

Converter Design

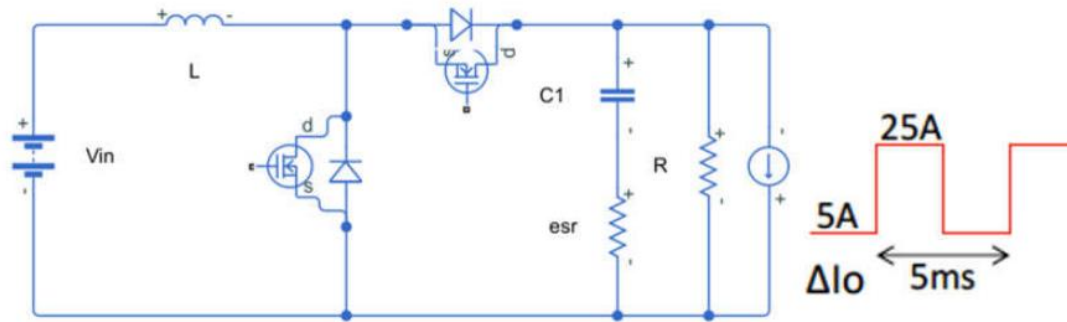
Assignment 3

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Q.1

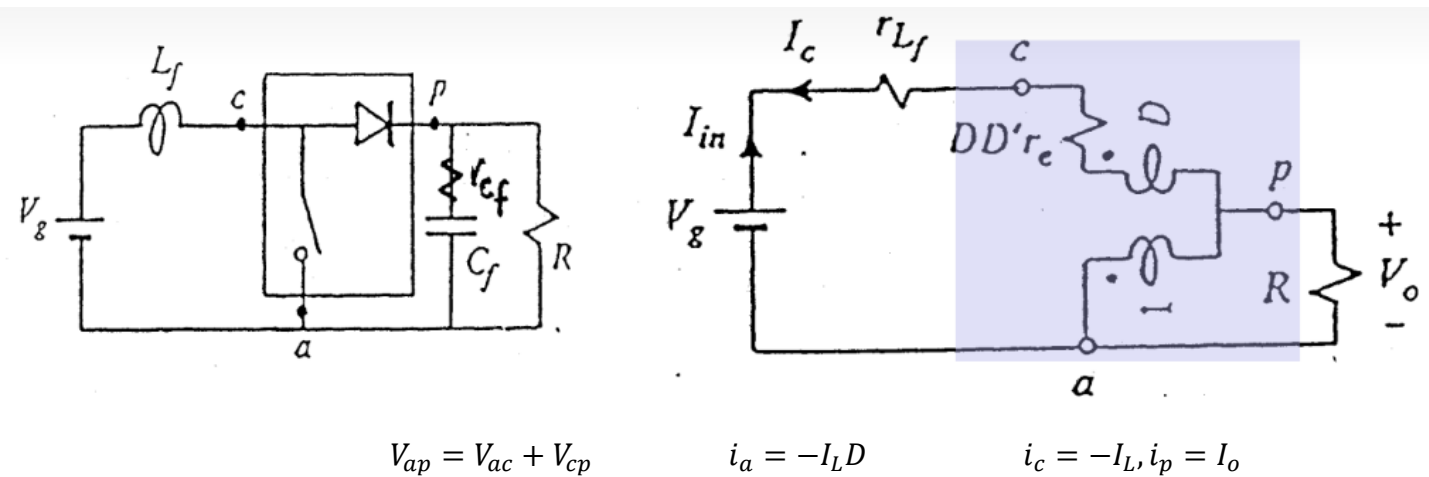
Bidirectional Boost converter is often used in renewable energy applications. It uses a synchronous rectifier (MOSFET) instead of the diode in the normal Boost topology. The two MOSFETs's gate signals are complementary. There is no DCM operation for this boost converter. The circuit diagram is illustrated below



The parameters of the above boost converter are:

$V_{in}=10V$, $V_o=16V$, $L=100nH$, $R_L=0.5m\Omega$, $R_c=4m\Omega$, $C_o=660\mu F$, $R_o=120m\Omega$, $f_s=1MHz$; As shown in the graph, load transient current step $\Delta I_o=20A$ (from 25A to 5A) with period $T_{load}=5ms$. Load transient is assumed to be an ideal square waveform (50% duty cycle) with 0 rising and falling time. In this exercise, we assume S_1 and S_2 are ideal switches and neglect the dead-time between S_1 and S_2 .

DC analysis



$$i_c = i_a - i_p = -I_L D - I_o = -I_L \quad I_L = \frac{I_o}{D'} = \frac{V_o}{D'R}$$

$$V_{ac} = -V_g + I_L r_L, V_{ap} = -V_o \quad V_{cp} = -V_o D + I_L D D' r_e$$

$$Z_p = \frac{R(R_c + (\frac{1}{SC}))}{R + R_c + (\frac{1}{SC})}$$

$$Z_p = \frac{R(SCR_c + 1)}{SCR_o + SCR_c + 1}$$

$$Z_o = -\frac{R_o(SCR_c + 1)}{SCR_o + SCR_c + 1} || \frac{(DD'R_e + R_L + sL)}{D'^2}$$

$$Z_o = \frac{-(DD'R_e + R_L + sL)R(R_cCs + 1)}{D'^2R(R_cCs + 1) + (DD'R_e + R_L + sL)RCs + (DD'R_e + R_L + sL)(R_cCs + 1)}$$

$$Z_o = \frac{-(DD'R_e + R_L + sL)R(R_cCs + 1)}{D'^2R(R_cCs + 1) + (DD'R_e + R_L + sL)RCs + (DD'R_e + R_L + sL)(R_cCs + 1)}$$

$$Z_o = \frac{-(DD'R_e + R_L + sL)R(R_cCs + 1)}{D'^2RR_cCs + D'^2R + (DD'R_e + R_L + sL)(C(R + R_c)s + 1)}$$

$$Z_o$$

$$= \frac{-(DD'R_e + R_L + sL)R(R_cCs + 1)}{(LC(R + R_c))s^2 + ((DD'R_e + R_L)(CR + CR_c) + L + D'^2RR_cC)s + (D'^2R + R_L + DD'R_e)}$$

$$Z_o$$

$$= \frac{-R \frac{(DD'R_e + R_L)}{D'^2} \left(1 + \frac{sL}{DD'R_e + R_L}\right) (R_cCs + 1)}{R + \frac{(R_L + DD'R_e)}{D'^2} \frac{LC(R + R_c)}{D'^2R + R_L + DD'R_e} s^2 + \frac{D'^2RCR_c + L + (DD'R_e + R_L)(CR + CR_c)}{D'^2R + R_L + DD'R_e} s + 1}$$

$$Z_o = -R_o \frac{\left(1 + \frac{s}{\omega_{zo}}\right) \left(1 + \frac{s}{\omega_{esr}}\right)}{1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}}$$

$$\omega_o = \frac{D'}{\sqrt{LC}} \sqrt{\frac{1 + \frac{r_L}{D'^2R} + \frac{Dr_e}{D'R}}{1 + \frac{r_c}{R}}}$$

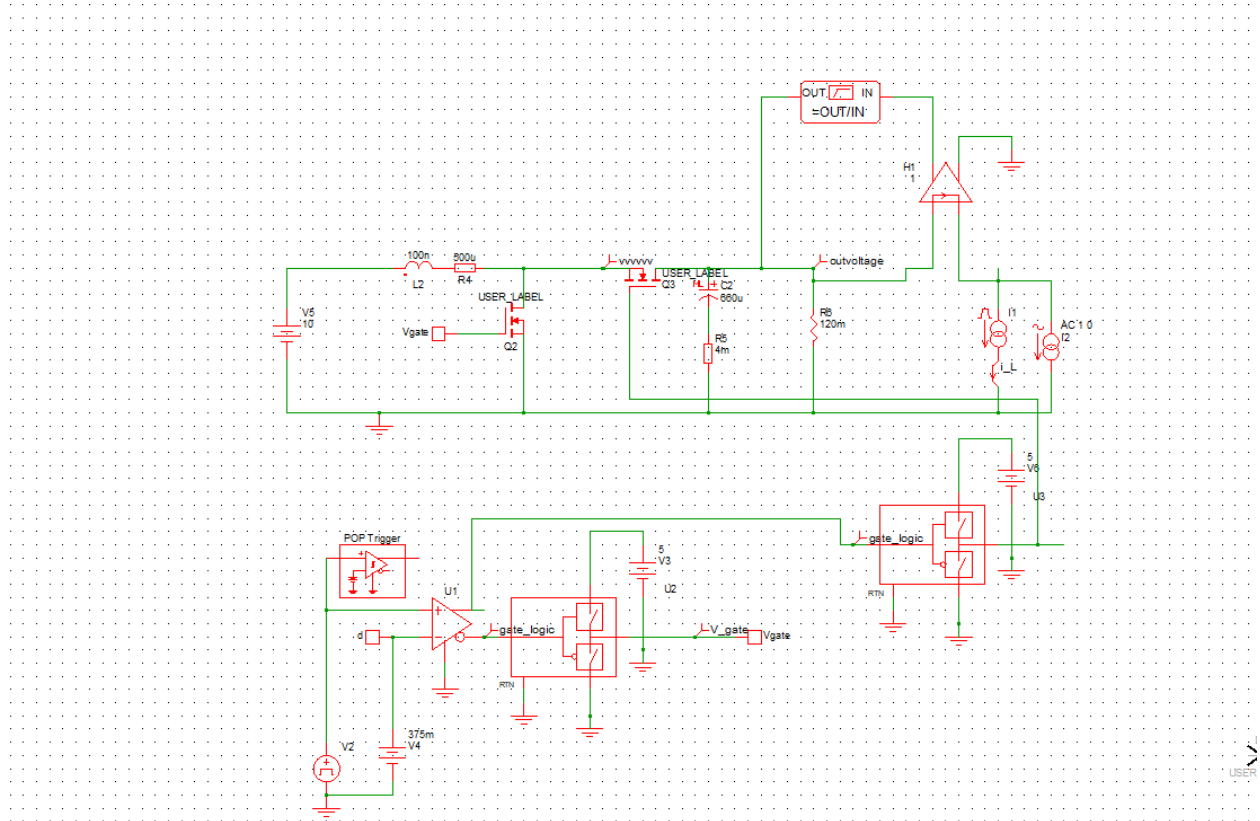
$$Q = \frac{\omega_o}{\frac{D'^2Rr_c}{L(R + r_c)} + \frac{1}{C(R + r_c)} + \frac{(DD'r_e + r_L)}{L}}$$

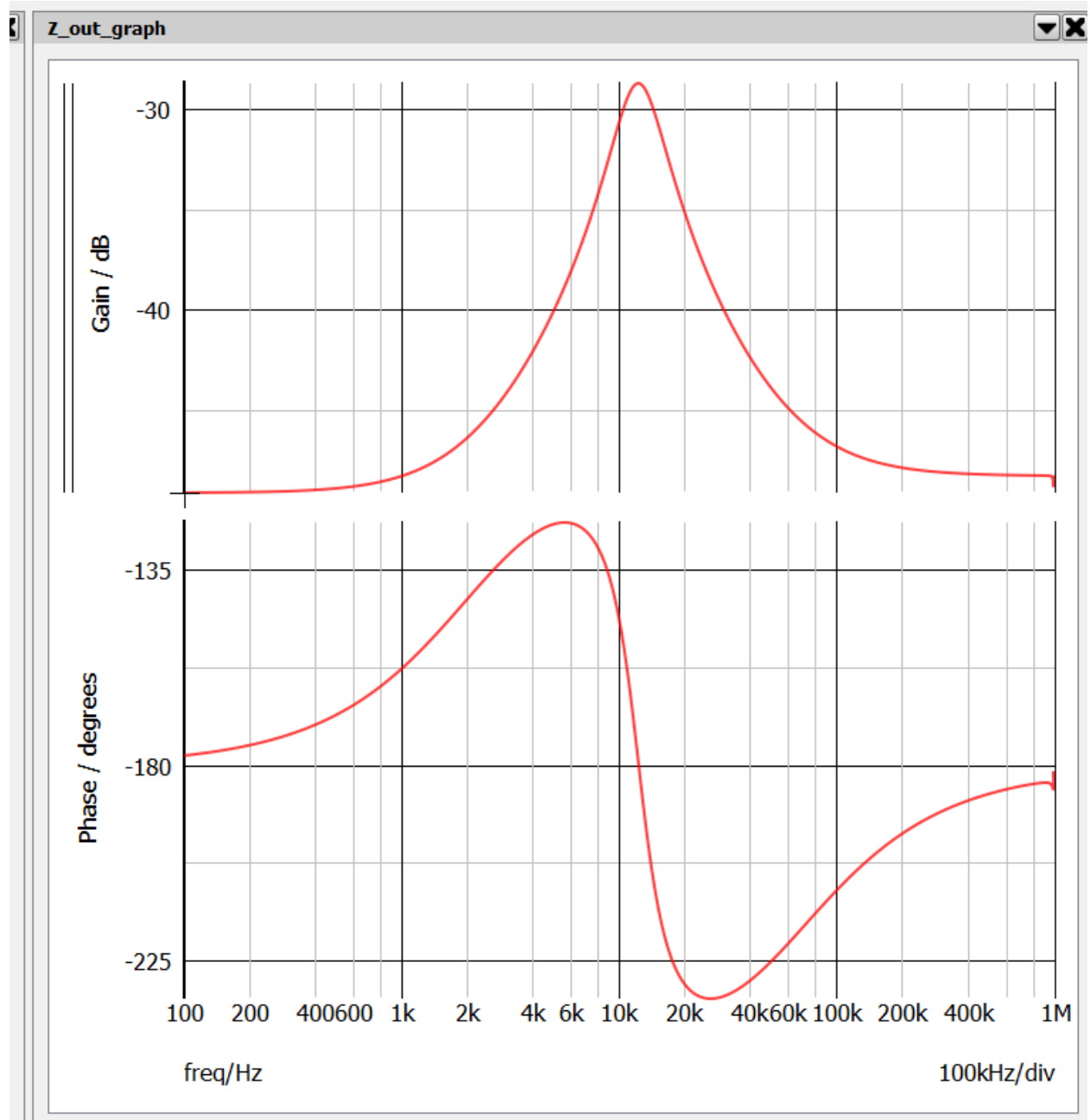
$$R_o = R || \frac{(DD'r_e + r_L)}{D'^2}$$

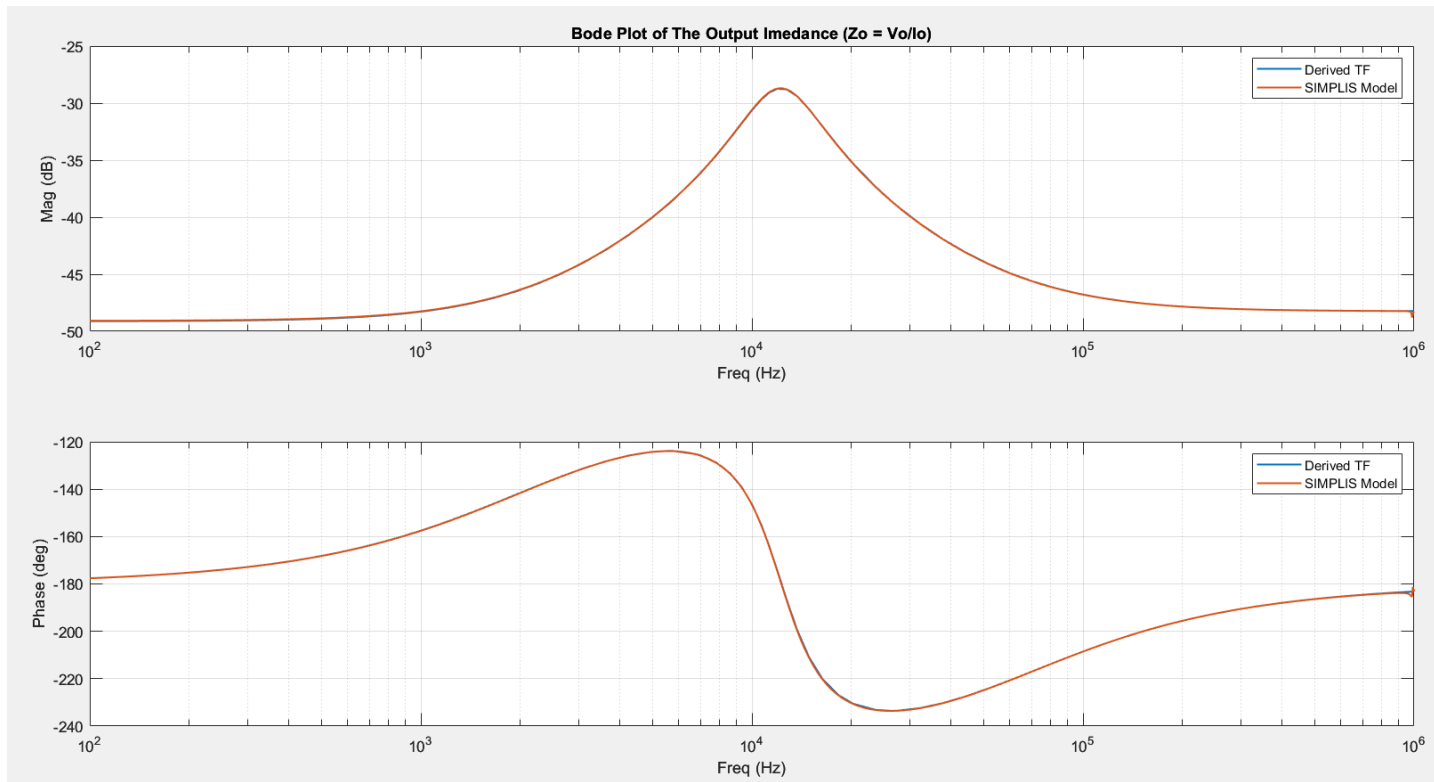
$$\omega_{zo} = \frac{(DD'r_e + r_L)}{L}$$

$$\omega_{esr} = \frac{1}{R_c C}$$

Simplis model







```

/MATLAB Drive/converters assignment 3/bode_boost.m
1 clear
2 clc
3 s=tf('s');
4
5 BB_data = xlsread('Z_out_boost.xlsx');
6
7 freq_1 = BB_data(:,1);
8 Gain_1 = BB_data(:,2);
9 phase_1 = BB_data(:,3);
10 Vin=10; %input voltage
11 Vo=16; %Output Voltage
12 L=100e-9; %Inductance
13 C=660e-6; %Capacitance
14 Rc=4e-3; %ESR
15 R=120e-3; %Load resistance
16 %Re=(R*(Rc+(1/(s*C))))/(R+(Rc+(1/(s*C))));
17 Re=(R*Rc)/(R+Rc);
18 RL=0.5e-3;
19 Fs=10^6; %Switching frequency
20 D=0.375; %Duty
21 D_ =1-D;
22 w={100*2*pi,10^6*2*pi}; %Frequency limit of Bode plot
23 Gzo=-(R*((RL+Re*D_)/(D_)^2))/(R+((RL+Re*D_)/(D_)^2));
24 Num_zo=-((s^2)*(C*L*Rc/(RL+Re*D_))+s*(C*Rc+(L/(RL+Re*D_))))+1;
25 Den_zo=(s^2)*(C*L*(R+Rc)/((D_)^2)*R+RL+Re*D_)+s*(L+(C*R+Rc)*(RL+Re*D_)+(D_)^2)*R+RL+Re*D_+1;
26 Zo=Gzo*Num_zo/Den_zo;
27 w0=((Re*D_)+RL)/L;
28
29 [mag,phase,wout]= bode(Zo,w);

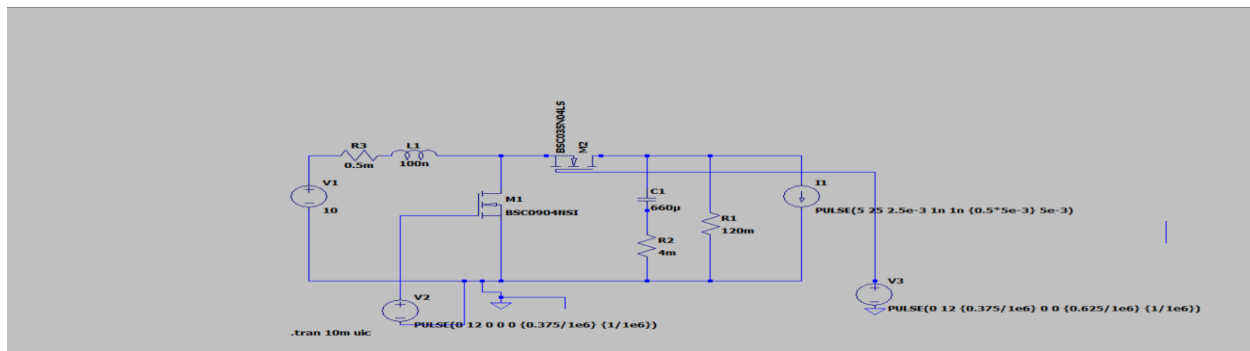
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TLAB Drive/converters assignment 3/bode_boost.m

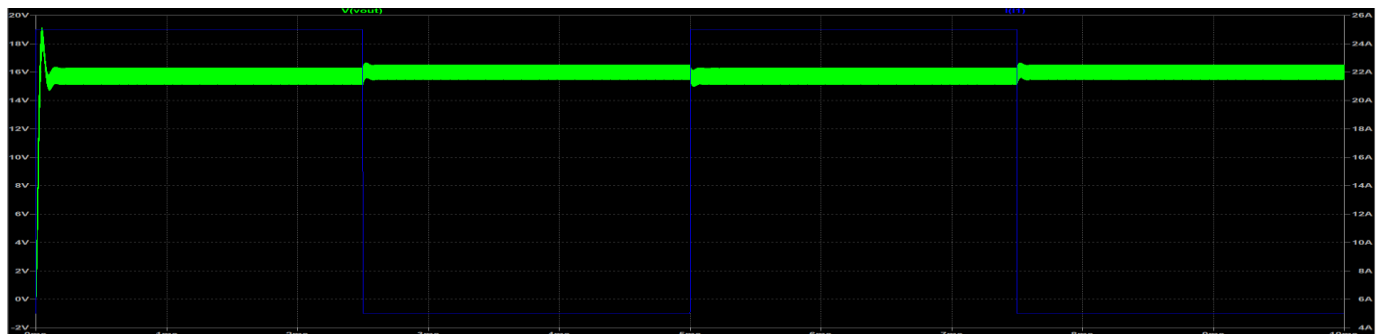
```
f_zo=wout/(2*pi);
figure(1);
subplot(2,1,1)
semilogx(f_zo,20*log10(mag_zo),freq_1,Gain_1,'LineWidth',1.2);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Bode Plot of The Control to Output ( Gzo = Vo / Io )');
legend('Transfer Function','SIMPLIS Model');

subplot(2,1,2)
semilogx(f_zo,ph_zo-180,freq_1,phase_1,'LineWidth',1.2);
xlabel('Frequency (Hz)');
ylabel('Phase (deg)');
legend('Transfer Function','SIMPLIS Model');
grid on;
```

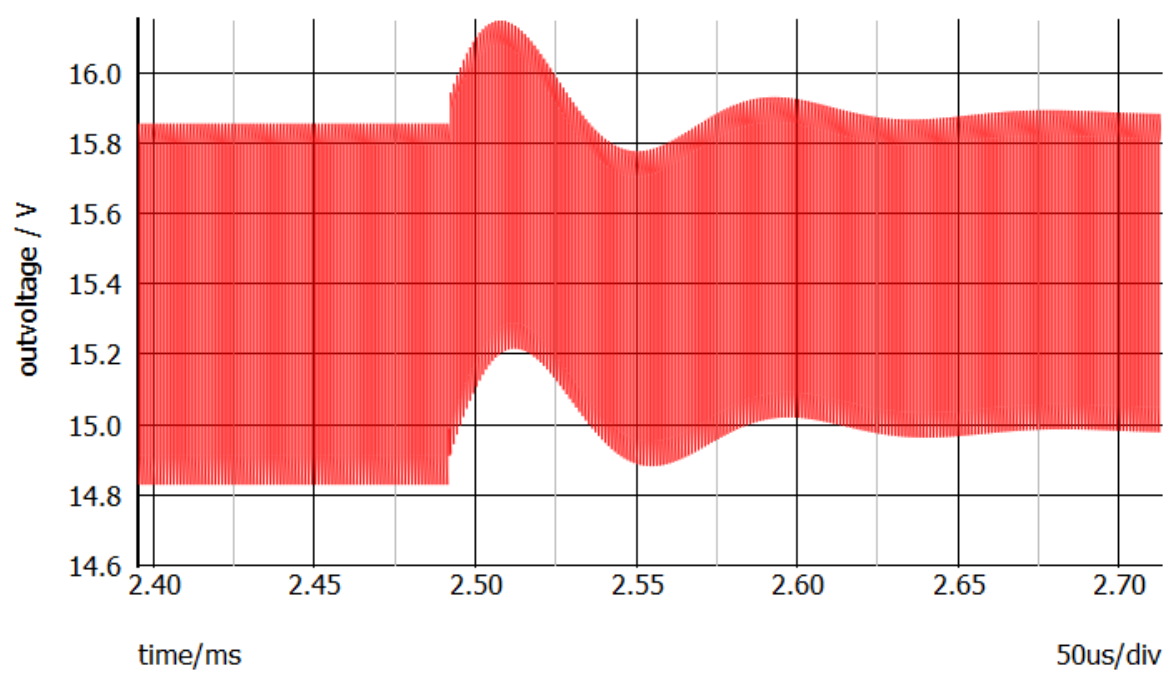
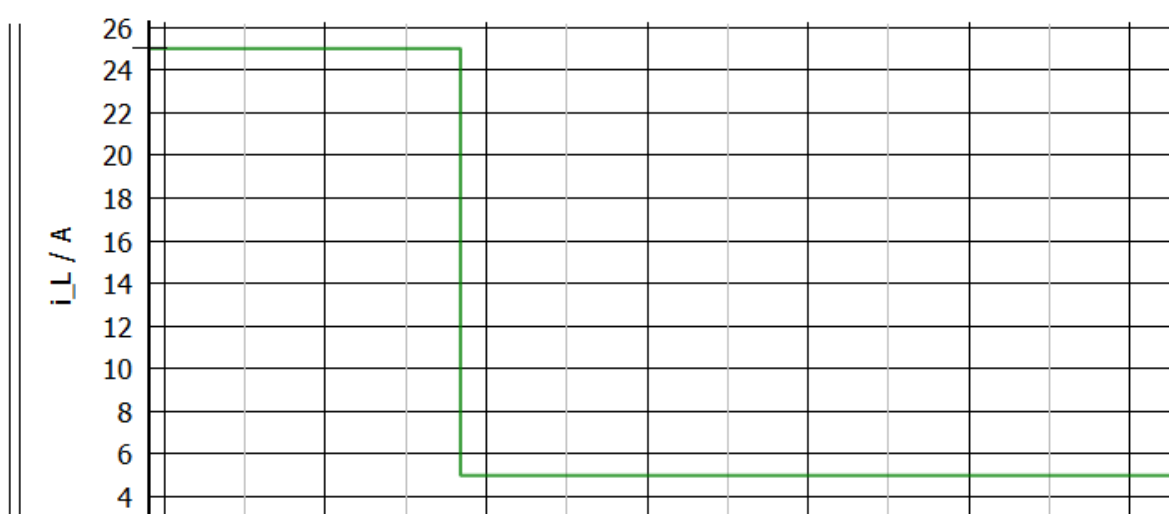
Ltspice model

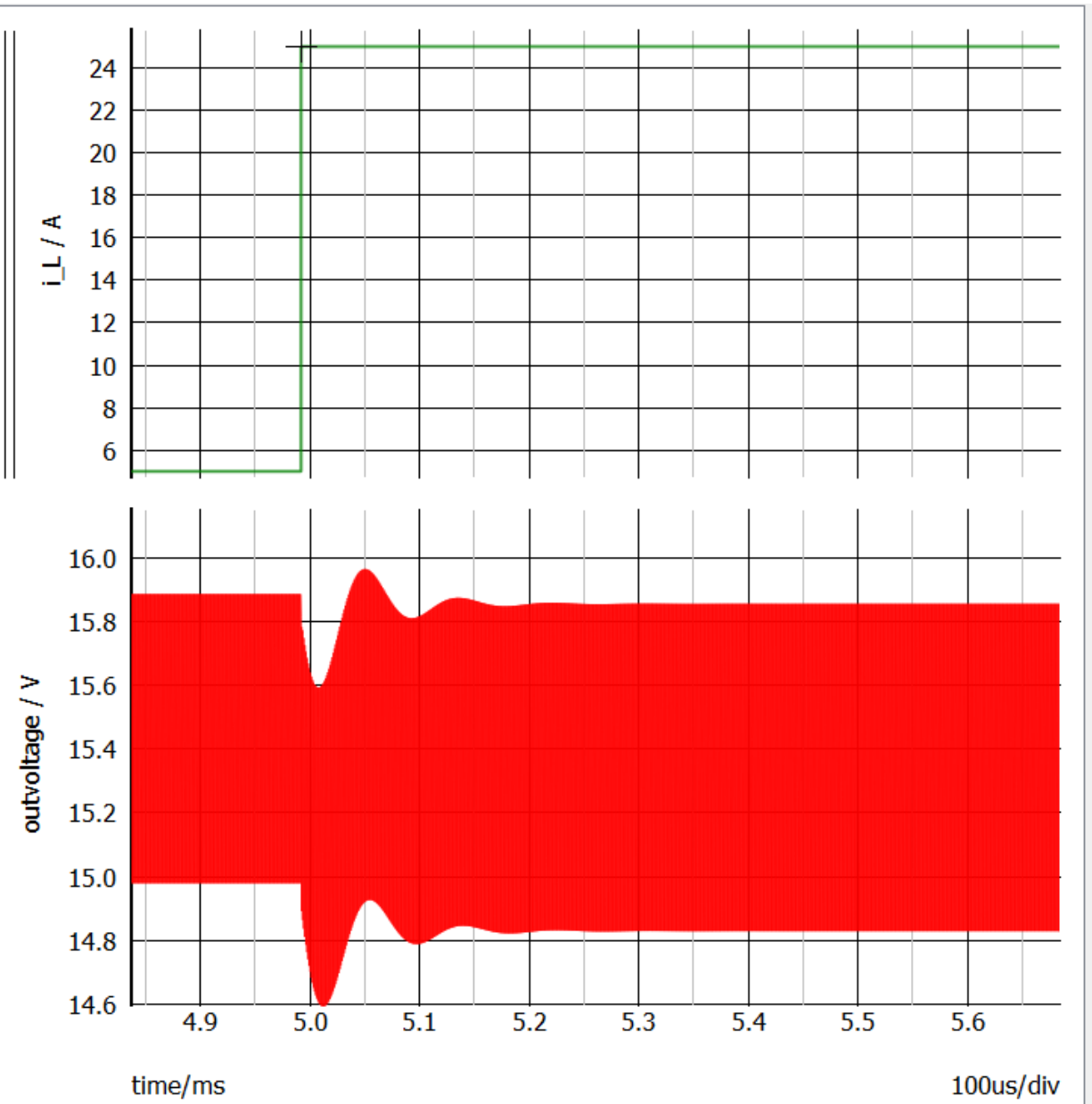


Transient response with Ltspice



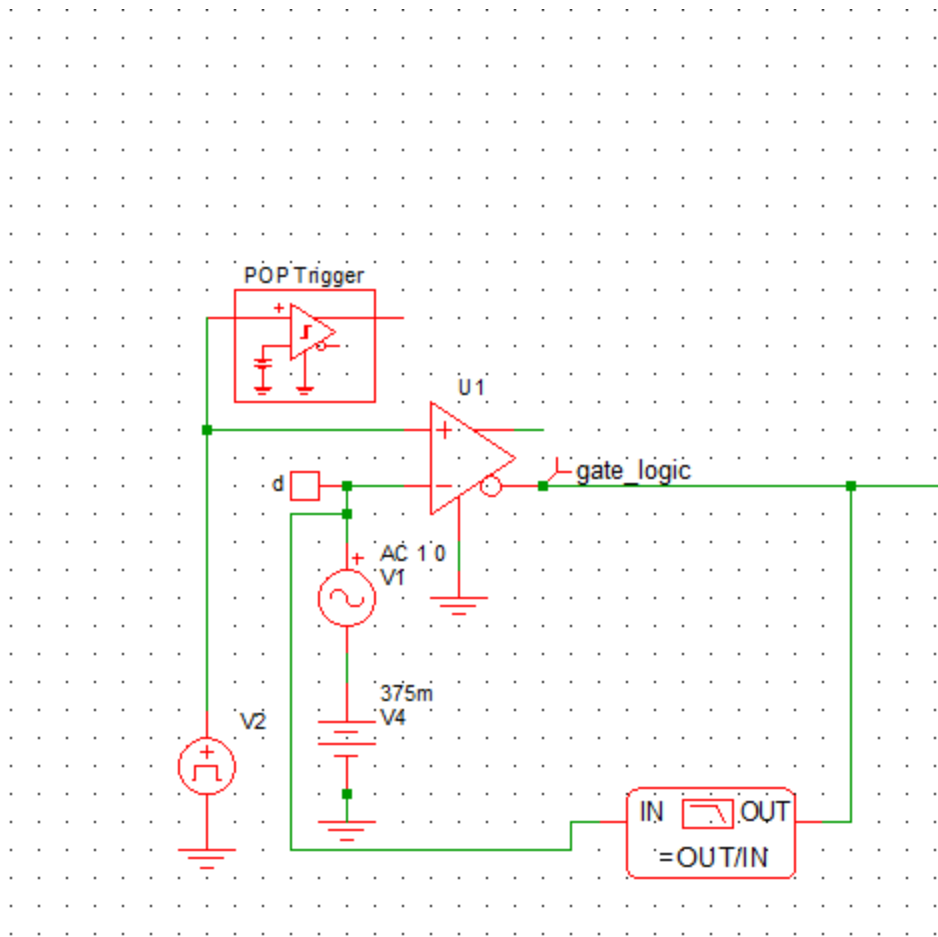
Transient response with Simplis





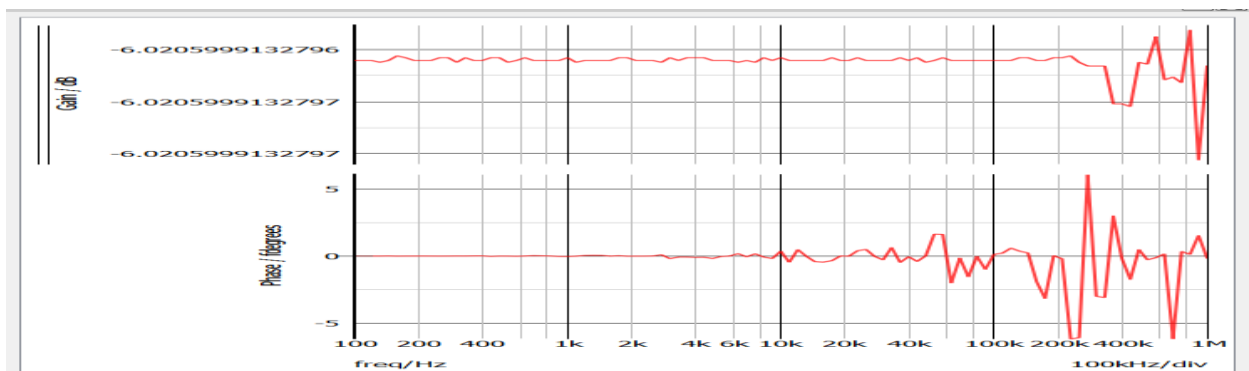
Q2

Finding $G_{PWM} = \frac{\hat{v}_c}{\hat{a}}$



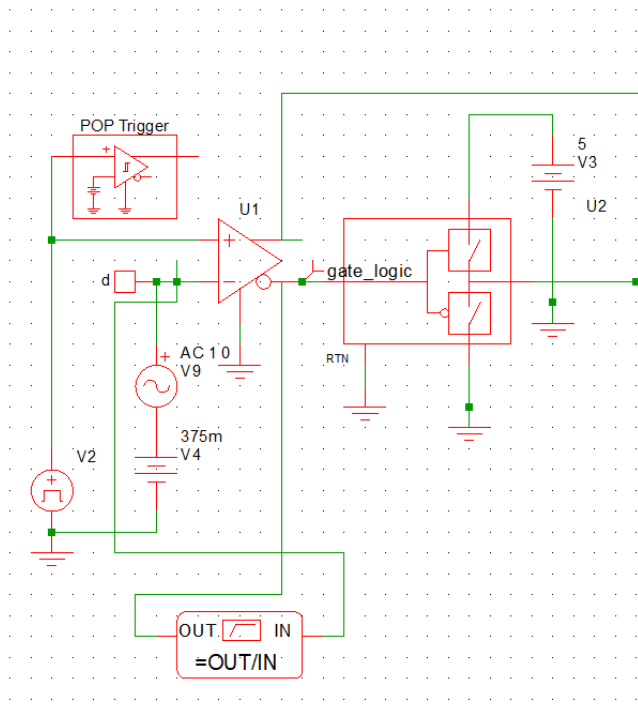
@ $V_p = 10$

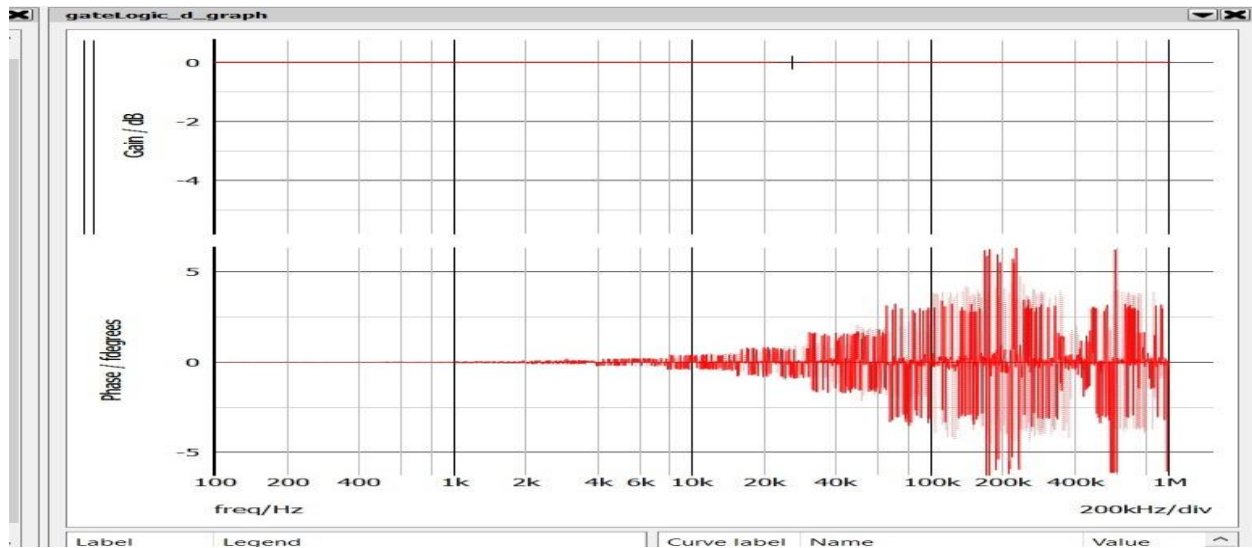
$$V_p = 10 \rightarrow |G_{PWMo}| = -6.3dB$$



@ $V_p = 1$

$$|G_{PWMo}| = 0 \text{ db}$$





$$gain = \frac{dD}{dV_{ctrl}}$$

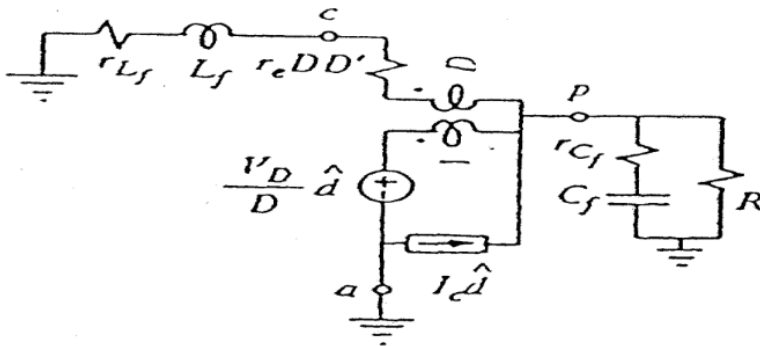
$$D = \frac{V_{ctrl}}{V_p}$$

$$\frac{dD}{dV_{ctrl}} = \frac{1}{V_p}$$

$$G_{pwm} = \frac{1}{V_p}$$

Q3 - a)

Obtaining control to output TF for controller design $G_{vd}(s) = \frac{\hat{v}_o(s)}{\hat{a}(s)} @ \hat{V}_g = 0$



$$V_{ap} = V_{ac} + V_{cp}$$

$$\hat{i}_a = \hat{i}_c D, \hat{i}_p = \frac{\hat{v}_o}{R} + \frac{\hat{v}_o}{r_c + \frac{1}{sC}}$$

$$\hat{i}_c = \hat{i}_a + I_c \hat{d} - \hat{i}_p = \hat{i}_c D + I_c \hat{d} - \frac{\hat{v}_o}{R || \left(r_c + \frac{1}{sC} \right)}$$

$$\hat{i}_c = \frac{1}{D'} \left(I_c \hat{d} - \frac{\hat{v}_o}{R || \left(r_c + \frac{1}{sC} \right)} \right)$$

$$V_{ac} = -\hat{i}_c (r_L + sL), V_{ap} = -\hat{v}_o, V_{cp} = -\hat{v}_o D - \hat{i}_c D D' r_e + V_D \hat{d}$$

$$\hat{v}_o = \hat{i}_c (r_L + sL) + \hat{v}_o D + \hat{i}_c D D' r_e - V_D \hat{d}$$

$$\hat{v}_o D' = \hat{i}_c (r_L + D D' r_e + sL) - V_D \hat{d}$$

$$\hat{v}_o = \frac{1}{D'^2} \left(I_c \hat{d} - \frac{\hat{v}_o}{R || \left(r_c + \frac{1}{sC} \right)} \right) (r_L + D D' r_e + sL) - \frac{V_D}{D'} \hat{d}$$

$$\hat{v}_o \left(1 + \frac{(r_L + D D' r_e + sL)}{D'^2 \left(R || \left(r_c + \frac{1}{sC} \right) \right)} \right) = \frac{1}{D'^2} I_c \hat{d} (r_L + D D' r_e + sL) - \frac{V_D}{D'} \hat{d}$$

$$V_D = -\hat{v}_o + I_c (D - D') r_e$$

$$\hat{v}_o \left(1 + \frac{(r_L + D D' r_e + sL)}{D'^2 \left(R || \left(r_c + \frac{1}{sC} \right) \right)} \right) = \frac{1}{D'^2} I_c \hat{d} (r_L + D D' r_e + sL) - \frac{(-V_o + I_c (D - D') r_e)}{D'} \hat{d}$$

$$I_C = -I_L = -\frac{I_o}{D'} = -\frac{V_o}{R D'} = -\frac{V_g}{R D'^2} \frac{1}{1 + \frac{r_L}{D'^2 R} + \frac{D r_e}{D' R}}, \text{ by using } M \text{ conversion ratio}$$

$$\hat{v}_o \left(1 + \frac{(r_L + D D' r_e + sL)}{D'^2 \left(R || \left(r_c + \frac{1}{sC} \right) \right)} \right) = \frac{1}{D'^2} \frac{V_o}{R D'} \left(-(r_L + D D' r_e + sL) + D D' r_e - D'^2 r_e \right) \hat{d} + \frac{V_o}{D'} \hat{d}$$

$$\hat{v}_o \left(\frac{D'^2 \left(R \parallel \left(r_c + \frac{1}{sC} \right) \right) + (r_L + DD'r_e + sL)}{D'^2 \left(R \parallel \left(r_c + \frac{1}{sC} \right) \right)} \right) = \frac{V_o}{D'} \left(\frac{1}{RD'^2} (-r_L - sL - D'^2 r_e + RD'^2) \right) \hat{d}$$

$$\frac{\hat{v}_o}{\hat{d}} = \frac{V_o D'^2}{D'} \frac{\left(\frac{1}{RD'^2} (-r_L - sL - D'^2 r_e + RD'^2) \right) \left(R \parallel \left(r_c + \frac{1}{sC} \right) \right)}{D'^2 \left(R \parallel \left(r_c + \frac{1}{sC} \right) \right) + (r_L + DD'r_e + sL)}$$

$$\frac{\hat{v}_o}{\hat{d}} = \frac{V_o}{D'} \frac{(-r_L - sL - D'^2 r_e + RD'^2)(r_c Cs + 1)}{D'^2 R r_c Cs + D'^2 R + (r_L + DD'r_e + sL)(C(R + r_c)s + 1)}$$

$$\frac{\hat{v}_o}{\hat{d}} = \frac{V_o}{D'} \frac{((RD'^2 - r_L - D'^2 r_e) - sL)(r_c Cs + 1)}{(LC(R + r_c))s^2 + ((DD'r_e + r_L)(CR + Cr_c) + L + D'^2 R r_c C)s + (D'^2 R + r_L + DD'r_e)}$$

$$\frac{\hat{v}_o}{\hat{d}}$$

$$= \frac{V_o}{D'(D'^2 R + r_L + DD'r_e)} \frac{((RD'^2 - r_L - D'^2 r_e) - sL)(r_c Cs + 1)}{\frac{LC(R + r_c)}{D'^2 R + r_L + DD'r_e} s^2 + \frac{D'^2 R Cr_c + L + (DD'r_e + r_L)(CR + Cr_c)}{D'^2 R + r_L + DD'r_e} s + 1}$$

$$\frac{\hat{v}_o}{\hat{d}}$$

$$= \frac{V_g RD'^2 (RD'^2 - r_L - D'^2 r_e)}{D'^2 (D'^2 R + r_L + DD'r_e)^2} \frac{\left(1 - \frac{sL}{RD'^2 - r_L - D'^2 r_e} \right) (r_c Cs + 1)}{\frac{LC(R + r_c)}{D'^2 R + r_L + DD'r_e} s^2 + \frac{D'^2 R Cr_c + L + (DD'r_e + r_L)(CR + Cr_c)}{D'^2 R + r_L + DD'r_e} s + 1}$$

$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = K \frac{\left(1 - \frac{s}{\omega_{rhp}} \right) \left(1 + \frac{s}{\omega_{esr}} \right)}{1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}}$$

$$K = \frac{V_g RD'^2 (RD'^2 - r_L - D'^2 r_e)}{D'^2 (D'^2 R + r_L + DD'r_e)^2}$$

$$\omega_{rhp} = \frac{(RD'^2 - r_L - D'^2 r_e)}{L}$$

From control to output TF we can get

$$f_o = 12.22kHz \quad f_{esr} = 60kHz \quad f_{rhp} = 71kHz \quad Q = 1.8547$$

Designing a type-3 compensator for $f_c = 50kHz \pm 5kHz$, $\varphi_m \geq 60^\circ$, $GM \geq 10dB$

- 1- Choosing $f_{P1} = f_{esr} = 60kHz$
- 2- Choosing $f_{P2} = 10f_c = 500kHz$
- 3- For the phase margin

$$\begin{aligned} \varphi_m = 180 - \tan^{-1}\left(\frac{f_c}{f_{P1}}\right) - \tan^{-1}\left(\frac{f_c}{f_{P2}}\right) - 90 + \tan^{-1}\left(\frac{f_c}{f_{esr}}\right) \\ + \tan^{-1}\left(\frac{f_c}{f_{Z1}}\right) + \tan^{-1}\left(\frac{f_c}{f_{Z2}}\right) + \tan^{-1}\left(\frac{f_c}{f_{rhp}}\right) \\ - \tan^{-1}\left(\frac{\frac{f_c}{Qf_o}}{1 - \left(\frac{f_c}{f_o}\right)^2}\right) \geq 60^\circ \end{aligned}$$

$$\text{Let } f_{Z1} = 5kHz \rightarrow f_{Z2} = 93kHz$$

- 4- For gain margin

$$|T| = 1 @ f_c \rightarrow 50kHz$$

$$\begin{aligned} |T| = \frac{\sqrt{1 + \left(\frac{f_c}{f_{Z1}}\right)^2} * \sqrt{1 + \left(\frac{f_c}{f_{Z2}}\right)^2} * \sqrt{1 + \left(\frac{f_c}{f_{rhp}}\right)^2}}{50 * 10^3 * 2\pi \sqrt{\left(1 - \left(\frac{f_c}{f_o}\right)^2\right)^2 + \left(\frac{f_c}{f_o Q}\right)^2} * \sqrt{1 + \left(\frac{f_c}{f_{P2}}\right)^2}} * V_g * FM \\ * \omega_I = 1 \end{aligned}$$

$$\omega_I = 36000$$

According to design notes in the lecture

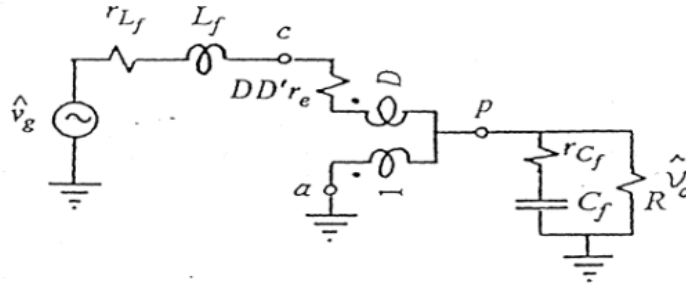
$$\omega_I = \frac{1}{R_1 C_1} \quad \omega_{Z1} = \frac{1}{R_2 C_1} \quad \omega_{Z2} = \frac{1}{R_1 C_2} \quad \omega_{P1} = \frac{1}{R_3 C_2} \quad \omega_{P2} = \frac{1}{R_2 C_3}$$

$$R_1 = 10K\Omega \quad R_2 = 13.8K\Omega \quad R_3 = 15.5K\Omega$$

$$C_1 = 2.7pF \quad C_2 = 171pF \quad C_3 = 23pF$$

continue with obtaining $G_{vg}(s) = \frac{\hat{v}_o(s)}{\hat{v}_g(s)}$, $G_{vd}(s) = \frac{\hat{v}_o(s)}{\hat{d}(s)}$

$$G_{vg}(s) = M \frac{\hat{v}_o(s)}{\hat{v}_g(s)} \text{ When } \hat{I}_o = \hat{d} = 0$$



Circuit Model

$$V_{ap} = V_{ac} + V_{cp}$$

$$i_a = \hat{i}_c D, i_p = \frac{\hat{v}_o}{R} + \frac{\hat{v}_o}{r_c + \frac{1}{sC}}$$

$$i_c = i_a - i_p = \hat{i}_c D - \left(\frac{\hat{v}_o}{R} + \frac{\hat{v}_o}{r_c + \frac{1}{sC}} \right)$$

$$i_c = -\frac{\hat{v}_o}{D'} \left(\frac{1}{R} + \frac{1}{r_c + \frac{1}{sC}} \right)$$

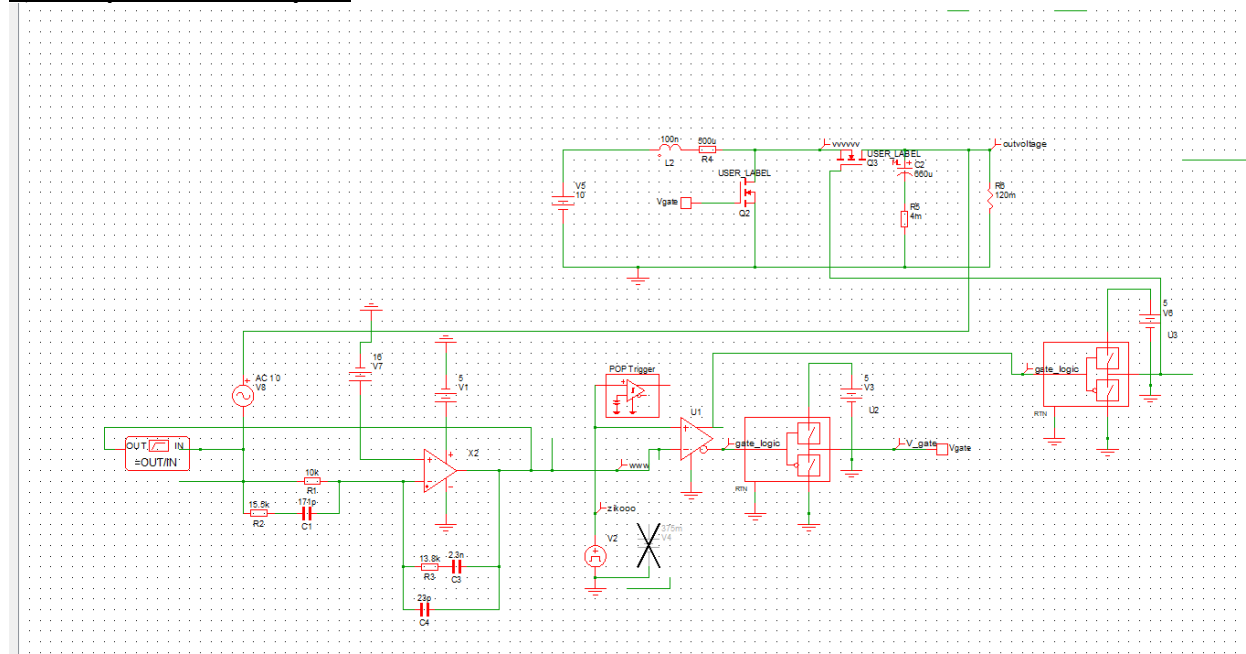
$$V_{ac} = -\hat{v}_g - \hat{i}_c(r_L + sL), V_{ap} = -\hat{v}_o, V_{cp} = -\hat{v}_o D - \hat{i}_c DD'r_e$$

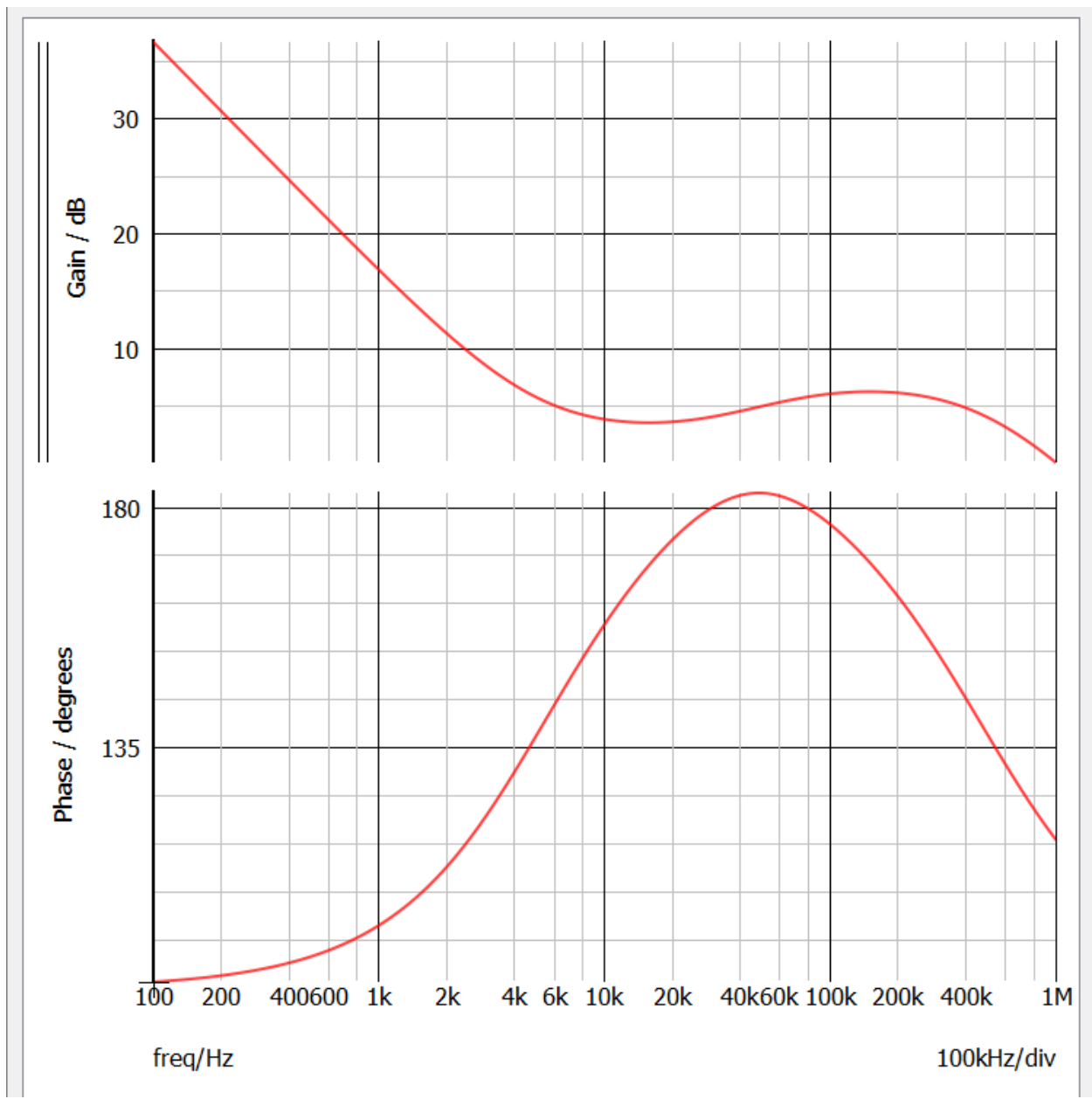
$$\hat{v}_o = \hat{v}_g + \hat{i}_c(r_L + sL) + \hat{v}_o D + \hat{i}_c DD'r_e$$

$$\begin{aligned}
\hat{v}_o D' &= \hat{v}_g + \hat{i}_c ((r_L + sL) + DD'r_e) \\
\hat{v}_o D' \left(\frac{D'^2 R r_c + \frac{D'^2 R}{sC} + \left(\left(r_c + \frac{1}{sC} + R \right) (r_L + sL + DD'r_e) \right)}{D'^2 R r_c + \frac{D'^2 R}{sC}} \right) &= \hat{v}_g \\
\frac{\hat{v}_o}{\hat{v}_g} &= \frac{1}{D'} \left(\frac{D'^2 R r_c + \frac{D'^2 R}{sC}}{D'^2 R r_c + \frac{D'^2 R}{sC} + \left(\left(r_c + \frac{1}{sC} + R \right) (r_L + sL + DD'r_e) \right)} \right) \\
\frac{\hat{v}_o}{\hat{v}_g} &= \frac{D'^2 R}{D'} \left(\frac{Cr_c s + 1}{(CR + Cr_c)Ls^2 + (D'^2 R Cr_c + L + (DD'r_e + r_L)(CR + Cr_c))s + (D'^2 R + r_L + DD'r_e)} \right) \\
\frac{\hat{v}_o}{\hat{v}_g} &= \frac{1}{D' \left(1 + \frac{r_L}{D'^2 R} + \frac{Dr_e}{D'R} \right)} \left(\frac{Cr_c s + 1}{\frac{(CR + Cr_c)L}{D'^2 R + r_L + DD'r_e} s^2 + \frac{D'^2 R Cr_c + L + (DD'r_e + r_L)(CR + Cr_c)}{D'^2 R + r_L + DD'r_e} s + 1} \right)
\end{aligned}$$

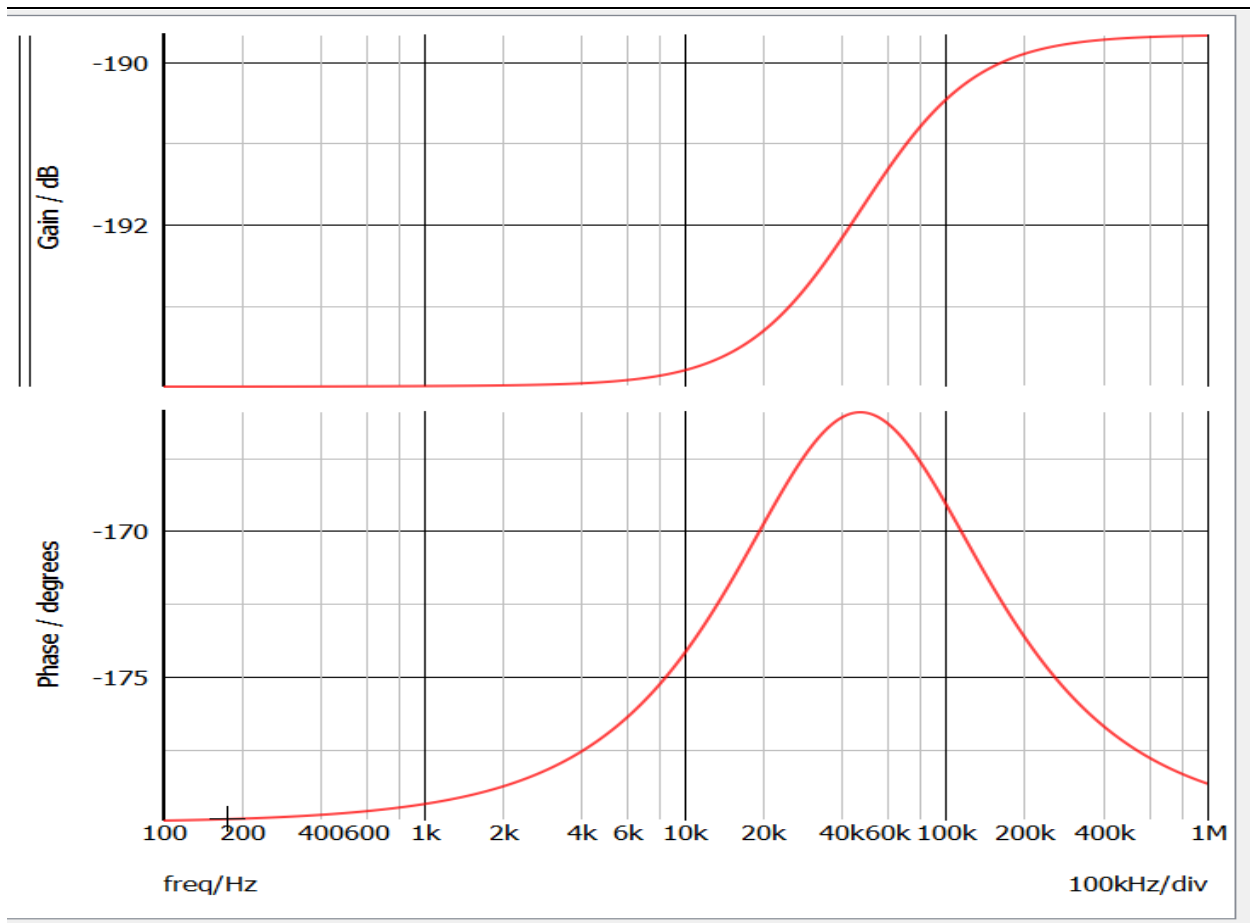
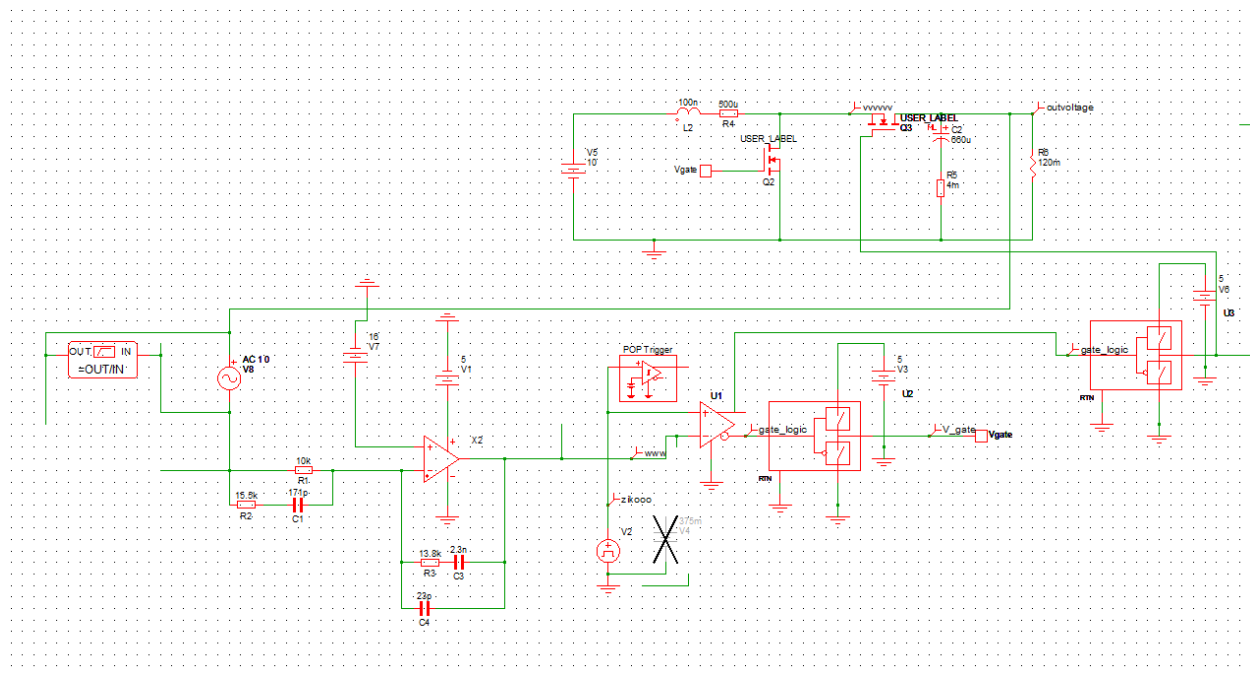
$$\begin{aligned}
\frac{\hat{v}_o(s)}{\hat{v}_g(s)} &= M \frac{1 + \frac{s}{\omega_{esr}}}{1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}} \\
\omega_o &= \sqrt{\frac{D'^2 R + r_L + DD'r_e}{LC(R + r_c)}} = \frac{D'}{\sqrt{LC}} \sqrt{\frac{1 + \frac{r_L}{D'^2 R} + \frac{Dr_e}{D'R}}{1 + \frac{r_c}{R}}} \\
Q &= \frac{\omega_o}{\frac{D'^2 R r_c}{L(R + r_c)} + \frac{1}{C(R + r_c)} + \frac{(DD'r_e + r_L)}{L}}, \quad \omega_{esr} = \frac{1}{r_c C}
\end{aligned}$$

Now we will obtain Control to Output Transfer Function

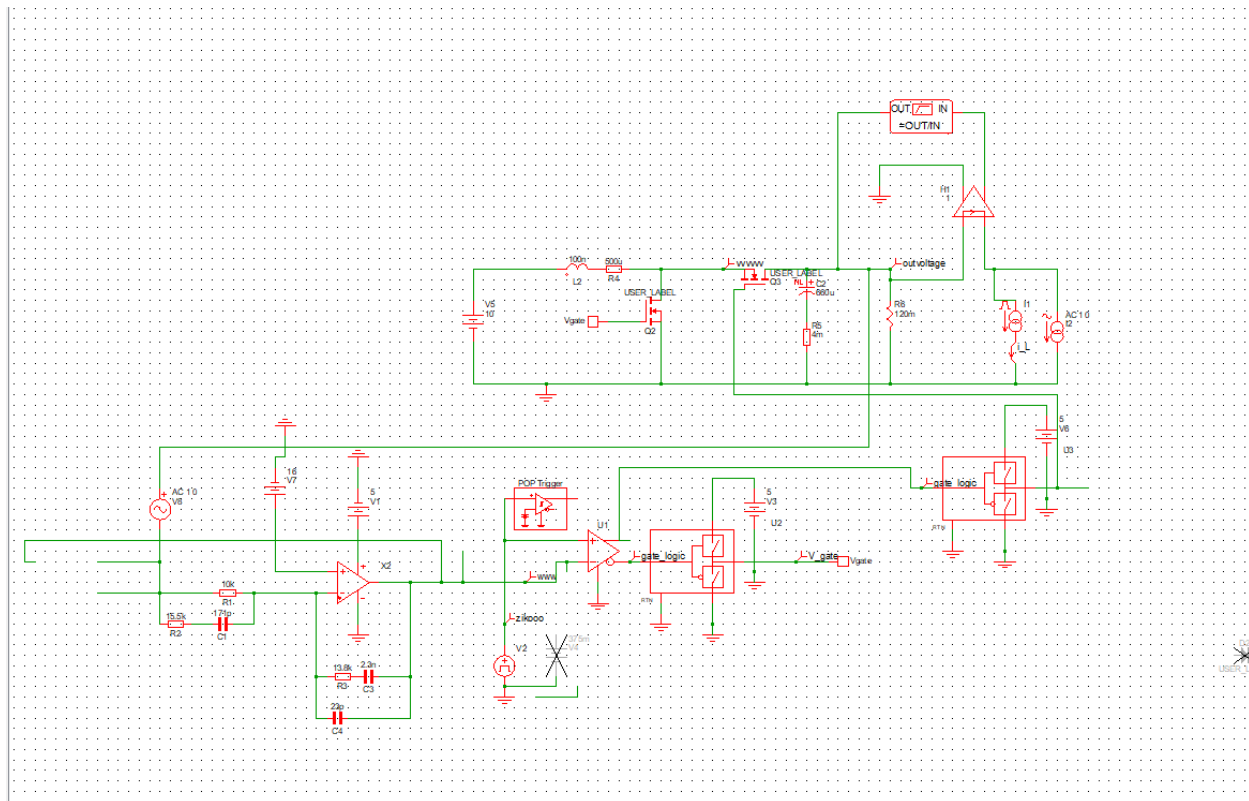


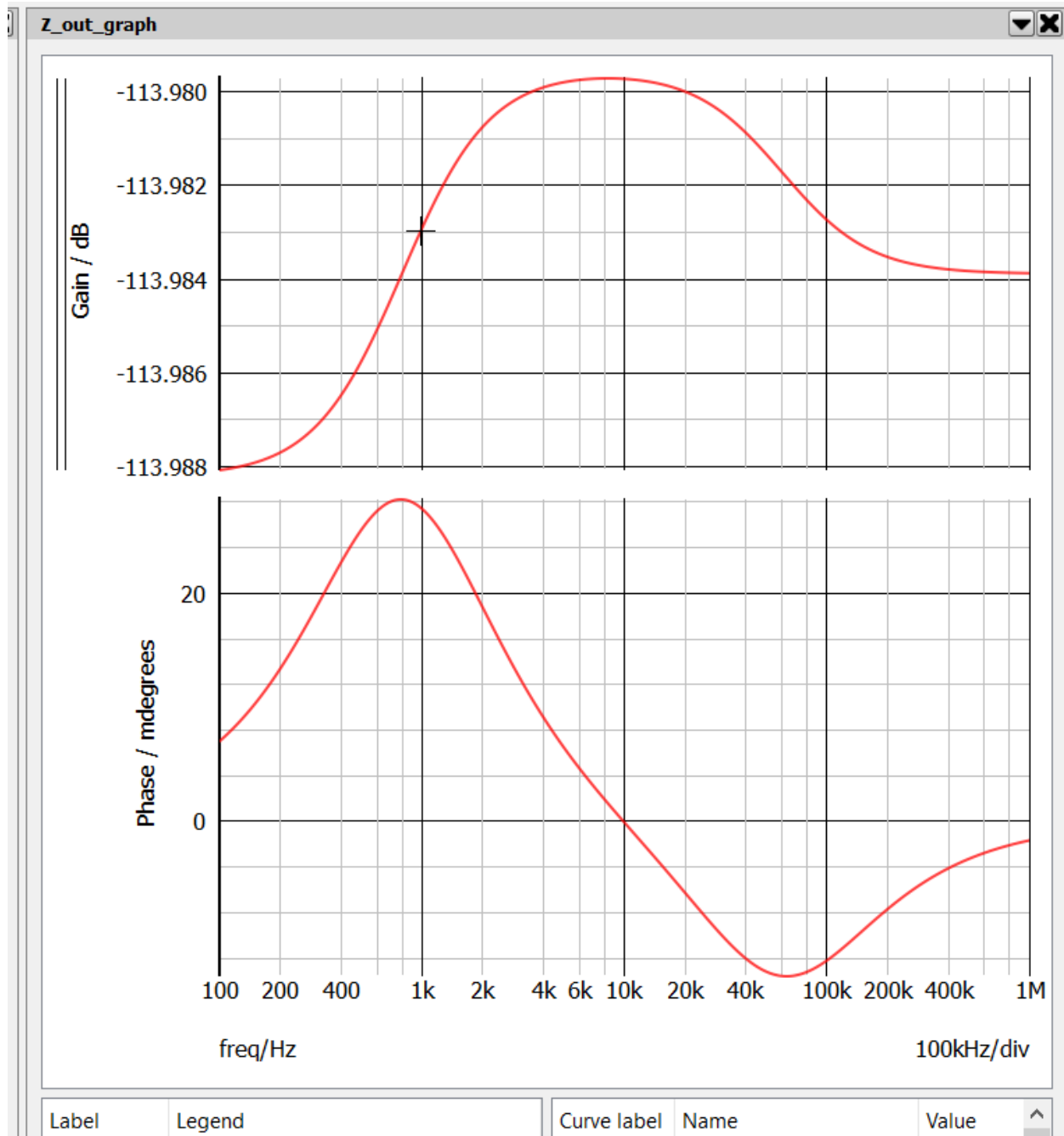


Loop gain bode plot



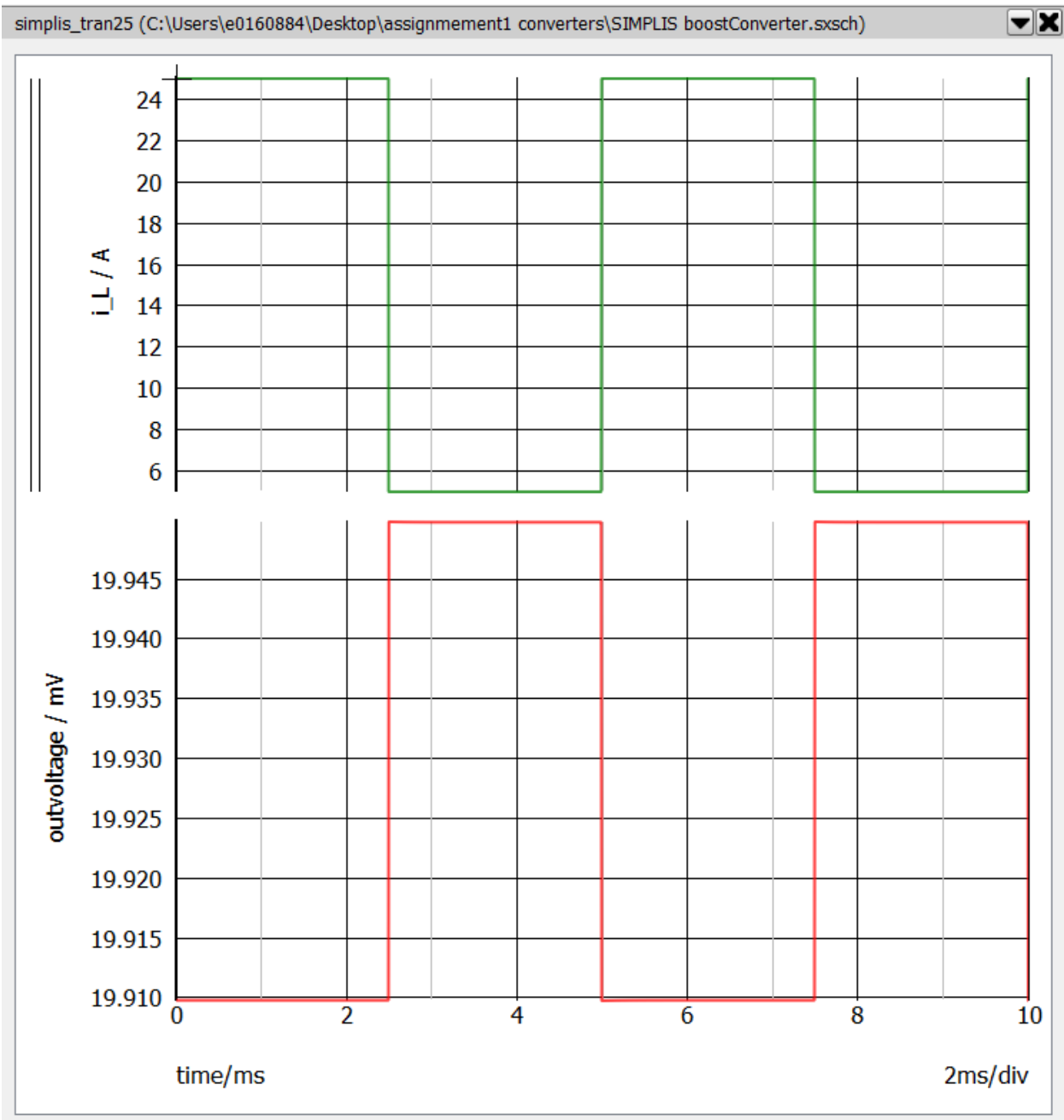
C- Closed loop response of output impedance



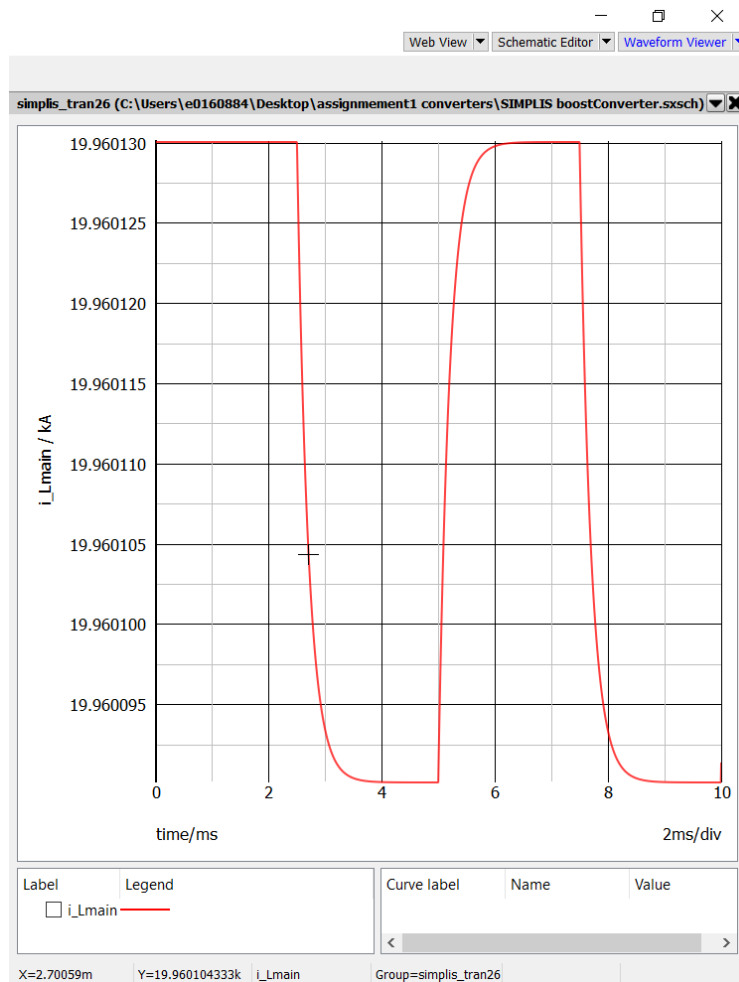


The controller failed to achieve the desired PM and GM because the location of the zeros as the compensator added an unstable zero which has a frequency higher than the cutoff frequency and near to the RHP zero and phase margin wouldn't be achieved and the controller hits -180

as a rule of thumb the gain cross over frequency should be lower than $\frac{W_{rhp}}{5}$



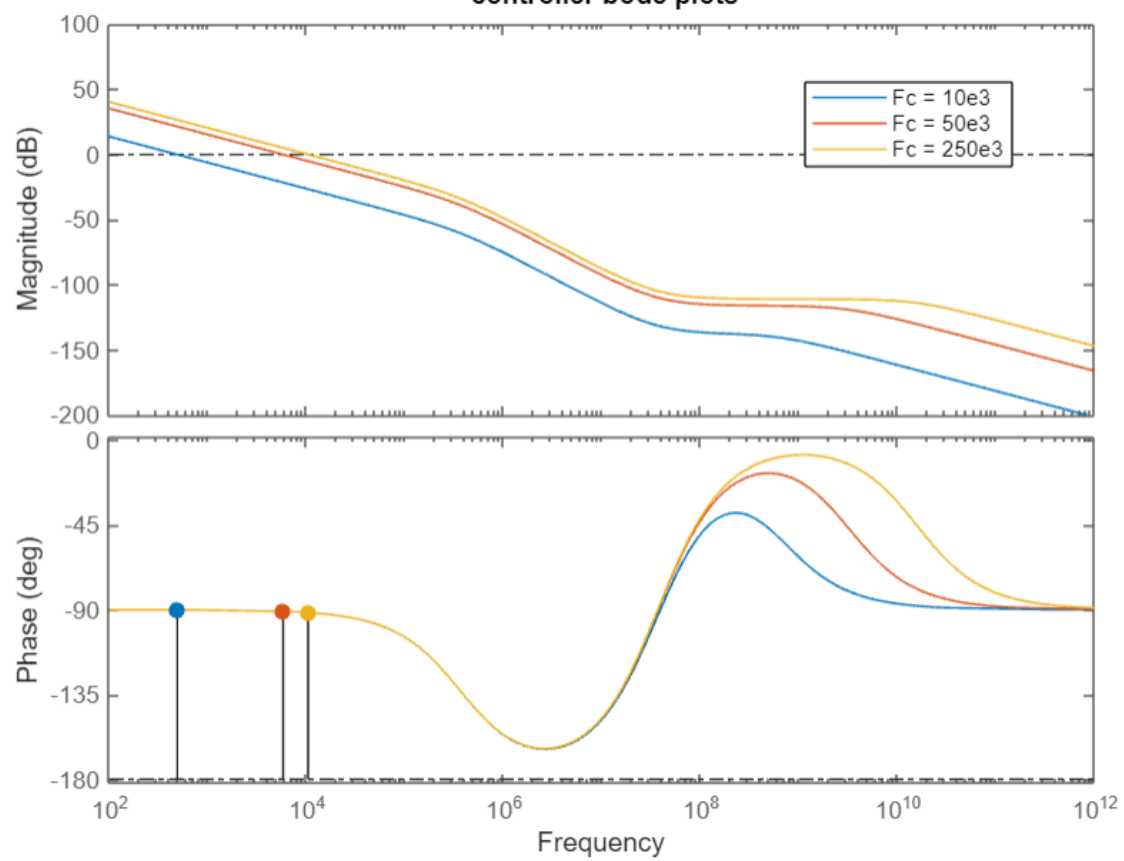
As also resulted from this the high inductor current due to higher duty ratio so it's also mandatory to apply saturation before the PWM modulator

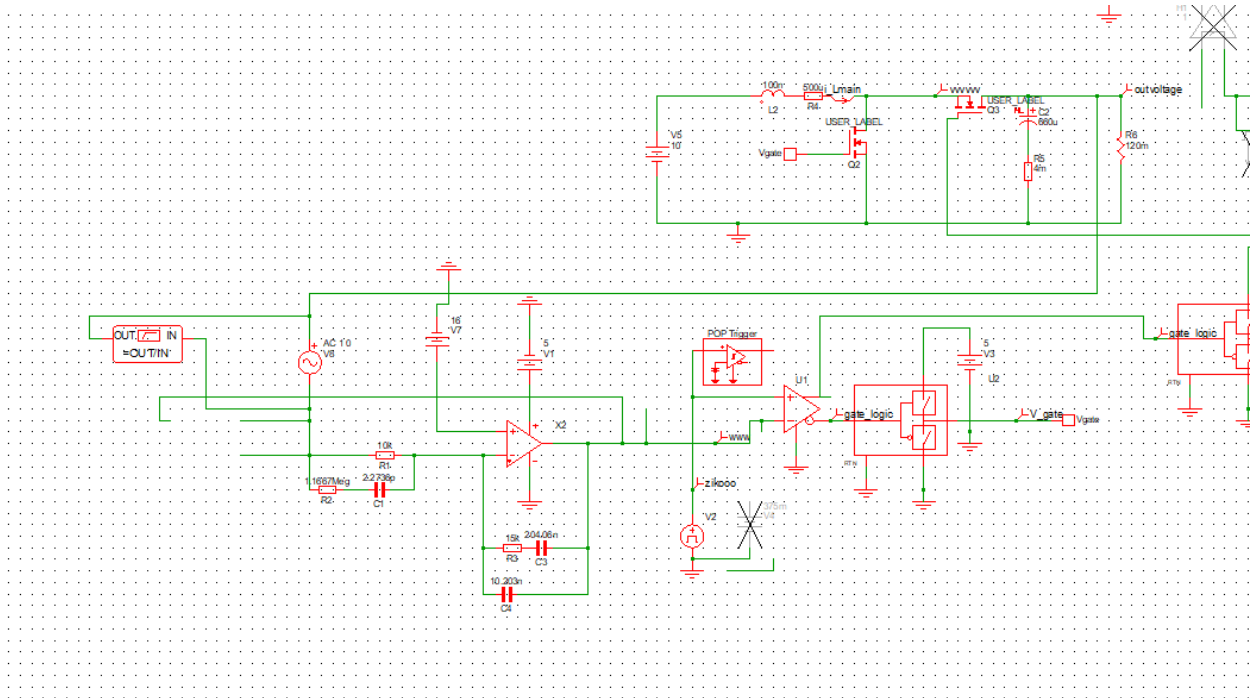


The controller failed to compensate and achieve target voltage.

Q4)

controller bode plots





Increasing the bandwidth by Increasing the integrator gain and decreasing the first zero fz1 and the second zero Fz2 away from Fo.

High DC gain will lead to higher bandwidth but poor PM and the same thing for the pwm modulator gain.

The bandwidth is also limited by the RHP zero as $\omega_{rhp} = \frac{(RD'^2 - r_L - D'^2 r_e)}{L}$

when the load current is high , the R will be small which means ω_{rhp}

Will be smaller and near to the imaginary axis which will leads to undershoot effect.

ω_{ESR} has insufficient effect on BW.

/rive/converters assignment 3/Q4.m

```
sw = [10e3 50e3 250e3];
```

```
s = tf('s');  
for i = 1 : length(sw)  
    fc = sw(i);  
    fz1 = 5e3;  
    fz2 = 7e3;  
    frhp = 71e3;  
    fo = 12e3;  
    q = 1.8547;  
    fp2 = 10*fc;  
    R1 = 10e3;
```

```
Wz1=fz1*(10^3)*2*pi;  
Wz2=fz2*(10^3)*2*pi;  
Wp1=60*(10^3)*2*pi;  
Wp2=10*fc*(10^3)*2*pi;
```

```
x1 = sqrt(1 + (fc/fz1)^2);  
x2 = sqrt(1 + (fc/fz2)^2);  
x3 = sqrt(1 + (fc/frhp)^2);  
x4 = sqrt((1 - (fc/fo)^2)^2 + (fc/(fo*q)^2));
```

_AB Drive/converters assignement 3/Q4.m

```
T = 10*(x1 * x2 * x3)/(x4 * x5 * 2*pi*fc);

Wi = 1/T;

C1 = 1/(R1*Wi);
R2 = 1/(C1*Wz1);
C2 = 1/(R1*Wz2);
R3 = 1/(C2*Wp1);
C3 = 1/(R2*Wp2);

A=(Wi/s)*(1+s/Wz1)*(1+s/Wz2)/((1+s/Wp1)*(1+s/Wp2));

bode(A)
title(' controller bode plots');
legend ('Fc = 10e3','Fc = 50e3','Fc = 250e3')
hold on

end
```