ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

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Talk Overview

Outline

Relation

Relation

A binary relation between two sets A and B is a subset R of A x B.

Ex.

- S is the set of students at a university and C is the set of classes offered by the university.
- The relation E between S and C indicates whether a student is enrolled in a given class.
- Usually we can denote this relation as sEc.

Relations and Function

Recall that functions have more restrictions on the connection between the domain and the target as follows.

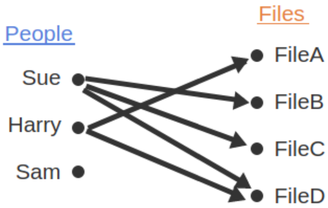
- Each element in the domain should point to one and only one element in the target.
- This is not the case in the relations

Arrow diagram for a relation

People = { Sue, Harry, Sam }

Files = { FileA, FileB, FileC, FileD }

Relation A: pAf if person p has access to file f



A = { (Sue, FileB) , (Sue, FileC) , (Sue, FileD) , (Harry, FileA) , (Harry, FileD) }

Matrix representation for a relation

People = { Sue, Harry, Sam }

Files = { File A, File B, File C, File D }

Relation A: pAf if person p has access to file f

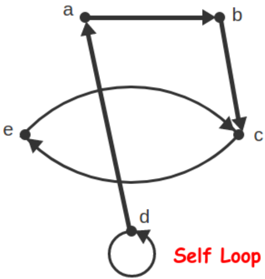
$$\begin{array}{c} \\ \text{Sue} \\ \text{Harry} \\ \text{Sam} \end{array} \begin{array}{c} \begin{array}{cccc} \text{File A} & \text{File B} & \text{File C} & \text{File D} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

A = { (Sue, File B) (Sue, File C) (Sue, File D)
(Harry, File A) (Harry, File D) }

Binary Relation on a Set

- We can have a binary relation between a set A and itself.
- In this case we call it a binary relation on the set A.
- The result is a subset of A x A.
- The set A is called the domain of the binary relation.

Ex.



A = { a, b, c, d, e }

R $\subseteq$ A x A

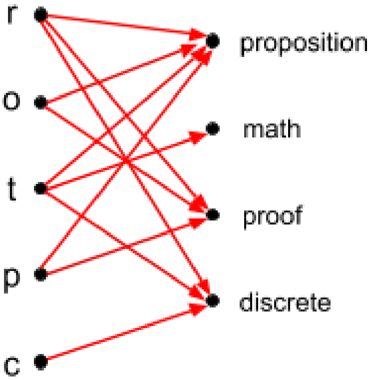
R = { (a, b) (b, c) (e, c) (c, e) (d, a) (d, d) }

Excercise 1

Draw the arrow diagram and the matrix representation for the following relation. Define the set A $=$ {r, o, t, p, c} and B $=$ {discrete, math, proof, proposition}. Define the relation R $\subseteq$ A x B such that (letter, word) is in the relation if that letter occurs somewhere in the word.

Excercise 1

Draw the arrow diagram and the matrix representation for the following relation. Define the set A = {r, o, t, p, c} and B = {discrete, math, proof, proposition}. Define the relation R ⊆ A x B such that (letter, word) is in the relation if that letter occurs somewhere in the word.



$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Excercise 2

Draw the arrow diagram for the following relation. The domain of relation D is {2, 3, 12, 16, 27, 48}. For x, y in the domain, xDy if y is an integer multiple of x.

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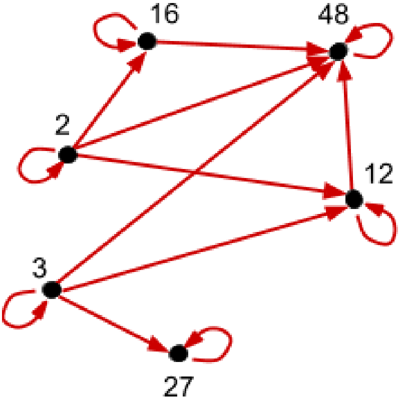
Draw the arrow diagram for the following relation. The domain of relation D is {2, 3, 12, 16, 27, 48}. For x, y in the domain, xDy if y is an integer multiple of x.

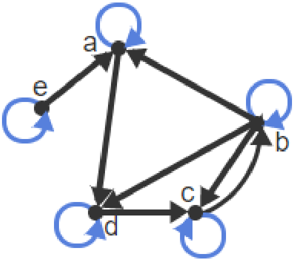


Outline

Binary relation R can be characterized by six properties. The properties are defined and illustrated using arrow diagrams.

- The relation R can be either reflexive or anti-reflexive or neither.
- The relation R can be either symmetric or anti-symmetric or neither.
- The relation R can be either transitive or not transitive.

Reflexive Relation

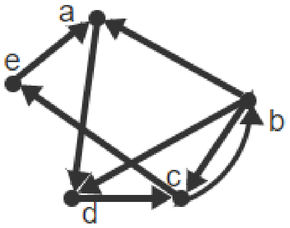


A = { a, b, c, d, e }

Relation R is reflexive if for all x ∈ A x R x

a R a, b R b, c R c, d R d, and e R e

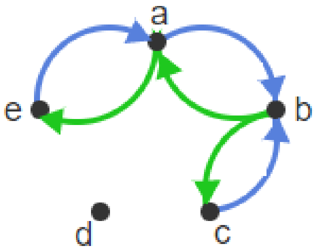
Anti-Reflexive Relation



A = { a, b, c, d, e }

Relation R is
anti-reflexive
if for all $x \in A$
it is not true that
x R x

Symmetric Relation

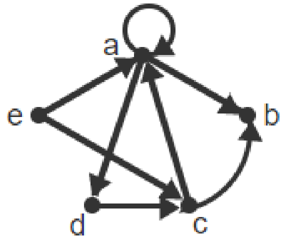


$A = \{ a, b, c, d, e \}$

Relation R on A is
symmetric
if for all $x, y \in A$
$x R y \leftrightarrow y R x$

$x R y \;\; \leftrightarrow \;\; y R x$

Anti-Symmetric Relation

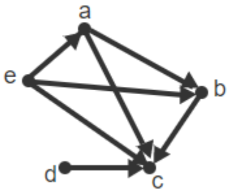


A = { a, b, c, d, e }

Relation R is
anti-symmetric
if for all $x, y \in A$
$x \mathrel{R} y$ and $y \mathrel{R} x \to x = y$

Note: there is no 

Transitive Relation



$A = \{ a, b, c, d, e \}$

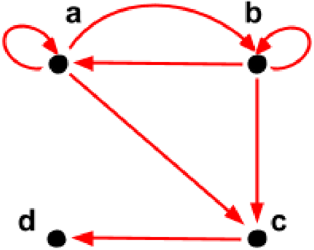
Relation R on A is
transitive if
for all $x, y, z \in A$
if $x \mathrel{R} y$ and $y \mathrel{R} z$,
then $x \mathrel{R} z$

$e \mathrel{R} a$ and $a \mathrel{R} b \longrightarrow e \mathrel{R} b$
$e \mathrel{R} b$ and $b \mathrel{R} c \longrightarrow e \mathrel{R} c$
$e \mathrel{R} a$ and $a \mathrel{R} c \longrightarrow e \mathrel{R} c$
$a \mathrel{R} b$ and $b \mathrel{R} c \longrightarrow a \mathrel{R} c$

Excercise 1

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
- symmetric, anti-symmetric, or neither
- transitive or not transitive



Excercise 1

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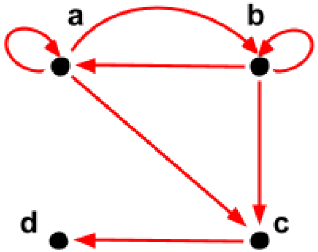


Answer.

- neither reflexive nor anti-reflexive.
- neither symmetric nor anti-symmetric.
- not transitive.

Excercise 2

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
- symmetric, anti-symmetric, or neither
- transitive or not transitive

The domain of the relation L is the set of all real numbers. For $x, y \in R$, xLy if $x < y$.

Excercise 2

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
- symmetric, anti-symmetric, or neither
- transitive or not transitive

The domain of the relation L is the set of all real numbers. For $x, y \in R$, xLy if $x < y$.

Answer.

- anti-reflexive: For any real number x, it is always false that $x < x$.
- anti-symmetric: For any two real numbers x and y, it can not be true that $x < y$ and $y < x$.
- transitive: If $x < y$ and $y < z$, then $x < z$.

Excercise 3

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
- symmetric, anti-symmetric, or neither
- transitive or not transitive

The domain of the relation L is the set of all real numbers. For $x, y \in R$, xLy if $x \leq y$.

Excercise 3

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
- symmetric, anti-symmetric, or neither
- transitive or not transitive

The domain of the relation L is the set of all real numbers. For $x, y \in R$, xLy if $x \leq y$.

Answer.

- reflexive: For any real number x, it is always true that $x \leq x$.
- anti-symmetric: For any two real numbers x and y, if $x \leq y$ and $y \leq x$, then $x = y$.
- transitive: If $x \leq y$ and $y \leq z$, then $x \leq z$.

Outline

Graph

Graph is simply a relation over set. It is used widely in computer science topics.

Ex.

- Internet pages.
- Friends on facebook.
- Train/Bus stations.
- Communication network.
- etc.

Directed Graph (digraph)

Digraph

A directed graph (or digraph, for short) consists of a pair (V, E). V is a set of vertices, and E, a set of directed edges.

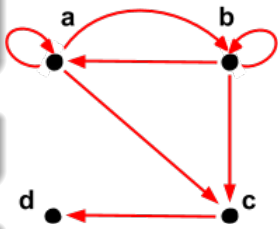
Vertex

An individual element of V is called a vertex.

Edge

A connection between two vertices.

Example

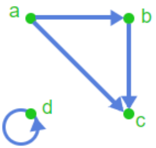


Graph Example

Graph G = (V, E)

V = { a, b, c, d }

E = { (a, b), (b, c), (a, c), (d, d) }



a is the tail of edge (a, b)
b is the head of edge (a, b)

The in-degree of c is 2
The out-degree of b is 1
The in-degree of d is 1

In-degree

The in-degree of a vertex is the number of edges pointing into it.

Out-degree

The out-degree of a vertex is the number of edges pointing out of it.

Walks and directed Graph

A walk in a directed graph G is a sequence of alternating vertices and edges that starts and ends with a vertex.

$$< v_0, v_1, v_2, ..., v_n >$$

Walk length

The length of a walk is l, the number of edges in the walk.

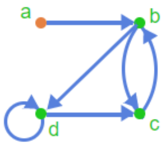
Open walk

An open walk is a walk in which the first and last vertices are not the same.

Closed walk

A closed walk is a walk in which the first and last vertices are the same.

Example



Walk:

⟨ a, b, c, b, d ⟩ walk length = 4

The walk is open because
the first and last vertices are not the same

Walk:

⟨ b, d, c, b, c, b ⟩ walk length = 5

Edge (c, b) occurs twice
so (c, b) is counted twice

This walk closed because the
first and last vertices are the same.

Closed walk of length 1: ⟨ d, d ⟩

Closed walk of length 0: ⟨ a ⟩

Defination

Trail

A trail is an open walk in which no edge occurs more than once.

Path

A path is a trail in which no vertex occurs more than once.

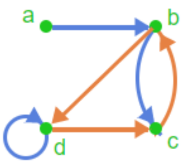
Circuit

A circuit is a closed walk in which no edge occurs more than once.

Cycle

A cycle is a circuit in which no vertex occurs more than once, except the first and last vertices which are the same.

Example



Walk:

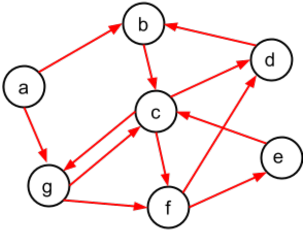
⟨ a, b, c, b, d ⟩ No edge occurs more than once. So this open walk is a trail.

b is reached twice
So this trail is not a path

⟨ b, d, c, b ⟩ No edge occurs more than once. So this closed walk is a circuit.

The circuit is a cycle because only the first and last vertices are repeated.

Excercise



- What is the in-degree of vertex d?
- What is the out-degree of vertex c?
- What is the head of edge (b, c)?
- What is the tail of edge (g, f)?
- List all the self-loops in the graph.
- Is <a, g, f, c, d> a walk in the graph? Is it a trail? Is it a path?
- Is <a, g, f, d, b> a walk in the graph? Is it a trail? Is it a path?
- Is <c, g, f, e> a circuit in the graph? Is it a cycle?

Questions