ANN Sheet-1

التاريخ .... الموضوع ....

*** Given Values**

$$W_1 = \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} = \begin{bmatrix} 0.15 & 0.20 \\ .25 & 0.30 \end{bmatrix}, \quad b_1 = [0.35]$$

$$W_2 = \begin{bmatrix} w_5 & w_6 \\ w_7 & w_8 \end{bmatrix} = \begin{bmatrix} .40 & .45 \\ .50 & .55 \end{bmatrix}, \quad b_2 = [0.60]$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.10 \end{bmatrix}$$

$$y_{actual} = \begin{bmatrix} o_1 \\ o_2 \end{bmatrix} = \begin{bmatrix} .01 \\ 0.99 \end{bmatrix}$$

*** forward propagation**

$$Z_1 = W_1 * X + b_1 = \left(\begin{bmatrix} 0.15 & 0.20 \\ .25 & .30 \end{bmatrix} * \begin{bmatrix} 0.05 \\ 0.10 \end{bmatrix} \right) + \begin{bmatrix} 1 \end{bmatrix} 0.35$$

$$Z_1 = \begin{bmatrix} 0.3775 \\ 0.3925 \end{bmatrix} \quad \cancel{\Rightarrow output}$$

$$A_1 = \sigma(Z_1) = \frac{1}{1+e^{-Z_1}} = \begin{bmatrix} 0.593 \\ 0.596 \end{bmatrix} \Rightarrow \text{output for } h_1, h_2$$

$$Z_2 = W_2 A_1 + b_2 = \begin{bmatrix} .40 & 0.45 \\ .50 & .55 \end{bmatrix} \begin{bmatrix} 0.593 \\ 0.596 \end{bmatrix} + 0.60$$

$$Z_2 = \begin{bmatrix} 1.1054 \\ 1.2243 \end{bmatrix}, \quad A_2 = \sigma(Z_2) = \begin{bmatrix} 0.7513 \\ 0.7728 \end{bmatrix}$$

output for O_1, O_2

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→ using RMS error

* ~~Calculating the~~ cost (error) — $e(y_{act}, y_{target})$

$$E = \sum \frac{1}{2}(y_{target} - y_{actual})^2$$

$$E = \sum \frac{1}{2}\left(\begin{bmatrix} .7513 \\ .7728 \end{bmatrix} - \begin{bmatrix} 0.01 \\ .99 \end{bmatrix}\right)^2 = \boxed{0.29838}$$

—*—*—

* Backward propagation & update parameters

Let the calculated values be:

~~$dz_2 = A_2 -$~~

$$z_1 = \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix}, \quad A_1 = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, \quad z_2 = \begin{bmatrix} O_1^* \\ O_2^* \end{bmatrix}, \quad A_2 = \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}$$

~~$\frac{\partial E}{\partial w}$~~

using the chain rule

$$\frac{\partial E}{\partial w_i} = \frac{\partial E}{\partial O_j} * \frac{\partial O_j}{\partial O_j^*} * \frac{\partial O_j^*}{\partial w_i} \Rightarrow \text{for } i = 5-8,\ j = 1, 2$$

* for $i = 5, j = 1$

we have $E = \frac{1}{2}(O_1 - \hat{O}_1)^2 + \frac{1}{2}(O_2 - \hat{O}_2)^2$ → 0

$$\therefore \frac{\partial E}{\partial O_1} = -2 * \frac{1}{2}(\hat{O}_1 - O_1) = O_1 - \hat{O}_1$$

$$= (0.7513 - 0.01) = \boxed{0.7413}$$

—*—*—

we have $O_1 = \frac{1}{1 + e^{-O_1^*}}$

$$\Rightarrow \frac{\partial O_1}{\partial O_1^*} = O_1(1 - O_1) = 0.7513(1 - 0.7513)$$

$$= \boxed{0.1868}$$

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we have $O_1^* = w_5 \hat{h}_1 + w_6 \hat{h}_2 + b_2$

$$\frac{\partial O_1^*}{\partial w_5} = \hat{h}_1 = \boxed{0.5932}$$

$$\therefore \frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial \hat{O}} * \frac{\partial \hat{O}_1}{\partial O_1^*} * \frac{\partial O_1^*}{\partial w_5}$$

$\boxed{\text{take } \alpha = 0.5}$

$$= 0.7413 * 0.1868 * 0.5932 = \boxed{0.08216}$$

$$\therefore w_5 = w_5 - \frac{\partial E}{\partial w_5} * \alpha = 0.40 - 0.5 * 0.08216 = \boxed{0.35}$$

for $i = 6, j = 1$ we only calc the third term

$$\frac{\partial O_1^*}{\partial w_6} = \hat{h}_2 = 0.7728$$

then $\frac{\partial E}{\partial w_6} = 0.7413 * 0.1868 * 0.7728 = \boxed{.107102}$

$$w_6 = w_6 - \frac{\partial E}{\partial w_6} * \alpha = 0.45 - 0.5 * .10712 = \boxed{0.39}$$

for $i = 7, j = 2$

$$\frac{\partial E}{\partial w_7} = \frac{\partial E}{\partial \hat{O}_2} * \frac{\partial \hat{O}_2}{\partial O_2^*} * \frac{\partial O_2^*}{\partial O_2^*}$$

$$\frac{\partial E}{\partial \hat{O}_2} = \hat{O}_2 - O_2 = 0.99 - 0.7728 = \boxed{0.2172}$$

$$\frac{\partial O_2^-}{\partial O_2^*} = \grave{O}_2(1-\grave{O}_2) = 0.7728(1-0.7728)$$

$$= \boxed{0.1755}$$

we have $O_2^* = w_7 \grave{h}_1 + w_8 \grave{h}_2 + b_2$

$$\frac{\partial O_2^*}{\partial w_7} = \grave{h}_1 = \boxed{0.5932}$$

$$\therefore \frac{\partial E}{\partial w_7} = 0.2172 \times 0.1755 \times 0.5932$$

$$= \boxed{0.02261}$$

$$w_7 = w_7 - \alpha \frac{\partial E}{\partial w_7} = .50 - (.5 \times 0.226) = \boxed{0.48}$$

for $i=8$ $j=2$ Change only the third part

$$\frac{\partial O_2^*}{\partial w_8} = \grave{h}_2 = \boxed{.7728}$$

$$\frac{\partial E}{\partial w_8} = 0.2172 \times 0.1755 \times 0.7728$$

$$= \boxed{0.02945}$$

$$w_8 = w_8 - \alpha \frac{\partial E}{\partial w_8} = \boxed{0.53}$$

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for $i = 1 \to 4$
$j = 1, 2$

$$\frac{\partial E}{\partial w_i} = \frac{\partial E}{\partial h_j} * \frac{\partial h_j}{\partial h_j^*} * \frac{\partial h_j^*}{\partial w_i}$$

for $i = 1, j = 1$ $\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial h_1} * \frac{\partial h_1}{\partial h_1^*} * \frac{\partial h_1^*}{\partial w_1}$

we have $E = E_{O1} + E_{O2}$

$$\frac{\partial E}{\partial h_1} = \frac{\partial E_{O1}}{\partial h_1} + \frac{\partial E_{O2}}{\partial h_1}$$

$$\frac{\partial E_{O1}}{\partial h_1} = \frac{\partial E_{O1}}{\partial O_1^*} * \frac{\partial O_1^*}{\partial h_1}$$

we have $E_{O1} = \frac{1}{2}(\hat{O}_1 - O_1)^2$, $\hat{O}_1 = \frac{1}{1+e^{-O_1^*}}$

$$\frac{\partial}{\partial O_1^*}\left(\frac{1}{2}\left(\frac{1}{1+e^{O_1^*}} - O_1\right)\right)^2$$

$= (\hat{O}_1 - O_1) * \hat{O}_1 (1 - \hat{O}_1)$ → Calculated earlier

$0.7413 * 0.1868 = 0.1384$

we have $O_1^* = w_5 * h_1 + \cancel{w_6 * h_2 + b}$

$= w_5 = 0.40$ → old value

$\therefore \frac{\partial E_{O1}}{\partial h_1} = 0.1384 * 0.40 = \boxed{0.0553}$

by applying the same on $\frac{\partial E_{O2}}{\partial h_1}$

we got $\frac{\partial E_{O2}}{\partial h_1} = \boxed{-0.0190}$

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$$\frac{\partial E}{\partial h_1'} = 0.0553 + 0.0190 = \boxed{0.03635}$$

$$\frac{\partial h_1'}{\partial h_1^*} = h_1'(1 - h_1') = \boxed{0.02413}$$

$$\frac{\partial h_1^*}{\partial w_1} = i_1 = \boxed{0.05}$$

$$\frac{\partial E}{\partial w_1} = 0.03635 * 0.2413 * 0.05 = \boxed{0.438 * 10^{-3}}$$

$$w_1 = w_1 - \alpha \frac{\partial E}{\partial w_1} = 0.15(0.5) * \quad = \boxed{0.14}$$

$i = 2 \quad j = 1$

$$\frac{\partial h_1^*}{\partial w_2} = i_2 = 0.10$$

$$\therefore \frac{\partial E}{\partial w_2} = 0.0363 * 0.2413 * .10 = \boxed{0.875 * 10^{-3}}$$

$$w_2 = w_2 - \alpha \frac{\partial E}{\partial w_2} = \boxed{0.19}$$

* the same as before

$i = 3 \quad j = 2$

* Calculate the new loss (error)

$$Z_1 = W_1 X + b_1 = \begin{bmatrix} .14 & .19 \\ .25 & 0.29 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.10 \end{bmatrix} + .35 = \begin{bmatrix} 0.376 \\ 0.391 \end{bmatrix}$$

$$A_1 = \sigma(Z_1) = \begin{bmatrix} 1.6866 \\ 1.6760 \end{bmatrix}$$

$$Z_2 = W_2 A_1 + b_2 = \begin{bmatrix} .35 & 0.39 \\ .48 & 0.53 \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix} + 0.60 = \begin{bmatrix} 1.843 \\ 2.297 \end{bmatrix}$$

$$A_2 = \sigma(Z_2) = \begin{bmatrix} 1.158 \\ 1.100 \end{bmatrix}$$

the new error value = $\boxed{0.2910}$

* Conclusion *

$w_1 = 0.14$
$w_2 = 0.19$
$w_3 = 0.25$
$w_4 = 0.29$
$w_5 = 0.35$
$w_6 = 0.39$
$w_7 = 0.48$
$w_8 = 0.53$

old error = 0.2983

New error = 0.2910

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