ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

Dr. Mahmoud Nabil
North Carolina A & T State University

August 26, 2019

Overview

1. Laws of propositional logic
2. Predicates and quantifiers
3. Quantified statements

Outline

Laws of propositional logic

- Used to get a simplified form of a complex compoud proposition.
- Used to show logical equivalence.

Idempotent laws:	$p \lor p \equiv p$	$p \land p \equiv p$
Associative laws:	$(p \lor q) \lor r \equiv p \lor (q \lor r)$	$(p \land q) \land r \equiv p \land (q \land r)$
Commutative laws:	$p \lor q \equiv q \lor p$	$p \land q \equiv q \land p$
Distributive laws:	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
Identity laws:	$p \lor F \equiv p$	$p \land T \equiv p$
Domination laws:	$p \land F \equiv F$	$p \lor T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \land \neg p \equiv F$ $\neg T \equiv F$	$p \lor \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \lor q) \equiv \neg p \land \neg q$	$\neg(p \land q) \equiv \neg p \lor \neg q$
Absorption laws:	$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$
Conditional identities:	$p \to q \equiv \neg p \lor q$	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

Excercise 1

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$

Excercise 1

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan

Excercise 1

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive

Excercise 1

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive
- $\neg p \wedge (q \vee \neg q)$ Commutative

Excercise 1

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive
- $\neg p \wedge (q \vee \neg q)$ Commutative
- $\neg p \wedge T$ Complement

Excercise 1

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive
- $\neg p \wedge (q \vee \neg q)$ Commutative
- $\neg p \wedge T$ Complement
- $\neg p$ Identity

Excercise 2

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \to q) \wedge (q \vee p)$

Excercise 2

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \to q) \land (q \lor p)$
- $(\neg p \lor q) \land (q \lor p)$ Conditional Identity

Excercise 2

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \rightarrow q) \land (q \lor p)$
- $(\neg p \lor q) \land (q \lor p)$ Conditional Identity
- $(q \lor \neg p) \land (q \lor p)$ Commutative Law

Excercise 2

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \rightarrow q) \land (q \lor p)$
- $(\neg p \lor q) \land (q \lor p)$ Conditional Identity
- $(q \lor \neg p) \land (q \lor p)$ Commutative Law
- $q \lor (\neg p \land p)$ Distributive Law

Excercise 2

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \rightarrow q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law
- $q \vee (\neg p \wedge p)$ Distributive Law
- $q \vee (p \wedge \neg p)$ Commutative Law

Excercise 2

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \rightarrow q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law
- $q \vee (\neg p \wedge p)$ Distributive Law
- $q \vee (p \wedge \neg p)$ Commutative Law
- $q \vee F$ Complement Law

Excercise 2

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \to q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law
- $q \vee (\neg p \wedge p)$ Distributive Law
- $q \vee (p \wedge \neg p)$ Commutative Law
- $q \vee F$ Complement Law
- q Identity Law

Excercise 3

Ex. Show the logical equivalence: $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

- $(p \to q) \wedge (p \to r)$

Excercise 3

Ex. Show the logical equivalence: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

- $(p \rightarrow q) \wedge (p \rightarrow r)$
- $(\neg p \vee q) \wedge (p \rightarrow r)$ Conditional Law

Excercise 3

Ex. Show the logical equivalence: $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

- $(p \to q) \wedge (p \to r)$
- $(\neg p \vee q) \wedge (p \to r)$ Conditional Law
- $(\neg p \vee q) \wedge (\neg p \vee r)$ Conditional Law

Excercise 3

Ex. Show the logical equivalence: $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

- $(p \to q) \wedge (p \to r)$
- $(\neg p \vee q) \wedge (p \to r)$ Conditional Law
- $(\neg p \vee q) \wedge (\neg p \vee r)$ Conditional Law
- $\neg p \vee (q \wedge r)$ Distributive Law
- $p \to (q \wedge r)$ Conditional Law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$

Excercise 4

Ex. Show the logical equivalence: $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

- $(p \rightarrow r) \vee (q \rightarrow r)$
- $(\neg p \vee r) \vee (q \rightarrow r)$ Conditional identity

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee (r \vee (\neg q \vee r))$ Associative law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee (r \vee (\neg q \vee r))$ Associative law
- $\neg p \vee ((r \vee \neg q) \vee r)$ Associative law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee (r \vee (\neg q \vee r))$ Associative law
- $\neg p \vee ((r \vee \neg q) \vee r)$ Associative law
- $\neg p \vee ((\neg q \vee r) \vee r)$ Commutative law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee (r \vee (\neg q \vee r))$ Associative law
- $\neg p \vee ((r \vee \neg q) \vee r)$ Associative law
- $\neg p \vee ((\neg q \vee r) \vee r)$ Commutative law
- $\neg p \vee (\neg q \vee (r \vee r))$ Associative law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee (r \vee (\neg q \vee r))$ Associative law
- $\neg p \vee ((r \vee \neg q) \vee r)$ Associative law
- $\neg p \vee ((\neg q \vee r) \vee r)$ Commutative law
- $\neg p \vee (\neg q \vee (r \vee r))$ Associative law
- $\neg p \vee (\neg q \vee r)$ Idempotent law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee (r \vee (\neg q \vee r))$ Associative law
- $\neg p \vee ((r \vee \neg q) \vee r)$ Associative law
- $\neg p \vee ((\neg q \vee r) \vee r)$ Commutative law
- $\neg p \vee (\neg q \vee (r \vee r))$ Associative law
- $\neg p \vee (\neg q \vee r)$ Idempotent law
- $(\neg p \vee \neg q) \vee r$ Associative law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee (r \vee (\neg q \vee r))$ Associative law
- $\neg p \vee ((r \vee \neg q) \vee r)$ Associative law
- $\neg p \vee ((\neg q \vee r) \vee r)$ Commutative law
- $\neg p \vee (\neg q \vee (r \vee r))$ Associative law
- $\neg p \vee (\neg q \vee r)$ Idempotent law
- $(\neg p \vee \neg q) \vee r$ Associative law
- $\neg(p \wedge q) \vee r$ De Morgan's law

Excercise 4

Ex. Show the logical equivalence: $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

- $(p \to r) \lor (q \to r)$
- $(\neg p \lor r) \lor (q \to r)$ Conditional identity
- $(\neg p \lor r) \lor (\neg q \lor r)$ Conditional identity
- $\neg p \lor (r \lor (\neg q \lor r))$ Associative law
- $\neg p \lor ((r \lor \neg q) \lor r)$ Associative law
- $\neg p \lor ((\neg q \lor r) \lor r)$ Commutative law
- $\neg p \lor (\neg q \lor (r \lor r))$ Associative law
- $\neg p \lor (\neg q \lor r)$ Idempotent law
- $(\neg p \lor \neg q) \lor r$ Associative law
- $\neg(p \land q) \lor r$ De Morgan's law
- $(p \land q) \to r$ Conditional identity

Outline

Predicates and quantifiers

Consider the following logical statment:

x is an odd number

- If x = 5, then the truth value is True.
- If x = 4, then the truth value is False.
- Hence, this statment is function of x. It can be denoted as $P(x)$.
- We call it a *predicate*.

Predicates and quantifiers

Predicate

Predicate is a logical statement whose truth value is a function of one or more variables.

Ex. $Q(x, y) : x^2 = y$

$Q(5, 25)$ is true because $5^2 = 25$

$Q(7, 51)$ is false because $7^2 \neq 51$

Predicates and quantifiers

Predicate Domain

It is the set of all possible values for the variable in the logical statement.

Ex.

- $P(x) : x + 1 > 1$ Domain(x) :all integers.
  - P(5) is True
  - P(-5) is False
- $P(\text{city})$: city has a population over 5,000,000 Domain(x): US cities
  - P(New York) is True
  - P(Greensboro) is False

Predicates and quantifiers

Predicate Domain

It is the set of all possible values for the variable in the logical statement.

Ex.

- $P(x) : x + 1 > 1$ Domain(x) :all integers.
  - P(5) is True
  - P(-5) is False
- $P(\text{city})$: city has a population over 5,000,000 Domain(x): US cities
  - P(New York) is True
  - P(Greensboro) is False

Note That: Once all the variables within the predicate are assigned values from the domain, then the predicate is truned to a proposition.

Predicates and quantifiers

Quantifiers

It is another way to turn a predicate into a proposition.

- Two types of quantifiers:
    - Universal Quantifiers
    - Existential Quantifiers

Universal Quantifier

- The logical statement $\forall x\ P(x)$ is read "for all x P(x) is true".

Universal Quantifier

- The logical statement $\forall x \ P(x)$ is read "for all x P(x) is true".
- It asserts that that P(x) is true for every possible value for x in its domain.

Universal Quantifier

- The logical statement $\forall x \ P(x)$ is read "for all x P(x) is true".
- It asserts that that P(x) is true for every possible value for x in its domain.
- $\forall x \ P(x)$ is a proposition.

Universal Quantifier

- The logical statement $\forall x \; P(x)$ is read "for all x P(x) is true".
- It asserts that that P(x) is true for every possible value for x in its domain.
- $\forall x \; P(x)$ is a proposition.

$$\forall x \; P(x) \equiv P(a_1) \wedge P(a_2) \wedge \cdots \wedge P(a_k) \qquad \text{Domain } \{a_1 \ldots a_k\}$$

Ex.

- P(x): Student x in the class completed the assignment.
- $\forall \; x \; P(x)$: Every student in the class completed the assignment

Universal Quantifier

- To show a statment with universal quatifier is false only a counter example is needed.

Ex. $\forall\, x\ (x+1) > 0$ Domain is all integers.

$P(-5)$ is false.

Universal Quantifier

- To show a statment with universal quatifier is false only a counter example is needed.

  Ex. $\forall\, x\ (x+1) > 0$ Domain is all integers.
  $P(-5)$ is false.

- To show a statment with universal quatifier is true a proof is needed. (OR show that it true for every value in the domain)

Universal Quantifier

- To show a statment with universal quatifier is false only a counter example is needed.

  Ex. $\forall\, x\ (x+1) > 0$ Domain is all integers.
  $P(-5)$ is false.

- To show a statment with universal quatifier is true a proof is needed. (OR show that it true for every value in the domain)

Universal Quantifier

- To show a statment with universal quatifier is false only a counter example is needed.

  Ex. $\forall\, x\ (x+1) > 0$ Domain is all integers.
  $P(-5)$ is false.

- To show a statment with universal quatifier is true a proof is needed. (OR show that it true for every value in the domain)

  Ex. $\forall\, x\ \frac{1}{x+1} < 1$ Domain is positive integers.

  Proof.

$$0 < x$$

$$1 < x + 1 \qquad \text{divided both sides by x+1}$$

$$\frac{1}{x+1} < 1$$

Existential Quantifier

- The logical statement $\exists x \ P(x)$ is read "There exists an x, such that P(x) is true".

Existential Quantifier

- The logical statement $\exists x \; P(x)$ is read "There exists an x, such that P(x) is true".
- $\exists x \; P(x)$ is a proposition.

Existential Quantifier

- The logical statement $\exists x\ P(x)$ is read "There exists an x, such that P(x) is true".
- $\exists x\ P(x)$ is a proposition.

Existential Quantifier

- The logical statement $\exists x \ P(x)$ is read "There exists an x, such that P(x) is true".
- $\exists x \ P(x)$ is a proposition.

$$\exists x \ P(x) \equiv P(a_1) \vee P(a_2) \vee \cdots \vee P(a_k) \qquad \text{Domain } \{a_1 \ldots a_k\}$$

Ex.

- P(x): Student x in the class completed the assignment.
- $\exists \ x \ P(x)$: There is a student in the class completed the assignment

Existential Quantifier

- To show a statment with existential quatifier is true only one counter example is needed.

Ex. $\exists\ x\ x+1>0$ Domain is all integers.
$P(5)$ is true.

Existential Quantifier

- To show a statment with existential quatifier is true only one counter example is needed.

  Ex. $\exists\, x\;\; x + 1 > 0$ Domain is all integers.
  $P(5)$ is true.

- To show a statment with existential quatifier is false a proof is needed. (OR show that it false for every value in the domain)

Existential Quantifier

- To show a statment with existential quatifier is true only one counter example is needed.

  Ex. $\exists\, x\;\; x + 1 > 0$ Domain is all integers.
  $P(5)$ is true.

- To show a statment with existential quatifier is false a proof is needed. (OR show that it false for every value in the domain)

Existential Quantifier

- To show a statment with existential quatifier is true only one counter example is needed.

  Ex. $\exists\ x\ x+1>0$ Domain is all integers.
  $P(5)$ is true.

- To show a statment with existential quatifier is false a proof is needed. (OR show that it false for every value in the domain)

  Ex. $P(x) : \exists\ x\ x+1<x$ Domain is positive integers.

  Proof.

$$x+1<x$$

$$1<0$$ both minus x

Outline

Quantified statements

- Logical operators $\neg, \wedge, \vee$ can be used to bind universally and existentially quantified statements.

Ex.

P(x): x is prime.
O(x): x is odd.

1. $\exists x(P(x) \wedge \neg O(x))$

Quantified statements

- Logical operators $\neg, \wedge, \vee$ can be used to bind universally and existentially quantified statements.

Ex.

P(x): x is prime.
O(x): x is odd.

1. $\exists x(P(x) \wedge \neg O(x))$ True x = 2
2. $\forall x(P(x) \rightarrow O(x))$

Quantified statements

- Logical operators $\neg, \wedge, \vee$ can be used to bind universally and existentially quantified statements.

Ex.

P(x): x is prime.
O(x): x is odd.

1. $\exists x(P(x) \wedge \neg O(x))$ True x = 2
2. $\forall x(P(x) \rightarrow O(x))$ False x = 2

Free and Bounded Variables

- A variable x in the predicate $P(x)$ is called a free variable.
- A variable x in the statement $\forall x\ P(x)$ is called a bounded variable.
- If all the variables in a statement are bounded, then a predicate is truned to a proposition.

Ex.

$\forall x(P(x) \wedge Q(x))$ Proposition

$\forall x(P(x)) \wedge Q(x)$ Not a Proposition

Excercise 1

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.

Excercise 1

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \wedge \neg D(x)) \equiv \mathit{True}$ (Sam)

Excercise 1

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x (N(x) \land \neg D(x)) \equiv \mathit{True}$ (Sam)
- Everyone missed the deadline or is a new employee.

Excercise 1

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \wedge \neg D(x)) \equiv \mathit{True}$ (Sam)
- Everyone missed the deadline or is a new employee.
  - $\forall x(D(x) \vee N(x)) \equiv \mathit{True}$ (Prove for every one)

Excercise 1

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \wedge \neg D(x)) \equiv \mathit{True}$ (Sam)
- Everyone missed the deadline or is a new employee.
  - $\forall x(D(x) \vee N(x)) \equiv \mathit{True}$ (Prove for every one)
- $\forall x((x \neq \text{Sam} \rightarrow N(x)))$.

Excercise 1

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \land \neg D(x)) \equiv \mathit{True}$ (Sam)
- Everyone missed the deadline or is a new employee.
  - $\forall x(D(x) \lor N(x)) \equiv \mathit{True}$ (Prove for every one)
- $\forall x((x \neq \text{Sam} \rightarrow N(x)))$.
  - Everyone except Sam is a new employee. False (Melanie, Bert)

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone did not miss the deadline and is a new employee.

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone did not miss the deadline and is a new employee.
  - $\exists x(\neg D(x) \wedge N(x)) \equiv \mathit{True}$ (Sam)

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone did not miss the deadline and is a new employee.
  - $\exists x(\neg D(x) \land N(x)) \equiv \mathit{True}$ (Sam)
- $\forall x(\neg D(x) \to \neg N(x))$

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone did not miss the deadline and is a new employee.
  - $\exists x(\neg D(x) \wedge N(x)) \equiv$ *True* (Sam)
- $\forall x(\neg D(x) \rightarrow \neg N(x))$
  - Everyone who did not miss the deadline is not a new employee. *False* (Sam)

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone did not miss the deadline and is a new employee.
    - $\exists x(\neg D(x) \wedge N(x)) \equiv \mathit{True}$ (Sam)
- $\forall x(\neg D(x) \rightarrow \neg N(x))$
    - Everyone who did not miss the deadline is not a new employee. *False* (Sam)
- $N(Bert) \rightarrow D(Bert)$.

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone did not miss the deadline and is a new employee.
  - $\exists x(\neg D(x) \land N(x)) \equiv \mathit{True}$ (Sam)
- $\forall x(\neg D(x) \to \neg N(x))$
  - Everyone who did not miss the deadline is not a new employee. *False* (Sam)
- $N(Bert) \to D(Bert)$.
  - If Bert is a new employee then he missed the deadline. True.

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- $\forall x(D(x) \leftrightarrow N(x))$

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- $\forall x(D(x) \leftrightarrow N(x))$
  - Everyone who missed the deadline is a new employee and vice versa. *False* (Melanie and Bert)

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- $\forall x(D(x) \leftrightarrow N(x))$
  - Everyone who missed the deadline is a new employee and vice versa. *False* (Melanie and Bert)
- If there is a new employee except Sam, then he missed the deadline.

Excercise 1 (continue)

For a group of employees

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- $\forall x(D(x) \leftrightarrow N(x))$
  - Everyone who missed the deadline is a new employee and vice versa. *False* (Melanie and Bert)
- If there is a new employee except Sam, then he missed the deadline.
  - $\exists x(N(x) \wedge (x \neq Sam)) \rightarrow D(x)$. True. (Beth and Al)

Questions