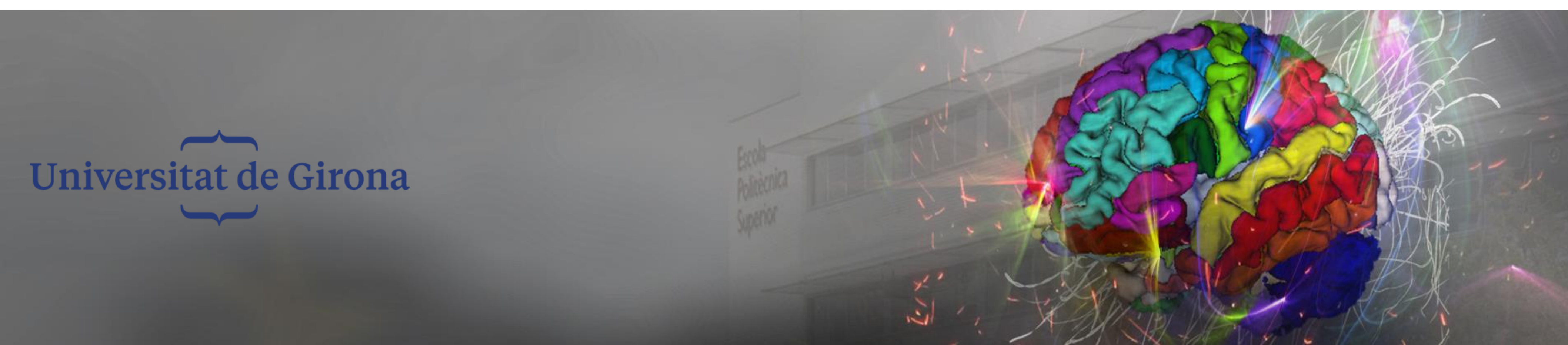
MISA
Image Pre-processing

Robert Martí



Medical Image Analysis

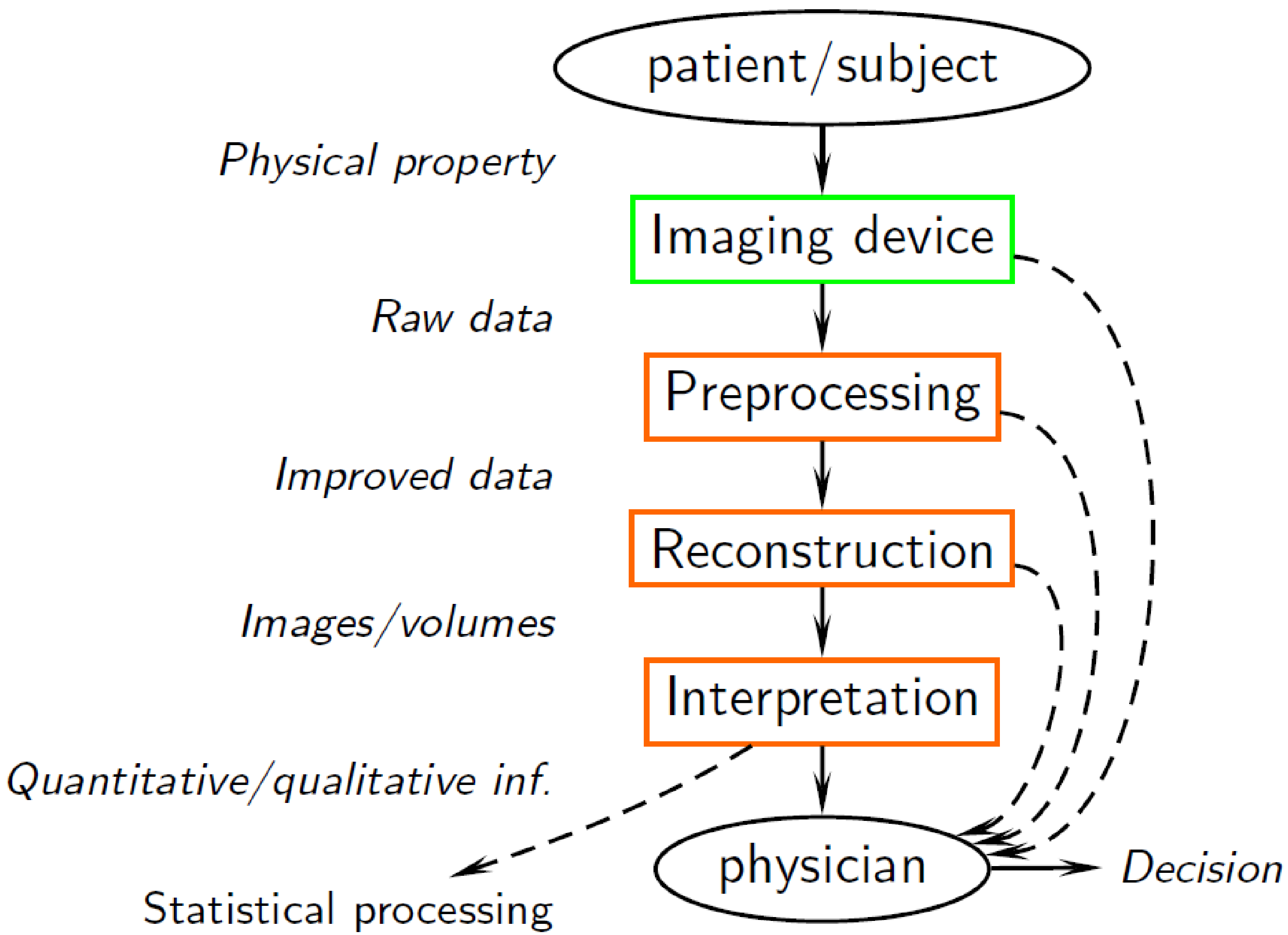
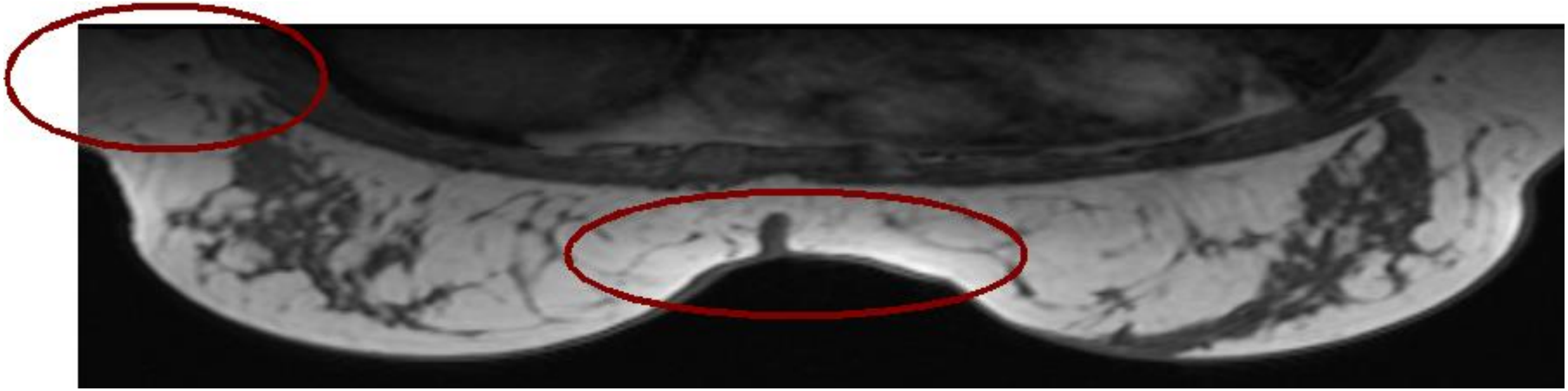


Image Pre-processing

- Filtering or image enhancement
- Enhance the quality of the images prior to segmentation or registration
- Reduce the *uninteresting* variability on the data
- Many different approaches (non-exclusive)
  - Spatial and temporal filtering and smoothing. Improve signal to noise ratio or enhance specific features
  - Distortion and motion correction
  - Normalization. Make images look more similar (bias correction, histogram matching)

Examples. Bias Correction



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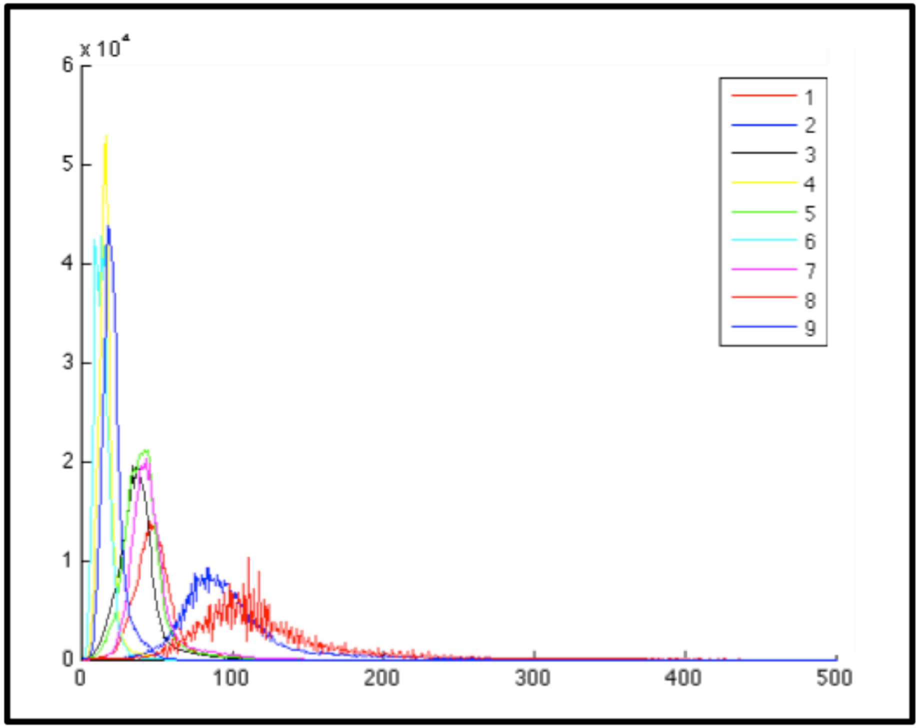
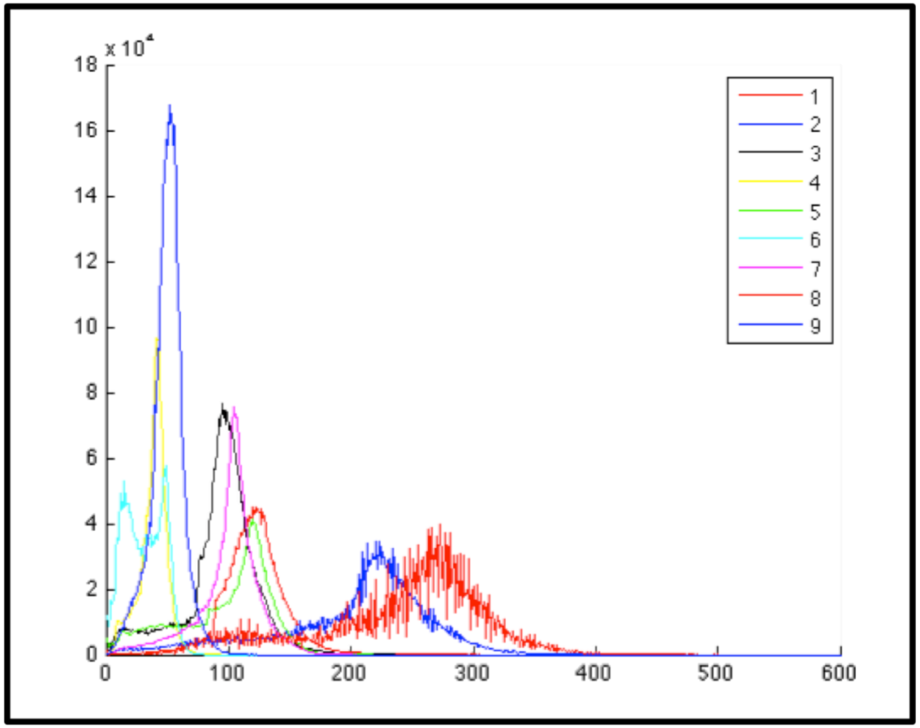
Examples. Interpatient normalization



Pectoral muscle

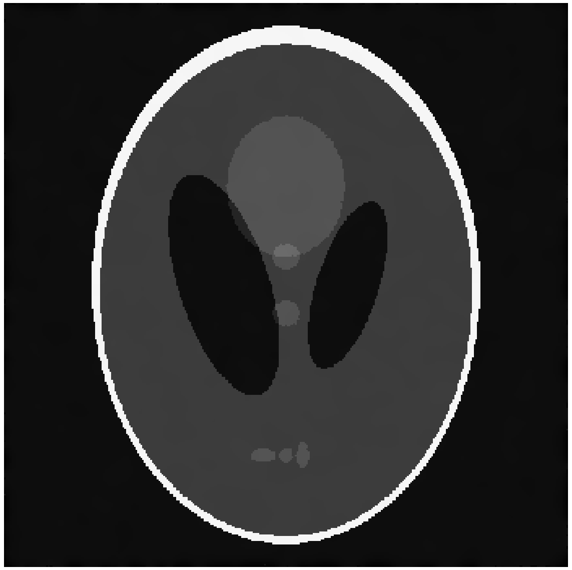


Fatty tissue

Examples. Spatial Smoothing

- Spatial and temporal filtering and smoothing.
  - Spatial filtering using anisotropic diffusion

Original Image



Anisotropic Diffusion

Gaussian Blurring

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Bias-field

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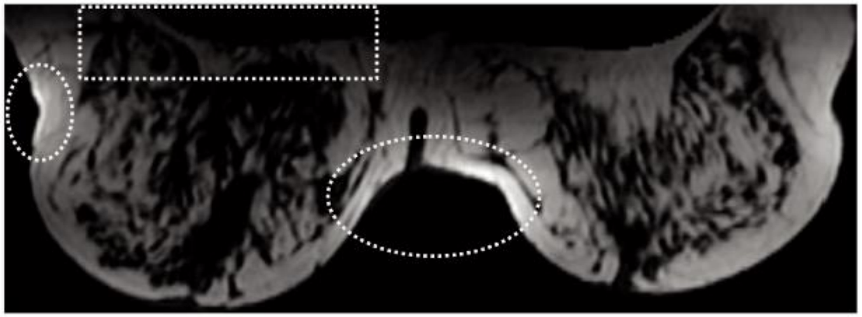
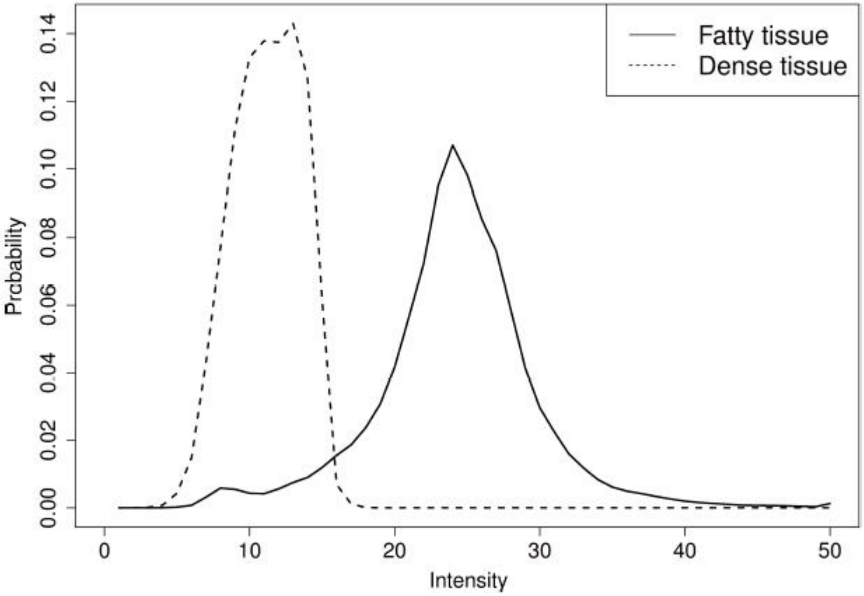
Bias Field correction

- Some modalities (MRI) add a non homogeneous noise in the images.
- In the same image/volume, the same tissue type has a different grey-level





Bias Field correction

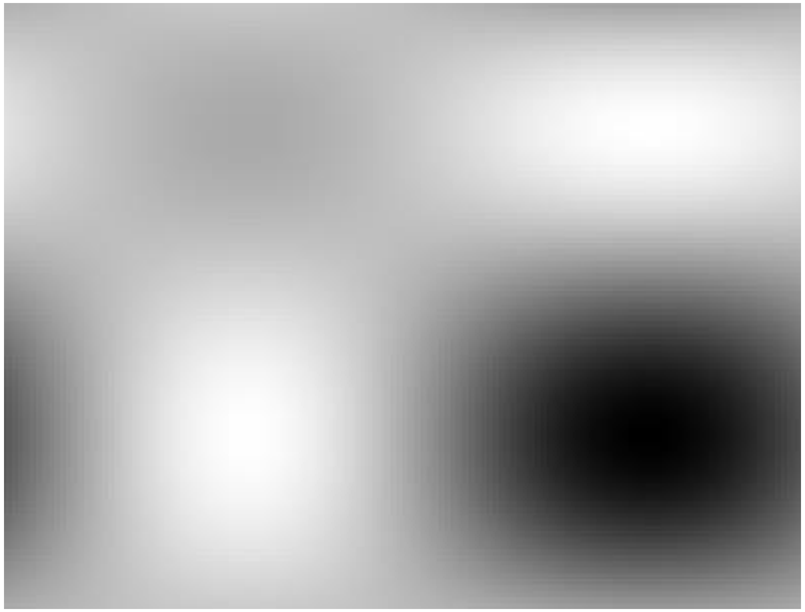
- Different approaches (just a few!)
  - N3: Sled, J. G., Zijdenbos, A. P., & Evans, A. C. (1998). A nonparametric method for automatic correction of intensity nonuniformity in MRI data. IEEE transactions on medical imaging, 17(1), 87-97.
  - Ahmed, M. N., Yamany, S. M., Mohamed, N., Farag, A. A., & Moriarty, T. (2002). A modified fuzzy c-means algorithm for bias field estimation and segmentation of MRI data. IEEE transactions on medical imaging, 21(3), 193-199.
  - Van Leemput, K., Maes, F., Vandermeulen, D., & Suetens, P. (1999). Automated model-based bias field correction of MR images of the brain. IEEE transactions on medical imaging, 18(10), 885-896.
  - Zhang, Y., Brady, M., & Smith, S. (2001). Segmentation of brain MR images through a hidden Markov random field model and the expectation-maximization algorithm. IEEE transactions on medical imaging, 20(1), 45-57.
  - Li, C., Gore, J. C., & Davatzikos, C. (2014). Multiplicative intrinsic component optimization (MICO) for MRI bias field estimation and tissue segmentation. *Magnetic resonance imaging*, *32*(7), 913-923.

Bias Fields

- Bias field model

$$v(\mathbf{x}) = u(\mathbf{x})f(\mathbf{x}) + n(\mathbf{x})$$

- u is true voxel value
- v is measured voxel value
- f is local varying multiplicative bias
- n is white Gaussian noise



Slice of sinusoidal bias field

N3 Algorithm

- Discarding noise, If we take the log of the equation

$$\hat{v}(\mathbf{x}) = \hat{u}(\mathbf{x}) + \hat{f}(\mathbf{x}).$$

we can consider them as probability distribution (i.e. if f is linearly increasing in a ROI it will follow a uniform distribution)

- Being V,F,U prob distributions, the distribution of the sum is the convolution,

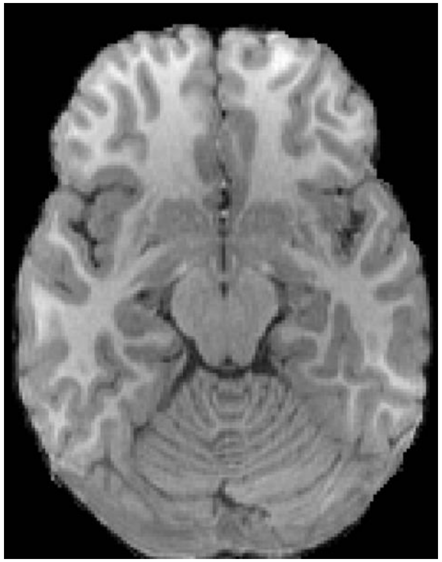
$$V(\hat{v}) = F(\hat{v}) * U(\hat{v})$$

https://en.wikipedia.org/wiki/Convolution_of_probability_distributions

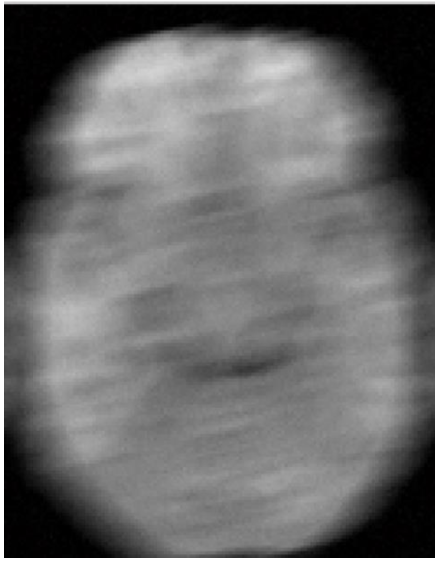
F (bias field) can be thought as a blurring effect to the original image. Blurring removes high frequency information!

N3 Algorithm

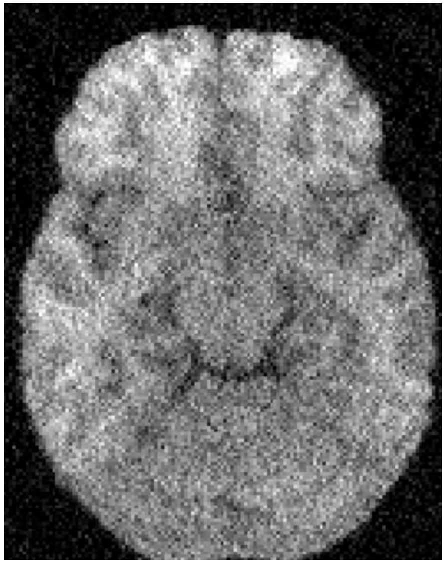
- Main idea: find a smooth and slowly varying field (f) that restores the high frequency in u.
- or… find u by sharpening v in order to find a smooth f function.
- Bias field smooths image (Smoothing = convolution)
- Sharpening = deconvolution (Wiener deconvolution)



Original

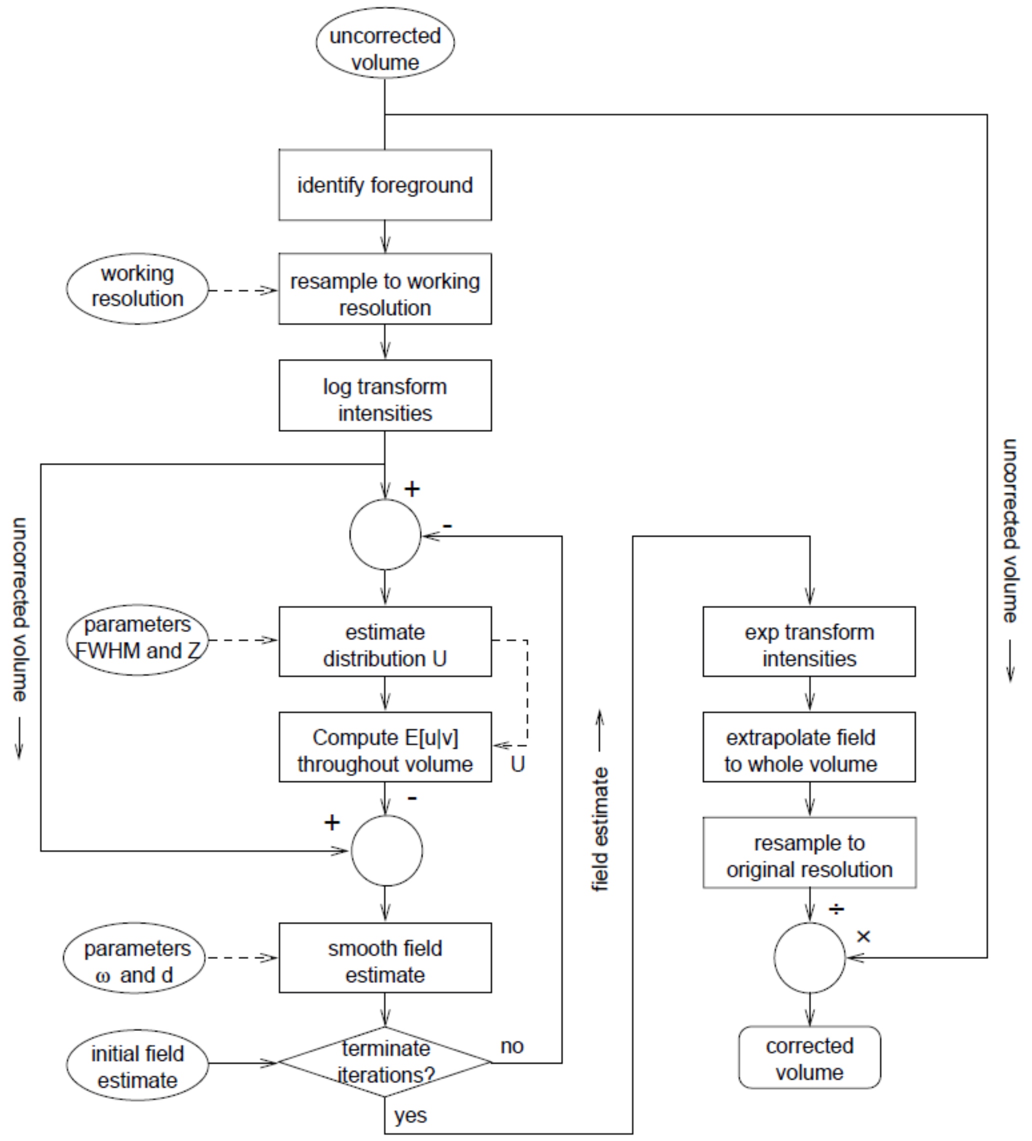


Blurred

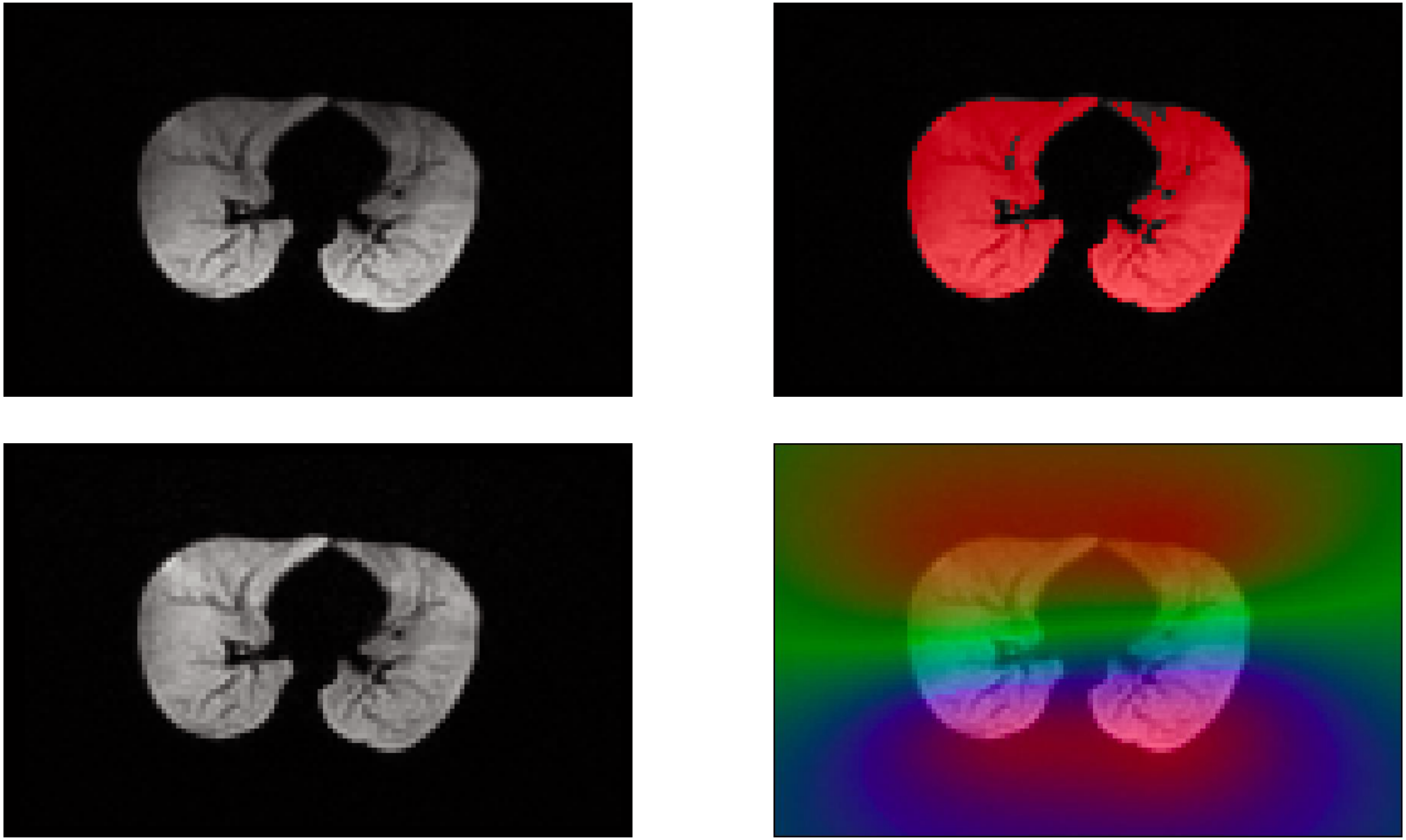


Deconvolved

N3 Algorithm



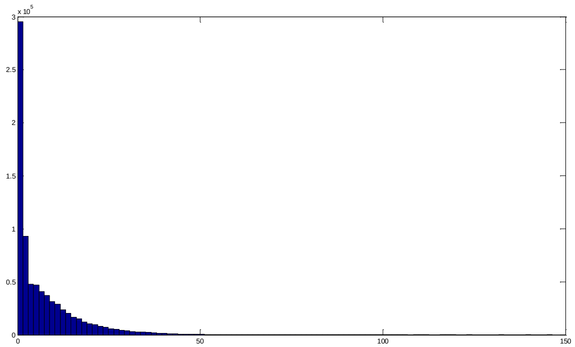
N3 Algorithm

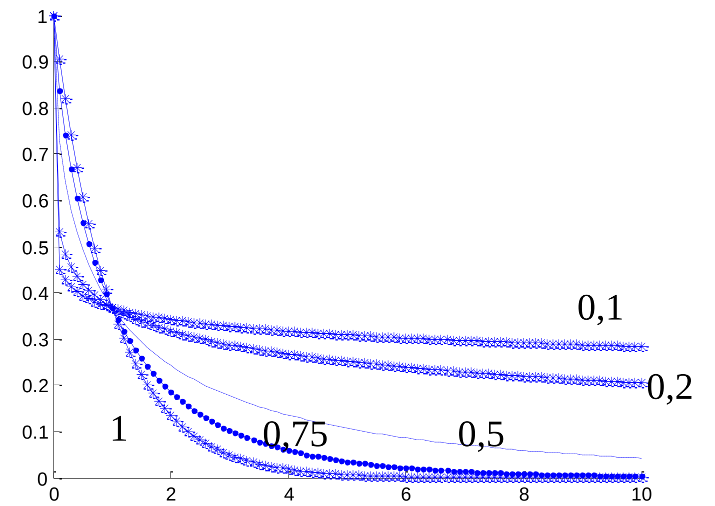


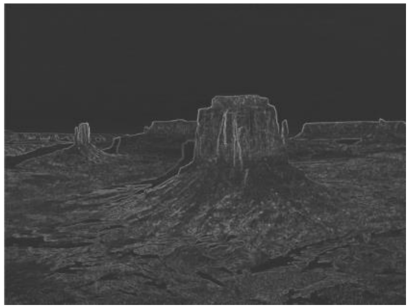
Gradient Distribution

- Explore the sparseness distribution of the intensity gradients.

$$p(x) = e^{-|x|^{\alpha}}$$







```
imatge = imread('Desert.jpg');
[gx, gy] = gradient(double(rgb2gray(imatge)));
mag = sqrt (gx.^2 + gy.^2);
imshow (uint8(mag));
hist(mag(:),1000)
```

Yuanjie Zheng et al, MICCAI 2009. **Automatic Correction of Intensity Nonuniformity from Sparseness of Gradient Distribution in Medical Images.**

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Gradient Distribution

$$Z(i,j) = I(i,j)B(i,j) \qquad \mathcal{Z}(i,j) = \mathcal{I}(i,j) + \mathcal{B}(i,j).$$

Z is the obtained image, *I* the non-biased image and *B* the bias, or take the log version $\mathcal{Z} = \ln Z$.

- Gradients of each image $\psi^{\mathcal{Z}}(i,j) = \psi^{\mathcal{I}}(i,j) + \psi^{\mathcal{B}}(i,j)$
- Given an image Z, we want to find the Bias which maximizes the P(B|Z).

$$\mathcal{B} = \arg\max_{\mathcal{B}} P(\mathcal{B}|\mathcal{Z}) \propto \arg\max_{\mathcal{B}} P(\mathcal{Z}|\mathcal{B})P(\mathcal{B}).$$

$$P(\mathcal{Z}|\mathcal{B}) = P\left(\psi^{\mathcal{I}}\right) = e^{-\left|\psi^{\mathcal{I}}\right|^{\alpha}}, \quad \alpha < 1.$$

$$P(\mathcal{Z}|\mathcal{B}) = e^{-\sum_{(i,j)}\left|\psi^{\mathcal{Z}}(i,j)-\psi^{\mathcal{B}}(i,j)\right|^{\alpha}} \qquad P(\mathcal{B}) = e^{-\lambda_s \sum_{(i,j)}\left(\mathcal{B}_{xx}(i,j)^2+\mathcal{B}_{yy}(i,j)^2\right)}$$

Data driven term Smoothness term

Yuanjie Zheng et al, MICCAI 2009. **Automatic Correction of Intensity Nonuniformity from Sparseness of Gradient Distribution in Medical Images.**

Gradient Distribution

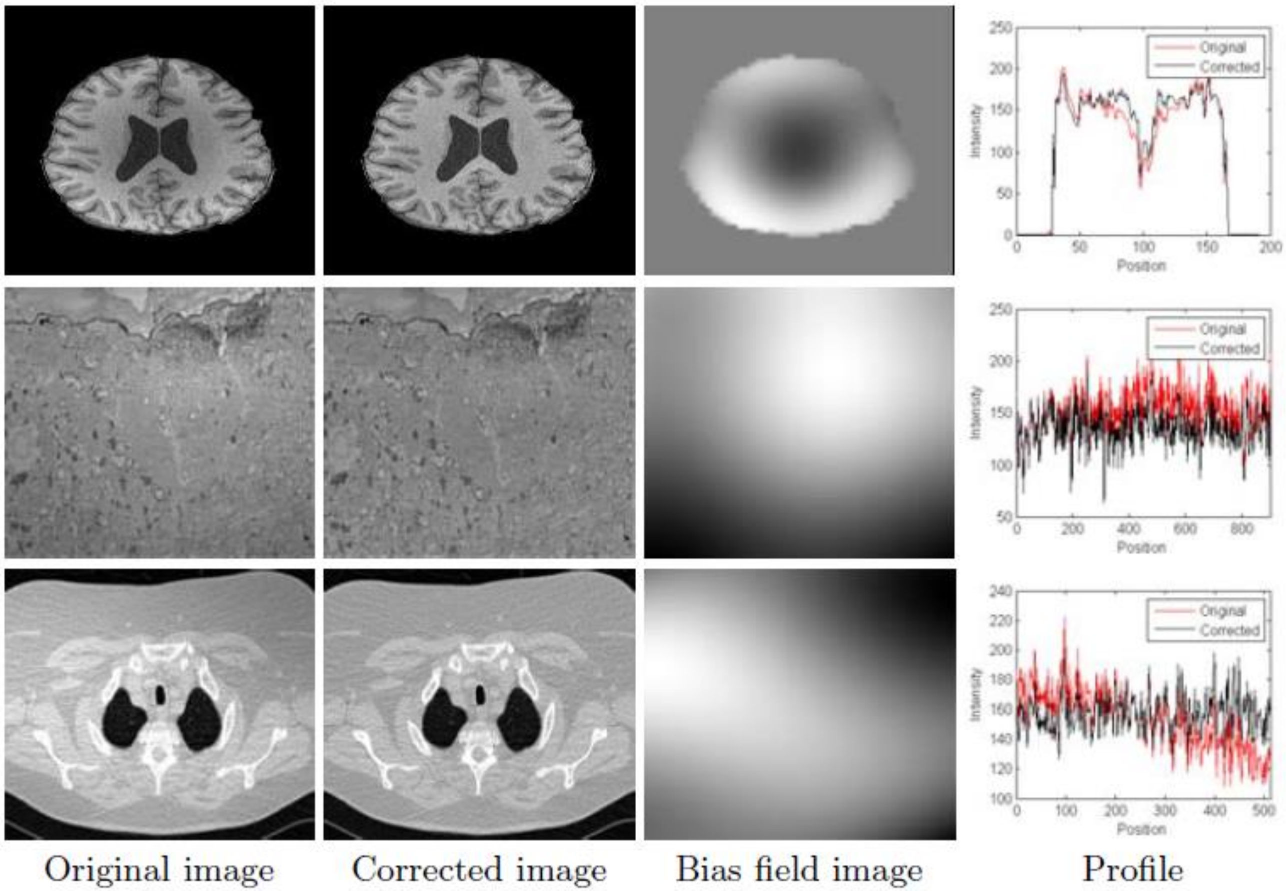

Fig. 3. Corrections of the bias field by our non-parametric method on one MR brain image (up), one TEM image (middle) from rabbit retina, and one CT lung image (down). The profiles are drawn on a horizontal line of the image.

Yuanjie Zheng et al, MICCAI 2009. **Automatic Correction of Intensity Nonuniformity from Sparseness of Gradient Distribution in Medical Images.**

MICO

- Express the image as multiplicative factor, similar to estimating reflectance and illumintaion in conventional images.

$$I(x) = b(x)J(x) + n(x)$$

- Assumption
  - Constant intensity of J(x) (as it is the same region), modelled as fuzzy membership functions (similar to FCM). Assuming N types of tissue.

$$J(x) = \sum_{i=1}^{N} c_i u_i(x).$$

  - Smooth varying of b(x), modelled as polynomials of 3 degrees (M basis functions, G).

$$b(x) = \mathbf{w}^T G(x)$$

Li, C., Gore, J. C., & Davatzikos, C. (2014). Multiplicative intrinsic component optimization (MICO) for MRI bias field estimation and tissue segmentation. *Magnetic resonance imaging*, *32*(7), 913-923

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MICO

- Minimise

$$F(b,J) = \int_\Omega |I(x) - b(x)J(x)|^2 dx.$$

- Using the models

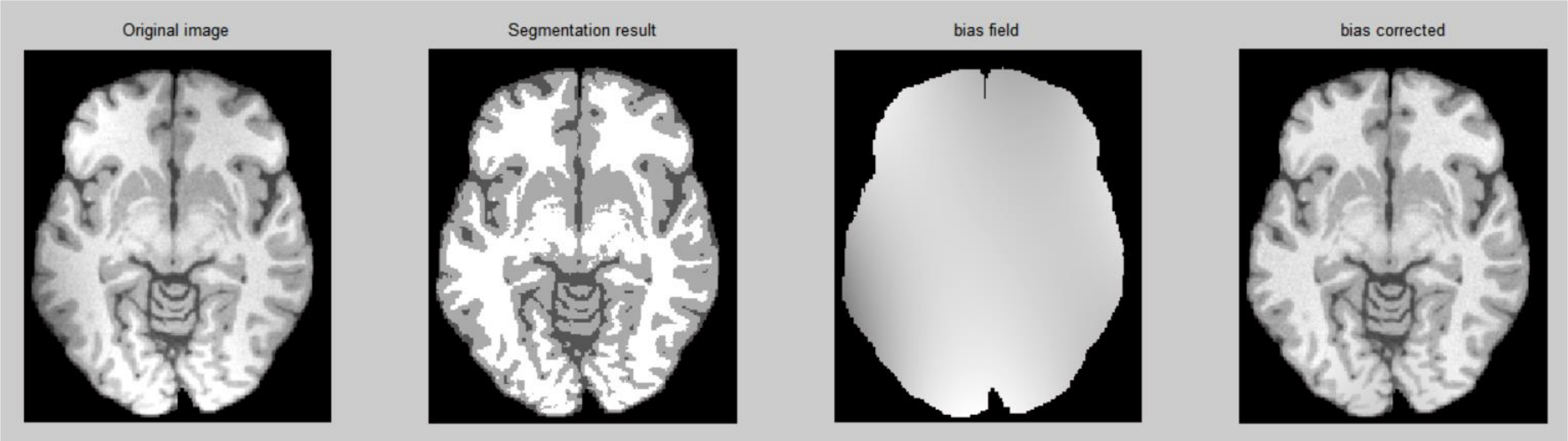
$$F(\mathbf{u},\mathbf{c},\mathbf{w}) = \int_\Omega \left| I(x) - \mathbf{w}^T G(x) \sum_{i=1}^{N} c_i u_i(x) \right|^2 dx$$

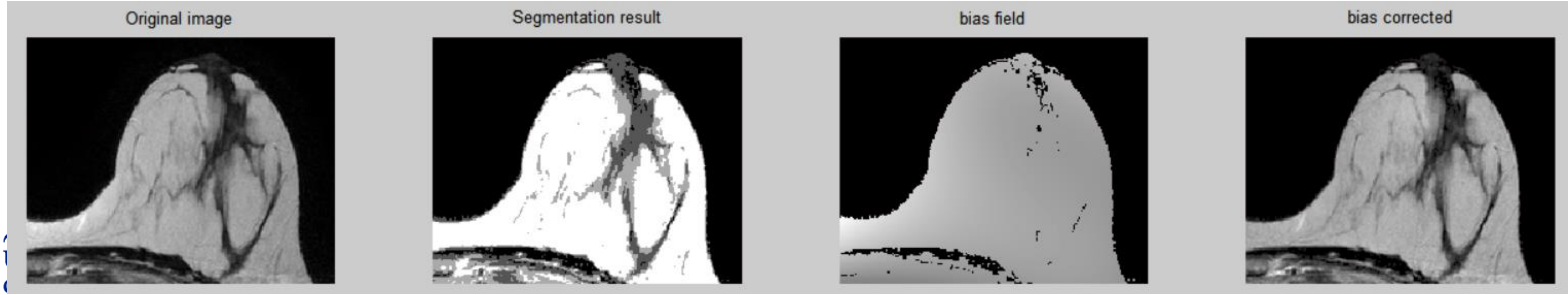
Minimisation of each of the parameters separately, u, c and w.

Li, C., Gore, J. C., & Davatzikos, C. (2014). Multiplicative intrinsic component optimization (MICO) for MRI bias field estimation and tissue segmentation. *Magnetic resonance imaging*, *32*(7), 913-923

MICO

- Examples. Segmentation and bias field.





To know more…

- M. Styner, C. Brechbhler, G. Szkely, and G. Gerig. Parametric estimate of intensity inhomogeneities applied to mri. IEEE Trans Med Imaging, 19(3):153–165, Mar 2000.
- Yuanjie Zheng et al. Automatic Correction of Intensity Nonuniformity from Sparseness of Gradient Distribution in Medical Images, MICCAI 2009.
- Sled, J. G., Zijdenbos, A. P., & Evans, A. C. (1998). A nonparametric method for automatic correction of intensity nonuniformity in MRI data. IEEE transactions on medical imaging, 17(1), 87-97.
- Ahmed, M. N., Yamany, S. M., Mohamed, N., Farag, A. A., & Moriarty, T. (2002). A modified fuzzy c-means algorithm for bias field estimation and segmentation of MRI data. IEEE transactions on medical imaging, 21(3), 193-199.
- Van Leemput, K., Maes, F., Vandermeulen, D., & Suetens, P. (1999). Automated model-based bias field correction of MR images of the brain. IEEE transactions on medical imaging, 18(10), 885-896.
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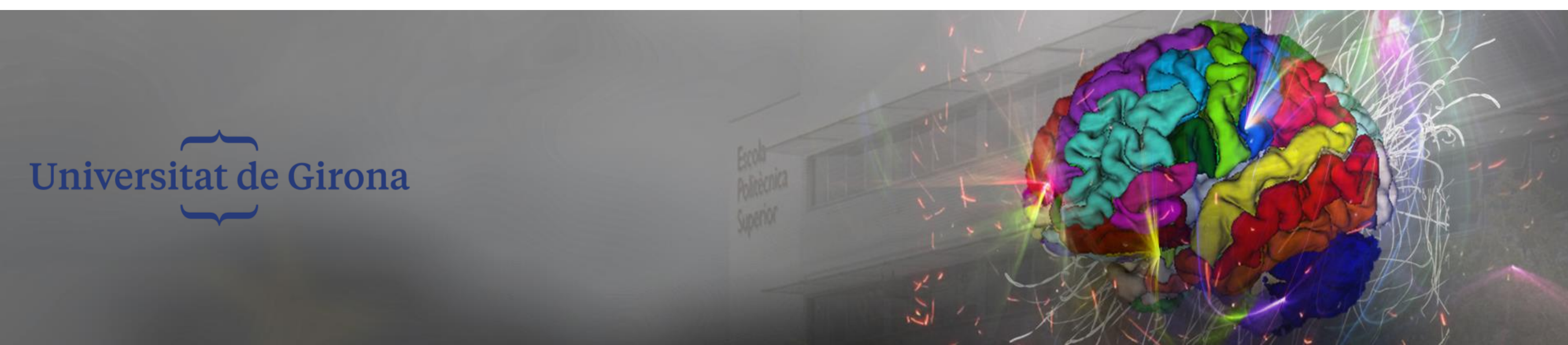
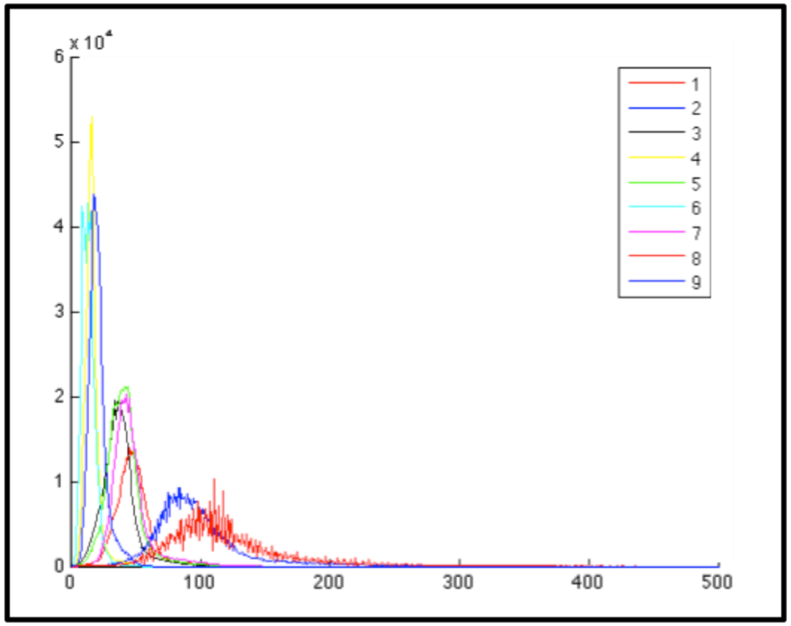
Intensity Normalisation

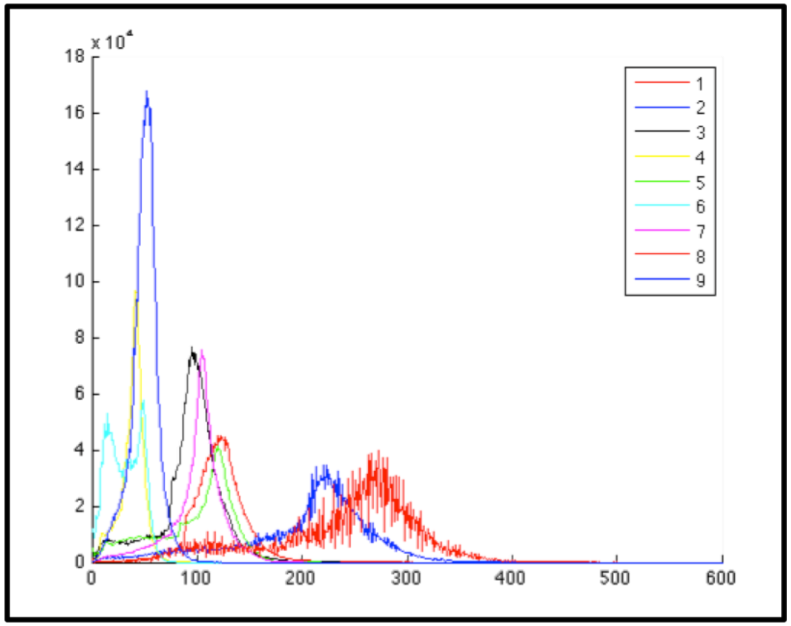


Normalization

- Tissue appearance normalization between different patients in MRI
  - Inter-patient differences
  - Same structure different values in different patients



Pectoral muscle



Fatty tissue

Normalization

- Sensitivity of the coil & tissue structure are important factors that cause a global differences between signal intensities
- Difference can be regarded as a global parameter γ different for each patient/acquisition.

- For an image at position r, we are looking at a tissue t. Should look like St but is not because of the unkown γ

$$I(\boldsymbol{r}|x_{\boldsymbol{r}} = t) = \gamma\, S_t$$

Normalization

- Supose that you know a certain in the image ("an easy to extract tissue type").

- You can normalise the image taking the reference tissue into account.

$$\hat{I}(r|x_{\boldsymbol{r}} = t) = \hat{I}_t = \frac{\gamma\, S_t}{\gamma\, S_{ref}} = \frac{I_t}{I_{ref}}$$

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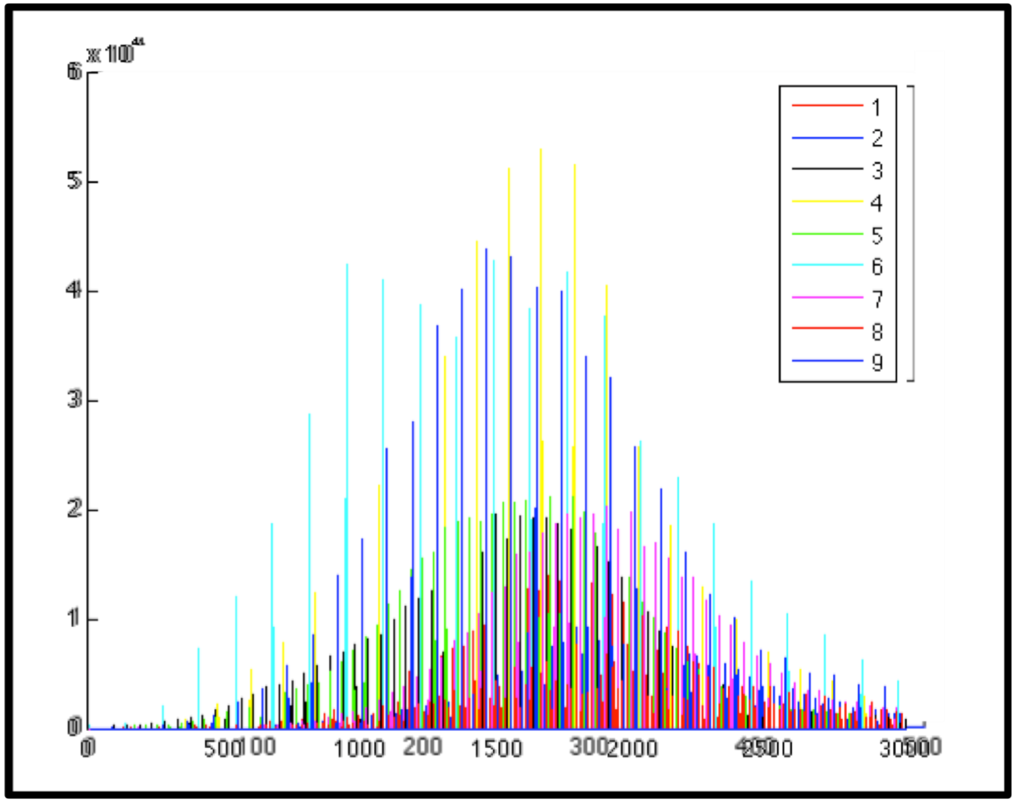
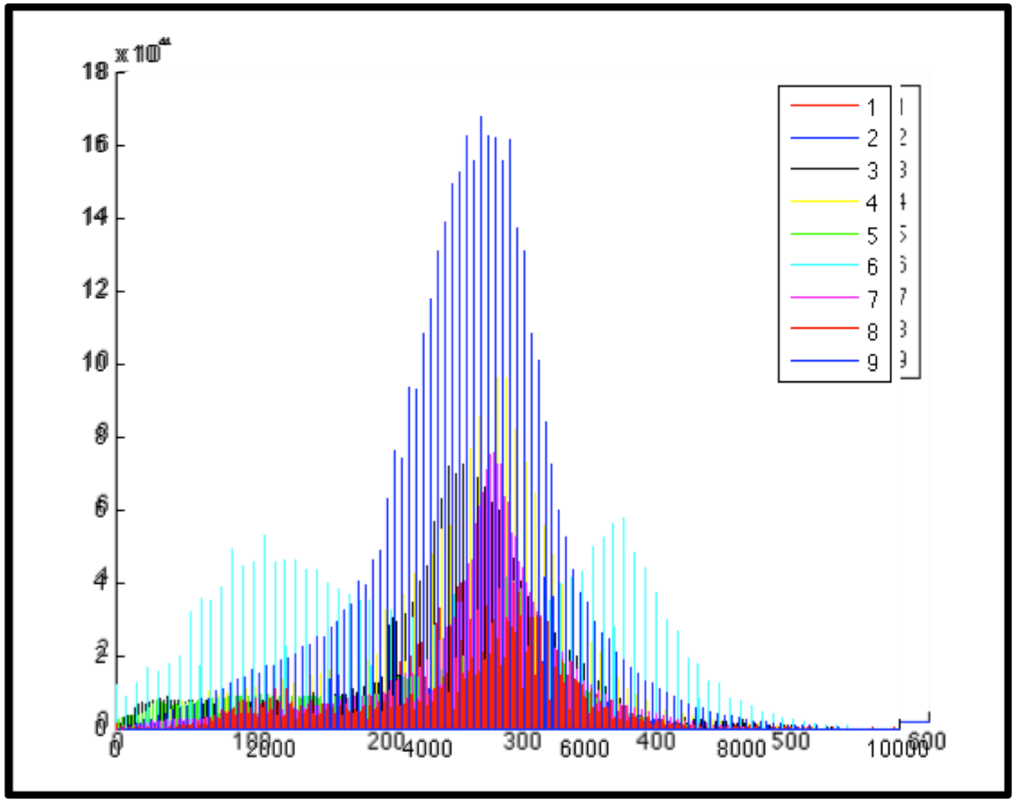
Normalization



Pectoral muscle



Fatty tissue

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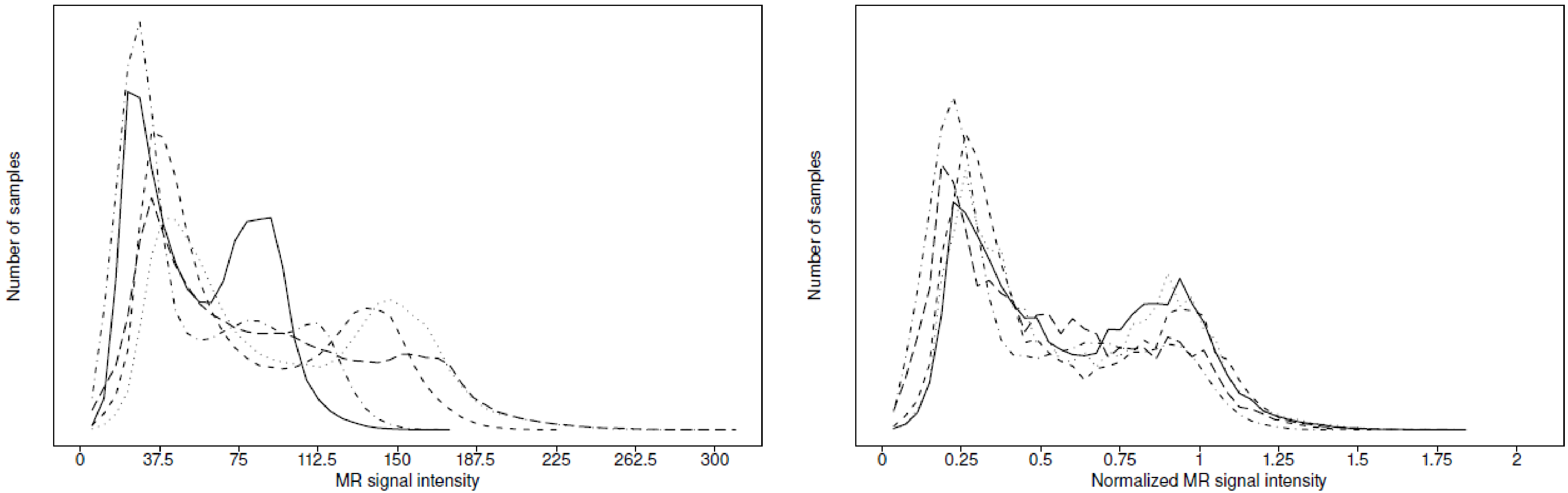
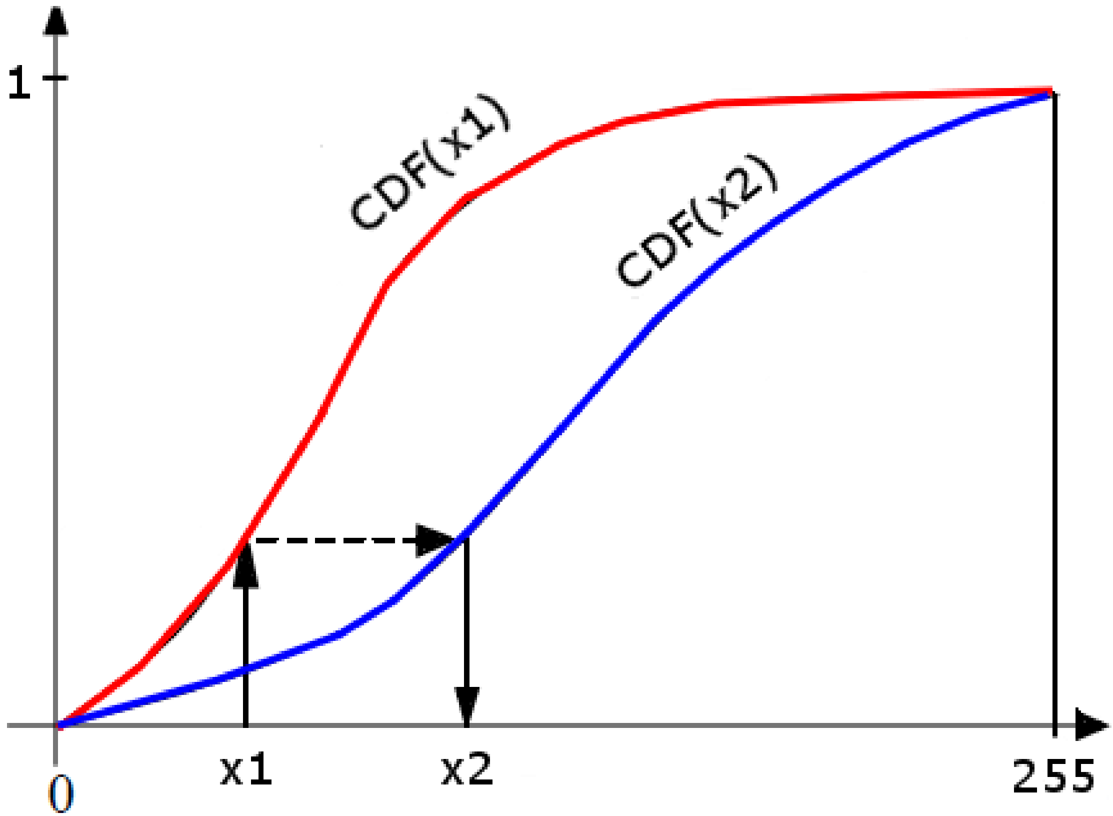
Normalization



(a) (b)

Histogram matching

- Find a mapping function based on F(x1) = F (x2)



Histogram matching

- Matlab

```
M = zeros(256,1,'uint8'); %// Store mapping - Cast to
uint8 to respect data type
hist1 = imhist(im1); %// Compute histograms
hist2 = imhist(im2);
cdf1 = cumsum(hist1) / numel(im1); %// Compute CDFs
cdf2 = cumsum(hist2) / numel(im2);

%// Compute the mapping
for idx = 1 : 256
    [~,ind] = min(abs(cdf1(idx) - cdf2));
    M(idx) = ind-1;
end

%// Now apply the mapping to get first image to make
%// the image look like the distribution of the second
image
out = M(double(im1)+1);
```

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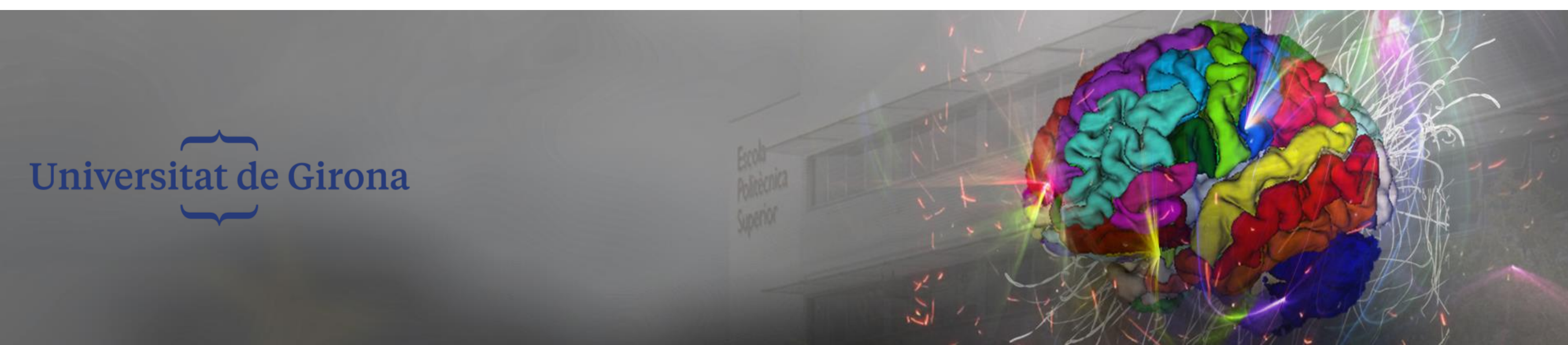
Normalization

- Simple method, yet effective.
- There are other approaches
  - A. Kalemis, D.M. Binnie, M.A. Flower, R.J. Ott. Image intensity normalisation by maximising the Siddon line integral in the joint intensity distribution space. MIA 13(6), 2009.
  - J.A Dauguet, JF Mangin, T. Delzescaux, V. Frouin. Robust Inter-slice Intensity Normalization Using Histogram Scale-Space Analysis. MICCAI 2004.
  - R. Philipsen, P. Maduskar, L. Hogeweg and B. van Ginneken. "Normalization of Chest Radiographs", in: Medical Imaging, volume 8670 of Proceedings of the SPIE, 2013, page 86700G

Anisotropic diffussion



Scale Space: Gaussian Pyramids

- Gaussian Pyramid
- Laplacian of Gaussian (LoG)
- Difference of Gaussians (DoG)

Gaussian pyramid

$$I(x,y,t) = I_0(x,y) * G(x,y,t)$$

$I_0(x,y)$: Original noisy image $G(x,y,t)$: Gaussian with variance t

Laplacian of Gaussians (LoG)

- Where do Laplacian masks come from?
  - Computation:
    - Gaussian smoothing (minimise noise effects)

$$g(r)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{r^2}{2\sigma^2}}$$

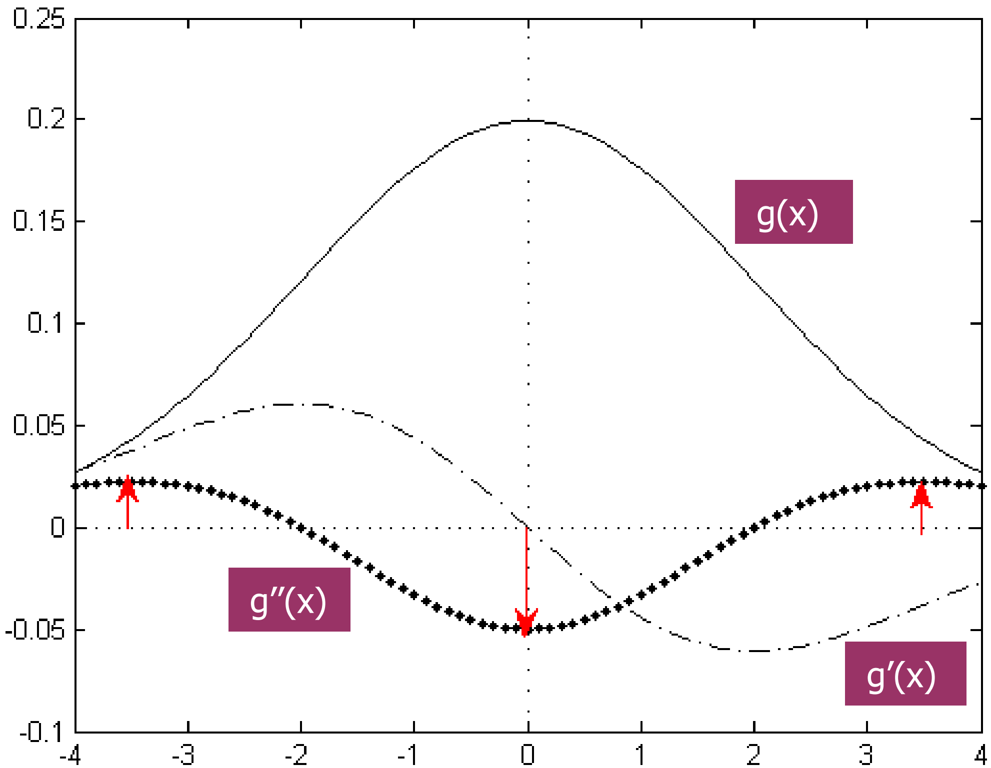
    - Compute second derivative (Laplacian)

$$g(r)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{r^2}{2\sigma^2}} \qquad g'(r)=\frac{-1}{\sqrt{2\pi}\sigma^3}r\,e^{-\frac{r^2}{2\sigma^2}}=\frac{-r}{\sigma^2}g(r)$$

$$g''(r)=\left(\frac{r^2}{\sqrt{2\pi}\sigma^5}-\frac{1}{\sqrt{2\pi}\sigma^3}\right)e^{-\frac{r^2}{2\sigma^2}}=\left(\frac{r^2}{\sigma^4}-\frac{1}{\sigma^2}\right)g(r)$$

Laplacian masks (LoG)

- Where do Laplacian masks come from?




```
sigma = 2;
nb = 4
x = [-nb:0.1:nb];
y = [-0.1:0.01:0.25]
mu = 0;
% gaussian
gauss_fun = normpdf(x,mu,sigma);
plot(x,gauss_fun,'k-');
hold on;

%first derivative
deriv_1st = (- x / sigma^2) .* gauss_fun;
plot(x,deriv_1st,'k-.');

%second derivative
deriv_2nd =((x.^2 / sigma^4)-(1 / sigma^2))
   .* gauss_fun;

plot(x,deriv_2nd,'k.');
```

Matlab Code

Laplacian masks (LoG)

- Edges in the Scale Space. Definitions

$$L(x, y, \sigma) = f(x, y) \cdot G(x, y, \sigma)$$

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma} \exp^{(x^2+y^2)/2\sigma}$$

$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f \quad = L_{xx} + L_{yy}$$

$$L_{xx} = \frac{\partial^2 L}{\partial^2 x} = \frac{\partial L_x}{\partial x} \qquad L_{yy} = \frac{\partial^2 L}{\partial^2 y} = \frac{\partial L_y}{\partial y},$$

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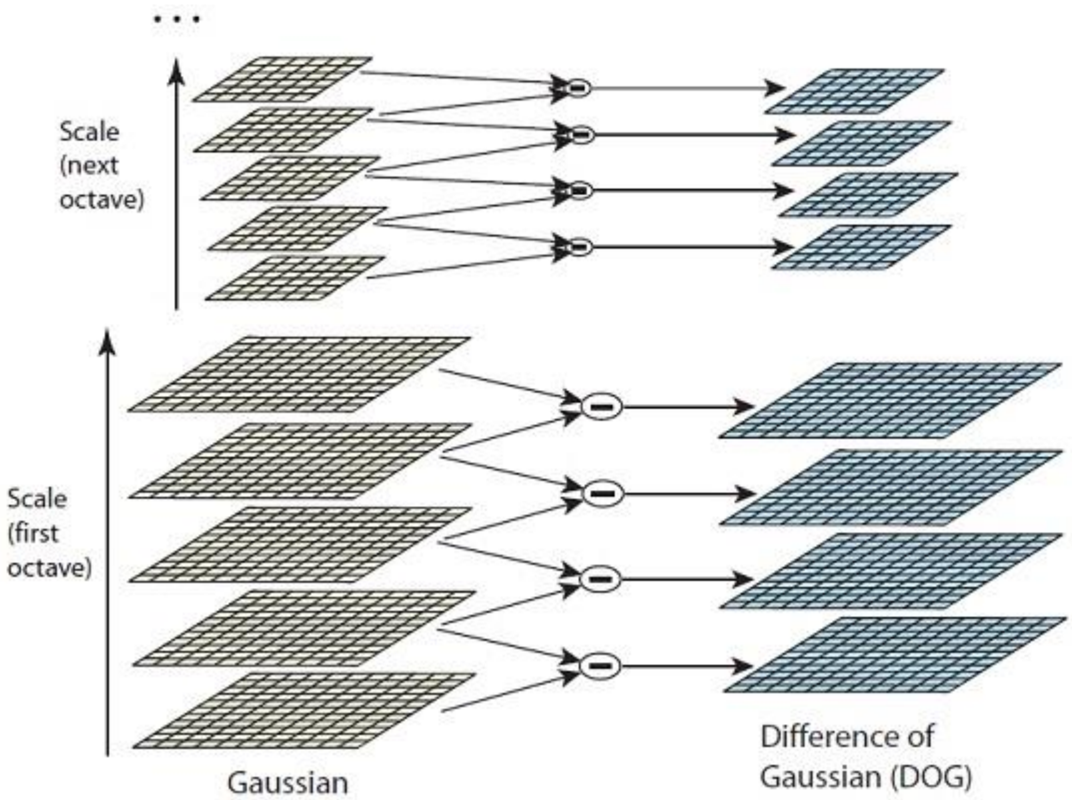
Difference of Gaussians (DoG).

- Approximation (under certain parameters) of the LoG.
- Used in many methods (i.e. SIFT).



Scale Space

- Representation of an image at different scales
  - Gaussian Pyramids

$$I(x, y, t) = I_0(x, y) * G(x, y, t)$$

  - LoG (DoG)

$$I_t(x, y, t) = \Delta I(x, y, t) = I_{xx} + I_{yy}$$
$$I(x, y, 0) = I_0(x, y)$$

Anisotropic Diffusion

- Aim:
  - Image enhancement without blurring the edges.

- Anisotropic Diffusion equation

$$I_t(x,y,t) = \nabla\left(c(x,y,t)\nabla I(x,y,t)\right)$$

if c(x,y,t) = constant, Isotropic diffusion

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Anisotropic Diffusion

$$I_t(x, y, t) = \nabla\left(c(x, y, t)\nabla I(x, y, t)\right)$$

- If c is
  - 0: pixel is at edge location (low diffusion)
  - 1: pixel is inside the region
- How do we estimate edge/region positions?
  - gradient!

$$c(x, y, t) = g\left(\left|\nabla I(x, y, t)\right|\right)$$

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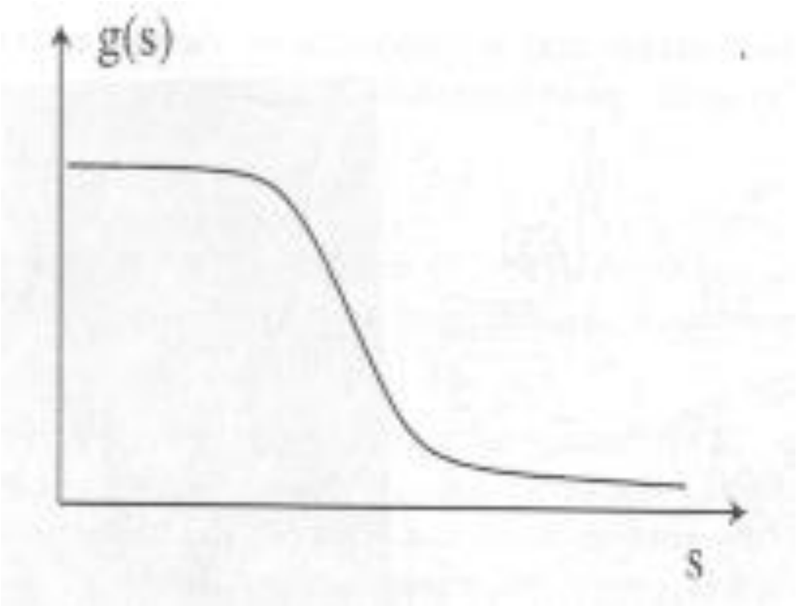
Coefficient Selection

$$c(x, y, t) = g\left(\left|\nabla I\left(x, y, t\right)\right|\right)$$

$g(s) \to 0$ $\quad$ $s \to \infty$

$g(s) \to 1$ $\quad$ $s \to 0$



Gradient Descriptors

- Leclerc
  - Favours large contrasted edges

$$g\left(\left|\nabla I\right|\right) = e^{-\frac{\left|\nabla I\right|^2}{k^2}}$$

- Lorentz
  - Favours large regions over the smaller ones.

$$g\left(\left|\nabla I\right|\right) = \frac{1}{1 + \frac{\left|\nabla I\right|^2}{k^2}}$$

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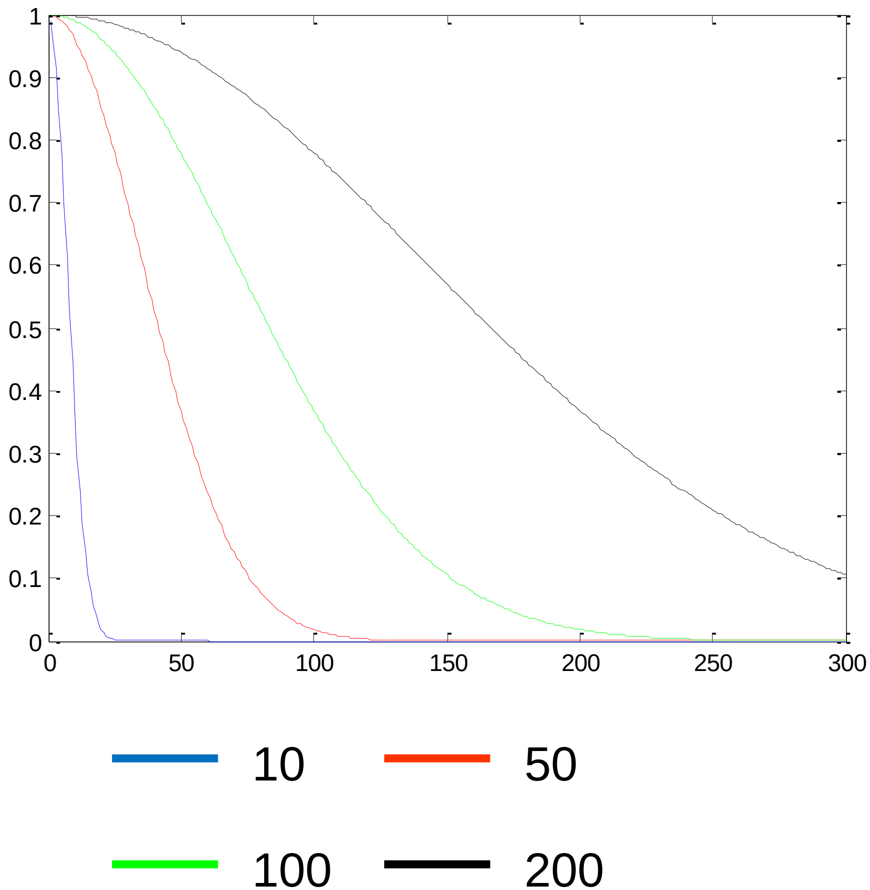
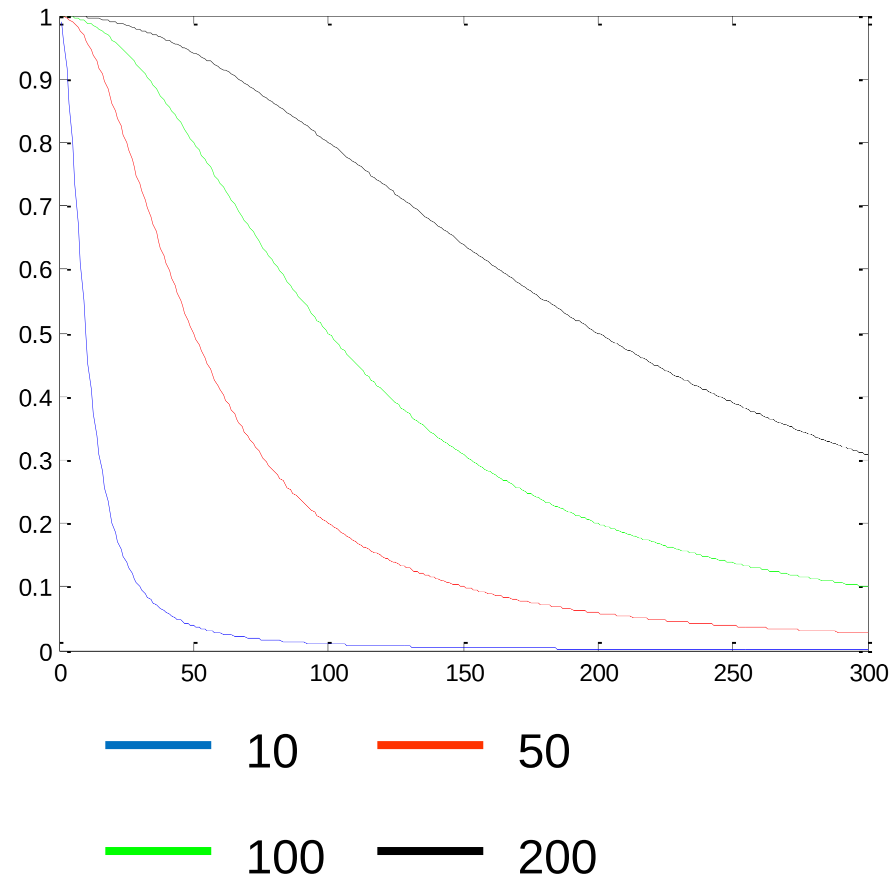
Gradient Descriptors

$$g(|\nabla I|) = e^{-\frac{|\nabla I|^2}{k^2}}$$



$$g(|\nabla I|) = \frac{1}{1+\frac{|\nabla I|^2}{k^2}}$$



Gradient Descriptors

- K is the diffusion constant or flow constant.

$$\phi(x,y,t) = c(x,y,t)\nabla I\left(x,y,t\right)$$

- Rewrite the equation

$$I_t\left(x,y,t\right) = \nabla\left(\phi\left(x,y,t\right)\right)$$

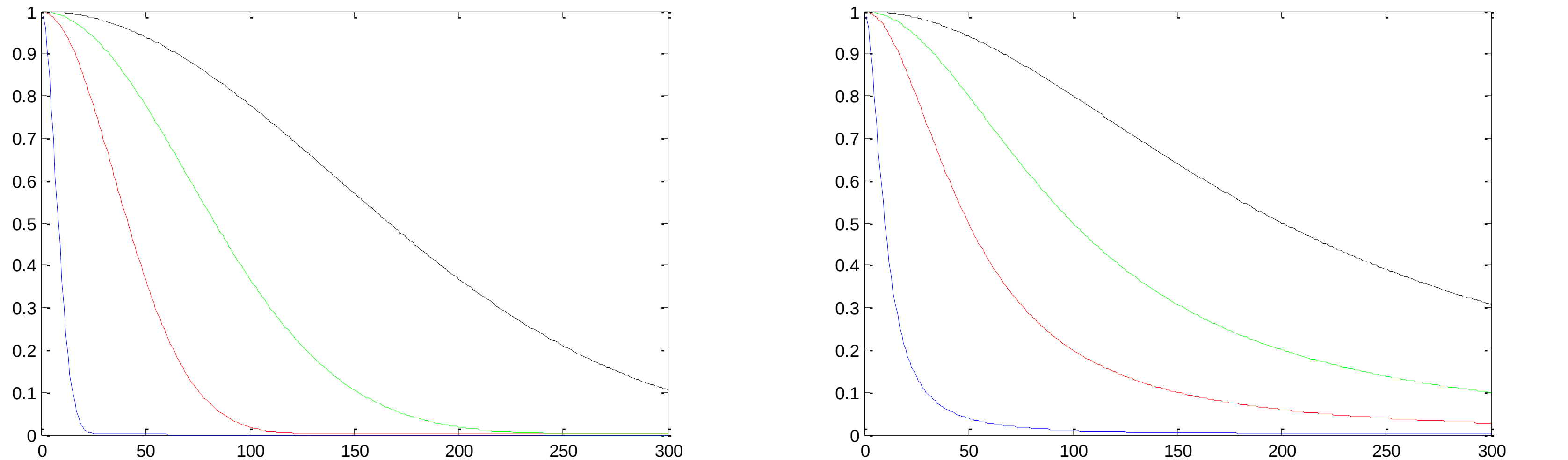
Flow

$$g(|\nabla I|) = e^{-\frac{|\nabla I|^2}{k^2}}$$

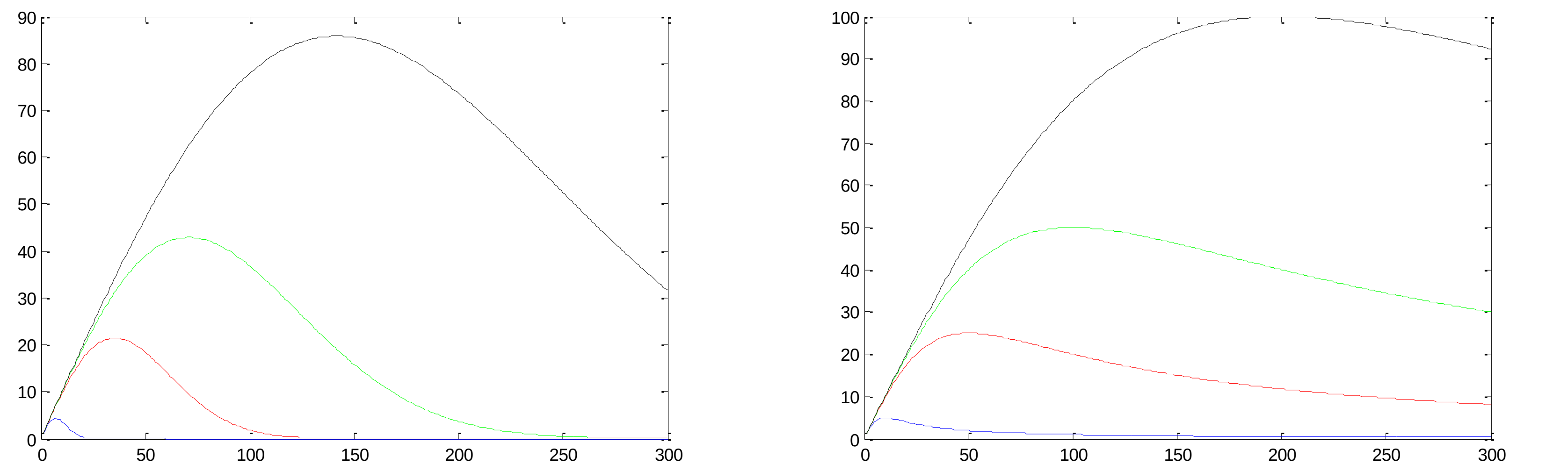
10 50

100 200

$$g(|\nabla I|) = \frac{1}{1+\frac{|\nabla I|^2}{k^2}}$$



$$\phi(x,y,t) = c(x,y,t)\nabla I(x,y,t)$$



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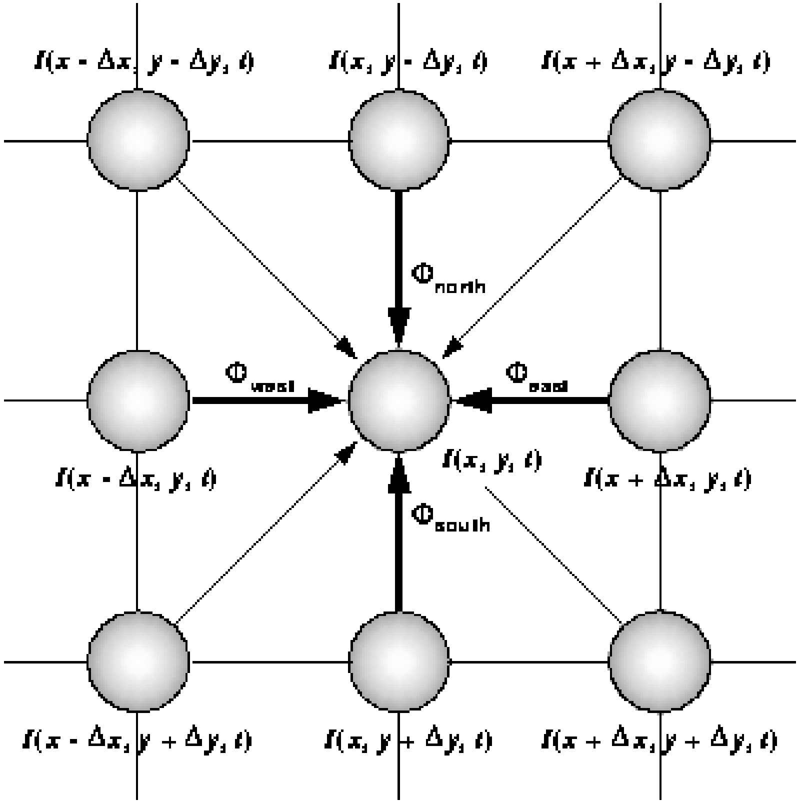
Discretizing the Diffusion Equation



$$I_{ij}^{t+1} = I_{ij}^{t} + \lambda\left(c_N D_N I + c_S D_S I + c_E D_E I + c_O D_O I\right)_{ij}^{t}$$

$$D_N I_{ij} = I_{i-1,j} - I_{i,j} \qquad c_{N_{ij}} = g\left(\left|D_N I_{ij}^{t}\right|\right)$$

Dimensions	Neighbors	Maximum Δt
1D	2	1/3
2D	4	1/5
	8	1/7
3D	6	1/7
	26	3/44

Example: Matlab Implementation

```
function diff = anisodiff(im, niter, kappa, lambda, option)

im = double(im);
[rows,cols] = size(im);
diff = im;

for i = 1:niter
%  fprintf('\rIteration %d',i);

  % Construct diffl which is the same as diff but
  % has an extra padding of zeros around it.
  diffl = zeros(rows+2, cols+2);
  diffl(2:rows+1, 2:cols+1) = diff;

  % North, South, East and West differences
  deltaN = diffl(1:rows,2:cols+1)   - diff;
  deltaS = diffl(3:rows+2,2:cols+1) - diff;
  deltaE = diffl(2:rows+1,3:cols+2) - diff;
  deltaW = diffl(2:rows+1,1:cols)   - diff;
```

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Example: Matlab Implementation

```
% Conduction
if option == 1
  cN = exp(-(deltaN/kappa).^2);
  cS = exp(-(deltaS/kappa).^2);
  cE = exp(-(deltaE/kappa).^2);
  cW = exp(-(deltaW/kappa).^2);
elseif option == 2
  cN = 1./(1 + (deltaN/kappa).^2);
  cS = 1./(1 + (deltaS/kappa).^2);
  cE = 1./(1 + (deltaE/kappa).^2);
  cW = 1./(1 + (deltaW/kappa).^2);
end

diff = diff + lambda*(cN.*deltaN + cS.*deltaS +
cE.*deltaE + cW.*deltaW);

end
```

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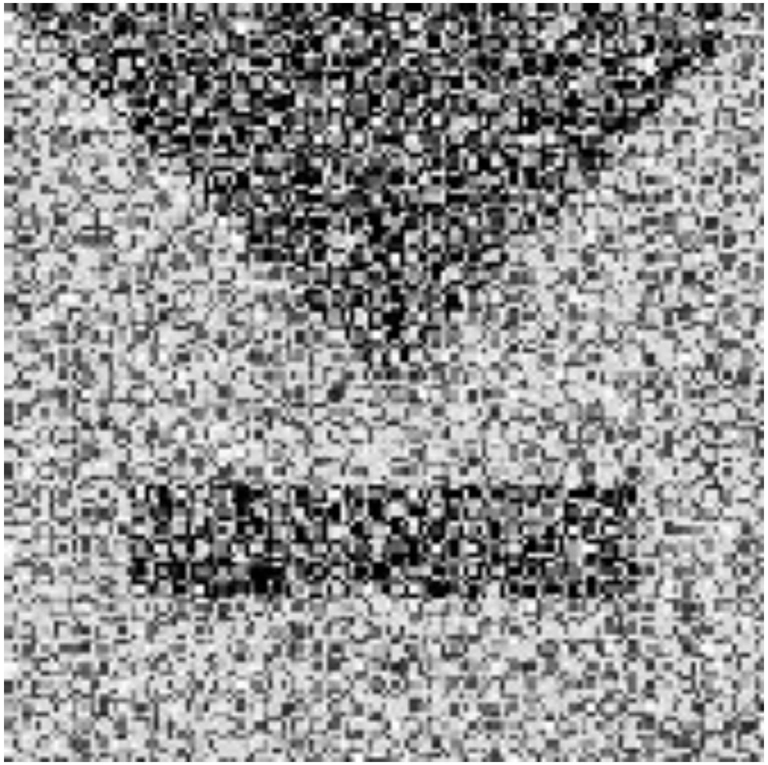
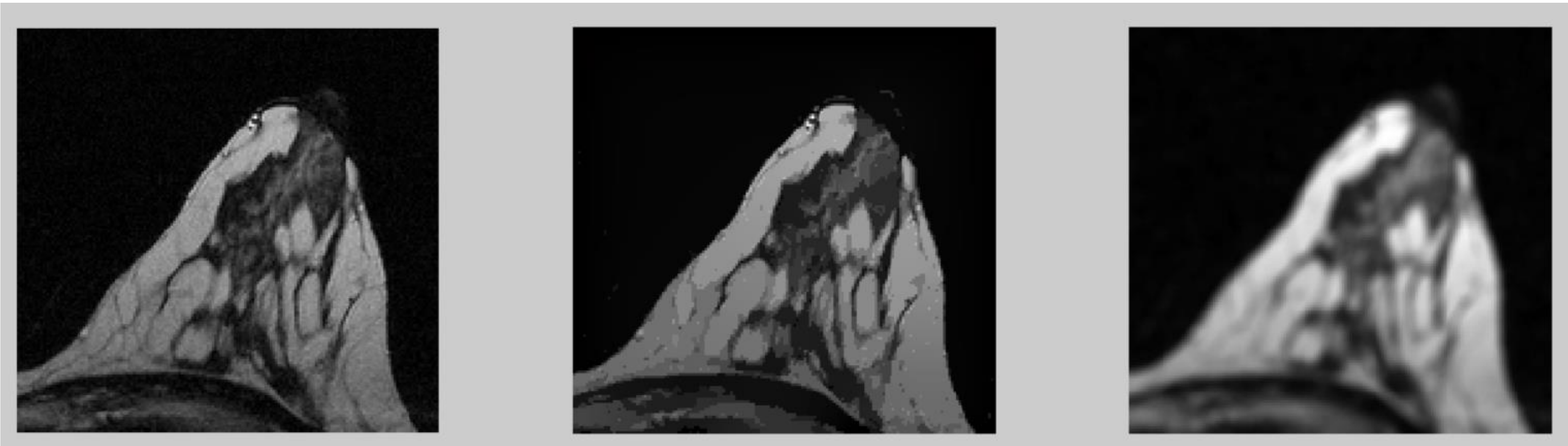
Noise removal example



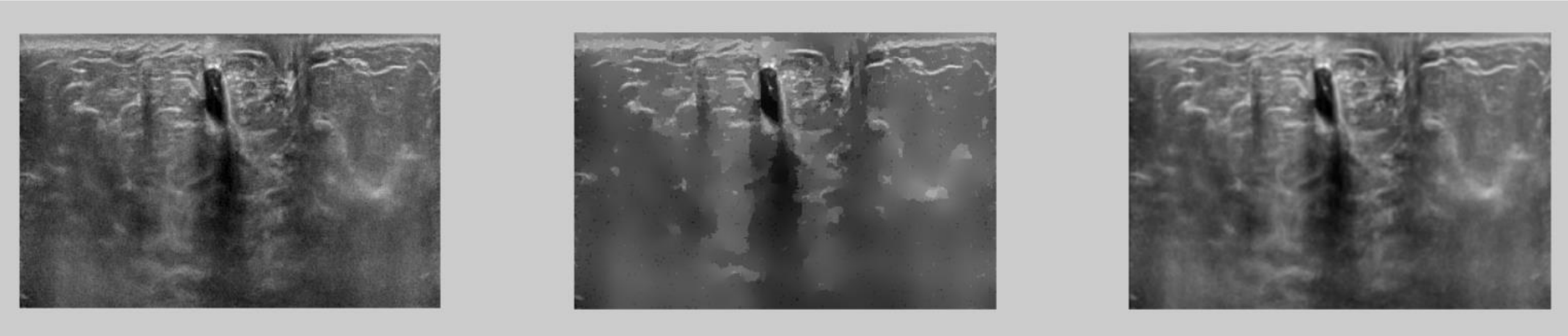
Noise Removal results



ORIGINAL

Anisotropic Diffusion
(500 iterations, k=5, Leclerc)

Gaussian
Smoothing



Noise removal results

ORIGINAL

Anisotropic Diffusion
(500 iterations, k=5, Lorenz)

Gaussian
Smoothing

Other applications: gap completion

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Demo code

Matlab Demo from : % Diffusion filtering toolbox.
% Version 1.1 Mar-2004
% Frederico D'Almeida - DEE/Federal University of Bahia - Brazil

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To know more…

- Perona & Malik. Scale-Space and Edge Detection using anisotropic diffusion. Earlier version: http://www.eecs.berkeley.edu/Pubs/TechRpts/1988/CSD-88-483.pdf
- J. Weickert. Anisotropic diffusion in image processing, Ph.D. thesis, Dept. of Mathematics, University of Kaiserslautern, Germany, January 1996.
- T. Lindeberg: Principles for automatic scale selection', Handbook on Computer Vision and Applications, volume 2, pp 239--274, Academic Press, Boston, USA, 1999.