ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

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Overview

1. Propositions and logical operations
2. Evaluating compound propositions
3. Conditional statements
4. Logical equivalence

Outline

What is logic?

Logic

Logic is the study of formal reasoning.

- Logic statement always has a well defined meaning.
- Logic used in
    - Artificial intelligence for automated reasoning.
    - Embedded systems for designing digital circuits.
    - Laws logic for defining the implications of a particular law.
    - Medicine for conditions and diagnosis.

Proposition (1/3)

Proposition

Proposition is a statement that is either evaluated to true or false.

Truth Value

It is a value indicating whether the proposition is actually true or false

Propositions Examples:

- There are an infinite number of prime numbers. $\rightarrow$ True
- The Declaration of Independence was signed on July 4,1812. $\rightarrow$ False

Proposition (2/3)

- Propositions are declarative sentences.
  Not Propositions Examples:
  - What time is it? → Question
  - Have a nice day. → Command
- Proposition truth value can be true, false, unknown, or a matter of opinion.
  Examples:
  - Monday will be cloudy. → Unknown
  - The movie was funny. → A matter of opinion
  - The extinction of the dinosaurs was caused by a meteor. → Unknown

Proposition (3/3)

- Variables names such as p and q can be used to denote arbitrary propositions.
  Example:
  - **p:** January has 31 days.
  - **q:** February has 33 days.

Compound Proposition

It is created by connecting individual propositions with logical operations.

Types of logical operations:

- Conjunction. **Ex.** $p \text{ and } q \equiv p \wedge q$
- Disjunction. **Ex.** $p \text{ or } q \equiv p \vee q$
- Negation. **Ex.** $\text{not } p \equiv \neg p$

Conjunction Operation

- The proposition $p \wedge q$ is read "p and q".
- $p \wedge q$ is true if both p is true and q is true.
- $p \wedge q$ is false if p is false, q is false, or both are false.

Example:

- **p:** January has 31 days.
- **q:** February has 33 days.
- $\mathbf{p} \wedge \mathbf{q}$: January has 31 days and February has 33 days.

Given the truth values of "p" and "q", what is the truth value of $p \wedge q$?

Disjunction Operation

- The proposition $p \vee q$ is read "p or q".
- $p \vee q$ is true if either one of p or q is true.
- $p \vee q$ is false if both p and q are false.

Example:

- **p:** January has 31 days.
- **q:** February has 33 days.
- **p** $\vee$ **q**: January has 31 days or February has 33 days.

Given the truth values of "p" and "q", what is the truth value of $p \vee q$?

Types of OR

Inclusive or

The inclusive or is the same as the disjunction $\vee$ operation and evaluates to true when one or both of the propositions are true.

Example: "Lucy opens the windows or doors when warm"

Exclusive or

The exclusive or of p and q evaluates to true only when p is true and q is false or when q is true and p is false.

Example: "Lucy is going to the park or the movie".

Denoted as $p \oplus q$

Negation Operation

- The negation operation acts on just one proposition.
- It has the effect of reversing the truth value of the proposition.
- It is denoted as $\neg p$ and read as "not p".

Example:

- **p:** The patient has diabetes.
- $\neg$ **p:** The patient does not have diabetes.

Truth Table (1/2)

Truth table

It shows the truth value of a compound proposition for every possible combination of truth values for the variables contained in the compound proposition.

Conjunction

p	q	p $\wedge$ q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	p $\wedge$ q
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	$\neg$ p
T	F
F	T

Truth Table (2/2)

- **Q:** How to fill the truth table for a compound proposition?
- **A:** If there are **n** variables, there are 2^n rows.
- The T and F values for each row are unique.
- Note that: The column of the first varible on the right alternates T F T F..., the column for the second variable alternates T T F F ..., etc.
- Can you fill the following truth table?

p	**q**	**p** $\oplus$ **q**
T	T	
T	F	
F	T	
F	F	

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T	T	
T	F	
F	T	
F	F	

Outline

Evaluation Order

- Since the compound proposition can contain many variables and many operations, the order of evaluating the operations matters.
- Order of operations in absence of parentheses.
  1. $\neg$ not
  2. $\wedge$ and
  3. $\vee$ or

Example:

- $p : T,\ q : F,\ r : T$

1. $p \wedge \neg(q \vee r)$
2. $T \wedge \neg \underbrace{(F \vee T)}$
3. $T \wedge \underbrace{\neg T}$
4. $\underbrace{T \wedge F}$
5. F

Try this!

1. $p \vee \neg q \wedge r$

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Example:

- $p : T, \; q : F, \; r : T$

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Try this!

1. $p \vee \neg q \wedge r$

Excercise 1

Write the truth table for $r \vee (p \wedge \neg q)$.

p	q	r	$r \vee (p \wedge \neg q)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

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F	T	F	
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Excercise 2

Consider the following pieces of identification a person might have in order to apply for a credit card:

- B: Applicant presents a birth certificate.
- D: Applicant presents a driver's license.
- M: Applicant presents a marriage license.

Questions

1. The applicant must present either a birth certificate, a driver's license or a marriage license.
2. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.
3. Applicant must present either a birth certificate or both a driver's license and a marriage license.

Outline

Conditional Operation (1/2)

- The proposition $p \to q$ is read "if p then q".
- The proposition $p \to q$ is false if p is true and q is false; otherwise, $p \to q$ is true.

Example:

- **p:** There is a traffic jam today.
- **q:** I will be late for work.
- $\mathbf{p} \to \mathbf{q}$: If there is a traffic jam today, then I will be late for work.

Truth Table:

p	**q**	$\mathbf{p} \to \mathbf{q}$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional Operation (2/2)

- In $p \to q$, proposition p is called the hypothesis, and the proposition q is called the conclusion.
- A conditional proposition can be thought of like a contract between two parties.

Ex: If $\underbrace{\text{you mow Mr. Smith's lawn}}_{p}$, then $\underbrace{\text{he will pay you}}_{q}$.

	q is true: Mr. Smith pays you.	**q is false:** Mr. Smith does not pay you.
p is true: You mow Mr. Smith's lawn	p→q = T	p→q = F
p is false: You do not mow Mr. Smith's lawn	p→q = T	p→q = T

The only scenario in which the contract is broken when p is true and q is false.

Examples on Conditional Operations

1. **s**: If it rains today, I will have my umbrella.
   It is raining today.
   I do not have my umbrella. False
2. **s**: If Sally took too long getting ready, she missed the bus.
   Sally did not take too long getting ready.
   Sally missed the bus. True
3. **s**: If it is sunny out, I ride my bike.
   It is not sunny out.
   I am not riding my bike. True

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English expressions of the Conditional Operations

Ex: If $\underbrace{\text{you mow Mr. Smith's lawn}}_{p\ (\textit{hypothesis})}$, then $\underbrace{\text{he will pay you}}_{q\ (\textit{conclusion})}$.

If p, then q.	If you mow Mr. Smith's lawn, then he will pay you.
If p, q.	If you mow Mr. Smith's lawn, he will pay you.
q if p	Mr. Smith will pay you if you mow his lawn.
p implies q.	Mowing Mr. Smith's lawn implies that he will pay you.
p only if q.	You will mow Mr. Smith's lawn only if he pays you.
p is sufficient for q.	Mowing Mr. Smith's lawn is sufficient for him to pay you.
q is necessary for p.	Mr. Smith's paying you is necessary for you to mow his lawn.

The Converse,Contrapositive, and Inverse

Proposition:	$p \rightarrow q$	Ex: If it is raining today, the game will be cancelled.
Converse:	$q \rightarrow p$	If the game is cancelled, it is raining today.
Contrapositive:	$\neg q \rightarrow \neg p$	If the game is not cancelled, then it is not raining today.
Inverse:	$\neg p \rightarrow \neg q$	If it is not raining today, the game will not be cancelled.

Note that: The contrapositive is the inverse of the converse!

The Converse,Contrapositive, and Inverse

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Contrapositive:	$\neg q \to \neg p$	If the game is not cancelled, then it is not raining today.
Inverse:	$\neg p \to \neg q$	If it is not raining today, the game will not be cancelled.

Note that: The contrapositive is the inverse of the converse!

The Biconditional Operation

- The proposition "p if and only if q" is expressed with the biconditional operation and is denoted $p \leftrightarrow q$.
- It is true when p and q have the same truth value and is false when p and q have different truth values.

Other meanings includes:

- p is necessary and sufficient for q.
- if p then q, and conversely.
- **iff** is an abbreviation of the expression "if and only if".

Truth Table:

p	**q**	$\mathbf{p \leftrightarrow q}$
T	T	T
T	F	F
F	T	F
F	F	T

Evaluation Order Now

- Order of operations in absence of parentheses.
  1. $\neg$ not
  2. $\wedge$ and
  3. $\vee$ or
  4. $\rightarrow$ if
  5. $\leftrightarrow$ if and only if

Exercise 1

Example:

- $p : T$, $q : T$, $r : F$

1. $p \vee \neg(q \leftrightarrow r)$
2. $T \vee \neg \underbrace{(T \leftrightarrow F)}$
3. $T \vee \underbrace{\neg F}$
4. $\underbrace{T \vee T}$
5. T

Try this!

1. $(p \leftrightarrow r) \wedge \neg q$

Exercise 1

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2. $T \vee \neg \underbrace{(T \leftrightarrow F)}$
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4. $\underbrace{T \vee T}$
5. T

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Exercise 2

For a degree in Computer Science, a student must take one of three project courses, P1, P2, or P3. The student must also take one of two theory courses, T1 or T2. Furthermore, if the student is an honors student, he or she must take the honors seminar S. Let H be the proposition indicating whether the student is an honors student.

- Formulate the previous statements using logical propositions.

Exercise 3

Give the inverse, converse and contrapositive for each of the following statements.

- **Q:** If the patient took the medicine, then she had side effects.
- **A:**
  - **Inverse:** If the patient didn't take the medicine, then she didn't have side effects.
  - **Contrapositive:** If the patient didn't have side effects, then she didn't take the medicine.
  - **Converse:** If the patient had side effects, then she took the medicine.

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- **A:**
  - **Inverse:** If the patient didn't take the medicine, then she didn't have side effects.
  - **Contrapositive:** If the patient didn't have side effects, then she didn't take the medicine.
  - **Converse:** If the patient had side effects, then she took the medicine.

Exercise 4

s: A person is a senior
y: A person is at least 17 years of age
p: A person is allowed to park in the school parking lot

Express in logic form

- A person is allowed to park in the school parking lot only if they are a senior and at least seventeen years of age.
- A person can park in the school parking lot if they are a senior or at least seventeen years of age.
- Being 17 years of age is a necessary condition for being able to park in the school parking lot.
- A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

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Exercise 5

The variable p is true, q is false, and the truth value for variable r is unknown. **Indicate** whether the truth value of each logical expression is true, false, or unknown.

- $p \to (q \wedge r)$
- $(p \wedge r) \leftrightarrow (q \wedge r)$
- $(p \wedge q) \to r$

Exercise 5

The variable p is true, q is false, and the truth value for variable r is unknown. **Indicate** whether the truth value of each logical expression is true, false, or unknown.

- $p \to (q \land r)$
- $(p \land r) \leftrightarrow (q \land r)$
- $(p \land q) \to r$

Exercise 5

The variable p is true, q is false, and the truth value for variable r is unknown. **Indicate** whether the truth value of each logical expression is true, false, or unknown.

- $p \to (q \wedge r)$
- $(p \wedge r) \leftrightarrow (q \wedge r)$
- $(p \wedge q) \to r$

Tutology and Contradiction

- **Tutology:** A proposition is always true.
  - **Ex.** $p \vee \neg p$
- **Contradiction:** A proposition is always false.
  - **Ex.** $p \wedge \neg p$

Is this statment a tutology, contradiction, or neither. $p \wedge q \rightarrow p$

Outline

Logical Equivalence

Logical Equivalence

Two compound propositions are logically equivalent if they have the same truth value regardless of the truth values of their individual propositions.

- The notation $s \equiv r$ is used to indicate that r and s are logically equivalent.
- Propositions s and r are logically equivalent if and only if the proposition $s \leftrightarrow r$ is a tautology.
- Logical equivalence can be proved with either the truth table or the laws of propositional logic (Next week).

Example 1

Show logical equivalence of $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \vee \neg q$
T	T	F	F	T	**F**	**F**
T	F	F	T	T	**F**	**F**
F	T	T	F	T	**F**	**F**
F	F	T	T	F	**T**	**T**

- Also known as the first De Morgan's law.
- When the negation operation is distributed inside the parentheses, the disjunction operation changes to a conjunction operation.

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p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \vee \neg q$
T	T	F	F	T	**F**	**F**
T	F	F	T	T	**F**	**F**
F	T	T	F	T	**F**	**F**
F	F	T	T	F	**T**	**T**

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- When the negation operation is distributed inside the parentheses, the disjunction operation changes to a conjunction operation.

Example 2

Show logical equivalence of $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	**F**	**F**
T	F	F	T	F	**T**	**T**
F	T	T	F	F	**T**	**T**
F	F	T	T	F	**T**	**T**

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Show logical equivalence of $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	**F**	**F**
T	F	F	T	F	**T**	**T**
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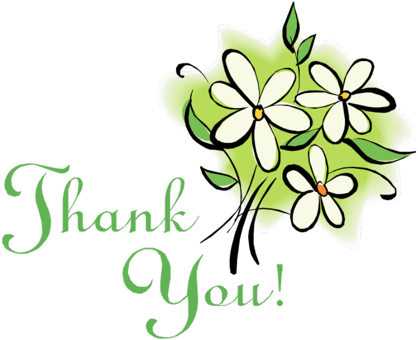

Questions