

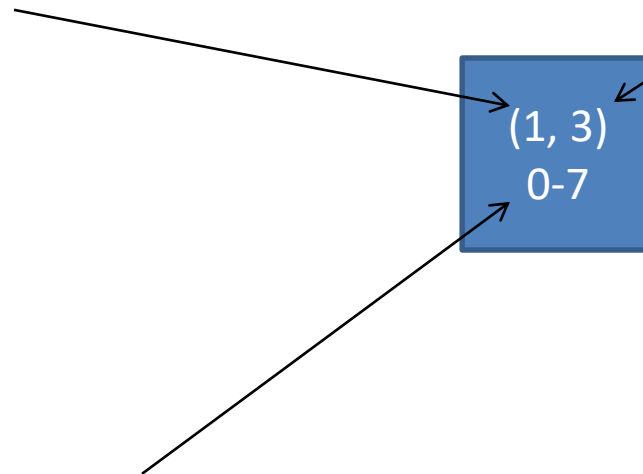
# Segment Tree

- Assume  $N = 8 \rightarrow$  values  $[0-7]$
- Then we will need 3 levels ( $2^3$ ). Total nodes 15 ( $-1+2^4$ ). We build an array for these nodes.
- For an easier code, we start indexing in the segment tree array from 1.
- Assume initial input is array: 2 5 6, the leaves of tree

# Let's represent tree node

First number represents  
Position in segment array  
Root start from 1

Second is the # of numbers  
in this interval = 3



Each node has an interval that it covers.  
0-7 is this node interval  
Typically root node is **whole range**  
and leaf node is interval of a **specific index**

(1, 0)  
0-7

Build the initial tree in  
Top down approach

Left child pos = is  $2 * \text{parent pos}$

Right child pos = is  $1 + 2 * \text{parent pos}$

Left child interval =  
 $(\text{start}, (\text{start}+\text{end})/2)$

Right child interval =  
 $(1+(\text{start}+\text{end})/2, \text{end})$

(2, 0)  
0-3

(3, 0)  
4-7

Keep branching till  $\text{start}=\text{end} \rightarrow$  branch node.  $0 \leq \text{start} < N$ . start is an input array index

0

1

2

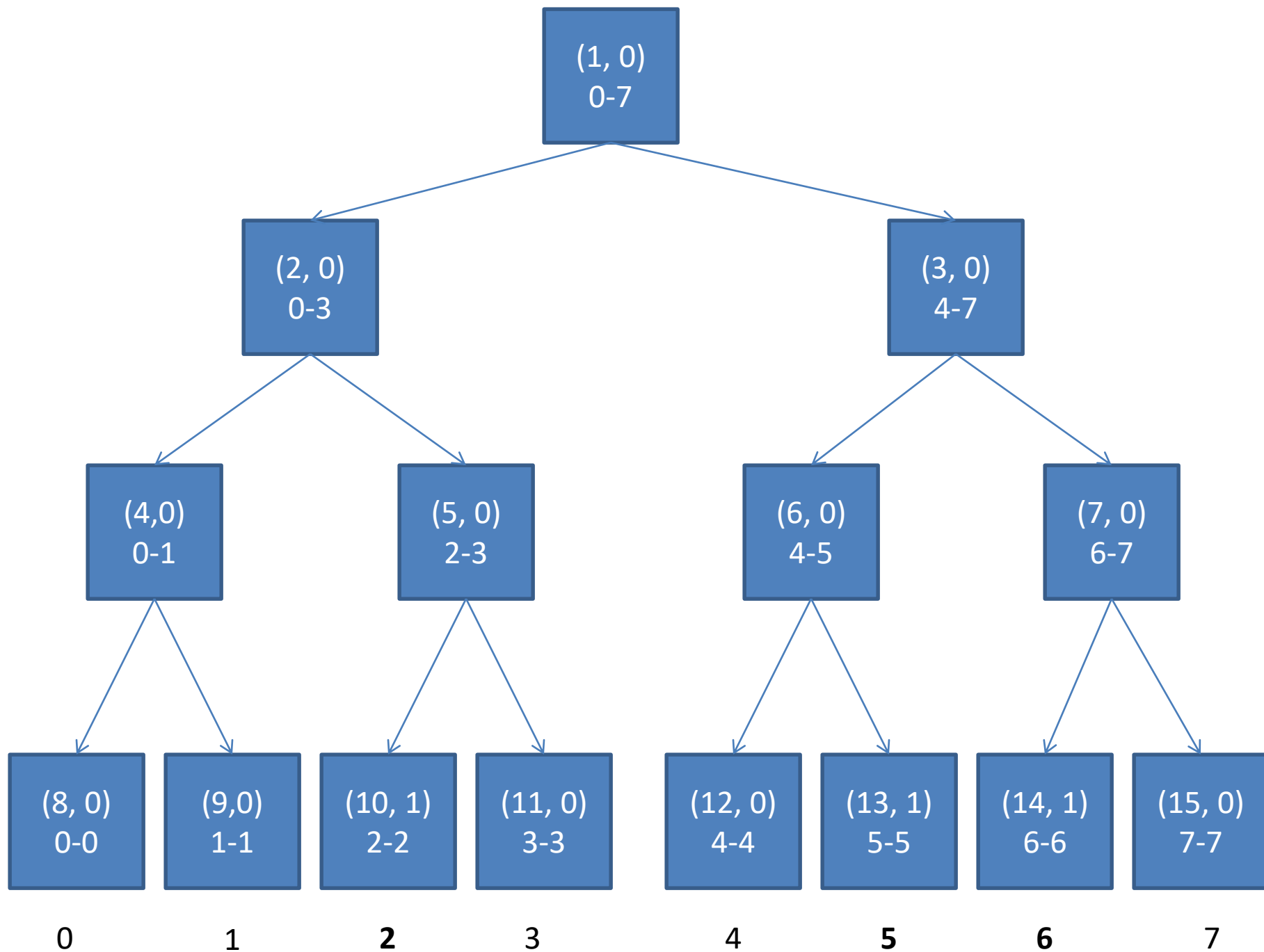
3

4

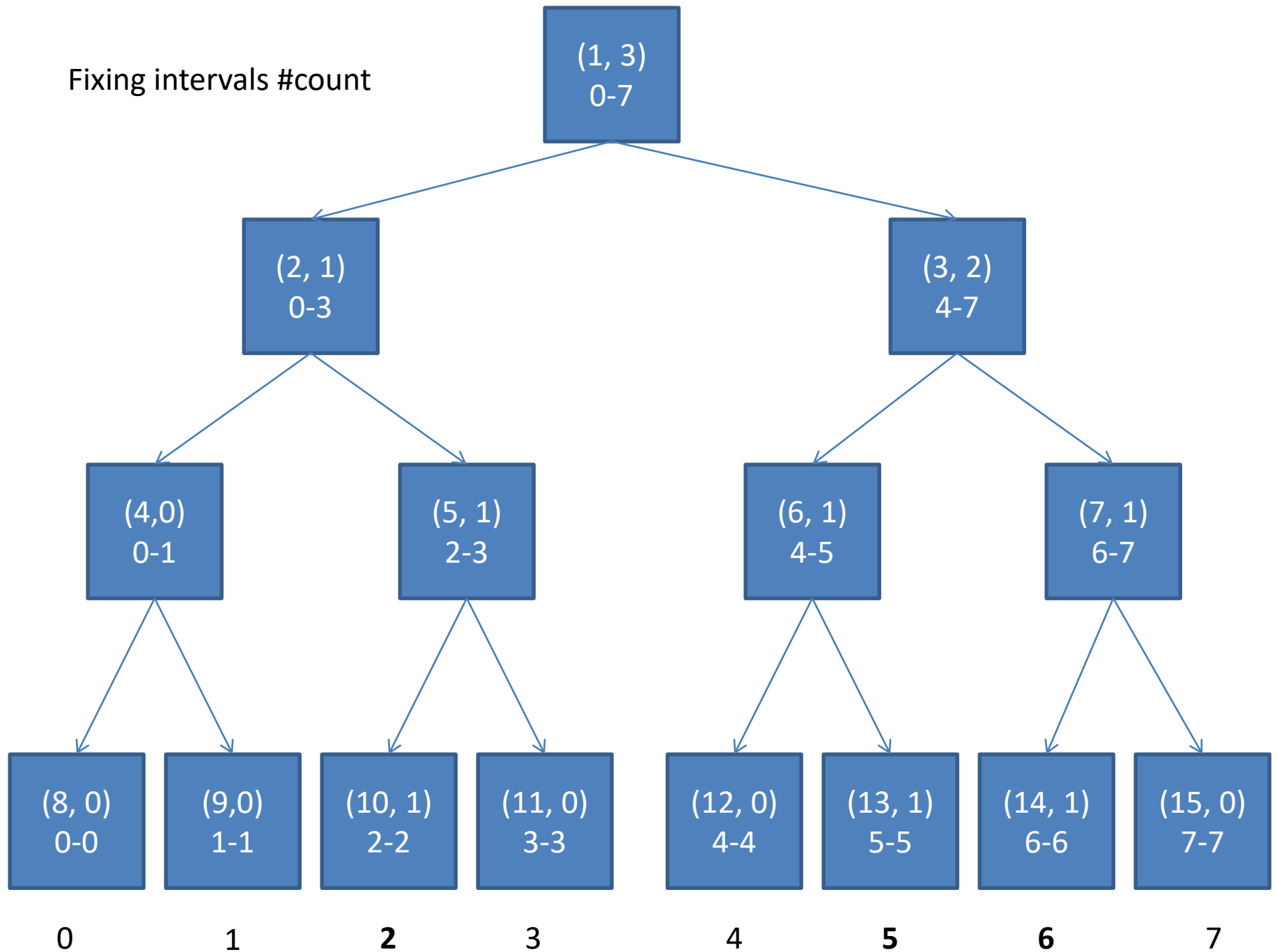
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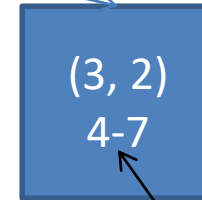
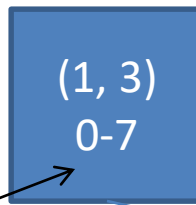
6

7



Fixing intervals #count





We have 3 numbers  
in range [0-7]

Generally, if we have N nodes, then we have N intervals  
Each count is HOW many numbers inside this range

What about an Adhock interval! E.g. interval (2, 6)?

Any interval could be constructed from merge of others!

We have 2 numbers  
in range [4-7]

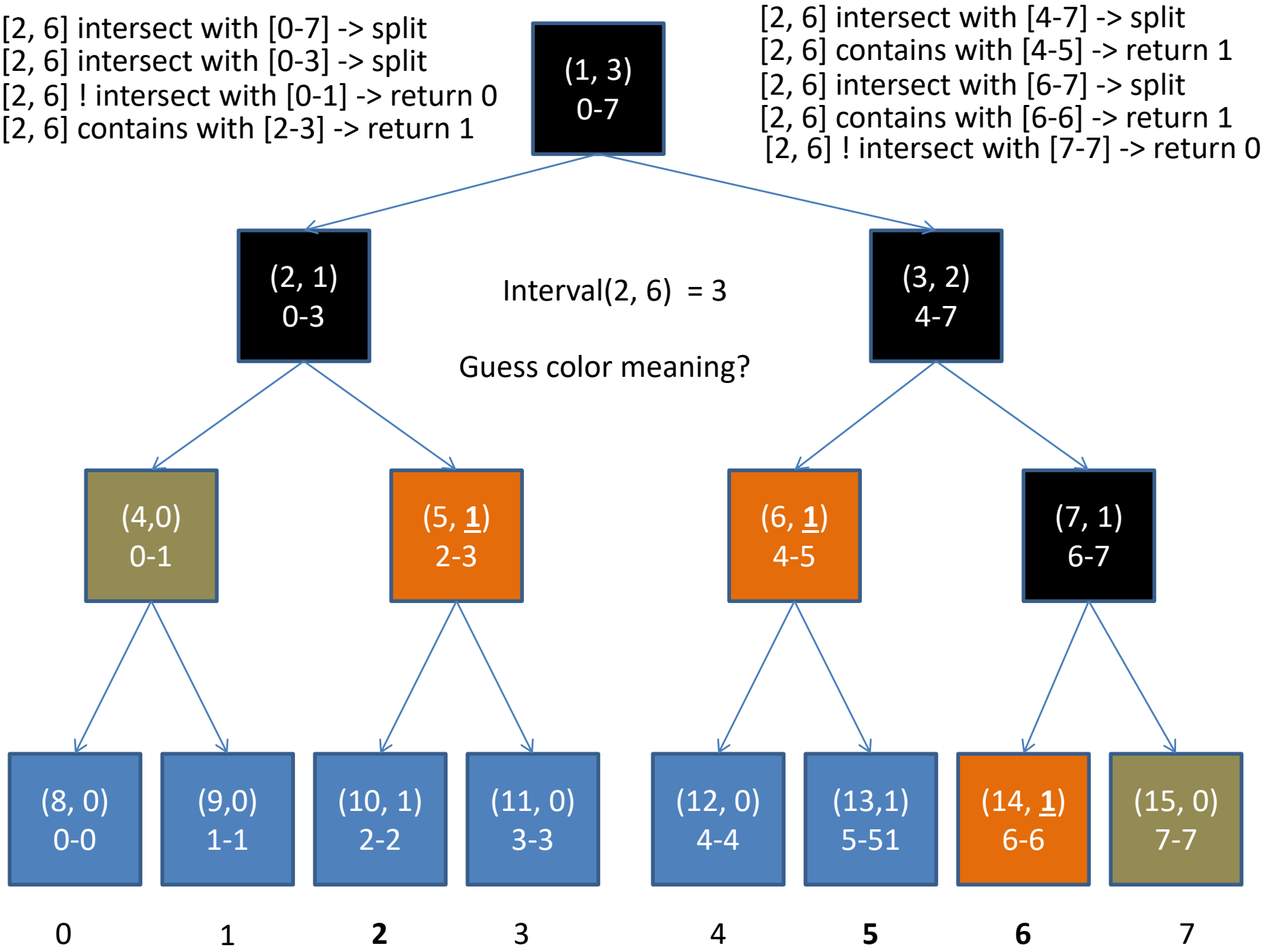
$$(2, 6) = (2, 3) + (4, 5) + (6, 6)$$

So smartly, move top down to sum these ranges

If no intersect, return 0

If interval is part of me, return its count.

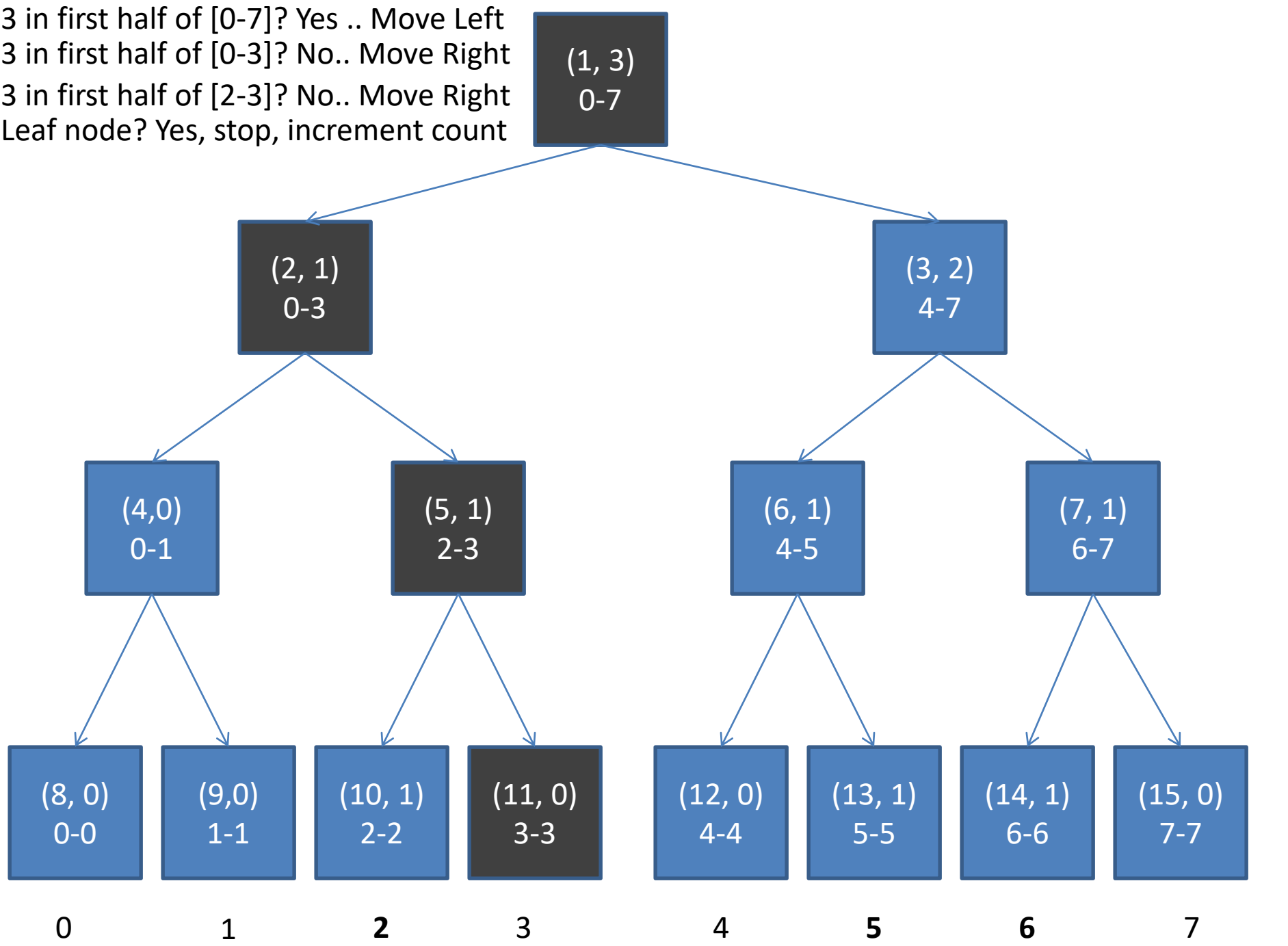
If not, call left and call right



- In last Interval **Query**, we have to **split** to 2 calls, but we stop once found covering interval  
→  $O(n)$
- Note that, building the initial segment array needs accessing all nodes, which are  $n \log n$ , So Order  $O(n \log n)$ .
- What about inserting new number? E.g. 3
- We just need to find the interval that contains 3, and update it. So  $O(\log n)$  only



3 in first half of [0-7]? Yes .. Move Left  
3 in first half of [0-3]? No.. Move Right  
3 in first half of [2-3]? No.. Move Right  
Leaf node? Yes, stop, increment count



Count update

