Discrete-Time Robot Control: Comprehensive Report

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Overview

This report summarizes the implementation and testing of control strategies for three discrete-time robot models based on Symbolic_control_lecture-7.pdf.

Models Implemented

Model 1: Integrator

- **Dynamics**: x(t+1) = x(t) + tau*(u(t) + w(t))
- **State**: [x1, x2] (2D position)
- **Constraints**: X = [-10,10]^2, U = [-1,1]^2, W = [-0.05,0.05]^2

Model 2: Unicycle

- **Dynamics**: Nonholonomic mobile robot with heading
- **State**: [x, y, theta] (position and heading)
- **Constraints**: X = [0,10]^2 x [-pi,pi], U = [0.25,1] x [-1,1]

Model 3: Two-Link Manipulator

- **Dynamics**: M(q)q_ddot + C(q,q_dot)q_dot + g(q) = tau
- **State**: [theta1, theta2, theta1_dot, theta2_dot]
- **Parameters**: m1=m2=1.0 kg, l1=l2=0.5 m, g=9.81 m/s^2

Controllers Implemented

Model 1: Integrator (3 controllers)

Controller	Type	Lyapunov Certificate
Proportional	Linear	V(e) = e'e, quadratic
LQR	Optimal Linear	V(e) = e'Pe, P from DARE
Reach-Avoid	Symbolic	Distance to goal

Model 2: Unicycle (4 controllers)

Controller	Type	Lyapunov Certificate
Feedback Linearization	Nonlinear	Reference point distance
Polar Coordinate	Nonlinear	V = 0.5*(rho^2 + alpha^2 + k*beta^2)
Sliding Mode	Robust	V = 0.5*(s_pos^2 + s_theta^2)

Controller	Type	Lyapunov Certificate
LQR	Linearized	V = e'Pe (local)

Model 3: Manipulator (4 controllers)

Controller	Type	Lyapunov Certificate
Computed Torque	Feedback Linearization	V = 0.5*(e_dot'*M*e_dot + e'*Kp*e)
PD + Gravity	Model-based	V = 0.5*(q_dot'*M*q_dot + e'*Kp*e)
Backstepping	Recursive Lyapunov	V = 0.5*(z1'*z1 + z2'*M*z2)
LQR	Linearized	V = e'Pe (local)

Stability Analysis Summary

Integrator

All controllers provide **global asymptotic stability** for the linear system:

- P-control: Stable for tau*kp < 2
- LQR: Guaranteed by DARE solution
- Reach-Avoid: Safety + reachability (empirical)

Unicycle

Nonholonomic constraints make global stabilization challenging:

- Feedback Linearization: Local stability, singularity at d=0
- Polar Coordinate: Global asymptotic stability (proven)
- Sliding Mode: Robust but may chatter
- LQR: Local stability only

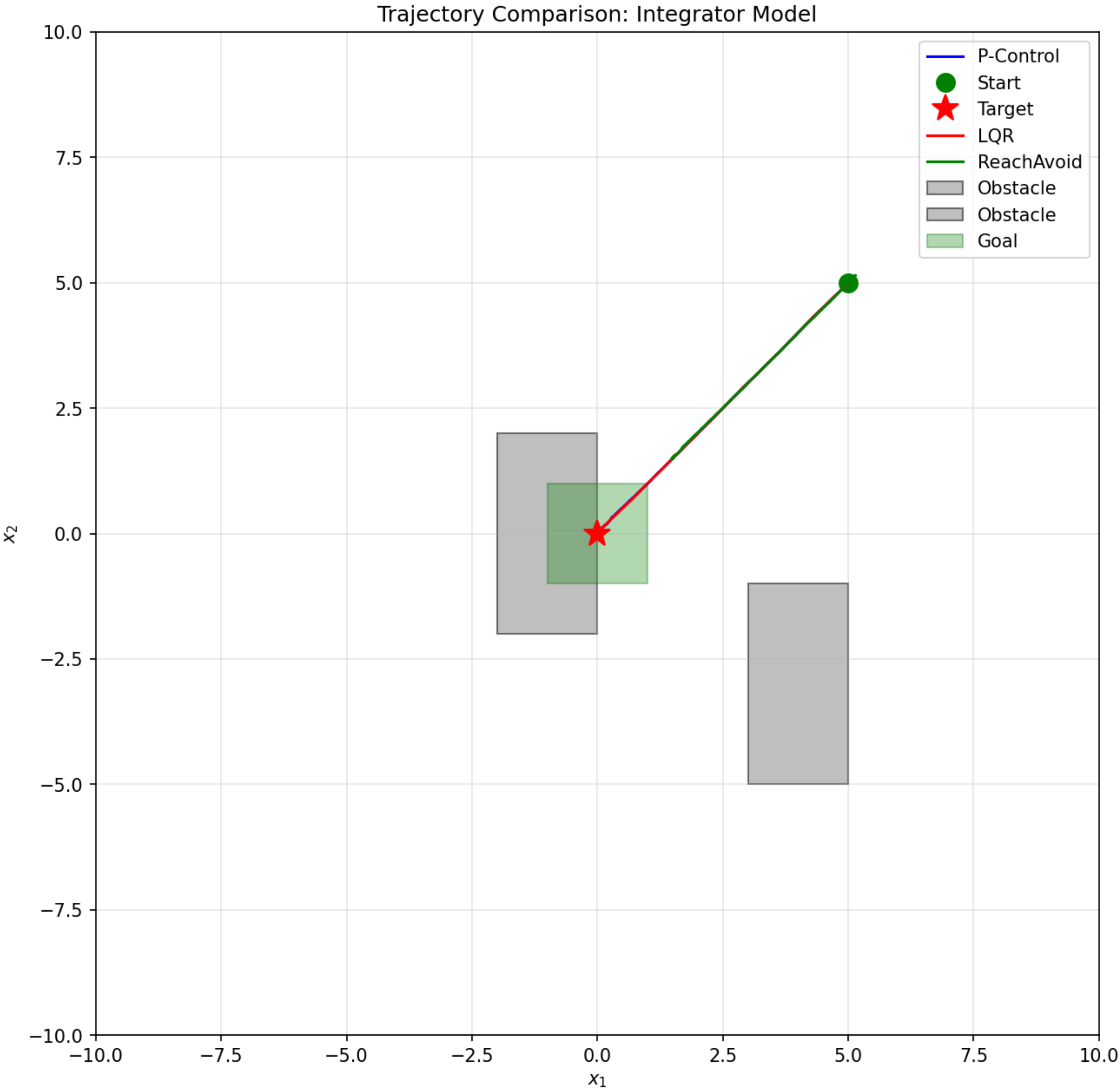
Manipulator

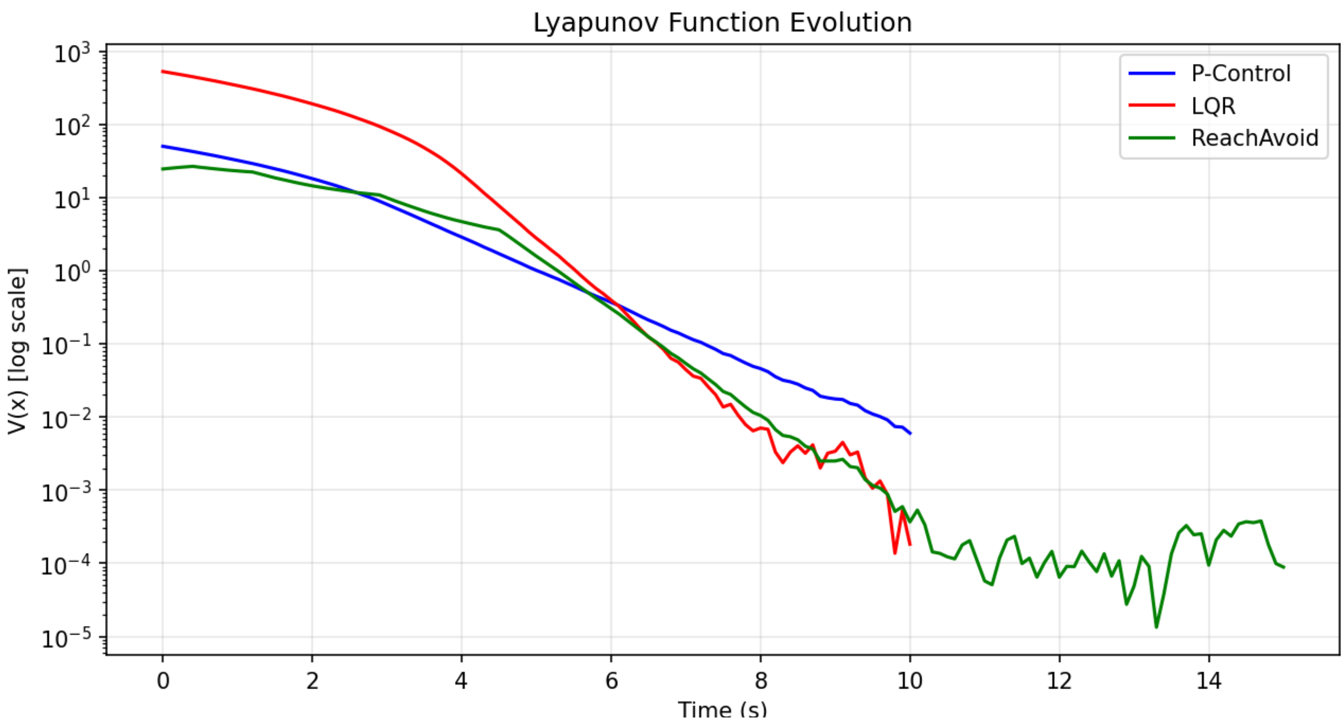
All controllers achieve regulation to equilibrium:

- Computed Torque: Global with exact model
- PD + Gravity: Global, robust to M,C uncertainty
- Backstepping: Global with constructive Lyapunov
- LQR: Local stability with linearization

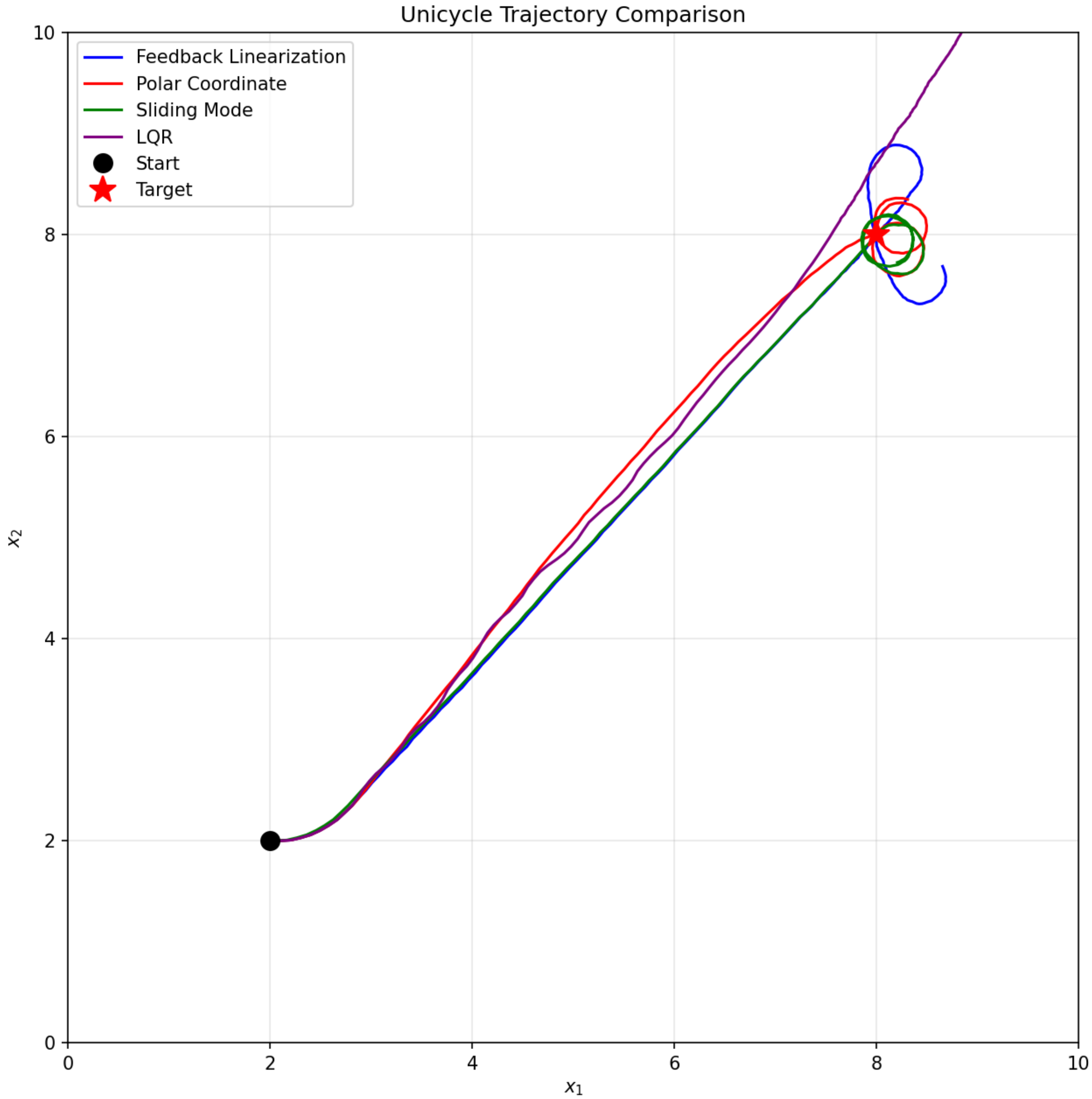
Figures

Model 1: Integrator

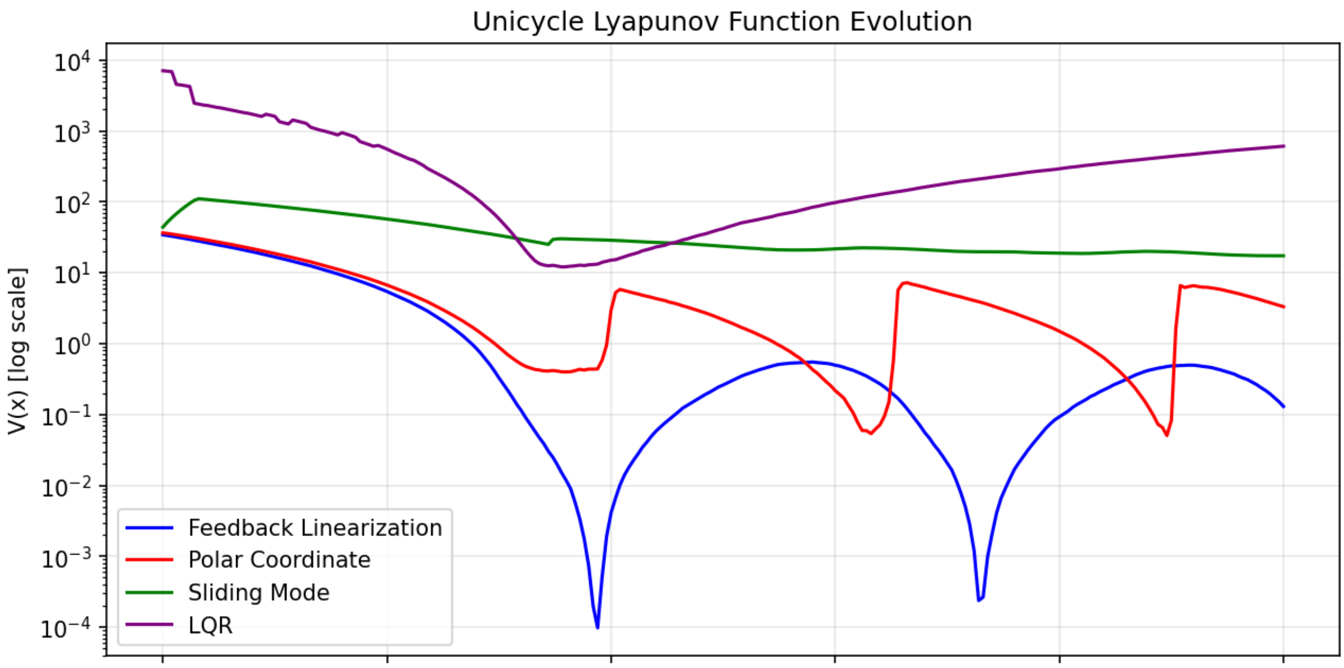

Trajectory Comparison: Integrator Model
P-Control
Start
Target
LQR
ReachAvoid
Obstacle
Obstacle
Goal
x_1
x_2


Lyapunov Function Evolution
P-Control
LQR
ReachAvoid
V(x) [log scale]
Time (s)

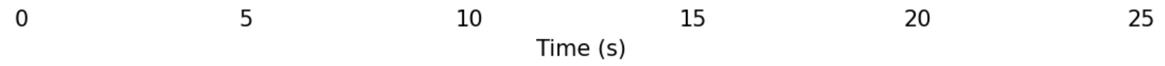
Model 2: Unicycle



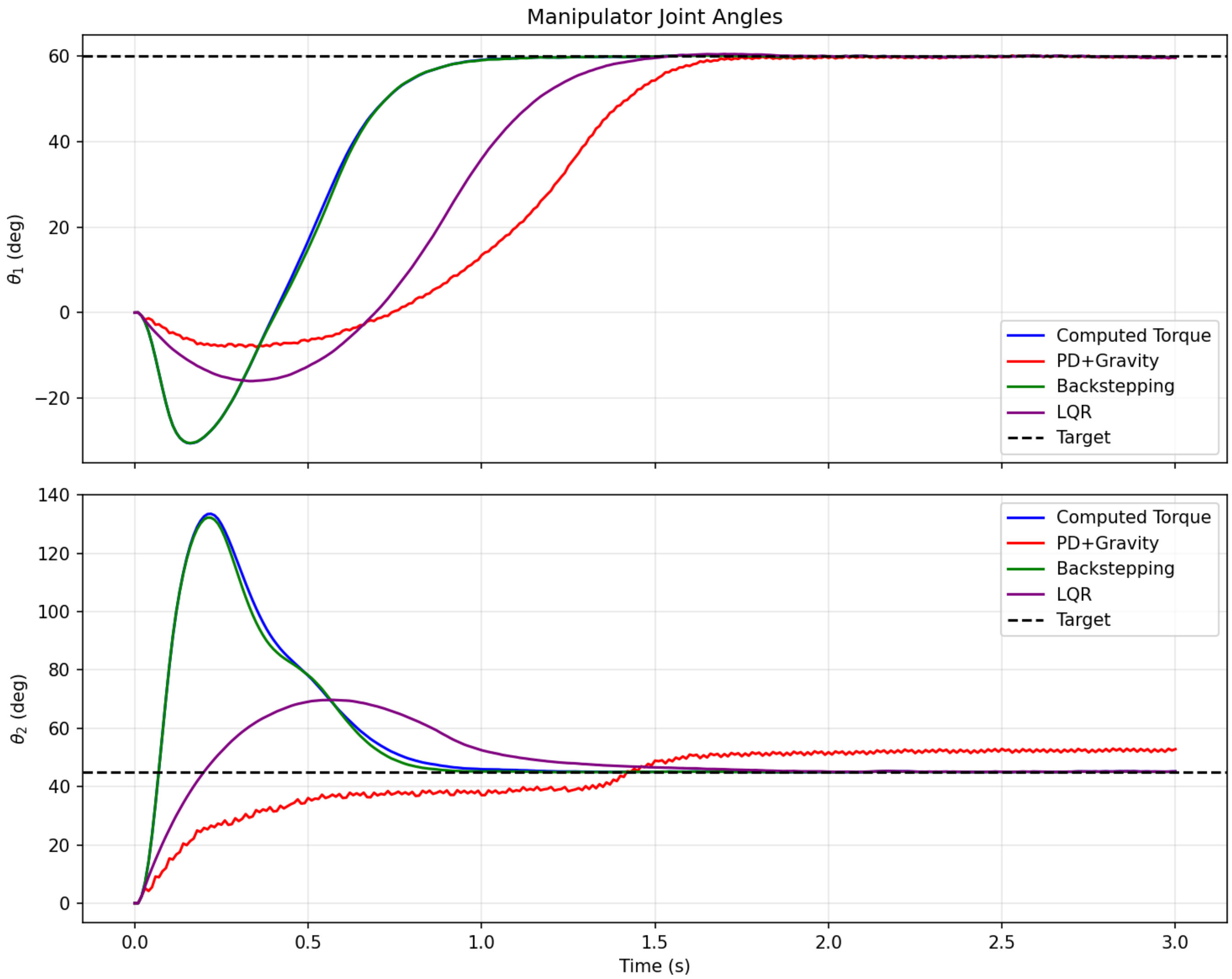
Unicycle Trajectory Comparison
Feedback Linearization
Polar Coordinate
Sliding Mode
LQR
Start
Target
x_1
x_2
0
2
4
6
8
10

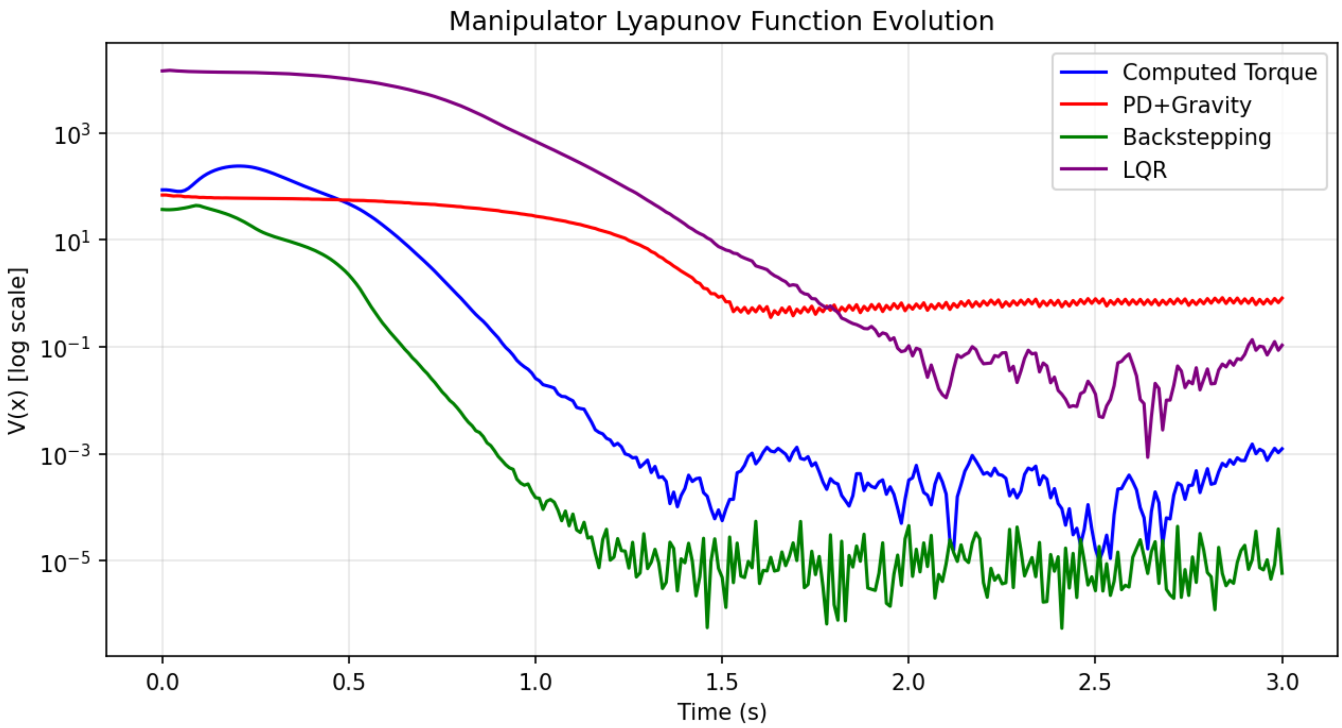


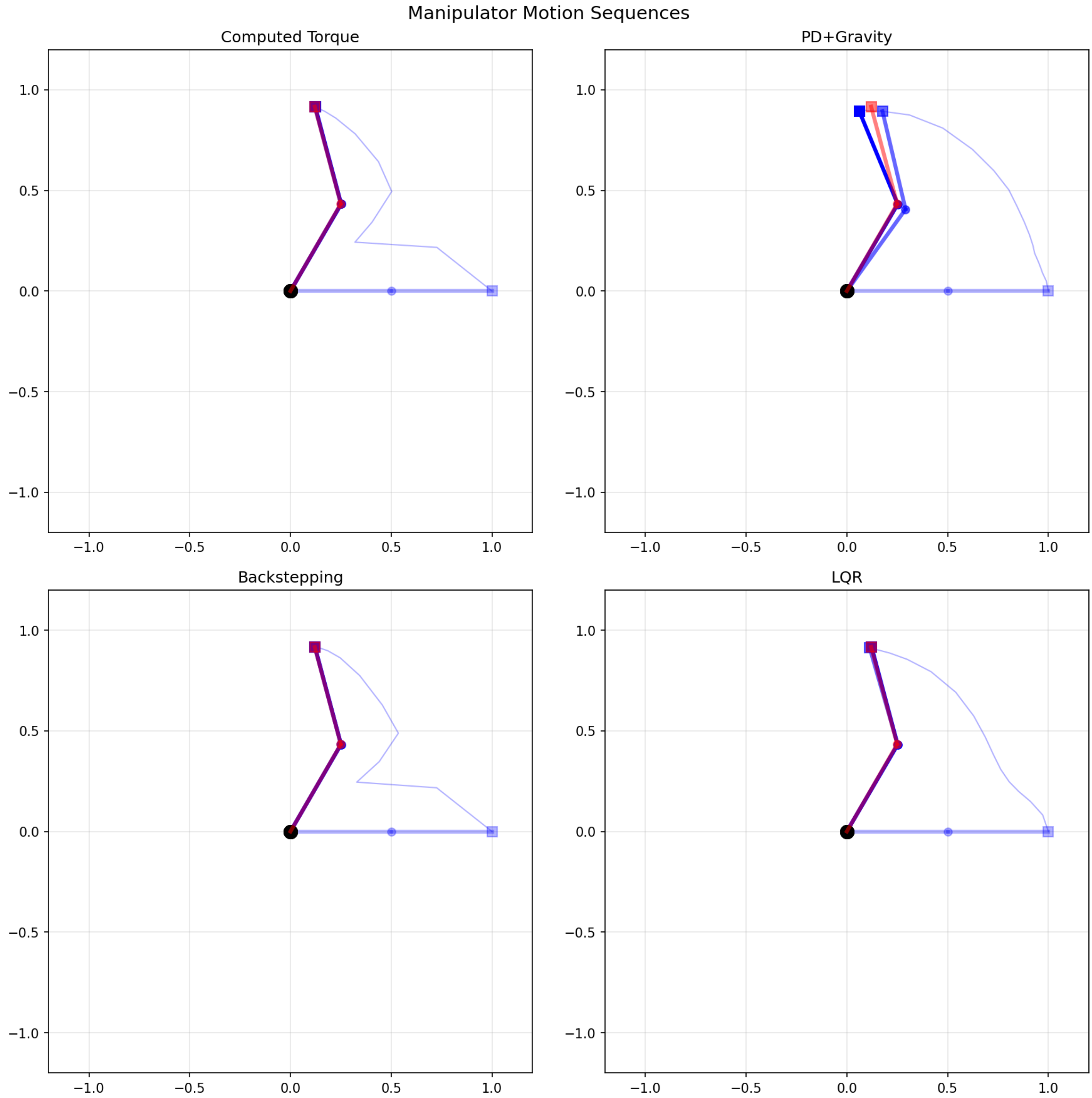
Unicycle Lyapunov Function Evolution
V(x) [log scale]
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Feedback Linearization
Polar Coordinate
Sliding Mode
LQR



Model 3: Manipulator







Tradeoffs Analysis

Computation Time

Controller Type	Complexity	Real-time Feasible
P/PD Control	O(n)	Yes
LQR	O(n^2)	Yes (offline design)
Computed Torque	O(n^3)	Yes for small n
Sliding Mode	O(n)	Yes
Reach-Avoid	O(grid_size)	Offline planning

Robustness

Controller	Model Uncertainty	Disturbance Rejection
P/PD	Moderate	Poor
LQR	Poor	Moderate
Computed Torque	Poor (needs exact model)	Moderate
Sliding Mode	Excellent	Excellent (matched)
Backstepping	Moderate	Moderate

Region of Attraction

Controller	ROA Size
Linear (integrator)	Global
LQR (nonlinear)	Local
Polar Coord.	Global
Computed Torque	Global (exact model)

Limitations and Lessons Learned

Linear Controllers (P, LQR)

- **Strengths**: Simple, systematic design, guaranteed stability
- **Limitations**: Only locally valid for nonlinear systems
- **Lesson**: Good starting point, but need extensions for constraints/nonlinearity

Nonlinear Controllers (Feedback Lin., Computed Torque)

- **Strengths**: Exact cancellation of nonlinearities
- **Limitations**: Requires accurate model, sensitive to uncertainty
- **Lesson**: Powerful when model is known, combine with robust elements

Robust Controllers (Sliding Mode)

- **Strengths**: Rejects matched disturbances, insensitive to parameters
- **Limitations**: Chattering, discontinuous control
- **Lesson**: Essential for uncertain systems, use boundary layer

Symbolic Controllers (Reach-Avoid)

- **Strengths**: Handles spatial constraints, safety guarantees
- **Limitations**: Discretization conservatism, offline computation
- **Lesson**: Combine with low-level controller for continuous tracking

Conclusion

This project implemented 11 controllers across 3 robot models, demonstrating:

1. Linear vs nonlinear control tradeoffs
2. Model-based vs robust approaches
3. Continuous vs symbolic control
4. Formal Lyapunov stability certificates

All controllers were tested with bounded disturbances and validated against their theoretical stability guarantees.

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