ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

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Talk Overview

1. Propositions and logical operations
2. Evaluating compound propositions
3. Conditional Operation
4. Logical Equivalence
5. Laws of propositional logic
6. Predicates and quantifiers
7. Quantified statements

Outline

What is logic?

Logic

Logic is the study of formal reasoning.

- Logic statement always has a well defined meaning.
- Logic used in
  - Artificial intelligence for automated reasoning.
  - Embedded systems for designing digital circuits.
  - Laws logic for defining the implications of a particular law.
  - Medicine for conditions and diagnosis.

Proposition(1/2)

Proposition

Proposition is a statement that is either evaluated to true or false.

Truth Value

It is a value indicating whether the proposition is actually true or false.

Propositions Examples:

- There are an infinite number of prime numbers. True
- The Declaration of Independence was signed on July 4,1812. False

Preposition(2/2)

- Propositions are declarative sentences.
  Not Propositions Examples:
  - What time is it? Question
  - Have a nice day. Command
- Proposition truth value can be true, false, unknown, or a matter of opinion. **Examples:**
  - Monday will be cloudy. Unknown
  - The movie was funny. A matter of opinion
  - The extinction of the dinosaurs was caused by a meteor. Unknown

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.
  - Command, not a proposition.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.
  - Command, not a proposition.
- The soup is cold.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.
  - Command, not a proposition.
- The soup is cold.
  - Proposition. Negation: The soup is not cold.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.
  - Command, not a proposition.
- The soup is cold.
  - Proposition. Negation: The soup is not cold.
- The patient has diabetes.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.
  - Command, not a proposition.
- The soup is cold.
  - Proposition. Negation: The soup is not cold.
- The patient has diabetes.
  - Proposition. Negation: The patient does not have diabetes.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.
  - Command, not a proposition.
- The soup is cold.
  - Proposition. Negation: The soup is not cold.
- The patient has diabetes.
  - Proposition. Negation: The patient does not have diabetes.
- The light is on.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

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- The patient has diabetes.
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- The light is on.
  - Proposition. Negation: The light is off.
- It's a beautiful day.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

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  - Proposition. Negation: The soup is not cold.
- The patient has diabetes.
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- The light is on.
  - Proposition. Negation: The light is off.
- It's a beautiful day.
  - Proposition. Negation: It is not a beautiful day.

Excercise

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  - Proposition. Negation: The soup is not cold.
- The patient has diabetes.
  - Proposition. Negation: The patient does not have diabetes.
- The light is on.
  - Proposition. Negation: The light is off.
- It's a beautiful day.
  - Proposition. Negation: It is not a beautiful day.
- $2 + 3 = 6$

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

- Have a nice day.
  - Command, not a proposition.
- The soup is cold.
  - Proposition. Negation: The soup is not cold.
- The patient has diabetes.
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- The light is on.
  - Proposition. Negation: The light is off.
- It's a beautiful day.
  - Proposition. Negation: It is not a beautiful day.
- $2 + 3 = 6$
  - Proposition. Negation: $2 + 3 \neq 6$.

Excercise

Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

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- $2 + 3 = 6$
  - Proposition. Negation: $2 + 3 \neq 6$.
- All politicians are dishonest.

Excercise

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- The light is on.
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- It's a beautiful day.
  - Proposition. Negation: It is not a beautiful day.
- $2 + 3 = 6$
  - Proposition. Negation: $2 + 3 \neq 6$.
- All politicians are dishonest.
  - Proposition. Negation: All politicians are honest

Variables

- Variables names such as p and q can be used to denote arbitrary propositions.

Example:

- **p:** January has 31 days.
- **q:** February has 33 days.

Compound Proposition

It is created by connecting individual propositions with logical operations.

Types of logical operations:

- Negation **Ex.** not p $\equiv \neg$ p
- Conjunction. **Ex.** p and q $\equiv$ p $\wedge$ q
- Disjunction. **Ex.** p and q $\equiv$ p $\vee$ q

Negation Operation

- The negation operation acts on just one proposition.
- It has the effect of reversing the truth value of the proposition.
- It is denoted as $\neg$p and read as not p.

Example:

- **p:** The patient has diabetes.
- $\neg$ **p:** The patient does not have diabetes

Conjunction Operation

- The proposition $p \wedge q$ is read p and q. $p \wedge q$ is true if both p is true and q is true.
- $p \wedge q$ is false if p is false, q is false, or both are false

Ex.

- **p:** January has 31 days.
- **q:** February has 33 days.
- **p $\wedge$ q:**January has 31 days and February has 33 days.

Given the truth values of "p" and "q", what is the truth value of $p \wedge q$?

Disjunction Operation

- The proposition $p \vee q$ is read p or q. $p \vee q$ is true if either p is true or q is true.
- $p \vee q$ is false if both p and q are false.

Ex.

- **p:** January has 31 days.
- **q:** February has 33 days.
- **p $\vee$ q:**January has 31 days or February has 33 days.

Given the truth values of "p" and "q", what is the truth value of $p \vee q$?

Types of OR

Inclusive or

The inclusive or is the same as the disjunction $\vee$ operation and evaluates to true when one or both of the propositions are true

Example: Lucy opens the windows or doors when warm

Exclusive or

The exclusive or of p and q evaluates to true only when p is true and q is false or when q is true and p is false.

Example: Lucy is going to the park or the movie

Denoted as $p \oplus q$

Excercise

Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or.

- February has 31 days or the number 5 is an integer.

Excercise

Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or.

- February has 31 days or the number 5 is an integer.
  - Inclusive or: True. Exclusive or: True.

Excercise

Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or.

- February has 31 days or the number 5 is an integer.
  - Inclusive or: True. Exclusive or: True.
- The number π is an integer or the sun revolves around the earth.

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- February has 31 days or the number 5 is an integer.
  - Inclusive or: True. Exclusive or: True.
- The number π is an integer or the sun revolves around the earth.
  - Inclusive or: False. Exclusive or: False.

Excercise

Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or.

- February has 31 days or the number 5 is an integer.
  - Inclusive or: True. Exclusive or: True.
- The number π is an integer or the sun revolves around the earth.
  - Inclusive or: False. Exclusive or: False.
- 20 nickels are worth one dollar or whales are mammals.

Excercise

Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or.

- February has 31 days or the number 5 is an integer.
  - Inclusive or: True. Exclusive or: True.
- The number π is an integer or the sun revolves around the earth.
  - Inclusive or: False. Exclusive or: False.
- 20 nickels are worth one dollar or whales are mammals.
  - Inclusive or: True. Exclusive or: False.

Excercise

Assume the propositions p, q, r, and s have the following truth values:

- p is false
- q is true
- r is false
- s is true

Ex.

- $\neg p$

Excercise

Assume the propositions p, q, r, and s have the following truth values:

- p is false
- q is true
- r is false
- s is true

Ex.

- $\neg p$
    - True

Excercise

Assume the propositions p, q, r, and s have the following truth values:

- p is false
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- s is true

Ex.

- $\neg p$
  - True
- $q \wedge s$

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Assume the propositions p, q, r, and s have the following truth values:

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Ex.

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  - True
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Ex.

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  - True
- $q \wedge s$
  - True
- $q \vee s$

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- $q \wedge s$
  - True
- $q \vee s$
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Assume the propositions p, q, r, and s have the following truth values:

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Ex.

- $\neg p$
  - True
- $q \wedge s$
  - True
- $q \vee s$
  - True
- $q \oplus s$

Excercise

Assume the propositions p, q, r, and s have the following truth values:

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Ex.

- $\neg p$
  - True
- $q \wedge s$
  - True
- $q \vee s$
  - True
- $q \oplus s$
  - False

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  - True
- $q \wedge s$
  - True
- $q \vee s$
  - True
- $q \oplus s$
  - False
- $q \oplus r$

Excercise

Assume the propositions p, q, r, and s have the following truth values:

- p is false
- q is true
- r is false
- s is true

Ex.

- $\neg p$
  - True
- $q \wedge s$
  - True
- $q \vee s$
  - True
- $q \oplus s$
  - False
- $q \oplus r$
  - True

Excercise

Express each English statement using only logical operations $\land$, $\lor$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.

Excercise

Express each English statement using only logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

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m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \wedge m$

Excercise

Express each English statement using only logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \wedge m$
- The patient took the medication, but still had migraines.

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Express each English statement using only logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below.

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n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \wedge m$
- The patient took the medication, but still had migraines.
  - $t \wedge m$

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Express each English statement using only logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \wedge m$
- The patient took the medication, but still had migraines.
  - $t \wedge m$
- The patient did not have migraines.

Excercise

Express each English statement using only logical operations $\land$, $\lor$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \land m$
- The patient took the medication, but still had migraines.
  - $t \land m$
- The patient did not have migraines.
  - $\neg m$

Excercise

Express each English statement using only logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \wedge m$
- The patient took the medication, but still had migraines.
  - $t \wedge m$
- The patient did not have migraines.
  - $\neg m$
- Despite the fact that the patient took the medication, the patient had nausea.

Excercise

Express each English statement using only logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \wedge m$
- The patient took the medication, but still had migraines.
  - $t \wedge m$
- The patient did not have migraines.
  - $\neg m$
- Despite the fact that the patient took the medication, the patient had nausea.
  - $t \wedge n$

Excercise

Express each English statement using only logical operations $\land$, $\lor$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \land m$
- The patient took the medication, but still had migraines.
  - $t \land m$
- The patient did not have migraines.
  - $\neg m$
- Despite the fact that the patient took the medication, the patient had nausea.
  - $t \land n$
- There is no way that the patient took the medication.

Excercise

Express each English statement using only logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below.

t: The patient took the medication.

n: The patient had nausea.

m: The patient had migraines.

Ex.

- The patient had nausea and migraines.
  - $n \wedge m$
- The patient took the medication, but still had migraines.
  - $t \wedge m$
- The patient did not have migraines.
  - $\neg m$
- Despite the fact that the patient took the medication, the patient had nausea.
  - $t \wedge n$
- There is no way that the patient took the medication.
  - $\neg t$

Truth Table(1/2)

Truth Table

It shows the truth value of a compound proposition for every possible combination of truth values for the variables contained in the compound proposition.

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	$\neg$ p
T	F
F	T

Truth Table(2/2)

How to fill the truth table for a compound proposition?

- If there are **n** variables, there are 2^n. rows.
- The T and F values for each row are unique.
- Note that: The column of the first varible on the right alternates T F T F..., the column for the second variable alternates T T F F ..., etc.
- Can you fill the following truth table?

p	**q**	**p $\oplus$ q**
T	T	
T	F	
F	T	
F	F	

Outline

Evaluation Order

Since the compound proposition can contain many variables and many operations, the order of evaluating the operations matters. Order of operations in absence of parentheses.

1. $\neg$ not
2. $\wedge$ and
3. $\vee$ or

Example:

- p : T, q : F, r : T

1. $p \wedge \neg(q \vee r)$

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2. $\wedge$ and
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Example:

- p : T, q : F, r : T

1. $p \wedge \neg(q \vee r)$
2. $T \wedge \underbrace{\neg(F \vee T)}$
3. $T \wedge \underbrace{\neg T}$
4. $T \wedge \underbrace{F}$
5. F

Evaluation Order

Since the compound proposition can contain many variables and many operations, the order of evaluating the operations matters. Order of operations in absence of parentheses.

1. $\neg$ not
2. $\wedge$ and
3. $\vee$ or

Example:

- p : T, q : F, r : T

1. p $\wedge$ $\neg$(q $\vee$ r)
2. T $\wedge$ $\underbrace{\neg(F \vee T)}$
3. T $\wedge$ $\underbrace{\neg T}$
4. T $\wedge$ $\underbrace{F}$
5. F

Try this!

- $p \vee \neg q \wedge r$

Excercise

- Write the truth table for $r \vee (p \wedge \neg q)$.

Excercise

- Write the truth table for $r \vee (p \wedge \neg q)$.

p	q	r	$r \vee (p \wedge \neg q)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Excercise

Consider the following identification a person might have to apply for credit:

- B: Applicant presents a birth certificate.
- D: Applicant presents a drivers license.
- M: Applicant presents a marriage license.

Questions

1. The applicant must present either a birth certificate, a drivers license or a marriage license.

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Consider the following identification a person might have to apply for credit:

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1. The applicant must present either a birth certificate, a drivers license or a marriage license.
    - $B \vee D \vee M$

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Questions

1. The applicant must present either a birth certificate, a drivers license or a marriage license.
   - $B \vee D \vee M$
2. The applicant must present at least two of the following forms of identification: birth certificate, drivers license, marriage license.

Excercise

Consider the following identification a person might have to apply for credit:

- B: Applicant presents a birth certificate.
- D: Applicant presents a drivers license.
- M: Applicant presents a marriage license.

Questions

1. The applicant must present either a birth certificate, a drivers license or a marriage license.
    - $B \lor D \lor M$
2. The applicant must present at least two of the following forms of identification: birth certificate, drivers license, marriage license.
    - $(B \land M) \lor (B \land D) \lor (M \land D)$

Excercise

Consider the following identification a person might have to apply for credit:

- B: Applicant presents a birth certificate.
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Questions

1. The applicant must present either a birth certificate, a drivers license or a marriage license.
   - $B \vee D \vee M$
2. The applicant must present at least two of the following forms of identification: birth certificate, drivers license, marriage license.
   - $(B \wedge M) \vee (B \wedge D) \vee (M \wedge D)$
3. Applicant must present either a birth certificate or both a drivers license and a marriage license.

Excercise

Consider the following identification a person might have to apply for credit:

- B: Applicant presents a birth certificate.
- D: Applicant presents a drivers license.
- M: Applicant presents a marriage license.

Questions

1. The applicant must present either a birth certificate, a drivers license or a marriage license.
   - $\text{B} \vee \text{D} \vee \text{M}$
2. The applicant must present at least two of the following forms of identification: birth certificate, drivers license, marriage license.
   - $(B \wedge M) \vee (B \wedge D) \vee (M \wedge D)$
3. Applicant must present either a birth certificate or both a drivers license and a marriage license.
   - $B \vee (D \wedge M)$

Outline

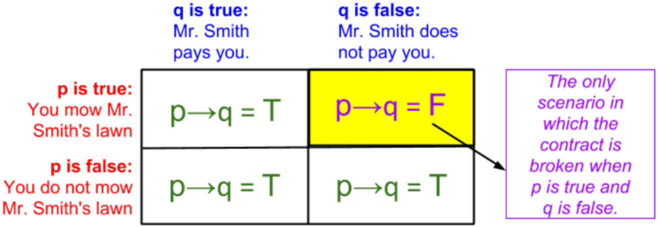
Conditional Operation (1/2)

- The proposition $p \to q$ is read "if p then q".
- In $p \to q$, proposition p is called the hypothesis, and the proposition q is called the conclusion.
- A conditional proposition can be thought of like a contract between two parties.

Conditional Operation (1/2)

- The proposition p → q is read "if p then q".
- In p → q, proposition p is called the hypothesis, and the proposition q is called the conclusion.
- A conditional proposition can be thought of like a contract between two parties.

Ex: If $\underbrace{\text{you mow Mr. Smith's lawn}}_{p}$, then $\underbrace{\text{he will pay you}}_{q}$.



Conditional Operation (2/2)

- The proposition p → q is false if p is true and q is false; otherwise, $p \rightarrow q$ is true.

Truth Table

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Ex.

- **p:** There is a traffic jam today.
- **q:** I will be late for work.
- **p → q:** If there is a traffic jam today, then I will be late for work.

Examples on Conditional Operations

Think of the following as contracts and find truth values

- **s:** If it rains today, I will have my umbrella.
  It is raining today.
  I do not have my umbrella.

Examples on Conditional Operations

Think of the following as contracts and find truth values

- **s:** If it rains today, I will have my umbrella.
  It is raining today.
  I do not have my umbrella.
  - False

Examples on Conditional Operations

Think of the following as contracts and find truth values

- **s:** If it rains today, I will have my umbrella.
  It is raining today.
  I do not have my umbrella.
  - False
- **s:** If Sally took too long getting ready, she missed the bus.
  Sally did not take too long getting ready.
  Sally missed the bus.

Examples on Conditional Operations

Think of the following as contracts and find truth values

- **s:** If it rains today, I will have my umbrella.
  It is raining today.
  I do not have my umbrella.
  - False
- **s:** If Sally took too long getting ready, she missed the bus.
  Sally did not take too long getting ready.
  Sally missed the bus.
  - True

Examples on Conditional Operations

Think of the following as contracts and find truth values

- **s:** If it rains today, I will have my umbrella.
  It is raining today.
  I do not have my umbrella.
  - False
- **s:** If Sally took too long getting ready, she missed the bus.
  Sally did not take too long getting ready.
  Sally missed the bus.
  - True
- **s:** If it is sunny out, I ride my bike.
  It is not sunny out.
  I am not riding my bike.

Examples on Conditional Operations

Think of the following as contracts and find truth values

- **s:** If it rains today, I will have my umbrella.
  It is raining today.
  I do not have my umbrella.
  - False
- **s:** If Sally took too long getting ready, she missed the bus.
  Sally did not take too long getting ready.
  Sally missed the bus.
  - True
- **s:** If it is sunny out, I ride my bike.
  It is not sunny out.
  I am not riding my bike.
  - True

English expressions of the Conditional Operations

Ex: If $\underbrace{\text{you mow Mr. Smith's lawn}}_{p\ (hypothesis)}$, then $\underbrace{\text{he will pay you}}_{q\ (conclusion)}$.

If p, then q.	If you mow Mr. Smith's lawn, then he will pay you.
If p, q.	If you mow Mr. Smith's lawn, he will pay you.
q if p	Mr. Smith will pay you if you mow his lawn.
p implies q.	Mowing Mr. Smith's lawn implies that he will pay you.
p only if q.	You will mow Mr. Smith's lawn only if he pays you.
p is sufficient for q.	Mowing Mr. Smith's lawn is sufficient for him to pay you.
q is necessary for p.	Mr. Smith's paying you is necessary for you to mow his lawn.

Excercise

Define the following propositions:

w: the roads were wet

a: there was an accident

h: traffic was heavy

Express in english form

- $w \rightarrow h$

Excercise

Define the following propositions:

w: the roads were wet

a: there was an accident

h: traffic was heavy

Express in english form

- $w \rightarrow h$
  - "If the roads were wet then traffic was heavy."

Excercise

Define the following propositions:

w: the roads were wet

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h: traffic was heavy

Express in english form

- $w \rightarrow h$
  - "If the roads were wet then traffic was heavy."
- $w \wedge h$

Excercise

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h: traffic was heavy

Express in english form

- $w \rightarrow h$
  - "If the roads were wet then traffic was heavy."
- $w \wedge h$
  - "If the roads were wet then traffic was heavy."

Excercise

Define the following propositions:

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a: there was an accident

h: traffic was heavy

Express in english form

- w $\rightarrow$ h
  - "If the roads were wet then traffic was heavy."
- w $\wedge$ h
  - "If the roads were wet then traffic was heavy."
- $\neg(a \wedge h)$

Excercise

Define the following propositions:

w: the roads were wet

a: there was an accident

h: traffic was heavy

Express in english form

- w $\rightarrow$ h
  - "If the roads were wet then traffic was heavy."
- w $\wedge$ h
  - "If the roads were wet then traffic was heavy."
- $\neg(a \wedge h)$
  - "It is not true that there was an accident and traffic was heavy."

Excercise

Define the following propositions:

w: the roads were wet

a: there was an accident

h: traffic was heavy

Express in english form

- w → h
    - "If the roads were wet then traffic was heavy."
- w ∧ h
    - "If the roads were wet then traffic was heavy."
- $\neg(a \wedge h)$
    - "It is not true that there was an accident and traffic was heavy."
- $h \rightarrow (a \vee w)$

Excercise

Define the following propositions:

w: the roads were wet

a: there was an accident

h: traffic was heavy

Express in english form

- w → h
  - "If the roads were wet then traffic was heavy."
- w ∧ h
  - "If the roads were wet then traffic was heavy."
- $\neg(a \wedge h)$
  - "It is not true that there was an accident and traffic was heavy."
- $h \rightarrow (a \vee w)$
  - "If traffic was heavy then there was an accident or the roads were wet."

Excercise

Define the following propositions:

w: the roads were wet

a: there was an accident

h: traffic was heavy

Express in english form

- $\mathrm{w} \to \mathrm{h}$
  - "If the roads were wet then traffic was heavy."
- $\mathrm{w} \wedge \mathrm{h}$
  - "If the roads were wet then traffic was heavy."
- $\neg(a \wedge h)$
  - "It is not true that there was an accident and traffic was heavy."
- $h \to (a \vee w)$
  - "If traffic was heavy then there was an accident or the roads were wet."
- $w \wedge \neg h$

Excercise

Define the following propositions:

w: the roads were wet

a: there was an accident

h: traffic was heavy

Express in english form

- w → h
  - "If the roads were wet then traffic was heavy."
- w ∧ h
  - "If the roads were wet then traffic was heavy."
- $\neg(a \wedge h)$
  - "It is not true that there was an accident and traffic was heavy."
- $h \rightarrow (a \vee w)$
  - "If traffic was heavy then there was an accident or the roads were wet."
- $w \wedge \neg h$
  - "The roads were wet but traffic was not heavy."

Excercise

For a degree in Computer Science, a student must take one of three project courses, P1, P2, or P3. The student must also take one of two theory courses, T1 or T2. Furthermore, if the student is an honors student, he or she must take the honors seminar S. Let H be the proposition indicating whether the student is an honors student.

- Formulate the previous statements using logical propositions.

Excercise

For a degree in Computer Science, a student must take one of three project courses, P1, P2, or P3. The student must also take one of two theory courses, T1 or T2. Furthermore, if the student is an honors student, he or she must take the honors seminar S. Let H be the proposition indicating whether the student is an honors student.

- Formulate the previous statements using logical propositions.
  - $(P1 \vee P2 \vee P3) \wedge (T1 \vee T2) \wedge (H \rightarrow S)$

The Converse,Contrapositive, and Inverse

Proposition:	$p \rightarrow q$	Ex: If it is raining today, the game will be cancelled.
Inverse:	$\neg p \rightarrow \neg q$	If it is not raining today, the game will not be cancelled.
Converse:	$q \rightarrow p$	If the game is cancelled, it is raining today.
Contrapositive:	$\neg q \rightarrow \neg p$	If the game is not cancelled, then it is not raining today.

Excercise

Give the inverse, converse and contrapositive for each of the following statements.

- **Q:** If the patient took the medicine, then she had side effects.

Excercise

Give the inverse, converse and contrapositive for each of the following statements.

- **Q:** If the patient took the medicine, then she had side effects.
- **A:**

Excercise

Give the inverse, converse and contrapositive for each of the following statements.

- **Q:** If the patient took the medicine, then she had side effects.
- **A:**
  - **Inverse:** If the patient didnt take the medicine, then she didnt have side effects.

Excercise

Give the inverse, converse and contrapositive for each of the following statements.

- **Q:** If the patient took the medicine, then she had side effects.
- **A:**
  - **Inverse:** If the patient didnt take the medicine, then she didnt have side effects.
  - **Converse:** If the patient had side effects, then she took the medicine.

Excercise

Give the inverse, converse and contrapositive for each of the following statements.

- **Q:** If the patient took the medicine, then she had side effects.
- **A:**
  - **Inverse:** If the patient didnt take the medicine, then she didnt have side effects.
  - **Converse:** If the patient had side effects, then she took the medicine.
  - **Contrapositive:** If the patient didnt have side effects, then she didnt take the medicine.

The Biconditional Operation

- The proposition "p if and only if q" is expressed with the biconditional operation and is denoted $p \leftrightarrow q$.
- It is true when p and q have the same truth value and is false when p and q have different truth values.

Other meanings includes:

- p is necessary and sufficient for q.
- if p then q, and conversely.
- **iff** is an abbreviation of the expression "if and only if".

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Evaluation Order Now

Order of operations in absence of parentheses.

1. $\neg$ not
2. $\wedge$ and
3. $\vee$ or
4. $\rightarrow$ if
5. $\leftrightarrow$ if and only if

Exercise

- **Evaluate:** $p \vee \neg(q \leftrightarrow r)$
- **Given** p : T, q : T, r : F

1. $T \vee \neg\underbrace{(T \leftrightarrow F)}$

Exercise

- **Evaluate:** $p \vee \neg(q \leftrightarrow r)$
- **Given** p : T, q : T, r : F

1. $T \vee \neg\underbrace{(T \leftrightarrow F)}$
2. $T \vee \underbrace{\neg F}$

Exercise

- **Evaluate:** $p \vee \neg(q \leftrightarrow r)$
- **Given** p : T, q : T, r : F

1. $T \vee \neg \underbrace{(T \leftrightarrow F)}$
2. $T \vee \underbrace{\neg F}$
3. $T \vee \underbrace{T}$

Exercise

- **Evaluate:** $p \vee \neg(q \leftrightarrow r)$
- **Given** p : T, q : T, r : F

1. $T \vee \neg\underbrace{(T \leftrightarrow F)}$
2. $T \vee \underbrace{\neg F}$
3. $T \vee \underbrace{T}$
4. T

Excercise

Give a truth table for each expression.

$(\neg p \wedge q) \rightarrow p$ $\quad$ $(p \rightarrow q) \rightarrow (q \rightarrow p)$ $\quad$ $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

Excercise

Give a truth table for each expression.

$(\neg p \wedge q) \to p$ $\quad$ $(p \to q) \to (q \to p)$ $\quad$ $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	(¬p ∧ q) → p
T	T	T
T	F	T
F	T	F
F	F	T

p	q	(p → q) → (q → p)
T	T	T
T	F	T
F	T	F
F	F	T

p	q	(p ↔ q) ⊕ (p ↔ ¬q)
T	T	T
T	F	T
F	T	T
F	F	T

Excercise

s: A person is a senior
y: A person is at least 17 years of age
p: A person is allowed to park in the school parking lot

Express in logic form

- A person can park in the school parking lot if they are a senior or at least seventeen years of age.

Excercise

s: A person is a senior

y: A person is at least 17 years of age

p: A person is allowed to park in the school parking lot

Express in logic form

- A person can park in the school parking lot if they are a senior or at least seventeen years of age.
  - $(s \vee y) \rightarrow p$

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s: A person is a senior
y: A person is at least 17 years of age
p: A person is allowed to park in the school parking lot

Express in logic form

- A person can park in the school parking lot if they are a senior or at least seventeen years of age.
  - $(s \vee y) \rightarrow p$
- Being 17 years of age is a necessary condition for being able to park in the school parking lot.

Excercise

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  - $(s \vee y) \rightarrow p$
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  - $p \rightarrow y$

Excercise

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Express in logic form

- A person can park in the school parking lot if they are a senior or at least seventeen years of age.
  - $(s \vee y) \rightarrow p$
- Being 17 years of age is a necessary condition for being able to park in the school parking lot.
  - $p \rightarrow y$
- A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

Excercise

s: A person is a senior
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Express in logic form

- A person can park in the school parking lot if they are a senior or at least seventeen years of age.
  - $(s \vee y) \rightarrow p$
- Being 17 years of age is a necessary condition for being able to park in the school parking lot.
  - $p \rightarrow y$
- A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.
  - $p \leftrightarrow (s \wedge y)$

Outline

Tutology and Contradiction

- **Tutology:** A compound proposition that is always true.
  - **Ex.** $p \vee \neg p$
- **Contradiction:** A compound proposition that is always false.
  - **Ex.** $p \wedge \neg p$

Is this statment a tutology, contradiction, or neither. $p \wedge q \rightarrow p$?

Excercise

Show whether each logical expression is a tautology, contradiction or neither.

1. $(p \vee q) \vee (q \rightarrow p)$

Excercise

Show whether each logical expression is a tautology, contradiction or neither.

1. $(p \vee q) \vee (q \rightarrow p)$

p	q	(p ∨ q) ∨ (q → p)
T	T	T
T	F	T
F	T	T
F	F	T

Excercise

Show whether each logical expression is a tautology, contradiction or neither.

1. $(p \vee q) \vee (q \rightarrow p)$

p	q	(p ∨ q) ∨ (q → p)
T	T	T
T	F	T
F	T	T
F	F	T

2. $(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$

Excercise

Show whether each logical expression is a tautology, contradiction or neither.

1. $(p \vee q) \vee (q \rightarrow p)$

p	q	(p ∨ q) ∨ (q → p)
T	T	T
T	F	T
F	T	T
F	F	T

2. $(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$

p	q	(p → q) ↔ (p ∧ ¬q)
T	T	F
T	F	F
F	T	F
F	F	F

Logical Equivalence

Logical Equivalence

Two compound propositions are logically equivalent if they have the same truth value regardless of the truth values of their individual propositions.

- The notation $s \equiv r$ is used to indicate that r and s are logically equivalent.
- Propositions s and r are logically equivalent if and only if the proposition $s \leftrightarrow r$ is a tautology

Example 1

Show logical equivalence of $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Example 1

Show logical equivalence of $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	**F**	**F**
T	F	F	T	T	**F**	**F**
F	T	T	F	T	**F**	**F**
F	F	T	T	F	**T**	**T**

- Also known as the De Morgans first law.
- When the negation operation is distributed inside the parentheses, the disjunction operation changes to a conjunction operation.

Example 2

Show logical equivalence of $\neg(p \wedge q) \equiv \neg p \vee \neg q$

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Show logical equivalence of $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	**F**	**F**
T	F	F	T	F	**T**	**T**
F	T	T	F	F	**T**	**T**
F	F	T	T	F	**T**	**T**

- Also known as the De Morgans second law.
- When the negation operation is distributed inside the parentheses, the conjunction operation changes to a disjunction operation.

Excercise

Show the logical equivalence using truth table

$$p \wedge (p \rightarrow q) \equiv p \wedge q$$

Excercise

Show the logical equivalence using truth table

$$p \wedge (p \rightarrow q) \equiv p \wedge q$$

p	q	$p \wedge (p \rightarrow q)$	$p \wedge q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Outline

Laws of Propositional Logic

- Used to get a simplified form of a complex compoud proposition.
- Used to show logical equivalence

Laws of Propositional Logic

- Used to get a simplified form of a complex compoud proposition.
- Used to show logical equivalence

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws:	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Excercise

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$

Excercise

Ex. Simplify the following compound proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan

Excercise

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive

Excercise

Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive
- $\neg p \wedge (q \vee \neg q)$ Commutative

Excercise

Ex. Simplify the following compoud proposition using propositional laws.

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- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive
- $\neg p \wedge (q \vee \neg q)$ Commutative
- $\neg p \wedge T$ Complement

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Ex. Simplify the following compoud proposition using propositional laws.

- $\neg(p \vee q) \vee (\neg p \wedge q)$
- $(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ Demorgan
- $\neg p \wedge (\neg q \vee q)$ Distributive
- $\neg p \wedge (q \vee \neg q)$ Commutative
- $\neg p \wedge T$ Complement
- $\neg p$ Identity

Excercise

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \to q) \land (q \lor p)$

Excercise

Ex. Simplify the following compound proposition using propositional laws.

- $(p \to q) \land (q \lor p)$
- $(\neg p \lor q) \land (q \lor p)$ Conditional Identity

Excercise

Ex. Simplify the following compound proposition using propositional laws.

- $(p \rightarrow q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law

Excercise

Ex. Simplify the following compound proposition using propositional laws.

- $(p \rightarrow q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law
- $q \vee (\neg p \wedge p)$ Distributive Law

Excercise

Ex. Simplify the following compound proposition using propositional laws.

- $(p \rightarrow q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law
- $q \vee (\neg p \wedge p)$ Distributive Law
- $q \vee (p \wedge \neg p)$ Commutative Law

Excercise

Ex. Simplify the following compoud proposition using propositional laws.

- $(p \rightarrow q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law
- $q \vee (\neg p \wedge p)$ Distributive Law
- $q \vee (p \wedge \neg p)$ Commutative Law
- $q \vee F$ Complement Law

Excercise

Ex. Simplify the following compound proposition using propositional laws.

- $(p \rightarrow q) \wedge (q \vee p)$
- $(\neg p \vee q) \wedge (q \vee p)$ Conditional Identity
- $(q \vee \neg p) \wedge (q \vee p)$ Commutative Law
- $q \vee (\neg p \wedge p)$ Distributive Law
- $q \vee (p \wedge \neg p)$ Commutative Law
- $q \vee F$ Complement Law
- q Identity Law

Excercise

Ex. Show the logical equivalence: $(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$

- $(p \rightarrow q) \land (p \rightarrow r)$

Excercise

Ex. Show the logical equivalence: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

- $(p \rightarrow q) \wedge (p \rightarrow r)$
- $(\neg p \vee q) \wedge (p \rightarrow r)$ Conditional Law

Excercise

Ex. Show the logical equivalence: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

- $(p \rightarrow q) \wedge (p \rightarrow r)$
- $(\neg p \vee q) \wedge (p \rightarrow r)$ Conditional Law
- $(\neg p \vee q) \wedge (\neg p \vee r)$ Conditional Law

Excercise

Ex. Show the logical equivalence: $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

- $(p \to q) \land (p \to r)$
- $(\neg p \lor q) \land (p \to r)$ Conditional Law
- $(\neg p \lor q) \land (\neg p \lor r)$ Conditional Law
- $\neg p \lor (q \land r)$ Distributive Law

Excercise

Ex. Show the logical equivalence: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

- $(p \rightarrow q) \wedge (p \rightarrow r)$
- $(\neg p \vee q) \wedge (p \rightarrow r)$ Conditional Law
- $(\neg p \vee q) \wedge (\neg p \vee r)$ Conditional Law
- $\neg p \vee (q \wedge r)$ Distributive Law
- $p \rightarrow (q \wedge r)$ Conditional Law

Excercise

Show the logical equivalence: $\neg p \rightarrow \neg q \equiv q \rightarrow p$

- $\neg$p $\rightarrow$ $\neg$q

Excercise

Show the logical equivalence: $\neg p \rightarrow \neg q \equiv q \rightarrow p$

- $\neg$p $\rightarrow$ $\neg$q
- $\neg\neg$p $\vee$ $\neg$q Conditional identity

Excercise

Show the logical equivalence: $\neg p \rightarrow \neg q \equiv q \rightarrow p$

- $\neg$p $\rightarrow$ $\neg$q
- $\neg\neg$p $\vee$ $\neg$q Conditional identity
- p $\vee$ $\neg$q Double negation law

Excercise

Show the logical equivalence: $\neg p \to \neg q \equiv q \to p$

- ¬p → ¬q
- ¬¬p ∨ ¬q Conditional identity
- p ∨ ¬q Double negation law
- ¬q ∨ p Commutative law

Excercise

Show the logical equivalence: $\neg p \to \neg q \equiv q \to p$

- $\neg p \to \neg q$
- $\neg\neg p \lor \neg q$ Conditional identity
- $p \lor \neg q$ Double negation law
- $\neg q \lor p$ Commutative law
- $q \to p$ Conditional identity

Excercise

Show the logical equivalence: $p \wedge (\neg p \rightarrow q) \equiv p$

- $p \wedge (\neg p \rightarrow q)$

Excercise

Show the logical equivalence: $p \wedge (\neg p \rightarrow q) \equiv p$

- $p \wedge (\neg p \rightarrow q)$
- $p \wedge (\neg\neg p \vee q)$ Conditional identity

Excercise

Show the logical equivalence: $p \wedge (\neg p \rightarrow q) \equiv p$

- $p \wedge (\neg p \rightarrow q)$
- $p \wedge (\neg\neg p \vee q)$ Conditional identity
- $p \wedge (p \vee q)$ Double negation law

Excercise

Show the logical equivalence: $p \wedge (\neg p \rightarrow q) \equiv p$

- $p \wedge (\neg p \rightarrow q)$
- $p \wedge (\neg\neg p \vee q)$ Conditional identity
- $p \wedge (p \vee q)$ Double negation law
- p Absorption law

Excercise

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$

Excercise

Ex. Show the logical equivalence: $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

- $(p \to r) \lor (q \to r)$
- $(\neg p \lor r) \lor (q \to r)$ Conditional identity

Excercise

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity

Excercise

Ex. Show the logical equivalence: $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

- $(p \rightarrow r) \vee (q \rightarrow r)$
- $(\neg p \vee r) \vee (q \rightarrow r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee r \vee \neg q \vee r$ Associative law

Excercise

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee r \vee \neg q \vee r$ Associative law
- $\neg p \vee \neg q \vee (r \vee r)$ Commutative law

Excercise

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee r \vee \neg q \vee r$ Associative law
- $\neg p \vee \neg q \vee (r \vee r)$ Commutative law
- $\neg p \vee \neg q \vee r$ Idempotent law

Excercise

Ex. Show the logical equivalence: $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

- $(p \rightarrow r) \vee (q \rightarrow r)$
- $(\neg p \vee r) \vee (q \rightarrow r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee r \vee \neg q \vee r$ Associative law
- $\neg p \vee \neg q \vee (r \vee r)$ Commutative law
- $\neg p \vee \neg q \vee r$ Idempotent law
- $(\neg p \vee \neg q) \vee r$ Associative law

Excercise

Ex. Show the logical equivalence: $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

- $(p \rightarrow r) \vee (q \rightarrow r)$
- $(\neg p \vee r) \vee (q \rightarrow r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee r \vee \neg q \vee r$ Associative law
- $\neg p \vee \neg q \vee (r \vee r)$ Commutative law
- $\neg p \vee \neg q \vee r$ Idempotent law
- $(\neg p \vee \neg q) \vee r$ Associative law
- $\neg (p \wedge q) \vee r$ De Morgans law

Excercise

Ex. Show the logical equivalence: $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

- $(p \to r) \vee (q \to r)$
- $(\neg p \vee r) \vee (q \to r)$ Conditional identity
- $(\neg p \vee r) \vee (\neg q \vee r)$ Conditional identity
- $\neg p \vee r \vee \neg q \vee r$ Associative law
- $\neg p \vee \neg q \vee (r \vee r)$ Commutative law
- $\neg p \vee \neg q \vee r$ Idempotent law
- $(\neg p \vee \neg q) \vee r$ Associative law
- $\neg(p \wedge q) \vee r$ De Morgans law
- $(p \wedge q) \to r$ Conditional identity

Outline

Predicate

Consider the following logical statment:

x is an odd number

- If $x = 5$, then the truth value is True.
- If $x = 4$, then the truth value is False.
- Hence, this statment is function of x. It can be denoted as P(x). We call it a predicate.

Predicate

Predicate

Predicate is a logical statement whose truth value is a function of one or more variables.

Ex. Q(x, y) : $x^2 = y$

Q(5, 25) is true because $5^2 = 25$

Q(7, 51) is false because $7^2 = 51$

Predicate

Predicate Domain

It is the set of all possible values for the variable in the logical statement.

Ex.

- P(x) : $x + 1 > 1$ Domain(x) :all integers.
  - P(5) is True
  - P(-5) is False
- P(city) : city has a population over 5,000,000 Domain(x): US cities
  - P(New York) is True
  - P(Greensboro) is False

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- P(city) : city has a population over 5,000,000 Domain(x): US cities
  - P(New York) is True
  - P(Greensboro) is False

Note That: Once all the variables within the predicate are assigned values from the domain, then the predicate is truned to a proposition.

Excercise

Given the following pedicates

$$\text{P(x): x is even.}$$
$$T(x,y)\text{: } 2^x = y$$

Indicate the truth value

- P(3)

Excercise

Given the following pedicates

$$\text{P(x): x is even.}$$
$$\text{T(x,y): } 2^x = \text{y}$$

Indicate the truth value

- P(3)
  - False

Excercise

Given the following pedicates

$$P(x)\text{: x is even.}$$
$$T(x,y)\text{: } 2^x = y$$

Indicate the truth value

- P(3)
  - False
- ¬P(3)

Excercise

Given the following pedicates

$$P(x): \text{ x is even.}$$
$$T(x,y): \ 2^x = y$$

Indicate the truth value

- P(3)
  - False
- $\neg$P(3)
  - True

Excercise

Given the following pedicates

$$P(x)\text{: x is even.}$$
$$T(x,y)\text{: } 2^x = y$$

Indicate the truth value

- P(3)
  - False
- ¬P(3)
  - True
- T(5,32)

Excercise

Given the following pedicates

$$P(x)\text{: x is even.}$$
$$T(x,y)\text{: } 2^x = y$$

Indicate the truth value

- $P(3)$
  - False
- $\neg P(3)$
  - True
- $T(5,32)$
  - True

Excercise

Given the following pedicates

$$P(x)\text{: } x \text{ is even.}$$
$$T(x,y)\text{: } 2^x = y$$

Indicate the truth value

- P(3)
    - False
- ¬P(3)
    - True
- T(5,32)
    - True
- T(5,x)

Excercise

Given the following pedicates

$$P(x)\text{: x is even.}$$
$$T(x,y)\text{: } 2^x = y$$

Indicate the truth value

- P(3)
  - False
- $\neg$P(3)
  - True
- T(5,32)
  - True
- T(5,x)
  - Not a Proposition it is a predicate

Excercise

Given the following pedicates

P(x): x is even.
T(x,y): 2^x =y

Indicate the truth value

- P(3)
  - False
- ¬P(3)
  - True
- T(5,32)
  - True
- T(5,x)
  - Not a Proposition it is a predicate
- P(3) ∨ T(5,32)

Excercise

Given the following pedicates

P(x): x is even.
T(x,y): 2^x =y

Indicate the truth value

- P(3)
  - False
- ¬P(3)
  - True
- T(5,32)
  - True
- T(5,x)
  - Not a Proposition it is a predicate
- P(3) ∨ T(5,32)
  - True

Quantifiers

Quantifiers

It is another way to turn a predicate into a proposition.

- Two types of quantifiers:
    - Universal Quantifiers
    - Existential Quantifiers

Universal Quantifier

Ex.

- $P(x)$: Student x in the class completed the assignment.
- $\forall\ x\ P(x)$: Every student in the class completed the assignment

Universal Quantifier

Ex.

- P(x): Student x in the class completed the assignment.
- $\forall$ x P(x): Every student in the class completed the assignment

Definition.

- The logical statement $\forall$x P(x) is read "for all x P(x) is true".
- It asserts that that P(x) is true for every possible value for x in its domain.
- $\forall$x P(x) is a proposition.

$$\forall x P(x) \equiv P(a_1) \wedge P(a_2) \wedge \cdots \wedge P(a_k) \qquad \text{Domain}\{a_1 \ldots a_k\}$$

Universal Quantifier

- To show a statment with universal quatifier is **false** only a counter example is needed.

  Ex. $\forall\, x\, (x + 1) > 0$ Domain is all integers.

Universal Quantifier

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  P(-5) is false.

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  Ex. $\forall$ x (x + 1) > 0 Domain is all integers.
  P(-5) is false.

- To show a statment with universal quatifier is **true** a proof is needed. (OR show that it **true** for every value in the domain)

  Ex. $\forall x(\frac{1}{x+1}) < 1$ Domain is positive integers

Universal Quantifier

- To show a statment with universal quatifier is **false** only a counter example is needed.

  Ex. $\forall$ x (x + 1) > 0 Domain is all integers.
  P(-5) is false.

- To show a statment with universal quatifier is **true** a proof is needed. (OR show that it **true** for every value in the domain)

  Ex. $\forall x(\frac{1}{x+1}) < 1$ Domain is positive integers

Proof.

1. $0 < x$
2. $1 < x + 1$
3. $\frac{1}{x+1} < 1$ by divided both sides by x+1

$\square$

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\forall x \ (x^2 \geq 0)$

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\forall x \ (x^2 \geq 0)$
  - True.

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\forall x \ (x^2 \geq 0)$
  - True.
- $\forall x \ (x^2 - x \neq 0)$

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\forall x\ (x^2 \geq 0)$
  - True.
- $\forall x\ (x^2 - x \neq 0)$
  - False.

Existential Quantifier

Ex.

- $P(x)$: Student x in the class completed the assignment. .
- $\exists$ x $P(x)$: : There is a student in the class completed the assignment

Existential Quantifier

Ex.

- P(x): Student x in the class completed the assignment. .
- ∃ x P(x): : There is a student in the class completed the assignment

Definition.

- The logical statement ∃x P(x) is read "There exists an x, such that P(x) is true".
- It asserts that that P(x) is true for just one value in the domain.
- ∃x P(x) is a proposition.

$$\exists x P(x) \equiv P(a_1) \vee P(a_2) \vee \cdots \vee P(a_k) \quad \text{Domain}\{a_1 \ldots a_k\}$$

Existential Quantifier

- To show a statment with existential quatifier is **true** only a counter example is needed.

Ex. $\exists$ x (x + 1) > 0 Domain is all integers.
P(5) is true.

Existential Quantifier

- To show a statment with existential quatifier is **true** only a counter example is needed.

  Ex. $\exists$ x (x + 1) > 0 Domain is all integers.
  P(5) is true.

- To show a statment with existential quatifier is **false** a proof is needed. (OR show that it **false** for every value in the domain)

  Ex. $\exists x + 1 < x$ Domain is positive integers

Existential Quantifier

- To show a statment with existential quatifier is **true** only a counter example is needed.

Ex. $\exists$ x (x + 1) > 0 Domain is all integers.
P(5) is true.

- To show a statment with existential quatifier is **false** a proof is needed. (OR show that it **false** for every value in the domain)

Ex. $\exists x + 1 < x$ Domain is positive integers

Proof.

1. x+1<x
2. 1<0 both sides minus x

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\exists x \ (x + 2 = 1)$

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\exists x \; (x + 2 = 1)$
  - True.

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\exists x\ (x + 2 = 1)$
  - True.
- $\exists x\ (x + x = 1)$

Excercise

Given the domain is the set of all integers. Which statements are true?

- $\exists x\ (x + 2 = 1)$
  - True.
- $\exists x\ (x + x = 1)$
  - False.

Excercise

The domain for this problem is a set {a,b,c,d}. The table below shows the value of three predicates for each of the elements in the domain. For example, Q(b) is false because the truth value in row b, column Q is F.

Which statements are true? Justify your answer. Ex.

- $\forall x\ P(x)$

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

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The domain for this problem is a set {a,b,c,d}. The table below shows the value of three predicates for each of the elements in the domain. For example, Q(b) is false because the truth value in row b, column Q is F.

Which statements are true? Justify your answer. Ex.

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  - True.

	P	Q	R
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b	T	F	F
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d	T	F	F

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- $\forall x\ P(x)$
  - True.
- $\exists x\ P(x)$

	P	Q	R
a	T	T	F
b	T	F	F
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Which statements are true? Justify your answer. Ex.

- $\forall$x P(x)
  - True.
- $\exists$x P(x)
  - True.

	P	Q	R
a	T	T	F
b	T	F	F
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Which statements are true? Justify your answer. Ex.

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  - True.
- $\exists x\ P(x)$
  - True.
- $\forall x\ Q(x)$

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

Excercise

The domain for this problem is a set {a,b,c,d}. The table below shows the value of three predicates for each of the elements in the domain. For example, Q(b) is false because the truth value in row b, column Q is F.

Which statements are true? Justify your answer. Ex.

- $\forall x\ P(x)$
  - True.
- $\exists x\ P(x)$
  - True.
- $\forall x\ Q(x)$
  - False.

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

Excercise

The domain for this problem is a set {a,b,c,d}. The table below shows the value of three predicates for each of the elements in the domain. For example, Q(b) is false because the truth value in row b, column Q is F.

Which statements are true? Justify your answer. Ex.

- $\forall x \, P(x)$
  - True.
- $\exists x \, P(x)$
  - True.
- $\forall x \, Q(x)$
  - False.
- $\exists x \, Q(x)$

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

Excercise

The domain for this problem is a set {a,b,c,d}. The table below shows the value of three predicates for each of the elements in the domain. For example, Q(b) is false because the truth value in row b, column Q is F.

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  - True.
- $\exists x\, P(x)$
  - True.
- $\forall x\, Q(x)$
  - False.
- $\exists x\, Q(x)$
  - True.

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

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The domain for this problem is a set {a,b,c,d}. The table below shows the value of three predicates for each of the elements in the domain. For example, Q(b) is false because the truth value in row b, column Q is F.

Which statements are true? Justify your answer. Ex.

- $\forall$x P(x)
  - True.
- $\exists$x P(x)
  - True.
- $\forall$x Q(x)
  - False.
- $\exists$x Q(x)
  - True.
- $\exists$x R(x)

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

Excercise

The domain for this problem is a set {a,b,c,d}. The table below shows the value of three predicates for each of the elements in the domain. For example, Q(b) is false because the truth value in row b, column Q is F.

Which statements are true? Justify your answer. Ex.

- $\forall x\ P(x)$
  - True.
- $\exists x\ P(x)$
  - True.
- $\forall x\ Q(x)$
  - False.
- $\exists x\ Q(x)$
  - True.
- $\exists x\ R(x)$
  - False.

	P	Q	R
a	T	T	F
b	T	F	F
c	T	F	F
d	T	F	F

Outline

Quantified statements

- Logical operators $\neg, \wedge, \vee$ can be used to bind universally and existentially quantified statements.

Ex.

P(x): x is prime.
O(x): x is odd.

- $\exists x(P(x) \wedge \neg O(x))$

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  - True $x = 2$

Quantified statements

- Logical operators $\neg, \wedge, \vee$ can be used to bind universally and existentially quantified statements.

Ex.

P(x): x is prime.
O(x): x is odd.

- $\exists x(P(x) \wedge \neg O(x))$
  - True $x = 2$
- $\forall x(P(x) \to O(x))$

Quantified statements

- Logical operators $\neg, \wedge, \vee$ can be used to bind universally and existentially quantified statements.

Ex.

P(x): x is prime.
O(x): x is odd.

- $\exists x(P(x) \wedge \neg O(x))$
  - True $x = 2$
- $\forall x(P(x) \rightarrow O(x))$
  - False $x=2$

Free and Bounded Variables

- A variable x in the predicate $P(x)$ is called a free variable.
- A variable x in the statement $\forall x\, P(x)$ is called a bounded variable.
- If all the variables in a statement are bounded, then a predicate is truned to a proposition.

Ex.

$$\forall x (P(x) \wedge Q(x)) \text{ Proposition}$$
$$\forall x (P(x)) \wedge Q(x) \text{ Not a Proposition}$$

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x (N(x) \wedge \neg D(x)) \equiv True$ (Sam)

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \land \neg D(x)) \equiv True$ (Sam)
- Everyone missed the deadline or is a new employee.

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \wedge \neg D(x)) \equiv \mathit{True}$ (Sam)
- Everyone missed the deadline or is a new employee.
  - $\forall$ x(D(x) $\vee$ N(x)) $\equiv$ True (Prove for every one)

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \wedge \neg D(x)) \equiv True$ (Sam)
- Everyone missed the deadline or is a new employee.
  - $\forall$ x(D(x) $\vee$ N(x)) $\equiv$ True (Prove for every one)
- $\forall x((x \neq Sam \rightarrow N(x)))$

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- There is a new employee who met the deadline.
  - $\exists x(N(x) \wedge \neg D(x)) \equiv \mathit{True}$ (Sam)
- Everyone missed the deadline or is a new employee.
  - ∀ x(D(x) ∨ N(x)) ≡ True (Prove for every one)
- $\forall x((x \neq Sam \rightarrow N(x)))$
  - Everyone except Sam is a new employee. False (Melanie, Bert)

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone miss the deadline and is a new employee.

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone miss the deadline and is a new employee.
  - $\exists x(N(x) \wedge D(x)) \equiv True$ (Beth)

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone miss the deadline and is a new employee.
  - $\exists x(N(x) \wedge D(x)) \equiv True$ (Beth)
- $\forall x(\neg D(x) \rightarrow \neg N(x))$

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone miss the deadline and is a new employee.
  - $\exists x(N(x) \wedge D(x)) \equiv True$ (Beth)
- $\forall x(\neg D(x) \rightarrow \neg N(x))$
  - Everyone who did not miss the deadline is not a new employee. False (Sam)

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone miss the deadline and is a new employee.
  - $\exists x(N(x) \wedge D(x)) \equiv \mathit{True}$ (Beth)
- $\forall x(\neg D(x) \rightarrow \neg N(x))$
  - Everyone who did not miss the deadline is not a new employee. False (Sam)
- $N(Bert) \rightarrow D(Bert)$

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- Someone miss the deadline and is a new employee.
  - $\exists x(N(x) \wedge D(x)) \equiv True$ (Beth)
- $\forall x(\neg D(x) \rightarrow \neg N(x))$
  - Everyone who did not miss the deadline is not a new employee. False (Sam)
- $N(Bert) \rightarrow D(Bert)$
  - If Bert is a new employee then he missed the deadline. True.

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- $\forall x(D(x) \leftrightarrow N(x))$

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- $\forall x(D(x) \leftrightarrow N(x))$
  - Everyone who missed the deadline is a new employee and vice versa. False (Melanie and Bert)

Excercise

For a group of employee below, Convert English to proposition and vice versa, then find the truth value.

D(x): x missed the deadline.
N(x): x is a new employee.

Name	N(x)	D(x)
Sam	T	F
Beth	T	T
Melanie	F	T
Al	T	T
Bert	F	T

Ex.

- $\forall x(D(x) \leftrightarrow N(x))$
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- If there is a new employee except Sam, then he missed the deadline.

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  - Everyone who missed the deadline is a new employee and vice versa. False (Melanie and Bert)
- If there is a new employee except Sam, then he missed the deadline.
  - $\exists x((x \neq Sam) \wedge N(x)) \rightarrow D(x)$. True. (Beth and Al)

Questions