# Supplementary results associated with

Performance Evaluation of Complex-Valued Neural Networks on Real and

Complex-Valued Classification and Reconstruction Tasks

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## 1 Introduction

This document presents additional experimental results that support the findings in the main paper, "Performance Evaluation of Complex-Valued Neural Networks on Real and Complex-Valued Classification and Reconstruction Tasks." The results cover a broad range of configurations, including different activation functions, optimizers, network depths, and input types (real vs. complex). This supplementary document provides the full experimental evidence supporting the findings reported in the main paper.

Results are grouped by task (classification, reconstruction and beamforming), then by dataset, and finally by model family (RVNNs or CVNNs).

### 1.1 Abbreviations and Notation

| Symbol | Meaning |
|---|---|
| 4S, 5S, 4L, 5L | 4-/5-layer fully-connected classification networks with small or large number of parameters |
| L, M, S, XS | Autoencoders with a large to extra small number of parameters |
| ReLU, Swish, ELU, etc. | Activation functions; see main paper Section 3.1 |
| Adam, SGD | Optimization algorithms |
| TT, IT | Training Time, Inference Time |
| CI | Confidence Interval with 95% confidence level |

### 1.2 Implementation Notes

- **Frameworks**: TensorFlow 2.16 (`cvnn==1.0.4`), PyTorch 2.2 (`complex-pytorch==0.5.1`).
- **Hardware**: NVIDIA GeForce RTX 3060 (12GB).

## 2 Classification

We evaluated CVNNs and RVNNs models with a variety of activation functions and optimizers using two image datasets with **real-valued** input (the original data type) or **complex-valued** input (obtained by either FFT or combining every two real-valued features to be one complex feature).

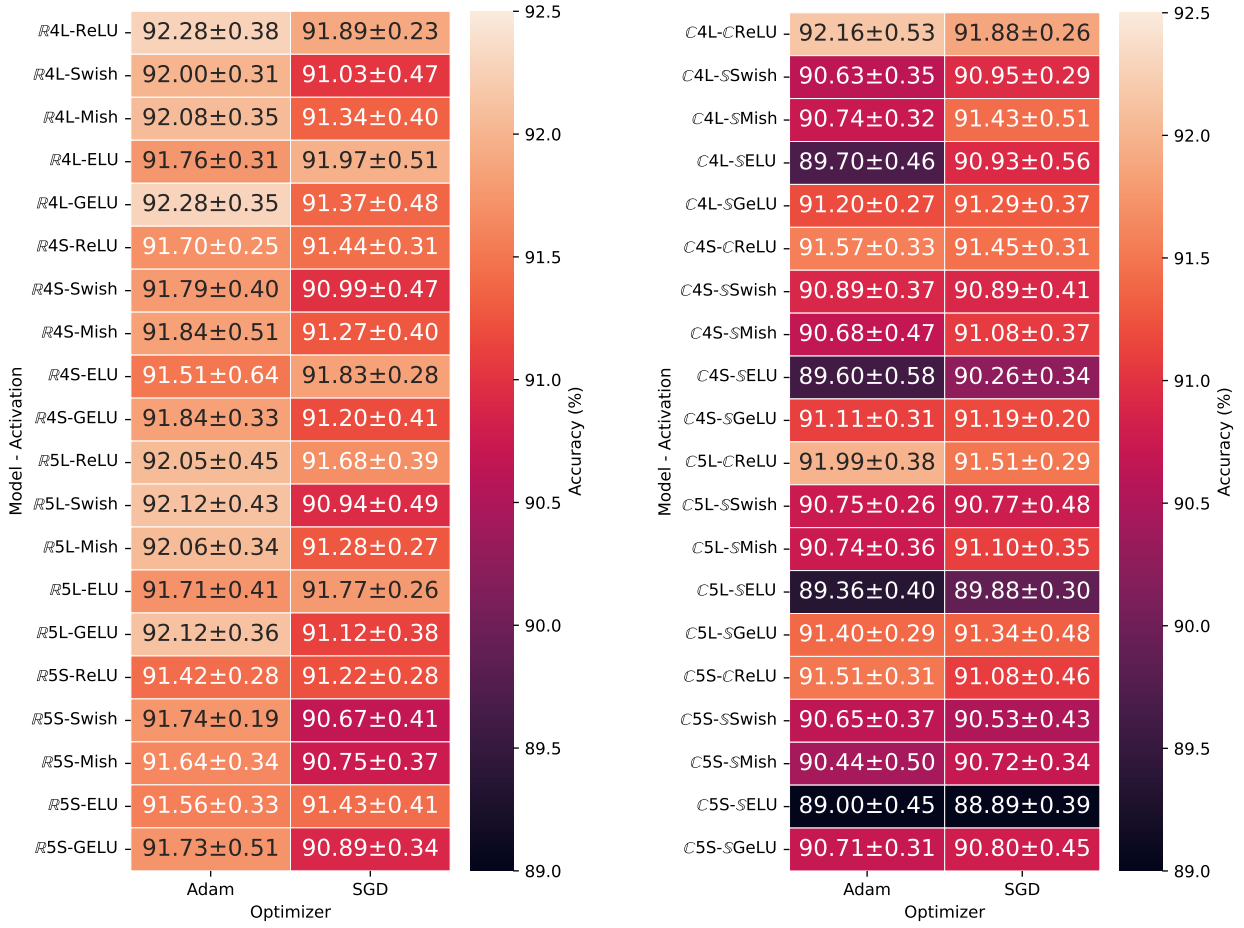
### 2.1 Real input

Figure 1 and 2 present the classification results for the experiments conducted on the Fashion MNIST and CIFAR-10 datasets, respectively. The input is real-valued for both types of neural networks using Tensorflow and the CVNNs library.

Figure 3 and 4 show similar experiments utilizing additional optimization algorithms (AMSGrad and NAdam) recommended by Reviewer 1.

Figure 5 and 6 show the results for the same experiments utilizing the PyTorch framework instead of the TensorFlow framework for both the RVNNs and the CVNNs.

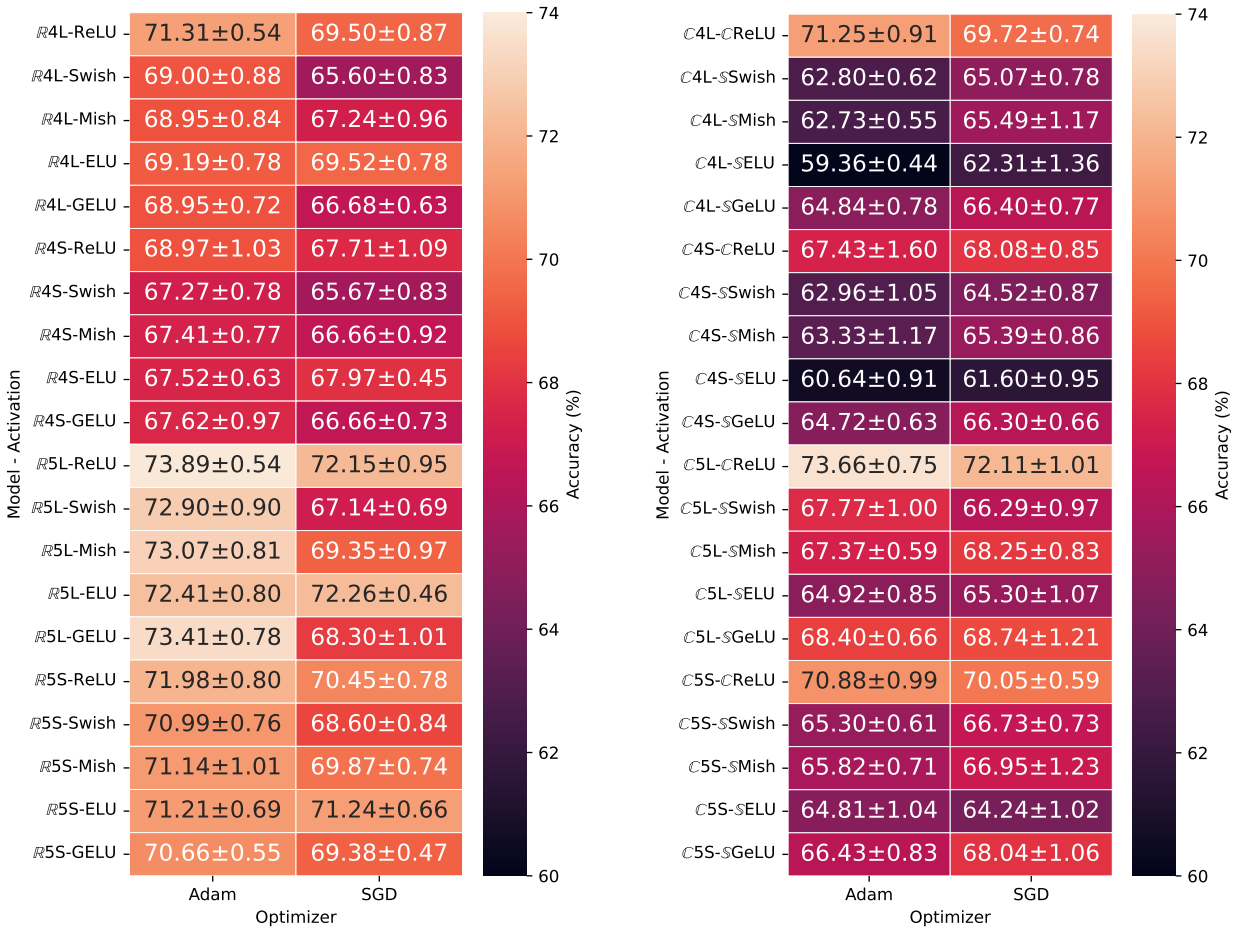
Table 1 collects a large set of results for the comparison of CVNNs and RVNNs when the input is real-valued, where the notations describing the various cases and the performance measures are provided in the list of abbreviations above.




(a) Real-Valued Neural Networks (RVNNs) Models.

(b) Complex-Valued Neural Networks (CVNNs) Models.

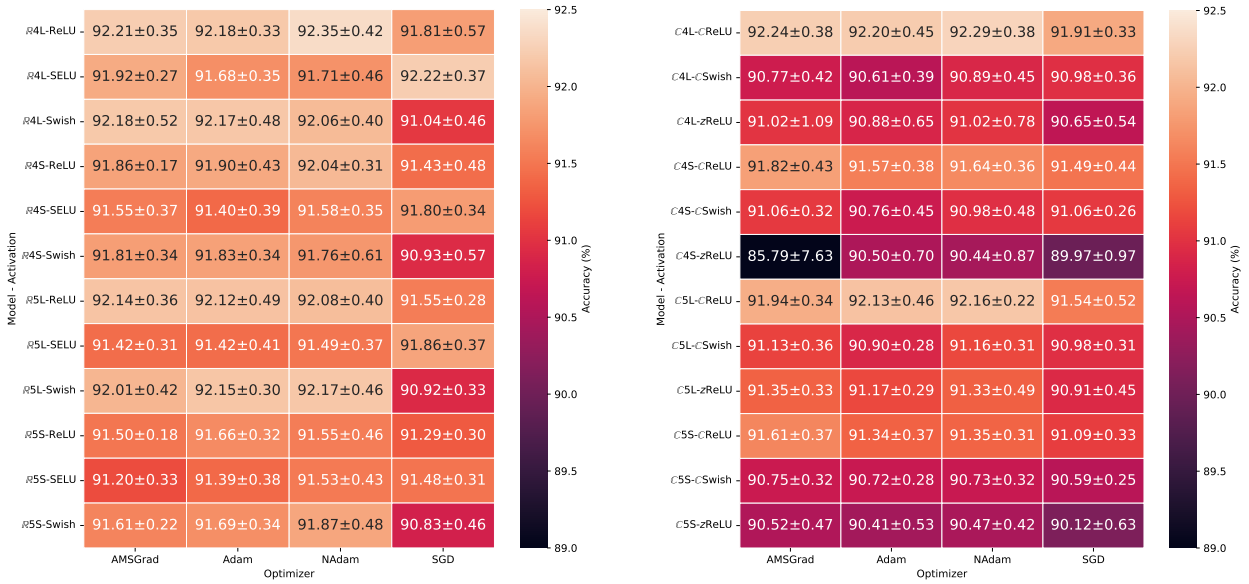
Figure 1: Mean accuracy comparison between CVNNs with real input (original data type) and RVNNs with real input (original data type) for Fashion MNIST datasets using different activations and Adam/SGD optimizer using TensorFlow for real models and CVNN library for complex models.




(a) RVNNs Models.

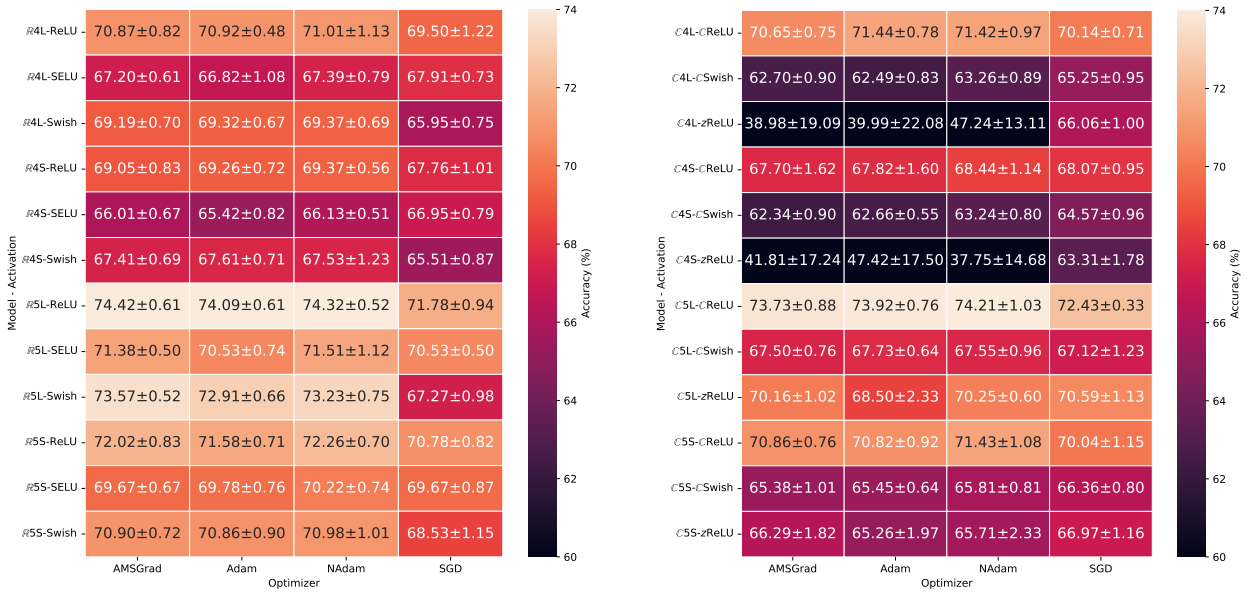
(b) CVNNs Models.

Figure 2: Mean accuracy comparison between CVNNs with real input (original data type) and RVNNs with real input (original data type) for CIFAR10 datasets using different activations and Adam/SGD optimizer using TensorFlow for real models and CVNN library for complex models.



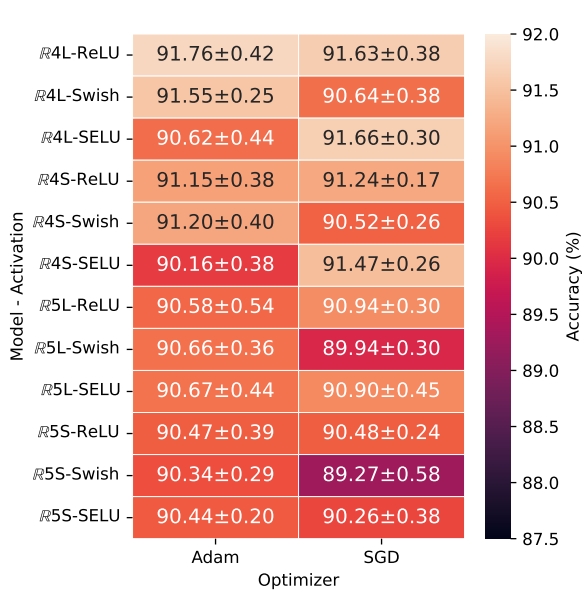

(a) RVNNs Models. (b) CVNNs Models.

Figure 3: Mean accuracy comparison between CVNNs with real input (original data type) and RVNNs with real input (original data type) for Fashion MNIST datasets using different activations and optimizers using TensorFlow for real models and CVNN library for complex models.

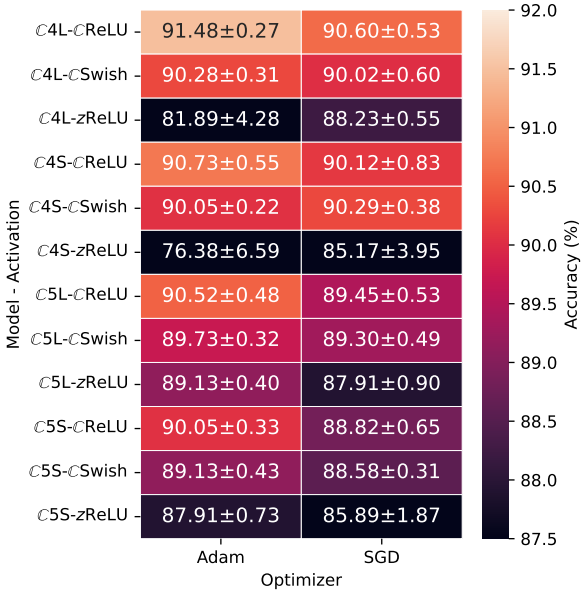



(a) RVNNs Models. (b) CVNNs Models.

Figure 4: Mean accuracy comparison between CVNNs with real input (original data type) and RVNNs with real input (original data type) for CIFAR10 datasets using different activations and optimizers using TensorFlow for real models and CVNN library for complex models.

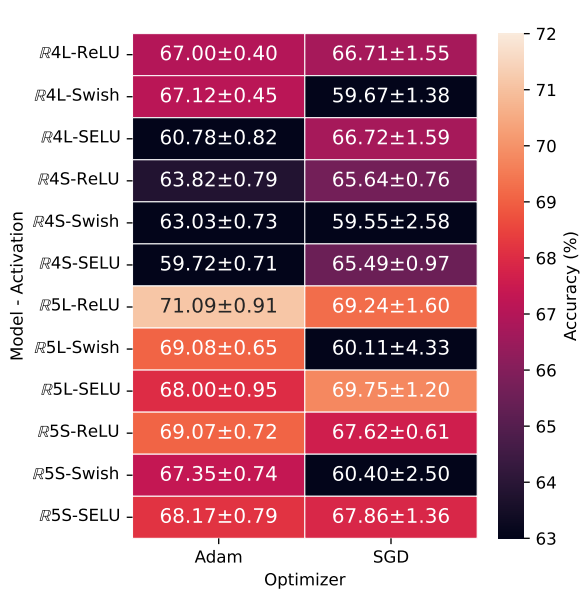



(a) RVNNs Models.

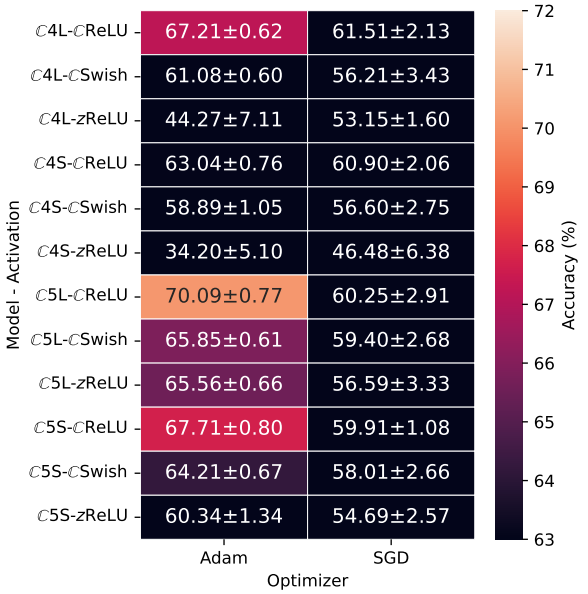



(b) CVNNs Models.

Figure 5: Mean accuracy comparison between CVNNs with real input (original data type) and RVNNs with real input (original data type) for Fashion MNIST datasets using different activations and Adam/SGD optimizer using PyTorch for real models and complexPyTorch library for complex models.




(a) RVNNs Models.




(b) CVNNs Models.

Figure 6: Mean accuracy comparison between CVNNs with real input (original data type) and RVNNs with real input (original data type) for CIFAR10 datasets using different activations and Adam/SGD optimizer using PyTorch for real models and complexPyTorch library for complex models.

Table 1: Results of classification experiments comparing RVNNs and CVNNs on the Fashion-MNIST and CIFAR-10 datasets, using real-valued inputs (original data type)

| Dataset | Act. | Opt. | Model | Accuracy±CI | TT±CI | IT±CI | epochs±CI |
|---|---|---|---|---|---|---|---|
| CIFAR10 | ℂReLU | Adam | ℂ4l | 71.25 ± 0.65 | 128.03 ± 6.68 | 0.58 ± 0.02 | 15.30 ± 0.83 |
| CIFAR10 | ℂMish | Adam | ℂ4l | 29.96 ± 10.01 | 277.38 ± 24.74 | 2.07 ± 0.11 | 11.60 ± 0.77 |
| CIFAR10 | 𝕊ELU | Adam | ℂ4l | 59.36 ± 0.31 | 132.61 ± 5.02 | 0.57 ± 0.00 | 15.90 ± 0.63 |
| CIFAR10 | 𝕊GELU | Adam | ℂ4l | 64.84 ± 0.56 | 150.94 ± 6.59 | 0.63 ± 0.00 | 15.10 ± 0.63 |
| CIFAR10 | 𝕊Mish | Adam | ℂ4l | 62.73 ± 0.40 | 139.49 ± 4.46 | 0.60 ± 0.00 | 15.00 ± 0.48 |
| CIFAR10 | splitSwish | Adam | ℂ4l | 62.80 ± 0.44 | 135.17 ± 3.89 | 0.59 ± 0.00 | 15.20 ± 0.45 |
| CIFAR10 | zReLU | Adam | ℂ4l | 51.49 ± 10.44 | 331.45 ± 58.17 | 0.60 ± 0.00 | 31.60 ± 5.59 |
| FMNIST | ℂReLU | Adam | ℂ4l | 92.16 ± 0.38 | 132.59 ± 6.04 | 0.52 ± 0.01 | 16.20 ± 0.74 |
| FMNIST | ℂMish | Adam | ℂ4l | 68.01 ± 22.06 | 266.35 ± 38.73 | 1.84 ± 0.12 | 11.90 ± 1.45 |
| FMNIST | 𝕊ELU | Adam | ℂ4l | 89.70 ± 0.33 | 115.77 ± 4.22 | 0.53 ± 0.00 | 14.00 ± 0.48 |
| FMNIST | 𝕊GELU | Adam | ℂ4l | 91.20 ± 0.19 | 142.39 ± 4.79 | 0.56 ± 0.00 | 15.10 ± 0.53 |
| FMNIST | 𝕊Mish | Adam | ℂ4l | 90.74 ± 0.23 | 137.73 ± 5.12 | 0.54 ± 0.00 | 15.10 ± 0.63 |
| FMNIST | splitSwish | Adam | ℂ4l | 90.63 ± 0.25 | 126.97 ± 6.38 | 0.53 ± 0.00 | 14.70 ± 0.76 |
| FMNIST | zReLU | Adam | ℂ4l | 90.94 ± 0.71 | 178.83 ± 23.85 | 0.53 ± 0.00 | 23.10 ± 3.12 |
| CIFAR10 | ℂReLU | SGD | ℂ4l | 69.72 ± 0.53 | 235.00 ± 9.92 | 0.57 ± 0.00 | 28.80 ± 1.21 |
| CIFAR10 | ℂMish | SGD | ℂ4l | 15.65 ± 6.80 | 233.98 ± 10.01 | 1.81 ± 0.16 | 10.30 ± 0.35 |
| CIFAR10 | 𝕊ELU | SGD | ℂ4l | 62.31 ± 0.97 | 204.06 ± 14.29 | 0.58 ± 0.00 | 25.10 ± 1.86 |
| CIFAR10 | 𝕊GELU | SGD | ℂ4l | 66.40 ± 0.55 | 284.79 ± 9.41 | 0.62 ± 0.00 | 29.60 ± 0.97 |
| CIFAR10 | 𝕊Mish | SGD | ℂ4l | 65.49 ± 0.84 | 263.95 ± 14.02 | 0.60 ± 0.00 | 29.10 ± 1.56 |
| CIFAR10 | splitSwish | SGD | ℂ4l | 65.07 ± 0.56 | 285.89 ± 17.35 | 0.60 ± 0.01 | 32.40 ± 2.00 |
| CIFAR10 | zReLU | SGD | ℂ4l | 41.34 ± 15.48 | 237.94 ± 56.43 | 0.60 ± 0.00 | 23.20 ± 5.49 |
| FMNIST | ℂReLU | SGD | ℂ4l | 91.88 ± 0.19 | 380.78 ± 40.42 | 0.52 ± 0.00 | 48.30 ± 5.01 |
| FMNIST | ℂMish | SGD | ℂ4l | 10.00 ± 0.00 | 221.32 ± 25.25 | 1.54 ± 0.04 | 10.40 ± 0.90 |
| FMNIST | 𝕊ELU | SGD | ℂ4l | 90.93 ± 0.40 | 307.91 ± 24.03 | 0.52 ± 0.00 | 39.00 ± 3.07 |
| FMNIST | 𝕊GELU | SGD | ℂ4l | 91.29 ± 0.26 | 415.42 ± 26.57 | 0.56 ± 0.00 | 44.80 ± 2.86 |
| FMNIST | 𝕊Mish | SGD | ℂ4l | 91.43 ± 0.36 | 403.99 ± 31.58 | 0.54 ± 0.01 | 46.30 ± 3.57 |
| FMNIST | splitSwish | SGD | ℂ4l | 90.95 ± 0.20 | 359.63 ± 26.93 | 0.53 ± 0.00 | 42.70 ± 3.22 |
| FMNIST | zReLU | SGD | ℂ4l | 90.71 ± 0.33 | 435.95 ± 53.40 | 0.53 ± 0.00 | 58.20 ± 7.22 |
| CIFAR10 | ℂReLU | Adam | ℂ4s | 67.43 ± 1.15 | 110.84 ± 6.54 | 0.45 ± 0.03 | 18.90 ± 1.14 |
| CIFAR10 | ℂMish | Adam | ℂ4s | 44.39 ± 5.91 | 241.56 ± 78.16 | 1.30 ± 0.03 | 15.20 ± 4.67 |
| CIFAR10 | 𝕊ELU | Adam | ℂ4s | 60.64 ± 0.65 | 152.74 ± 10.77 | 0.45 ± 0.02 | 25.70 ± 1.85 |
| CIFAR10 | 𝕊GELU | Adam | ℂ4s | 64.72 ± 0.45 | 125.41 ± 4.87 | 0.47 ± 0.00 | 19.10 ± 0.79 |
| CIFAR10 | 𝕊Mish | Adam | ℂ4s | 63.33 ± 0.84 | 139.35 ± 10.92 | 0.46 ± 0.00 | 21.90 ± 1.73 |
| CIFAR10 | splitSwish | Adam | ℂ4s | 62.96 ± 0.75 | 128.19 ± 9.84 | 0.44 ± 0.00 | 21.10 ± 1.63 |
| CIFAR10 | zReLU | Adam | ℂ4s | 40.84 ± 15.23 | 210.32 ± 54.02 | 0.46 ± 0.00 | 29.60 ± 7.60 |
| FMNIST | ℂReLU | Adam | ℂ4s | 91.57 ± 0.24 | 113.18 ± 3.71 | 0.40 ± 0.01 | 18.80 ± 0.56 |
| FMNIST | ℂMish | Adam | ℂ4s | 84.74 ± 1.87 | 206.04 ± 31.41 | 1.18 ± 0.03 | 14.10 ± 2.04 |
| FMNIST | 𝕊ELU | Adam | ℂ4s | 89.60 ± 0.42 | 113.94 ± 8.54 | 0.42 ± 0.01 | 18.10 ± 1.24 |
| FMNIST | 𝕊GELU | Adam | ℂ4s | 91.11 ± 0.22 | 117.74 ± 6.25 | 0.43 ± 0.00 | 17.80 ± 0.94 |
| FMNIST | 𝕊Mish | Adam | ℂ4s | 90.68 ± 0.34 | 124.86 ± 8.26 | 0.42 ± 0.01 | 19.00 ± 1.26 |
| FMNIST | splitSwish | Adam | ℂ4s | 90.89 ± 0.27 | 123.82 ± 10.10 | 0.41 ± 0.00 | 19.90 ± 1.63 |
| FMNIST | zReLU | Adam | ℂ4s | 88.38 ± 3.03 | 198.77 ± 50.61 | 0.42 ± 0.00 | 34.30 ± 8.78 |
| CIFAR10 | ℂReLU | SGD | ℂ4s | 68.08 ± 0.61 | 200.23 ± 12.16 | 0.44 ± 0.00 | 35.30 ± 2.13 |
| CIFAR10 | ℂMish | SGD | ℂ4s | 16.70 ± 7.74 | 191.60 ± 68.84 | 1.12 ± 0.07 | 13.50 ± 4.74 |
| CIFAR10 | 𝕊ELU | SGD | ℂ4s | 61.60 ± 0.68 | 207.88 ± 17.79 | 0.44 ± 0.00 | 37.00 ± 3.22 |
| CIFAR10 | 𝕊GELU | SGD | ℂ4s | 66.30 ± 0.47 | 253.46 ± 12.91 | 0.47 ± 0.00 | 39.60 ± 2.03 |
| CIFAR10 | 𝕊Mish | SGD | ℂ4s | 65.39 ± 0.62 | 250.44 ± 13.61 | 0.46 ± 0.00 | 40.40 ± 2.19 |

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| Dataset | Act. | Opt. | Model | Accuracy±CI | TT±CI | IT±CI | epochs±CI |
|---|---|---|---|---|---|---|---|
| CIFAR10 | splitSwish | SGD | ℂ4s | 64.52 ± 0.62 | 259.16 ± 17.83 | 0.45 ± 0.00 | 43.10 ± 2.93 |
| CIFAR10 | zReLU | SGD | ℂ4s | 35.87 ± 15.94 | 227.65 ± 61.63 | 0.45 ± 0.00 | 34.10 ± 9.24 |
| FMNIST | ℂReLU | SGD | ℂ4s | 91.45 ± 0.22 | 309.27 ± 25.85 | 0.40 ± 0.00 | 53.60 ± 4.31 |
| FMNIST | ℂMish | SGD | ℂ4s | 16.59 ± 14.90 | 164.27 ± 63.24 | 0.99 ± 0.05 | 12.00 ± 4.28 |
| FMNIST | 𝕊ELU | SGD | ℂ4s | 90.26 ± 0.24 | 261.99 ± 26.63 | 0.40 ± 0.00 | 45.30 ± 4.60 |
| FMNIST | 𝕊GELU | SGD | ℂ4s | 91.19 ± 0.14 | 340.28 ± 35.29 | 0.44 ± 0.00 | 52.70 ± 5.44 |
| FMNIST | 𝕊Mish | SGD | ℂ4s | 91.08 ± 0.27 | 315.43 ± 35.27 | 0.41 ± 0.00 | 50.80 ± 5.68 |
| FMNIST | splitSwish | SGD | ℂ4s | 90.89 ± 0.29 | 323.03 ± 34.26 | 0.41 ± 0.00 | 53.40 ± 5.66 |
| FMNIST | zReLU | SGD | ℂ4s | 89.67 ± 0.76 | 335.12 ± 74.84 | 0.41 ± 0.00 | 60.20 ± 13.53 |
| CIFAR10 | ℂReLU | Adam | ℂ5l | 73.66 ± 0.54 | 162.50 ± 8.02 | 0.68 ± 0.01 | 16.70 ± 0.83 |
| CIFAR10 | ℂMish | Adam | ℂ5l | 13.01 ± 6.81 | 259.68 ± 6.88 | 1.93 ± 0.11 | 10.10 ± 0.23 |
| CIFAR10 | 𝕊ELU | Adam | ℂ5l | 64.92 ± 0.61 | 205.82 ± 17.23 | 0.68 ± 0.01 | 21.30 ± 1.79 |
| CIFAR10 | 𝕊GELU | Adam | ℂ5l | 68.40 ± 0.47 | 181.45 ± 5.97 | 0.74 ± 0.00 | 15.90 ± 0.53 |
| CIFAR10 | 𝕊Mish | Adam | ℂ5l | 67.37 ± 0.42 | 177.94 ± 8.28 | 0.71 ± 0.00 | 16.50 ± 0.77 |
| CIFAR10 | splitSwish | Adam | ℂ5l | 67.77 ± 0.71 | 174.30 ± 10.86 | 0.71 ± 0.01 | 17.00 ± 1.07 |
| CIFAR10 | zReLU | Adam | ℂ5l | 46.37 ± 13.73 | 341.84 ± 83.80 | 0.71 ± 0.00 | 29.00 ± 7.07 |
| FMNIST | ℂReLU | Adam | ℂ5l | 91.99 ± 0.27 | 163.01 ± 6.94 | 0.64 ± 0.01 | 16.60 ± 0.69 |
| FMNIST | ℂMish | Adam | ℂ5l | 36.23 ± 24.26 | 256.11 ± 11.17 | 1.90 ± 0.15 | 10.40 ± 0.37 |
| FMNIST | 𝕊ELU | Adam | ℂ5l | 89.36 ± 0.29 | 197.24 ± 19.45 | 0.62 ± 0.00 | 20.50 ± 2.06 |
| FMNIST | 𝕊GELU | Adam | ℂ5l | 91.40 ± 0.21 | 187.32 ± 12.28 | 0.68 ± 0.00 | 17.00 ± 1.12 |
| FMNIST | 𝕊Mish | Adam | ℂ5l | 90.74 ± 0.26 | 181.40 ± 10.34 | 0.66 ± 0.00 | 17.10 ± 0.98 |
| FMNIST | splitSwish | Adam | ℂ5l | 90.75 ± 0.19 | 169.70 ± 9.72 | 0.65 ± 0.00 | 16.70 ± 0.96 |
| FMNIST | zReLU | Adam | ℂ5l | 91.32 ± 0.32 | 256.01 ± 40.20 | 0.64 ± 0.00 | 28.00 ± 4.44 |
| CIFAR10 | ℂReLU | SGD | ℂ5l | 72.11 ± 0.72 | 338.43 ± 16.51 | 0.69 ± 0.01 | 35.30 ± 1.88 |
| CIFAR10 | ℂMish | SGD | ℂ5l | 12.07 ± 4.67 | 267.54 ± 23.43 | 1.91 ± 0.11 | 10.50 ± 0.91 |
| CIFAR10 | 𝕊ELU | SGD | ℂ5l | 65.30 ± 0.77 | 394.89 ± 17.23 | 0.68 ± 0.01 | 41.70 ± 1.82 |
| CIFAR10 | 𝕊GELU | SGD | ℂ5l | 68.74 ± 0.87 | 433.06 ± 25.22 | 0.74 ± 0.00 | 39.00 ± 2.24 |
| CIFAR10 | 𝕊Mish | SGD | ℂ5l | 68.25 ± 0.59 | 402.92 ± 15.09 | 0.71 ± 0.00 | 38.30 ± 1.43 |
| CIFAR10 | splitSwish | SGD | ℂ5l | 66.29 ± 0.70 | 412.44 ± 22.68 | 0.71 ± 0.01 | 40.10 ± 2.09 |
| CIFAR10 | zReLU | SGD | ℂ5l | 48.08 ± 14.37 | 517.43 ± 117.70 | 0.71 ± 0.01 | 44.50 ± 10.03 |
| FMNIST | ℂReLU | SGD | ℂ5l | 91.51 ± 0.21 | 429.75 ± 39.68 | 0.63 ± 0.00 | 45.20 ± 4.34 |
| FMNIST | ℂMish | SGD | ℂ5l | 10.00 ± 0.00 | 239.77 ± 0.28 | 1.73 ± 0.01 | 10.00 ± 0.00 |
| FMNIST | 𝕊ELU | SGD | ℂ5l | 89.88 ± 0.21 | 455.37 ± 61.09 | 0.63 ± 0.00 | 48.50 ± 6.59 |
| FMNIST | 𝕊GELU | SGD | ℂ5l | 91.34 ± 0.34 | 522.93 ± 41.92 | 0.68 ± 0.00 | 48.80 ± 3.96 |
| FMNIST | 𝕊Mish | SGD | ℂ5l | 91.10 ± 0.25 | 501.80 ± 37.92 | 0.66 ± 0.01 | 48.00 ± 3.62 |
| FMNIST | splitSwish | SGD | ℂ5l | 90.77 ± 0.34 | 500.49 ± 30.37 | 0.65 ± 0.00 | 50.50 ± 3.08 |
| FMNIST | zReLU | SGD | ℂ5l | 90.86 ± 0.36 | 688.40 ± 125.58 | 0.64 ± 0.00 | 76.40 ± 13.97 |
| CIFAR10 | ℂReLU | Adam | ℂ5s | 70.88 ± 0.71 | 174.43 ± 15.74 | 0.53 ± 0.01 | 24.60 ± 2.24 |
| CIFAR10 | ℂMish | Adam | ℂ5s | 35.64 ± 9.40 | 184.79 ± 12.39 | 1.40 ± 0.05 | 11.20 ± 0.66 |
| CIFAR10 | 𝕊ELU | Adam | ℂ5s | 64.81 ± 0.75 | 235.75 ± 27.61 | 0.52 ± 0.00 | 34.30 ± 4.02 |
| CIFAR10 | 𝕊GELU | Adam | ℂ5s | 66.43 ± 0.59 | 190.83 ± 12.62 | 0.57 ± 0.00 | 24.40 ± 1.62 |
| CIFAR10 | 𝕊Mish | Adam | ℂ5s | 65.82 ± 0.51 | 195.49 ± 14.73 | 0.55 ± 0.00 | 25.80 ± 1.96 |
| CIFAR10 | splitSwish | Adam | ℂ5s | 65.30 ± 0.44 | 180.57 ± 14.75 | 0.53 ± 0.00 | 25.10 ± 2.06 |
| CIFAR10 | zReLU | Adam | ℂ5s | 44.17 ± 12.89 | 259.31 ± 59.11 | 0.54 ± 0.00 | 32.10 ± 7.33 |
| FMNIST | ℂReLU | Adam | ℂ5s | 91.51 ± 0.22 | 162.68 ± 6.78 | 0.49 ± 0.00 | 22.20 ± 0.88 |
| FMNIST | ℂMish | Adam | ℂ5s | 57.13 ± 23.41 | 197.62 ± 29.01 | 1.28 ± 0.09 | 12.00 ± 1.55 |
| FMNIST | 𝕊ELU | Adam | ℂ5s | 89.00 ± 0.32 | 222.94 ± 32.65 | 0.49 ± 0.00 | 30.50 ± 4.53 |
| FMNIST | 𝕊GELU | Adam | ℂ5s | 90.71 ± 0.22 | 171.16 ± 14.56 | 0.53 ± 0.00 | 21.40 ± 1.85 |
| FMNIST | 𝕊Mish | Adam | ℂ5s | 90.44 ± 0.36 | 184.72 ± 15.11 | 0.51 ± 0.01 | 23.20 ± 1.87 |
| FMNIST | splitSwish | Adam | ℂ5s | 90.65 ± 0.27 | 181.02 ± 13.58 | 0.50 ± 0.00 | 24.20 ± 1.84 |

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| Dataset | Act. | Opt. | Model | Accuracy±CI | TT±CI | IT±CI | epochs±CI |
|---|---|---|---|---|---|---|---|
| FMNIST | zReLU | Adam | $\mathbb{C}$5s | 90.35 ± 0.32 | 285.20 ± 52.78 | 0.50 ± 0.00 | 41.10 ± 7.66 |
| CIFAR10 | $\mathbb{C}$ReLU | SGD | $\mathbb{C}$5s | 70.05 ± 0.42 | 335.41 ± 31.37 | 0.54 ± 0.02 | 49.00 ± 5.02 |
| CIFAR10 | $\mathbb{C}$Mish | SGD | $\mathbb{C}$5s | 14.46 ± 6.73 | 162.74 ± 5.31 | 1.24 ± 0.07 | 10.20 ± 0.30 |
| CIFAR10 | $\mathbb{S}$ELU | SGD | $\mathbb{C}$5s | 64.24 ± 0.73 | 448.65 ± 61.45 | 0.53 ± 0.00 | 66.60 ± 9.16 |
| CIFAR10 | $\mathbb{S}$GELU | SGD | $\mathbb{C}$5s | 68.04 ± 0.76 | 404.46 ± 22.73 | 0.57 ± 0.00 | 53.20 ± 2.94 |
| CIFAR10 | $\mathbb{S}$Mish | SGD | $\mathbb{C}$5s | 66.95 ± 0.88 | 408.59 ± 44.88 | 0.55 ± 0.00 | 55.80 ± 6.12 |
| CIFAR10 | splitSwish | SGD | $\mathbb{C}$5s | 66.73 ± 0.52 | 438.81 ± 23.49 | 0.54 ± 0.00 | 60.80 ± 3.27 |
| CIFAR10 | zReLU | SGD | $\mathbb{C}$5s | 48.99 ± 9.84 | 393.93 ± 91.36 | 0.53 ± 0.00 | 50.20 ± 11.68 |
| FMNIST | $\mathbb{C}$ReLU | SGD | $\mathbb{C}$5s | 91.08 ± 0.33 | 386.16 ± 29.71 | 0.49 ± 0.01 | 54.60 ± 4.29 |
| FMNIST | $\mathbb{C}$Mish | SGD | $\mathbb{C}$5s | 10.00 ± 0.00 | 176.93 ± 43.31 | 1.13 ± 0.01 | 11.20 ± 2.71 |
| FMNIST | $\mathbb{S}$ELU | SGD | $\mathbb{C}$5s | 88.89 ± 0.28 | 480.96 ± 63.01 | 0.49 ± 0.00 | 68.70 ± 8.91 |
| FMNIST | $\mathbb{S}$GELU | SGD | $\mathbb{C}$5s | 90.80 ± 0.32 | 475.93 ± 64.27 | 0.53 ± 0.00 | 61.20 ± 8.32 |
| FMNIST | $\mathbb{S}$Mish | SGD | $\mathbb{C}$5s | 90.72 ± 0.24 | 446.75 ± 54.44 | 0.51 ± 0.00 | 58.80 ± 7.25 |
| FMNIST | splitSwish | SGD | $\mathbb{C}$5s | 90.53 ± 0.31 | 451.12 ± 45.98 | 0.50 ± 0.00 | 62.20 ± 6.29 |
| FMNIST | zReLU | SGD | $\mathbb{C}$5s | 89.57 ± 0.60 | 571.27 ± 107.90 | 0.50 ± 0.00 | 84.00 ± 15.86 |
| CIFAR10 | GELU | Adam | $\mathbb{R}$4l | 68.95 ± 0.51 | 29.93 ± 0.95 | 0.20 ± 0.00 | 13.70 ± 0.48 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$4l | 68.95 ± 0.60 | 30.89 ± 1.12 | 0.20 ± 0.00 | 13.50 ± 0.51 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$4l | 69.19 ± 0.56 | 30.10 ± 1.03 | 0.20 ± 0.00 | 13.90 ± 0.53 |
| CIFAR10 | ReLU | Adam | $\mathbb{R}$4l | 71.31 ± 0.38 | 37.82 ± 1.27 | 0.19 ± 0.00 | 16.80 ± 0.56 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$4l | 67.44 ± 0.52 | 48.94 ± 1.68 | 0.24 ± 0.00 | 14.40 ± 0.50 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$4l | 69.00 ± 0.63 | 42.24 ± 0.84 | 0.21 ± 0.00 | 14.20 ± 0.30 |
| FMNIST | GELU | Adam | $\mathbb{R}$4l | 92.28 ± 0.25 | 33.46 ± 1.00 | 0.16 ± 0.00 | 15.20 ± 0.45 |
| FMNIST | Mish | Adam | $\mathbb{R}$4l | 92.08 ± 0.25 | 33.37 ± 1.58 | 0.16 ± 0.00 | 14.80 ± 0.74 |
| FMNIST | ELU | Adam | $\mathbb{R}$4l | 91.76 ± 0.22 | 29.96 ± 1.48 | 0.16 ± 0.00 | 14.00 ± 0.75 |
| FMNIST | ReLU | Adam | $\mathbb{R}$4l | 92.28 ± 0.27 | 36.75 ± 2.49 | 0.15 ± 0.00 | 15.90 ± 1.09 |
| FMNIST | SELU | Adam | $\mathbb{R}$4l | 91.66 ± 0.36 | 47.10 ± 2.83 | 0.19 ± 0.00 | 14.20 ± 0.88 |
| FMNIST | Swish | Adam | $\mathbb{R}$4l | 92.00 ± 0.22 | 43.73 ± 2.22 | 0.17 ± 0.00 | 15.10 ± 0.79 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$4l | 66.68 ± 0.45 | 52.20 ± 2.18 | 0.20 ± 0.00 | 26.10 ± 1.14 |
| CIFAR10 | Mish | SGD | $\mathbb{R}$4l | 67.24 ± 0.69 | 56.34 ± 2.60 | 0.20 ± 0.00 | 26.40 ± 1.27 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$4l | 69.52 ± 0.56 | 49.54 ± 1.91 | 0.19 ± 0.00 | 24.80 ± 1.00 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$4l | 69.50 ± 0.63 | 56.69 ± 2.45 | 0.20 ± 0.00 | 26.50 ± 1.18 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$4l | 67.86 ± 0.43 | 72.28 ± 4.10 | 0.24 ± 0.00 | 22.00 ± 1.26 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$4l | 65.60 ± 0.60 | 78.22 ± 3.83 | 0.21 ± 0.00 | 27.30 ± 1.35 |
| FMNIST | GELU | SGD | $\mathbb{R}$4l | 91.37 ± 0.34 | 81.69 ± 6.39 | 0.16 ± 0.00 | 40.80 ± 3.20 |
| FMNIST | Mish | SGD | $\mathbb{R}$4l | 91.34 ± 0.29 | 77.78 ± 8.22 | 0.16 ± 0.00 | 37.40 ± 4.02 |
| FMNIST | ELU | SGD | $\mathbb{R}$4l | 91.97 ± 0.36 | 78.55 ± 6.64 | 0.16 ± 0.00 | 40.20 ± 3.45 |
| FMNIST | ReLU | SGD | $\mathbb{R}$4l | 91.89 ± 0.16 | 94.93 ± 8.65 | 0.15 ± 0.00 | 43.20 ± 4.00 |
| FMNIST | SELU | SGD | $\mathbb{R}$4l | 92.11 ± 0.24 | 104.43 ± 9.19 | 0.19 ± 0.00 | 32.60 ± 2.90 |
| FMNIST | Swish | SGD | $\mathbb{R}$4l | 91.03 ± 0.34 | 112.08 ± 7.95 | 0.17 ± 0.00 | 40.40 ± 2.89 |
| CIFAR10 | GELU | Adam | $\mathbb{R}$4s | 67.62 ± 0.69 | 28.68 ± 1.23 | 0.19 ± 0.00 | 16.90 ± 0.79 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$4s | 67.41 ± 0.55 | 31.04 ± 1.58 | 0.19 ± 0.00 | 16.50 ± 0.91 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$4s | 67.52 ± 0.45 | 26.72 ± 1.04 | 0.19 ± 0.00 | 15.90 ± 0.71 |
| CIFAR10 | ReLU | Adam | $\mathbb{R}$4s | 68.97 ± 0.74 | 40.31 ± 6.15 | 0.18 ± 0.00 | 22.50 ± 3.57 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$4s | 65.92 ± 0.40 | 38.38 ± 1.34 | 0.20 ± 0.00 | 15.80 ± 0.56 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$4s | 67.27 ± 0.55 | 37.31 ± 1.00 | 0.19 ± 0.00 | 17.20 ± 0.45 |
| FMNIST | GELU | Adam | $\mathbb{R}$4s | 91.84 ± 0.24 | 31.54 ± 1.72 | 0.16 ± 0.00 | 17.50 ± 0.97 |
| FMNIST | Mish | Adam | $\mathbb{R}$4s | 91.84 ± 0.36 | 32.93 ± 1.51 | 0.16 ± 0.00 | 17.20 ± 0.81 |
| FMNIST | ELU | Adam | $\mathbb{R}$4s | 91.51 ± 0.46 | 29.69 ± 2.06 | 0.16 ± 0.00 | 16.40 ± 1.27 |
| FMNIST | ReLU | Adam | $\mathbb{R}$4s | 91.70 ± 0.18 | 35.56 ± 2.55 | 0.14 ± 0.00 | 17.90 ± 1.28 |
| FMNIST | SELU | Adam | $\mathbb{R}$4s | 91.31 ± 0.32 | 37.65 ± 1.94 | 0.16 ± 0.00 | 15.60 ± 0.84 |

Continued on next page

| Dataset | Act. | Opt. | Model | Accuracy±CI | TT±CI | IT±CI | epochs±CI |
|---|---|---|---|---|---|---|---|
| FMNIST | Swish | Adam | $\mathbb{R}$4s | 91.79 ± 0.29 | 38.93 ± 1.65 | 0.15 ± 0.00 | 18.00 ± 0.75 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$4s | 66.66 ± 0.52 | 51.07 ± 2.89 | 0.19 ± 0.00 | 33.00 ± 1.91 |
| CIFAR10 | Mish | SGD | $\mathbb{R}$4s | 66.66 ± 0.66 | 51.66 ± 1.76 | 0.19 ± 0.00 | 29.50 ± 1.03 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$4s | 67.97 ± 0.32 | 44.84 ± 2.73 | 0.19 ± 0.00 | 29.30 ± 1.85 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$4s | 67.71 ± 0.78 | 55.51 ± 2.44 | 0.18 ± 0.00 | 32.40 ± 1.48 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$4s | 66.85 ± 0.61 | 59.49 ± 3.05 | 0.20 ± 0.00 | 25.60 ± 1.31 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$4s | 65.67 ± 0.60 | 67.78 ± 4.75 | 0.19 ± 0.00 | 32.80 ± 2.36 |
| FMNIST | GELU | SGD | $\mathbb{R}$4s | 91.20 ± 0.30 | 75.24 ± 6.28 | 0.16 ± 0.00 | 45.50 ± 3.80 |
| FMNIST | Mish | SGD | $\mathbb{R}$4s | 91.27 ± 0.29 | 80.24 ± 7.25 | 0.16 ± 0.00 | 44.90 ± 4.05 |
| FMNIST | ELU | SGD | $\mathbb{R}$4s | 91.83 ± 0.20 | 73.03 ± 5.98 | 0.16 ± 0.00 | 45.20 ± 3.74 |
| FMNIST | ReLU | SGD | $\mathbb{R}$4s | 91.44 ± 0.22 | 91.01 ± 6.39 | 0.15 ± 0.00 | 48.70 ± 3.41 |
| FMNIST | SELU | SGD | $\mathbb{R}$4s | 91.72 ± 0.34 | 86.30 ± 8.43 | 0.16 ± 0.00 | 37.80 ± 3.71 |
| FMNIST | Swish | SGD | $\mathbb{R}$4s | 90.99 ± 0.33 | 97.25 ± 9.76 | 0.15 ± 0.00 | 47.00 ± 4.82 |
| CIFAR10 | GELU | Adam | $\mathbb{R}$5l | 73.41 ± 0.56 | 36.64 ± 1.12 | 0.20 ± 0.00 | 15.00 ± 0.48 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$5l | 73.07 ± 0.58 | 37.05 ± 1.23 | 0.20 ± 0.00 | 14.30 ± 0.48 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$5l | 72.41 ± 0.57 | 33.81 ± 1.77 | 0.20 ± 0.00 | 13.90 ± 0.79 |
| CIFAR10 | ReLU | Adam | $\mathbb{R}$5l | 73.89 ± 0.39 | 42.65 ± 2.75 | 0.20 ± 0.00 | 17.10 ± 1.09 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$5l | 70.65 ± 0.62 | 56.14 ± 1.74 | 0.24 ± 0.00 | 14.80 ± 0.45 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$5l | 72.90 ± 0.65 | 48.11 ± 1.94 | 0.23 ± 0.00 | 14.70 ± 0.59 |
| FMNIST | GELU | Adam | $\mathbb{R}$5l | 92.12 ± 0.26 | 39.97 ± 1.10 | 0.17 ± 0.00 | 16.30 ± 0.48 |
| FMNIST | Mish | Adam | $\mathbb{R}$5l | 92.06 ± 0.24 | 41.25 ± 1.75 | 0.18 ± 0.00 | 16.20 ± 0.74 |
| FMNIST | ELU | Adam | $\mathbb{R}$5l | 91.71 ± 0.29 | 35.94 ± 1.42 | 0.17 ± 0.00 | 14.80 ± 0.66 |
| FMNIST | ReLU | Adam | $\mathbb{R}$5l | 92.05 ± 0.32 | 42.74 ± 2.35 | 0.17 ± 0.00 | 16.40 ± 0.90 |
| FMNIST | SELU | Adam | $\mathbb{R}$5l | 91.33 ± 0.30 | 53.50 ± 2.21 | 0.21 ± 0.00 | 14.40 ± 0.60 |
| FMNIST | Swish | Adam | $\mathbb{R}$5l | 92.12 ± 0.31 | 52.87 ± 1.16 | 0.19 ± 0.00 | 16.50 ± 0.38 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$5l | 68.30 ± 0.73 | 65.61 ± 2.85 | 0.20 ± 0.00 | 29.20 ± 1.30 |
| CIFAR10 | Mish | SGD | $\mathbb{R}$5l | 69.35 ± 0.69 | 66.72 ± 3.73 | 0.20 ± 0.00 | 28.10 ± 1.60 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$5l | 72.26 ± 0.33 | 61.05 ± 2.73 | 0.20 ± 0.00 | 27.40 ± 1.27 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$5l | 72.15 ± 0.68 | 71.69 ± 2.76 | 0.20 ± 0.00 | 30.30 ± 1.17 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$5l | 70.67 ± 0.52 | 81.59 ± 4.43 | 0.25 ± 0.00 | 22.30 ± 1.22 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$5l | 67.14 ± 0.50 | 96.89 ± 3.46 | 0.22 ± 0.00 | 30.80 ± 1.11 |
| FMNIST | GELU | SGD | $\mathbb{R}$5l | 91.12 ± 0.27 | 95.68 ± 12.16 | 0.17 ± 0.00 | 42.20 ± 5.42 |
| FMNIST | Mish | SGD | $\mathbb{R}$5l | 91.28 ± 0.19 | 96.55 ± 6.20 | 0.18 ± 0.00 | 40.90 ± 2.69 |
| FMNIST | ELU | SGD | $\mathbb{R}$5l | 91.77 ± 0.18 | 87.74 ± 6.54 | 0.17 ± 0.00 | 39.40 ± 2.94 |
| FMNIST | ReLU | SGD | $\mathbb{R}$5l | 91.68 ± 0.28 | 104.42 ± 10.86 | 0.17 ± 0.00 | 42.20 ± 4.41 |
| FMNIST | SELU | SGD | $\mathbb{R}$5l | 91.77 ± 0.30 | 107.45 ± 7.96 | 0.20 ± 0.00 | 30.00 ± 2.24 |
| FMNIST | Swish | SGD | $\mathbb{R}$5l | 90.94 ± 0.35 | 133.43 ± 8.07 | 0.19 ± 0.00 | 43.60 ± 2.64 |
| CIFAR10 | GELU | Adam | $\mathbb{R}$5s | 70.66 ± 0.40 | 37.83 ± 2.27 | 0.19 ± 0.00 | 20.30 ± 1.31 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$5s | 71.14 ± 0.72 | 40.69 ± 3.16 | 0.19 ± 0.00 | 19.80 ± 1.61 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$5s | 71.21 ± 0.49 | 34.82 ± 1.94 | 0.19 ± 0.00 | 18.80 ± 1.11 |
| CIFAR10 | ReLU | Adam | $\mathbb{R}$5s | 71.98 ± 0.57 | 48.82 ± 3.82 | 0.18 ± 0.00 | 24.00 ± 1.94 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$5s | 69.82 ± 0.65 | 54.86 ± 3.13 | 0.21 ± 0.00 | 20.40 ± 1.23 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$5s | 70.99 ± 0.54 | 46.91 ± 1.43 | 0.19 ± 0.00 | 19.60 ± 0.60 |
| FMNIST | GELU | Adam | $\mathbb{R}$5s | 91.73 ± 0.37 | 37.11 ± 1.32 | 0.16 ± 0.00 | 19.10 ± 0.71 |
| FMNIST | Mish | Adam | $\mathbb{R}$5s | 91.64 ± 0.24 | 38.61 ± 1.71 | 0.16 ± 0.00 | 18.40 ± 0.84 |
| FMNIST | ELU | Adam | $\mathbb{R}$5s | 91.56 ± 0.23 | 35.72 ± 2.94 | 0.16 ± 0.00 | 18.60 ± 1.59 |
| FMNIST | ReLU | Adam | $\mathbb{R}$5s | 91.42 ± 0.20 | 44.26 ± 3.58 | 0.15 ± 0.00 | 20.50 ± 1.69 |
| FMNIST | SELU | Adam | $\mathbb{R}$5s | 91.42 ± 0.30 | 46.88 ± 2.67 | 0.16 ± 0.00 | 17.40 ± 1.02 |
| FMNIST | Swish | Adam | $\mathbb{R}$5s | 91.74 ± 0.13 | 48.84 ± 2.10 | 0.16 ± 0.00 | 20.00 ± 0.89 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$5s | 69.38 ± 0.33 | 77.49 ± 6.00 | 0.19 ± 0.00 | 45.00 ± 3.54 |

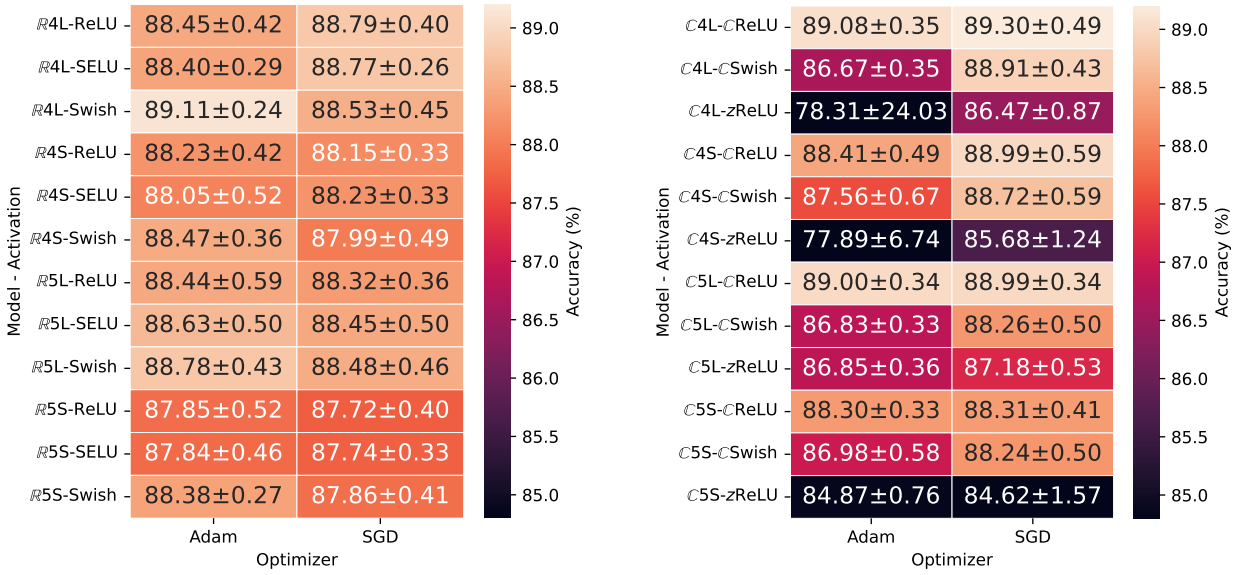
Continued on next page

| Dataset | Act. | Opt. | Model | Accuracy±CI | TT±CI | IT±CI | epochs±CI |
|---|---|---|---|---|---|---|---|
| CIFAR10 | Mish | SGD | $\mathbb{R}$5s | 69.87 ± 0.53 | 78.01 ± 4.18 | 0.19 ± 0.00 | 41.30 ± 2.26 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$5s | 71.24 ± 0.47 | 65.37 ± 4.25 | 0.19 ± 0.00 | 38.40 ± 2.53 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$5s | 70.45 ± 0.56 | 82.32 ± 5.36 | 0.19 ± 0.00 | 42.40 ± 2.83 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$5s | 69.64 ± 0.66 | 80.43 ± 6.58 | 0.21 ± 0.00 | 31.20 ± 2.56 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$5s | 68.60 ± 0.60 | 104.66 ± 3.90 | 0.19 ± 0.00 | 45.90 ± 1.67 |
| FMNIST | GELU | SGD | $\mathbb{R}$5s | 90.89 ± 0.24 | 87.09 ± 7.82 | 0.16 ± 0.00 | 49.00 ± 4.46 |
| FMNIST | Mish | SGD | $\mathbb{R}$5s | 90.75 ± 0.27 | 88.22 ± 10.01 | 0.16 ± 0.00 | 45.80 ± 5.27 |
| FMNIST | ELU | SGD | $\mathbb{R}$5s | 91.43 ± 0.29 | 78.16 ± 6.23 | 0.16 ± 0.00 | 45.00 ± 3.65 |
| FMNIST | ReLU | SGD | $\mathbb{R}$5s | 91.22 ± 0.20 | 100.03 ± 5.51 | 0.15 ± 0.00 | 48.90 ± 2.69 |
| FMNIST | SELU | SGD | $\mathbb{R}$5s | 91.49 ± 0.25 | 98.26 ± 8.02 | 0.16 ± 0.00 | 38.20 ± 3.12 |
| FMNIST | Swish | SGD | $\mathbb{R}$5s | 90.67 ± 0.29 | 120.02 ± 14.51 | 0.15 ± 0.00 | 52.10 ± 6.34 |

## 2.2 FFT Input

This section illustrates the results of the classification experiments where the input is complex-valued for the CVNNs models obtained by applying FFT and real-valued for the RVNNs models obtained by splitting the FFT output into real and imaginary parts.

Figure 7 shows the classification results of Fashion MNIST datasets after applying FFT using multiple activation functions and optimization algorithms.




(a) RVNNs Models.

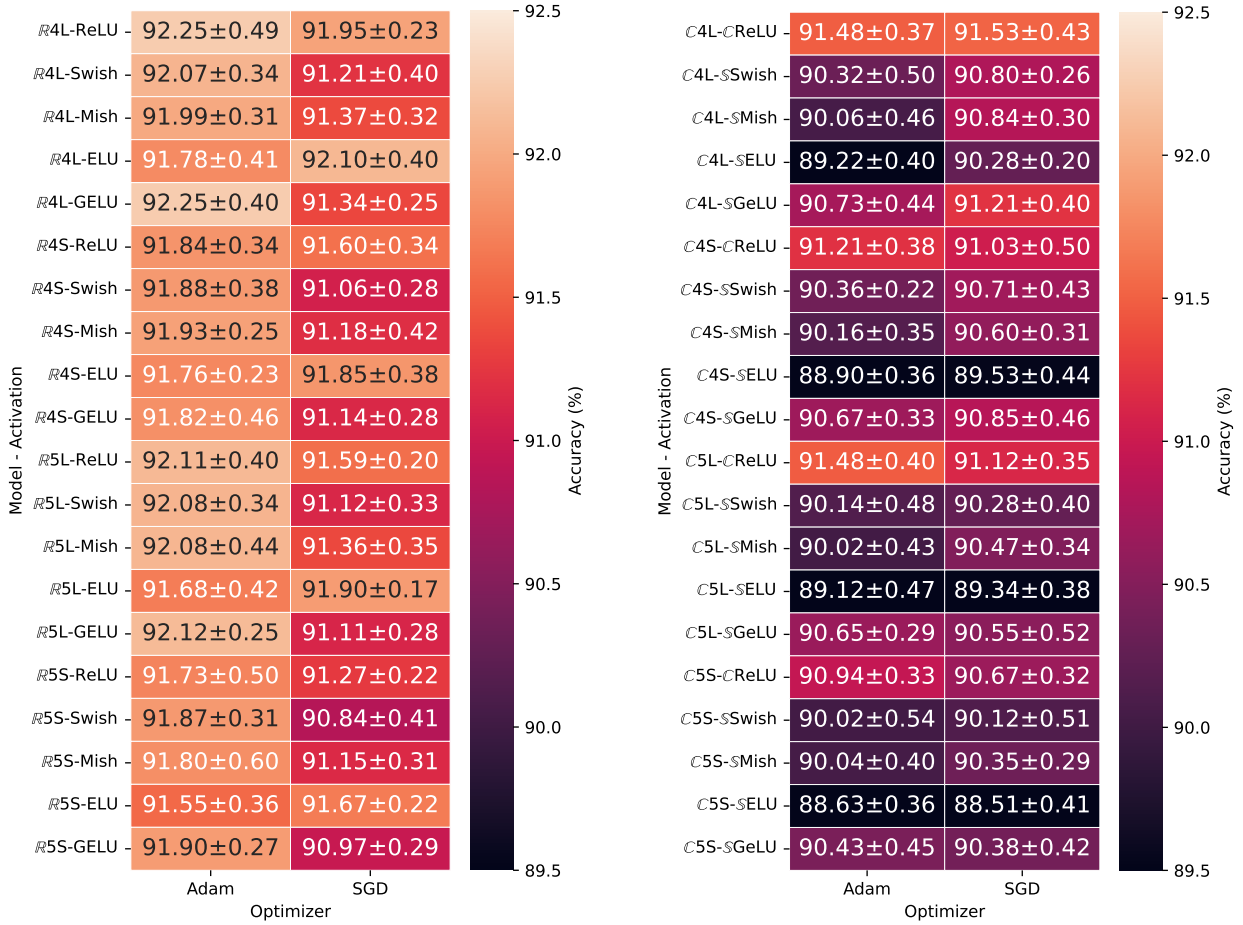
(b) CVNNs Models.

Figure 7: Mean accuracy comparison between CVNNs with complex input (obtained by applying **FFT** to the original real-valued input) and RVNNs with real input (obtained by splitting the output of Fast Fourier Transform (FFT) output) for Fashion MNIST dataset using different activations and Adam/SGD optimizer.

## 2.3 Combined Input

Figures 8 and 9 report the classification accuracy of the CVNN and RVNN models on the Fashion-MNIST and CIFAR-10 datasets, together with their 95 % confidence intervals. Complementary scatter plots in Figures 10 and 11 relate accuracy to training and inference times, respectively, highlighting the computational trade-offs of each architecture.

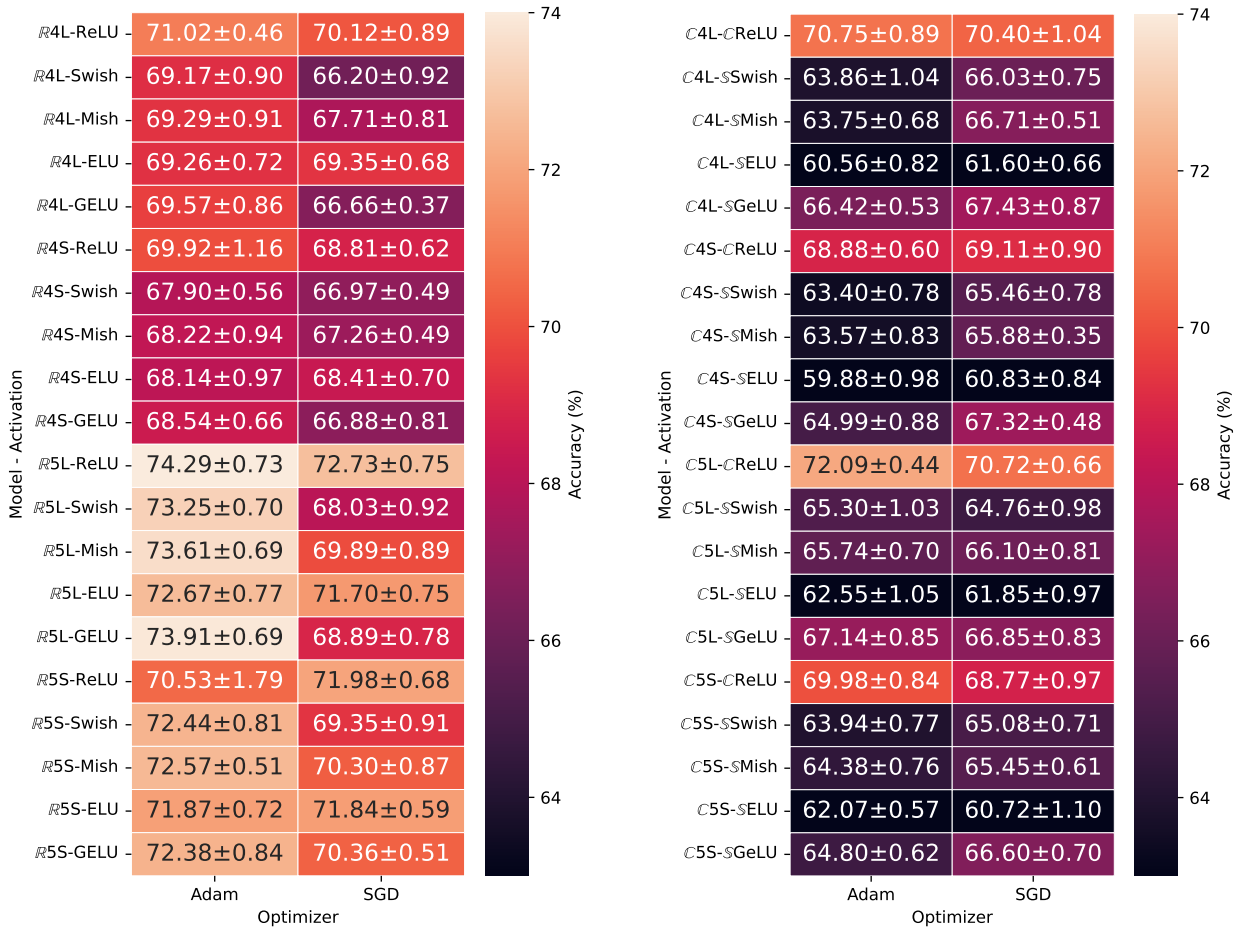
Table 2 collect presents the results for the cases when the input is complex-valued, such that a pair of real valued input forms the complex input value.




(a) RVNNs Models.

(b) CVNNs Models.

Figure 8: Comparison of mean accuracy between CVNNs using complex inputs (formed by pairing every two original features into one complex number) and RVNNs using the original real-valued inputs. The evaluation is conducted on the Fashion MNIST dataset, exploring various activation functions and optimization algorithms.




(a) RVNNs Models.

(b) CVNNs Models.

Figure 9: Comparison of mean accuracy between CVNNs using complex inputs (formed by pairing every two original features into one complex number) and RVNNs using the original real-valued inputs. The evaluation is conducted on the CIFAR-10 dataset, exploring various activation functions and optimization algorithms.

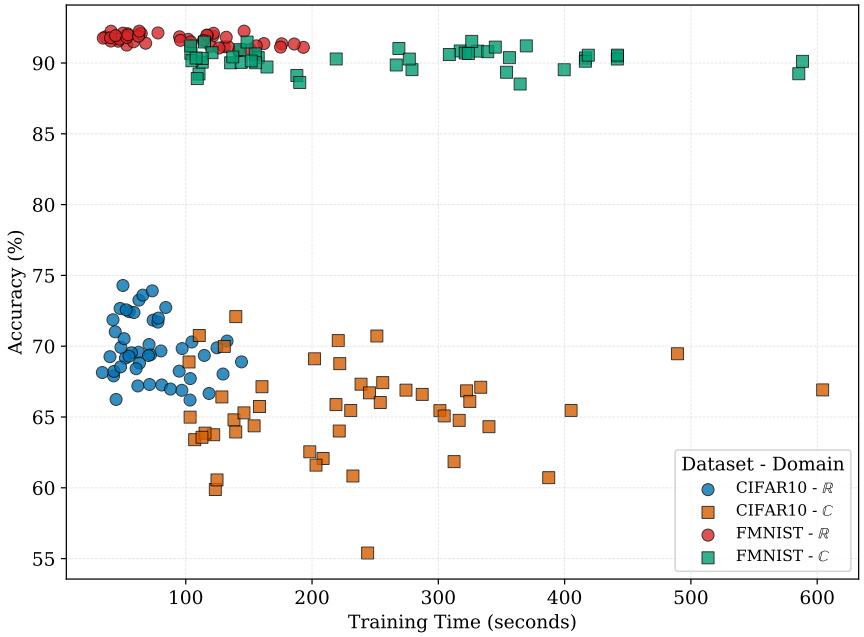



Figure 10: Comparison of mean accuracy vs training time between CVNNs using complex inputs (formed by pairing every two original features into one complex number) and RVNNs using the original real-valued inputs. The evaluation is conducted on the FMNIST and CIFAR10 datasets, exploring various activation functions and optimization algorithms.

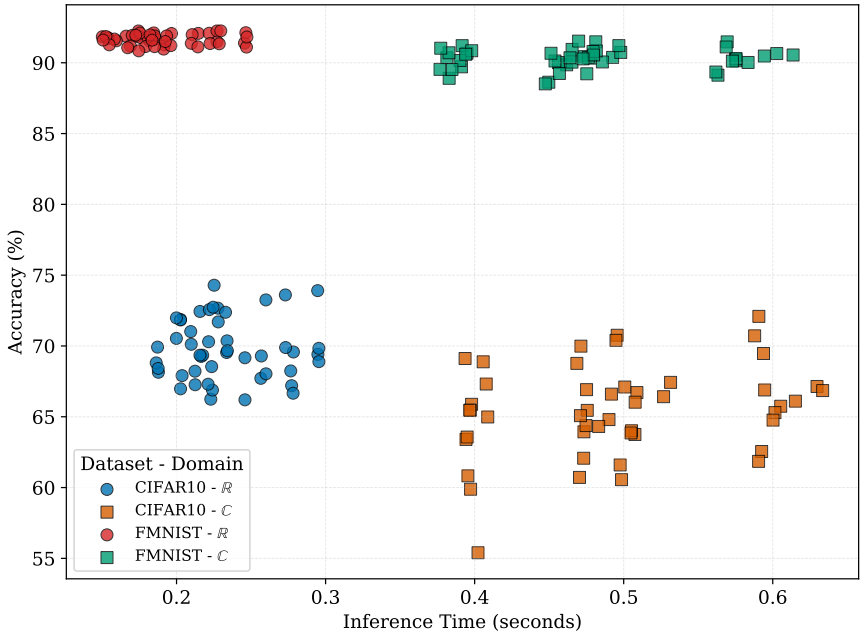



Figure 11: Comparison of mean accuracy vs inference time between CVNNs using complex inputs (formed by pairing every two original features into one complex number) and RVNNs using the original real-valued inputs. The evaluation is conducted on the FMNIST and CIFAR10 datasets, exploring various activation functions and optimization algorithms.

Table 2: Results of classification experiments comparing RVNNs and CVNNs on the Fashion-MNIST and CIFAR-10 datasets, using complex-valued inputs created by merging every two real-valued features into a single complex feature.

| Dataset | Act. | Opt. | Model | Accuracy±CI | TT | IT | epochs |
|---|---|---|---|---|---|---|---|
| CIFAR10 | GELU | Adam | $\mathbb{R}$4lc | 69.57 ± 0.61 | 62.63 ± 1.69 | 0.28 ± 0.00 | 13.60 ± 0.37 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$5lc | 69.41 ± 0.79 | 72.16 ± 3.38 | 0.29 ± 0.00 | 14.30 ± 0.68 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$5sc | 69.54 ± 0.54 | 57.06 ± 4.20 | 0.23 ± 0.00 | 17.20 ± 1.30 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$4lc | 69.17 ± 0.64 | 52.24 ± 1.37 | 0.25 ± 0.00 | 13.60 ± 0.37 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$4sc | 67.90 ± 0.40 | 42.86 ± 2.11 | 0.20 ± 0.00 | 16.40 ± 0.84 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$5lc | 73.25 ± 0.50 | 62.93 ± 1.64 | 0.26 ± 0.00 | 14.50 ± 0.38 |
| CIFAR10 | Swish | Adam | $\mathbb{R}$5sc | 72.44 ± 0.58 | 55.29 ± 1.97 | 0.22 ± 0.00 | 18.70 ± 0.68 |
| FMNIST | GELU | Adam | $\mathbb{R}$4sc | 91.82 ± 0.33 | 48.91 ± 2.64 | 0.18 ± 0.00 | 16.30 ± 0.90 |
| FMNIST | Swish | Adam | $\mathbb{R}$5sc | 91.87 ± 0.22 | 54.29 ± 2.76 | 0.17 ± 0.00 | 18.60 ± 0.97 |
| FMNIST | Swish | Adam | $\mathbb{R}$5lc | 92.08 ± 0.24 | 63.39 ± 2.76 | 0.21 ± 0.00 | 15.30 ± 0.68 |
| FMNIST | Swish | Adam | $\mathbb{R}$4sc | 91.88 ± 0.27 | 44.12 ± 1.66 | 0.17 ± 0.00 | 16.90 ± 0.63 |
| FMNIST | Swish | Adam | $\mathbb{R}$4lc | 92.07 ± 0.24 | 53.99 ± 1.83 | 0.20 ± 0.00 | 14.60 ± 0.50 |
| FMNIST | SELU | Adam | $\mathbb{R}$5sc | 91.27 ± 0.33 | 53.26 ± 3.07 | 0.19 ± 0.00 | 16.30 ± 0.96 |
| FMNIST | SELU | Adam | $\mathbb{R}$5lc | 91.40 ± 0.24 | 68.29 ± 3.93 | 0.25 ± 0.00 | 14.30 ± 0.83 |
| FMNIST | SELU | Adam | $\mathbb{R}$4sc | 91.54 ± 0.34 | 46.56 ± 1.89 | 0.18 ± 0.00 | 15.90 ± 0.63 |
| FMNIST | SELU | Adam | $\mathbb{R}$4lc | 91.50 ± 0.35 | 58.61 ± 2.52 | 0.23 ± 0.00 | 13.70 ± 0.59 |
| FMNIST | ReLU | Adam | $\mathbb{R}$5sc | 91.73 ± 0.36 | 44.02 ± 2.11 | 0.15 ± 0.00 | 18.70 ± 0.90 |
| FMNIST | ReLU | Adam | $\mathbb{R}$5lc | 92.11 ± 0.29 | 50.31 ± 2.96 | 0.18 ± 0.00 | 16.00 ± 0.95 |
| FMNIST | ReLU | Adam | $\mathbb{R}$4sc | 91.84 ± 0.24 | 36.36 ± 2.14 | 0.15 ± 0.00 | 17.80 ± 1.00 |
| FMNIST | ReLU | Adam | $\mathbb{R}$4lc | 92.25 ± 0.35 | 40.88 ± 1.96 | 0.17 ± 0.00 | 15.00 ± 0.75 |
| FMNIST | ELU | Adam | $\mathbb{R}$5sc | 91.55 ± 0.26 | 40.66 ± 1.85 | 0.16 ± 0.00 | 16.40 ± 0.77 |
| FMNIST | ELU | Adam | $\mathbb{R}$5lc | 91.68 ± 0.30 | 47.32 ± 1.47 | 0.19 ± 0.00 | 14.20 ± 0.45 |
| FMNIST | ELU | Adam | $\mathbb{R}$4sc | 91.76 ± 0.16 | 34.69 ± 1.25 | 0.15 ± 0.00 | 15.60 ± 0.50 |
| FMNIST | ELU | Adam | $\mathbb{R}$4lc | 91.78 ± 0.29 | 40.42 ± 1.72 | 0.18 ± 0.00 | 13.70 ± 0.59 |
| FMNIST | Mish | Adam | $\mathbb{R}$5sc | 91.80 ± 0.43 | 53.30 ± 4.07 | 0.18 ± 0.00 | 17.70 ± 1.35 |
| FMNIST | Mish | Adam | $\mathbb{R}$5lc | 92.08 ± 0.31 | 64.76 ± 2.96 | 0.22 ± 0.00 | 14.70 ± 0.68 |
| FMNIST | Mish | Adam | $\mathbb{R}$4sc | 91.93 ± 0.18 | 44.39 ± 2.25 | 0.17 ± 0.00 | 16.50 ± 0.84 |
| FMNIST | Mish | Adam | $\mathbb{R}$4lc | 91.99 ± 0.22 | 54.16 ± 2.05 | 0.21 ± 0.00 | 13.90 ± 0.53 |
| FMNIST | GELU | Adam | $\mathbb{R}$5sc | 91.90 ± 0.19 | 62.56 ± 4.13 | 0.19 ± 0.00 | 18.80 ± 1.25 |
| FMNIST | GELU | Adam | $\mathbb{R}$5lc | 92.12 ± 0.18 | 77.99 ± 4.19 | 0.25 ± 0.00 | 15.90 ± 0.86 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$4sc | 66.24 ± 0.55 | 44.84 ± 2.19 | 0.22 ± 0.00 | 15.00 ± 0.75 |
| CIFAR10 | SELU | Adam | $\mathbb{R}$4lc | 67.19 ± 0.79 | 62.06 ± 2.53 | 0.28 ± 0.00 | 13.70 ± 0.59 |
| FMNIST | GELU | Adam | $\mathbb{R}$4lc | 92.25 ± 0.28 | 63.16 ± 2.65 | 0.23 ± 0.00 | 14.40 ± 0.60 |
| CIFAR10 | ReLU | Adam | $\mathbb{R}$4lc | 71.02 ± 0.33 | 44.16 ± 3.10 | 0.21 ± 0.00 | 16.30 ± 1.22 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$5sc | 71.87 ± 0.52 | 42.36 ± 2.66 | 0.20 ± 0.00 | 17.10 ± 1.09 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$5lc | 73.61 ± 0.50 | 66.02 ± 1.63 | 0.27 ± 0.00 | 14.40 ± 0.37 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$5lc | 72.67 ± 0.55 | 48.03 ± 1.16 | 0.23 ± 0.00 | 14.30 ± 0.35 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$4sc | 68.14 ± 0.70 | 33.96 ± 1.82 | 0.19 ± 0.00 | 15.50 ± 0.84 |
| CIFAR10 | GELU | Adam | $\mathbb{R}$5sc | 72.38 ± 0.60 | 58.99 ± 2.82 | 0.23 ± 0.00 | 17.50 ± 0.84 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$4sc | 68.22 ± 0.67 | 43.16 ± 1.30 | 0.21 ± 0.00 | 15.70 ± 0.48 |
| CIFAR10 | ReLU | Adam | $\mathbb{R}$4sc | 69.92 ± 0.83 | 48.65 ± 6.73 | 0.19 ± 0.00 | 23.70 ± 3.39 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$4lc | 69.29 ± 0.65 | 55.06 ± 1.55 | 0.26 ± 0.00 | 13.50 ± 0.38 |
| CIFAR10 | ReLU | Adam | $\mathbb{R}$5sc | 70.53 ± 1.28 | 51.20 ± 4.74 | 0.20 ± 0.00 | 22.20 ± 2.02 |
| CIFAR10 | ELU | Adam | $\mathbb{R}$4lc | 69.26 ± 0.51 | 40.00 ± 1.07 | 0.22 ± 0.00 | 13.40 ± 0.37 |
| CIFAR10 | Mish | Adam | $\mathbb{R}$5sc | 72.57 ± 0.36 | 52.86 ± 2.58 | 0.22 ± 0.00 | 17.30 ± 0.90 |
| CIFAR10 | GELU | Adam | $\mathbb{R}$5lc | 73.91 ± 0.49 | 73.49 ± 1.71 | 0.29 ± 0.00 | 14.30 ± 0.35 |
| CIFAR10 | GELU | Adam | $\mathbb{R}$4sc | 68.54 ± 0.47 | 48.44 ± 1.94 | 0.22 ± 0.00 | 16.00 ± 0.67 |

Continued on next page

| Dataset | Act. | Opt. | Model | Accuracy±CI | TT | IT | epochs |
|---|---|---|---|---|---|---|---|
| CIFAR10 | ReLU | Adam | $\mathbb{R}$5lc | 74.29 ± 0.52 | 50.25 ± 1.63 | 0.23 ± 0.00 | 16.10 ± 0.53 |
| FMNIST | ELU | SGD | $\mathbb{R}$5sc | 91.67 ± 0.16 | 101.96 ± 3.83 | 0.16 ± 0.00 | 43.40 ± 1.66 |
| CIFAR10 | Mish | SGD | $\mathbb{R}$4lc | 67.71 ± 0.58 | 103.61 ± 2.93 | 0.26 ± 0.00 | 26.30 ± 0.76 |
| FMNIST | ELU | SGD | $\mathbb{R}$5lc | 91.90 ± 0.12 | 121.55 ± 5.47 | 0.19 ± 0.00 | 38.00 ± 1.72 |
| FMNIST | ELU | SGD | $\mathbb{R}$4sc | 91.85 ± 0.27 | 95.10 ± 8.88 | 0.15 ± 0.00 | 45.10 ± 4.24 |
| FMNIST | ELU | SGD | $\mathbb{R}$4lc | 92.10 ± 0.29 | 121.94 ± 7.76 | 0.18 ± 0.00 | 43.10 ± 2.75 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$5lc | 72.73 ± 0.54 | 84.17 ± 6.01 | 0.22 ± 0.00 | 28.30 ± 1.97 |
| FMNIST | Mish | SGD | $\mathbb{R}$5lc | 91.36 ± 0.25 | 176.01 ± 11.75 | 0.22 ± 0.00 | 41.40 ± 2.76 |
| FMNIST | Mish | SGD | $\mathbb{R}$5sc | 91.15 ± 0.22 | 130.52 ± 9.69 | 0.18 ± 0.00 | 45.40 ± 3.39 |
| CIFAR10 | Mish | SGD | $\mathbb{R}$4sc | 67.26 ± 0.35 | 80.97 ± 2.47 | 0.21 ± 0.00 | 30.70 ± 0.96 |
| FMNIST | Mish | SGD | $\mathbb{R}$4sc | 91.18 ± 0.30 | 119.73 ± 12.23 | 0.17 ± 0.00 | 46.30 ± 4.77 |
| FMNIST | Mish | SGD | $\mathbb{R}$4lc | 91.37 ± 0.23 | 161.85 ± 3.99 | 0.21 ± 0.00 | 42.80 ± 1.06 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$5lc | 68.89 ± 0.56 | 144.15 ± 7.12 | 0.30 ± 0.00 | 28.80 ± 1.42 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$4sc | 66.88 ± 0.58 | 96.91 ± 3.57 | 0.22 ± 0.00 | 33.10 ± 1.28 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$4lc | 66.66 ± 0.26 | 118.47 ± 4.78 | 0.28 ± 0.00 | 26.40 ± 1.08 |
| FMNIST | GELU | SGD | $\mathbb{R}$5sc | 90.97 ± 0.21 | 151.21 ± 15.01 | 0.19 ± 0.00 | 47.20 ± 4.71 |
| FMNIST | GELU | SGD | $\mathbb{R}$5lc | 91.11 ± 0.20 | 193.15 ± 15.53 | 0.25 ± 0.00 | 40.60 ± 3.27 |
| FMNIST | GELU | SGD | $\mathbb{R}$4sc | 91.14 ± 0.20 | 133.72 ± 8.93 | 0.19 ± 0.00 | 46.20 ± 3.11 |
| FMNIST | GELU | SGD | $\mathbb{R}$4lc | 91.34 ± 0.18 | 185.90 ± 11.70 | 0.23 ± 0.00 | 43.60 ± 2.74 |
| CIFAR10 | GELU | SGD | $\mathbb{R}$5sc | 70.36 ± 0.36 | 132.72 ± 5.44 | 0.23 ± 0.00 | 40.80 ± 1.71 |
| CIFAR10 | Mish | SGD | $\mathbb{R}$5lc | 69.89 ± 0.64 | 124.72 ± 5.12 | 0.27 ± 0.00 | 28.00 ± 1.17 |
| FMNIST | ReLU | SGD | $\mathbb{R}$5sc | 91.27 ± 0.16 | 103.70 ± 9.73 | 0.15 ± 0.00 | 47.00 ± 4.42 |
| FMNIST | ReLU | SGD | $\mathbb{R}$4sc | 91.60 ± 0.25 | 95.83 ± 8.44 | 0.15 ± 0.00 | 49.20 ± 4.47 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$4sc | 68.81 ± 0.44 | 63.40 ± 4.75 | 0.19 ± 0.00 | 32.10 ± 2.37 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$4lc | 70.12 ± 0.64 | 70.94 ± 2.62 | 0.21 ± 0.00 | 27.40 ± 1.02 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$4lc | 68.24 ± 0.58 | 94.81 ± 7.38 | 0.28 ± 0.00 | 21.50 ± 1.69 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$4sc | 67.30 ± 0.67 | 71.33 ± 3.19 | 0.22 ± 0.00 | 24.80 ± 1.11 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$5lc | 69.83 ± 0.66 | 97.18 ± 4.04 | 0.30 ± 0.00 | 19.70 ± 0.83 |
| CIFAR10 | SELU | SGD | $\mathbb{R}$5sc | 69.66 ± 0.54 | 80.35 ± 3.83 | 0.23 ± 0.00 | 25.20 ± 1.21 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$4lc | 66.20 ± 0.66 | 103.39 ± 3.57 | 0.25 ± 0.00 | 27.90 ± 0.98 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$4sc | 66.97 ± 0.35 | 87.87 ± 2.95 | 0.20 ± 0.00 | 35.10 ± 1.19 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$5lc | 68.03 ± 0.66 | 129.53 ± 6.18 | 0.26 ± 0.00 | 30.80 ± 1.46 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$5sc | 71.84 ± 0.42 | 74.18 ± 4.25 | 0.20 ± 0.00 | 31.50 ± 1.82 |
| FMNIST | Swish | SGD | $\mathbb{R}$5sc | 90.84 ± 0.29 | 141.90 ± 14.42 | 0.17 ± 0.00 | 50.50 ± 5.17 |
| FMNIST | ReLU | SGD | $\mathbb{R}$4lc | 91.95 ± 0.16 | 117.04 ± 8.50 | 0.17 ± 0.00 | 44.90 ± 3.26 |
| FMNIST | Swish | SGD | $\mathbb{R}$5lc | 91.12 ± 0.23 | 175.08 ± 16.14 | 0.21 ± 0.00 | 43.50 ± 3.97 |
| FMNIST | Swish | SGD | $\mathbb{R}$4lc | 91.21 ± 0.28 | 155.86 ± 10.60 | 0.20 ± 0.00 | 43.60 ± 2.96 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$5lc | 71.70 ± 0.54 | 77.96 ± 4.29 | 0.23 ± 0.00 | 24.10 ± 1.37 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$4sc | 68.41 ± 0.50 | 60.73 ± 3.93 | 0.19 ± 0.00 | 29.10 ± 1.95 |
| CIFAR10 | ELU | SGD | $\mathbb{R}$4lc | 69.35 ± 0.49 | 70.50 ± 2.79 | 0.22 ± 0.00 | 24.60 ± 0.97 |
| FMNIST | SELU | SGD | $\mathbb{R}$5sc | 91.50 ± 0.22 | 104.03 ± 6.03 | 0.19 ± 0.00 | 33.10 ± 1.92 |
| FMNIST | SELU | SGD | $\mathbb{R}$5lc | 91.82 ± 0.22 | 132.06 ± 11.25 | 0.25 ± 0.00 | 28.40 ± 2.44 |
| FMNIST | SELU | SGD | $\mathbb{R}$4sc | 91.97 ± 0.21 | 116.68 ± 10.81 | 0.18 ± 0.00 | 41.00 ± 3.84 |
| FMNIST | SELU | SGD | $\mathbb{R}$4lc | 92.25 ± 0.18 | 146.22 ± 8.05 | 0.23 ± 0.00 | 35.00 ± 1.94 |
| CIFAR10 | Mish | SGD | $\mathbb{R}$5sc | 70.30 ± 0.62 | 104.82 ± 8.71 | 0.22 ± 0.00 | 35.80 ± 2.99 |
| CIFAR10 | ReLU | SGD | $\mathbb{R}$5sc | 71.98 ± 0.49 | 78.33 ± 5.05 | 0.20 ± 0.00 | 35.80 ± 2.26 |
| FMNIST | ReLU | SGD | $\mathbb{R}$5lc | 91.59 ± 0.14 | 114.00 ± 10.01 | 0.18 ± 0.00 | 38.50 ± 3.43 |
| FMNIST | Swish | SGD | $\mathbb{R}$4sc | 91.06 ± 0.20 | 126.37 ± 9.54 | 0.17 ± 0.00 | 50.20 ± 3.84 |
| CIFAR10 | Swish | SGD | $\mathbb{R}$5sc | 69.35 ± 0.65 | 114.70 ± 6.76 | 0.22 ± 0.00 | 40.50 ± 2.48 |
| CIFAR10 | $\mathbb{S}$Mish | Adam | $\mathbb{C}$4lc | 63.75 ± 0.48 | 122.11 ± 5.37 | 0.51 ± 0.00 | 15.30 ± 0.68 |

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| Dataset | Act. | Opt. | Model | Accuracy±CI | TT | IT | epochs |
|---|---|---|---|---|---|---|---|
| FMNIST | ℂReLU | Adam | ℂ5sc | 90.94 ± 0.23 | 143.04 ± 10.76 | 0.47 ± 0.00 | 20.40 ± 1.62 |
| FMNIST | 𝕊ELU | Adam | ℂ4lc | 89.22 ± 0.29 | 110.53 ± 5.81 | 0.48 ± 0.01 | 15.10 ± 0.86 |
| CIFAR10 | 𝕊GELU | Adam | ℂ5sc | 64.80 ± 0.44 | 138.15 ± 7.56 | 0.49 ± 0.00 | 19.50 ± 1.08 |
| CIFAR10 | 𝕊GELU | Adam | ℂ5lc | 67.14 ± 0.61 | 160.33 ± 7.26 | 0.63 ± 0.00 | 16.20 ± 0.74 |
| CIFAR10 | 𝕊GELU | Adam | ℂ4sc | 64.99 ± 0.63 | 103.48 ± 4.51 | 0.41 ± 0.00 | 17.40 ± 0.77 |
| CIFAR10 | 𝕊GELU | Adam | ℂ4lc | 66.42 ± 0.38 | 128.57 ± 4.08 | 0.53 ± 0.00 | 15.40 ± 0.50 |
| FMNIST | 𝕊ELU | Adam | ℂ4sc | 88.90 ± 0.25 | 109.13 ± 7.66 | 0.38 ± 0.00 | 19.20 ± 1.30 |
| FMNIST | 𝕊ELU | Adam | ℂ5lc | 89.12 ± 0.33 | 187.95 ± 20.05 | 0.56 ± 0.00 | 21.70 ± 2.34 |
| FMNIST | zReLU | Adam | ℂ4lc | 90.38 ± 0.23 | 157.41 ± 26.85 | 0.49 ± 0.01 | 22.00 ± 3.79 |
| FMNIST | zReLU | Adam | ℂ4sc | 89.72 ± 0.36 | 164.30 ± 32.53 | 0.39 ± 0.00 | 29.60 ± 6.06 |
| CIFAR10 | 𝕊ELU | Adam | ℂ5sc | 62.07 ± 0.40 | 208.79 ± 12.05 | 0.47 ± 0.00 | 32.00 ± 1.85 |
| CIFAR10 | 𝕊ELU | Adam | ℂ5lc | 62.55 ± 0.75 | 198.20 ± 12.67 | 0.59 ± 0.00 | 22.40 ± 1.44 |
| CIFAR10 | 𝕊ELU | Adam | ℂ4sc | 59.88 ± 0.70 | 123.44 ± 9.66 | 0.40 ± 0.00 | 22.60 ± 1.76 |
| CIFAR10 | 𝕊ELU | Adam | ℂ4lc | 60.56 ± 0.59 | 124.75 ± 4.82 | 0.50 ± 0.01 | 16.80 ± 0.66 |
| FMNIST | zReLU | Adam | ℂ5lc | 90.28 ± 0.25 | 219.17 ± 18.25 | 0.58 ± 0.00 | 26.30 ± 2.21 |
| FMNIST | zReLU | Adam | ℂ5sc | 89.86 ± 0.28 | 266.72 ± 43.75 | 0.46 ± 0.00 | 39.80 ± 6.37 |
| FMNIST | 𝕊ELU | Adam | ℂ5sc | 88.63 ± 0.26 | 190.26 ± 23.13 | 0.45 ± 0.00 | 28.00 ± 3.42 |
| FMNIST | 𝕊GELU | Adam | ℂ4sc | 90.67 ± 0.24 | 104.01 ± 5.66 | 0.39 ± 0.00 | 17.30 ± 0.96 |
| CIFAR10 | ℂReLU | Adam | ℂ5sc | 69.98 ± 0.60 | 130.54 ± 9.00 | 0.47 ± 0.00 | 20.10 ± 1.41 |
| CIFAR10 | ℂReLU | Adam | ℂ5lc | 72.09 ± 0.32 | 139.62 ± 5.04 | 0.59 ± 0.00 | 15.80 ± 0.56 |
| CIFAR10 | ℂReLU | Adam | ℂ4sc | 68.88 ± 0.43 | 102.62 ± 6.00 | 0.41 ± 0.02 | 18.70 ± 1.26 |
| CIFAR10 | ℂReLU | Adam | ℂ4lc | 70.75 ± 0.64 | 110.74 ± 4.96 | 0.50 ± 0.01 | 15.00 ± 0.67 |
| FMNIST | 𝕊GELU | Adam | ℂ5lc | 90.65 ± 0.21 | 155.10 ± 8.18 | 0.60 ± 0.00 | 16.10 ± 0.86 |
| FMNIST | ℂReLU | Adam | ℂ5lc | 91.48 ± 0.28 | 148.60 ± 7.09 | 0.57 ± 0.00 | 16.70 ± 0.83 |
| FMNIST | ℂSwish | Adam | ℂ5sc | 90.02 ± 0.38 | 135.53 ± 11.18 | 0.46 ± 0.00 | 19.50 ± 1.63 |
| FMNIST | ℂReLU | Adam | ℂ4lc | 91.48 ± 0.26 | 114.84 ± 7.39 | 0.48 ± 0.01 | 15.60 ± 1.02 |
| FMNIST | 𝕊Mish | Adam | ℂ4lc | 90.06 ± 0.33 | 113.07 ± 6.54 | 0.49 ± 0.00 | 14.30 ± 0.83 |
| FMNIST | 𝕊Mish | Adam | ℂ4sc | 90.16 ± 0.25 | 104.84 ± 6.65 | 0.39 ± 0.00 | 17.70 ± 1.12 |
| CIFAR10 | zReLU | Adam | ℂ5sc | 64.32 ± 1.77 | 340.06 ± 67.46 | 0.48 ± 0.00 | 53.40 ± 10.65 |
| CIFAR10 | zReLU | Adam | ℂ5lc | 66.89 ± 0.96 | 274.35 ± 48.37 | 0.59 ± 0.00 | 32.80 ± 5.80 |
| CIFAR10 | zReLU | Adam | ℂ4sc | 55.40 ± 5.22 | 244.01 ± 98.06 | 0.40 ± 0.00 | 46.20 ± 18.64 |
| CIFAR10 | zReLU | Adam | ℂ4lc | 64.01 ± 4.07 | 221.56 ± 49.74 | 0.51 ± 0.00 | 31.70 ± 7.18 |
| FMNIST | 𝕊Mish | Adam | ℂ5lc | 90.02 ± 0.31 | 155.33 ± 9.37 | 0.58 ± 0.00 | 16.50 ± 1.03 |
| FMNIST | 𝕊Mish | Adam | ℂ5sc | 90.04 ± 0.29 | 143.92 ± 6.94 | 0.47 ± 0.00 | 19.90 ± 0.98 |
| FMNIST | ℂSwish | Adam | ℂ5lc | 90.14 ± 0.34 | 151.88 ± 13.22 | 0.58 ± 0.00 | 16.80 ± 1.46 |
| CIFAR10 | ℂSwish | Adam | ℂ5sc | 63.94 ± 0.55 | 139.57 ± 7.96 | 0.47 ± 0.00 | 20.90 ± 1.19 |
| CIFAR10 | ℂSwish | Adam | ℂ4sc | 63.40 ± 0.56 | 107.05 ± 5.03 | 0.39 ± 0.00 | 19.10 ± 0.92 |
| CIFAR10 | ℂSwish | Adam | ℂ4lc | 63.86 ± 0.75 | 115.32 ± 3.99 | 0.50 ± 0.00 | 14.90 ± 0.53 |
| FMNIST | 𝕊GELU | Adam | ℂ4lc | 90.73 ± 0.31 | 120.77 ± 4.27 | 0.50 ± 0.00 | 14.90 ± 0.53 |
| FMNIST | 𝕊GELU | Adam | ℂ5sc | 90.43 ± 0.32 | 137.21 ± 9.60 | 0.47 ± 0.00 | 18.70 ± 1.31 |
| FMNIST | ℂSwish | Adam | ℂ4lc | 90.32 ± 0.35 | 113.07 ± 5.80 | 0.48 ± 0.00 | 14.90 ± 0.79 |
| FMNIST | ℂSwish | Adam | ℂ4sc | 90.36 ± 0.15 | 108.25 ± 3.34 | 0.38 ± 0.00 | 19.00 ± 0.58 |
| CIFAR10 | 𝕊Mish | Adam | ℂ5sc | 64.38 ± 0.54 | 154.17 ± 8.72 | 0.47 ± 0.00 | 22.40 ± 1.31 |
| CIFAR10 | 𝕊Mish | Adam | ℂ5lc | 65.74 ± 0.50 | 158.38 ± 7.57 | 0.61 ± 0.00 | 16.70 ± 0.83 |
| CIFAR10 | 𝕊Mish | Adam | ℂ4sc | 63.57 ± 0.59 | 112.90 ± 6.42 | 0.39 ± 0.00 | 19.60 ± 1.13 |
| CIFAR10 | ℂSwish | Adam | ℂ5lc | 65.30 ± 0.74 | 146.14 ± 5.68 | 0.60 ± 0.00 | 15.90 ± 0.63 |
| FMNIST | ℂReLU | Adam | ℂ4sc | 91.21 ± 0.27 | 104.01 ± 8.80 | 0.39 ± 0.01 | 18.50 ± 1.52 |
| FMNIST | 𝕊GELU | SGD | ℂ4lc | 91.21 ± 0.28 | 369.68 ± 27.99 | 0.50 ± 0.00 | 46.80 ± 3.53 |
| FMNIST | 𝕊GELU | SGD | ℂ4sc | 90.85 ± 0.33 | 317.37 ± 26.66 | 0.40 ± 0.00 | 54.50 ± 4.58 |
| FMNIST | 𝕊Mish | SGD | ℂ5lc | 90.47 ± 0.24 | 441.86 ± 30.77 | 0.59 ± 0.01 | 47.40 ± 3.32 |

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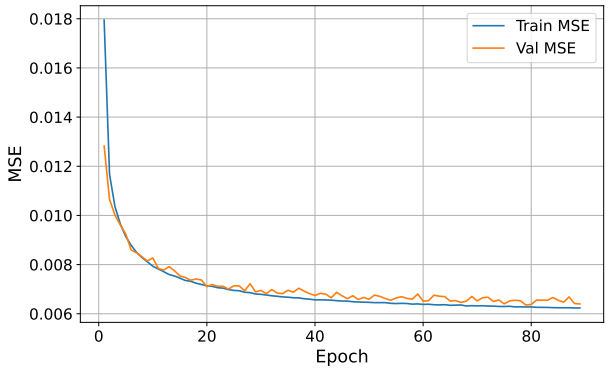
| Dataset | Act. | Opt. | Model | Accuracy±CI | TT | IT | epochs |
|---|---|---|---|---|---|---|---|
| FMNIST | 𝕊GELU | SGD | ℂ5sc | 90.38 ± 0.30 | 356.30 ± 23.33 | 0.48 ± 0.00 | 50.10 ± 3.24 |
| FMNIST | 𝕊Mish | SGD | ℂ4lc | 90.84 ± 0.22 | 330.99 ± 19.72 | 0.48 ± 0.00 | 43.20 ± 2.65 |
| FMNIST | 𝕊Mish | SGD | ℂ4sc | 90.60 ± 0.22 | 308.75 ± 22.68 | 0.39 ± 0.00 | 53.00 ± 3.90 |
| FMNIST | 𝕊Mish | SGD | ℂ5sc | 90.35 ± 0.21 | 416.56 ± 37.81 | 0.46 ± 0.00 | 58.90 ± 5.10 |
| FMNIST | ℂSwish | SGD | ℂ4lc | 90.80 ± 0.19 | 339.11 ± 27.65 | 0.48 ± 0.00 | 45.90 ± 3.78 |
| FMNIST | ℂSwish | SGD | ℂ4sc | 90.71 ± 0.31 | 321.33 ± 22.62 | 0.38 ± 0.00 | 58.30 ± 4.12 |
| FMNIST | ℂSwish | SGD | ℂ5lc | 90.28 ± 0.29 | 441.79 ± 17.11 | 0.58 ± 0.00 | 50.40 ± 1.97 |
| FMNIST | ℂSwish | SGD | ℂ5sc | 90.12 ± 0.36 | 416.43 ± 38.25 | 0.45 ± 0.00 | 62.00 ± 5.77 |
| FMNIST | zReLU | SGD | ℂ4lc | 90.55 ± 0.14 | 441.87 ± 31.07 | 0.48 ± 0.00 | 65.80 ± 4.61 |
| FMNIST | zReLU | SGD | ℂ4sc | 89.53 ± 0.57 | 399.71 ± 45.63 | 0.38 ± 0.00 | 75.60 ± 8.57 |
| FMNIST | 𝕊GELU | SGD | ℂ5lc | 90.55 ± 0.37 | 418.79 ± 47.35 | 0.61 ± 0.01 | 43.50 ± 4.78 |
| FMNIST | 𝕊ELU | SGD | ℂ5sc | 88.51 ± 0.29 | 364.81 ± 42.82 | 0.45 ± 0.00 | 55.50 ± 6.55 |
| CIFAR10 | zReLU | SGD | ℂ5sc | 66.92 ± 0.66 | 604.31 ± 61.87 | 0.48 ± 0.01 | 99.40 ± 10.16 |
| FMNIST | 𝕊ELU | SGD | ℂ4sc | 89.53 ± 0.31 | 279.11 ± 27.05 | 0.38 ± 0.00 | 51.70 ± 5.03 |
| CIFAR10 | ℂReLU | SGD | ℂ4lc | 70.40 ± 0.74 | 220.78 ± 15.16 | 0.49 ± 0.00 | 30.80 ± 2.10 |
| CIFAR10 | ℂReLU | SGD | ℂ4sc | 69.11 ± 0.65 | 201.99 ± 17.49 | 0.39 ± 0.00 | 38.50 ± 3.34 |
| CIFAR10 | ℂReLU | SGD | ℂ5lc | 70.72 ± 0.47 | 251.31 ± 14.02 | 0.59 ± 0.00 | 29.40 ± 1.66 |
| CIFAR10 | ℂReLU | SGD | ℂ5sc | 68.77 ± 0.70 | 221.94 ± 20.97 | 0.47 ± 0.00 | 35.50 ± 3.34 |
| CIFAR10 | 𝕊ELU | SGD | ℂ4lc | 61.60 ± 0.47 | 203.12 ± 7.87 | 0.50 ± 0.00 | 28.10 ± 1.09 |
| CIFAR10 | 𝕊ELU | SGD | ℂ4sc | 60.83 ± 0.60 | 232.34 ± 18.35 | 0.40 ± 0.00 | 44.10 ± 3.48 |
| CIFAR10 | 𝕊ELU | SGD | ℂ5lc | 61.85 ± 0.69 | 312.49 ± 19.08 | 0.59 ± 0.00 | 36.30 ± 2.21 |
| CIFAR10 | 𝕊ELU | SGD | ℂ5sc | 60.72 ± 0.78 | 387.40 ± 58.52 | 0.47 ± 0.00 | 61.60 ± 9.22 |
| CIFAR10 | 𝕊GELU | SGD | ℂ4lc | 67.43 ± 0.62 | 255.96 ± 13.83 | 0.53 ± 0.01 | 31.40 ± 1.66 |
| CIFAR10 | 𝕊GELU | SGD | ℂ4sc | 67.32 ± 0.34 | 238.85 ± 6.58 | 0.41 ± 0.00 | 41.50 ± 1.13 |
| CIFAR10 | 𝕊GELU | SGD | ℂ5lc | 66.85 ± 0.59 | 322.36 ± 15.83 | 0.63 ± 0.01 | 33.40 ± 1.62 |
| CIFAR10 | 𝕊GELU | SGD | ℂ5sc | 66.60 ± 0.50 | 287.26 ± 14.79 | 0.49 ± 0.00 | 42.00 ± 2.16 |
| CIFAR10 | 𝕊Mish | SGD | ℂ4lc | 66.71 ± 0.37 | 245.48 ± 12.14 | 0.51 ± 0.00 | 31.50 ± 1.55 |
| CIFAR10 | 𝕊Mish | SGD | ℂ4sc | 65.88 ± 0.25 | 219.00 ± 9.35 | 0.40 ± 0.00 | 39.20 ± 1.68 |
| CIFAR10 | 𝕊Mish | SGD | ℂ5lc | 66.10 ± 0.58 | 324.91 ± 12.31 | 0.62 ± 0.00 | 34.30 ± 1.39 |
| CIFAR10 | 𝕊Mish | SGD | ℂ5sc | 65.45 ± 0.43 | 301.22 ± 11.91 | 0.48 ± 0.01 | 45.20 ± 1.75 |
| CIFAR10 | ℂSwish | SGD | ℂ4lc | 66.03 ± 0.53 | 254.04 ± 12.42 | 0.51 ± 0.00 | 33.70 ± 1.65 |
| CIFAR10 | ℂSwish | SGD | ℂ4sc | 65.46 ± 0.55 | 230.62 ± 12.17 | 0.40 ± 0.00 | 42.50 ± 2.22 |
| CIFAR10 | ℂSwish | SGD | ℂ5lc | 64.76 ± 0.70 | 316.55 ± 13.57 | 0.60 ± 0.00 | 35.50 ± 1.55 |
| CIFAR10 | ℂSwish | SGD | ℂ5sc | 65.08 ± 0.51 | 304.67 ± 14.31 | 0.47 ± 0.00 | 47.20 ± 2.21 |
| CIFAR10 | zReLU | SGD | ℂ4lc | 67.10 ± 0.88 | 333.56 ± 28.29 | 0.50 ± 0.01 | 49.20 ± 4.20 |
| CIFAR10 | zReLU | SGD | ℂ4sc | 65.46 ± 0.67 | 405.15 ± 49.29 | 0.40 ± 0.00 | 79.80 ± 9.74 |
| CIFAR10 | zReLU | SGD | ℂ5lc | 69.47 ± 0.73 | 489.35 ± 21.69 | 0.59 ± 0.00 | 59.90 ± 2.64 |
| FMNIST | zReLU | SGD | ℂ5lc | 90.12 ± 0.29 | 588.39 ± 84.19 | 0.57 ± 0.00 | 72.40 ± 10.34 |
| FMNIST | ℂReLU | SGD | ℂ4lc | 91.53 ± 0.31 | 326.44 ± 15.54 | 0.47 ± 0.00 | 46.60 ± 2.19 |
| FMNIST | ℂReLU | SGD | ℂ4sc | 91.03 ± 0.36 | 268.75 ± 24.53 | 0.38 ± 0.00 | 49.70 ± 4.65 |
| FMNIST | ℂReLU | SGD | ℂ5lc | 91.12 ± 0.25 | 345.12 ± 21.32 | 0.57 ± 0.01 | 40.50 ± 2.41 |
| FMNIST | ℂReLU | SGD | ℂ5sc | 90.67 ± 0.23 | 323.81 ± 20.80 | 0.45 ± 0.01 | 49.50 ± 3.11 |
| FMNIST | 𝕊ELU | SGD | ℂ4lc | 90.28 ± 0.14 | 277.05 ± 23.44 | 0.47 ± 0.00 | 39.30 ± 3.34 |
| FMNIST | 𝕊ELU | SGD | ℂ5lc | 89.34 ± 0.27 | 354.02 ± 29.01 | 0.56 ± 0.00 | 41.90 ± 3.43 |
| FMNIST | zReLU | SGD | ℂ5sc | 89.24 ± 0.36 | 585.40 ± 124.01 | 0.46 ± 0.00 | 91.00 ± 19.28 |

# 3 Reconstuction task

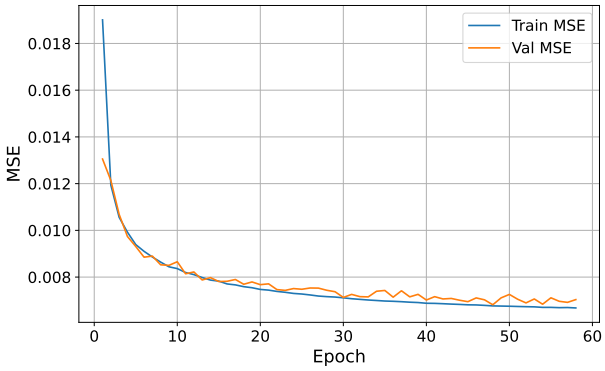
Figure 12 shows the reconstruction MSE for the CVNNs and RVNNs large models on the Fashion MNIST datasets. Figures 13 and 14 report the mean-squared-error (MSE) obtained in the image-reconstruction task on the Fashion-MNIST and CIFAR-10 datasets, respectively. Each panel compares three model families:

1. RVNNs (real inputs, original data type with $P^{\mathbb{R}}$ real-valued parameters)
2. CVNNs ($P^{\mathbb{C}} \approx P^{\mathbb{R}}$) – complex-valued networks whose total parameter count is matched to the corresponding RVNN; and
3. CVNNs ($P^{\mathbb{C}} = 2 \times P^{\mathbb{R}}$) – complex networks whose parameter count is doubled the corresponding RVNNs.

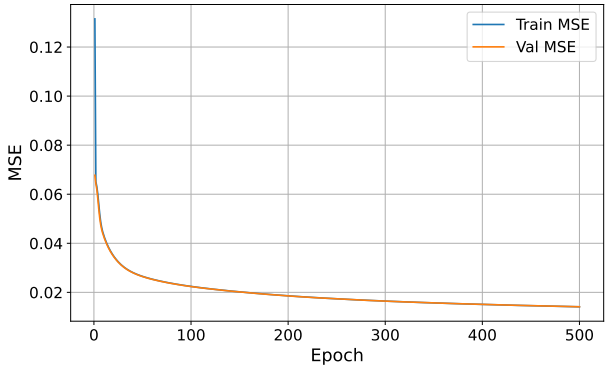
Across both datasets, the RVNNs achieve the lowest errors when parameter budgets are identical. However, when CVNNs are allowed twice as many parameters, their MSE drops, narrowing the performance gap with RVNNs.



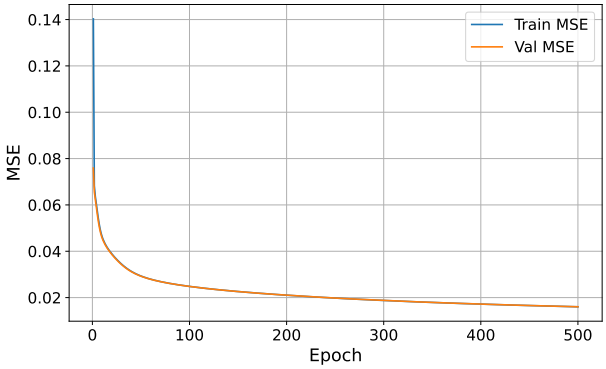

(a) RVNNs Models (ReLU - Adam).




(b) CVNNs Models (ℂReLU-Adam)

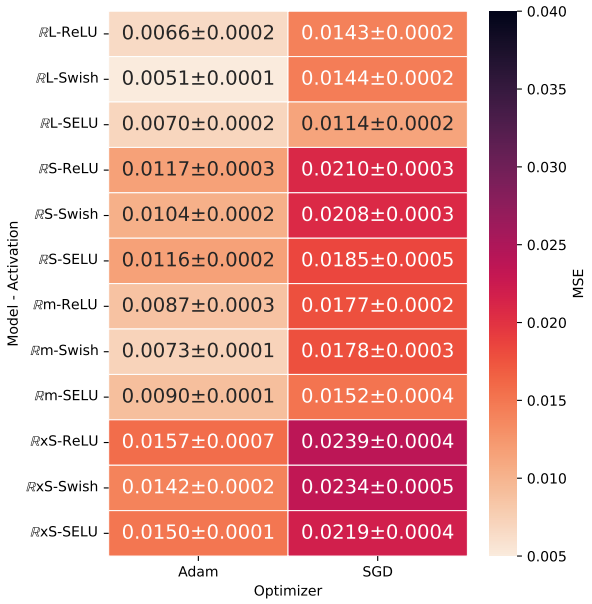



(c) RVNNs Models (ReLU-SGD).

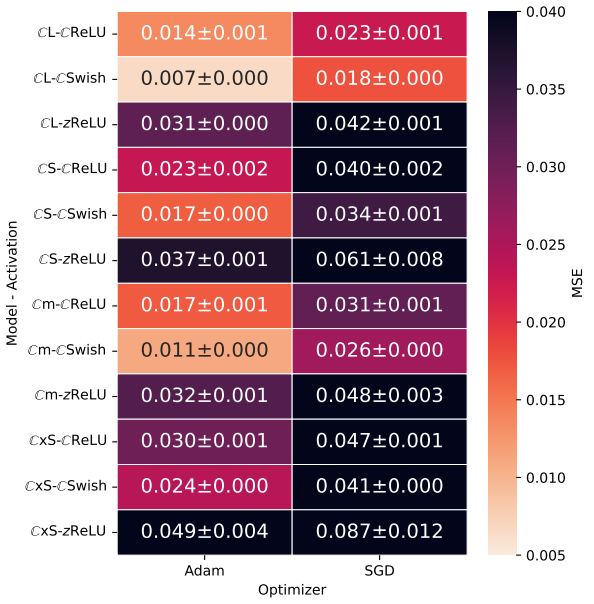



(d) CVNNs Models (ℂReLU-SGD)

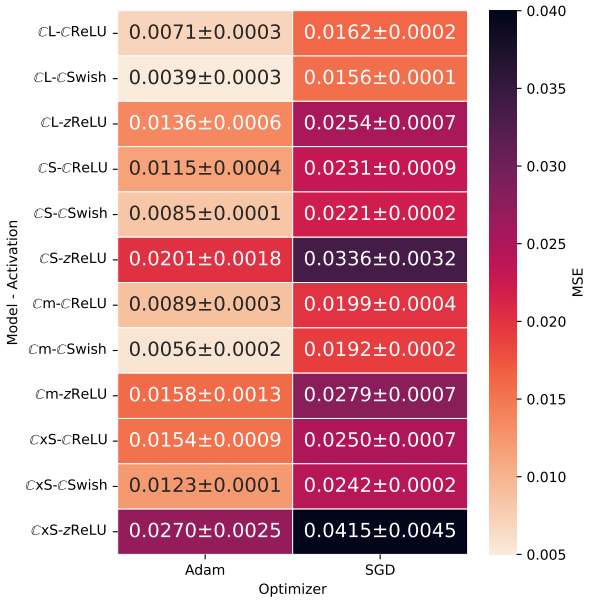
Figure 12: Training and validation reconstruction MSE curves on the Fashion MNIST dataset.




(a) RVNNs Models.

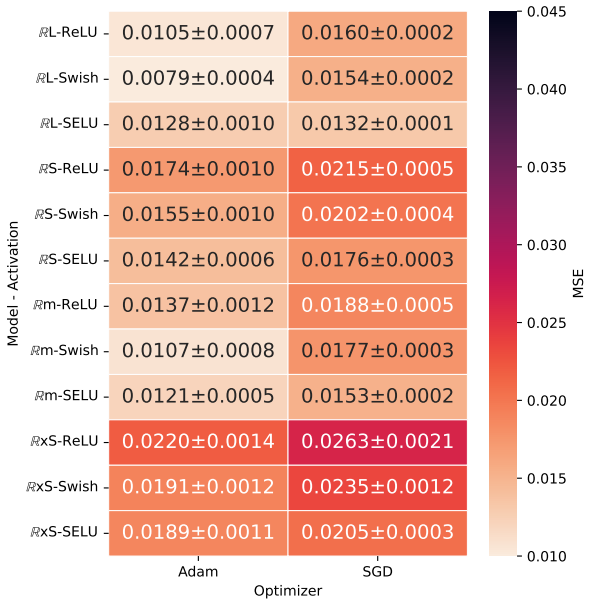



(b) CVNNs Models with $P^{\mathbb{C}} \approx P^{\mathbb{R}}$.

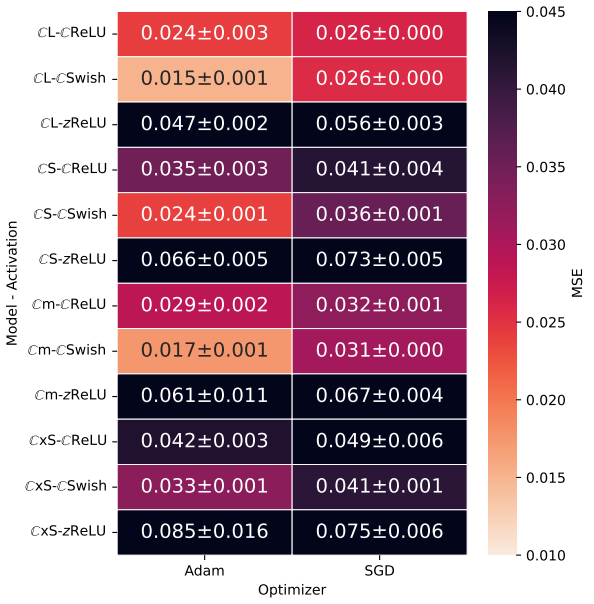



(c) CVNNs Models with $P^{\mathbb{C}} = 2 \times P^{\mathbb{R}}$.

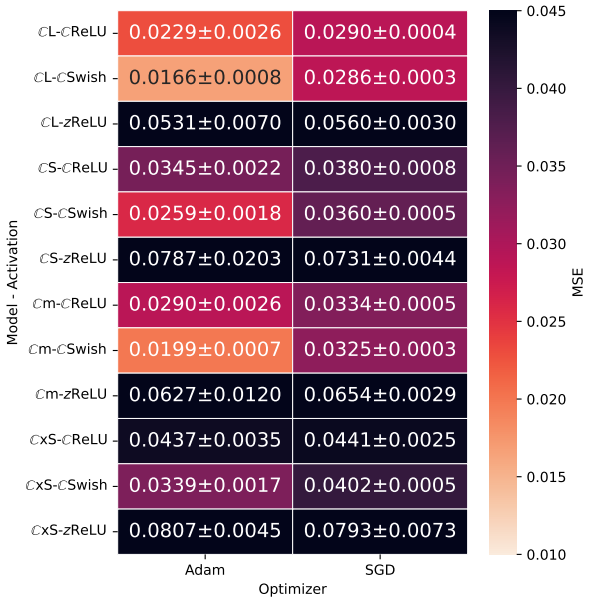
Figure 13: MSE comparison of CVNNs ((b) with complex input obtained by combining every two real features to be one complex number; (c) with real input;) and RVNNs (real input; original data type) on FMINST dataset using different activations and Adam/SGD optimizer.




(a) RVNNs Models.




(b) CVNNs Models with $P^{\mathbb{C}} \approx P^{\mathbb{R}}$.




(c) CVNNs Models with $P^{\mathbb{C}} = 2 \times P^{\mathbb{R}}$.

Figure 14: MSE comparison of CVNNs ((b) with complex input obtained by combining every two real features to be one complex number; (c) with real input;) and RVNNs (real input; original data type) on Cifar10 dataset using different activations and Adam/SGD optimizer.

# 4 Beamforming task

This section shows the results for the beamforming task. Table 3 provides the results of all beamforming experiments. It is clear that the RVNNs model outperforms the CVNNs counterpart model in terms of the MSE and training time for almost all settings. Figure 15 illustrates the MSE vs. the training time for the real and the complex models across multiple activations.

Table 3: Average MSE, training time (TT) in seconds, and the number of epochs along with the confidence interval with 95% confidence level.

| Model | Act | MSE | TT (s) | Epochs |
|---|---|---|---|---|
| $\mathbb{C}$ | $\mathbb{S}$ELU | 1.098e-05 ± 2.842e-06 | 653.231 ± 99.510 | 105.900 ± 16.227 |
| $\mathbb{C}$ | $\mathbb{S}$GELU | 1.575e-05 ± 5.640e-06 | 559.725 ± 60.747 | 90.100 ± 9.511 |
| $\mathbb{C}$ | $\mathbb{S}$Mish | 1.147e-05 ± 3.479e-06 | 678.647 ± 100.010 | 109.100 ± 15.948 |
| $\mathbb{C}$ | $\mathbb{C}$ReLU | 2.382e-05 ± 1.064e-05 | 580.758 ± 86.388 | 94.200 ± 14.171 |
| $\mathbb{C}$ | $\mathbb{S}$SELU | 5.056e-05 ± 6.495e-05 | 1266.906 ± 214.649 | 205.900 ± 34.851 |
| $\mathbb{C}$ | $\mathbb{C}$Swish | 1.058e-05 ± 4.164e-06 | 797.470 ± 121.189 | 129.700 ± 19.689 |
| $\mathbb{R}$ | ELU | 4.499e-06 ± 1.324e-06 | 148.785 ± 18.420 | 91.300 ± 11.326 |
| $\mathbb{R}$ | GELU | 4.442e-06 ± 5.872e-07 | 144.082 ± 13.165 | 84.900 ± 7.708 |
| $\mathbb{R}$ | Mish | 3.453e-06 ± 9.281e-07 | 164.808 ± 26.953 | 96.900 ± 15.743 |
| $\mathbb{R}$ | ReLU | 3.435e-06 ± 6.699e-07 | 171.714 ± 13.289 | 104.600 ± 8.904 |
| $\mathbb{R}$ | SELU | 4.548e-06 ± 1.102e-06 | 148.825 ± 25.792 | 91.900 ± 15.926 |
| $\mathbb{R}$ | Swish | 4.138e-06 ± 8.995e-07 | 151.001 ± 23.202 | 93.400 ± 14.537 |

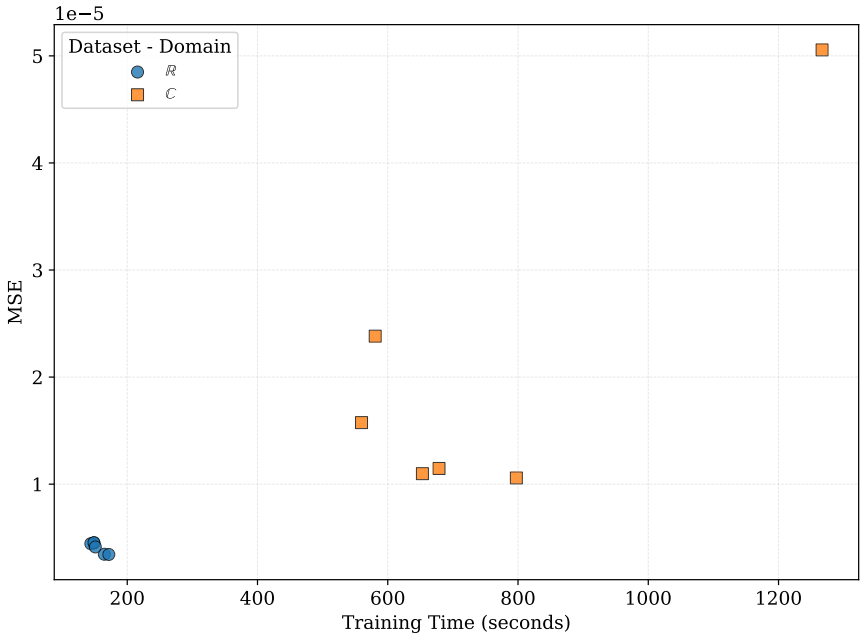



Figure 15: Comparison of average MSE vs training time between CVNNs and RVNNs on the beamforming dataset.