UNIT : I SYSTEMS AND THEIR REPRESENTATION.

Components of Control systems :

I/P → force $f(t)$

O/P → Displacement x (or) Velocity v

Components:

Mass → M

Friction → B

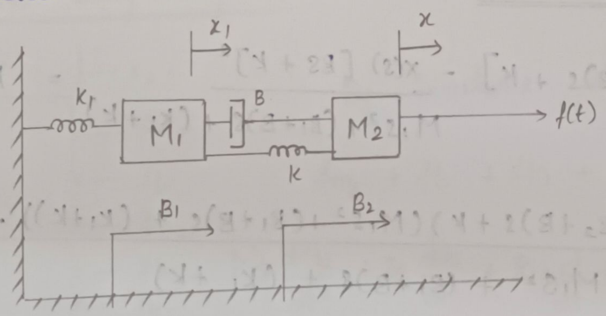
Spring / elasticity → K

Mathematical Models of Control system.

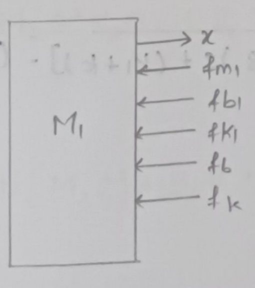
1) Mechanical translation / linear System

2) Mechanical Rotational system.

1) Write the differential equation for the Mechanical system shown below and determine the transfer function.



M1 ⇒ Free body diagram



By Newton's Second law,

$$f_{m_1} + f_{b_1} + f_{k_1} + f_b + f_k = 0$$

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 x_1 + B\frac{d}{dt}(x_1 - x) + k(x_1 - x) = 0 \quad \text{——— (1)}$$

Taking Laplace transform for eqn ①,

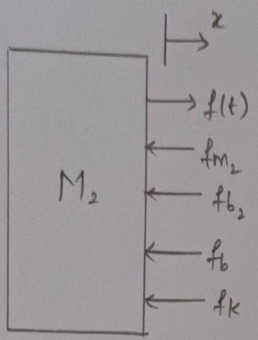
$$\overbrace{BS[X_1(S) - X(S)]}\qquad \overbrace{K[X_1(S) - X(S)]}$$

$$M_1 S^2 X_1(S) + B_1 S X_1(S) + K_1 X_1(S) + BS X_1(S) - BS X(S) + K X_1(S) - K X(S) = 0$$

$$X_1(S)[M_1 S^2 + (B_1 + B)S + (K_1 + K)] = X(S)[BS + K]$$

$$X_1(S) = \frac{X(S)[BS + K]}{M_1 S^2 + (B_1 + B)S + (K_1 + K)} \quad \text{——} ②$$

M_2 => Free body diagram



By Newton's Second law,

$$f_{m_2} + f_{b_2} + f_b + f_k = f(t)$$

$$M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1) = f(t) \quad \text{——} ③$$

Taking laplace transform for eqn ③

$$M_2 S^2 X(S) + B_2 S X(S) + BS[X(S) - X_1(S)] + K[X(S) - X_1(S)] = F(S)$$

$$X(S)[M_2 S^2 + (B_2 + B)S + K] - X_1(S)[BS + K] = F(S) \quad \text{——} ④$$

Substitute eqn ② in ④

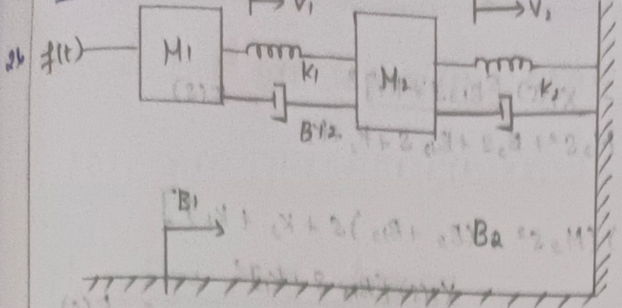
$$X(S)[M_2 S^2 + (B_2 + B)S + K] - \frac{X(S)[BS + K]^2}{M_1 S^2 + (B_1 + B)S + (K_1 + K)} = F(S)$$

$$X(S)\frac{[(M_2 S^2 + (B_2 + B)S + K)(M_1 S^2 + (B_1 + B)S + (K_1 + K)) - (BS + K)^2]}{M_1 S^2 + (B_1 + B)S + (K_1 + K)} = F(S)$$

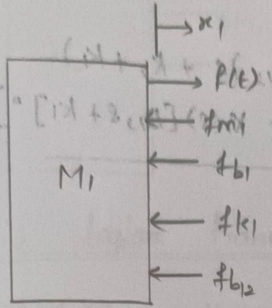
Transfer function;

$$\frac{X(S)}{F(S)} = \frac{M_1 S^2 + (B_1 + B)S + (K_1 + K)}{[(M_2 S^2 + (B_2 + B)S + K)(M_1 S^2 + (B_1 + B)S + (K_1 + K))] - (BS + K)^2}$$

H/w:



$M_1 \Rightarrow$ Free body diagram:



By Newton's 2nd law,

$$f_{m_1} + f_{b_1} + f_{k_1} + f_{b_{12}} = f(t)$$

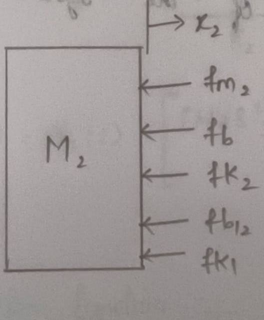
$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 (x_1 - x_2) + B_{12} \frac{d}{dt}(x_1 - x_2) = f(t) \quad \text{——} ①$$

Taking Laplace transform for eqn ①,

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + k_1 X_1(s) + B_{12} s X_1(s) - B_{12} s X_2(s) - k_1 X_2(s) = f(t)$$

$$X_1(s)\left[M_1 s^2 + B_1 s + k_1 + B_{12} s\right] - X_2(s)\left[B_{12} s + k_1\right] = F(s) \quad \text{——} ②$$

$M_2 \Rightarrow$ Free body diagram



By Newton's 2nd law,

$$f_{m_2} + f_b + f_{k_1} + f_{b_{12}} + f_{K_2} = 0$$

$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2 + K_1(x_2 - x_1) = 0 \quad \text{——} ③$$

Taking Laplace Transform for eqn ③

$$M_2 s^2 X_2(s) + B_2 s X_2(s) + B_{12} s X_2(s) - B_{12} s X_1(s) + k_2 X_2(s) + k_1 X_2(s) - k_1 X_1(s) = 0$$

$$X_2(s)\left[M_2 s^2 + B_2 s + B_{12} s + k_2 + k_1\right] - X_1(s)\left[B_{12} s + k_1\right] = 0$$

$$X_2(s) = \frac{X_1(s)\left[B_{12} s + k_1\right]}{M_2 s^2 + B_2 s + B_{12} s + k_2 + k_1} \quad \text{——} ④$$

Substitute (1) in (2),

$$X_1(s)\left[M_1 s^2 + B_1 s + K_1 + B_{12} s\right] - \frac{X_1(s)\left[B_{12} s + K_1\right]^2}{M_2 s^2 + B_2 s + B_{12} s + K_2 + K_1} = F(s)$$

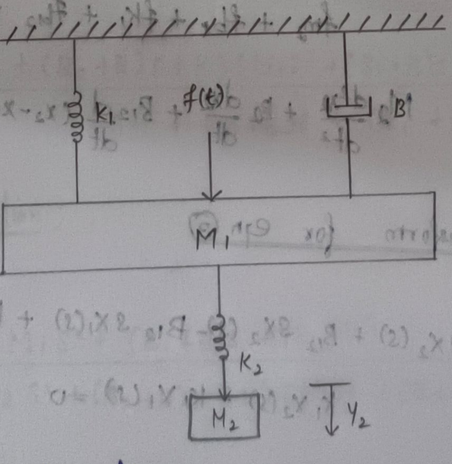
$$\frac{X_1(s)\left[(M_1 s^2 + (B_1 + B_{12})s + k_1)(M_2 s^2 + (B_2 + B_{12})s + K_2 + k_1)\right] - X_1(s)\left[B_{12} s + k_1\right]^2}{M_2 s^2 + (B_2 + B_{12})s + k_2 + k_1} = F(s)$$

$$\frac{X_1(s)\left[(M_1 s^2 + (B_1 + B_{12})s + k_1)(M_2 s^2 + (B_2 + B_{12})s + k_2 + k_1) - X_1(s)\left[B_{12} s + k_1\right]^2\right]}{M_2 s^2 + (B_2 + B_{12})s + K_2 + K_1} = F(s)$$

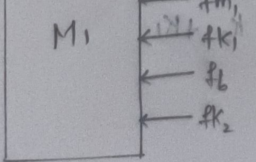
Transfer Function:-

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + (B_2 + B_{12})s + K_2 + K_1}{X_1(s)\left[(M_1 s^2 + (B_1 + B_{12})s + k_1)(M_2 s^2 + (B_2 + B_{12})s + K_2 + K_1) - X_1(s)\left[B_{12} s + K_1\right]^2\right]}$$

3. Determine the transfer function $\frac{Y_2(s)}{F(s)}$ of the system shown in the figure.



$M_1 \Rightarrow$ Free body diagram.



By Newton's 2nd law,

$$f(t) = f_{m_1} + f_{k_1} + f_b + f_{k_2}$$

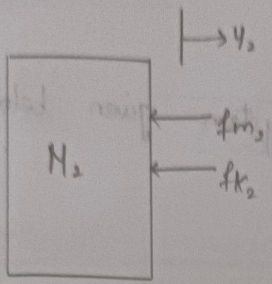
$$f(t) = M_1 \frac{d^2 y_1}{dt^2} + K_1 y_1 + B\frac{dy_1}{dt} + K_2(y_1 - y_2) \quad \text{— (1)}$$

Taking laplace transform for eqn ①,

$$F(s) = M_1 s^2 Y_1(s) + K_1 Y_1(s) + BsY_1(s) + K_2 Y_1(s) - K_2 Y_2(s)$$

$$F(s) = Y_1(s)\left[M_1 s^2 + Bs + (K_1 + K_2)\right] - Y_2(s) K_2 \quad \text{—— ②}$$

M_2 → Free body diagram



By Newton's 2nd law,

$$f_{m_2} + f_{k_2} = 0$$

$$M_2 \frac{d^2 y_2}{dt^2} + K_2 (y_2 - y_1) = 0 \quad \text{—— ③}$$

Taking laplace transform for eqn ③

$$M_2 S^2 Y_2(s) + K_2 Y_2(s) - K_2 Y_1(s) = 0$$

$$Y_2(s)\left[M_2 S^2 + K_2\right] = K_2 Y_1(s)$$

$$Y_1(s) = \frac{Y_2(s)\left[M_2 s^2 + K_2\right]}{K_2} \quad \text{—— ④}$$

Substitute ④ in ②,

$$F(s) = Y_2(s) \frac{\left[M_2 S^2 + K_2\right]}{K_2} \left[M_1 S^2 + Bs + (K_1 + K_2)\right] - Y_2(s) K_2$$

$$F(s) = Y_2(s) \left[\frac{(M_2 S^2 + K_2)(M_1 S^2 + Bs + (K_1 + K_2)) - K_2^2}{K_2}\right]$$

Transfer function :-

$$\frac{Y_2(s)}{F(s)} = \frac{K_2}{\left[(M_2 S^2 + K_2)(M_1 S^2 + Bs + (K_1 + K_2))\right] - K_2^2}$$

Mechanical Rotational System :-

$M \Rightarrow J$ (Moment of inertia)

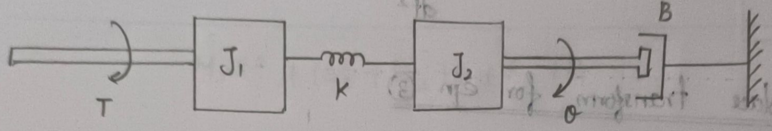
$B \Rightarrow B$

$K \Rightarrow K$

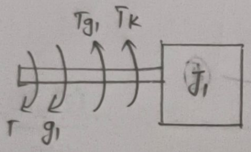
$F \Rightarrow$ Torque T

$x \Rightarrow \theta$ angular displacement.

1) Write differential equations for the system given below. Obtain Transfer function.



Free body diagram of J_1 :-



$$T = T_{J_1} + T_K$$

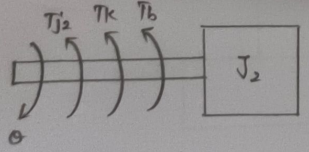
$$T = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta)$$

Taking Laplace Transform:

$$T(s) = J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s)$$

$$T(s) = \theta_1(s)\left[J_1 s^2 + K\right] - K\theta(s) \quad \text{—— (1)}$$

Free body diagram of J_2 :-



$$0 = J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1)$$

Taking Laplace transform:

$$0 = J_2 s^2 \theta(s) + Bs\,\theta(s) + K\theta(s) - K\theta_1(s)$$

$$\theta(s)\left[J_2 s^2 + Bs + K\right] = K\theta_1(s)$$

$$\theta_1(s) = \theta(s)\frac{\left[J_2 + Bs + K\right]}{K} \quad \text{—— (2)}$$

Subtitute ② in ①,

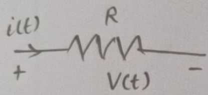
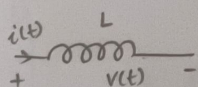
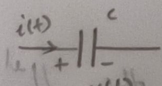
$$T(s) = \theta(s)\,\frac{[J_2 + Bs + k]}{k}\,(J_1 s^2 + k) - k\theta(s)$$

$$= \theta(s)\,\frac{[(J_2 s^2 + Bs + k)(J_1 s^2 + k) - k^2]}{k}$$

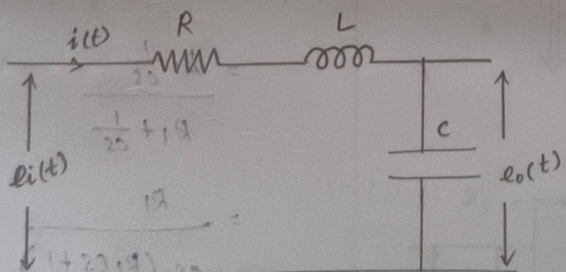
Transfer function :-

$$\frac{\theta(s)}{T(s)} = \frac{k}{[(J_2 s^2 + Bs + k)(J_1 s^2 + k)] - k^2}$$

Mathematical modeling of Electrical system

Element	Voltage across the element	Current through the element
	$V(t) = R\,i(t)$	$i(t) = \frac{V(t)}{R}$
	$V(t) = L\,\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int v(t)\,dt$
	$V(t) = \frac{1}{C}\int i(t)\,dt$	$i(t) = C\,\frac{dv(t)}{dt}$

Derive Transfer function $\frac{E_o(s)}{E_i(s)}$ for the given Electrical system.



Apply KVL at I/P side

$$e_i(t) = R\,i(t) + L\,\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

Taking L.T

$$E_i(s) = R\,I(s) + LS\,I(s) + \frac{1}{CS} I(s)$$

$$= I(s)\left[R + LS + \frac{1}{CS}\right] \quad \text{—— ①}$$

Apply KVL for o/p side,

$$e_0(t) = \frac{1}{c}\int i(t)\,dt$$

Apply L.T $\Rightarrow$ $E_0(s) = \frac{1}{CS} I(s)$

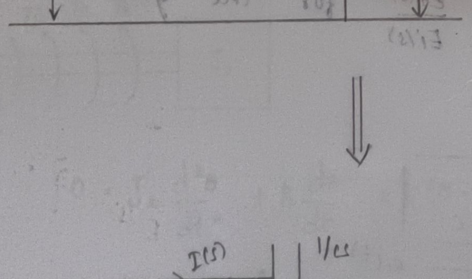
Transfer function:

$$\frac{E_0(s)}{E_i(s)} = \frac{\frac{1}{CS}\cancel{I(s)}}{\cancel{I(s)}\left[R+LS+\frac{1}{CS}\right]} = \frac{1}{CS\left[R+LS+\frac{1}{CS}\right]}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{\left[\frac{RCS + LCS^2 + 1}{\cancel{CS}}\right]\cancel{CS}}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{RCS + LCS^2 + 1}$$

2)



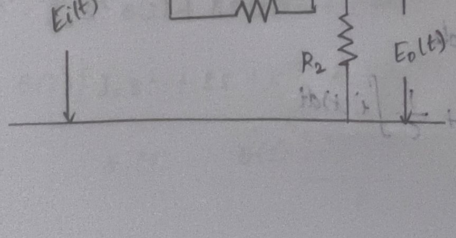
Assuming the $\|^{el}$ combinations as Z_1,

$$Z_1 = R_1 \,\|^{l}\, \frac{1}{CS}$$

$$= \frac{R_1 \frac{1}{CS}}{R_1 + \frac{1}{CS}}$$

$$= \frac{R_1}{CS\left(\frac{R_1CS+1}{CS}\right)}$$

$$\boxed{Z_1 = \frac{R_1}{R_1CS+1}}$$



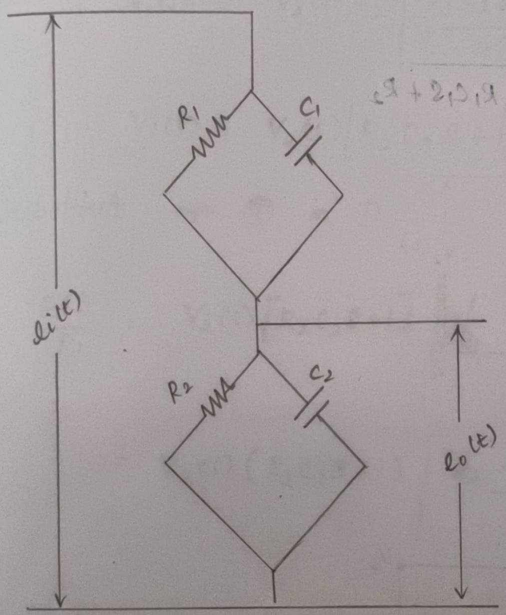
By Applying voltage division rule,

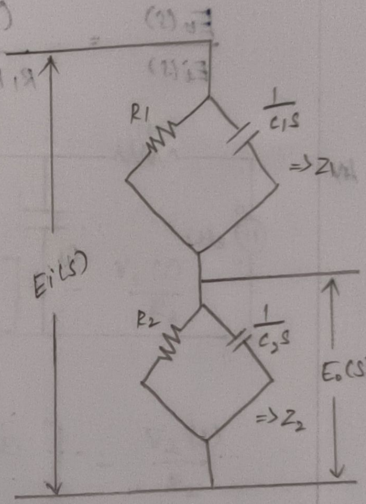
$$E_o(s) = \frac{E_i(s)\, R_2}{R_2 + Z_1}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2}{R_2 + \frac{R_1}{R_1Cs+1}}$$

$$= \frac{R_2}{\frac{R_1R_2Cs + R_2 + R_1}{R_1Cs+1}}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2R_1Cs + R_2}{R_1R_2Cs + R_2 + R_1}$$





$$Z_1 = R_1 \,||^l\, \frac{1}{C_1 s}$$

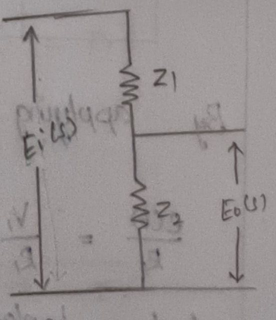
$$= \frac{R_1 \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}}$$

$$Z_1 = \frac{R_1}{R_1C_1s+1}$$

$$Z_2 = R_2 \,||^l\, \frac{1}{C_2 s}$$

$$= \frac{R_2 \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}}$$

$$Z_2 = \frac{R_2}{R_2C_2s+1}$$



By Applying Voltage division rule,

$$E_0(s) = \frac{E_i(s)\, Z_2}{Z_1 + Z_2}$$

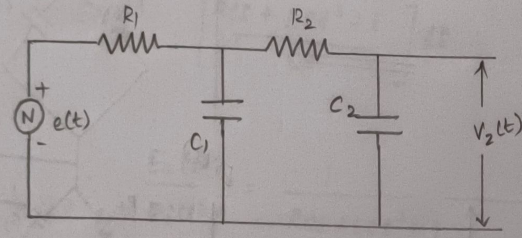
Transfer function,

$$\frac{E_0(s)}{E_i(s)} = \frac{R_2 / R_2 C_2 s + 1}{\frac{R_1}{R_1 C_1 s + 1} + \frac{R_2}{R_2 C_2 s + 1}}$$

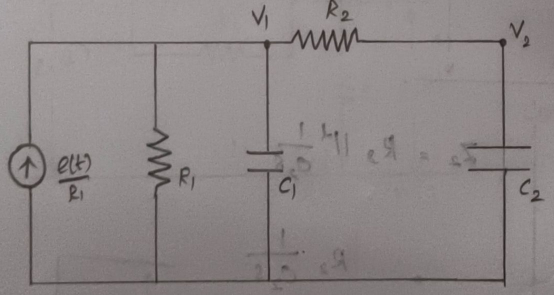
$$= \frac{R_2}{R_2 C_2 s + 1 \left[\frac{R_1 R_2 C_2 s + R_1 + R_2 R_1 C_1 s + R_2}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)} \right]}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{(R_1 C_1 s + 1) R_2}{R_1 R_2 C_2 S + R_1 + R_2 R_1 C_1 S + R_2}$$

4)



$\Downarrow$



By applying Kirchoff's Current rule for Node 1.

$$\frac{e(t)}{R_1} = \frac{V_1(t)}{R_1} + C_1 \frac{dv_1(t)}{dt} + \frac{V_1(t) - V_2(t)}{R_2}$$

Apply laplace transform,

$$\frac{E(s)}{R_1} = \frac{V_1(s)}{R_1} + C_1 S V_1(s) + \frac{V_1(s)}{R_2} - \frac{V_2(s)}{R_2}$$

$$= V_1(s) \left[\frac{1}{R_1} + C_1 s + \frac{1}{R_2} \right] - \frac{V_2(s)}{R_2} \quad \text{——— (1)}$$

KCL for Node 2,

$$0 = \frac{V_2(t) - V_1(t)}{R_2} + C_2 \frac{dV_2(t)}{dt}$$

Taking L.T;

$$0 = \frac{V_2(S)}{R_2} - \frac{V_1(S)}{R_2} + C_2 S V_2(S)$$

$$\frac{V_1(S)}{R_2} = V_2(S)\left[\frac{1}{R_2} + C_2 S\right] = V_2(S)\left[\frac{R_2 C_2 S + 1}{R_2}\right]$$

$$\frac{V_1(S)}{R_2} = V_2(S)\left[\frac{R_2 C_2 S + 1}{R_2}\right]$$

$$V_1(S) = V_2(S) \cancel{R_2} \left[\frac{R_2 C_2 S + 1}{\cancel{R_2}}\right]$$

$$V_1(S) = V_2(S)\left[R_2 C_2 S + 1\right] \quad \text{—— ②}$$

Substitute eqn ② in ①,

$$\frac{E(S)}{R_1} = V_2(S)\left[R_2 C_2 S + 1\right]\left[\frac{1}{R_1} + C_1 S + \frac{1}{R_2}\right] - \frac{V_2(S)}{R_2}$$

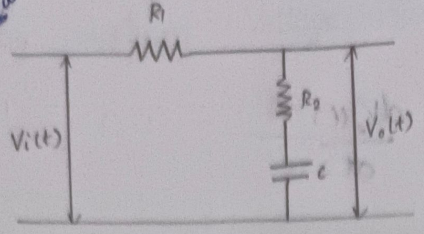
$$= V_2(S)(R_2 C_2 S + 1)\left[\frac{R_2 + R_1 R_2 C_1 S + R_1}{R_1 R_2}\right] - \frac{V_2(S)}{R_2}$$

$$= V_2(S)\left[\frac{(R_2 C_2 S + 1)(R_1 + R_2 + R_1 R_2 C_1 S)}{R_1 R_2} - \frac{1}{R_2}\right]$$

$$\frac{E(S)}{\cancel{R_1}} = V_2(S)\left[\frac{(R_2 C_2 S + 1)(R_1 + R_2 + R_1 R_2 C_1 S) - R_1}{\cancel{R_1} R_2}\right]$$

$$\therefore \frac{V_2(S)}{E(S)} = \frac{R_2}{(R_2 C_2 S + 1)(R_1 + R_2 + R_1 R_2 C_1 S) - R_1}$$

5) H/w



Applying KVL at i/p side:

$$e_i(t) = R_1 i(t) + R_2 i(t) + \frac{1}{C}\int i(t)\,dt$$

applying laplace:

$$E_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{Cs} I(s)$$

$$E_i(s) = I(s)\left[R_1 + R_2 + \frac{1}{Cs}\right] \quad \text{——— ①}$$

Applying KVL at o/p side:

$$e_o(t) = R_2 i(t) + \frac{1}{Cs}\int i(t)\,dt$$

applying laplace:

$$E_o(s) = I(s)\left[R_2 + \frac{1}{Cs}\right] \quad \text{——— ②}$$

<u>Transfer function</u>:-

$$\frac{E_o(s)}{E_i(s)} = \frac{\cancel{I(s)}\left[R_2 + \frac{1}{Cs}\right]}{\cancel{I(s)}\left[R_1 + R_2 + \frac{1}{Cs}\right]}$$

$$= \frac{R_2Cs + 1/\cancel{Cs}}{R_1Cs + R_2Cs + 1/\cancel{Cs}}$$

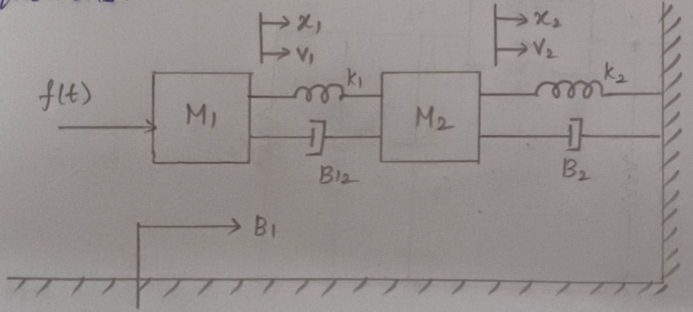
$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{R_2Cs + 1}{R_1Cs + R_2Cs + 1}$$

Electrical analogous of Mechanical translational system:

Mechanical System	Electrical System	
	Force - Voltage	Force - current
1) I/P → Force f	Voltage (e)	Current (i)
2) O/P → displacement (x) Velocity (v)	current (i)	Voltage (v)
3) Mass (M) $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$	Inductance (L) $V = L\frac{di}{dt}$	Capacitance (c) $i = c\frac{dv}{dt}$
4) Friction (B) $f = B\frac{dx}{dt} = Bv$	Resistance (R) $V = Ri$	Resistance (1/R) $i = \frac{V}{R}$
5) Spring (K) $f = kx = k\int v\,dt$	capacitance (1/c) $V = \frac{1}{c}\int i\,dt$	Inductance (1/L) $i = \frac{1}{L}\int v\,dt$

Write differential equations governing mechanical systems shown in the figure.

Draw force voltage, force current electrical analogous circuits and verify by writing mesh and node equations.



$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B_{12}\frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = f(t)$$

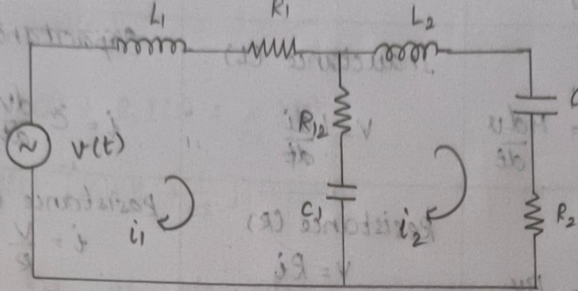
$$M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + k_2x_2 + B_{12}\frac{d}{dt}(x_2 - x_1) + k_1(x_2 - x_1) = 0$$

Write the above equations in terms of velocity.

$$M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + k\int(v_1 - v_2)\,dt = f(t)$$

$$M_2 \frac{dv_2}{dt} + B_2 v_2 + k_2\int v_2\,dt + B_{12}(v_2 - v_1) + k_1\int(v_2 - v_1) = 0$$

Force - Voltage analogous

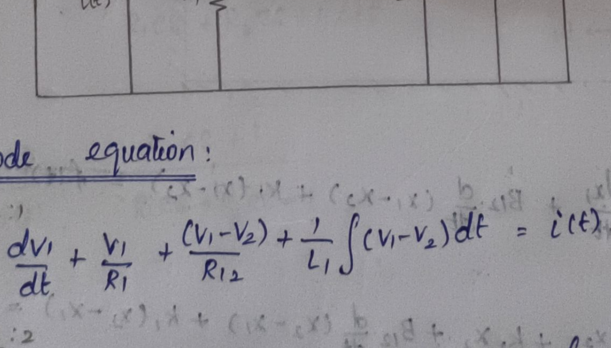


Mesh equation:

$$L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int(i_1 - i_2)\,dt = V(t)$$

$$L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt + R_2 i_2 + R_{12}(i_2 - i_1) + \frac{1}{C_1}\int(i_2 - i_1)\,dt = 0$$

Force - Current analogous



Node equation:

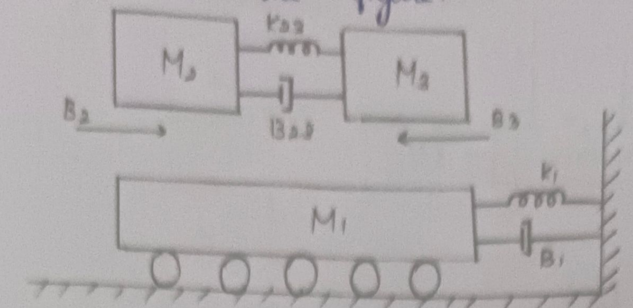
Node : 1

$$C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{(V_1 - V_2)}{R_{12}} + \frac{1}{L_1}\int(V_1 - V_2)\,dt = i(t)$$

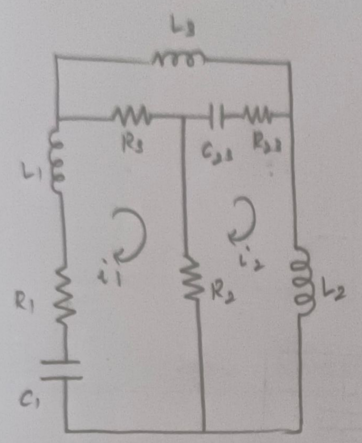
Node : 2

$$C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\,dt + \frac{V_2}{R_2} + \frac{V_2 - V_1}{R_{12}} + \frac{1}{L_1}\int(V_2 - V_1)\,dt = 0$$

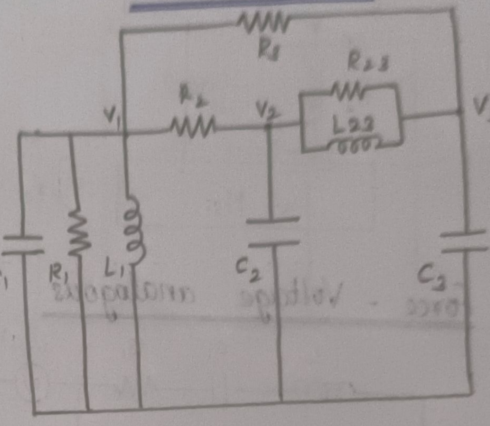
1) Draw an electrical analogous for the Mechanical system shown in the figure.



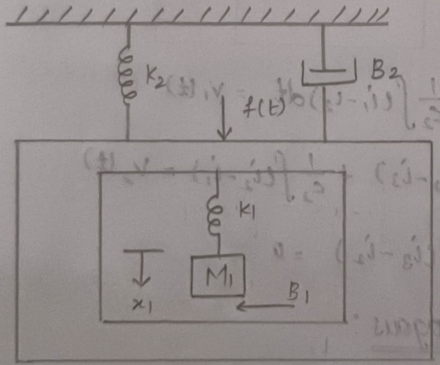
Force - voltage



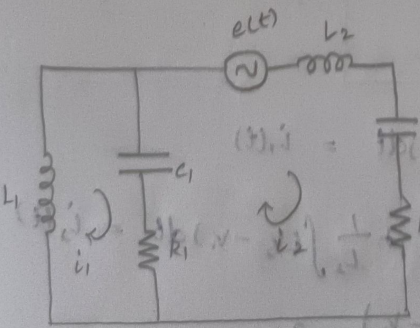
Force - current



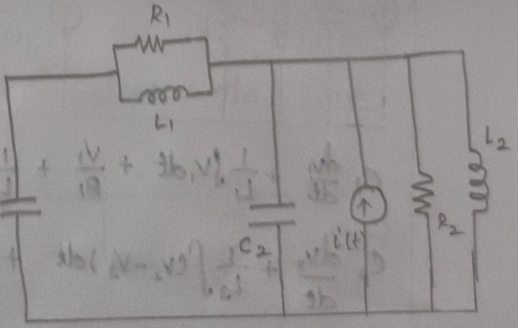
2)



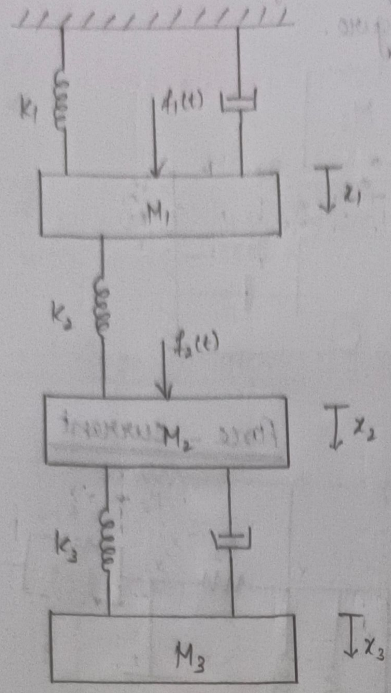
Force - voltage



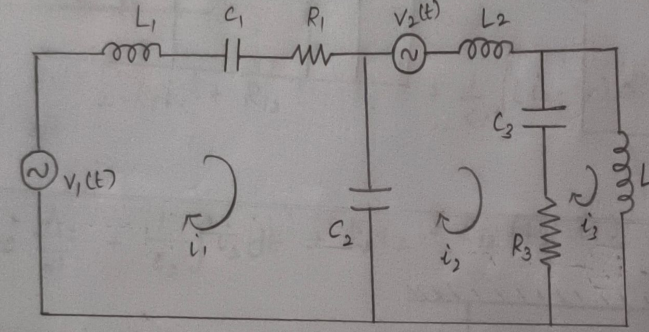
Force - Current



3)



Force - Voltage analogous

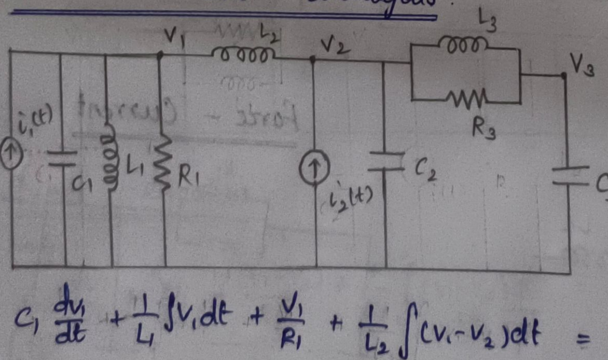


$$L_1\frac{di_1}{dt} + \frac{1}{C_1}\int i_1 dt + R i_1 + \frac{1}{C_2}\int (i_1-i_2)dt = V_1(t)$$

$$L_2\frac{di_2}{dt} + \frac{1}{C_3}\int (i_2-i_3) + R_3(i_2-i_3) + \frac{1}{C_2}\int (i_2-i_1) = V_2(t)$$

$$L_3\frac{di_3}{dt} + R_3(i_3-i_2) + \frac{1}{C_3}\int (i_3-i_2) = 0$$

Force - current analogous:



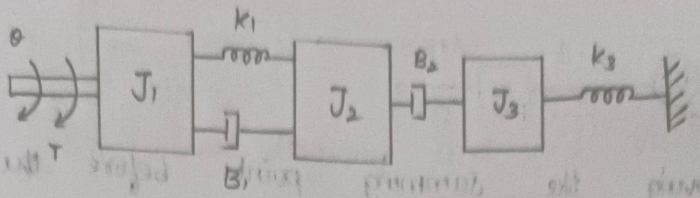
$$C_1\frac{dv_1}{dt} + \frac{1}{L_1}\int v_1 dt + \frac{V_1}{R_1} + \frac{1}{L_2}\int (V_1-V_2)dt = i_1(t)$$

$$C_2\frac{dv_2}{dt} + \frac{1}{L_3}\int (V_2-V_3)dt + \frac{(V_2-V_3)}{R_3} + \frac{1}{L_2}\int (V_2-V_1)dt = i_2(t)$$

$$C_3\frac{dv_3}{dt} + \frac{1}{L_3}\int (V_3-V_2)dt + \frac{(V_3-V_2)}{R_3} = 0$$

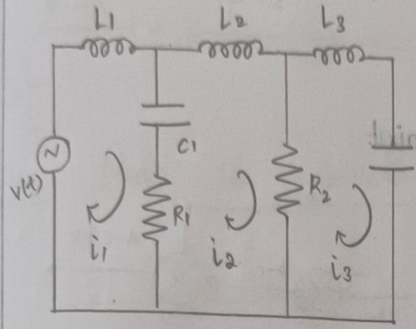
H/W

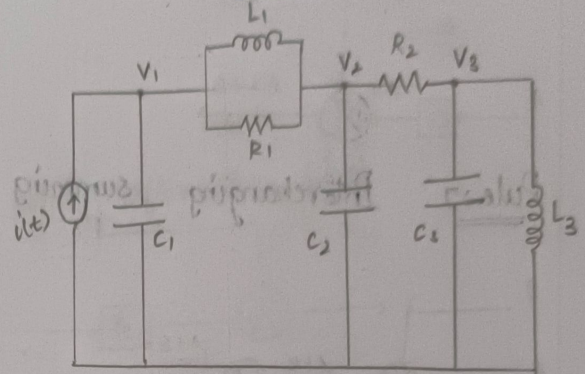
47 Draw force - current, force - voltage analogous circuit for Mechanical rotation system shown in the figure.



Force - voltage

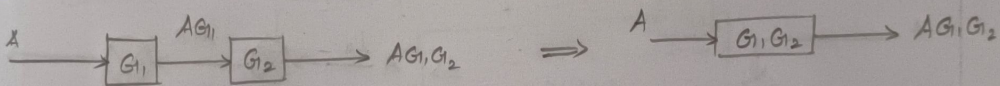
Force - Current



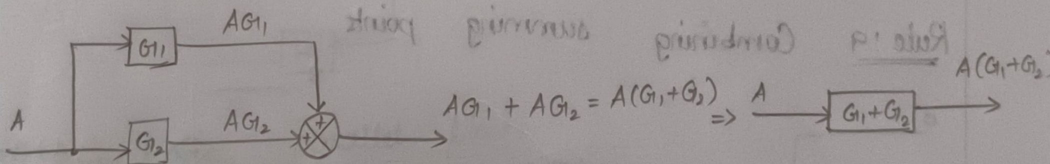


Block Diagram Reduction:

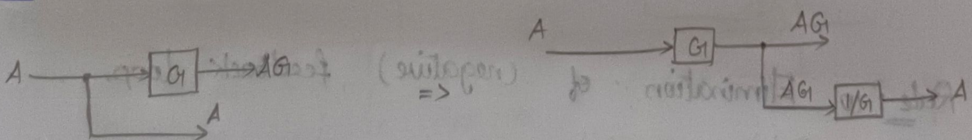
Rule: 1 Combining the blocks in Cascade.



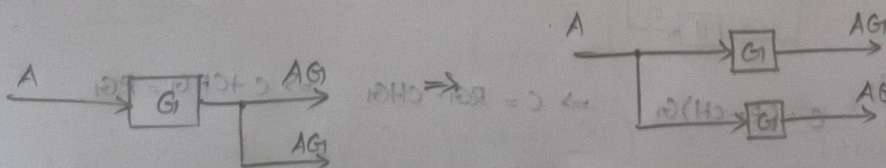
Rule: 2 Combining parallel blocks (or combining feed forward paths)



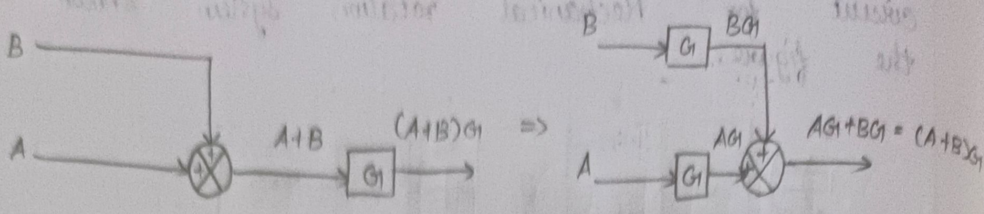
Rule: 3 Moving the branch point ahead of the block.



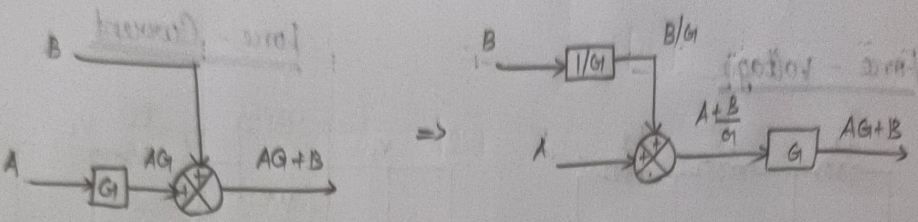
Rule: 4 Moving the branch point before the block.



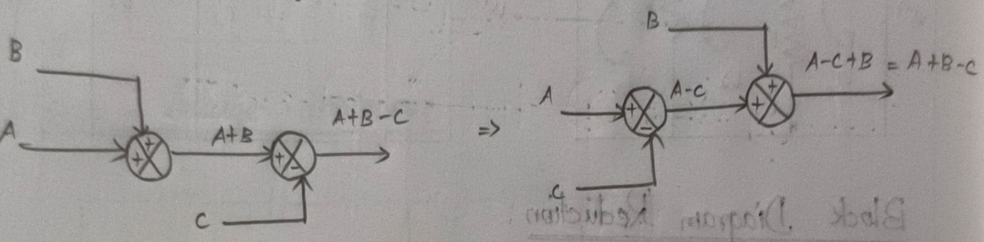
Rule:5 Moving the summing point ahead of the block



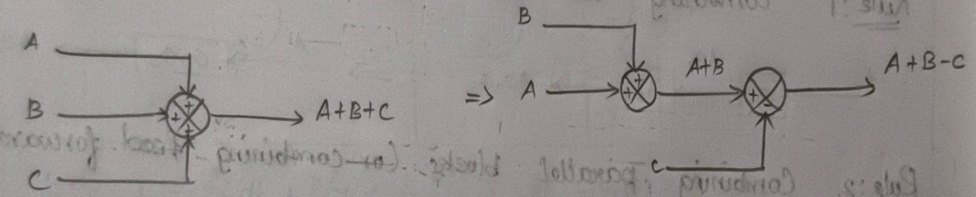
Rule:6 Moving the summing point before the block



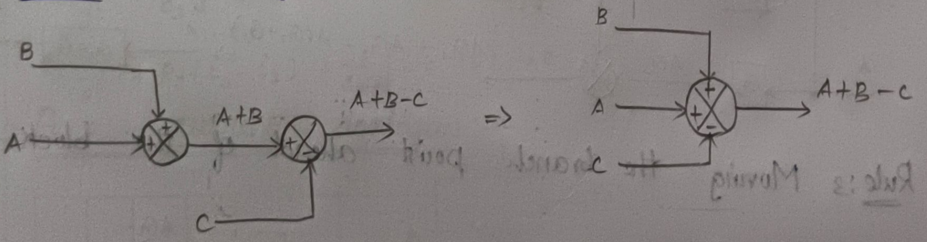
Rule:7 Interchanging summing point



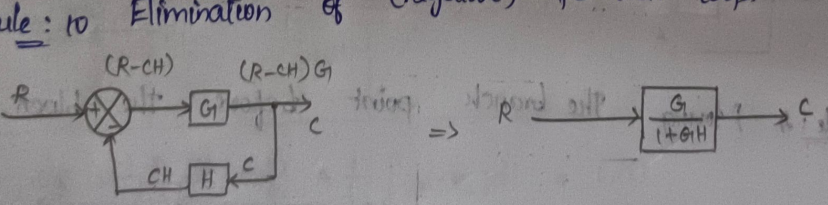
Rule:8 Splitting summing points



Rule:9 Combining summing points



Rule: 10 Elimination of (negative) feedback loop.

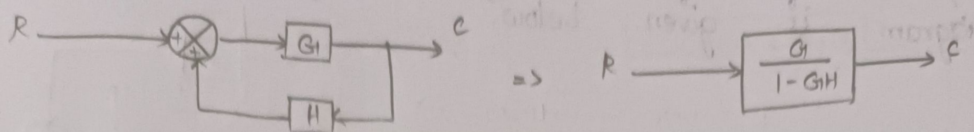


Proof: $C = (R - CH)G$ => $C = RG - CHG$ => $C + CHG = RG$

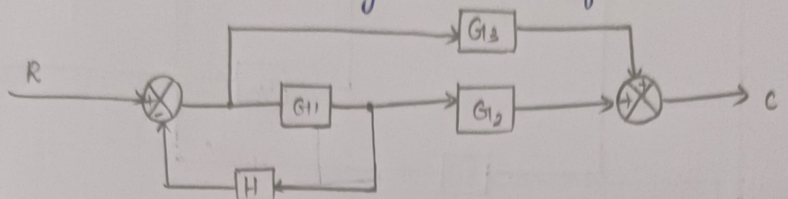
$\therefore \; C(1 + HG) = RG$

$$\frac{C}{R} = \frac{G}{1 + HG} \Rightarrow \frac{C}{R} = \frac{G}{1 + GH}$$

Rule : 11. Elimination of (positive) feedback loop.

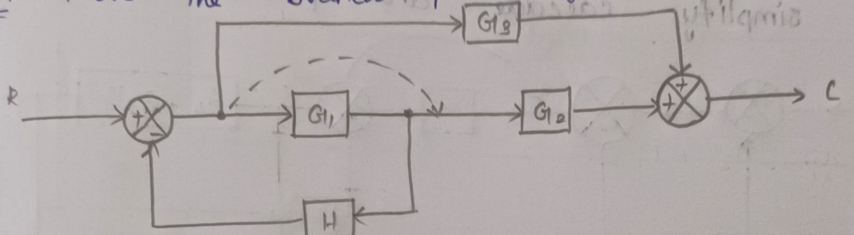


Eg: 1. Reduce the block diagram and find C/R.

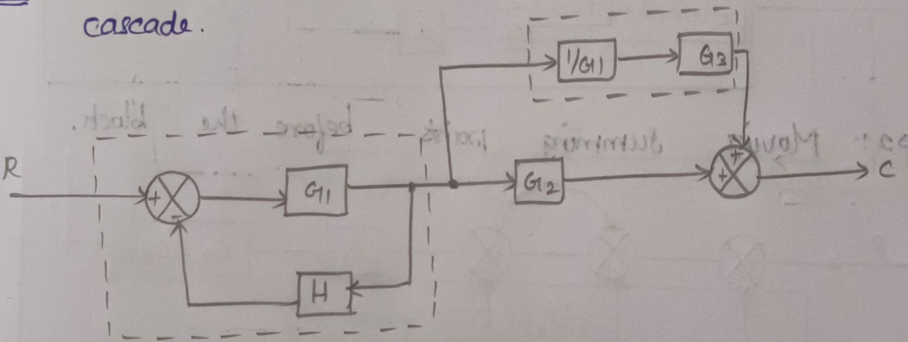


Soln:

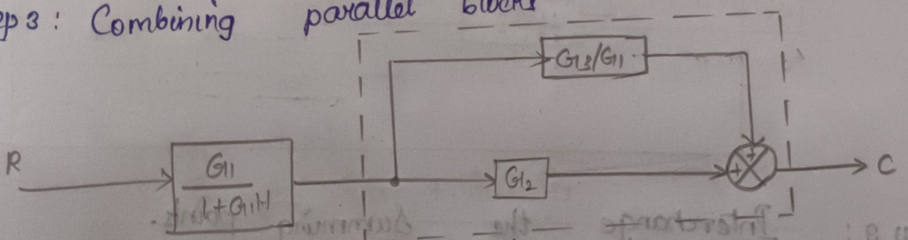
Step 1: Move the branch point after the block.



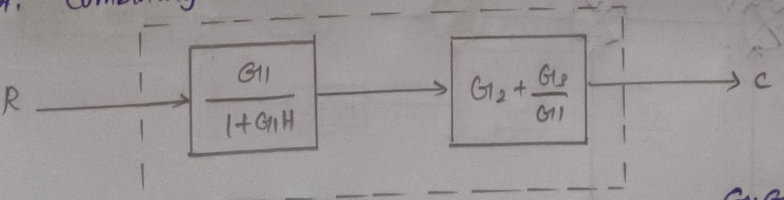
Step 2: Eliminate the feedback path and combining blocks in cascade.



Step 3: Combining parallel blocks



Step 4: Combining block in cascade.



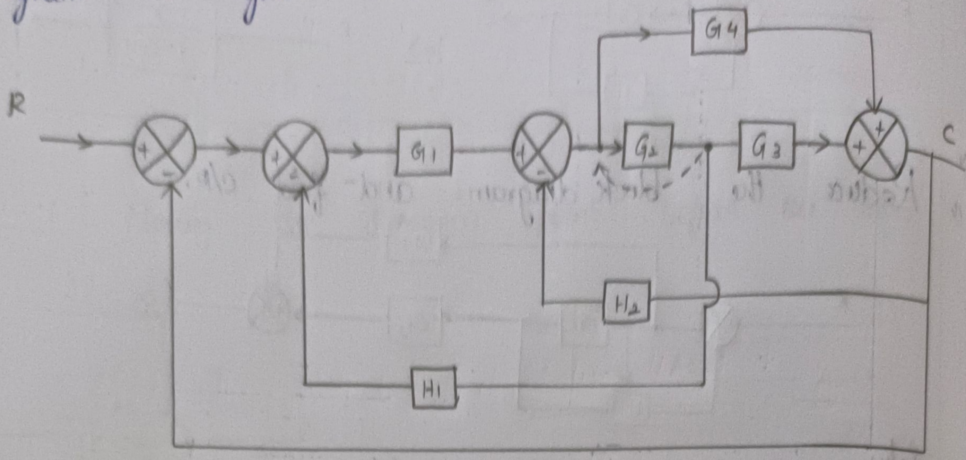
$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right) = \left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right) = \frac{G_1G_2+G_3}{1+G_1H}$$

RESULT:

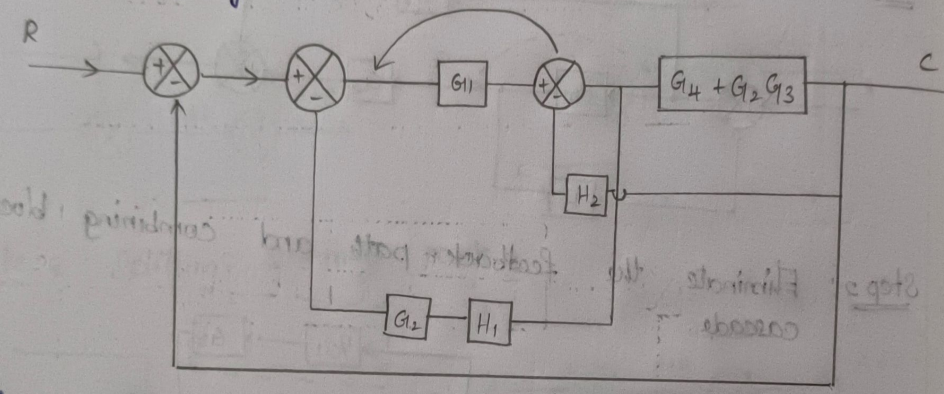
The overall transfer function of the system,

$$\frac{C}{R} = \frac{G_1G_2+G_3}{1+G_1H}$$

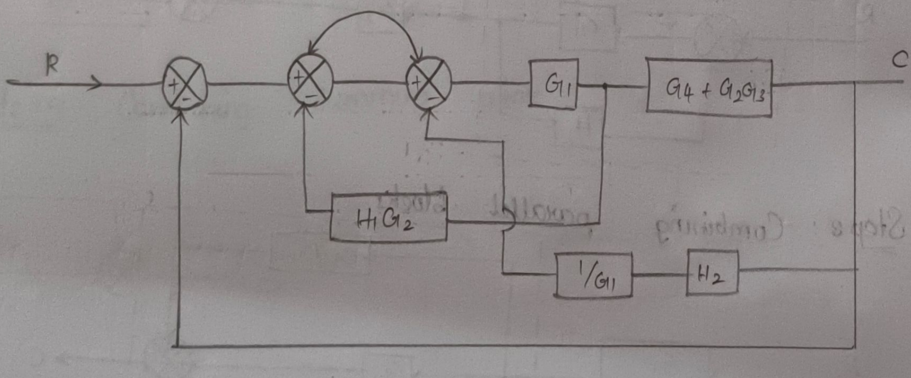
Eg: 1.1 Using Block diagram reduction technique find close loop transfer function of the system. Whose block diagram is given below.



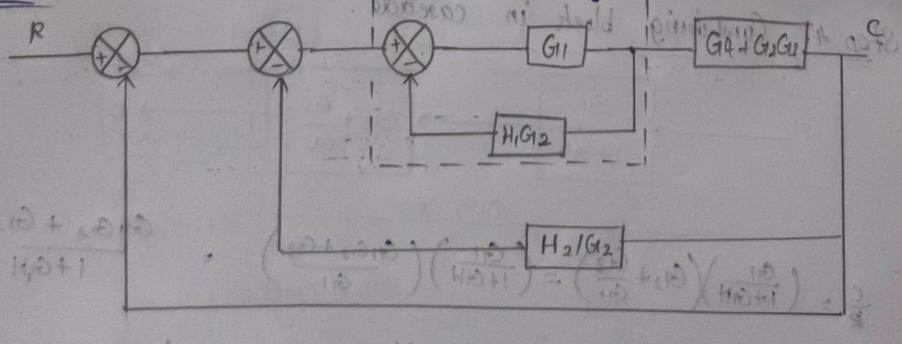
Step 1: Moving the branch before the block and simplify cascade and parallel blocks.



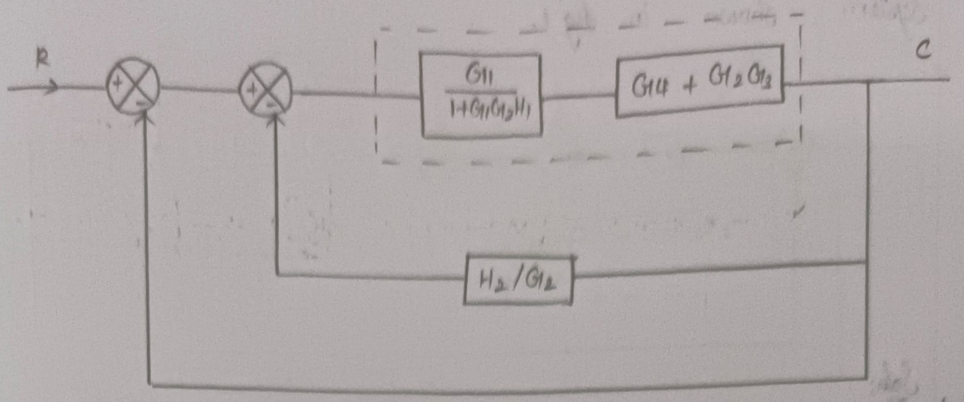
Step 2: Moving summing points before the block.



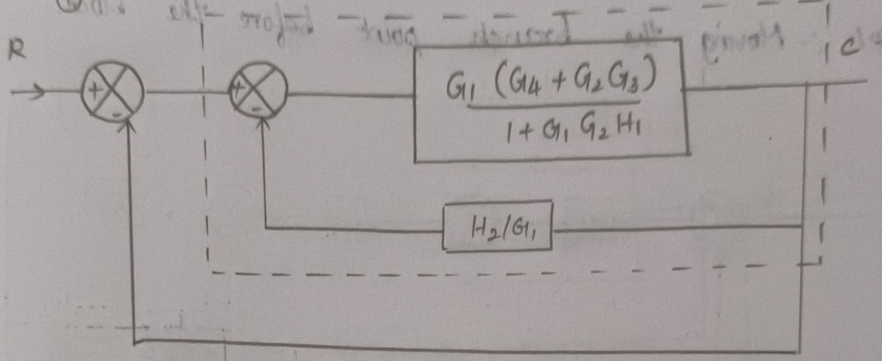
Step 3: Interchange the Summing points.



Step 4: Simplify feedback paths.

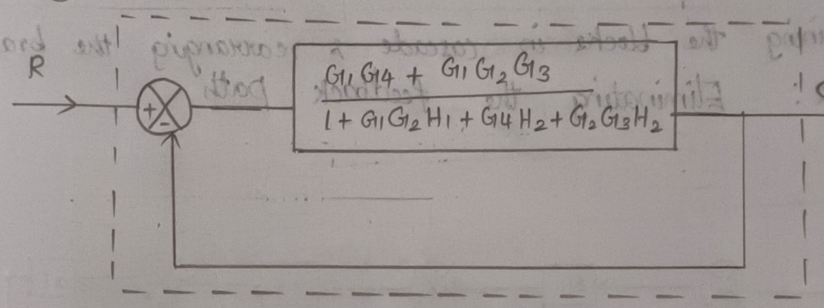


①

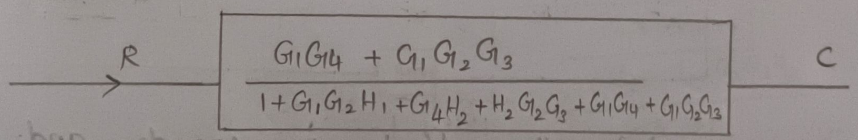


1.18
1.19
1.21
1.22
1.23

②



③



∴ Transfer function

$$C/R = \frac{G_1G_4 + G_1G_2G_3}{1+G_1G_2H_1+G_4H_2+H_2G_2G_3+G_1G_4+G_1G_2G_3}$$

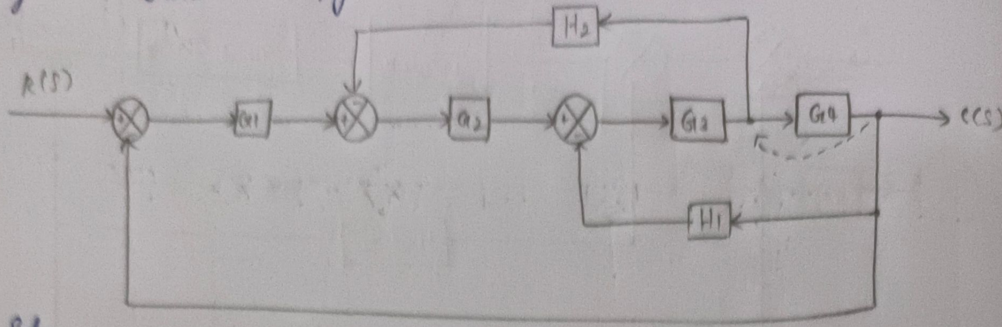
① $\frac{G_1(G_4+G_2G_3)}{1+G_1G_2H_1}$

② $$\frac{\frac{G_1(G_4+G_2G_3)}{1+G_1G_2H_1}}{1+\left(\frac{G_1(G_4+G_2G_3)}{1+G_1G_2H_1}\right)\frac{H_2}{G_1}} = \frac{G_1(G_4+G_2G_3)}{(1+G_1G_2H_1)+H_2(G_4+G_2G_3)}$$

$$= \frac{G_1G_4+G_1G_2G_3}{1+G_1G_2H_1+G_4H_2+G_2H_2G_3H_2}$$

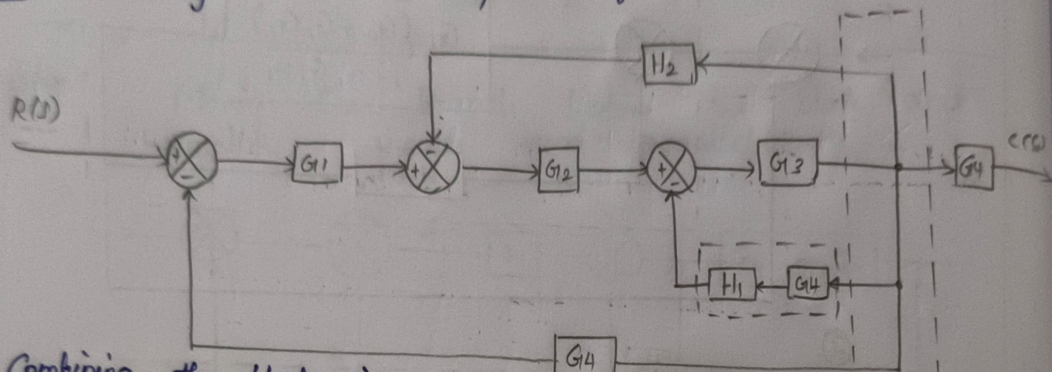
③ $$\frac{\frac{G_1G_4+G_1G_2G_3}{1+G_1G_2H_1+G_4H_2+H_2G_2G_3}}{1+\frac{G_1G_4+G_1G_2G_3}{1+G_1G_2H_1+G_4H_2+H_2G_2G_3}} = \frac{G_1G_4+G_1G_2G_3}{1+G_1G_2H_1+G_4H_2+H_2G_2G_3+G_1G_4+G_1G_2G_3}$$

Eg: 1.18 Determine the overall transfer function $\frac{C(s)}{R(s)}$ for the system shown in fig 1.



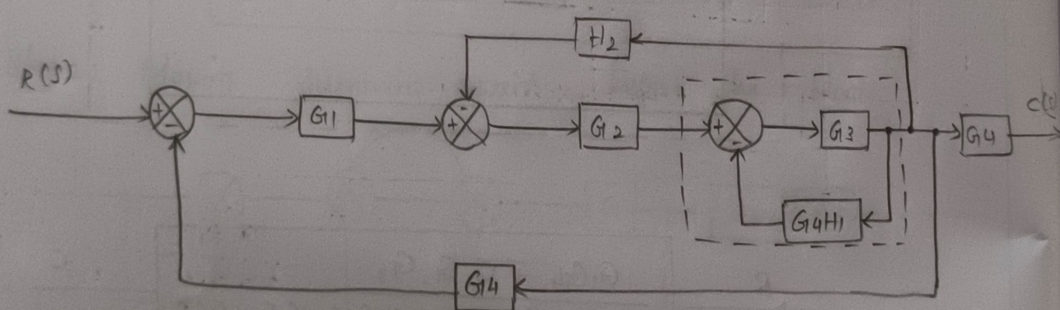
Soln:

Step 1 :- Moving the branch point before the block.

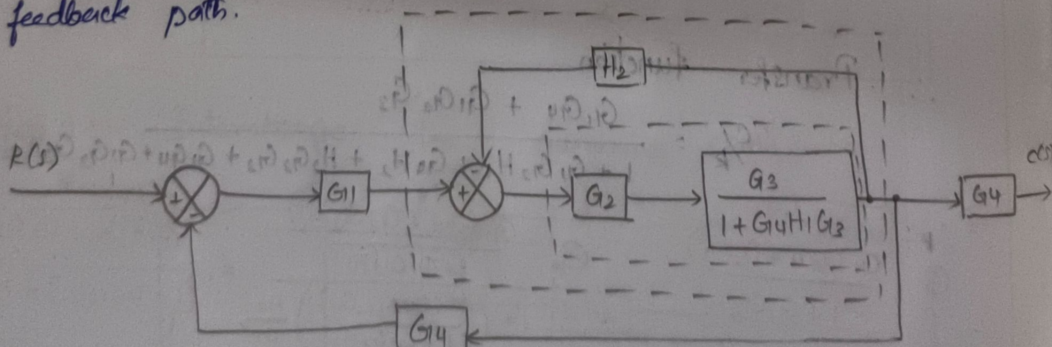


Combining the blocks in cascade & rearranging the branch point

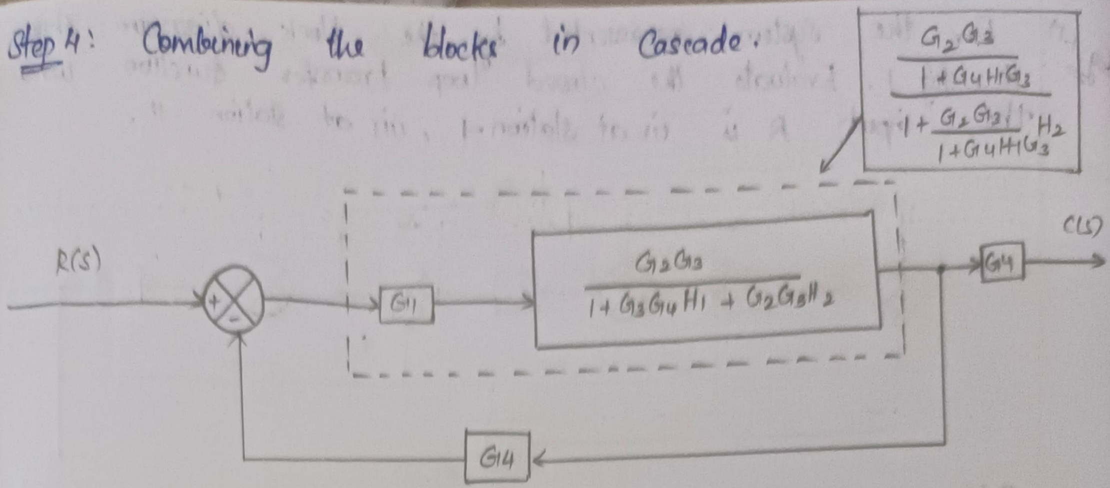
Step 2 : Eliminating the feedback path



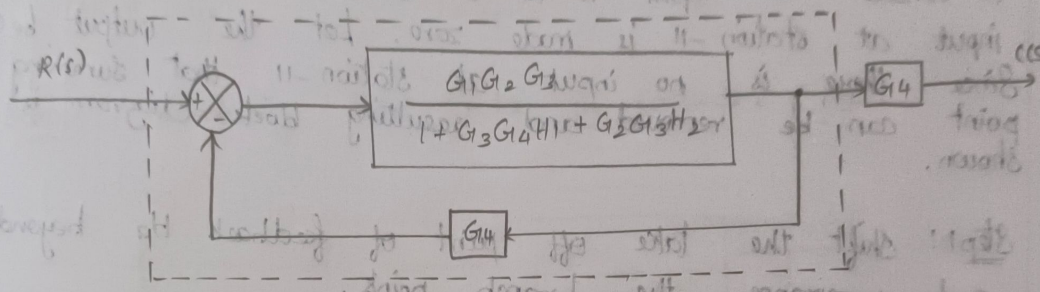
Step 3 : Combining the blocks in cascade and eliminating feedback path.



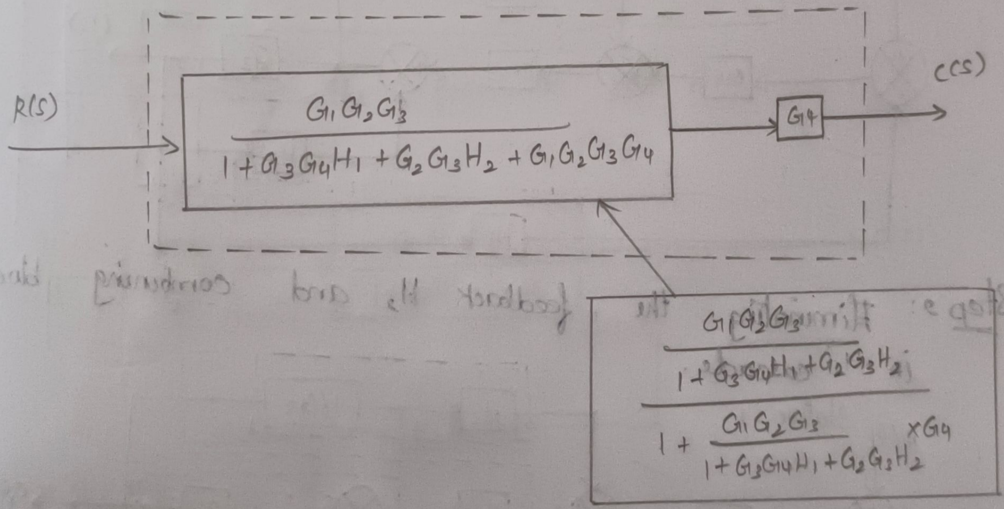
Step 4: Combining the blocks in Cascade.



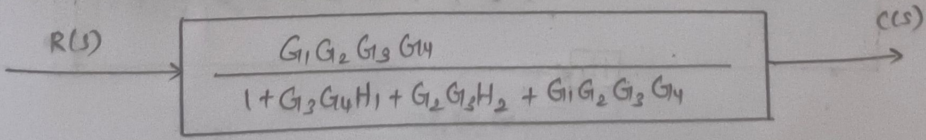
Step 5: Eliminating the feedback path.



Step 6: Combining the blocks in cascade.



$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

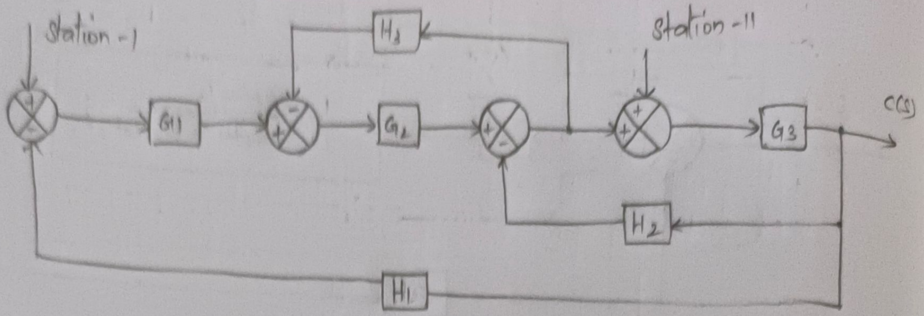


Result:

The Overall transfer function of the system is given by,

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

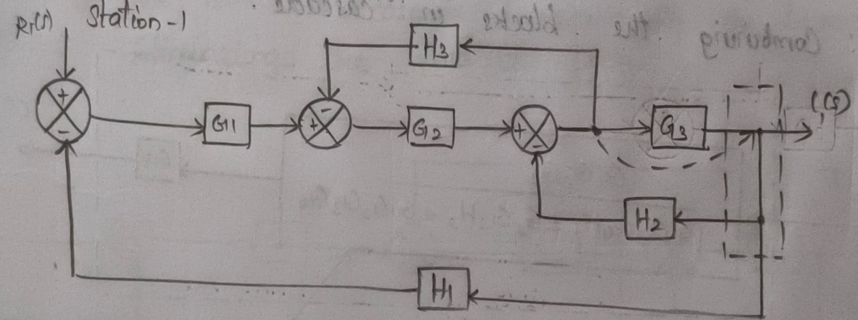
Eg: 1.19 For the system represented by the block diagram shown in fig 1. Evaluate the closed loop transfer function when the input R is (i) at station-I, (ii) at station-II.



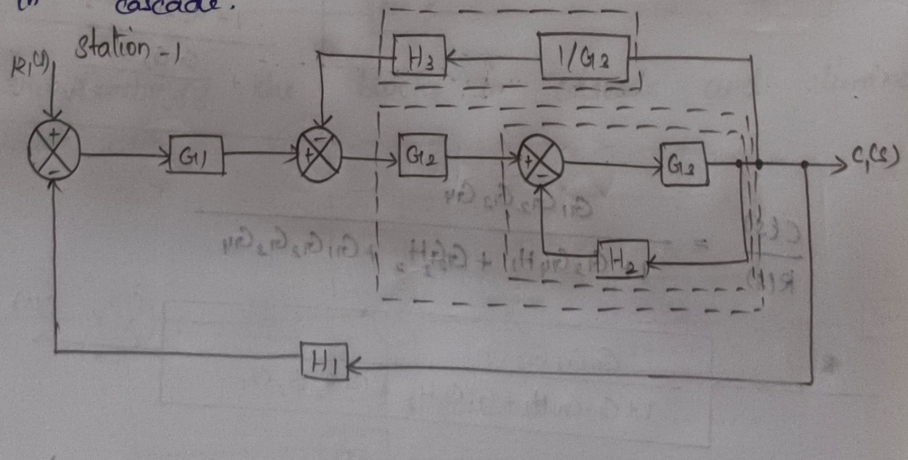
Soln:

(i) Consider the input R is at station-I and so the input at station-II is made zero. Let the output be C_1. Since there is no input at station-II that summing point can be removed and resulting block diagram is shown.

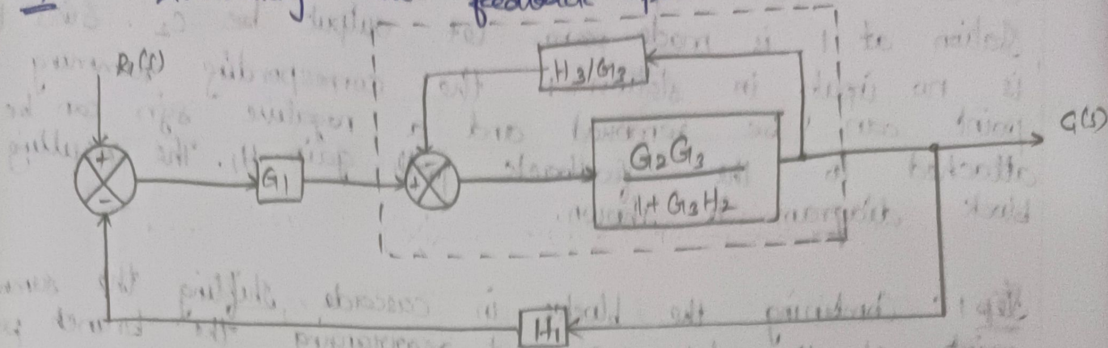
Step 1: Shift the take off point of feedback H_2 beyond G_3 and rearrange the branch point.



Step 2: Eliminating the feedback H_2 and combining blocks in cascade.

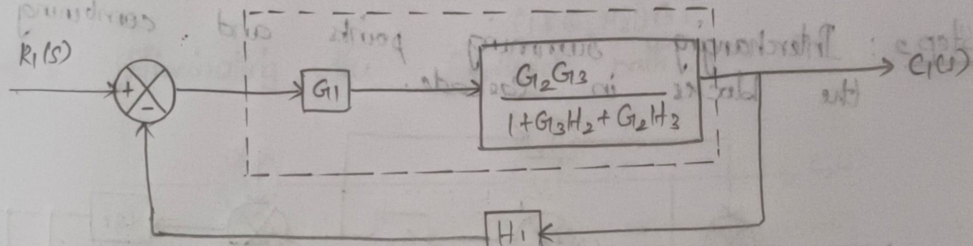


Step 3: Eliminating the feedback path.

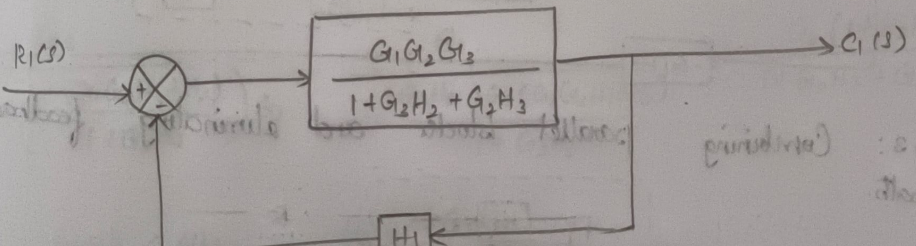


$$\frac{\dfrac{G_2G_3}{1+G_3H_2}}{1+\dfrac{G_2G_3}{1+G_3H_2}\times\dfrac{H_3}{G_3}} = \frac{\dfrac{G_2G_3}{1+G_3H_2}}{1+\dfrac{G_2H_3}{1+G_3H_2}} = \frac{G_2G_3}{1+G_3H_2+G_2H_3}$$

Step 4: Combining the blocks in Cascade.



Step 5: Eliminating the feedback path H_1

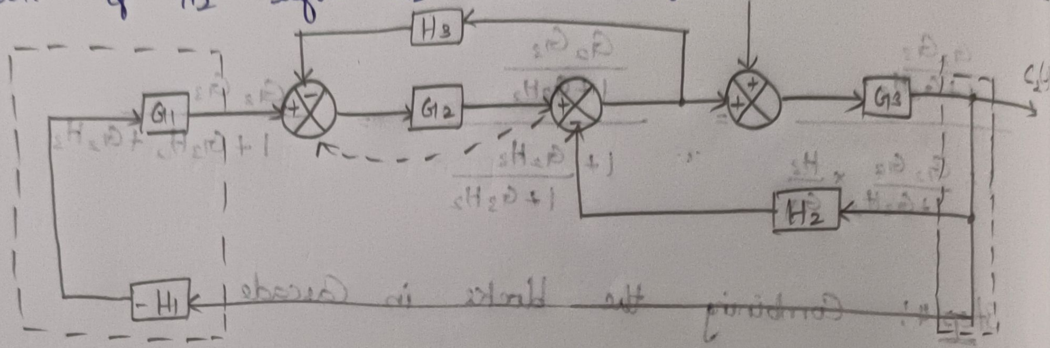


$$\frac{\dfrac{G_1G_2G_3}{1+G_3H_2+G_2H_3}}{1+\dfrac{G_1G_2G_3}{1+G_3H_2+G_2H_3}\times H_1} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

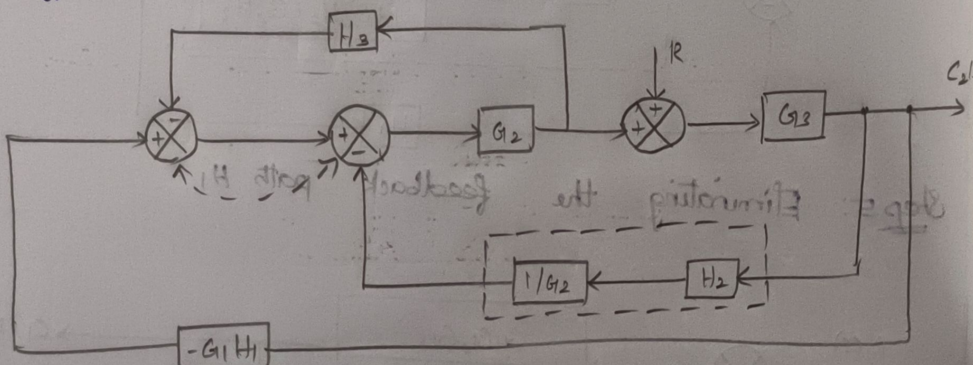
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

(ii) Consider the input R at station -II, the input station at I is made zero. Let output be C_2. Since there is no input in station -I the corresponding summing point can be removed and a negative sign can be attached to the feedback path gain H_1. The resulting block diagram is shown.

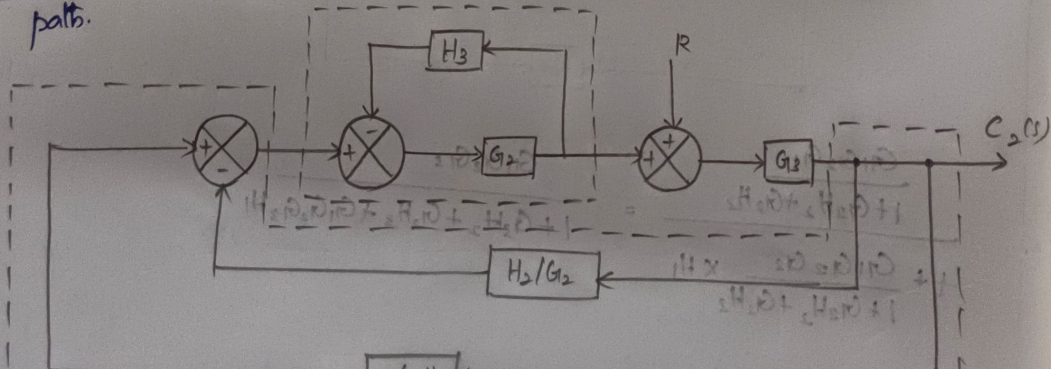
Step 1: Combining the blocks in cascade, shifting the summing point of H_2 before G_2 and rearranging the branch points.



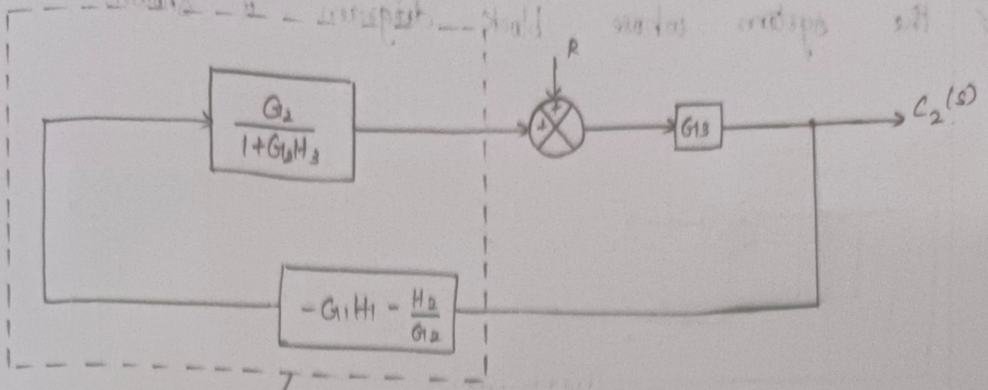
Step 2: Interchanging summing points and combining the blocks in cascade.



Step 3: Combining parallel blocks and eliminating feedback path.



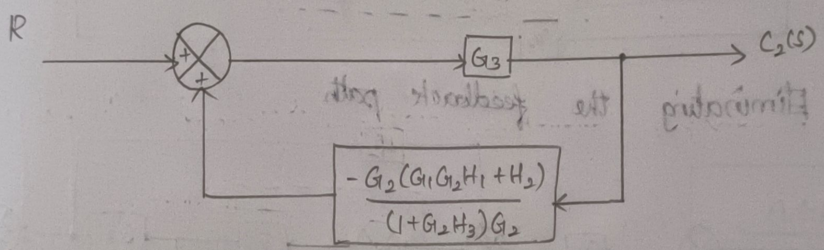
Step 4: Combining the blocks in cascade.



$$\left(\frac{G_2}{1+G_2H_3}\right) \times \left(-G_1H_1 - \frac{H_2}{G_2}\right) = \left(\frac{G_2}{1+G_2H_3}\right) \times \left(-\frac{G_1H_1G_2 - H_2}{G_2}\right)$$

$$= \frac{-G_2(G_1G_2H_1 + H_2)}{(1+G_2H_3)G_2}$$

Step 5: Eliminating the feedback path.



$$\frac{G_3}{1 - \left(\frac{-(G_1G_2H_1 + H_2)}{1+G_2H_3}\right)G_3} = \frac{G_3}{\frac{1+G_2H_3 + G_3(G_1G_2H_1 + H_2)}{1+G_2H_3}} = \frac{G_3(1+G_2H_3)}{1+G_2H_3 + G_3(G_1G_2H_1 + H_2)}$$

$$\therefore \frac{C_2(s)}{R} = \frac{G_3(1+G_2H_3)}{1+G_2H_3 + G_3(G_1G_2H_1 + H_2)}$$

Result:

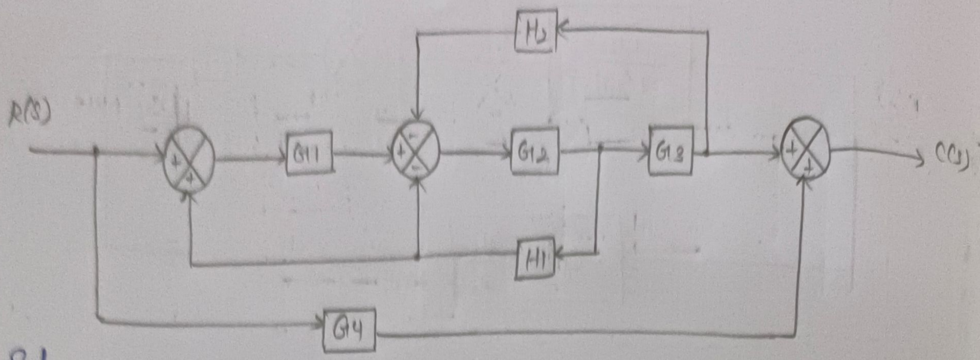
The transfer function of the system with input at station-I is,

$$\frac{C_1}{R} = \frac{G_1G_2G_3}{1 + G_3H_2 + G_2H_3 + G_1G_2G_3H_1}$$

station-II is,

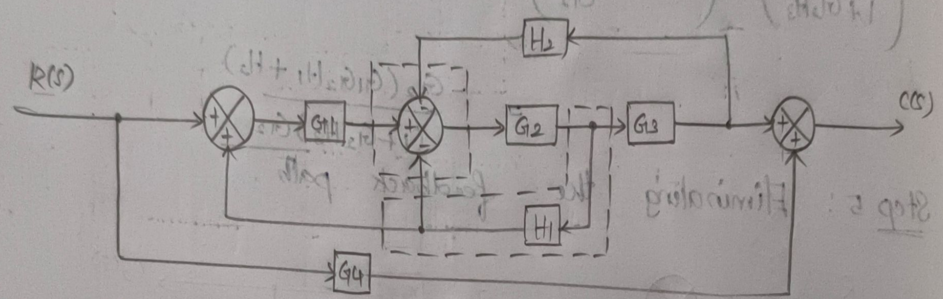
$$\frac{C_2}{R} = \frac{G_3(1+G_2H_3)}{1+G_2H_3 + G_3(G_1G_2H_1 + H_2)}$$

Eg 1.21 Obtain the closed loop transfer function $C(s)/R(s)$ of the system whose block diagram is shown.

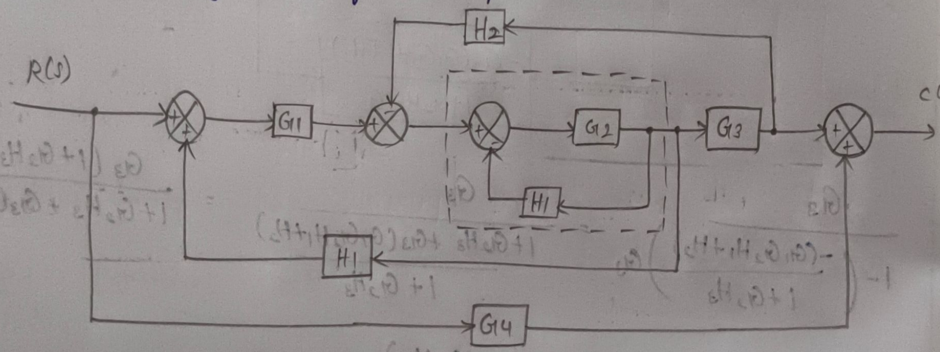


Soln:

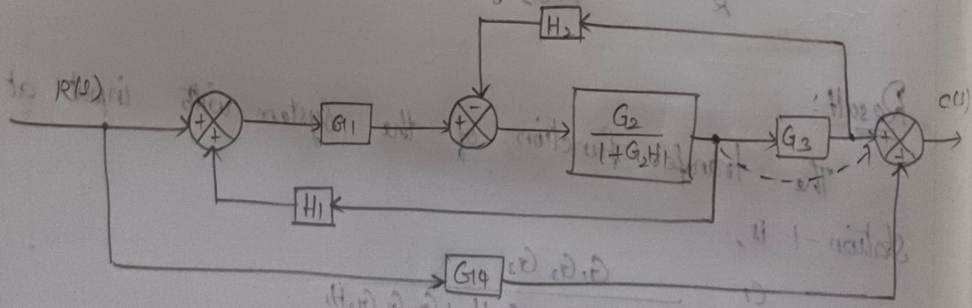
Step 1: Splitting the summing point and rearranging the branch points.



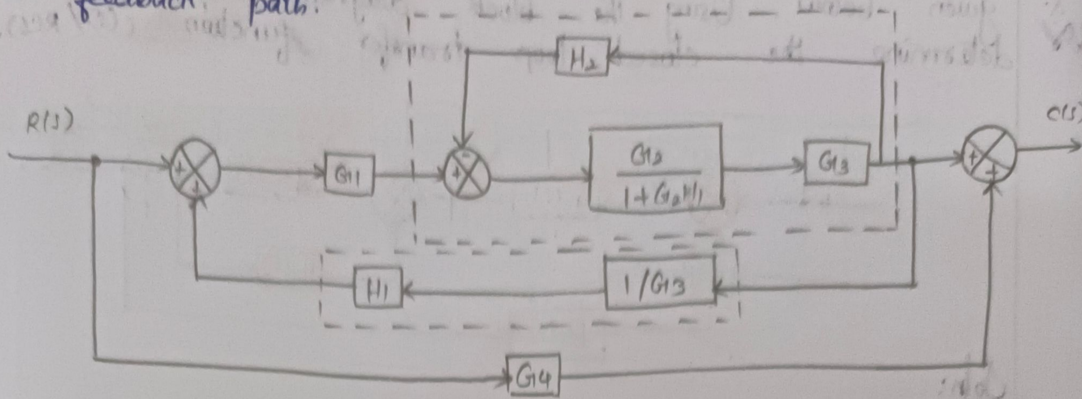
Step 2: Eliminating the feedback path



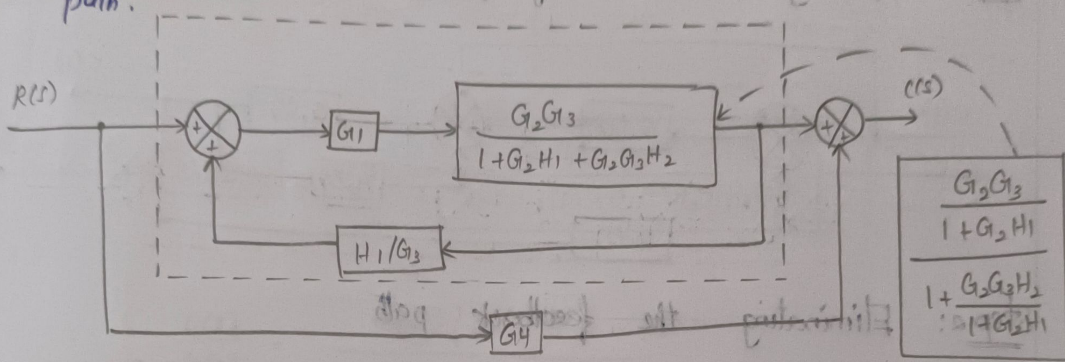
Step 3: Shifting the branch point after the block.



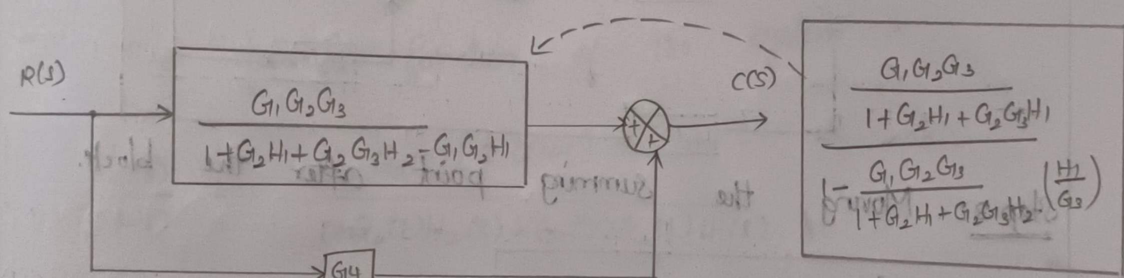
Step 4: Combining the blocks in cascade and eliminating feedback path.



Step 5: Combining the blocks in cascade and eliminating feedback path.



Step 6: Eliminating forward path.



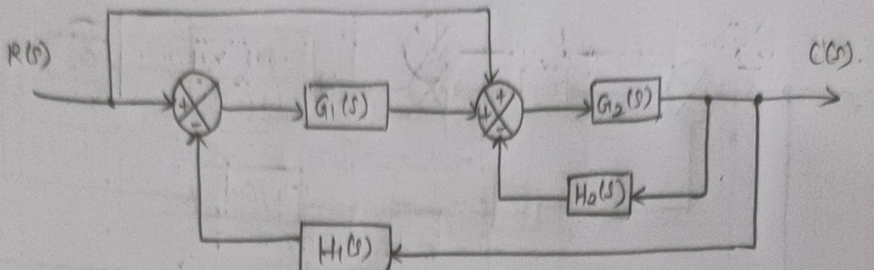
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1} + G_4$$

Result:

The transfer function of the system is

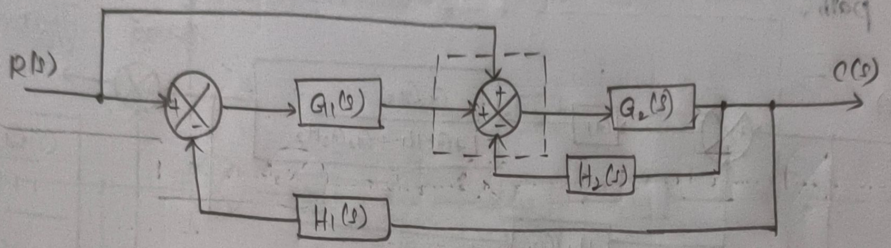
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1} + G_4$$

Eg: 1.22 The block diagram of a closed loop system is given below. Using the block diagram reduction technique determine the closed loop transfer function C(S)/R(S).

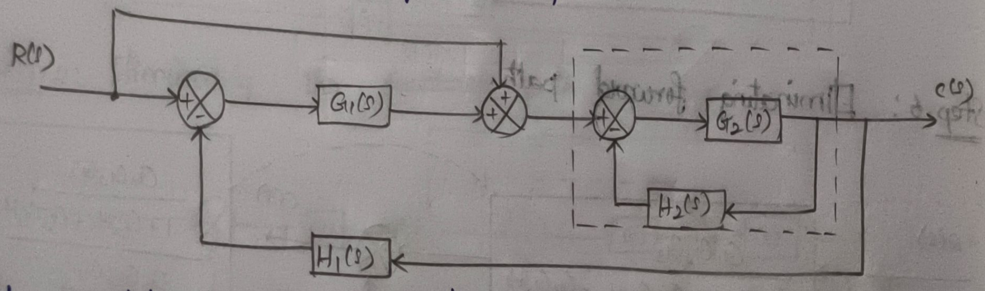


Soln:

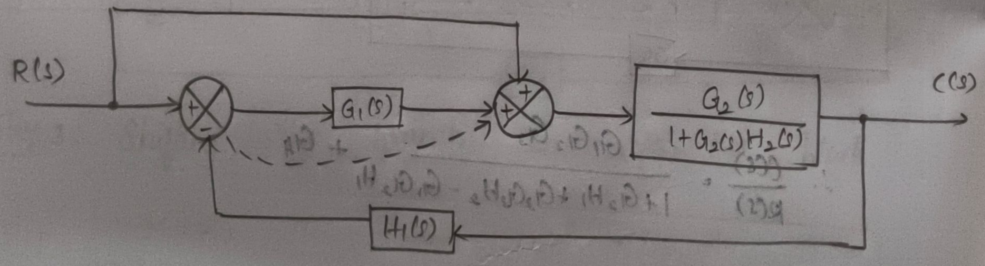
Step 1: Splitting the summing point



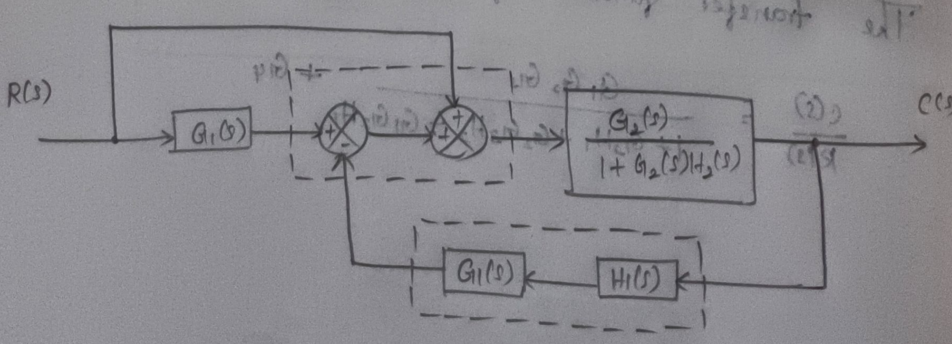
Step 2: Eliminating the feedback path



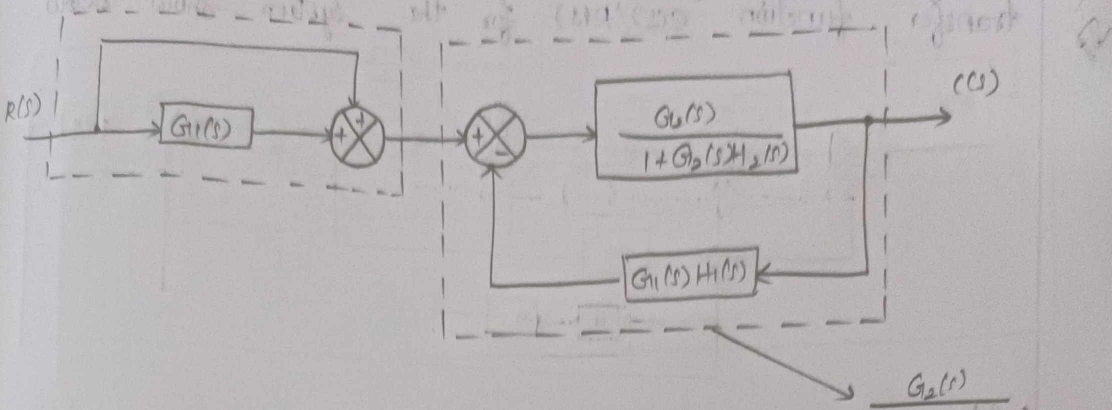
Step 3: Moving the summing point after the block.



Step 4: Interchanging the summing points and combining the blocks in cascade.

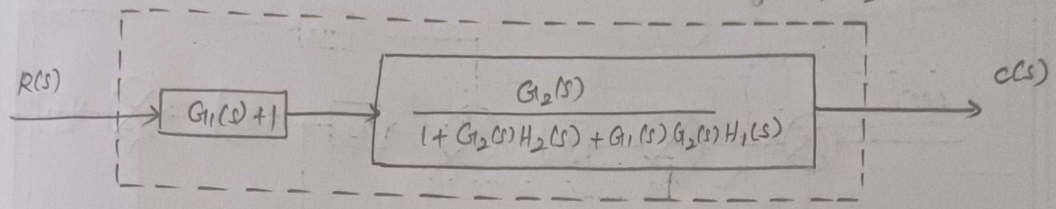


Step 5: Eliminating the feedback path and feed forward path.



$$\frac{\dfrac{G_2(s)}{1+G_2(s)H_2(s)}}{1+\dfrac{G_2(s)}{1+G_2(s)H_2(s)}\,G_1(s)H_1(s)}$$

Step 6: Combining the blocks in cascade.



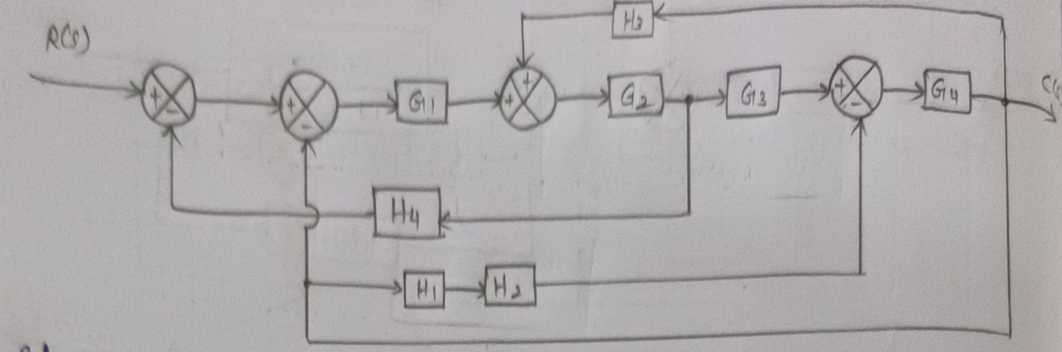
$$\therefore \frac{C(s)}{R(s)} = \frac{G_2(s)\,[G_1(s)+1]}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)}$$

Result:

The transfer function of the system is,

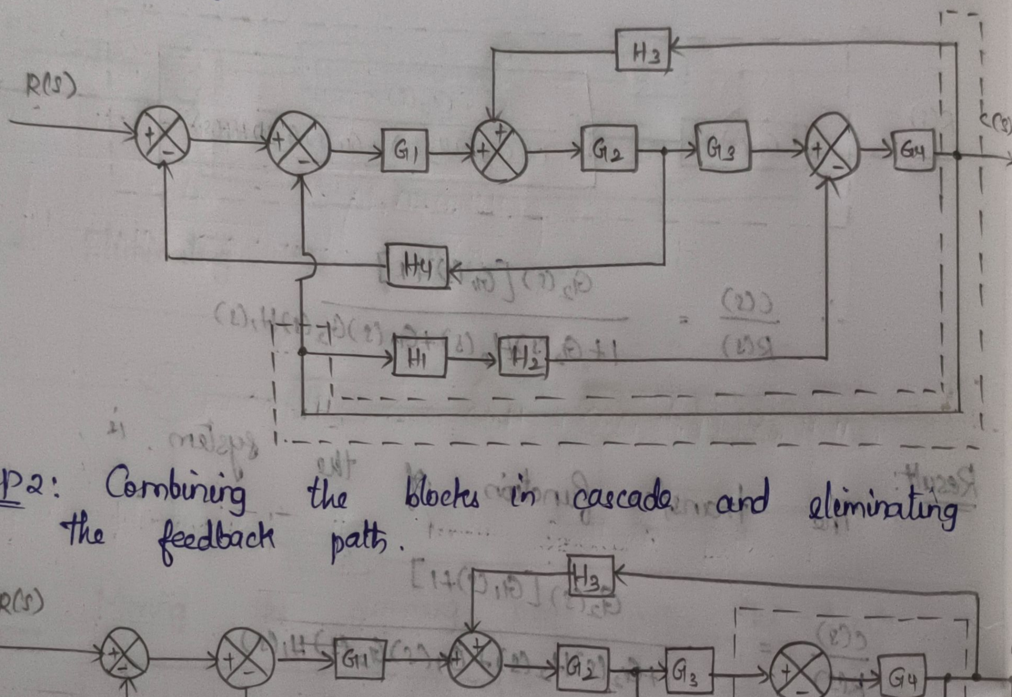
$$\frac{C(s)}{R(s)} = \frac{G_2(s)\,[G_1(s)+1]}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)}$$

Eg: 1.23. Using block diagram reduction technique find the transfer function C(s)/R(s) for the system show below.

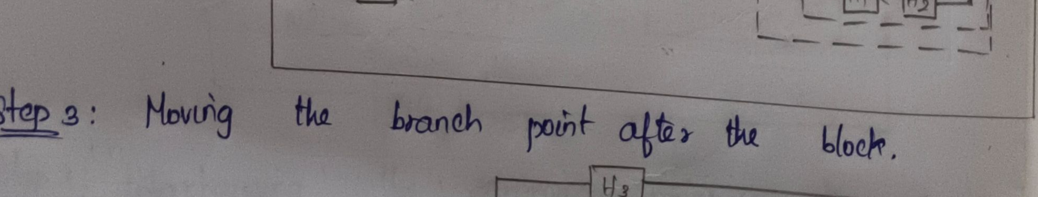


Soln:

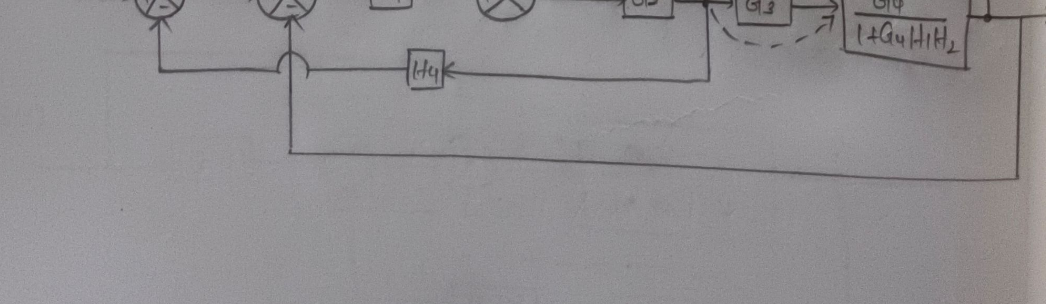
Step 1: Rearranging the branch points.



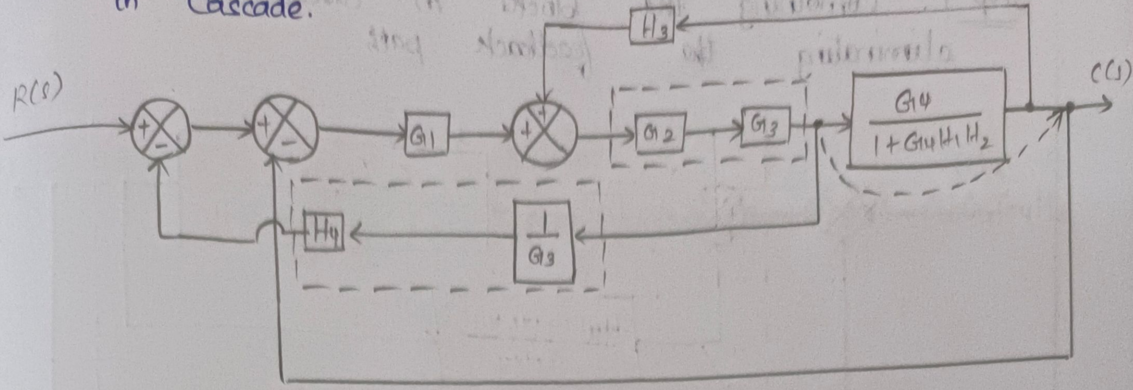
Step 2: Combining the blocks in cascade and eliminating the feedback path.



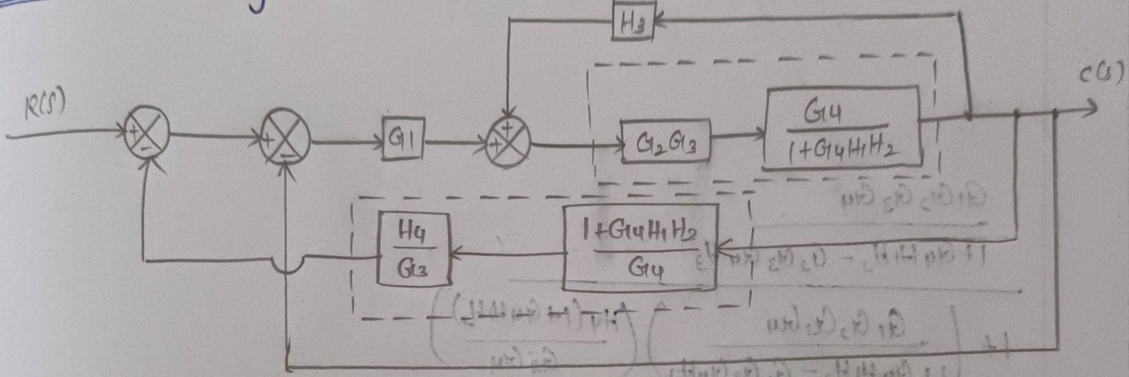
Step 3: Moving the branch point after the block.



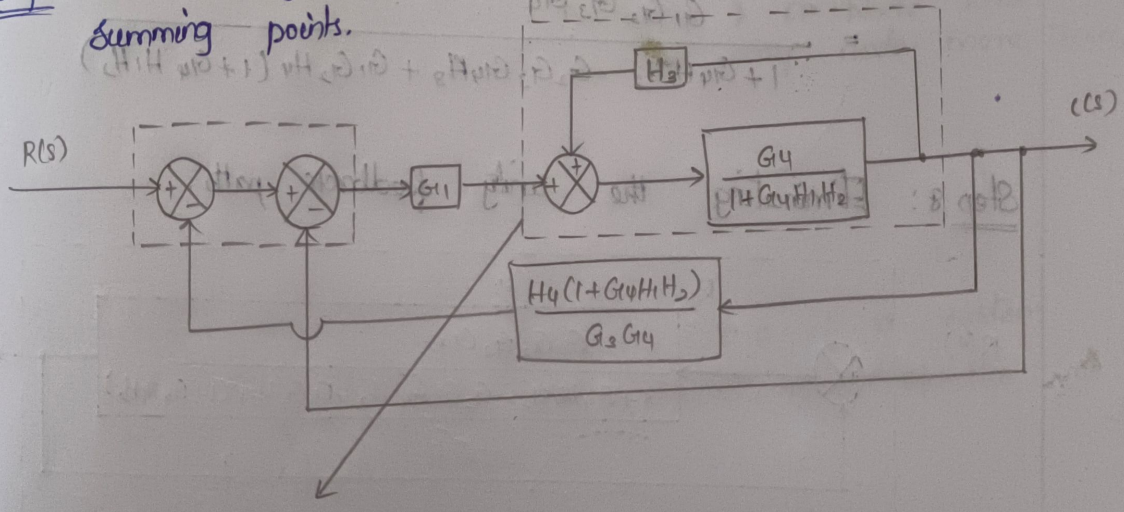
Step 4: Moving the branch point and combining the blocks in Cascade.



Step 5: Combining the blocks in Cascade.

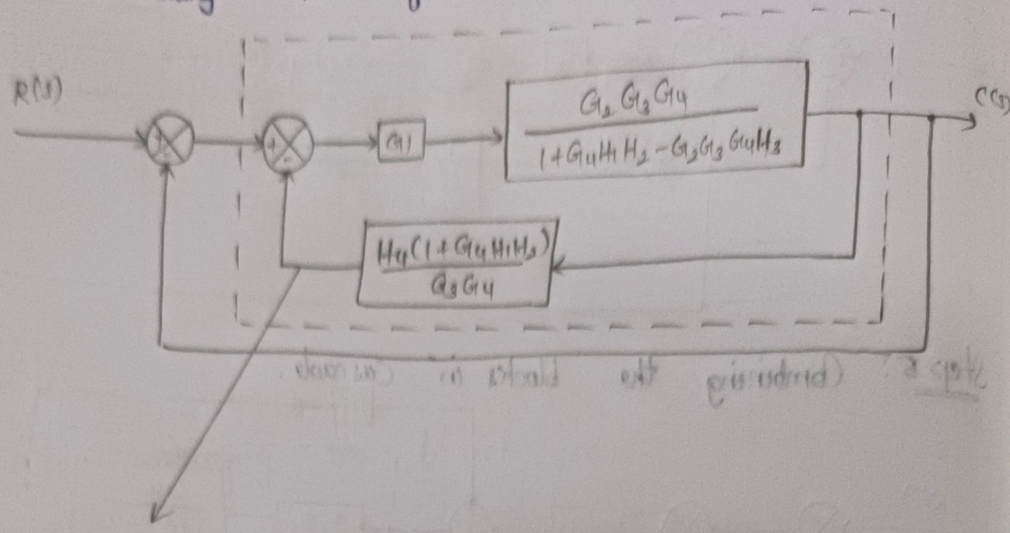


Step 6: Eliminating feedback path and interchanging the summing points.



$$\frac{\dfrac{G_2G_3G_4}{1+G_4H_1H_2}}{1-\dfrac{G_2G_3G_4H_3}{1+G_4H_1H_2}} = \frac{G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}$$

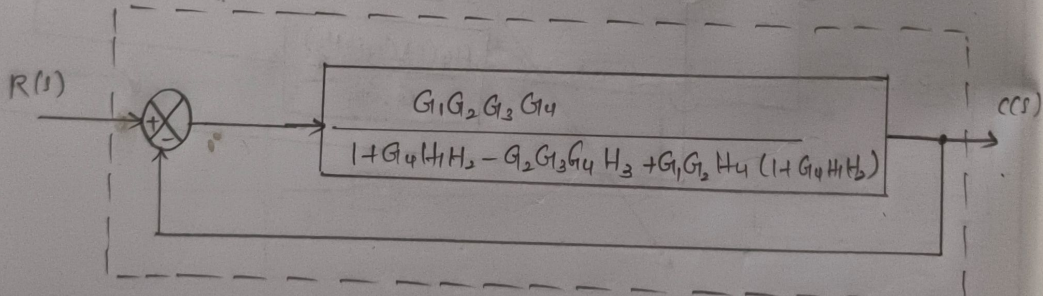
Step 7: Combining the blocks in cascade and eliminating the feedback path.



$$\frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}}{1+\left(\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}\right)\left(\dfrac{H_4(1+G_4H_1H_2)}{G_3G_4}\right)}$$

$$= \frac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}$$

Step 8: Eliminating the unity feedback path.



$$\therefore \frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}}{1+\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}}$$

$$= \frac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)+G_1G_2G_3G_4}$$

$$= \frac{G_1G_2G_3G_4}{1+H_1H_2(G_4+G_1G_2G_4H_4)+G_1G_2(H_4+G_3G_4)-G_2G_3G_4H_3}$$

Result :

The transfer function of the system is,

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + H_1H_2(G_4 + G_1G_2G_4H_4) + G_1G_2(H_4 + G_3G_4) - G_2G_3G_4H_3}$$

SIGNAL FLOW GRAPH

Node $\longrightarrow$ •

Branch $\longrightarrow$ •⟶

Transmittance / gain $\longrightarrow$ •⟶•

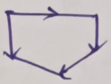
IP Node $\longrightarrow$ •⟶

OP Node $\longrightarrow$ ⟶•

Path $\longrightarrow$ Connecting branch

closed path $\longrightarrow$ starts and ends at same node.

Forward path $\longrightarrow$ path between IP Node & OP Node end and does not cross any node more than one.

Individual loop $\longrightarrow$ 

MASON'S GAIN FORMULA

It used to determine T.F of a system using signal flow graph. The overall gain of the system is given as follows.

$$\text{Gain } T = \frac{1}{\Delta} \Sigma P_K \Delta_K$$

Where ;

$K \rightarrow$ no. of forward path

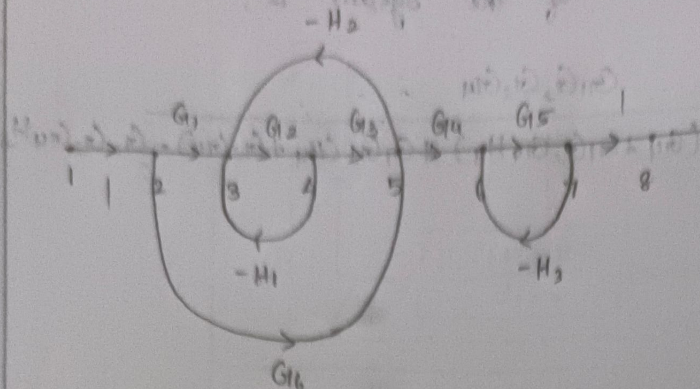
$P_K \rightarrow$ Forward gain of K^{th} path

$\Delta = 1 -$ (Sum of individual loop gain) + (Sum of gain of path of all two non touchy loop) − etc...

$\Delta_K = \Delta$ for that part of the graph when is not touchy K^{th} forward path.

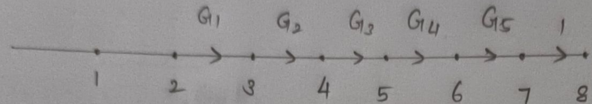
9/1/25

1) Find the transfer function of the system



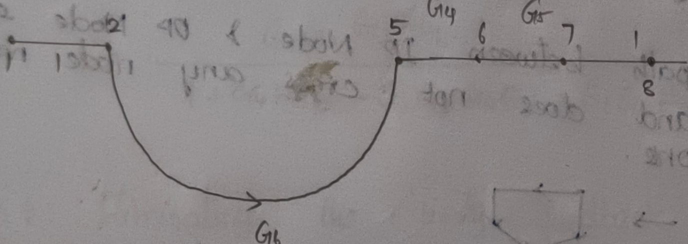
I) Forward path gain

FP1 :-



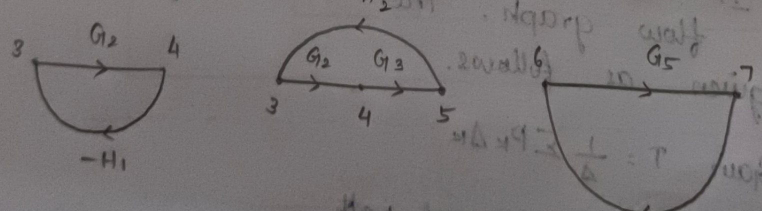
Gain of $P_1 = G_1 + G_2 + G_3 + G_4 + G_5$

FP2 :-



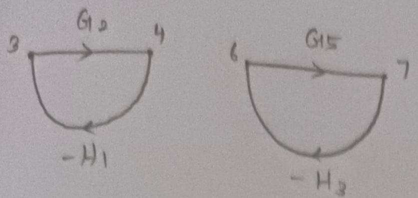
Gain of $P_2 = G_6 + G_4 + G_5$

II) Individual loop gain

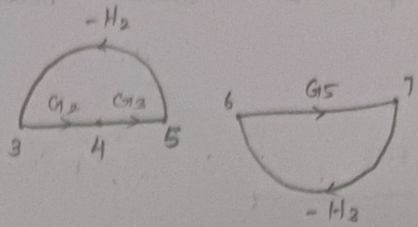


$P_{11} = -G_2 H_1$ $P_{21} = -G_2 G_3 H_2$ $P_{31} = -G_5 H_3$

iii) Gain of two non-touching loop.



$$P_{12} = (-G_2H_1)(-G_5H_3)$$
$$= G_2G_5H_1H_3$$



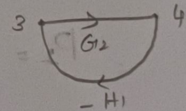
$$P_{22} = (-G_2H_1)(-G_5H_3)$$
$$= G_2G_3G_5H_2H_3$$

IV) Calculation of Δ and Δ_k

$$\Delta = 1 + (G_2H_1 + G_2G_3H_2 + G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$
$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3.$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward path.

∴ The part of the graph which is non touching with second forward path is shown.



$$\Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$$

V) Transfer function, T.

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

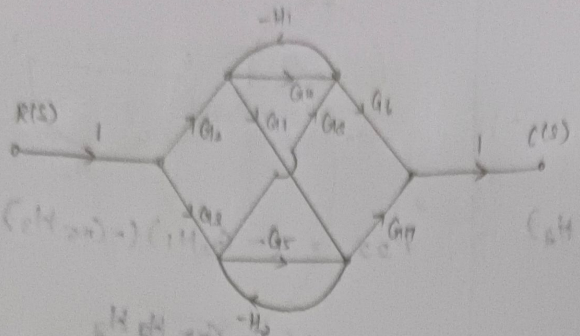
(No. of forward paths is 2 and so K = 2).

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1 + G_2H_1)}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

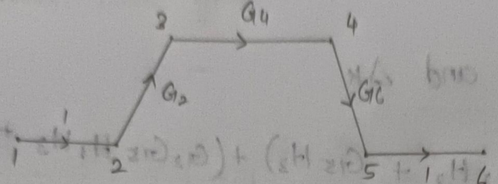
$$= \frac{G_2G_4G_5\,[G_1G_3 + G_6/G_2 + G_6H_1]}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

Eg. 1.26 Find the overall gain of the system whose signal flow graph is shown below.



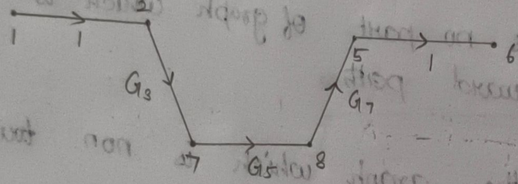
I) Forward path gain:

FP1 :-



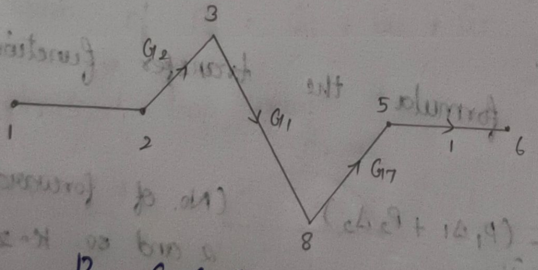
$P_1 = G_2 G_4 G_6$

FP2 :-



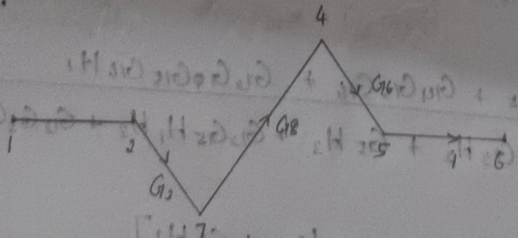
$P_2 = G_3 G_5 G_7$

FP3 :-



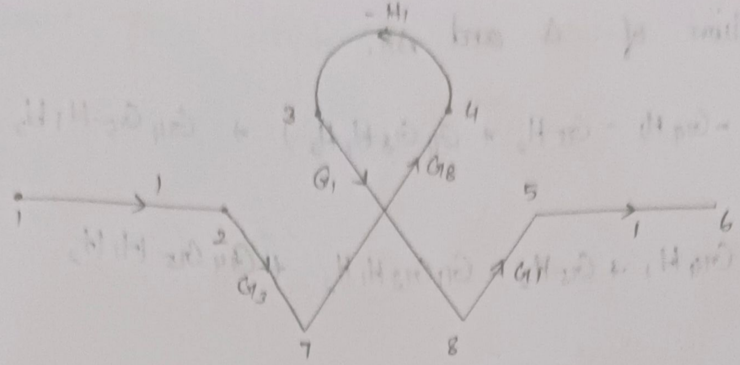
$P_3 = G_2 G_1 G_7$

FP4 :-



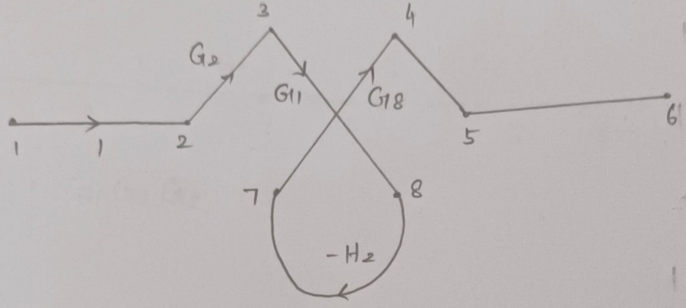
$P_4 = G_3 G_6 G_8$

FP 5 :-



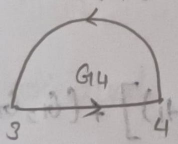
$$P_5 = -G_3 G_1 G_8 G_7 H_1$$

FP6 :-

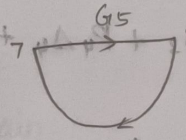


$$P_6 = -G_1 G_2 G_6 G_8 H_2$$

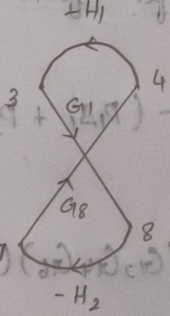
ii) Individual loop Gain.



$$P_{11} = -G_4 H_1$$

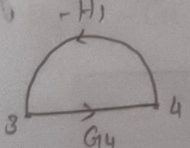


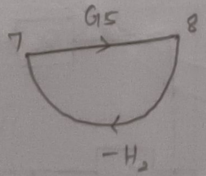
$$P_{21} = (-G_5 H_2)$$



$$P_{31} = G_1 G_8 H_2 H_1$$

iii) Gain product of two non touchy loop:





$$P_{12} = (-G_4 H_1)(-G_5 H_2)$$

$$= G_4 G_5 H_1 H_2$$

IV) Calculation of Δ and Δ_K.

$$\Delta = 1 - (-G_4H_1 - G_5H_2 + G_1G_8H_1H_2) + G_4G_5H_1H_2$$

$$= 1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2$$

$$\Delta_1 = 1 - (-G_5H_2) = 1 + G_5H_2$$

$$\Delta_2 = 1 - (-G_4H_1) = 1 + G_4H_1$$

$$\Delta_3 = 1 - 0 = 1$$

$$\Delta_4 = 1 - 0 = 1$$

$$\Delta_5 = 1 - 0 = 1$$

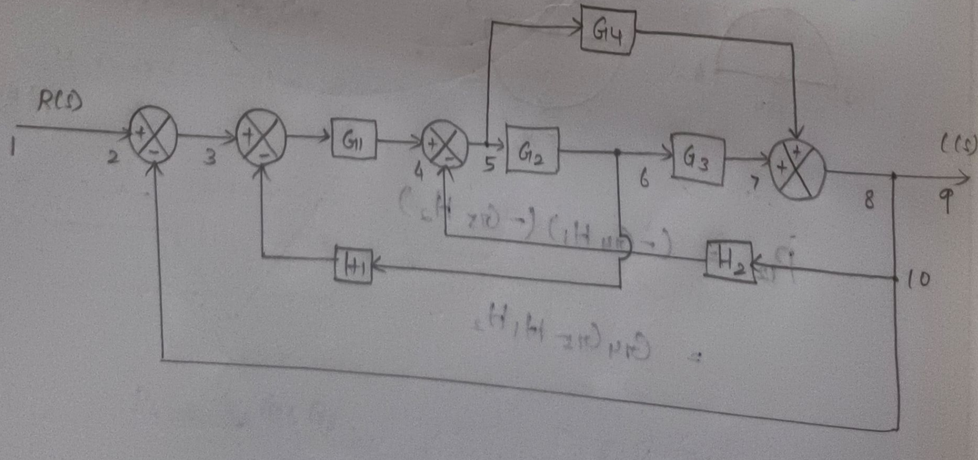
$$\Delta_6 = 1 - 0 = 1$$

V) Transfer function using Mason's gain formula.

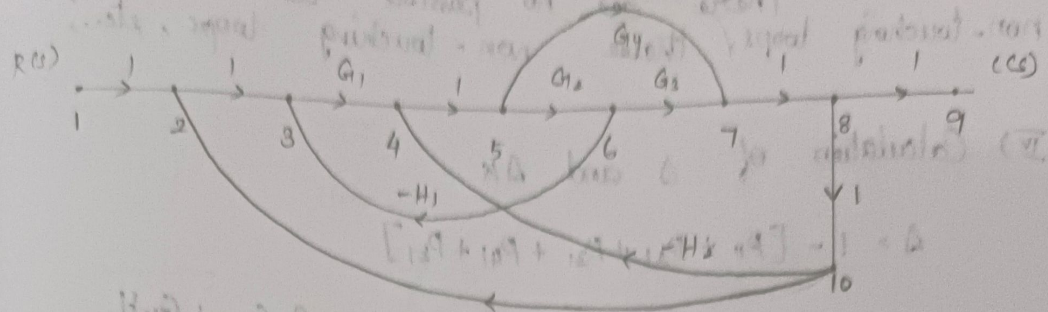
$$T = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6)$$

$$= \frac{[(G_2G_4G_6)(1+G_5H_2)] + [(G_3G_5G_7)(1+G_4H_1)] + (G_2G_1G_7) + (G_3G_6G_8) + (-G_3G_1G_8G_7H_1) + (-G_1G_2G_6G_8H_2)}{1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2}$$

Eg 1.31 Convert the block diagram to signal flow graph and determine the transfer function using Mason's gain formula.

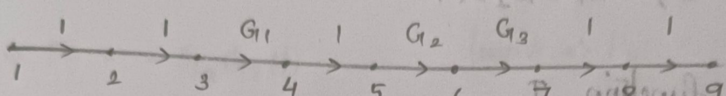


The signal flow graph for the above block diagram.



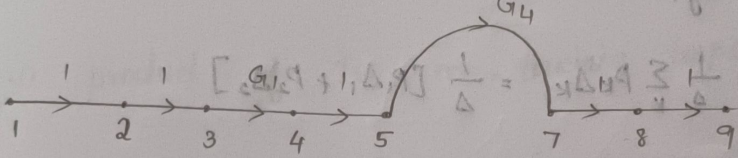
I) Forward path gain :

FP1 :-



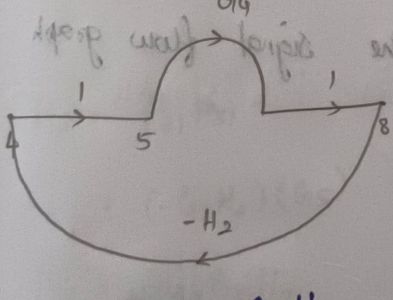
$P_1 = G_1 G_2 G_3$

FP2 :-

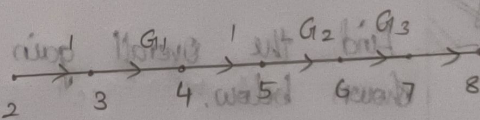


$P_2 = G_1 G_4$

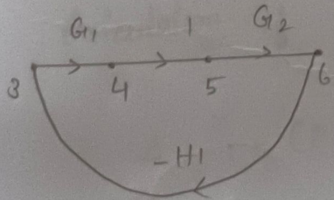
II) Individual loop gain :



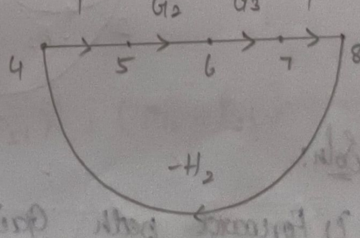
$P_{11} = -G_4 H_2$



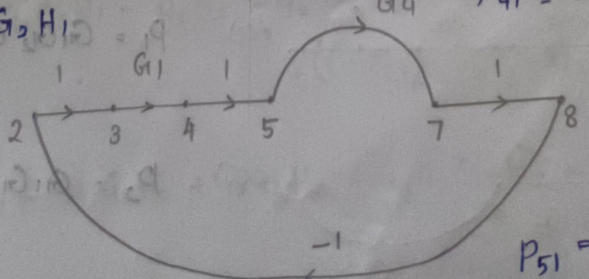
$P_{21} = -G_1 G_2 G_3$



$P_{31} = -G_1 G_2 H_1$



$P_{41} = -G_2 G_3 H_2$



$P_{51} = -G_1 G_4$

iii) Gain Products of Two Non-touching loops.

There are no possible combinations of two non-touching loops, three non-touching loops, etc..,

IV) Calculation of Δ and ΔK.

$$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51}]$$

$$= 1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2$$

Since no part of graph is non-touching with forward paths -1 and 2, $\Delta_1 = \Delta_2 = 1$.

V) Transfer function T,

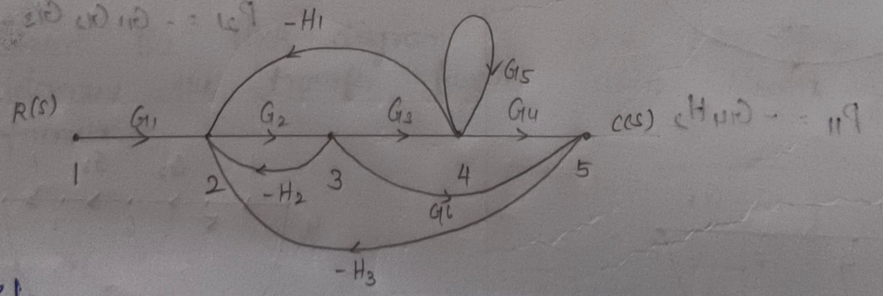
By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K \Delta_K = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2}$$

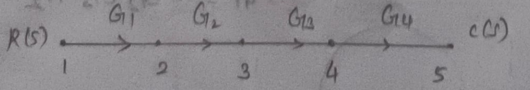
Eg: 1.27 Hw.

Find the overall gain $C(s)/R(s)$ for the signal flow graph shown below.

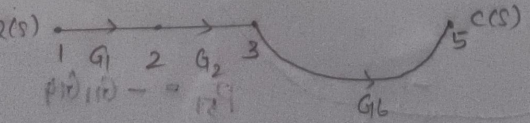


Soln:

I) Forward path Gains.

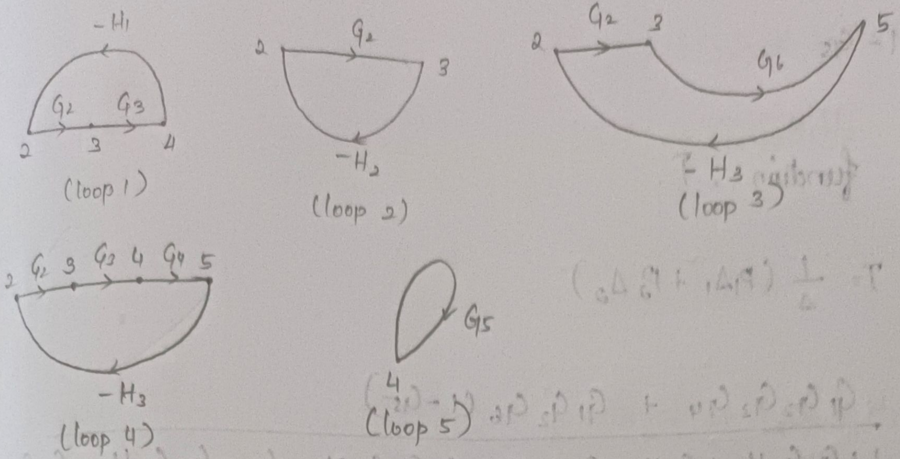


$P_1 = G_1G_2G_3G_4$



$P_2 = G_1G_2G_6$

ii) Individual loop Gain.



loop 1 $\Rightarrow P_{11} = -G_2G_3H_1$

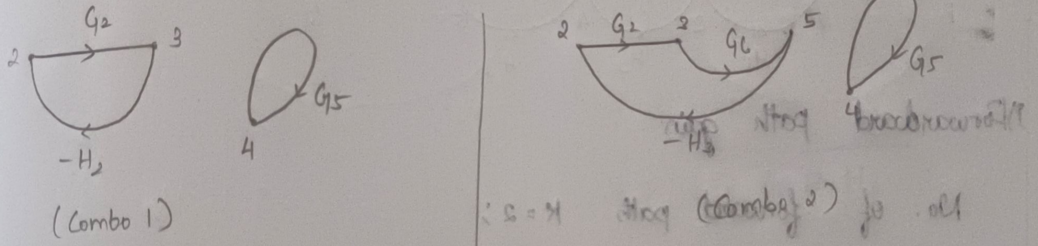
loop 2 $\Rightarrow P_{21} = -G_2H_2$

loop 3 $\Rightarrow P_{31} = -G_2G_6H_3$

loop 4 $\Rightarrow P_{41} = -G_2G_3G_4H_3$

loop 5 $\Rightarrow P_{51} = G_5$

iii) Gain product of two non-touching loops.



$P_{12} = P_{21}P_{51}$

$= (-G_2H_2)(G_5)$

$P_{12} = -G_2G_5H_2$

$P_{22} = P_{31}P_{51}$

$= (-G_2G_6H_3)(G_5)$

$P_{22} = -G_2G_5G_6H_3$

iv) Calculation of Δ and Δ_K.

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2G_3H_1 - G_2H_2 - G_2G_6H_3 - G_2G_3G_4H_3 + G_5) + ((-G_2G_5H_2) - G_2G_5G_6H_3)$$

$$= 1 + G_2G_3H_1 + G_2H_2 + G_2G_6H_3 + G_2G_3G_4H_3 - G_2G_5H_2 - G_2G_5G_6H_3 - G_5$$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - G_5$

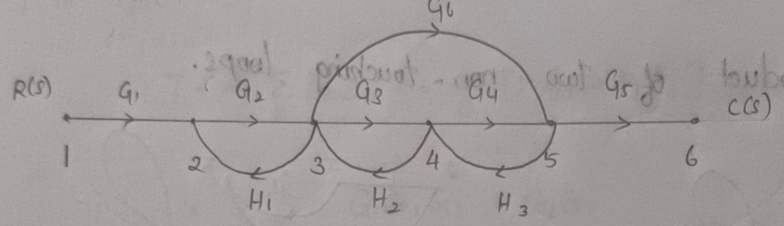
Transfer function, T.

$$T = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6(1-G_5)}{1 + G_2G_3H_1 + G_2H_2 + G_2G_6H_3 + G_2G_3G_4H_3 - G_5 - G_2G_5H_2 - G_2G_5G_6H}$$

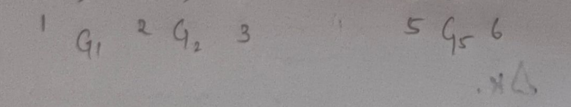
$$T = \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + G_2G_3H_1 + G_2H_2 + G_2G_6H_3 + G_2G_3G_4H_3 - G_5 - G_2G_5H_2 - G_2G_5G_6H}$$

Eg: 1.29

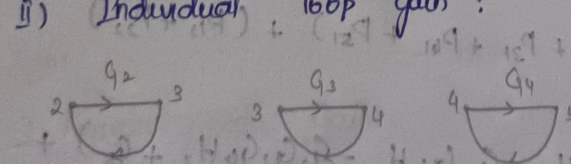


I) Forward path gain

No. of forward path $K = 2$;

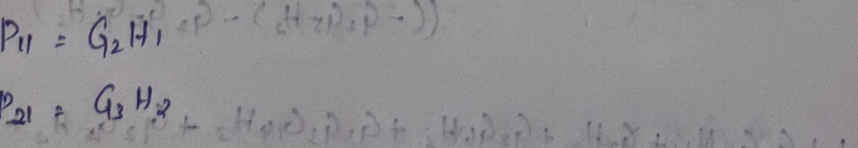


$P_1 = G_1G_2G_3G_4G_5$



$P_2 = G_1G_2G_6G_5$

II) Individual loop gain:



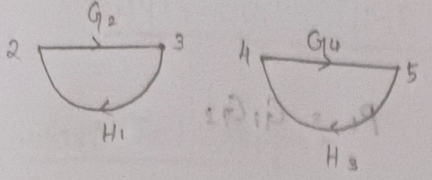
$P_{11} = G_2H_1$

$P_{21} = G_3H_2$

$P_{31} = G_4H_3$

$P_{41} = G_6H_2H_3$

iii) Gain product of two non-touching loops.



$$P_{12} = (G_2H_1)(G_4H_3)$$

$$P_{12} = G_2G_4H_1H_3$$

iv) Calculation of Δ and Δk.

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41}) + P_{12}$$

$$= 1 - (G_2H_1 + G_3H_2 + G_4H_3 + G_6H_2H_3) + G_2G_4H_1H_3$$

$$\Delta = 1 - G_2H_1 - G_2H_2 - G_4H_3 - G_6H_2H_3 + G_2G_4H_1H_3$$

v) Transfer function.

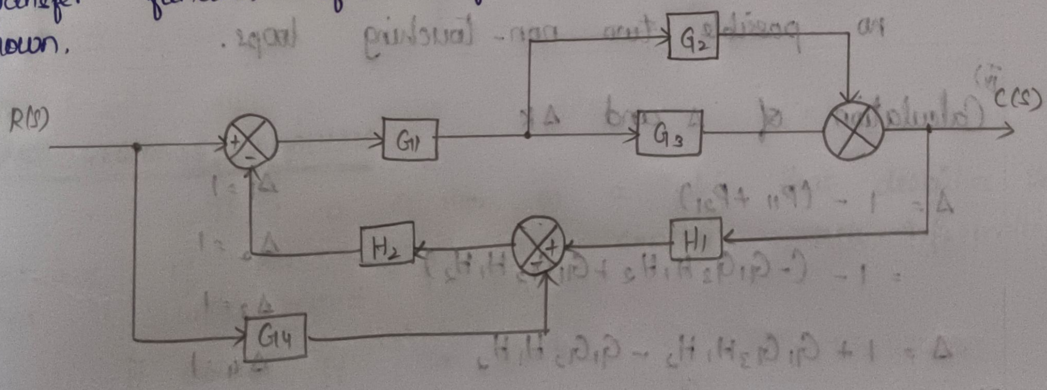
$$T = \frac{1}{\Delta} \Sigma P_k \Delta_k \qquad \because \Delta_1 = 1$$

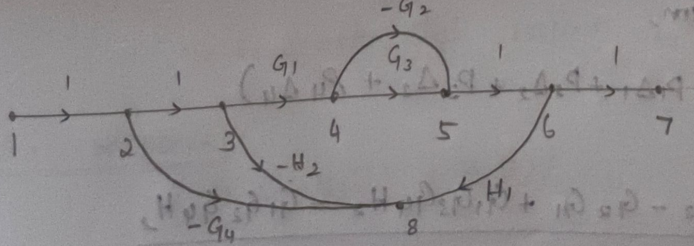
$$\Delta_2 = 1$$

$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

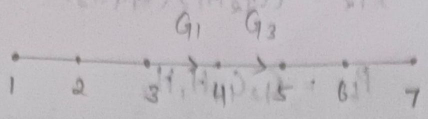
$$T = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_6G_5}{1 - G_2H_1 - G_3H_2 - G_4H_3 - G_6H_2H_3 + G_2G_4H_1H_3}$$

Draw a signal flow graph and evaluate the closed loop transfer function of a system whose block diagram is shown.

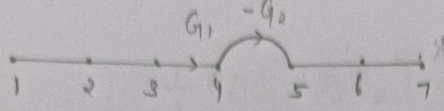




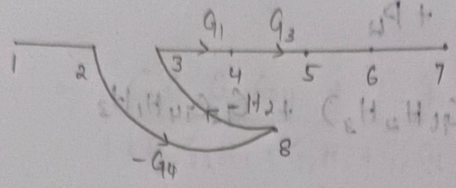
I) Forward path gain :



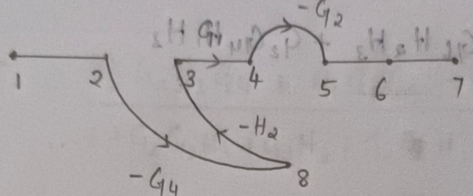
$P_1 = G_1 G_3$



$P_2 = -G_1 G_2$

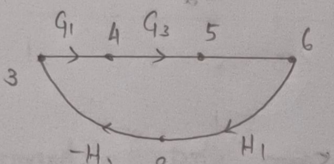


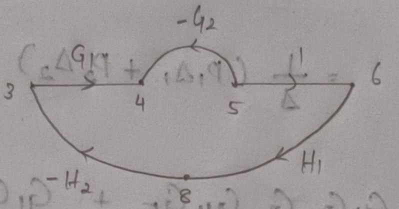
$P_3 = G_1 G_3 G_4 H_2$



$P_4 = -G_1 G_2 G_4 H_2$

ii) Individual loop gain :





$P_{11} = -G_1 G_3 H_1 H_2$ $\quad P_{21} = G_1 G_2 H_1 H_2$

iii) Gain product of two non-touching loops

no possible two non-touching loops.

iv) Calculation of Δ and Δk

$$\Delta = 1 - (P_{11} + P_{21})$$

$$= 1 - (-G_1 G_3 H_1 H_2 + G_1 G_2 H_1 H_2)$$

$$\Delta = 1 + G_1 G_3 H_1 H_2 - G_1 G_2 H_1 H_2$$

$\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1$

$\Delta_4 = 1$

v) Transfer function,

$$T = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4)$$

$$T = \frac{G_1G_3 - G_2G_1 + G_1G_3G_4H_2 - G_1G_2G_4H_2}{1 + G_1G_3H_1H_2 - G_1G_2H_1H_2}$$