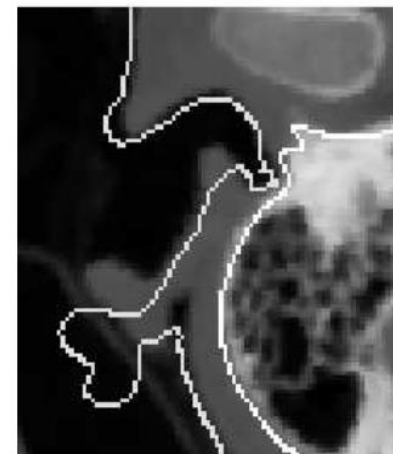
- Overlay



Checker board



Color (R & G)



Contour

Visual Assessment

- Example
 - Observer study

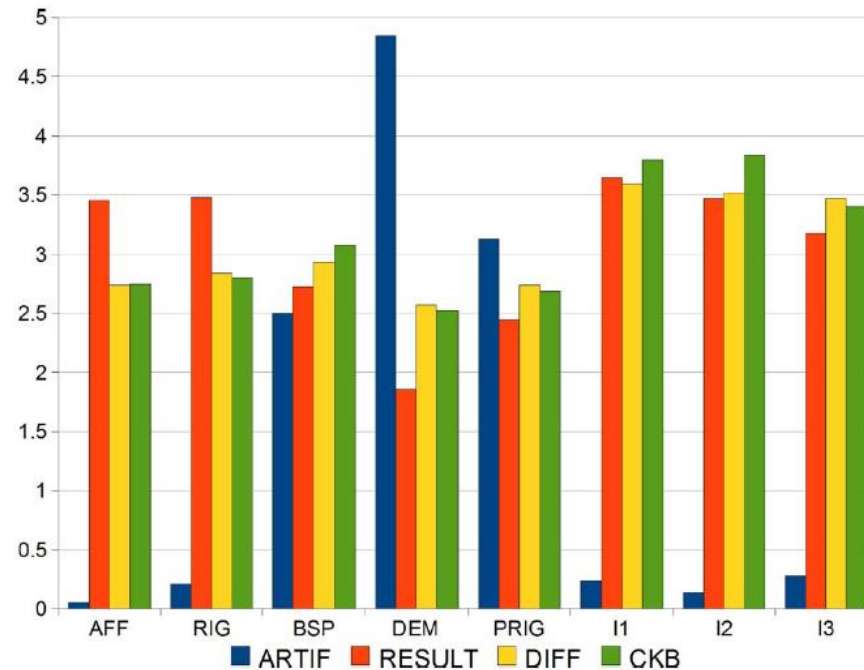
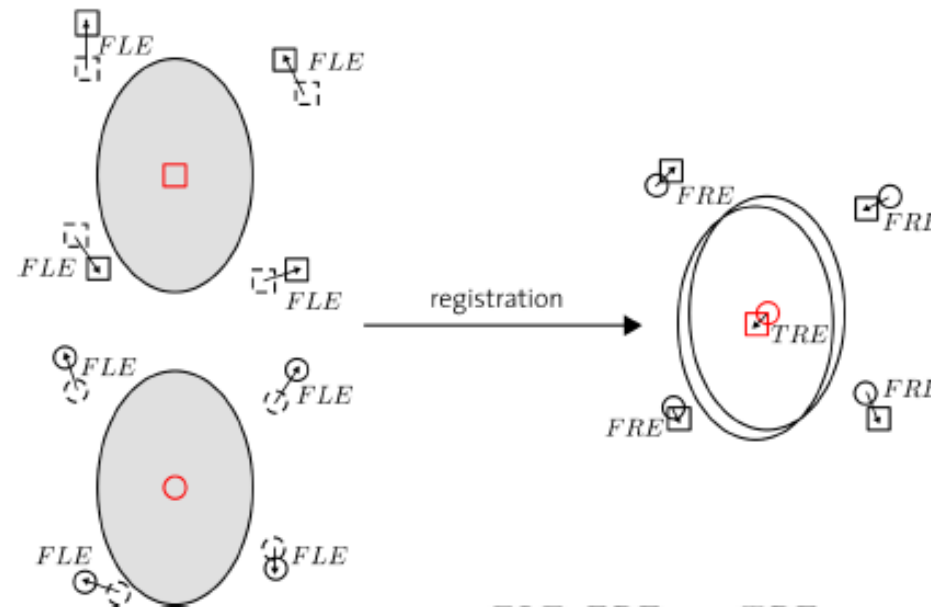
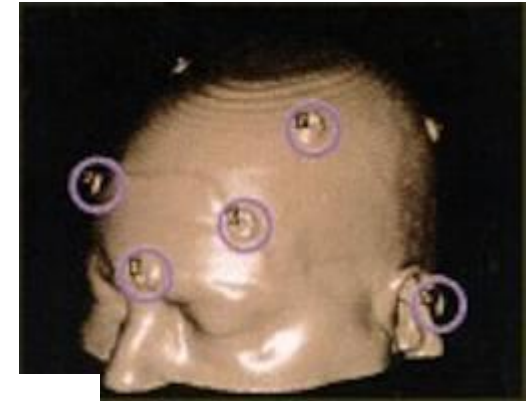


Fig. 4. Summary of the observer study. Bars show observer perception, in all cases higher means better except for the number of artifacts (ARTIF) were lower means better. AFF = affine, RIG = rigid, BSP = B-Splines FFD, DEM = Demons, PRIG = polyrigid, I1 = MR BSP, I2 = AFF + BSP, I3 = MR AFF + MR BSP.

Y.Díez, A.Oliver, X.Lladó, J.Freixenet, J.Martí, J.C.Vilanova, and R.Martí. Revisiting intensity-based image registration applied to mammography. IEEE Trans. on Information Technology in BioMedicine, 15(5), pp 716-725, 2011.

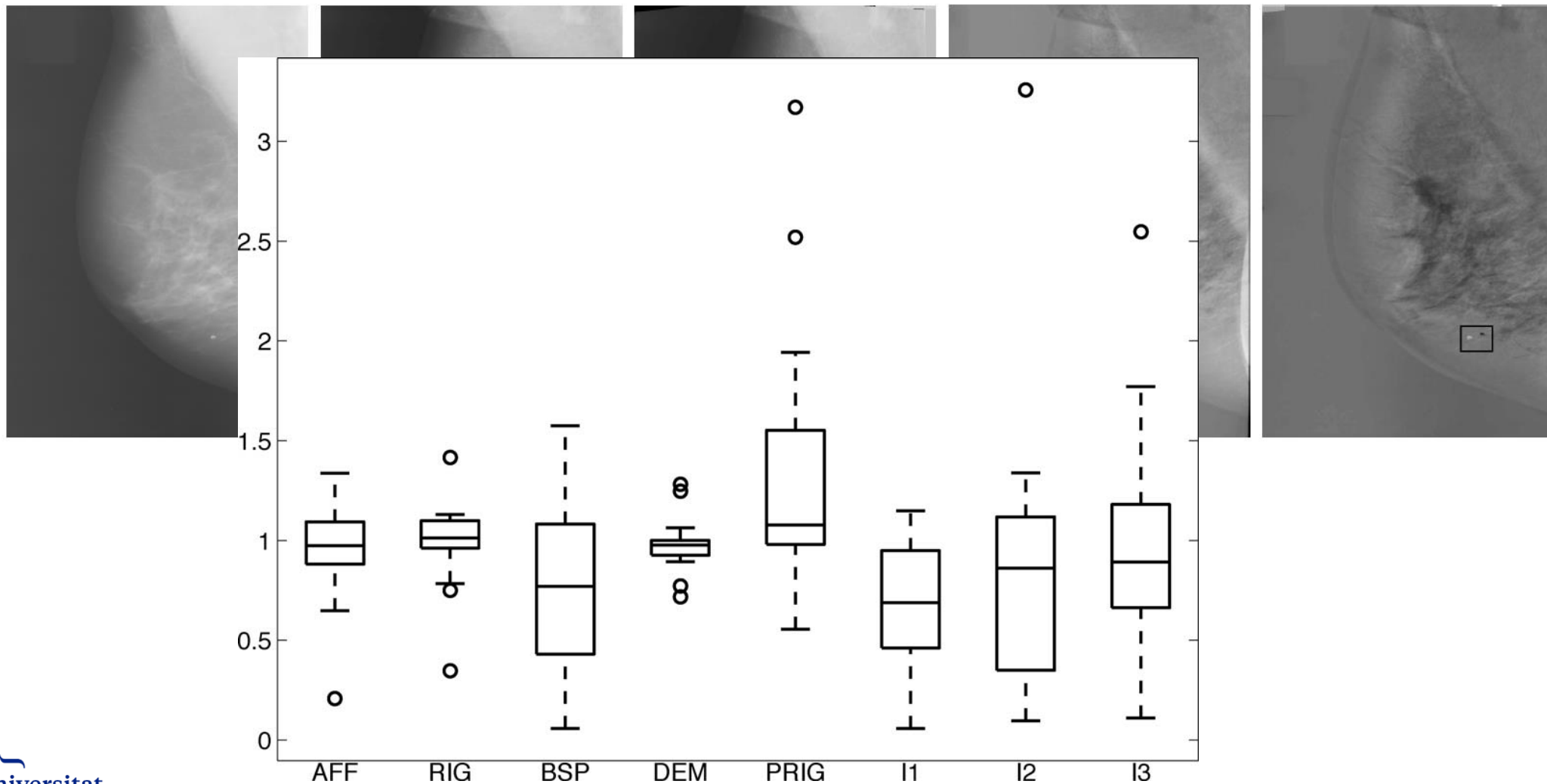
Errors

- Fiducial Localization Error (FLE)
- Fiducial Registration Error (FRE)
- Target Registration Error (TRE)



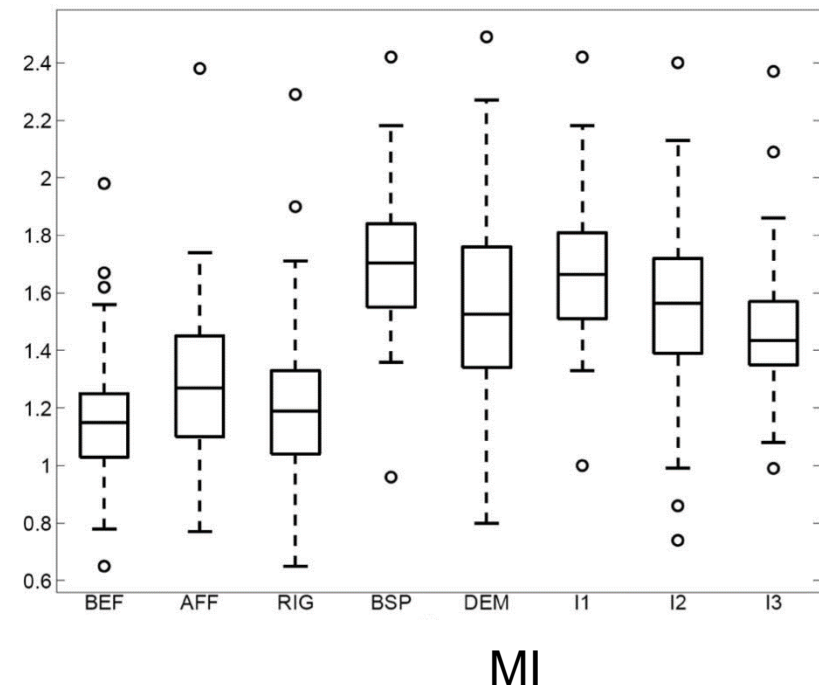
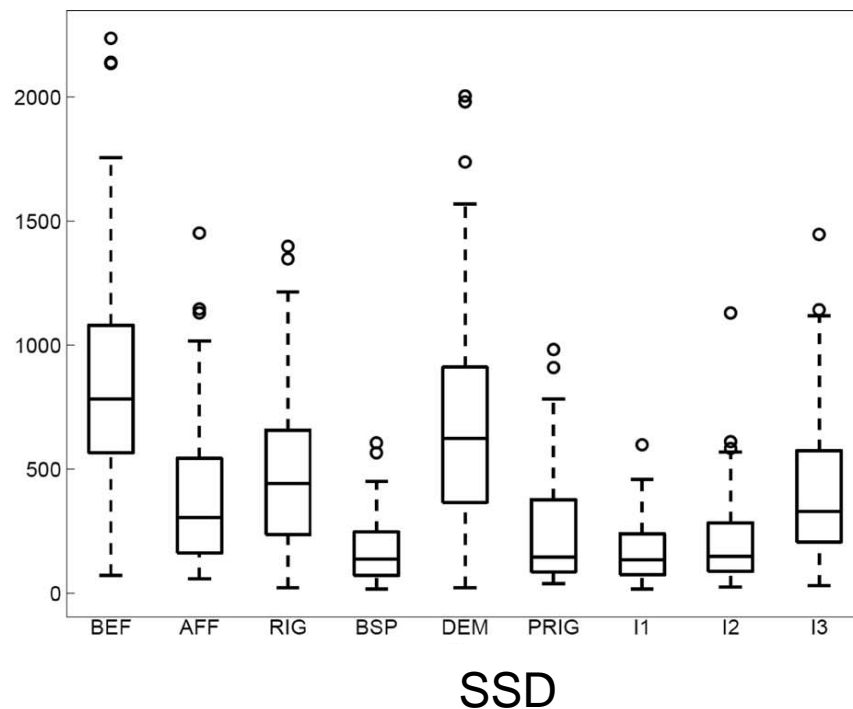
Errors. example

- TRE: Computing error after registration between microcalcifications.



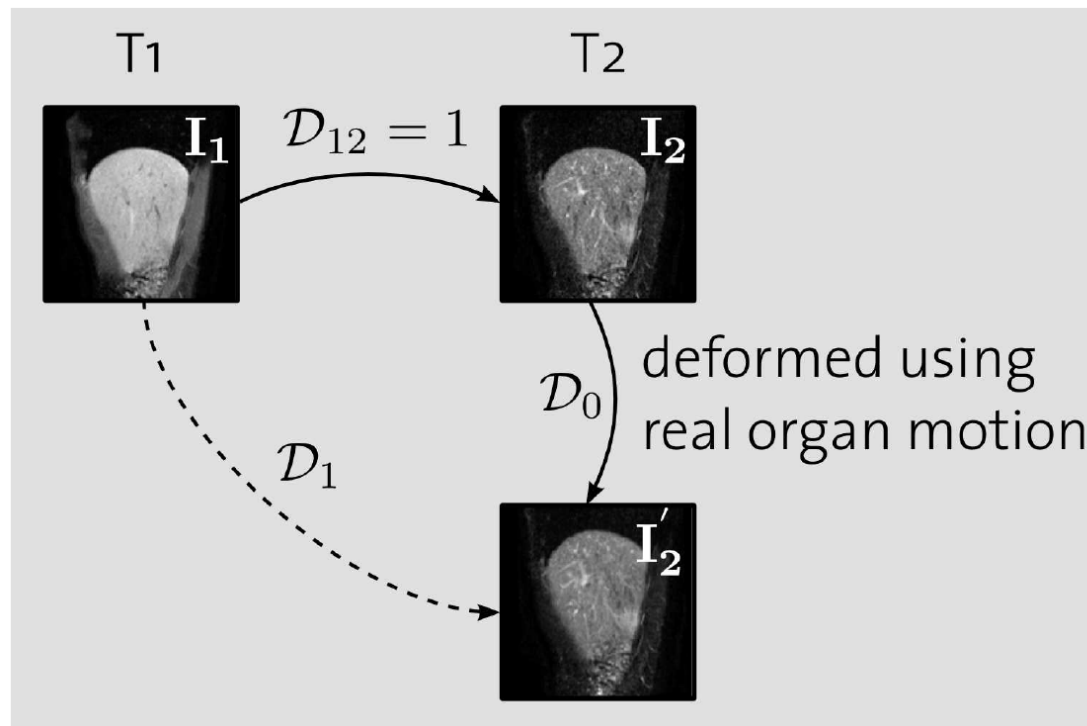
Similarity metrics

- Metric comparison
 - A higher metric DOES NOT always mean better registration

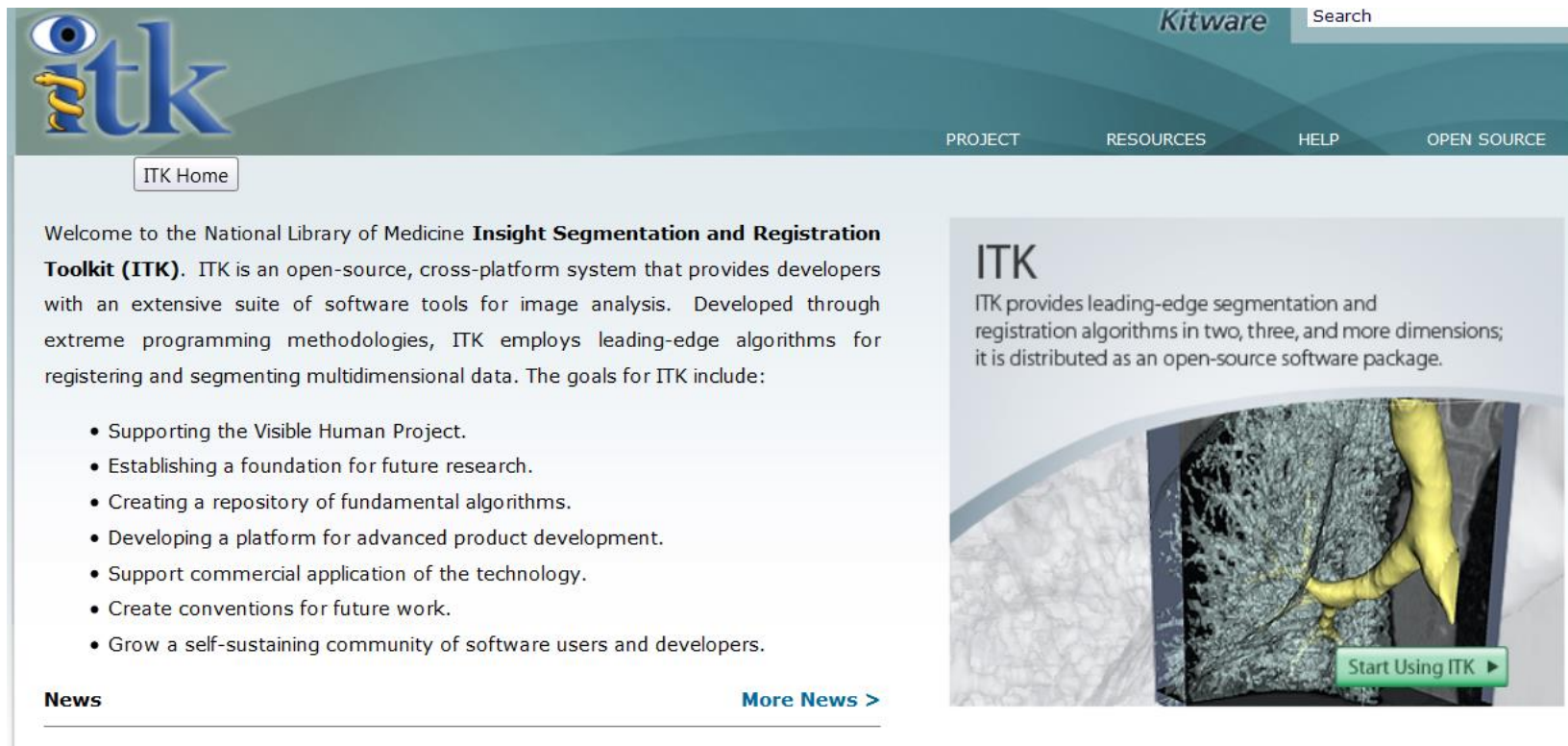


Synthetic / Simulated models

- Difficulty of obtaining the ground truth
- Synthetic/Simulated examples
- Simulate deformation and recover it with IR.



ITK & Image registration



The screenshot shows the ITK website with the following content:

ITK Home

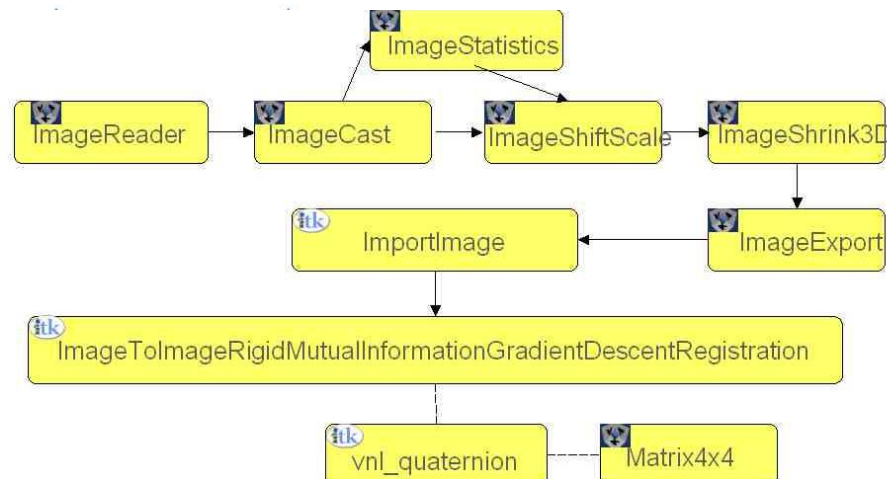
Welcome to the National Library of Medicine **Insight Segmentation and Registration Toolkit (ITK)**. ITK is an open-source, cross-platform system that provides developers with an extensive suite of software tools for image analysis. Developed through extreme programming methodologies, ITK employs leading-edge algorithms for registering and segmenting multidimensional data. The goals for ITK include:

- Supporting the Visible Human Project.
- Establishing a foundation for future research.
- Creating a repository of fundamental algorithms.
- Developing a platform for advanced product development.
- Support commercial application of the technology.
- Create conventions for future work.
- Grow a self-sustaining community of software users and developers.

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ITK
ITK provides leading-edge segmentation and registration algorithms in two, three, and more dimensions; it is distributed as an open-source software package.

[Start Using ITK ▶](#)



Matlab & Image registration

The `imregconfig` function makes it easy to pick the correct optimizer and metric configuration to use with `imregister`. These two images have different intensity distributions, which suggests a multimodal configuration.

```
[optimizer,metric] = imregconfig('multimodal');
```

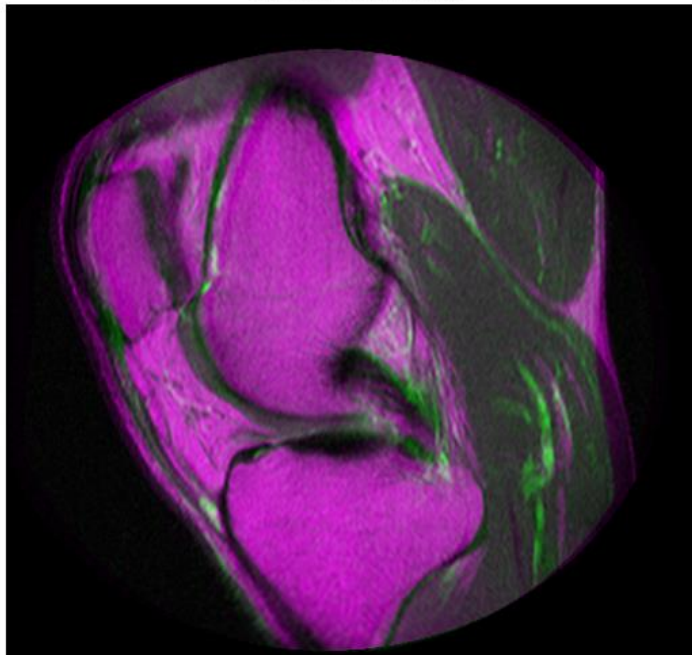
The distortion between the two images includes scaling, rotation, and (possibly) shear. Use an affine transformation to register the images.

It's very, very rare that `imregister` will align images perfectly with the default settings. Nevertheless, using them is a useful way to decide which properties to tune first.

```
movingRegisteredDefault = imregister(moving, fixed, 'affine', optimizer, metric);

f3 = figure;
imshowpair(movingRegisteredDefault, fixed);
figure(f3);
title('A: Default registration');
```

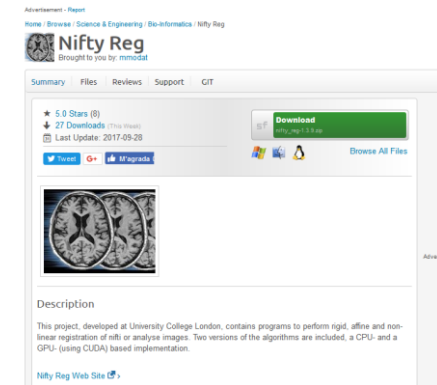
A: Default registration



Software & Libraries

- Niftyreg

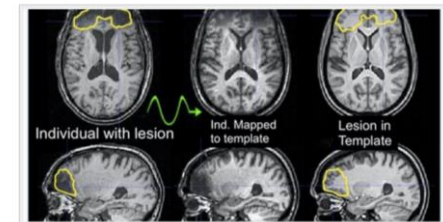
<https://sourceforge.net/projects/niftyreg/>



- ANTS

<http://picsl.upenn.edu/software/ants/>

ANTS

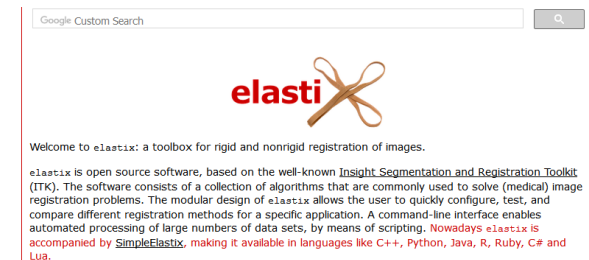


Advanced Normalization Tools (ANTs) extracts information from complex datasets that include imaging. ANTs development is led by Brian Avants and supported by other researchers and developers at PICS and other institutions.

ANTs is open source. Code, binaries and documentation are available at the [ANTs website](http://www.kitware.com/ANTs/).

- Elastix

<http://elastix.isi.uu.nl/>



- Getting Started
 - <http://www.itk.org/CourseWare/Training/GettingStarted-I.pdf>
 - <http://www.itk.org/CourseWare/Training/GettingStarted-III.pdf>
 - Example code: ImageRegistration1.cxx
- Software Guide <http://www.itk.org/ItkSoftwareGuide.pdf>

To know more...

- *Insight into Images: Principles and Practice for Segmentation, Registration and Image Analysis, Terry S. Yoo (Editor)*
- *Handbook of Medical Imaging: Processing and Analysis, Isaac Bankman (Editor)*
- Fundamentals of Medical Imaging, P. Suetens, Cambridge University Press 2002
- A. Melbourne, G. Ridgway, D. J. Hawkes. Image Similarity Metrics in Image Registration. CMIC, UCL London.
- Medical Image Analysis, A. Dhawan, Wiley 2003
- ITK Software Guide (www.itk.org)