ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

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Talk Overview

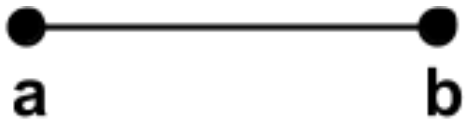
1. Inroduction to graphs
2. Graph Representation

Outline

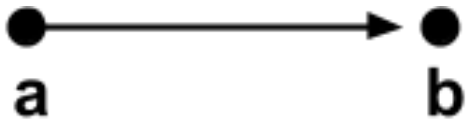
Undirected Graphs

Undirected graph

In an undirected graph, the edges are unordered pairs of vertices, which is useful for modeling relationships that are symmetric.



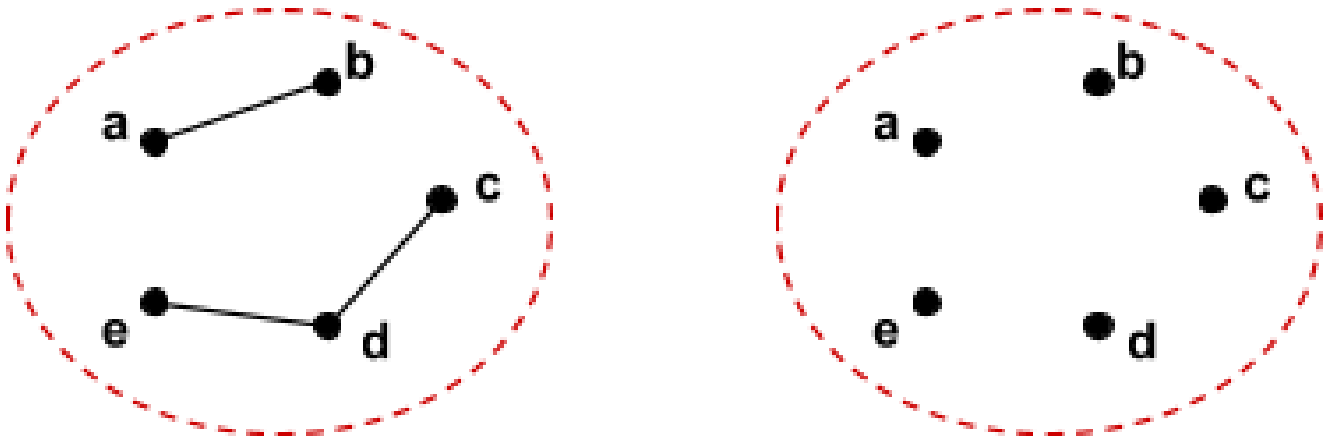
undirected edge
{a, b}



directed edge
(a, b)

Example

Can you list the vertices set and the edges set of the following two graphs?

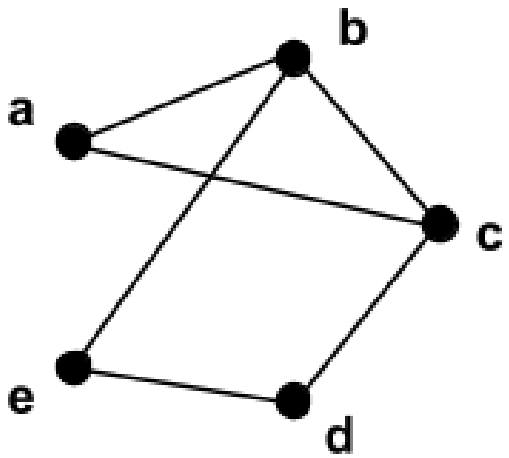


Adjacent/Neighbours Vertices

Adjacent/Neighbours Vertices

Two vertices are said to be adjacent (neighbours) if there is an edge between them.

Ex.



- a and b are neighbours.
- b and c are neighbours.

Parallel Edges and Self Loops

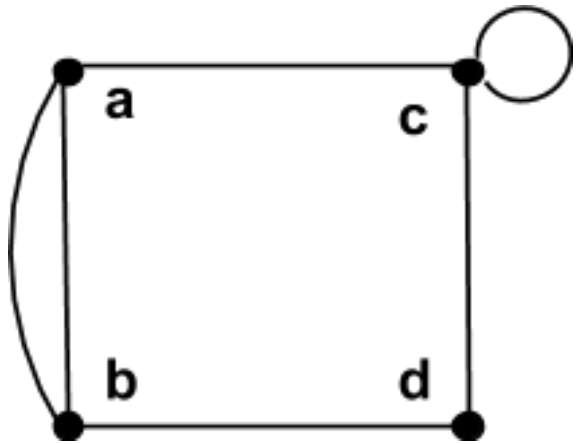
Parallel edges

Parallel edges are multiple edges between the same pair of vertices.

Self Loop

A graph can also have a self-loop which is an edge between a vertex and itself.

Ex.

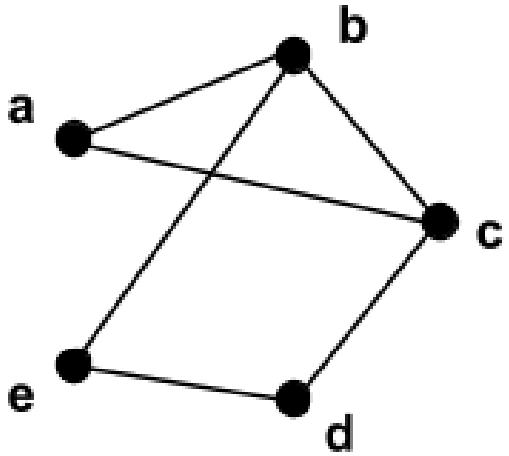


Simple Graph

Simple Graph

If a graph does not have parallel edges or self-loops, it is said to be a simple graph.

Ex.



Graph Total Degree and Regular Graphs

Total Degree

The total degree of a graph is the sum of the degrees of all of the vertices.

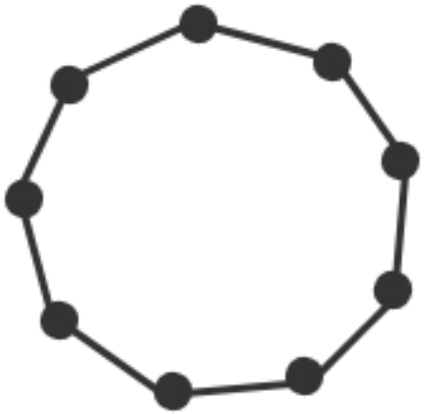
Regular Graph

In a regular graph, all the vertices have the same degree.

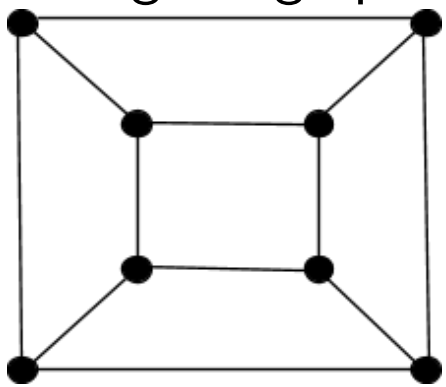
D-regular Graph

In a d-regular graph, all the vertices have degree d.

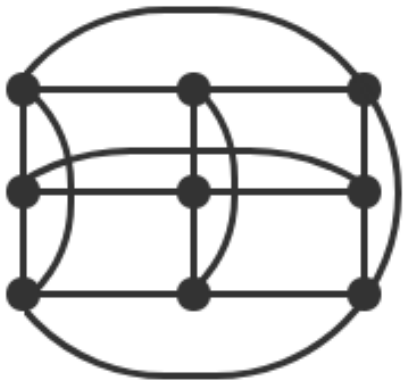
2-regular graph



3-regular graph



4-regular graph



Theorem on Number of Edges and Total Degree

Theorem

Twice the number of edges of a graph is equal to the total degree.

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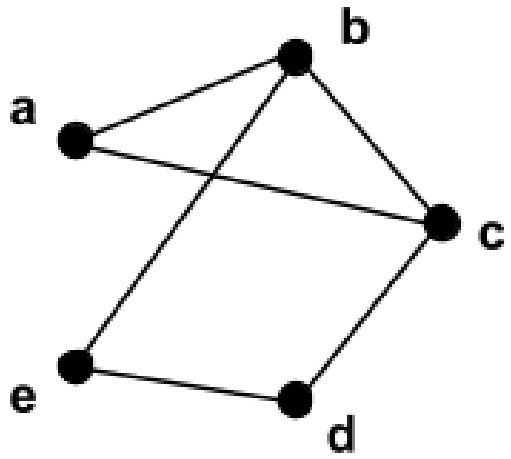
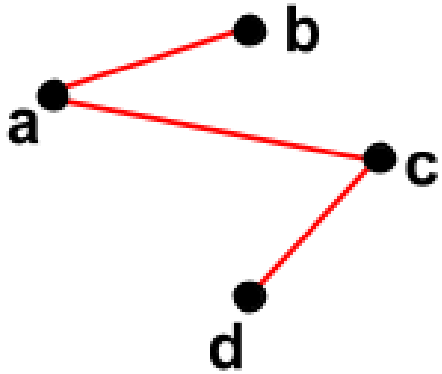
Ex.



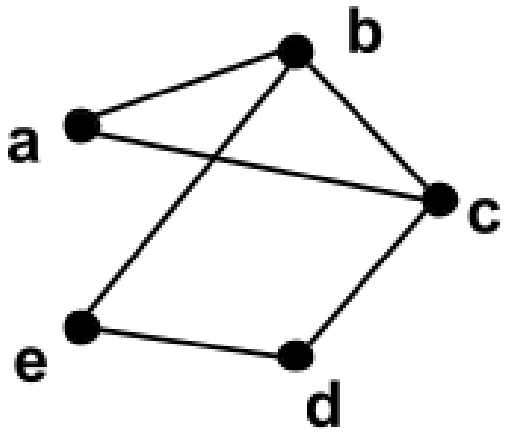
- 5 edges in the graph.
- The total degree is 5*2=10

Subgraph

A graph H $= (V_H, E_H)$ is a subgraph of a graph G $= (V_G, E_G)$ if VH $\subseteq$ VG and EH $\subseteq$ EG.



H



G

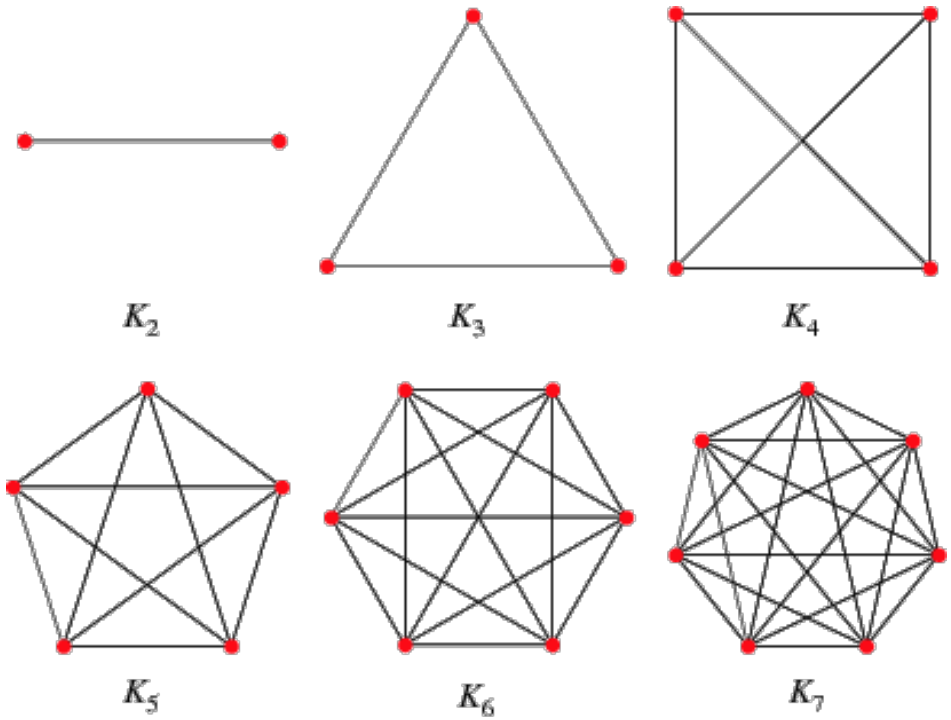
H is a *subgraph* of G

Complete Graphs

Complete Graph

A complete graph k_n with n vertices has an edge between every pair of vertices.

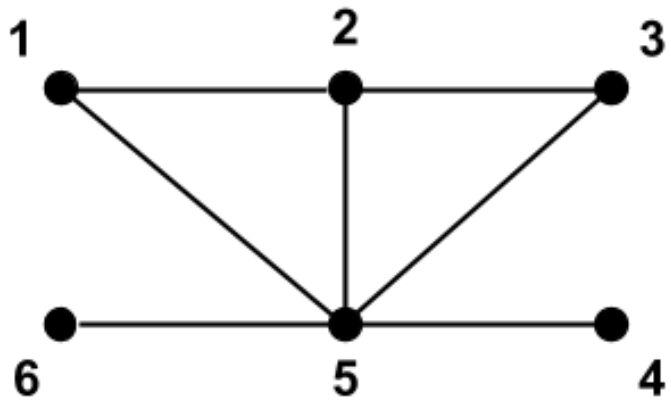
Ex.



Excercise

A graph G is depicted in the diagram on the right.

1. What is the total degree of G?
2. List the neighbors of vertex 5.
3. What is the degree of vertex 6?
4. Which vertices are adjacent to vertex 3?
5. Is G a regular graph? Why or why not?
6. Is K_3 a subgraph of G? If so, name the vertices in the subgraph.
7. Is K_4 a subgraph of G? If so, name the vertices in the subgraph.



Outline

How to represent the graph?

- Are the visual drawings a good way to represent the graph and process it with computer programs?

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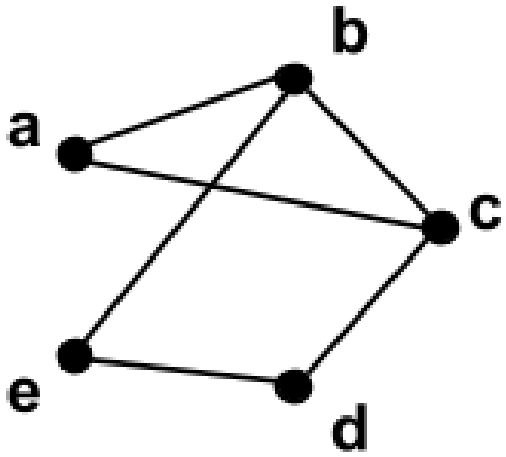
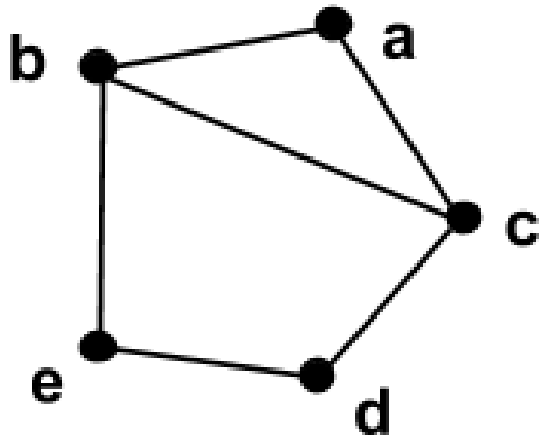
- Are these two graphs different or the same?





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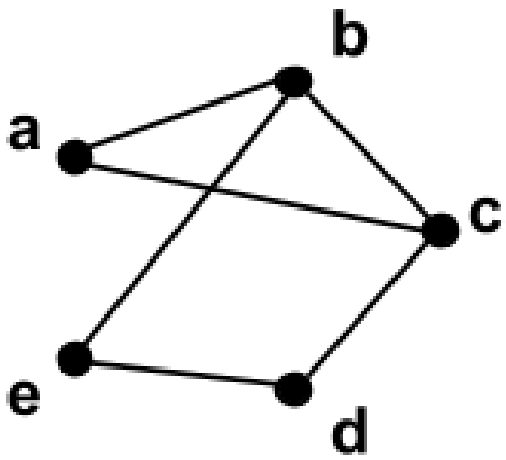

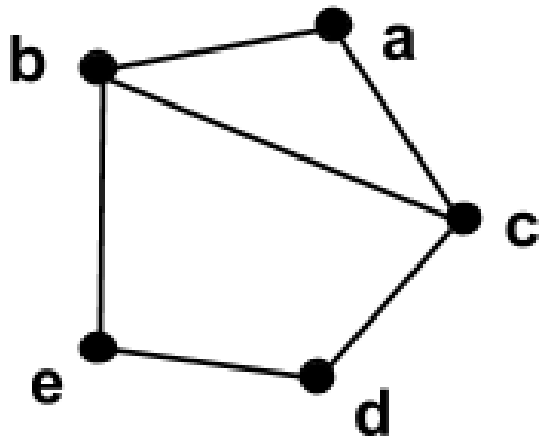

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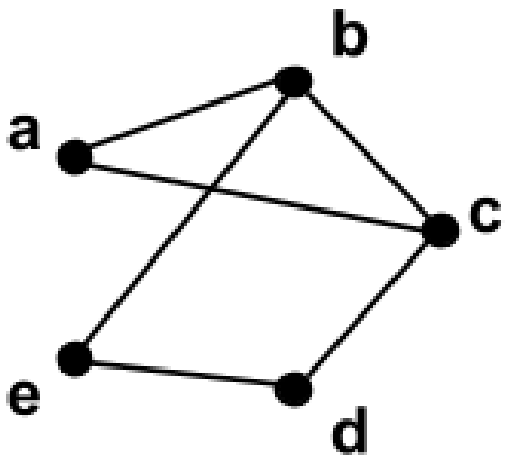
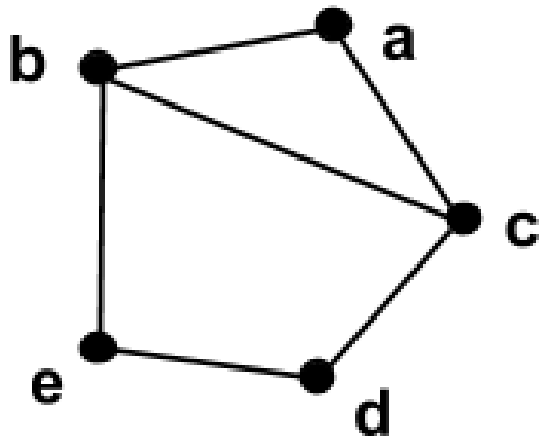




- Can you list the vertices and the edges of both graphs?

$$V = \{a, b, c, d, e\}$$

$$E = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, e\}, \{c, d\}, \{d, e\}\}$$

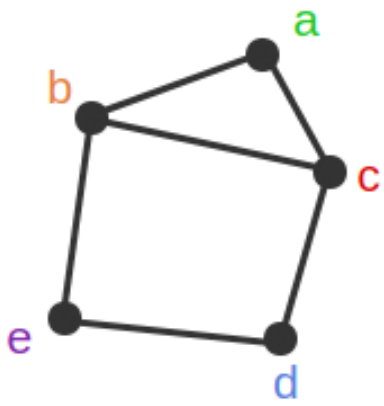
How to represent the graph?

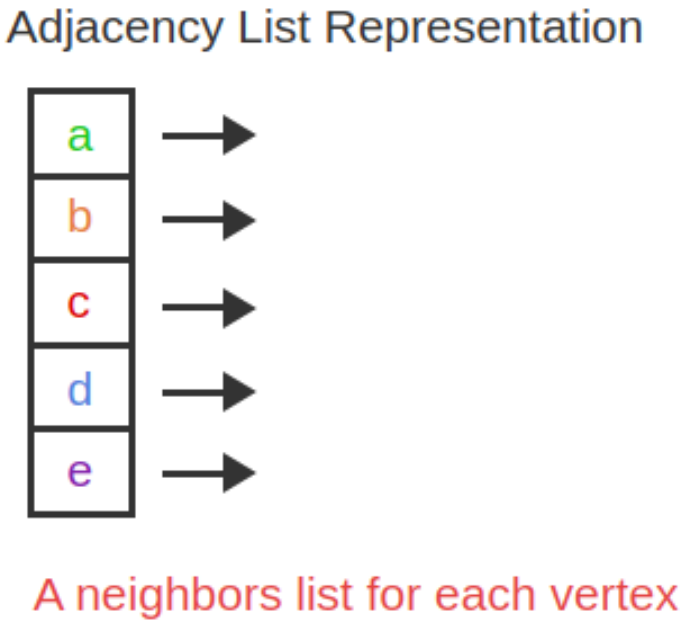
Two standard ways to represent graphs

- Adjacency list representation.
- Matrix representation.

Adjacency List Representation

In the adjacency list representation of a graph, each vertex has a list of all its neighbors.





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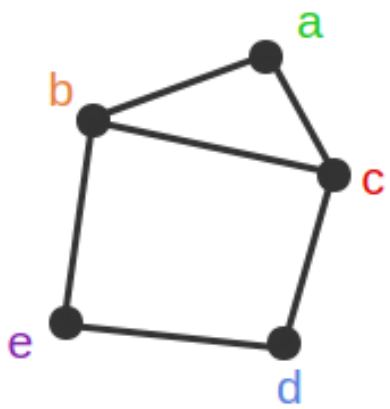
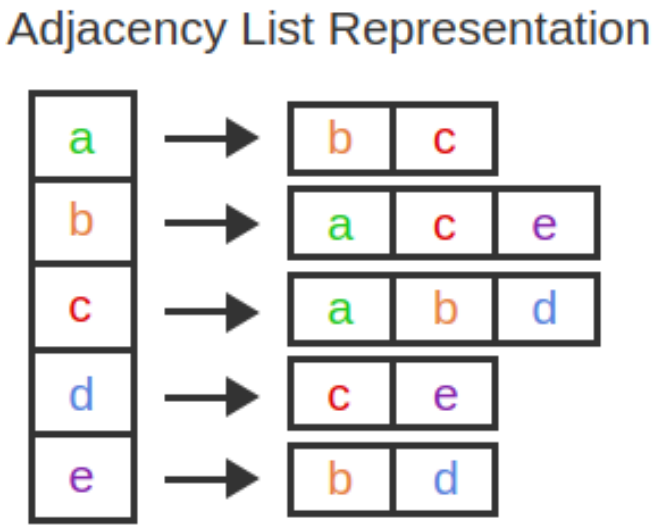
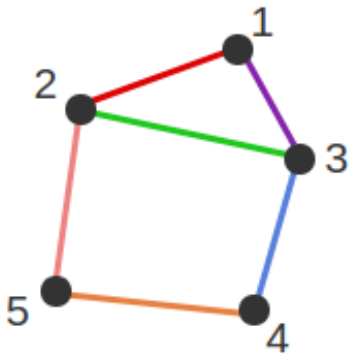




- Is b adjacent to c? Scan b's list to look for c. Vertex c is found in b's list, so yes, b is adjacent to c.
- Worst case time to scan b's list is proportional to deg(b). i.e., O(deg(b)).

Matrix Representation

The matrix representation for a graph with **n** vertices is an **n** by **n** matrix whose entries are all either 0 or 1, indicating whether or not each edge is present.



Matrix Representation

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	0	1
3	1	1	0	1	0
4	0	0	1	0	1
5	0	1	0	1	0

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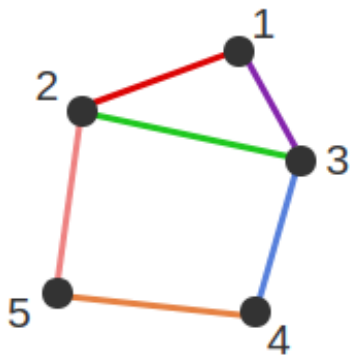

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4	0	0	1	0	1
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Undirected graph:
Matrix is symmetric
about the diagonal

The matrix representation of an undirected graph is symmetric about the diagonal.

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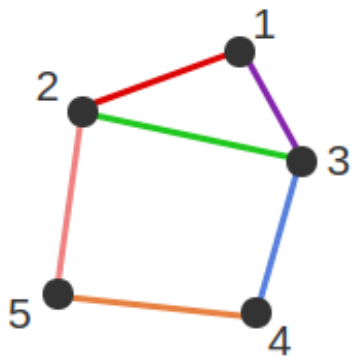

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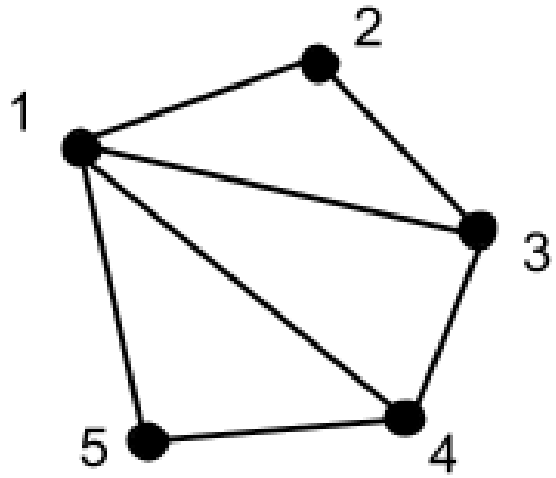
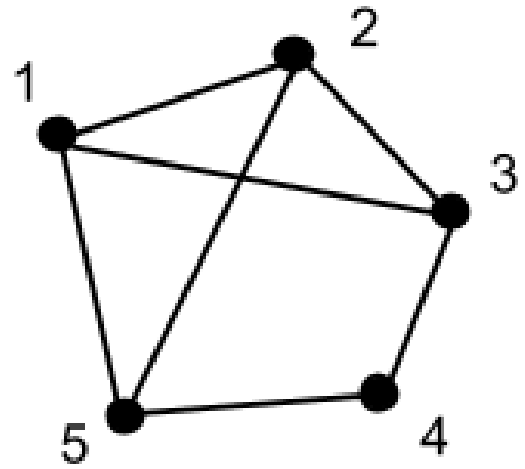
- Is 2 adjacent to 5? Look at , the entry in row 2, column 5. , so the answer is yes.
- O(1) time to answer.

Excercise

Give the adjacency list and the matrix representation of the below graphs.



G



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Questions