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Time Series with Applications on Big Data

Number of Aircraft Departing

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Masters Applied Data Science

Contents

1 Introduction

This report takes you on a comprehensive journey to develop the optimal model for analyzing time series data. The dataset used consists of the number of aircraft departures in Italy from 1948 to 2014 [Istituto Nazionale di Statistica (ISTAT)].

The process begins with an in-depth analysis of the dataset, assessing its stationarity and variance. If the series is found to be non-stationary, transformations are applied to achieve stationarity. Subsequently, model selection is performed using ACF and PACF plots, along with Information Criteria methods such as AIC, BIC, and HQC.

Once the most suitable models are identified, they are constructed and thoroughly evaluated, ensuring that the residuals conform to the characteristics of a Gaussian white noise process.

The final stage involves generating forecasts, with the model demonstrating the lowest RMSE being selected as the best-performing model. This model is then employed to forecast future values with enhanced accuracy.

All analyses and computations were conducted using Gretl, ensuring robust and reliable results.

2 Preliminary Analysis

Upon plotting the series, we obtained the following result:

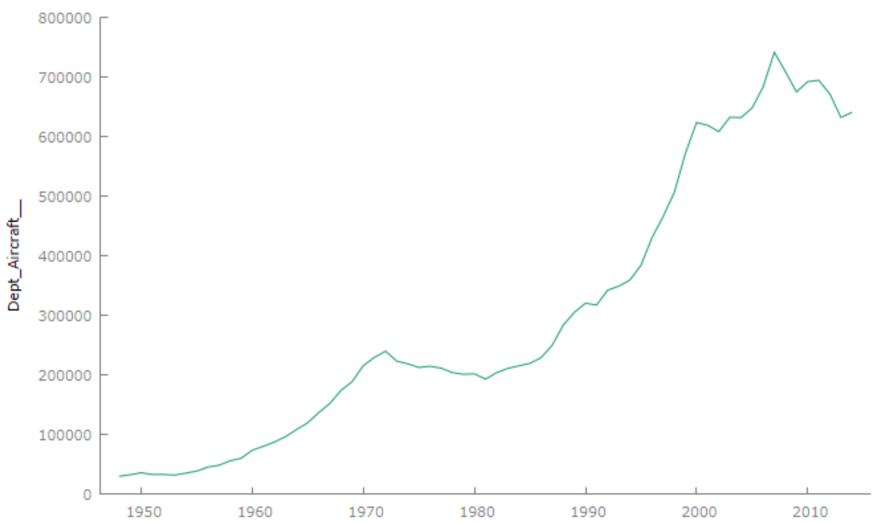

Figure 1: Number of aircraft departures in Italy from 1948 to 2014

Examining plot in Figure 1 reveals a clear upward trend. However, to build an ARIMA model, it is essential for the series to be stationary. Therefore, a transformation is required to achieve stationarity.

3 Data Transformation

If x_t represents our original time series and $\Delta = 1 - L$, where L is the lag operator, the transformed series is given by $y_t = \Delta x_t = x_t - x_{t-1}$. The plot y_t is shown in Figure 2

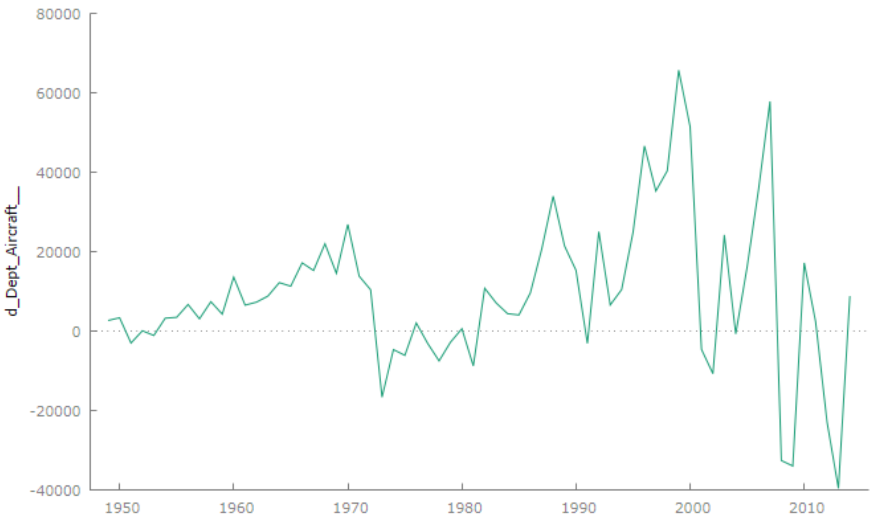

Figure 2: Plot of Δx_t, the first difference of the original plot

Parameter	**Value**
ADF Test Statistics	-4.88
p-value	3.90×10^{-5}
No. of Lags Used in ADF Test	0
No. of Obs in Time Series	65
Critical Values (1%,5%,10%)	-3.54, -2.91, -2.59
Max Eigenvalue Stats	1456.08

Table 1: ADF stationary test results of y_t

Summary Stats	**Value**
Mean	9258.28
Min	-39587.30
Max	65687.00
Standard Deviation	19279.20
Missing Obs	1

Table 2: Descriptive statistics of y_t

To assess stationarity, the Augmented Dickey-Fuller (ADF) test was conducted using Python, with the results displayed in Table 1. The evidence of stationarity is sufficient, as the ADF test statistic is negative and exceeds the critical values at the 1%, 5%, and 10% levels. Additionally, the p-value is less than 0.05, allowing us to reject the null hypothesis of non-stationarity.

However, Table 2 also reveals a higher standard deviation, indicating significant variation that needs to be addressed.

To address this, I will first apply a logarithmic transformation to the original dataset and then compute the first difference to stabilize variance and ensure stationarity.

If x_t represents our original time series, the new logarithmic series is given by $z_t = \log x_t$. The plot of z_t is in Figure 3

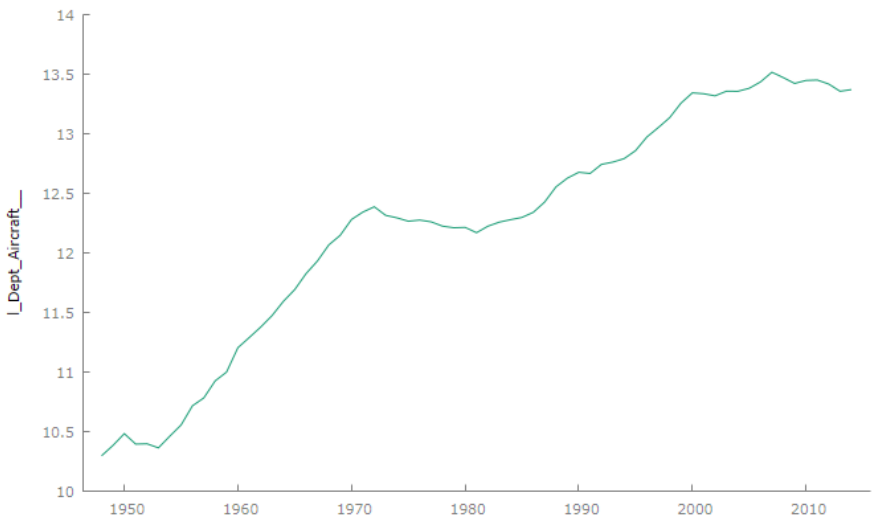

Figure 3: Plot of z_t, log of original plot

Similarly, if $\Delta = 1 - L$, where L is the lag operator, the new transformed logarithmic series is given by

$$\Delta z_t = \Delta \log x_t = \log x_t - \log x_{t-1}.$$

The plot of Δz_t is shown in the Figure 4

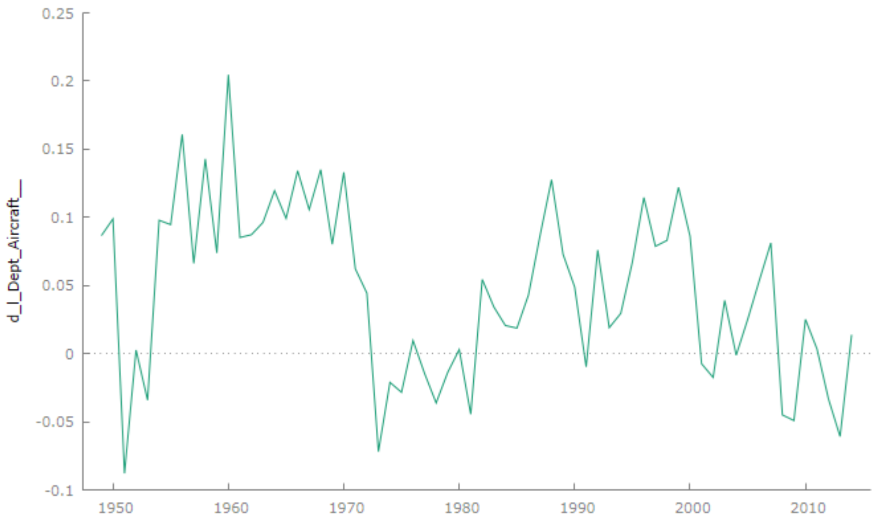

Figure 4: Plot of Δz_t, the first difference of the log of original values

Parameter	Value
ADF Test Statistics	-4.41
p-value	2.85×10^{-4}
No. of Lags Used in ADF Test	0
No. of Obs in Time Series	65
Critical Values (1%,5%,10%)	-3.54, -2.91, -2.59
Max Eigenvalue Stats	-193.31

Table 3: ADF stationary test results of Δz_t

Summary Stats	Value
Mean	0.0465
Min	-0.0878
Max	0.2046
Standard Deviation	0.0627
Missing Obs	1

Table 4: Description statistics of Δz_t

Examining the plot of the first difference of the logarithm of the original values (Δz_t) in Figure 4, it is evident that the variation around the mean has significantly decreased. This observation is supported by the statistics in Table 3, which indicate that the standard deviation is close to zero.

The graph also demonstrates stationarity, as confirmed by Table 4. The p-value is less than 0.05, rejecting the null hypothesis of non-stationarity. Furthermore, the ADF test statistic is negative and even more negative than the critical values at 1%, 5%, and 10%, further reinforcing confidence in the stationarity of the series.

This newly transformed series, z_t, will now be the focus for the next step: Model Identification.

Before proceeding, the data is split into training and testing sets. The training set includes values from 1948 to 2000, comprising 53 observations, while the testing set consists of values from 2001 to 2014, totaling 14 observations.

4 Model Identification

The initial analysis is conducted by examining the correlogram of Δz_t using the training dataset as shown in the Figure 5

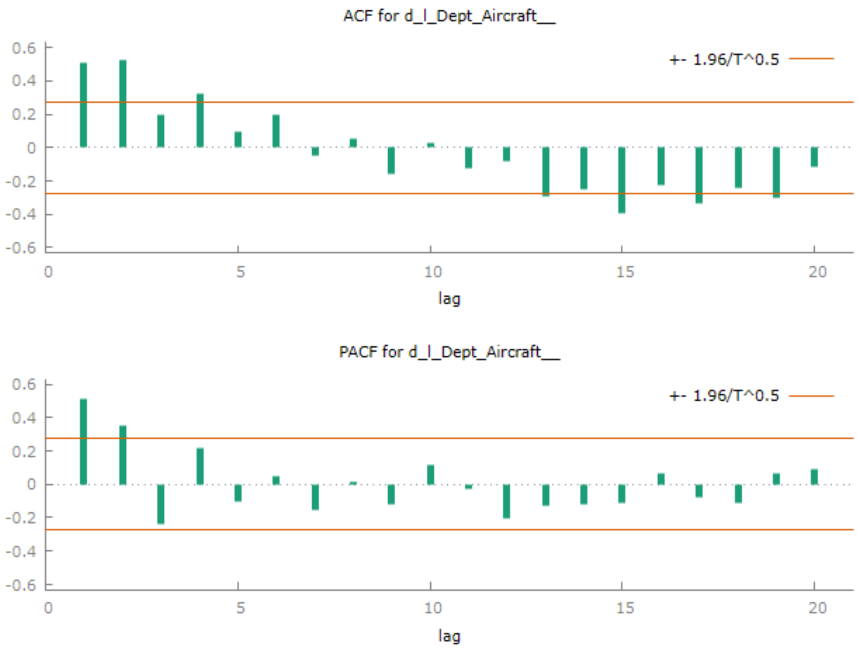

Figure 5: Autocorrelogram of Δz_t

Examining the correlogram suggests that a potential model for this series could be **AR**(2), as it exhibit a significant spike at the second lag in PACF, while for ACF there are many significant lags that occur at different orders and can be disregarded. Hence, we have:

1. $\Delta z_t \sim \text{ARMA}(2,0) \implies z_t \sim \text{ARIMA}(2,1,0)$

The model has the following equations:

$$(1-\phi_1 L - \phi_2 L^2)\Delta z_t = c + u_t \tag{4.1}$$

$$(1-\phi_1 L - \phi_2 L^2)(1-L) z_t = c + u_t \tag{4.2}$$

$$(1-\phi_1 L - \phi_2 L^2)(1-L)\log x_t = c + u_t \tag{4.3}$$

However, the correlogram alone does not provide sufficient certainty for building the ARIMA model. To ensure robustness, Information Criteria methods are applied using three estimators—AIC, BIC, and HQC—to identify the most appropriate model.

4.1 Information Criteria

The Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Criterion (HQC) are measures used to evaluate the fit of a statistical model while penalizing for model complexity to prevent overfitting. These criteria are particularly useful in selecting the best ARIMA model by balancing goodness-of-fit and simplicity. A description is provided in Table 5.

1. AIC	**2. BIC**	**3. HQC**
$-2\ln(L) + 2k$	$-2\ln(L) + k\ln(n)$	$-2\ln(L) + 2k\ln(\ln(n))$
• L: Maximum likelihood of the model. • k: Number of estimated parameters.	• n: Number of observations in the data. • L: Maximum likelihood of the model. • k: Number of estimated parameters.	• n: Number of observations. • L: Maximum likelihood of the model. • k: Number of estimated parameters.
The model with the lowest AIC is preferred. AIC penalizes complexity less heavily than BIC, making it more prone to overfitting.	BIC imposes a stronger penalty for model complexity compared to AIC, as the penalty term $k\ln(n)$ increases with the sample size.	HQC penalizes complexity less heavily than BIC but more than AIC, and is useful when the sample size n is large.

Table 5: Brief Description of Information Criteria Used

In Gretl, the `armax` function is utilized to identify the appropriate models. The parameter settings for this function are represented in Figure 6.

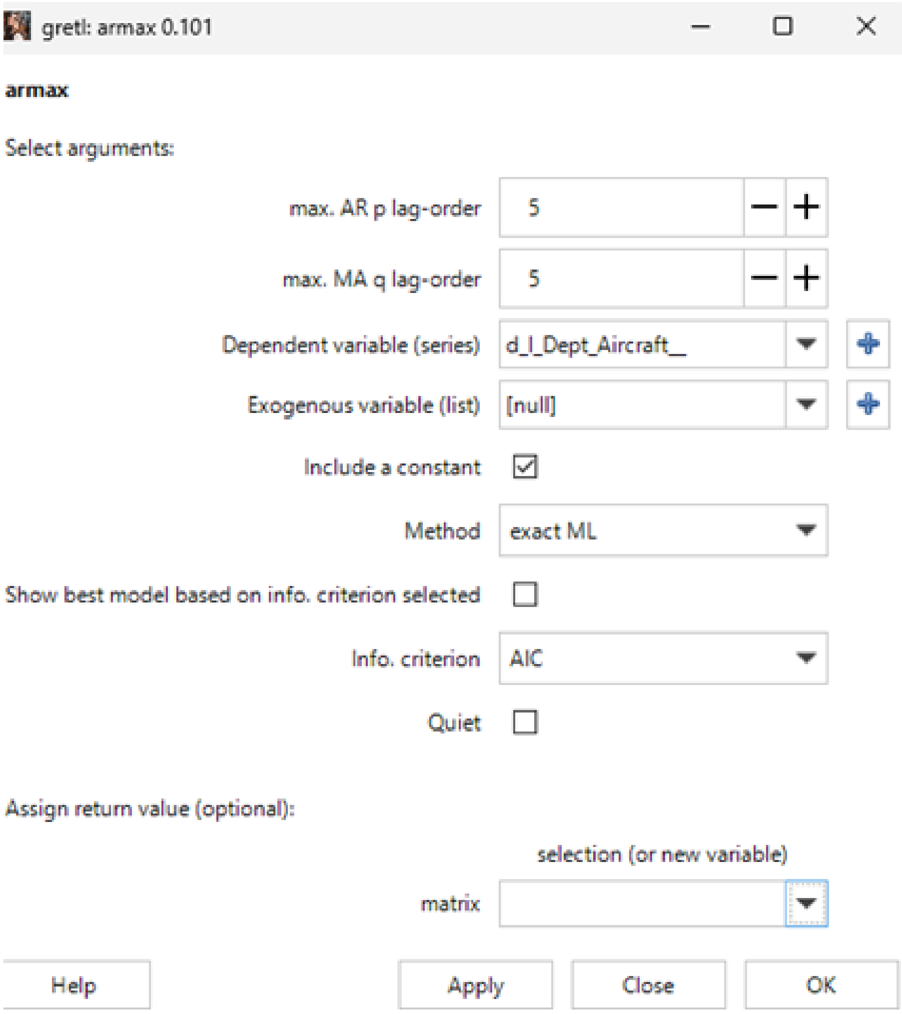

Figure 6: The maximum lags for p and q are set to 5, and a constant is included.

The obtained result is as follows:

```
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? armax(5, 5, d_l_Dept_Aircraft__, null, 1, 1, 0, 1, 0)
==============================================
 Information Criteria of ARMAX(p,q) for d_l_Dept_Aircraft__
----------------------------------------------
 p, q          AIC            BIC            HQC
----------------------------------------------
 0, 0    -137.9814     -134.0789     -136.4853
 0, 1    -143.7520     -137.8982     -141.5078
 0, 2    -158.1300     -150.3250     -155.1378
 0, 3    -156.1867     -146.4305     -152.4464
 0, 4    -157.3948     -145.6873     -152.9064
 0, 5    -155.4203     -141.7616     -150.1838
 1, 0    -151.3354     -145.4817     -149.0912
 1, 1    -153.1311     -145.3261     -150.1389
 1, 2    -156.7886     -147.0324     -153.0483
 1, 3    -155.1000     -143.3926     -150.6117
 1, 4    -155.5791     -141.9204     -150.3426
 1, 5    -153.7125     -138.1025     -147.7280
 2, 0    -156.6507     -148.8457     -153.6584
 2, 1    -166.2901*    -156.5338*    -162.5498*
 2, 2    -164.6634     -152.9559     -160.1750
 2, 3    -162.6714     -149.0127     -157.4350
 2, 4    -160.6725     -145.0626     -154.6880
 2, 5    -158.9551     -141.3939     -152.2226
 3, 0    -159.6044     -149.8482     -155.8641
 3, 1    -164.6678     -152.9603     -160.1794
 3, 2    -162.6684     -149.0097     -157.4320
 3, 3    -162.1728     -146.5628     -156.1883
 3, 4    -158.6958     -141.1346     -151.9632
 3, 5    -161.7937     -142.2813     -154.3131
 4, 0    -160.7662     -149.0588     -156.2779
 4, 1    -162.6688     -149.0101     -157.4324
 4, 2    -160.6834     -145.0734     -154.6989
 4, 3    -162.0562     -144.4950     -155.3237
 4, 4    -160.1370     -140.6246     -152.6564
 4, 5    -156.3631     -134.8994     -148.1344
 5, 0    -159.3898     -145.7311     -154.1534
 5, 1    -160.7066     -145.0967     -154.7221
 5, 2    -160.3528     -142.7916     -153.6203
 5, 3    -159.9257     -140.4132     -152.4451
 5, 4    -159.9902     -138.5265     -151.7615
 5, 5    -159.4006     -135.9857     -150.4239
==============================================
* indicates best models.
'9999.9999' suggests failures to estimate the models.
```

Figure 7: Result of `armax` function

By looking at the figure X, it's evident that the best model for all three criteria is an ARMA(2, 1).

$$\Delta z_t \sim \text{ARMA}(2,1) \Rightarrow z_t \sim \text{ARIMA}(2,1,1)$$

The model has the following equations:

$$(1-\phi_1 L-\phi_2 L^2)\Delta z_t = c+(1+\theta_1 L)u_t \tag{4.4}$$

$$(1-\phi_1 L-\phi_2 L^2)(1-L)z_t = c+(1+\theta_1 L)u_t \tag{4.5}$$

$$(1-\phi_1 L-\phi_2 L^2)(1-L)\log x_t = c+(1+\theta_1 L)u_t \tag{4.6}$$

So, now we have two models to examine: ARIMA (2,1,0) and ARIMA (2,1,1)

4.2 Model Estimation and Checking

4.2.1 First Model: ARIMA (2,1,0)

The estimation of this model yields the following results:

```
Function evaluations: 25
Evaluations of gradient: 7

Model 1: ARIMA, using observations 1949-2000 (T = 52)
Estimated using AS 197 (exact ML)
Dependent variable: (1-L) l_Dept_Aircraft__
Standard errors based on Hessian

             coefficient   std. error     z     p-value
  -----------------------------------------------------
  const       0.0625774    0.0203169    3.080   0.0021  ***
  phi_1       0.319861     0.126327     2.532   0.0113  **
  phi_2       0.360468     0.127728     2.822   0.0048  ***

Mean dependent var   0.058556   S.D. dependent var   0.062385
Mean of innovations -0.000769   S.D. of innovations  0.049412
R-squared            0.996584   Adjusted R-squared   0.996516
Log-likelihood       82.32533   Akaike criterion    -156.6507
Schwarz criterion   -148.8457   Hannan-Quinn        -153.6584

                        Real  Imaginary    Modulus  Frequency
  -----------------------------------------------------------
  AR
    Root  1           1.2800     0.0000     1.2800     0.0000
    Root  2          -2.1673     0.0000     2.1673     0.5000
  -----------------------------------------------------------
```

Figure 8: Model Estimation Results - ARIMA (2,1,0)

Thus, Equation (4.3) becomes:

$$(1 - 0.32L - 0.36L^2)(1 - L)\log x_t = 0.06 + u_t \tag{1}$$

The next step is *Model Checking*, which involves verifying whether the residuals can be considered a realization of Gaussian White Noise. To begin, let's examine the plot of residuals versus time.

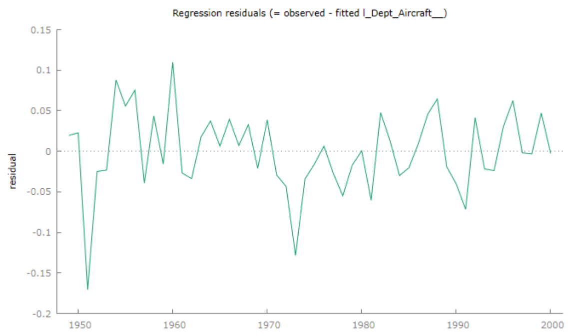

Figure 9: Residual Plot

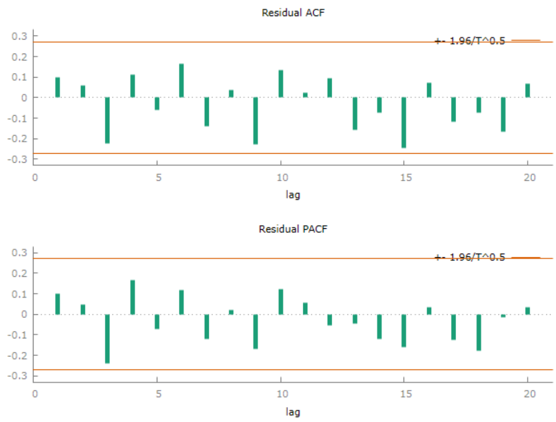

Figure 10: Correlogram of Residuals

From the plot in Figure 9, it can be observed that the residuals appear stationary, suggesting they may represent a realization of white noise. To further confirm this, we examine the correlogram of the residuals in Figure 10. Based on these observations, we conclude that the residuals are indeed a realization of white noise.

Next, to verify whether the residuals are a realization of Gaussian White Noise, we analyze their normality and review the Q-Q plot.

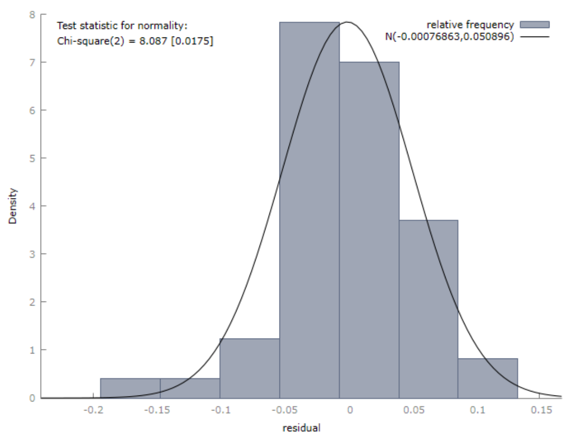

Figure 11: Histogram Plot

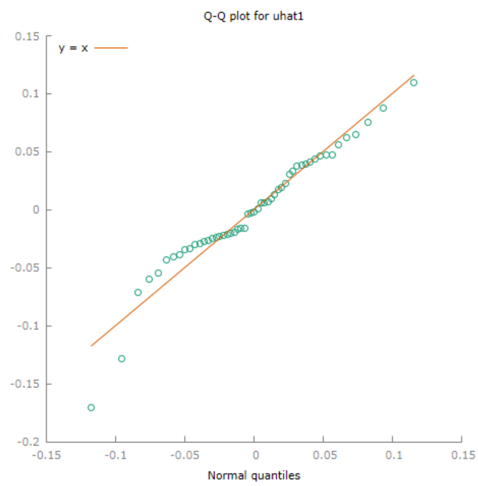

Figure 12: Q-Q Plot

From Figure 11, it is evident that the residuals do not follow a Gaussian distribution (the p-value = 0.0175, which is less than 0.05). Similarly, Figure 12, which shows the Q-Q plot of the residuals versus the actual data of the time series, suggests that the data points do not align perfectly with the straight line.

Overall, while the model isn't the Gaussian White Noise, it can still be considered a potential candidate for forecasting on the test set.

4.2.2 Second Model ARIMA (2,1,1)

The results of the second model estimation are shown in Figure 13.

```
Function evaluations: 109
Evaluations of gradient: 29

Model 2: ARIMA, using observations 1949-2000 (T = 52)
Estimated using AS 197 (exact ML)
Dependent variable: (1-L) l_Dept_Aircraft__
Standard errors based on Hessian

             coefficient   std. error     z       p-value
  ---------------------------------------------------------
  const       0.0622690    0.0185405     3.359    0.0008   ***
  phi_1      -0.286379     0.100217     -2.858    0.0043   ***
  phi_2       0.698265     0.0979702     7.127    1.02e-12 ***
  theta_1     0.877042     0.0708382    12.38     3.32e-35 ***

Mean dependent var   0.058556   S.D. dependent var   0.062385
Mean of innovations -0.000990   S.D. of innovations  0.043571
R-squared            0.997354   Adjusted R-squared   0.997246
Log-likelihood       88.14503   Akaike criterion    -166.2901
Schwarz criterion   -156.5338   Hannan-Quinn        -162.5498

                        Real  Imaginary    Modulus  Frequency
  -----------------------------------------------------------
  AR
    Root  1          -1.0091     0.0000     1.0091     0.5000
    Root  2           1.4192     0.0000     1.4192     0.0000
  MA
    Root  1          -1.1402     0.0000     1.1402     0.5000
  -----------------------------------------------------------
```

Figure 13: Model Estimation Results - ARIMA (2,1,1)

Thus, Equation (4.6) becomes:

$$(1 + 0.29L - 0.70L^2)(1 - L)\log x_t = 0.06 + (1 + 0.88L)u_t \tag{2}$$

The plot of the residuals versus time and its correlogram are shown below:

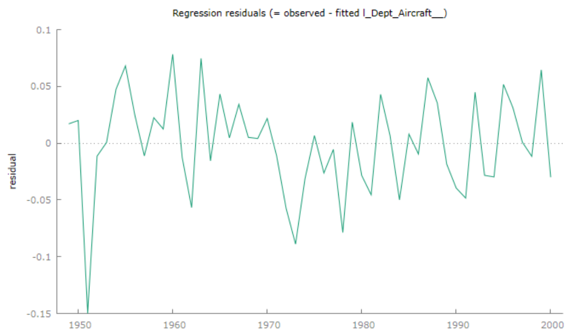

Figure 14: Residual Plot

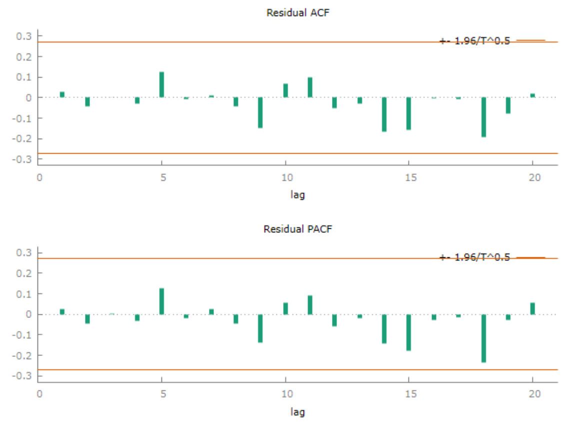

Figure 15: Correlogram of Residuals

From Figure 14, it is evident that the residuals appear to represent a realization of White Noise. To verify whether these residuals also follow a Gaussian distribution, we examine their normality using a histogram and Q-Q plot.

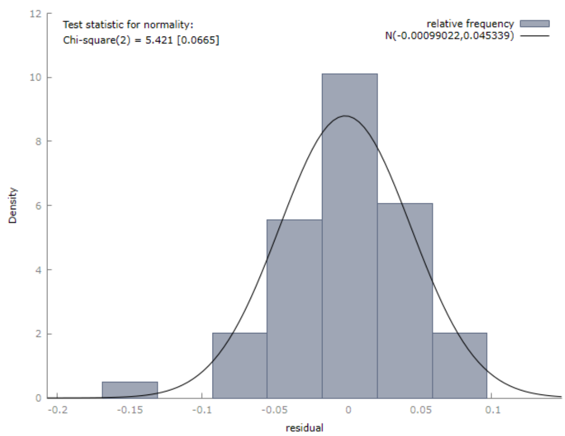

Figure 16: Histogram Plot

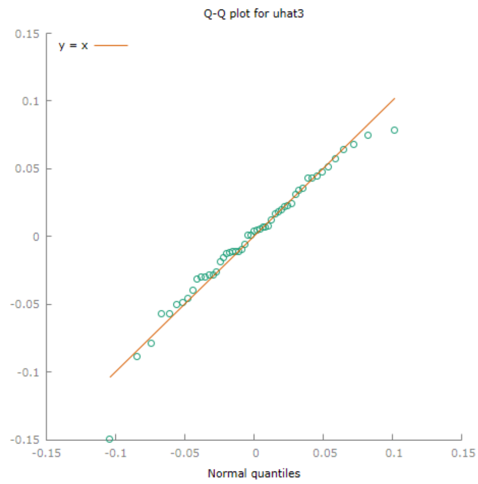

Figure 17: Q-Q Plot

From Figure 16, the p-value for the normality test is 0.0665, which is greater than the significance level of 0.05. This suggests that we accept the null hypothesis, meaning there is strong evidence against the assumption that the residuals follow a Gaussian distribution. Furthermore, as observed in the Q-Q plot in Figure 17, the data points closely align with the reference line, further supporting the conclusion that the residuals exhibit characteristics of Gaussian White Noise. Based on these findings, we consider this model to be a valid candidate for forecasting on the test set.

The next step is to use these selected models for forecasting on the test set.

4.3 Forecast on test set

The prediction on the test set ranges in our dataset from 2001-2014.

By doing the forecast on the test set, the following results are obtained:

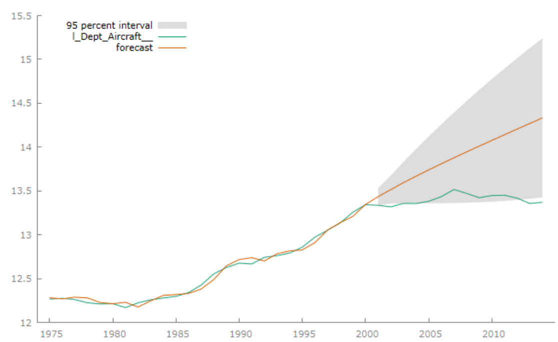

Figure 18: Prediction on Test set ARIMA (2,1,0)

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For 95% confidence intervals, z(0.025) - 1.96

	l_Dept_Aircraft__	prediction	std. error	95% interval
1975	12.265991	12.281713		
1976	12.275589	12.269331		
1977	12.261048	12.288442		
1978	12.224929	12.279861		
1979	12.211110	12.228138		
1980	12.214100	12.213674		
1981	12.169709	12.230080		
1982	12.224124	12.176593		
1983	12.258404	12.245531		
1984	12.279040	12.308988		
1985	12.297767	12.318001		
1986	12.340770	12.331200		
1987	12.427266	12.381279		
1988	12.554869	12.490438		
1989	12.627683	12.646867		
1990	12.676676	12.716975		
1991	12.667013	12.738599		
1992	12.742981	12.701587		
1993	12.762042	12.783801		
1994	12.791596	12.815527		
1995	12.858591	12.827924		
1996	12.973109	12.910677		
1997	13.051849	13.053893		
1998	13.134906	13.138319		
1999	13.256891	13.209861		
2000	13.343030	13.345853		
2001	13.335605	13.434559	0.049412	13.337713 - 13.531404
2002	13.318147	13.514890	0.081822	13.354522 - 13.675257
2003	13.357201	13.593562	0.120223	13.357949 - 13.829214
2004	13.356096	13.667713	0.157079	13.359844 - 13.975582
2005	13.381385	13.739795	0.193794	13.359966 - 14.119624
2006	13.434994	13.809577	0.229082	13.360585 - 14.258570
2007	13.516221	13.877885	0.263102	13.362216 - 14.393555
2008	13.471262	13.944893	0.295640	13.365449 - 14.524337
2009	13.422170	14.010953	0.326765	13.370505 - 14.651401
2010	13.447312	14.076242	0.356499	13.377515 - 14.774968
2011	13.450700	14.140942	0.384926	13.386501 - 14.895382
2012	13.417256	14.205175	0.412121	13.397433 - 15.012917
2013	13.356492	14.269047	0.438173	13.410244 - 15.127851
2014	13.370352	14.332636	0.463166	13.424848 - 15.240424

Forecast evaluation statistics using 14 observations

Mean Error	-0.49876
Root Mean Squared Error	0.56211
Mean Absolute Error	0.49876
Mean Percentage Error	-3.7198
Mean Absolute Percentage Error	3.7198
Theil's U2	14.166
Bias proportion, UM	0.7873
Regression proportion, UR	0.20448

Figure 19: Readings of Prediction on Test set ARIMA (2,1,0)

By looking at the prediction on test set Figure 18 we can clearly see that the readings of 2013 and 2014 are out of the confidence interval also highlighted the exact values in Figure 19. The registered RMSE value for z_t is 0.56.

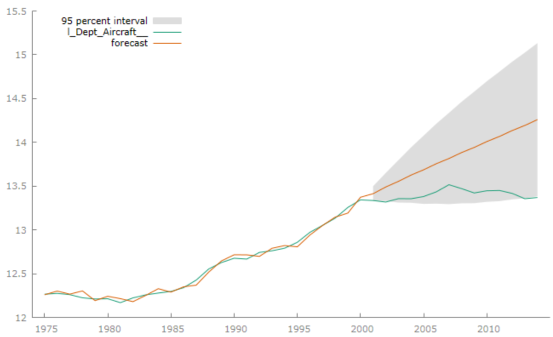
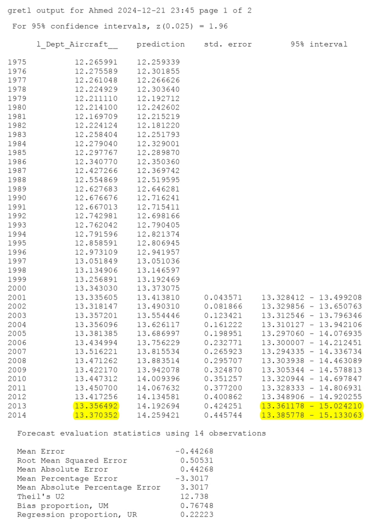


Figure 20: Prediction on Test set ARIMA (2,1,1)



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For 95% confidence intervals, z(0.025) = 1.96

	l_Dept_Aircraft__	prediction	std. error	95% interval
1975	12.265991	12.259339		
1976	12.275589	12.301855		
1977	12.261048	12.266626		
1978	12.224929	12.303640		
1979	12.211110	12.192712		
1980	12.214100	12.242602		
1981	12.169709	12.215219		
1982	12.224124	12.181220		
1983	12.258404	12.251793		
1984	12.279040	12.329001		
1985	12.297767	12.289870		
1986	12.340770	12.350360		
1987	12.427266	12.369742		
1988	12.554869	12.519595		
1989	12.627683	12.646281		
1990	12.676676	12.716241		
1991	12.667013	12.715411		
1992	12.742981	12.698166		
1993	12.762042	12.790405		
1994	12.791596	12.821374		
1995	12.858591	12.806945		
1996	12.973109	12.941957		
1997	13.051849	13.051036		
1998	13.134906	13.146597		
1999	13.256891	13.192469		
2000	13.343030	13.373075		
2001	13.335605	13.413810	0.043571	13.328412 - 13.499208
2002	13.318147	13.490310	0.081866	13.329856 - 13.650763
2003	13.357201	13.554446	0.123421	13.312546 - 13.796346
2004	13.356096	13.626117	0.161222	13.310127 - 13.942106
2005	13.381385	13.686997	0.198951	13.297060 - 14.076935
2006	13.434994	13.756229	0.232771	13.300007 - 14.212451
2007	13.516221	13.815534	0.265923	13.294335 - 14.336734
2008	13.471262	13.883514	0.295707	13.303938 - 14.463089
2009	13.422170	13.942078	0.324870	13.305344 - 14.578813
2010	13.447312	14.009396	0.351257	13.320944 - 14.697847
2011	13.450700	14.067632	0.377200	13.328333 - 14.806931
2012	13.417256	14.134581	0.400862	13.348906 - 14.920255
2013	13.356492	14.192694	0.424251	13.361178 - 15.024210
2014	13.370352	14.259421	0.445744	13.385778 - 15.133063

Forecast evaluation statistics using 14 observations

Mean Error	-0.44268
Root Mean Squared Error	0.50531
Mean Absolute Error	0.44268
Mean Percentage Error	-3.3017
Mean Absolute Percentage Error	3.3017
Theil's U2	12.738
Bias proportion, UM	0.76748
Regression proportion, UR	0.22223

Figure 21: Readings of Prediction on Test set ARIMA (2,1,1)

Examining the predictions on the test set in Figure 20, it is observed that the readings for 2013 and 2014 also fall outside the confidence interval similar to the previous. However, they are not as far outside as they were with the previous model. The actual values, highlighted along with their exact figures in Figure 21, indicate that the difference between the observed values and the lower bound of the confidence interval is smaller than in the first model. This model achieves an RMSE value of 0.50 for z_t, which is lower than that of the previous model.

In the evaluations above, the forecasts were based on $z_t = \log x_t$. However, to obtain forecasts for x_t, the values need to be transformed back using $x_t = \exp(z_t)$. To calculate the necessary indicators for x_t, an Excel sheet containing the predicted and actual values of z_t's forecasts on the test set was utilized, enabling the computation of x_t's indicators. This process was essential for selecting the best model.

CASE 1 =>	ARIMA(2,1,0)						
YEAR	yt	yt hat	Error = yt hat -yt		xt = exp(yt)	xt hat = exp (yt hat)	Error = xt hat - xt
2001	13.335605	13.434559	0.098954		618841.833729	683210.984949	64369.151220
2002	13.318147	13.514890	0.196743		608131.852438	740358.642647	132226.790209
2003	13.357201	13.593582	0.236381		632351.696402	800972.585403	168620.889000
2004	13.356096	13.667713	0.311617		631653.333694	862605.724926	230952.391232
2005	13.381385	13.739795	0.358410		647830.910048	927079.869545	279248.959496
2006	13.434994	13.809577	0.374583		683508.246377	994084.010825	310575.764448
2007	13.516221	13.877885	0.361664		741344.716087	1064360.811551	323016.095464
2008	13.471262	13.944893	0.473631		708752.740016	1138125.307904	429372.567888
2009	13.422170	14.010953	0.588783		674798.900310	1215848.810126	541049.909815
2010	13.447312	14.076242	0.628930		691979.769953	1297879.062867	605899.292914
2011	13.450700	14.140942	0.690242		694328.173362	1384627.903812	690299.730450
2012	13.417256	14.205175	0.787919		671491.072503	1476485.264358	804994.191855
2013	13.356492	14.269047	0.912555		631903.517948	1573868.249186	941964.731238
2014	13.370352	14.332636	0.962284		640722.676295	1677199.506436	1036476.830141
		ME	-0.498764			ME	-468504.806812
		MSE	0.315971			MSE	308035493491.921000
		RMSE	0.562113			RMSE	555009.453516
		MPE	-3.719818			MPE	-70.424716
		MAPE	3.719817912			MAPE	70.42471601

Figure 22: Excel ARIMA(2,1,0)

CASE 2 =>	ARIMA(2,1,1)						
YEAR	yt	yt hat	Error = yt hat -yt		xt = exp(yt)	xt hat = exp (yt hat)	Error = xt hat - xt
2001	13.335605	13.413810	0.078205		618841.833729	669181.096642	50339.262913
2002	13.318147	13.490310	0.172163		608131.852438	722382.459557	114250.607119
2003	13.357201	13.554446	0.197245		632351.696402	770231.193452	137879.497050
2004	13.356096	13.626117	0.270021		631653.333694	827460.788821	195807.455127
2005	13.381385	13.686997	0.305612		647830.910048	879401.639442	231570.729393
2006	13.434994	13.756229	0.321235		683508.246377	942441.379921	258933.133544
2007	13.516221	13.815534	0.299313		741344.716087	1000023.442310	258678.726224
2008	13.471262	13.883514	0.412252		708752.740016	1070368.992721	361616.252705
2009	13.422170	13.942078	0.519908		674798.900310	1134925.990314	460127.090004
2010	13.447312	14.009396	0.562084		691979.769953	1213957.206524	521977.436571
2011	13.450700	14.067632	0.616932		694328.173362	1286752.293615	592424.120252
2012	13.417256	14.134581	0.717325		671491.072503	1375848.239043	704357.166540
2013	13.356492	14.192694	0.836202		631903.517948	1458171.774837	826268.256889
2014	13.370352	14.259421	0.889069		640722.676295	1558790.877233	918068.200938
		ME	-0.442683			ME	-402306.995376
		MSE	0.255342			MSE	232008722804.419000
		RMSE	0.505313			RMSE	481672.837935
		MPE	-3.301688			MPE	-60.510770
		MAPE	3.301687817			MAPE	60.51077004

Figure 23: Excel ARIMA(2,1,1)

Indicator	Model 1 – ARIMA (2,1,0)	Model 2 – ARIMA (2,1,1)
Mean Error	$\mathbf{-4.69 \times 10^5}$	-4.02×10^5
Mean Square Error	3.08×10^{11}	$\mathbf{2.32 \times 10^{11}}$
Root Mean Square Error	5.55×10^5	$\mathbf{4.82 \times 10^5}$
Mean Percentage Error	$\mathbf{-70.4}$	-60.5
Mean Absolute Percentage Error	70.4	**60.5**

Table 6: Comparison of different models

Based on the comparison of the different indicators in Table 6, it is evident that the second model, ARIMA(2,1,1), achieves the best performance. The second model consistently has lower values across the majority of key indicators.

Thus, ARIMA(2,1,1) is selected as the better model and will be used to forecast new values.

5 Forecast of future values

After having restored the full range of the time series (from 1948 to 2014), it is possible to forecast 20 future values (from 2015 to 2034).

The result obtained is the following:

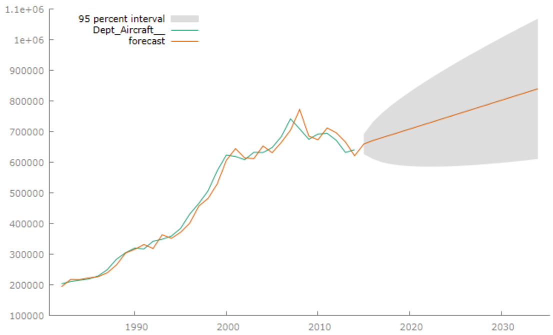

Figure 24: Next 20 years prediction plot

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For 95% confidence intervals, z(0.025) = 1.96

	Dept_Aircraft__	prediction	std. error	95% interval
1982	203643.	193737.		
1983	210745.	217630.		
1984	215139.	217351.		
1985	219206.	222782.		
1986	228838.	226527.		
1987	249513.	239346.		
1988	283472.	264749.		
1989	304883.	303831.		
1990	320192.	315864.		
1991	317113.	331546.		
1992	342142.	318626.		
1993	348726.	363404.		
1994	359186.	351851.		
1995	384074.	371283.		
1996	430675.	401085.		
1997	465957.	457170.		
1998	506311.	481842.		
1999	571998.	529025.		
2000	623454.	606217.		
2001	618842.	644593.		
2002	608132.	614086.		
2003	632352.	611748.		
2004	631653.	653022.		
2005	647831.	631003.		
2006	683509.	664729.		
2007	741345.	704598.		
2008	708753.	772051.		
2009	674799.	685200.		
2010	691980.	673284.		
2011	694328.	711901.		
2012	671491.	696054.		
2013	631904.	665959.		
2014	640723.	621060.		
2015		659299.	16700.3	626567. - 692031.
2016		670935.	30817.5	610534. - 731336.
2017		680251.	40981.1	599929. - 760572.
2018		689466.	48896.8	593630. - 785302.
2019		698811.	55624.2	589790. - 807832.
2020		708194.	61615.0	587431. - 828957.
2021		717577.	67075.4	586112. - 849043.
2022		726958.	72124.6	585597. - 868320.
2023		736339.	76842.8	585729. - 886948.
2024		745719.	81287.6	586398. - 905040.
2025		755099.	85501.6	587519. - 922679.
2026		764480.	89517.4	589029. - 939931.
2027		773860.	93360.7	590877. - 956844.
2028		783241.	97051.8	593023. - 973459.
2029		792621.	100608.	595434. - 989809.
2030		802002.	104042.	598083. - 1.00592e+06
2031		811382.	107367.	600947. - 1.02182e+06
2032		820763.	110591.	604007. - 1.03752e+06

Figure 25: Next 20 years prediction reading

Future forecasts indicate that the number of aircraft departures in Italy is expected to increase after 2014. However, this prediction does not account for exogenous factors that might influence this trend.

For instance, while the forecast suggests a "linear" growth, real-world events such as the SARS-CoV-2 pandemic in 2019 led to a drastic reduction in aircraft departures. Other factors that may significantly impact this trend include weather and climatic conditions, changes in aviation regulations, international relations, shifts in population mobility, and, most importantly, geopolitical conflicts and security issues.

Therefore, it is important to recognize that this growth trajectory will not continue indefinitely and is subject to substantial variability.

6 Conclusion

In this project, a time series analysis was conducted to study and forecast the number of aircraft departures in Italy from 1948 to 2014. The data was analyzed and modeled using various ARIMA configurations, and the best-performing model was selected based on evaluation criteria. The chosen model was then employed to predict the next 20 observations, offering insights into potential future trends in aircraft departures.

The forecast indicated an expected increase in departures after 2014, though it is important to note that this model does not account for exogenous factors. Nonetheless, the project provided a comprehensive understanding of historical trends and demonstrated the utility of ARIMA models for forecasting in aviation.

This project served as a valuable exercise in bridging theoretical knowledge with practical application, addressing challenges that often arise in real-world data analysis scenarios. The results highlight the importance of combining statistical modeling with domain knowledge to achieve robust and meaningful predictions.

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