**3rd Year Communications and Electronics Department**

**Antennas and Waveguides Engineering- ELC3050**

**Rectangular Cavity Resonator Assignment**

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| **Name** | **Section** | **Bench No.** |
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**Objective:**

We aim to identify the two first resonance Frequencies for a Rectangular Cavity Resonator, and calculate their respective bandwidth and Quality factor.

**Resonance Frequencies**

Generally the resonance frequency for or mode is:

We know that the first resonance frequency is always mode which is the dominant mode, the resonant frequency for is:

But the next resonance frequency depends on the ratios between the 3 dimensions of the cavity.

1-If and then the next resonance frequency corresponds to mode whose resonance frequency is

2-But if the three dimensions are close to each other, then the next mode is whose resonance frequency is:

**Quality Factor Calculations**

The Quality Factor:

1-Quality Factor Due to Conductor walls losses, it depends on the mode

2-Quality Factor Due to the Dielectric Filling: it doesn’t depend on the mode

So the Total Quality factor of the Cavity will be

We will return to the quality factor calculations in more detail.

**Bandwidth Calculations:**

We know that the fractional Bandwidth

Now we return to the Quality Factor Calculations:

Quality factor Calculations for modes

We get the field Expressions for the mode:

Then to get the stored Electric energy ():

Then to get the power losses due to conducting walls:

So we get

So finally we get the quality factor due to conductive walls to be for mode:

We will use this previous expression to calculate the quality factor for both and modes and hence get the BW.

Now we want to get the Q expression for the possible mode:

First we get the field expressions:

From the boundary conditions we get:

So the field expressions will be:

Now we will go through the same steps we went through with calculations:

To get the stored Electric energy ():

To get the power loss due to the conducting walls:

So finally we get the Quality factor due to conductive walls for mode to be:

Now we have the Quality factor due to conductive walls for all the possible modes

The other component of the quality factor is due to the dielectric filling which doesn’t depend on the mode and only depends on the dielectric filling and its loss tangent.

And the total Quality factor for every mode will be:

We can then easily calculate the bandwidth as previously stated:

Example:

Problem (3)  
A rectangular cavity having and a length of is filled with a dielectric of relative permittivity at the resonant frequency of the mode. The cavity is made of copper . Find the quality factor for the mode.

From the previous givens we get:

By inputting the previous parameters to our code we get:

tan(δ) : 4\*10^-5

Enter The Dielectric Constant :2.5

σ : 5.8\*10^7

a in Cm : 2

b in Cm : 1

d in Cm : 6

First Resonance Frequency corresponds to TE101 Mode

First Resonance Frequency = 5000000000 Hz

Its Quality Factor= 4.531296e+03

Its Fractional Bandwidth= 2.206874e-02 %

Its Actual Bandwidth : 1.103437e+06 Hz

Next Resonance frequency corresponds to TE102 Mode

Second Resonance Frequency = 5.700877e+09 Hz

Its Quality Factor = 5.073636e+03

Its Fractional Bandwidth : 1.970973e-02 %

Its Actual Bandwidth : 1.123628e+06 Hz

**Summary of formulae:**

For mode:

For mode:

For mode:

For all modes:

The used code:

%Defining the Speed of light and Constants

c=3\*(10^8);

Muo=4\*pi\*10^-7;

eo=(10^-9)/(36\*pi);

%\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*Taking Cavity Parameters as input\*\*\*\*\*\*\*\*\*\*\*\*%

%Enter the loss tangent and Calculating Qdielectric%

tandel=input("tan(?) : ");

Qd=1/tandel;

%Enter the Dielectric Constant%

er=input("Enter The Dielectric Constant :");

cr=c/sqrt(er);

%Enter The Conductivity of the walls Metal%

cond=input("? : ");

%Taking Cavity Dimensions as input

a=input("a in Cm : ")\*(10^-2);

b=input("b in Cm : ")\*(10^-2);

d=input("d in Cm : ")\*(10^-2);

%\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*%

%\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*Calculating for the first resonance Frequency\*\*\*\*\*\*\*\*\*\*\*\*\*\*%

%Calculating First Res Freq TE101 and its Qulaity Factor%

fc\_101=calcResFreq(a,b,d,1,0,1,cr);

fprintf("First Resonance Frequency corresponds to TE101 Mode\n");

fprintf("First Resonance Frequency = %d Hz\n",fc\_101);

k\_101=2\*pi\*fc\_101/cr;

Eta=sqrt(Muo/(eo\*er));

Rs\_101=sqrt(pi\*fc\_101\*Muo/cond);

Q\_101=((k\_101\*a\*d)^3)\*b\*Eta/((2\*Rs\_101\*(pi^2))\*(2\*(a^3)\*b+2\*b\*(d^3)+(a^3)\*d+a\*(d^3)));

Q=(Qd\*Q\_101)/(Qd+Q\_101);

fprintf("Its Quality Factor= %d\n",Q);

fprintf("Its Fractional Bandwidth= %d %% \n",100/Q);

fprintf("Its Actual Bandwidth : %d Hz\n", fc\_101/Q);

%Calculating for TE102%

fc\_102=calcResFreq(a,b,d,1,0,2,cr);

k\_102=2\*pi\*fc\_102/cr;

Rs\_102=sqrt(pi\*fc\_102\*Muo/cond);

Q\_102=((k\_102\*a\*d)^3)\*b\*Eta/((2\*Rs\_102\*(pi^2))\*(8\*(a^3)\*b+2\*b\*(d^3)+4\*(a^3)\*d+a\*(d^3)));

%Calculating for TE011%

fc\_011=calcResFreq(a,b,d,0,1,1,cr);

w\_011=2\*pi\*fc\_011;

Rs\_011=sqrt(pi\*fc\_011\*Muo/cond);

Q\_011=(w\_011^3)\*(Muo^2)\*eo\*er\*(b^3)\*a\*d/(2\*(pi^2)\*Rs\_011\*(2\*a\*d+b\*d+(b^3)/d+2\*(b^3)\*a/(d^2)));

if (fc\_102<=fc\_011)

%The next resonance is TE102%

fprintf("Next Resonance frequency corresponds to TE102 Mode\n");

fprintf("Second Resonance Frequency = %d Hz\n",fc\_102);

Q=(Qd\*Q\_102)/(Qd+Q\_102);

fprintf("Its Quality Factor = %d\n",Q);

fprintf("Its Fractional Bandwidth : %d %% \n", 100/Q);

fprintf("Its Actual Bandwidth : %d Hz\n", fc\_102/Q);

else

%The next resonance is TE011%

fprintf("Next Resonance frequency corresponds to TE011 Mode\n");

fprintf("Second Resonance Frequency = %d Hz\n",fc\_011);

Q=(Qd\*Q\_011)/(Qd+Q\_011);

fprintf("Its Quality Factor = %d\n",Q);

fprintf("Its Fractional Bandwidth : %d %% \n", 100/Q);

fprintf("Its Actual Bandwidth : %d Hz\n", fc\_011/Q);

end

function fc = calcResFreq(a,b,d,m,n,l,cr)

kx=(m\*pi/a);

ky=(n\*pi/b);

B=(l\*pi/d);

k=sqrt((kx^2)+(ky^2)+(B^2));

fc=cr\*k/(2\*pi);

end

References

DAVID M. POZAR, “Microwave Resonators,” in *Microwave engineering*, S.l.: JOHN WILEY & SONS, 2021, pp. 284–288.