

# Discrete-Time Robot Control: Design, Simulation & Comparison

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A comprehensive Python project for designing, implementing, simulating, and comparing control strategies for three discrete-time robot models.

## Models

1. **Integrator Model** - 2D double integrator with additive disturbances
2. **Unicycle Model** - Nonholonomic mobile robot with heading dynamics
3. **Two-Link Manipulator** - Planar robot arm with Euler discretization

## Controllers Implemented

### Linear Controllers

- Proportional (P) Control
- PID Control
- LQR (Linear Quadratic Regulator)
- State-feedback with Pole Placement

### Nonlinear Controllers

- Feedback Linearization
- Backstepping Control
- Sliding Mode Control (SMC)
- Computed Torque (for manipulator)

### Optimal/Constrained

- Model Predictive Control (MPC)

### Data-Driven

- Policy Gradient RL baseline

## Features

- **Lyapunov Stability Analysis:** Symbolic and numerical verification
- **Symbolic Control:** Reachability, avoidance, recurrence via grid abstraction and LTL/Buchi automata
- **Interactive Drawing Canvas:** Click to draw polygonal obstacles and goal regions
- **Animation Export:** Export trajectories as GIF or MP4
- **Automated Testing:** pytest suite with 26 tests
- **Comprehensive Reports:** Automatic markdown reports with numerical metrics and figures

## Interactive Canvas

Launch the interactive drawing canvas with:

```
python interactive_canvas.py
```

Features:

- **Draw obstacles:** Click to add vertices, right-click to finish polygon
- **Draw goal region:** Switch to Goal mode, draw polygon
- **Set start position:** Switch to Start mode, click to place
- **Select controller:** Choose from LQR, Proportional, MPC, or Reach-Avoid
- **Run simulation:** Click Run to see trajectory
- **Export animation:** Save as GIF

Keyboard shortcuts: **o**=obstacle, **g**=goal, **s**=start, **c**=clear, **r**=run

## Installation

```
pip install -r requirements.txt
```

## Quick Start

```
# Run smoke test for integrator model
python run_smoke_test.py

# Run all tests
pytest tests/

# Run full comparison with all models
python run_all.py

# Generate comprehensive comparison report with numerical metrics
python generate_report.py

# Launch interactive drawing canvas
python interactive_canvas.py

# Run interactive demo with user input
python interactive_demo.py
```

## Project Structure

```
├── models/                # Robot model definitions
│   ├── integrator.py     # 2D integrator
│   ├── unicycle.py       # Nonholonomic mobile robot
│   └── manipulator.py    # Two-link planar arm
└── controllers/          # Controller implementations
```

```

├── proportional.py # P control
├── pid.py          # PID control
├── lqr.py          # LQR control
├── mpc.py          # Model Predictive Control
├── rl_policy_gradient.py # REINFORCE RL
├── unicycle/       # Unicycle-specific controllers
├── manipulator/    # Manipulator-specific controllers
├── sim/           # Simulation engine & plotting
│   ├── simulator.py # Core simulation loop
│   ├── plotting.py  # Visualization utilities
│   └── animation.py # GIF/MP4 export
├── symbolic/      # Symbolic control
│   ├── grid_abstraction.py # Space discretization
│   ├── reach_avoid.py      # Reach-avoid planning
│   └── ltl_automata.py     # LTL/Buchi for patrolling
├── analysis/      # Stability analysis
│   └── lyapunov.py    # Lyapunov function computation
├── notebooks/     # Jupyter demos (3 notebooks)
├── tests/         # pytest test suite (26 tests)
├── report/        # Generated reports & figures
├── interactive_canvas.py # Draw obstacles/goals interactively
├── interactive_demo.py  # CLI demo with user input
├── generate_report.py   # Generate comparison report
├── run_all.py          # Run full test suite
├── requirements.txt
├── USAGE_GUIDE.md      # Detailed usage documentation
└── README.md

```

## Model Specifications

Based on discrete-time models from Symbolic\_control\_lecture-7.pdf.

### Model 1: Integrator

- State:  $x = [x_1, x_2]$
- Dynamics:  $x(t+1) = x(t) + \tau(u(t) + w(t))$
- State constraints:  $X = [-10, 10] \times [-10, 10]$
- Input constraints:  $U = [-1, 1] \times [-1, 1]$
- Disturbance:  $W = [-0.05, 0.05] \times [-0.05, 0.05]$

### Model 2: Unicycle

- State:  $x = [x_1, x_2, \theta]$
- Input constraints:  $U = [0.25, 1] \times [-1, 1]$
- Disturbance:  $W = [-0.05, 0.05]^3$

### Model 3: Two-Link Manipulator

- State:  $x = [\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2]$
- Parameters:  $m_1=m_2=1.0$  kg,  $\ell_1=\ell_2=0.5$  m,  $g=9.81$  m/s<sup>2</sup>

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