ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

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Talk Overview

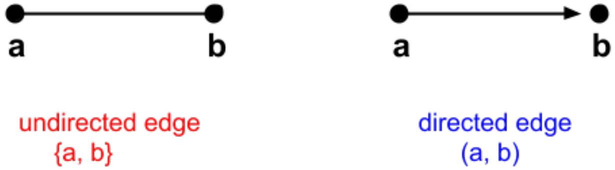
1. Inroduction to graphs
2. Graph representation
3. Graph isomorphism
4. Walks, trails, circuits, paths, and cycles

Outline

Undirected Graphs

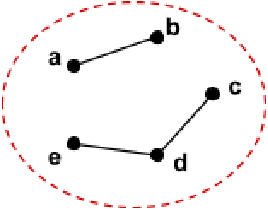
Undirected graph

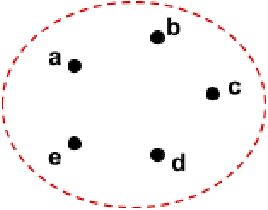
In an undirected graph, the edges are unordered pairs of vertices, which is useful for modeling relationships that are symmetric.



Example

Can you list the vertices set and the edges set of the following two graphs?



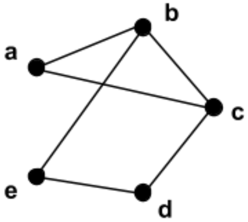


Adjacent/Neighbours Vertices

Adjacent/Neighbours Vertices

Two vertices are said to be adjacent (neighbours) if there is an edge between them.

Ex.



- a and b are neighbours.
- b and c are neighbours.

Parallel Edges and Self Loops

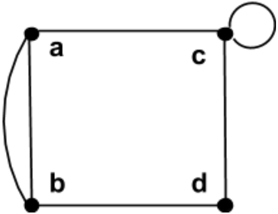
Parallel edges

Parallel edges are multiple edges between the same pair of vertices.

Self Loop

A graph can also have a self-loop which is an edge between a vertex and itself.

Ex.



Simple Graph

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If a graph does not have parallel edges or self-loops, it is said to be a simple graph.

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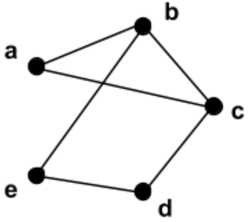

Graph Total Degree and Regular Graphs

Total Degree

The total degree of a graph is the sum of the degrees of all of the vertices.

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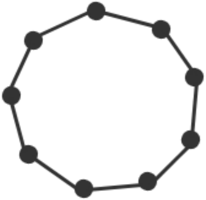
Regular Graph

In a regular graph, all the vertices have the same degree.

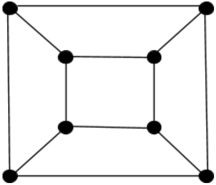
D-regular Graph

In a d-regular graph, all the vertices have degree d.

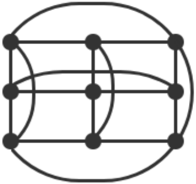
2-regular graph



3-regular graph



4-regular graph



Theorem on Number of Edges and Total Degree

Theorem

Twice the number of edges of a graph is equal to the total degree.

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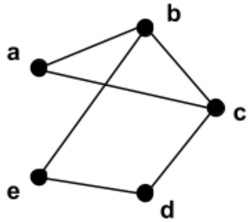
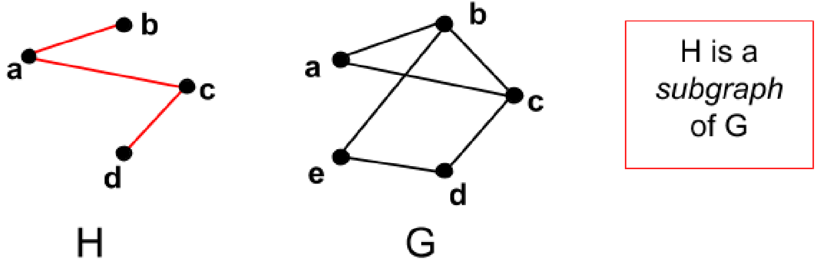
Ex.



- 6 edges in the graph.
- The total degree is 6*2=12

Subgraph

A graph H $= (V_H, E_H)$ is a subgraph of a graph G $= (V_G, E_G)$ if VH ⊆ VG and EH ⊆ EG.

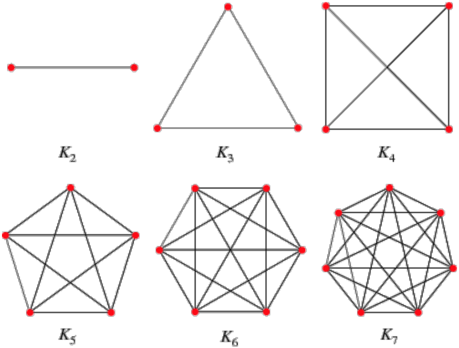


Complete Graphs

Complete Graph

A complete graph k_n with n vertices has an edge between every pair of vertices.

Ex.

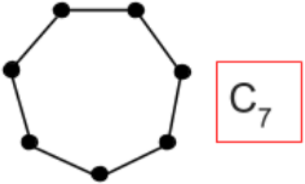


Cyclic Graph

Cyclic Graph

A cyclic graph C_n with n vertices have its edges connect the vertices in a ring shape.

Ex.

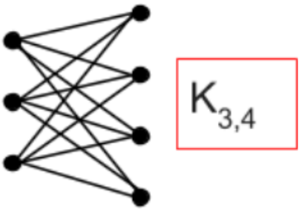


Bipartite Graph

Bipartite graph

$K_{n,m}$ has n+m vertices. The vertices are divided into two sets: one with m vertices and one set with n vertices. There are no edges between vertices in the same set, but there is an edge between every vertex in one set and every vertex in the other set.

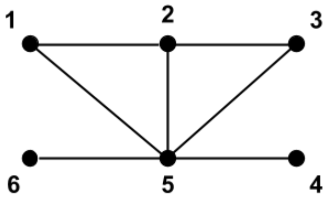
Ex.



Excercise

A graph G is depicted in the diagram on the right.

1. What is the total degree of G?
2. List the neighbors of vertex 5.
3. What is the degree of vertex 6?
4. Which vertices are adjacent to vertex 3?
5. Is G a regular graph? Why or why not?
6. Is K_3 a subgraph of G? If so, name the vertices in the subgraph.
7. Is K_4 a subgraph of G? If so, name the vertices in the subgraph.



Outline

How to represent the graph?

- Are the visual drawings a good way to represent the graph and process it with computer programs?

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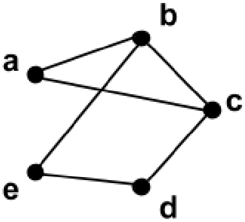

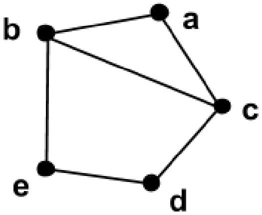

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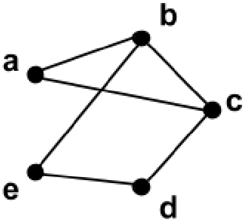

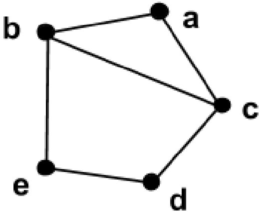

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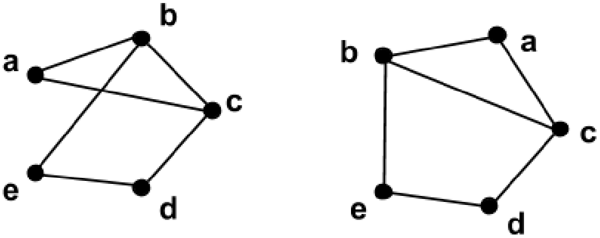


- Can you list the vertices and the edges of both graphs?

$$V = \{a, b, c, d, e\}$$

$$E = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, e\}, \{c, d\}, \{d, e\}\}$$

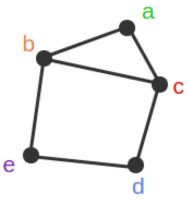
How to represent the graph?

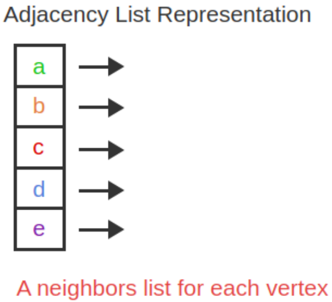
Two standard ways to represent graphs

- Adjacency list representation.
- Matrix representation.

Adjacency List Representation

In the adjacency list representation of a graph, each vertex has a list of all its neighbors.





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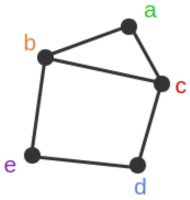
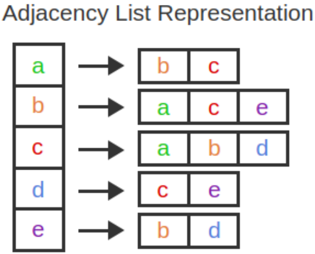
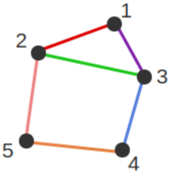




- Is b adjacent to c? Scan b's list to look for c. Vertex c is found in b's list, so yes, b is adjacent to c.
- Worst case time to scan b's list is proportional to deg(b). i.e., O(deg(b)).

Matrix Representation

The matrix representation for a graph with **n** vertices is an **n** by **n** matrix whose entries are all either 0 or 1, indicating whether or not each edge is present.



Matrix Representation

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	0	1
3	1	1	0	1	0
4	0	0	1	0	1
5	0	1	0	1	0

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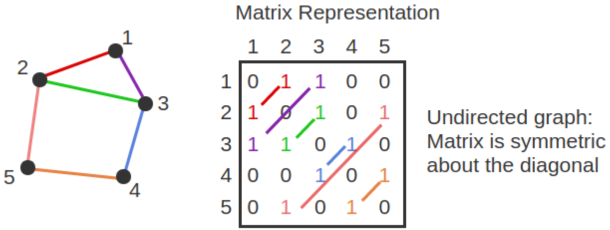

The matrix representation of an undirected graph is symmetric about the diagonal.

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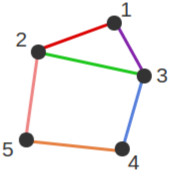

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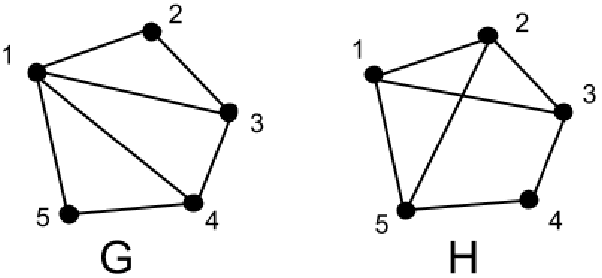
5	0	1	0	1	0

- Is 2 adjacent to 5? Look at , the entry in row 2, column 5. , so the answer is yes.
- O(1) time to answer.

Excercise

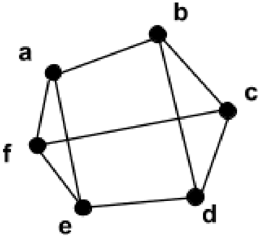
Give the adjacency list and the matrix representation of the below graphs.

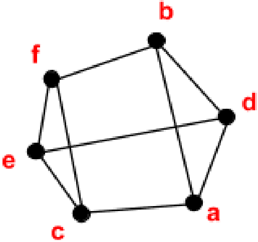


Outline

Graph Isomorphism

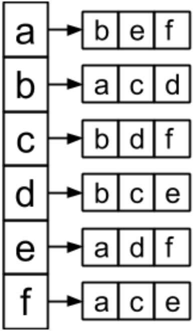
Consider the two similar graphs pictured below.

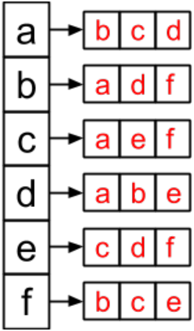




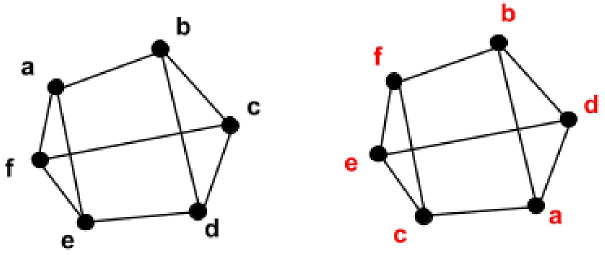
Graph Isomorphism

What about the two graphs adjacency list? Can you claim the similarity?





Graph Isomorphism



Two graphs are said to be isomorphic if there is a correspondence between the vertex sets of each graph such that there is an edge between two vertices of one graph if and only if there is an edge between the corresponding vertices of the second graph and vice versa.

Graph Isomorphism

Graph Isomorphism

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. G_1 and G_2 are isomorphic
if there is a bijection $f : V_1 \to V_2$ such that
for every pair of vertices x, y $\in V_1$ and $\{x, y\} \in E_1$
if and only if $\{f(x), f(y)\} \in E_2$.

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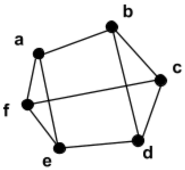
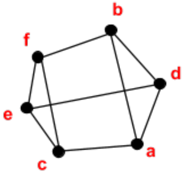
Ex.

- a $\leftrightarrow$ f; b $\leftrightarrow$ b; c $\leftrightarrow$ d; d $\leftrightarrow$ a; e $\leftrightarrow$ c; f $\leftrightarrow$ e;

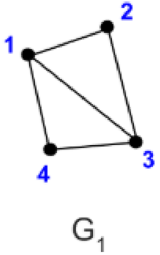


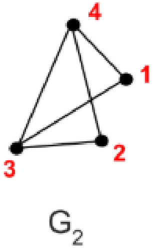


Excercise 1

Which of the following two funcions is an isomorphism from G_1 to G_2

1. $f(1) = 4$; $f(2) = 1$; $f(3) = 2$; $f(4) = 3$.
2. $f(1) = 4$; $f(2) = 1$; $f(3) = 3$; $f(4) = 2$.

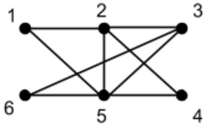


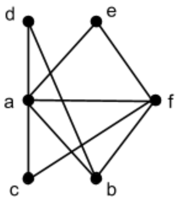


Excercise 2

The below graphs are isomorphic. Which of the following function is isomorphic for the two graphs shown.

1. g(5) = b; g(3) = a; g(2) = e; g(1) = f; g(6) = d; g(4) = c.
2. g(5) = e; g(3) = b; g(2) = f; g(1) = a; g(6) = c; g(4) = d.
3. g(5) = a; g(3) = b; g(2) = f; g(1) = e; g(6) = d; g(4) = c.





Preserved under Isomorphism

A property is said to be preserved under isomorphism if whenever two graphs are isomorphic, one graph has the property if and only if the other graph also has the property. The two properties we will study are:

- Vertex Degree.
- Degree sequence.

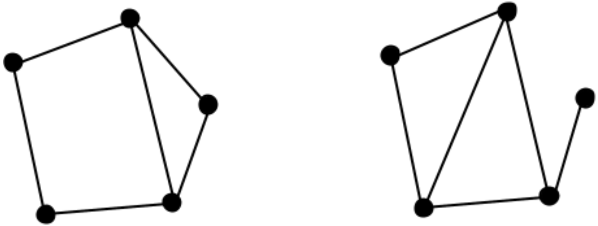
Vertex Degree and Isomorphism

Theorem

Vertex degree is preserved under isomorphism.

Ex.

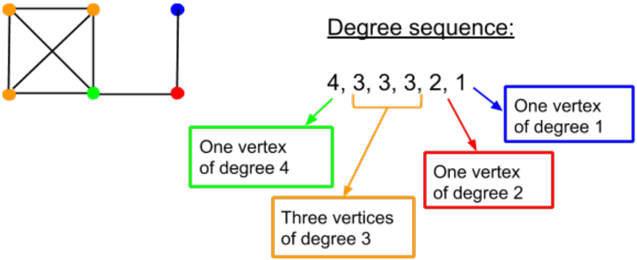
Prove the following two graph are not isomorphic using vertex degree property.



Degree Sequence

The degree sequence of a graph is a list of the degrees of all of the vertices in non-increasing order.

Ex.



Degree Sequence and Isomorphism

Theorem

The degree sequence of a graph is preserved under isomorphism.

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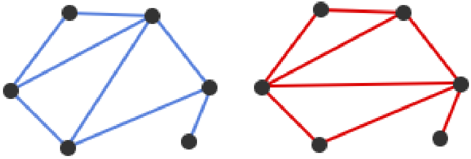

G_1 G_2

Other Properties Preserved Under Isomorphism

- Total degree.
- Total number of edges.
- Total number of vertices.
- Number of vertices whose degree is an even number.
- Number of vertices whose degree is an odd number.
- etc.

Notes on Isomorphism

- Determining graphs isomorphism is hard problem in graph theory.
- Satisfying some graph properties does not mean the two graphs are isomorphic.

Ex.

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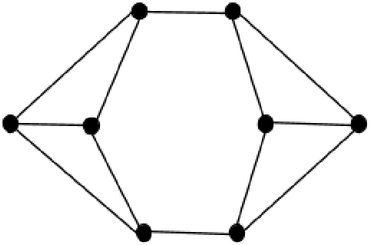

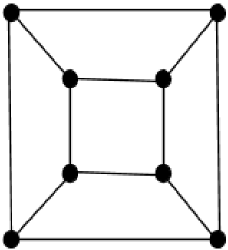

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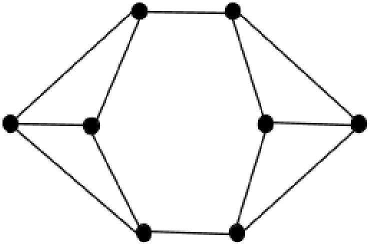
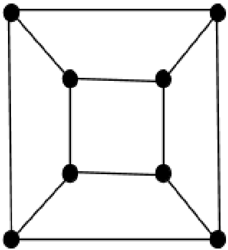
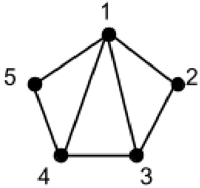


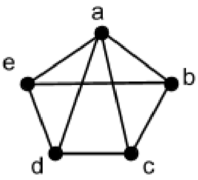


These two graphs have the same vertex degree and vertex sequence properties, however, not isomorphic.

Excercise 1

For the below graphs, show that they are not isomorphic by showing that there is a property that is preserved under isomorphism which one graph has and the other does not.





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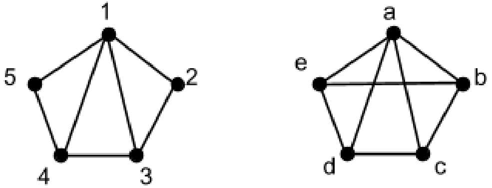
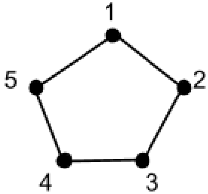
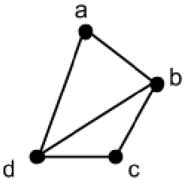


The graph on the left has seven edges and the graph on the right has eight edges.

Excercise 2

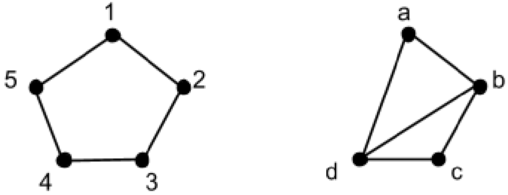
For the below graphs, show that they are not isomorphic by showing that there is a property that is preserved under isomorphism which one graph has and the other does not.





Excercise 2

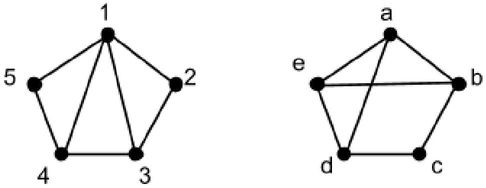
For the below graphs, show that they are not isomorphic by showing that there is a property that is preserved under isomorphism which one graph has and the other does not.



The graph on the left has five vertices and the graph on the right has four vertices.

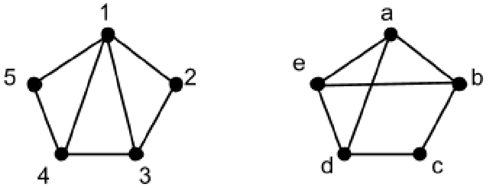
Excercise 3

For the below graphs, show that they are not isomorphic by showing that there is a property that is preserved under isomorphism which one graph has and the other does not.



Excercise 3

For the below graphs, show that they are not isomorphic by showing that there is a property that is preserved under isomorphism which one graph has and the other does not.



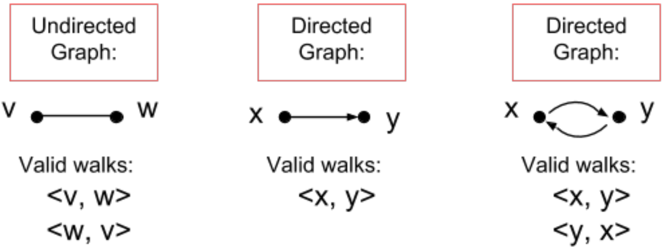
The graph on the left has a degree 4 vertex. The graph on the right does not have a degree four vertex.

Outline

Walks and directed Graph

A walk in a graph G is a sequence of alternating vertices and edges that starts and ends with a vertex.

$$< v_0, v_1, v_2, \ldots, v_n >$$



Open and Closed Walks

Open walk

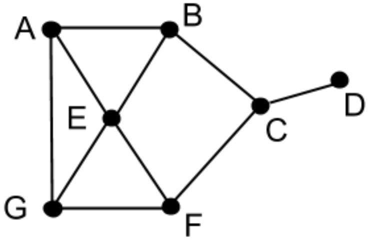
An open walk is a walk in which the first and last vertices are different.

Closed walk

A closed walk is a walk in which the first and last vertices are the same.

Ex.

- <A, E, F, G> Open walk
- <A, E, F, G, A> closed walk



Definations

Trail

A trail is an open walk in which no edge occurs more than once.

Path

A path is a trail in which no vertex occurs more than once.

Circuit

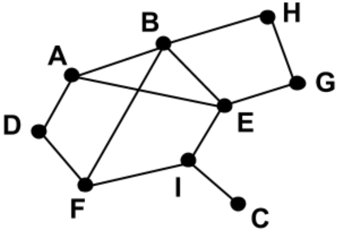
A circuit is a closed walk in which no edge occurs more than once.

Cycle

A cycle is a circuit in which no vertex occurs more than once, except the first and last vertices which are the same.

Excercise

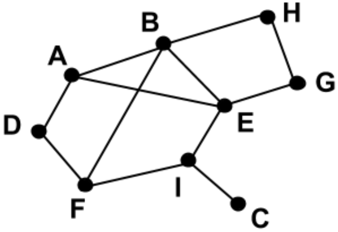
For the following graph.



1. What is the maximum length of a path in the graph? Give an example of a path of that length.

Excercise

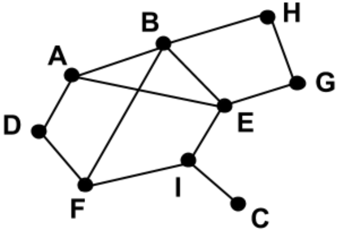
For the following graph.



1. What is the maximum length of a path in the graph? Give an example of a path of that length.
   - 8 <C, I, F, D, A, E, B, H, G>
2. What is the maximum length of a cycle in the graph? Give an example of a cycle of that length.

Excercise

For the following graph.



1. What is the maximum length of a path in the graph? Give an example of a path of that length.
   - 8 <C, I, F, D, A, E, B, H, G>
2. What is the maximum length of a cycle in the graph? Give an example of a cycle of that length.
   - 8 < I, F, D, A, B, H, G, E, I >

Questions