Time Series

Project I

Better Forecasting: CAPM vs ARMA

Mahmood Hasan ID: B00809823

February 25, 2019

DALHOUSIE UNIVERSITY

1 INTRODUCTION

This paper compares the precision of Capital Asset Pricing Model (CAPM) and Auto Regressive Moving Average (ARMA) model using logarithmic stock return data of BlackBerry Limited and Telus Corporation. Using data from December 1997 to February 2019, this paper starts with CAPM to find the relationship between return on stocks and return on market using the following CAPM model,

$$R_i - R_{RF} = \alpha + \beta(RM_i - R_{RF}) + \epsilon_t \quad (1)$$

where R_i denotes return of two differnt stocks(BB for BlackBerry Ltd and T for Telus Corporation), R_{RF} stands for Return of Risk Free stocks (in our case, Canada 30-year Bond Yield) and RM stands for Return of the Market portifolio which in our case is S&P/TSX composite index.All returns are logarithmic returns.

The paper then moves on with the discussion to establish appropriate ARMA model of each stock and carry on 12 period forecasting of each stock return and market returns. The paper concludes a comparison of stock return forcasting using estimated α & β from CAPM and forcasted values of ARMA(p,q) model of market returns.

2 CAPM

Estimation of CAPM of BB stock, where $ExpectedBB = R_{BB} - R_{RF}$ & $ExpectedM = RM_{BB} - R_{RF}$.

$$\widehat{\text{ExpectedBB}} = \underset{(0.011656)}{0.00363837} + \underset{(0.19318)}{1.47972}\,\text{ExpectedM}$$

$$T = 242 \quad \bar{R}^2 = 0.1931 \quad F(1,240) = 58.670 \quad \hat{\sigma} = 0.17990$$

(standard errors in parentheses)

From the above regression we can see a statistically significant β coefficient of 1.48 and statistically insignificant α value of 0.0036. β is a measure of a stock's volatility in relation to the market which means BB stock is in theory, 48% more volatile than the market.

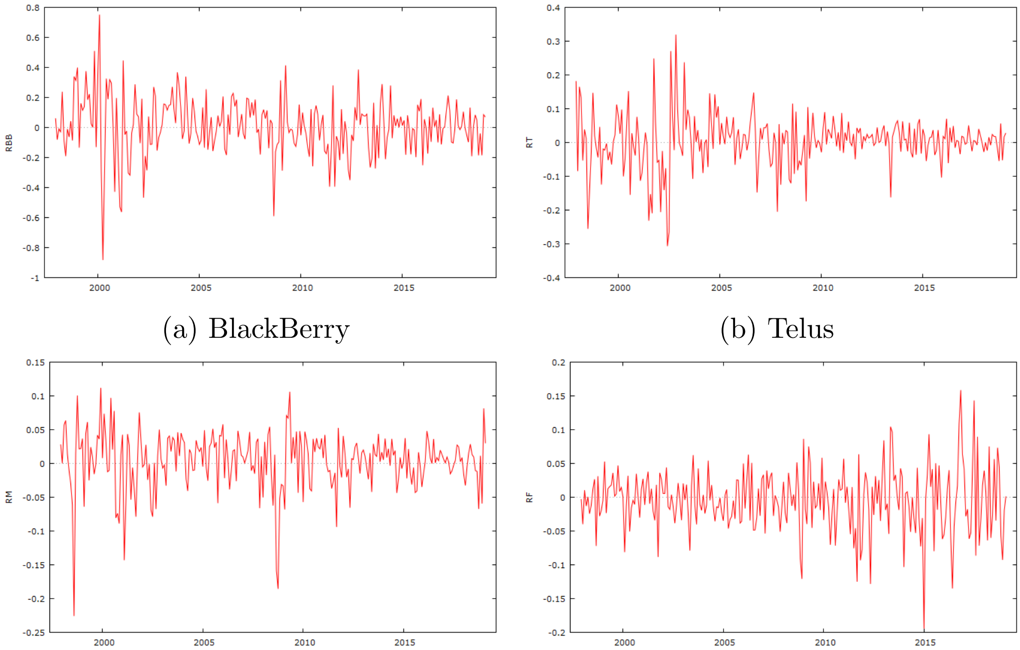
Estimation of CAPM of T stock, where $ExpectedT = R_T - R_{RF}$.

$$\widehat{\text{ExpectedT}} = \underset{(0.0049208)}{-2.66824\text{e–}005} + \underset{(0.081556)}{1.01400}\,\text{ExpectedM}$$

$$T = 242 \quad \bar{R}^2 = 0.3892 \quad F(1,240) = 154.58 \quad \hat{\sigma} = 0.075949$$

(standard errors in parentheses)

Telus stock also gives a statistically significant β coefficient of 1.041 which shows that this stock is not so volatile with respect to the market and can be taken as a less riskier stock than BlackBerry.α value is insignificant in this case aswell.



(a) BlackBerry (b) Telus

(c) Market (d) Risk Free

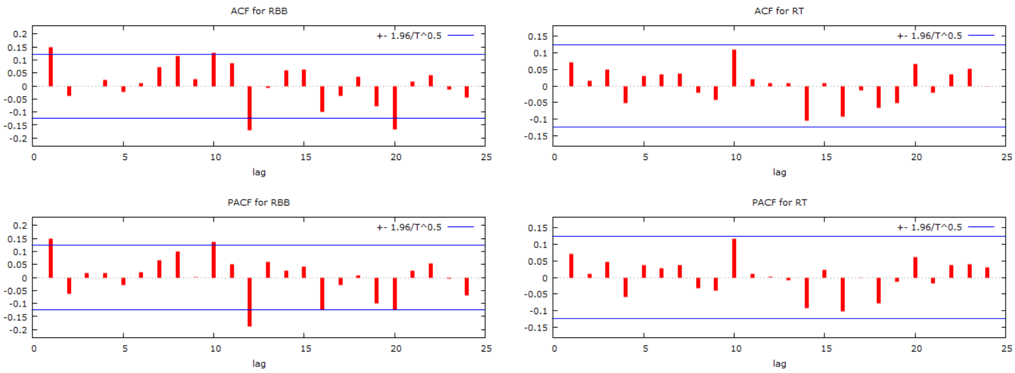
Figure 1: Time series Plots

3 ARMA model

Before we can move on to making a model, it is very important that the data we will use are stationary. Figure 1a to Figure 1d gives us the idea that the data are stationary. For a stronger proof,the Augmented Dickey Fuller tests for the all the variables asserts stationarity as well(shown in 8.1).

Next we look at the ACF and PACF plots to get some idea about the type of ARMA(p,q) model. These plots can often be used to see if there exists any kind of trend or seasonality in our data. For simplicity we do not test for any trend or seasonality. We will relly on ACF/PACF plots and on AIC and/or BIC to find the appropriate ARMA(p,q) model.

Even though we expect the ACF to die down at an exponential rate and PACF to have a sharpt fall, Figure 2a to 2b does not show such pattern. Instead we see an intial peak for B , suggesting ARMA(1,1) for BB stocks but we also see peaks at 12 and even 20. These peaks at times can suggest different kinds of patterns in data or outliers that we have ignored to study in this analysis. ACF and PACF of T stock data shows no sign of autocorrelation but it will not be just to only focus on these plots to come to conclusion about ARMA models.Figure 3 shows the ACF and PACF plots for the market returns which suggests a ARMA(1,1) model.



(a) BlackBerry (b) Telus

Figure 2: ACF & PACF plots

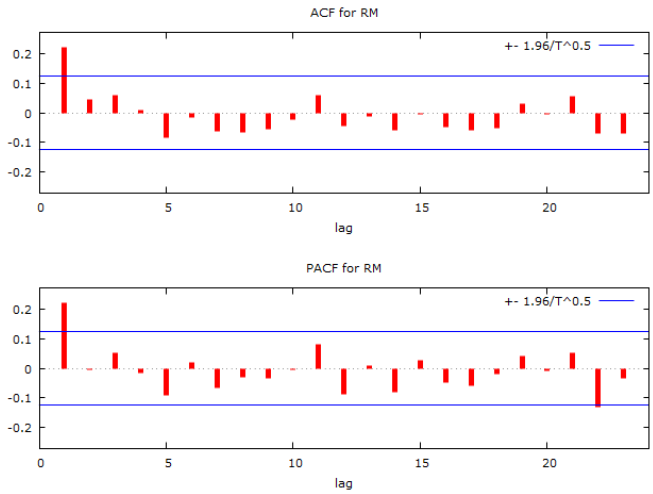
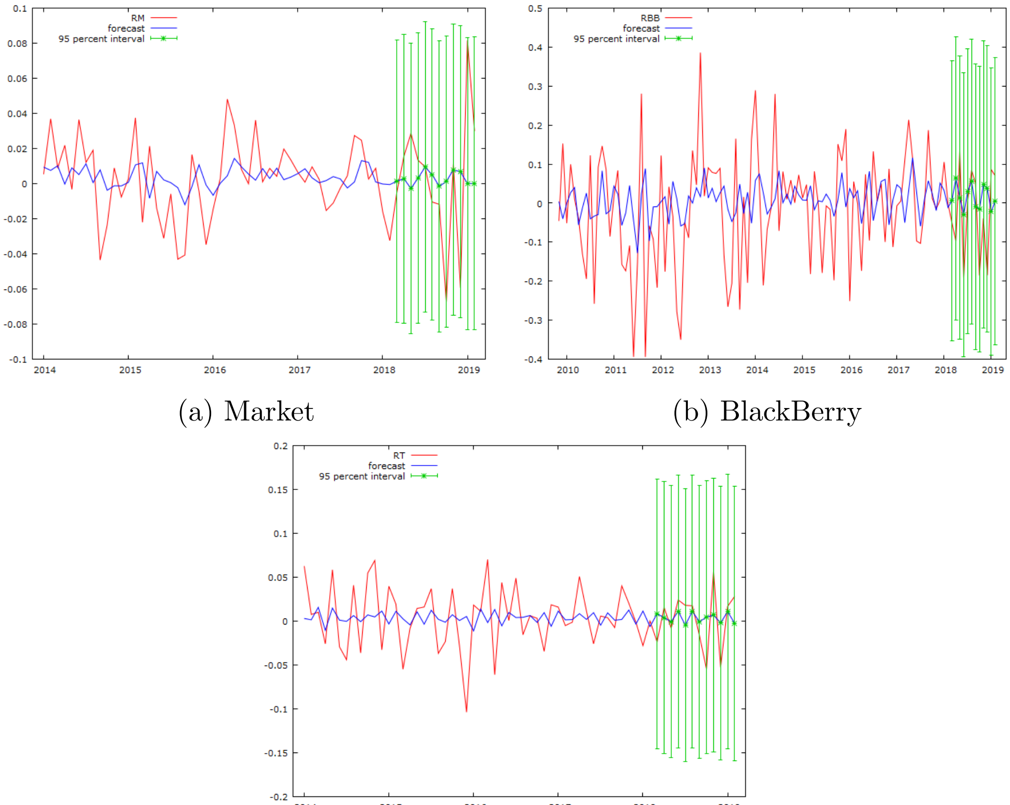


Figure 3: Market Return

If we use BIC of BB stock, we have to use a ARMA(1,1) model but given that our dataset is not significantly large and that AIC gives a ARMA(3,4), model we will go with ARMA(3,4) for BB. AIC and BIC for T stock tell us to set us a ARMA(3,2).Finally even though BIC and AIC both suggests a ARMA(2,2) for market returns we see that running ARMA(2,2) gives insignificant parameter estimates. We instead use ARMA(2,3) that gives rise to statistically similar result and forecasts with ARMA(2,2) but with significant parameters.All the ARMA model estimates are mentioned in (8.2).

4 Forecasting

Figure 4a shows the forecast made using ARMA(2,3) model for the Market stock.Figure 3b shows the forcast plot of BB. What is interesting about this forecast is that any model below ARMA(3,4) gives a horizontal forcast and many models above ARMA(3,4) gives similar predictions but with insignificant AR(p) and MA(q) paramters. In our case we see all ARMA(3,4) parameters estimated are significant.Figure 3c shows the forecast for T stock. All the plots are made using 95% confidence interval.



(a) Market (b) BlackBerry

(c) Telus

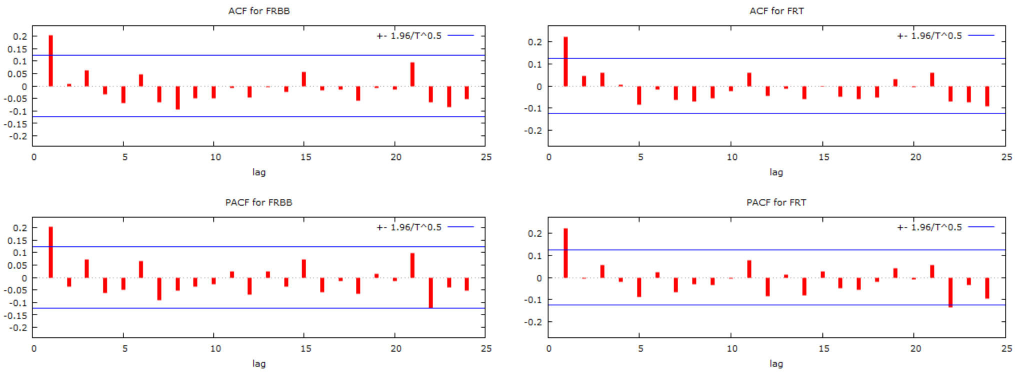
Figure 4: 12 period forecasat plots of each stocks

5 Forecast using CAPM and Market forecast values

Let us denote FRM, FRBB and FRT as forecasted data of the market returns, stock return of BB and T respectively, where stock returns of BB and T are calculated taking into account the values of ($\alpha = 0$ *as insignificant for both stocks*) β from the CAPM and 12period forecasted data of market return.

5.1 Blackberry

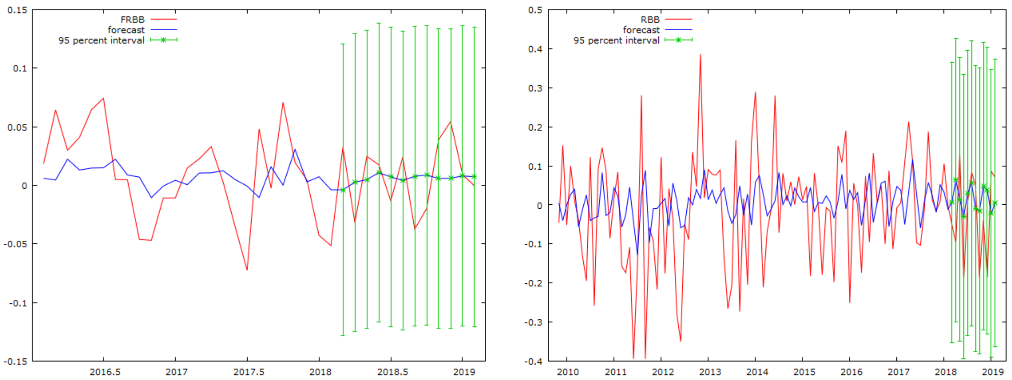
New forcasted stock return of BB is $FRBB = RF + 1.47972 * (FRM - RF)$ and the ACF and PACF plots are shown in Figure 5.



(a) BlackBerry (b) Telus

Figure 5: ACF & PACF plots

We can see a significant change in the ACF and PACF plots for both the plots. ACF and PACF plots of BB stocks shows a peak for 1 lags and no absurd peaks after that unlike before.AIC&BIC suggests a ARMA (2,3)model.Figure 6a shows the forcast of B stock using this new model(Appendix 8.4 presents estimated ARMA models and forecasted values).Figure 6b shows the forecast of old ARMA(3,4) model. From the information available in 8.4 and by looking at the plots we see a signifcant rise in error in prediction in the new model.Mean Percentage Error is 206.83 compared to the previous 82.164.Mean Absolute Error is 0.025189 compared to the previous Mean Absolute Error of 0.10447.



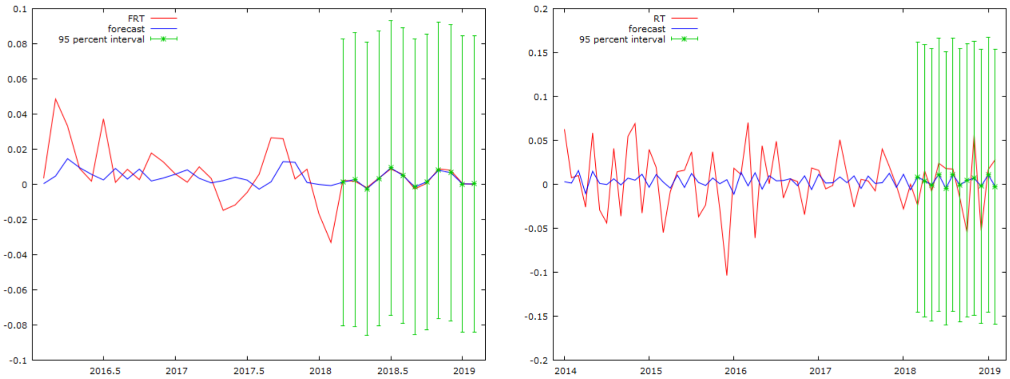
(a) New forecast (b) Old forecast

Figure 6: BlackBerry

5.2 Telus

Previously ACF&PACF of T stokcs suggested a ARMA(0,0) but this changed to a suggestion of ARMA(1,1).AIC of T stocks suggests Using ARMA(2,3).Using information from Appendix 8.4 we see a significantly diffent forecast than before. Mean

Absolute Error has dropped to 0.00073467 from 0.025359 and Mean Percentage Error dropped to -31.132 from 87.595. The adjustment brought about by the values from CAPM model brings about a better forecast in the case of Telus Corporation that has very strict market volatility unlike weak volatility of BlacKBerry stocks.



(a) New forecast

(b) Old forecast

Figure 7: Telus

6 Misspecification Test

All the models used in this paper showed normality of the residuals and Ljung-Box (1978) Q-statistic was used to test for no serial correlation of the errors.Given the all the models pass the misspecification test, it is safe to say the models are properly specified.

7 Conclusion

This paper focused on setting up ARMA(p,q) model using monthly data of BlackBery Ltd and Telus Corporation stocks from the period December 1997 to February 2019. What we see ACF& or PACF plots in our case did not helo much in determinig the final models. AIC& or BIC in most of the cases suggested models that did not give proper forecasts. We saw an exception in the case of the Telus stock which gave a very good forecast when CAPM β values and forcasted Market stocks was taken into consideration. It is very apparent from the plots presented in this papaer that the data has outliers and fluctates rapidly due to other factors not considered in our analogy. Hence the ARMA(p,q) models did not perform well enough in most cases but the models discussed can be improved by using different methods of forecasting or maybe simply by using ARIMA(p,d,q) instead of ARMA(p,q) models. Overall this paper can be used as a benchmark of how to perfor ARMA(p,q) models and do forecasting using ACF&PACF and AIC&BIC.

8 Appendix

8.1 Augmented Dickey-Fuller test for stationarity

```
Augmented Dickey-Fuller test for RBB
including 0 lags of (1-L)RBB
(max was 14, criterion AIC)
sample size 242
unit-root null hypothesis: a = 1

  test without constant
  model: (1-L)y = (a-1)*y(-1) + e
  estimated value of (a - 1): -0.842172
  test statistic: tau_nc(1) = -13.2427
  p-value 2.651e-026
  1st-order autocorrelation coeff. for e: 0.010
```

(a) BlackBerry

```
Augmented Dickey-Fuller test for RT
including 0 lags of (1-L)RT
(max was 14, criterion AIC)
sample size 242
unit-root null hypothesis: a = 1

  test without constant
  model: (1-L)y = (a-1)*y(-1) + e
  estimated value of (a - 1): -0.923947
  test statistic: tau_nc(1) = -14.5383
  p-value 4.696e-029
  1st-order autocorrelation coeff. for e: 0.011
```

(b) Telus

```
Augmented Dickey-Fuller test for RM
including 0 lags of (1-L)RM
(max was 14, criterion AIC)
sample size 242
unit-root null hypothesis: a = 1

  test without constant
  model: (1-L)y = (a-1)*y(-1) + e
  estimated value of (a - 1): -0.771784
  test statistic: tau_nc(1) = -12.3024
  p-value 3.811e-024
  1st-order autocorrelation coeff. for e: 0.000
```

(c) Market

```
Augmented Dickey-Fuller test for RF
including 0 lags of (1-L)RF
(max was 14, criterion AIC)
sample size 242
unit-root null hypothesis: a = 1

  test without constant
  model: (1-L)y = (a-1)*y(-1) + e
  estimated value of (a - 1): -0.956649
  test statistic: tau_nc(1) = -14.8645
  p-value 1.063e-029
  1st-order autocorrelation coeff. for e: 0.000
```

(d) Risk Free

Figure 8: Augmented Dickey-Fuller test

8.2 ARMA models

ARMA(3,4) of BlackBerry

Model 1: ARMA, using observations 1997:12–2018:02 ($T = 243$)
Dependent variable: RBB
Standard errors based on Hessian

	Coefficient	Std. Error	z	p-value
const	0.00964108	0.00852088	1.1315	0.2579
ϕ_1	0.713557	0.0244358	29.2013	0.0000
ϕ_2	−0.690027	0.0254535	−27.1093	0.0000
ϕ_3	0.951604	0.0257093	37.0139	0.0000
θ_1	−0.554872	0.0626619	−8.8550	0.0000
θ_2	0.604032	0.0523819	11.5313	0.0000
θ_3	−0.807241	0.0541859	−14.8976	0.0000
θ_4	−0.241919	0.0630472	−3.8371	0.0001

Mean dependent var	0.010905	S.D. dependent var	0.195641
Mean of innovations	0.004751	S.D. of innovations	0.183142
Log-likelihood	64.68858	Akaike criterion	−111.3772
Schwarz criterion	−79.93960	Hannan–Quinn	−98.71441

ARMA(3,2) of Telus

Model 2: ARMA, using observations 1997:12–2018:02 ($T = 243$)
Dependent variable: RT
Standard errors based on Hessian

	Coefficient	Std. Error	z	p-value
const	0.00385159	0.00550480	0.6997	0.4841
ϕ_1	−1.64752	0.0651691	−25.2807	0.0000
ϕ_2	−0.852685	0.111324	−7.6595	0.0000
ϕ_3	0.0526125	0.0663226	0.7933	0.4276
θ_1	1.77233	0.0251912	70.3552	0.0000
θ_2	1.00000	0.0254758	39.2529	0.0000

Mean dependent var	0.003766	S.D. dependent var	0.080905
Mean of innovations	−0.000080	S.D. of innovations	0.078441
Log-likelihood	271.8084	Akaike criterion	−529.6168
Schwarz criterion	−505.1654	Hannan–Quinn	−519.7680

		Real	Imaginary	Modulus	Frequency
AR					
	Root 1	−0.8989	−0.4976	1.0275	−0.4195
	Root 2	−0.8989	0.4976	1.0275	0.4195
	Root 3	18.0047	0.0000	18.0047	0.0000
MA					
	Root 1	−0.8862	−0.4634	1.0000	−0.4233
	Root 2	−0.8862	0.4634	1.0000	0.4233

		Real	Imaginary	Modulus	Frequency
AR					
	Root 1	1.0112	0.0000	1.0112	0.0000
	Root 2	−0.1431	−1.0093	1.0194	−0.2724
	Root 3	−0.1431	1.0093	1.0194	0.2724
MA					
	Root 1	1.0000	0.0000	1.0000	0.0000
	Root 2	−4.1336	0.0000	4.1336	0.5000
	Root 3	−0.1016	−0.9948	1.0000	−0.2662
	Root 4	−0.1016	0.9948	1.0000	0.2662

ARMA(2,3) of Market

Model 3: ARMA, using observations 1997:12–2018:02 (T = 243)
Dependent variable: RM
Standard errors based on Hessian

	Coefficient	Std. Error	z	p-value
const	0.00361192	0.00335865	1.0754	0.2822
ϕ_1	0.190096	0.0376848	5.0444	0.0000
ϕ_2	−0.934615	0.0305880	−30.5550	0.0000
θ_1	0.0260258	0.0685919	0.3794	0.7044
θ_2	0.940380	0.0261114	36.0142	0.0000
θ_3	0.257532	0.0653686	3.9397	0.0001

Mean dependent var	0.003553	S.D. dependent var	0.043032
Mean of innovations	−0.000063	S.D. of innovations	0.041105
Log-likelihood	428.9137	Akaike criterion	−843.8274
Schwarz criterion	−819.3760	Hannan–Quinn	−833.9786

		Real	Imaginary	Modulus	Frequency
AR					
	Root 1	0.1017	−1.0294	1.0344	−0.2343
	Root 2	0.1017	1.0294	1.0344	0.2343
MA					
	Root 1	0.1158	−0.9933	1.0000	−0.2315
	Root 2	0.1158	0.9933	1.0000	0.2315
	Root 3	−3.8830	0.0000	3.8830	0.5000

For 95% confidence intervals, $z(0.025) = 1.96$

For 95% confidence intervals, $z(0.025) = 1.96$

8.3 Forecasting Results

Forcasting of Market Returns

For 95% confidence intervals, $z(0.025) = 1.96$

Obs	RM	prediction	std. error	95% interval	
2018:03	−0.004894	0.001397	0.041105	−0.079168	0.081962
2018:04	0.015535	0.002805	0.042054	−0.079620	0.085229
2018:05	0.028649	−0.002686	0.042098	−0.085197	0.079825
2018:06	0.013373	0.003169	0.042182	−0.079505	0.085844
2018:07	0.009555	0.009414	0.042202	−0.073300	0.092127
2018:08	−0.010468	0.005129	0.042289	−0.077757	0.088014

2018:09	−0.011736	−0.001522	0.042295	−0.084419	0.081374
2018:10	−0.067282	0.001218	0.042379	−0.081844	0.084281
2018:11	0.011285	0.007956	0.042380	−0.075107	0.091018
2018:12	−0.059295	0.006675	0.042455	−0.076536	0.089885
2019:01	0.081599	0.000135	0.042456	−0.083079	0.083348
2019:02	0.029946	0.000088	0.042519	−0.083246	0.083423

Forecast evaluation statistics

Mean Error	0.00020749
Mean Squared Error	0.0015182
Root Mean Squared Error	0.038964
Mean Absolute Error	0.027969
Mean Percentage Error	89.648
Mean Absolute Percentage Error	89.648
Theil's U	0.96117
Bias proportion, U^M	2.8357e-005
Regression proportion, U^R	0.11592
Disturbance proportion, U^D	0.88406

Forecasting of BlackBerry

For 95% confidence intervals, $z(0.025) = 1.96$

Obs	RBB	prediction	std. error	95% interval	
2018:03	−0.050043	0.003981	0.183652	−0.355971	0.363933
2018:04	−0.097067	0.059931	0.186271	−0.305154	0.425016
2018:05	0.132880	0.010711	0.186524	−0.354870	0.376293
2018:06	−0.191090	−0.032991	0.187077	−0.399656	0.333674
2018:07	0.007073	0.026071	0.187260	−0.340951	0.393093
2018:08	0.084070	0.051564	0.187395	−0.315723	0.418852
2018:09	0.051221	−0.012687	0.187905	−0.380973	0.355599
2018:10	−0.185061	−0.019835	0.188017	−0.388341	0.348671
2018:11	−0.039451	0.043811	0.188406	−0.325458	0.413079
2018:12	−0.184722	0.032959	0.188406	−0.336310	0.402227
2019:01	0.085809	−0.025574	0.188930	−0.395869	0.344722
2019:02	0.070252	0.000840	0.188936	−0.369468	0.371148

Forecast evaluation statistics	
Mean Error	-0.037909
Mean Squared Error	0.014319
Root Mean Squared Error	0.11966
Mean Absolute Error	0.10447
Mean Percentage Error	82.164
Mean Absolute Percentage Error	126.93
Theil's U	0.67172
Bias proportion, U^M	0.10036
Regression proportion, U^R	0.032941
Disturbance proportion, U^D	0.8667

Forecasting of Telus

Obs	RT	prediction	std. error	95% interval	
2018:03	-0.023592	0.008434	0.078441	-0.145307	0.162174
2018:04	0.015572	0.004045	0.079049	-0.150888	0.158979
2018:05	-0.007865	-0.000566	0.079181	-0.155759	0.154627
2018:06	0.024056	0.011205	0.079251	-0.144123	0.166534
2018:07	0.018038	-0.004487	0.079258	-0.159829	0.150855
2018:08	0.017511	0.011087	0.079269	-0.144278	0.166451
2018:09	-0.016250	-0.000571	0.079337	-0.156069	0.154927
2018:10	-0.054604	0.004530	0.079454	-0.151197	0.160258
2018:11	0.056283	0.006885	0.079567	-0.149063	0.162834
2018:12	-0.052519	-0.001958	0.079629	-0.158028	0.154112
2019:01	0.016873	0.010872	0.079640	-0.145219	0.166963
2019:02	0.028280	-0.002601	0.079643	-0.158698	0.153497

Forecast evaluation statistics	
Mean Error	-0.0020911
Mean Squared Error	0.0009712
Root Mean Squared Error	0.031164
Mean Absolute Error	0.025359
Mean Percentage Error	87.595
Mean Absolute Percentage Error	87.595
Theil's U	0.75249
Bias proportion, U^M	0.0045025
Regression proportion, U^R	0.0057813
Disturbance proportion, U^D	0.98972

8.4 Forcasting using CAPM and forcasted Market reuturns

ARMA(2,3) model Estimation of BlackBerry

Model 30: ARMA, using observations 1997:12–2018:02 (T = 243)
Dependent variable: FRBB
Standard errors based on Hessian

	Coefficient	Std. Error	z	p-value
const	0.00703248	0.00512169	1.3731	0.1697
ϕ_1	−0.327209	0.286511	−1.1420	0.2534
ϕ_2	−0.722314	0.258054	−2.7991	0.0051
θ_1	0.547088	0.282063	1.9396	0.0524
θ_2	0.801056	0.264402	3.0297	0.0024
θ_3	0.236164	0.0701058	3.3687	0.0008

Mean dependent var	0.007078	S.D. dependent var	0.065471
Mean of innovations	−7.74e–06	S.D. of innovations	0.063379
Log-likelihood	325.4862	Akaike criterion	−636.9724
Schwarz criterion	−612.5210	Hannan–Quinn	−627.1236

		Real	Imaginary	Modulus	Frequency
AR					
	Root 1	−0.2265	−1.1546	1.1766	−0.2808
	Root 2	−0.2265	1.1546	1.1766	0.2808
MA					
	Root 1	−3.0859	0.0000	3.0859	0.5000
	Root 2	−0.1530	−1.1614	1.1714	−0.2709
	Root 3	−0.1530	1.1614	1.1714	0.2709

12 period Forecast of BlackBerry

For 95% confidence intervals, $z(0.025) = 1.96$

Obs	FRBB	prediction	std. error	95% interval	
2018:03	0.032718	−0.003796	0.063379	−0.128016	0.120424
2018:04	−0.031923	0.002723	0.064893	−0.124464	0.129910
2018:05	0.024834	0.004981	0.064894	−0.122209	0.132170
2018:06	0.017586	0.010817	0.065068	−0.116715	0.138348
2018:07	−0.013175	0.007276	0.065095	−0.120308	0.134861
2018:08	0.024137	0.004219	0.065157	−0.123485	0.131924
2018:09	−0.037377	0.007777	0.065196	−0.120006	0.135559
2018:10	−0.019694	0.008821	0.065209	−0.118987	0.136629
2018:11	0.039046	0.005910	0.065242	−0.121962	0.133782
2018:12	0.054378	0.006108	0.065242	−0.121765	0.133981
2019:01	0.009304	0.008146	0.065262	−0.119764	0.136056
2019:02	−0.000542	0.007336	0.065262	−0.120576	0.135248

Forecast evaluation statistics	
Mean Error	0.0024145
Mean Squared Error	0.00084433
Root Mean Squared Error	0.029057
Mean Absolute Error	0.025189
Mean Percentage Error	206.83
Mean Absolute Percentage Error	206.83
Theil's U	0.6337
Bias proportion, U^M	0.0069047
Regression proportion, U^R	0.13399
Disturbance proportion, U^D	0.85911

ARMA(2,3) Model of Telus

Model 4: ARMA, using observations 1997:12–2018:02 ($T = 243$)
Dependent variable: FRT
Standard errors based on Hessian

	Coefficient	Std. Error	z	p-value
const	0.00371478	0.00340365	1.0914	0.2751
ϕ_1	0.190146	0.0375444	5.0646	0.0000
ϕ_2	−0.935104	0.0304134	−30.7464	0.0000
θ_1	0.0272369	0.0681596	0.3996	0.6894
θ_2	0.940285	0.0261942	35.8967	0.0000
θ_3	0.258364	0.0650171	3.9738	0.0001

Mean dependent var	0.003656	S.D. dependent var	0.043582
Mean of innovations	−0.000063	S.D. of innovations	0.041630
Log-likelihood	425.8377	Akaike criterion	−837.6754
Schwarz criterion	−813.2240	Hannan–Quinn	−827.8266

		Real	Imaginary	Modulus	Frequency
AR					
	Root 1	0.1017	−1.0291	1.0341	−0.2343
	Root 2	0.1017	1.0291	1.0341	0.2343
MA					
	Root 1	0.1156	−0.9933	1.0000	−0.2316
	Root 2	0.1156	0.9933	1.0000	0.2316
	Root 3	−3.8705	0.0000	3.8705	0.5000

12 period forecast of Telus

For 95% confidence intervals, $z(0.025) = 1.96$

Obs	FRT	prediction	std. error	95% interval	
2018:03	0.002311	0.001211	0.041630	−0.080382	0.082805

2018:04	0.001792	0.002842	0.042602	−0.080657	0.086341
2018:05	−0.001883	−0.002458	0.042646	−0.086043	0.081127
2018:06	0.003590	0.003357	0.042729	−0.080391	0.087105
2018:07	0.008755	0.009419	0.042749	−0.074368	0.093206
2018:08	0.005684	0.005134	0.042837	−0.078824	0.089093
2018:09	−0.002568	−0.001349	0.042842	−0.085319	0.082620
2018:10	0.000608	0.001425	0.042927	−0.082711	0.085560
2018:11	0.008862	0.008015	0.042927	−0.076121	0.092151
2018:12	0.008067	0.006674	0.043003	−0.077610	0.090957
2019:01	0.000403	0.000256	0.043004	−0.084030	0.084543
2019:02	0.000070	0.000290	0.043066	−0.084118	0.084698

Forecast evaluation statistics

Mean Error	7.2752e-005
Mean Squared Error	6.9377e-007
Root Mean Squared Error	0.00083293
Mean Absolute Error	0.00073467
Mean Percentage Error	−31.132
Mean Absolute Percentage Error	60.189
Theil's U	0.11657
Bias proportion, U^M	0.0076291
Regression proportion, U^R	0.053738
Disturbance proportion, U^D	0.93863

9 DATA collection and preparation

Data collected from www.investing.com and adjusted to logarithmic returns using EXCEL.Estimation of ARMA models, forecasting and AIC & BIC calculation done using both Gretl and R.