Electricity Theft Detection with Privacy Preservation for Smart Grid AMI Networks Using Machine Learning

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Talk Overview

1. Demorgans' Law for Quantifiers
2. Nested Quantifiers
3. Logical Reasoning
4. Rules of inference with propositions
5. Rules of inference with quantifiers

Outline

Demorgans' Law for Universal Quantifiers

- $\neg\forall F(x) \equiv ?$

F(x) : bird x can fly

- $\forall F(x)$: every bird x can fly.
- $\neg\forall F(x) \equiv$ not every bird x can fly.
- $\neg\forall F(x) \equiv$ there exists a bird that can not fly $\equiv \exists\neg F(x)$.

Demorgans' Law for Universal Quantifiers

Domain of discourse = $\{a_1, a_2, \ldots, a_n\}$

$$\neg \forall x\, P(x) \quad \equiv \quad \exists x\, \neg P(x)$$

$$\equiv \qquad\qquad \equiv$$

$$\neg(P(a_1) \wedge P(a_2) \wedge \ldots \wedge P(a_n)) \equiv \neg P(a_1) \vee \neg P(a_2) \vee \ldots \vee \neg P(a_n)$$

Demorgans' Law for Exstential Quantifiers

- $\neg\exists F(x) \equiv ?$

F(x) : x is absent today

- $\exists F(x)$: there is a child in the class who is absent today.
- $\neg\exists F(x) \equiv$ It is not true that there is a child in the class who is absent today.
- $\neg\exists F(x) \equiv$ Every child in the class is not absent today $\equiv \forall\neg F(x)$.

Demorgans' Law for Exstential Quantifiers

Domain of discourse = $\{a_1, a_2, \ldots, a_n\}$

$$\neg \exists x \, P(x) \equiv \forall x \, \neg P(x)$$

$$\equiv \qquad\qquad \equiv$$

$$\neg(P(a_1) \lor P(a_2) \lor \ldots \lor P(a_n)) \equiv \neg P(a_1) \land \neg P(a_2) \land \ldots \land \neg P(a_n)$$

Outline

Bound and Free Variables

$\forall x \exists y P(x,y)$	x and y are both bound.
$\forall x P(x,y)$	x is bound and y is free.
$\exists y \exists z T(x,y,z)$	y and z are bound. x is free.

Nested Quatifiers

M(x, y): x sent an email to y

- $\forall x \forall y M(x, y) \equiv$ "Everyone sent an email to everyone."
- $\forall x \forall y M(x, y)$ will be false if only an employee does not send an email to himself.
- $\exists x \exists y M(x, y) \equiv$ There is a person who sent an email to someone.
- $\exists x \exists y M(x, y)$ will be true even if at least one employee sends an email to himself.
- $\forall x \exists y M(x, y) \equiv$ "Every person sent an email to someone"
- $\exists x \forall y M(x, y) \equiv$ "There is a person who sent an email to everyone"

Nested Quantifiers as a Two-person Game

Player	Action	Goal
Existential player	Selects values for existentially bound variables	Tries to make the expression true
Universal player	Selects values for universally bound variables	Tries to make the expression false

Nested Quantifiers as a Two-person Game

- $\forall x \exists y (x + y = 0)$
- The universal player first selects the value of x. Regardless of which value the universal player selects for x, the existential player can select y to be -x, which will cause the sum x + y to be 0.
- $\exists x \forall y (x + y = 0)$
- Now, the existential player goes first and selects a value for x. Regardless of the value chosen for x, the universal player can select some value for y that causes the predicate to be false.

Expressing Everyone Else in Quantified Statements

M(x, y): x sent an email to y

- $\forall x \forall y M(x, y) \equiv$ "Everyone sent an email to everyone including himself"
- How could we use logic to express that everyone sent an email to everyone else without including the case that everyone sent an email to himself or herself?
- $\forall x \forall y((x \neq y) \rightarrow M(x, y))$

Expressing Uniqueness in Quantified Statements

Express: "Exactly one person was late to the meeting"

L(x): x was late to meeting.

Someone was late to the meeting.

Need a way to express that x is the only person who was late to the meeting

For every y, if $y \neq x$ then y was not late for the meeting.

$\exists x\, (L(x) \land \forall y((x \neq y) \rightarrow \neg L(y))$

"Exactly one person was late to the meeting."

Note

- Quantifiers can be moved.
- $\exists x(L(x) \land \forall y((y \neq x) \rightarrow \neg L(y))) \equiv \exists x \forall y(L(x) \land ((y \neq x) \rightarrow \neg L(y)))$

De Morgan's Law with Nested Quantifiers

$\neg\forall x\, \forall y\, P(x, y) \equiv \exists x\, \exists y\, \neg P(x, y)$
$\neg\forall x\, \exists y\, P(x, y) \equiv \exists x\, \forall y\, \neg P(x, y)$
$\neg\exists x\, \forall y\, P(x, y) \equiv \forall x\, \exists y\, \neg P(x, y)$
$\neg\exists x\, \exists y\, P(x, y) \equiv \forall x\, \forall y\, \neg P(x, y)$

Example

M(x, y): x sent an email to y

- $\forall x \forall y M(x, y)$
- False. Counterexamples: x = y = 2 or x = y = 3.
- $\exists x \exists y M(x, y)$
- True. Counterexamples: x = y = 2 or x = y = 3.

M	1	2	3
1	T	T	T
2	T	F	T
3	T	T	F

Outline

Argument

- An argument is a sequence of propositions, called hypotheses, followed by a final proposition, called the conclusion.
- An argument is valid if the conclusion is true whenever the hypotheses are all true, otherwise the argument is invalid.

- $p_1 \ldots p_n$ are the hypotheses and c is the conclusion.
- The argument is valid whenever the proposition $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow c$ is a tautology.
- Order of $p_1 \ldots p_n$ does not matter.s

$$\begin{array}{l} p_1 \\ p_2 \\ \ldots. \\ \underline{p_n} \\ \therefore c \end{array}$$

Showing validity of an Argument using Truth Table

Truth table proof of
$$\begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

Fill in the truth table for $p \rightarrow q$

Hypotheses are: p, $p \rightarrow q$

Hyp		Hyp
p	q	$p \rightarrow q$
$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
T	F	F
F	T	T
F	F	T

$\Leftarrow$ There is only one row in which both hypotheses are true

Verify that the conclusion q is also true in this row.

$$\Rightarrow \begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$ is a valid argument

Showing invalidity of an Argument using Counter Example

- An argument can be shown to be invalid by showing an assignment of truth values to its variables that makes all the hypotheses true and the conclusion false.

Ex.

$$\begin{array}{l} \neg p \\ \underline{p \rightarrow q} \\ \therefore \neg q \end{array}$$

- when p = F and q = T, the hypotheses $p \rightarrow q$ and $\neg p$ are both true, but the conclusion q is false.

English Argument

Do not be Tricked

5 is not an even number. If 5 is an even number, then 7 is an even number. ∴ 7 is not an even number.	$\equiv$	$\neg p$ $p \rightarrow q$ $\therefore \neg q$

- In a valid argument, the conclusion must follow from the hypotheses for every possible combination of truth values for the individual propositions.

Outline

Rules

Rule of inference	Name
p $p \rightarrow q$ $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	Modus tollens
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

Excercise

Ex.

$$\begin{array}{l} (r \lor w) \to c \\ \neg c \\ \hline \neg w \end{array}$$

1.	$(r \lor w) \to c$	Hypothesis
2.	$\neg c$	Hypothesis
3.	$\neg(r \lor w)$	Modus tollens, 1, 2
4.	$\neg r \land \neg w$	De Morgan's law, 3
5.	$\neg w \land \neg r$	Commutative law, 4
6.	$\neg w$	Simplification, 5

Outline

Rules of inference with quantifiers

Every employee who received a large bonus works hard.
Linda is an employee at the company.
Linda received a large bonus.

∴ Some employee works hard.

- Rules of inference with quantifiers used
  - To get general rules from knowing specific information about the elements of the domain.
  - To get specific rules from knowing general information about the elements of the domain.

Elements of the domain

Arbitrary element

An arbitrary element of a domain has no special properties other than those shared by all the elements of the domain.

Particular element

A particular element of the domain may have properties that are not shared by all the elements of the domain.

Elements of the domain

Rule of Inference	Name	Example
c is an element (arbitrary or particular) $\underline{\forall x\, P(x)}$ $\therefore P(c)$	Universal instantiation	Sam is a student in the class. Every student in the class completed the assignment. Therefore, Sam completed his assignment.
c is an arbitrary element $\underline{P(c)\quad}$ $\therefore \forall x\, P(x)$	Universal generalization	Let c be an arbitrary integer. $c \leq c^2$ Therefore, every integer is less than or equal to its square.
$\underline{\exists x\, P(x)}$ $\therefore$ (c is a particular element) $\wedge$ P(c)	Existential instantiation*	There is an integer that is equal to its square. Therefore, $c^2 = c$, for some integer c.
c is an element (arbitrary or particular) $\underline{P(c)\quad}$ $\therefore \exists x\, P(x)$	Existential generalization	Sam is a particular student in the class. Sam completed the assignment. Therefore, there is a student in the class who completed the assignment.

- The rules existential instantiation and universal instantiation replace a quantified variable with an element of the domain.
- The rules existential generalization and universal generalization replace an element of the domain with a quantified variable.

Excercise

Ex.

$\exists x\, P(x)$
$\forall x\, Q(x)$
$\overline{\exists x\, (P(x) \wedge Q(x))}$

1. $\exists x\, P(x)$	Hypothesis
2. (c is a particular element) $\wedge$ P(c)	Reason A
3. P(c) $\wedge$ (c is a particular element)	Commutative law, 2
4. $\forall x\, Q(x)$	Hypothesis
5. c is a particular element	Simplification, 2
6. Q(c)	Reason B
7. P(c)	Reason C
8. P(c) $\wedge$ Q(c)	Reason D
9. $\exists x(P(x) \wedge Q(x))$	Reason E

- What are reasons A, B, C, D, and E?

Showing an argument with quantified statements is invalid

Ex.

$$\begin{array}{l} \exists x\, P(x) \\ \exists x\, Q(x) \\ \hline \exists x\, (P(x) \wedge Q(x)) \end{array}$$

	P	Q
c	T	F
d	F	T

- The two hypotheses, $\exists x P(x)$ and $\exists x Q(x)$, are both true for the values for P and Q on elements c and d given in the table.
- However, the conclusion $\exists x(P(x) \wedge Q(x))$ is false. No element satisfy them both at the same time

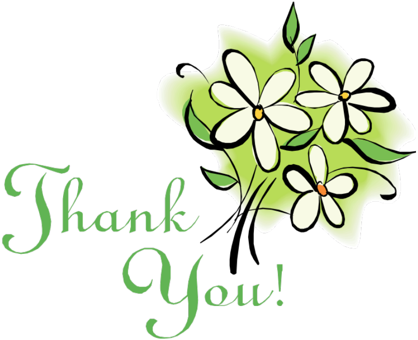
False Proof

Ex.

$\exists x\, P(x)$
$\exists x\, Q(x)$
$\overline{\exists x\, (P(x) \land Q(x))}$

- It is important to define a new particular element with a new name for each use of existential instantiation within the same logical proof

1.	$\exists x\, P(x)$	Hypothesis
2.	(c is a particular element) $\land$ P(c)	Existential instantiation, 1
3.	P(c) $\land$ (c is a particular element)	Commutative law, 2
4.	$\exists x\, Q(x)$	Hypothesis
5.	(c is a particular element) $\land$ Q(c)	Existential instantiation, 4
6.	Q(c) $\land$ (c is a particular element)	Commutative law, 5
7.	P(c)	Simplification, 3
8.	Q(c)	Simplification, 6
9.	P(c) $\land$ Q(c)	Conjunction, 7, 8
10.	c is a particular element	Simplification, 2
11.	$\exists x\, (P(x) \land Q(x))$	Existential generalization, 9, 10



Questions