ID: Home

- 1. Give the first ten terms of the following sequences. You can assume that the sequences start with an index of 1. Logs are to base 2. Indicate whether the sequence is increasing, decreasing, non-increasing, or non-decreasing.
  - (a) The first two terms in the sequence are 1. The rest of the terms are the sum of the two preceding terms.
  - (b) The  $n^{th}$  term is  $\frac{1}{n^2}$ .
  - (c) The  $n^{th}$  term is 5.
  - (d) The nth term is  $n^3$ .
  - (e) The nth term is  $-\lceil log(n) \rceil$
  - (f) The nth term is  $3^{\lceil log(n) \rceil}$
- 2. Suppose someone takes out a home improvement loan for \$30,000. The annual interest on the loan is 6% and is compounded monthly. The monthly payment is \$600. Let an denote the amount owed at the end of the nth month. The payments start in the first month and are due the last day of every month. Give a recurrence relation for  $a_n$
- 3. write down an equivalent expression where the last term in the sum is outside the summation.  $\sum_{k=0}^{k=n-1} (k^2 4k + 1)$
- 4. Substitute variable j for k, where j = k 1, in the summation:  $\sum_{j=0}^{j=n-1} (2j^2 4j + 2)$
- 5. A population of cows on a farm grows by 10% each year. Suppose that the population starts with 20 cows. If each cow consumes 5 pounds of food each year, then how much food is consumed in 5 years.
- 6. proof the following using mathematical induction
  - (a) Prove that for any positive integer n,  $\sum_{j=1}^{j=n} j^3 = (\frac{n(n+1)}{2})^2$
  - (b) Prove that for any positive integer n,  $\sum_{j=1}^{j=n} j(j-1) = (\frac{n(n^2-1)}{3})^2$
  - (c) Prove that for any positive integer n,  $\sum_{j=1}^{j=n} j \times 2^j = (n-1)2^{n+1} + 2$
  - (d) Prove that for  $n \ge 2$ ,  $3^n > 2^n + n^2$
  - (e) Prove that for  $n \ge 4$ ,  $n! \ge 2^n$