

Knapsack Approximation

Omar Youssef - Mahmoud Abdel-Ghaffar - Mohamed Ali

900232621 - 900232587 - 900232794

May 10, 2025

§1 Instance Generation

We generated **500k** diverse knapsack problem instances to evaluate the quality of approximation algorithms. To ensure variation and coverage of all test cases, the dataset includes: 1) Different item set sizes (small to large). 2) Randomized item weights and values. 3) Varying knapsack capacities proportional to total item weights. 4) A mix of item distributions: uniformly random, correlated, and high-value outliers, which tries all possible correlations between the items weight and their value.

§2 Approximation Algorithm

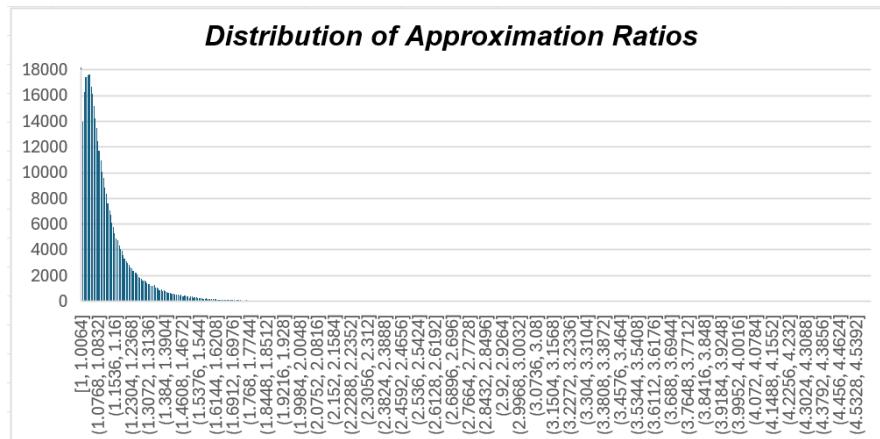
We designed a **greedy approximation** based on a custom parameter:

$$parameter[i] = \frac{value[i]}{\sqrt{weight[i]}}$$

This parameter balances item value against a softened penalty on weight. It often outperforms the traditional value/weight ratio, especially in instances where heavier items carry disproportionately high value. This greedy approach resembles a portion our code that handles cases with large input size (n) and capacity where $n > 30$ and $capacity > 200$. For $n \leq 30$ and $capacity \leq 200$, we find the optimal solution by using the known **DP** approach. Additionally, for $n \leq 15$, we find the exact solution using **Brute Force** by generating all subsets of the items and choosing the one that guarantee maximum profit. This algorithm ensures finding the optimal and exact solutions whenever possible while finding the closest approximation to the exact solutions in most cases.

§3 Performance Analysis

Below is the histogram of the approximation ratio along with statistics summary that elaborate the efficiency of our algorithm.

Figure 1: *Distribution of Approximation Ratios*

<i>Approximation Ratio</i>	
Mean	1.10208
Standard Error	0.00021
Median	1.0588
Mode	1
Standard Deviation	0.14533
Sample Variance	0.02112
Kurtosis	33.9304
Skewness	4.18766
Range	3.6
Minimum	1
Maximum	4.6
Sum	551052
Count	500010

Figure 2: *Summary Statistics*

§4 AI tool conversations

- <https://claude.ai/share/bc3c42ee-3951-4266-a43f-1b6e33fddd4e>
- <https://claude.ai/share/4736b640-bfbf-4491-ad73-98ba9ef18239>
- <https://claude.ai/share/44d48291-1724-44f3-aac1-61bf897cc75d>