

1. Calculate the following:

- $(99 \times (65) + 312) \bmod 7$
- $(82 \times (55) + 44 \times (15)) \bmod 7$
- $55^7 \bmod 3$
- $79^{12} \bmod 6$
- $2^{1000} \bmod 5$

2. Some numbers and their prime factorizations are given below.

- $140 = 2^2 \times 5 \times 7$
- $175 = 5^2 \times 7$
- $532 = 2^2 \times 7 \times 19$
- $648 = 2^3 \times 3^4$
- $1078 = 2 \times 7^2 \times 11$
- $1083 = 3 \times 19^2$
- $15435 = 3^2 \times 5 \times 7^3$
- $1078 = 2 \times 7^2 \times 11$
- $25480 = 2^3 \times 5 \times 7^2 \times 13$

Use these prime factorizations to compute the following quantities.

- (a)  $\gcd(532, 15435)$
- (b)  $\gcd(648, 1083)$
- (c)  $\text{lcm}(532, 1083)$
- (d)  $\text{lcm}(1083, 15435)$
- (e)  $\text{lcm}(648, 15435)$
- (f)  $\gcd(1078, 140)$
- (g)  $\gcd(1078, 25480)$
- (h)  $\text{lcm}(1078, 140)$
- (i)  $\text{lcm}(175, 25480)$
- (j)  $\text{lcm}(140, 25480)$

3. Suppose that the  $O(\sqrt{N})$  algorithm for primality test is given the number 653117 as input. How many numbers would the algorithm have to check to either find a factor or determine that the input is prime? (653117 happens to be a prime number).