



Competitive Programming

From Problem 2 Solution in $O(1)$

Algebra

Number Bases and Polynomials

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Number Bases

- Base 10 (Decimal) is the usual base in our life
- Base 2 (Binary) is in computer world (on/off)
- Base 8 (Octal): Christmas is Halloween?
- Base 16 (Hexadecimal) with A=10...F=15
- Base K has K digits: 0, 1, 2....K-1
- We can represent any base!
 - Up to 36, you can use Alphanumeric (A-Z), 26 letters
 - $X = T_n * b^n + \dots + T_2 * b^2 + T_1 * b^1 + T_0 * b^0$ (base b)
 - $5736 = 5 * 10^3 + 7 * 10^2 + 3 * 10^1 + 6 * 10^0$ (base 10)

Number Bases

Note : to convert from decimal to :
decimal, take sum all numbers as power of 2
Octal, take sum of every 3 numbers as power of 2
Hexadecimal, take sum of every 4 numbers as power of 2

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Octal	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

Src: Discrete Mathematics and Its Applications - Kenneth Rosen

Decimal Base

- $5736 = 5*10^3 + 7*10^2 + 3*10^1 + 6 * 10^0$
- $5736 = ((5*10+7)*10+3)*10+6$
- $5736 \% 10 = 6$ (**get** last digit)
- $5736 / 10 = 573$ (**remove** last digit)
- All properties we do in base 10, are same
 - $2AF3 = 2*16^3 + A * 16^2 + F * 16^1 + 3 * 16^0$
 - $2AF3 = 2*16^3 + 10 * 16^2 + 15 * 16^1 + 3 * 16^0$
 - $2AF3 = 10995$
- To convert X in base A to Base B
 - Convert X to base 10, then from base to base B

Bases Conversion

- To convert any base to decimal, just evaluate it: $2AF3 = (((2*16 + 10) * 16) + 15) * 16 + 3$
 - This evaluation is **faster** than computing 16^i for each term. Its ideas based on **Horner's** method
- To convert decimal to any base
 - Remember: $X = T_n*b^n + \dots + T_2*b^2 + T_1*b^1 + T_0*b^0$
 - $522 = a*16^2 + b*16 + c$ (in base 16)
 - Then $522 \% 16 = 0 + 0 + c$
 - Then $522 / 16 = a*16 + b + 0$
 - Fact: Get digit by $\%base$, remove it by $/base$

Bases Conversion

very Important

```
string letters = "0123456789ABCDEF";
int toInt(char c) {    return letters.find(c);    }

int FromAnyBasetoDecimal(string in, int base) {
    int res = 0;
    for (size_t i = 0; i < in.size(); ++i)
        res *= base, res += toInt(in[i]);
    return res;
}

string FromDecimaltoAnyBase(int number, int base) {
    if (number == 0)
        return "0";

    string res = "";

    for (; number; number /= base)
        res = letters[number % base] + res;
    return res;
}
```

<https://www.geeksforgeeks.org/convert-base-decimal-vice-versa/#>

Hex - Binary Conversions

- You can use previous way always. Just as fact
- Hex is 16 base. Each digit can be converted to 4 binary bits
 - E.g. 6 => 0100, F => 1111
- From Hex to Binary: replace each digit with 4 bits
- From Binary to Hex: replace each 4 bits (from left) to 1 Hex
 - 1011110101101010010 = 101 1110 1011 0101 0010
 - 1011110101101010010 = 5 E B 5 2 = **5EB52**

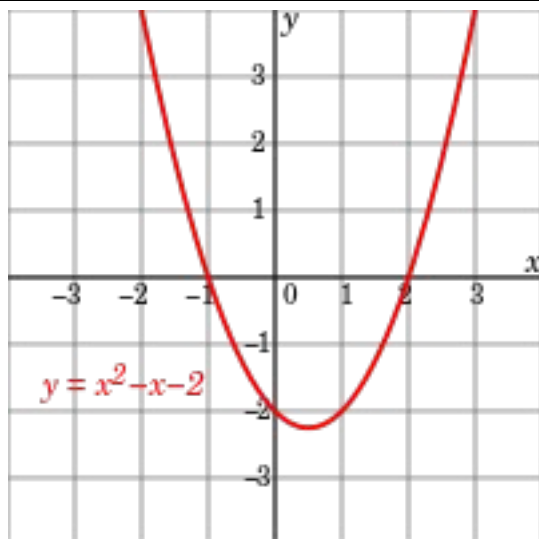
Polynomial

- Expression of variables and coefficients with the +, -, *, and non-negative integer exponents
- $x, x^2 - 4x + 7, x^3 + 2xyz^2 + yz + 1$
- $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0, \sum_{i=0}^n a_i x^i$
- To evaluate polynomial, just evaluate terms
- $f(x) = x^3 - x$ $f(2) = 8 - 2 = 6$
- $f(x, y) = 2x^3 + 4x^2 y + xy^5 + y^2 - 7.$ $f(3, 2) = ?$
- Polynomial degree is the highest term
 - Degree zero $f(x) = 0$ is just axis.
 - Degree 1 is line equation: $f(x) = 2x + 3$

Graphs of polynomial functions

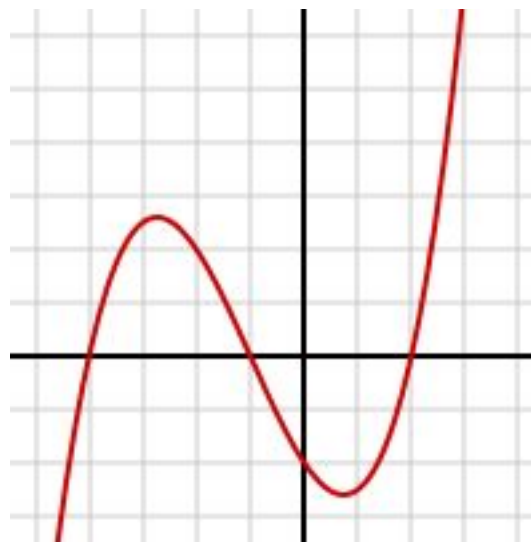
$$(x + 1)(x - 2)$$

2nd degree (parabola)



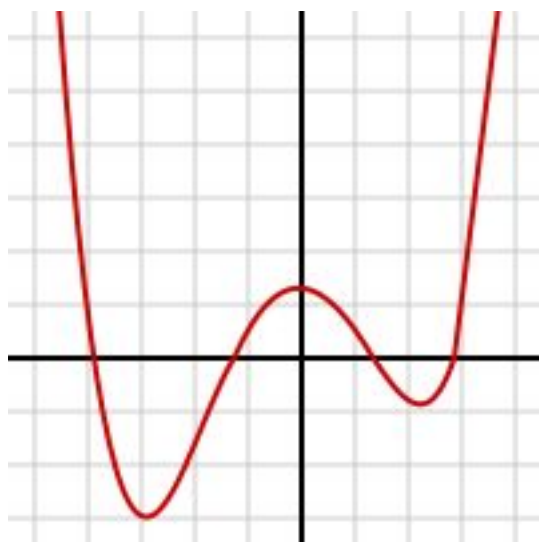
$$\frac{1}{4}(x + 4)(x + 1)(x - 2)$$

3rd degree



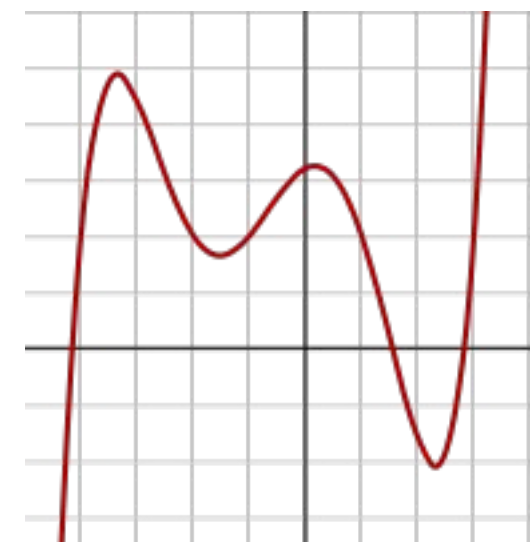
$$\frac{1}{14}(x + 4)(x + 1)(x - 1)(x - 3) + 0.5$$

4th degree



$$\frac{1}{20}(x + 4)(x + 2)(x + 1)(x - 1)(x - 3) + 2$$

5th degree



Arithmetic of polynomials

$$\begin{array}{l} P = 3x^2 - 2x + 5xy - 2 \\ Q = -3x^2 + 3x + 4y^2 + 8 \end{array} \quad \begin{array}{l} P + Q = 3x^2 - 2x + 5xy - 2 - 3x^2 + 3x + 4y^2 + 8 \\ P + Q = x + 5xy + 4y^2 + 6 \end{array}$$

$$\begin{array}{l} P = 2x + 3y + 5 \\ Q = 2x + 5y + xy + 1 \end{array} \quad \begin{array}{l} PQ = (2x \cdot 2x) + (2x \cdot 5y) + (2x \cdot xy) + (2x \cdot 1) \\ \quad + (3y \cdot 2x) + (3y \cdot 5y) + (3y \cdot xy) + (3y \cdot 1) \\ \quad + (5 \cdot 2x) + (5 \cdot 5y) + (5 \cdot xy) + (5 \cdot 1) \end{array}$$

$$PQ = 4x^2 + 21xy + 2x^2y + 12x + 15y^2 + 3xy^2 + 28y + 5$$

Solving polynomial equations

- To solve it (find roots) let $f(x) = 0$
 - E.g. find values that makes evaluation is zero
 - In other words, f meets the x -axis
- Linear case is trivial
 - $f(x)=2x - 6 = 0 \Rightarrow 2x = 6 \Rightarrow x = 6 / 2 = 3$
 - $f(x)=a(x-3)-a=0 \Rightarrow a(x-3) = a \Rightarrow x-3 = 1$ (Wrong $a = 0$?)
- 2nd degree (Quadratic equation)
 - $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 - $a \neq 0$. $a > 0$ graph opens up. $a < 0$ opens down
 - Min/Max point at $x = -b/2a$
- Practice for on solving equations

Polynomials: Misc

- Monomial is 1 term ($3xy$), Binomial is 2 terms ($4x+3$), Trinomial is 3 terms ($x+y^2-5$)
- $3x^2-6X-24 \Rightarrow$ Factorize $\Rightarrow (x-4)(3x+6)$
 - Roots: $x = 4, x = -2$
 - $x^2-5x \Rightarrow x(x-5) \Rightarrow$ roots $x = 0, x = 5$
- Derivative $aX^b = a*b*X^{(b-1)}$
 - Derivative $(3x^2-6X-24) = 3*2*X - 6 = 6X-6$
- Homework - Write code that:
 - represents and evaluates a polynomial in $O(n)$
 - multiply 2 polynomials / computes polynomial derivative
 - solves the quadratic equation

Polynomials: More later to study

- Some hard problems need more knowledge
- Learn how to solve roots of polynomial generally (integer roots, real roots)
- Some problems can be solved by Lagrange Interpolation

تم بحمد الله

علمكم الله ما ينفعكم

ونفعكم بما تعلمتم

وزادكم علماً