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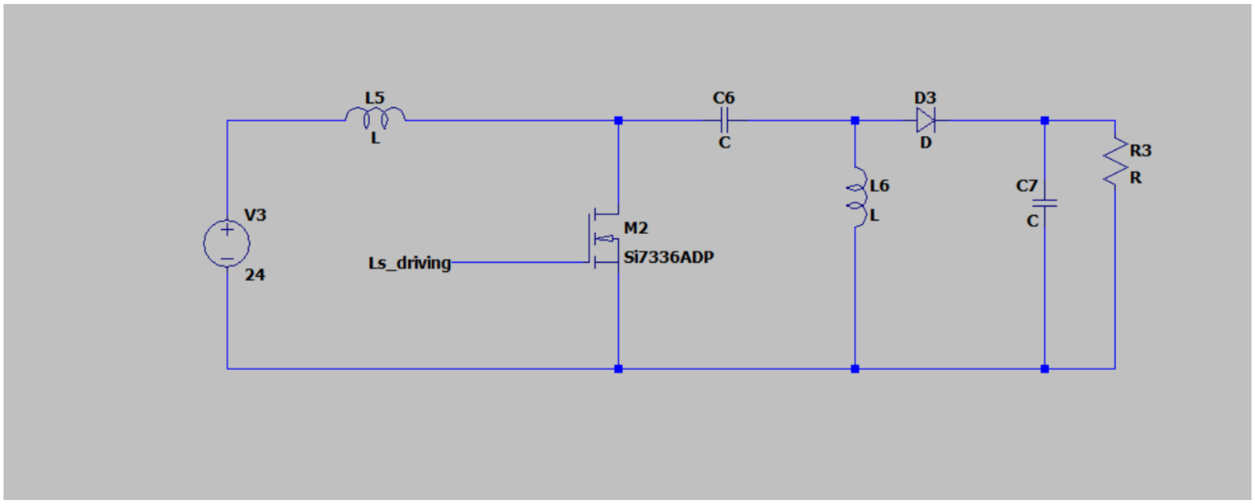
Submitted to: Prof. Abdelmomen Mahgoub

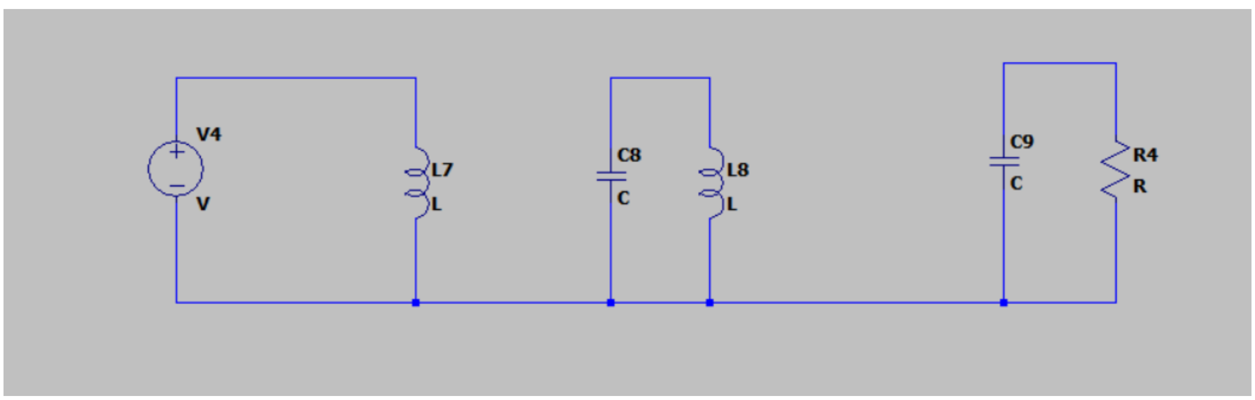
Assignment No.2

Q1 Sepic converter



- Steady state analysis

On state



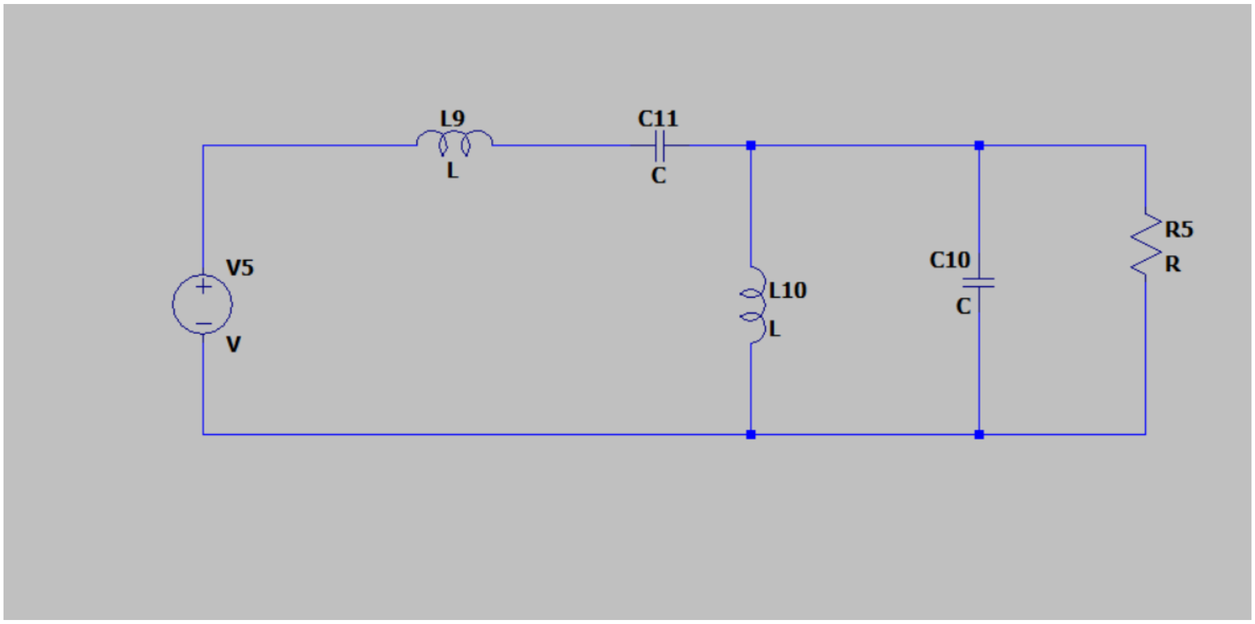
$$V_{L1} = V_{in}$$

$$V_{L2} = V_{C1}$$

$$I_{c1} = -I_{L2}$$

$$I_{C2} = -\frac{V_o}{R}$$

OFF State



Using KVL

$$V_{in} - V_{L1} - V_{C1} = V_{L2}$$

$$V_{L1} = V_{L2}$$

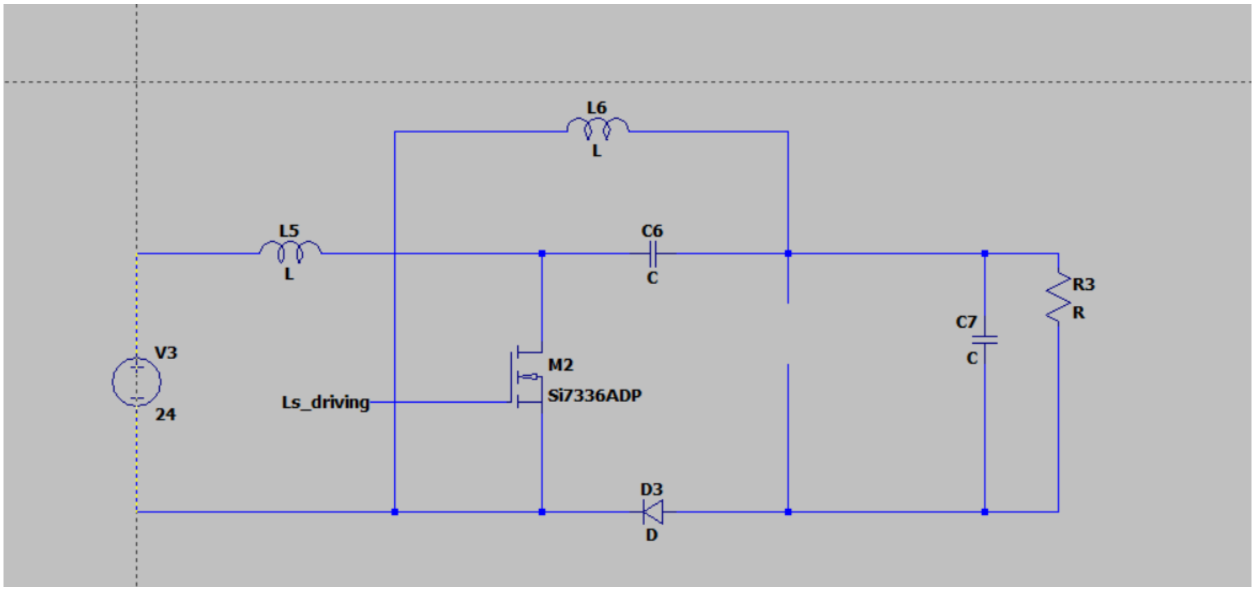
$$I_{C1} = I_{L1}$$

$$I_{C2} = I_{c1} + L_2 - \frac{V_O}{R}$$

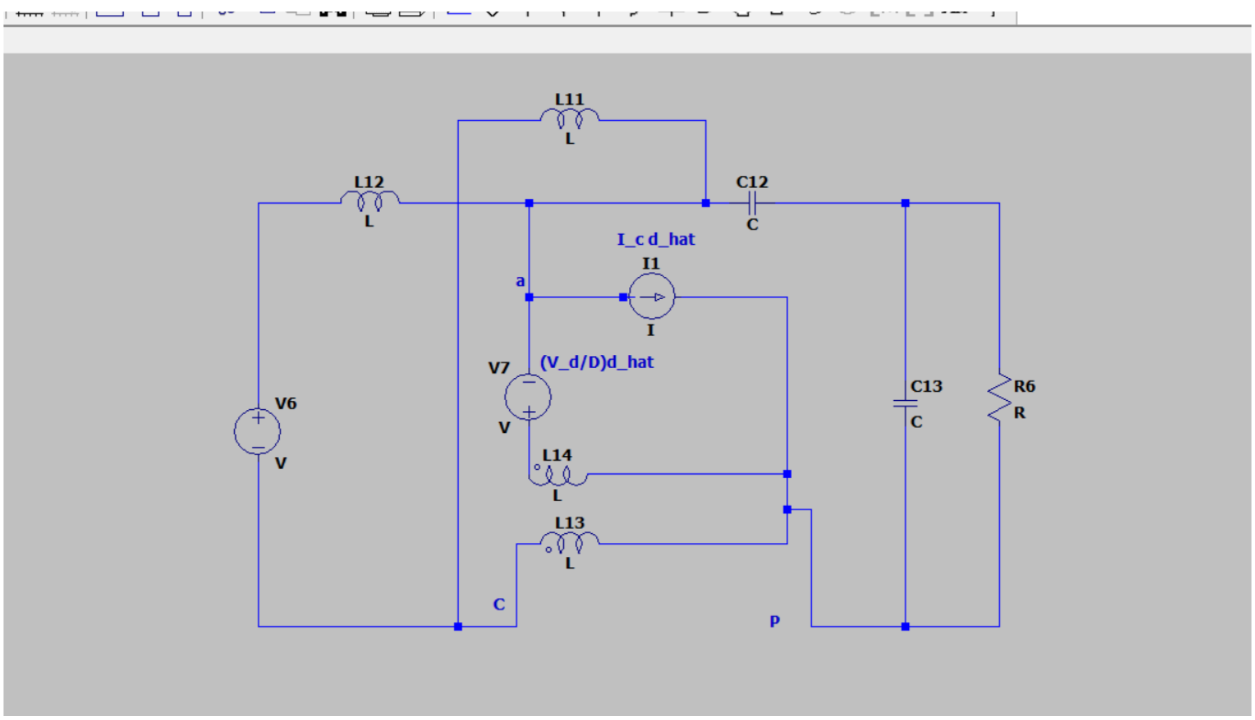
For $I_{c2} = I_{C1} = 0$

$$I_{L2} = \frac{V_o}{R}$$

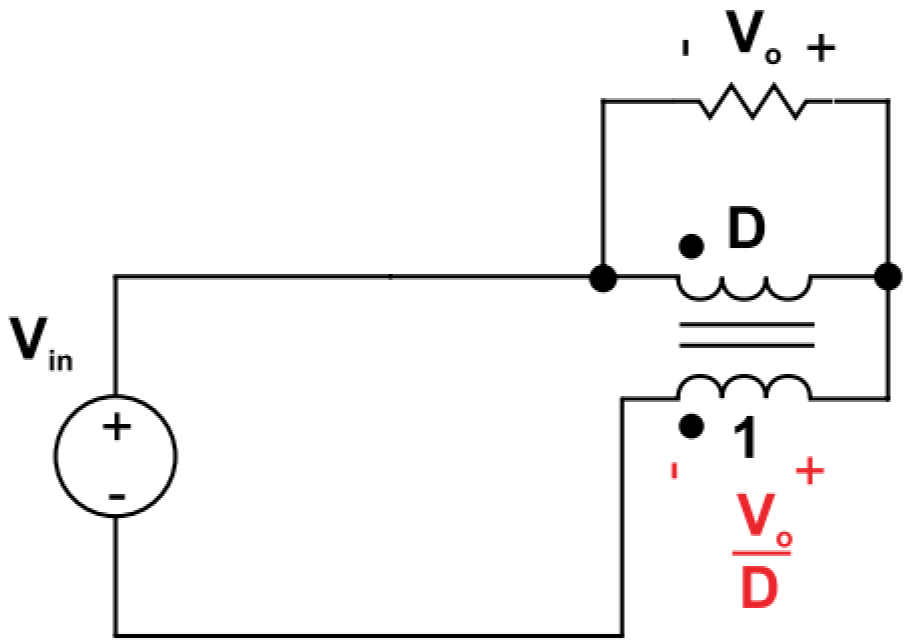
For applying three terminal pwm switch, we have to make some modifications.



This modification won't affect the operation and the Converter will have the same functionality.



To get DC operating point and Gain



By using KVL

$$V_g + V_o - \frac{1}{D}V_o = 0$$

$$V_g = \left(\frac{1}{D} - 1\right)V_o = \frac{D^{`}}{D}$$

$$V_o = \frac{D^{`}}{D}V_g$$

$$I_o = \frac{I_g}{D} - I_g$$

$$M(D) = \frac{V_o}{V_g}$$

For V_d

$$I_c = \frac{I_g}{D} = \frac{I_o}{D^{`}}$$

$$\mathrm{V_d} = \mathrm{V_{ap}} = \mathrm{V_g} + \mathrm{V_o} = \frac{\mathrm{V_g}}{\mathrm{D'}} = \frac{\mathrm{V_o}}{D}$$

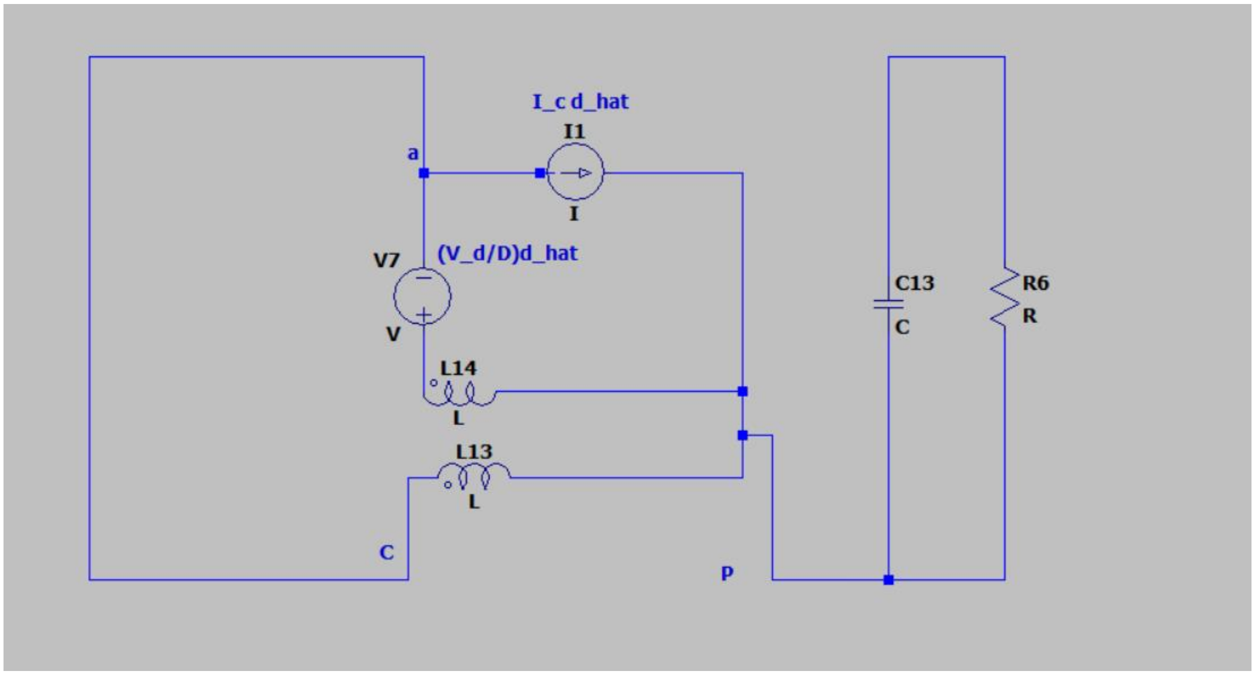
To find G_{vd}(control to output transfer function)

We will use N Extra element method to derive the TF

1- DC gain

$$G_{vd} = \frac{G_{vdo} N(s)}{D(s)}$$

To find DC gain we will have the passive elements in their steady state.



$$V_{ap} = V_{cp} + V_{ac}$$

Hence $V_{ac} = 0$

$$V_{ap} = \frac{\grave{v_o}}{D} - \frac{V_D}{D}\grave{d}$$

$$V_{cp} = \grave{v_o}$$

$$\frac{\grave{v_o}}{D} - \grave{v_o} = \frac{V_d}{D}\grave{d}$$

$$\frac{\grave{v_o}}{\grave{d}} = \frac{V_D}{\grave{D}}$$

$$V_D = \frac{V_g}{\grave{D}}$$

$$then\ the\ DC\ gain\ is \frac{V_g}{D`^2}$$

For the Extra element method

$$D(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4$$

$$D(s) = (1 + \frac{s}{W_{o1} Q_1} + \frac{s^2}{W_{o1}^2})(1 + \frac{s}{W_{o2} Q_2} + \frac{s^2}{W_{o2}^2})$$

$$a_1 = \frac{1}{W_{o1} Q_1}$$

$$a_2 = \frac{1}{W_{o1}^2}$$

$$a_3 = \frac{1}{W_{o1} Q_1 W_{o2}^2} + \frac{1}{W_{o2} Q_2 W_{o1}^2}$$

$$a_4 = \frac{1}{W_{o1}^2 W_{o2}^2}$$

- R(x element) means the resistance seen by this x element, when all other elements are in DC state and $d` = 0$
- R(X,Y) means the resistance seen by X when all other elements in Dc state except element Y is in high frequency state and $d` = 0$

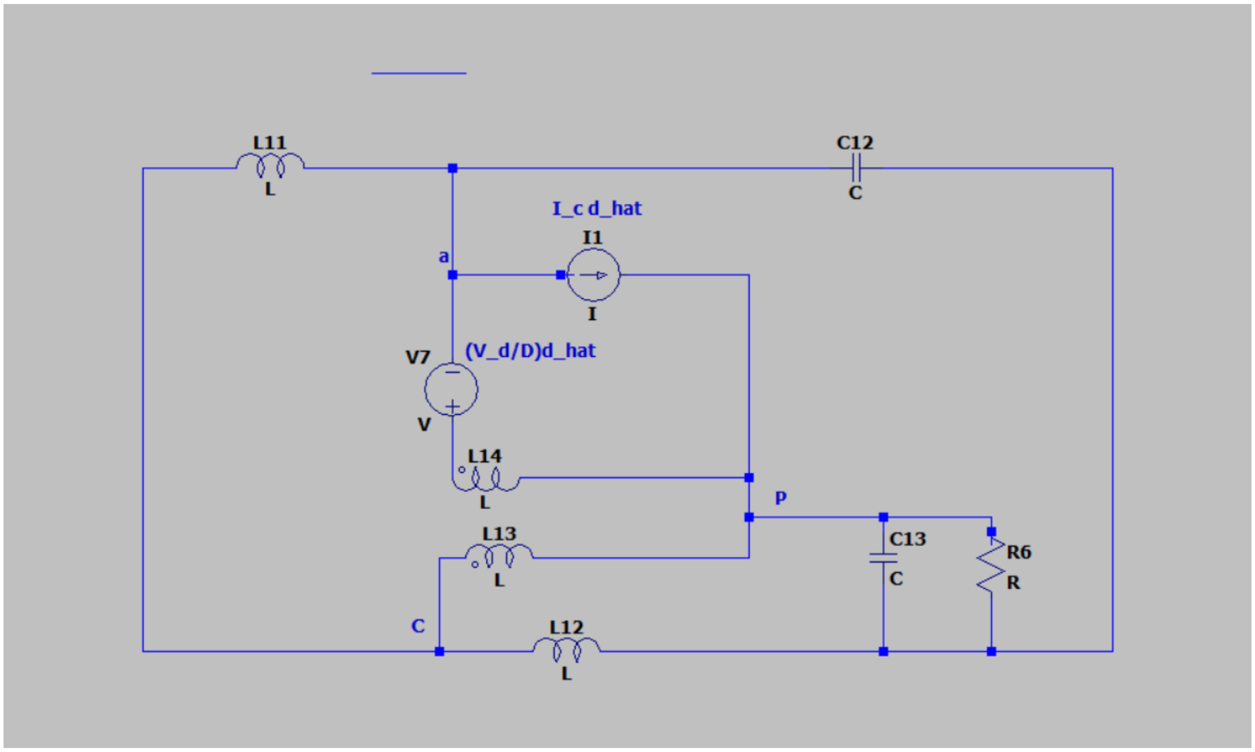
$$a_1 = \frac{L_1}{R(L_1)} + \frac{L_2}{R(L_2)} + C_1 R(C_1) + C_2 R(C_2)$$

$$a_2 = \frac{L_1}{R(L_1)} \frac{L_2}{R(L_2, L_1)} + \frac{L_1}{R(L_1)} \mathrm{C_1 R(C_1, L_1\ )} + \frac{L_1}{R(L_1)} \mathrm{C_2 R(C_2, L_1\ )} + + \frac{L_2}{R(L_2)} \mathrm{C_1 R(C_1, L_2\ )} + \frac{L_2}{R(L_2)} \mathrm{C_2 R(C_2, L_2\ )} + C_1 R(C_1)\, C_2 R(C_2, C_1)$$

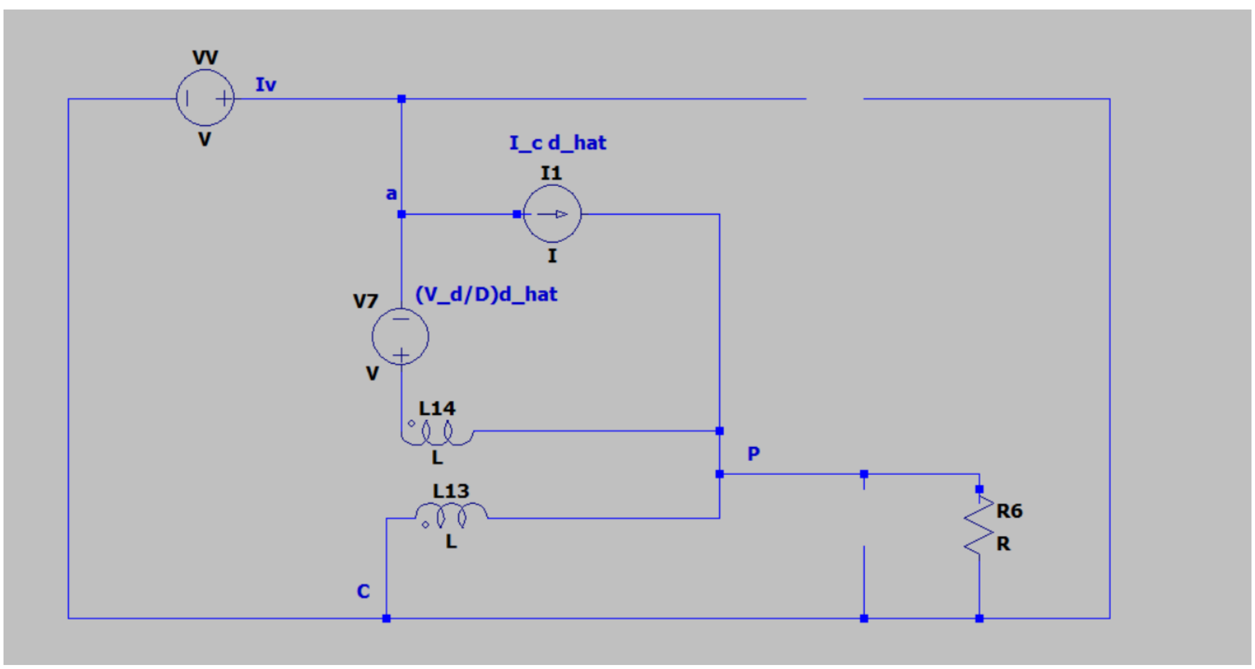
$$a_3 = \frac{L_1}{R(L_1)} \mathrm{C_1 R(C_1, L_1\ )} \frac{L_2}{R(L_2, C_1, L_1)} + \frac{L_1}{R(L_1)} \mathrm{C_1 R(C_1, L_1\ )}\, C_2 R(C_2, C_1, L_1) + \frac{L_2}{R(L_2)} \mathrm{C_1 R(C_1, L_2\ )} C_2 R(C_2, C_1, L_2) + \frac{L_1}{R(L_1)} \mathrm{C_2 R(C_2, L_1\ )} \frac{L_2}{R(L_2, C_2, L_1)}$$

$$a_4 = \frac{L_1}{R(L_1)} \mathrm{C_1 R(C_1, L_1\ )} \frac{L_2}{R(L_2, C_1, L_1)} \mathrm{C_2 R(C_2, L_1, C_1\ , L_2)}$$

Rearranging the circuit


L11
L
C12
C
I_c d_hat
I1
a
I
V7
(V_d/D)d_hat
V
L14
L
P
C13
C
R6
R
L13
L
C
L12
L

$for\ R(L_1)$


VV
Iv
V
I_c d_hat
I1
a
I
V7
(V_d/D)d_hat
V
L14
L
P
L13
L
R6
R
C

$I_a = I_v$

$$I_c = \frac{I_v}{D}$$

$$I_p = I_c - I_a = \frac{D^` I_v}{D}$$

$$V_{ac} = V_{vv}$$

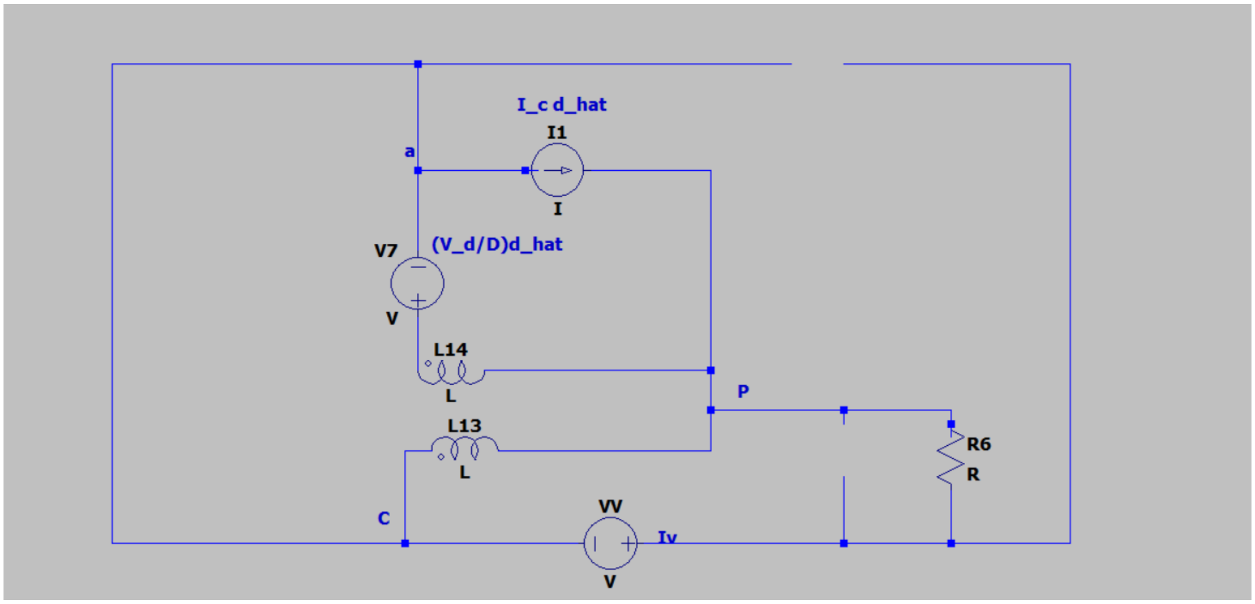
$$V_{cp} = \frac{D^` i_v}{D} R \, , V_{ap} = \frac{V_{cp}}{D}$$

$$V_{ac} = V_{vv} = V_{ac} - V_{cp}$$

$$V_{vv} = \frac{V_{cp}}{D} - V_{cp} = \frac{D^{`2}}{D^2} i_v R$$

$$R(L_1) = \frac{D^{`2}}{D^2} R$$

$$For \ R(L2)$$



$$V_{cp} = v_{vv} - I_v R \, , V_{ac} = 0$$

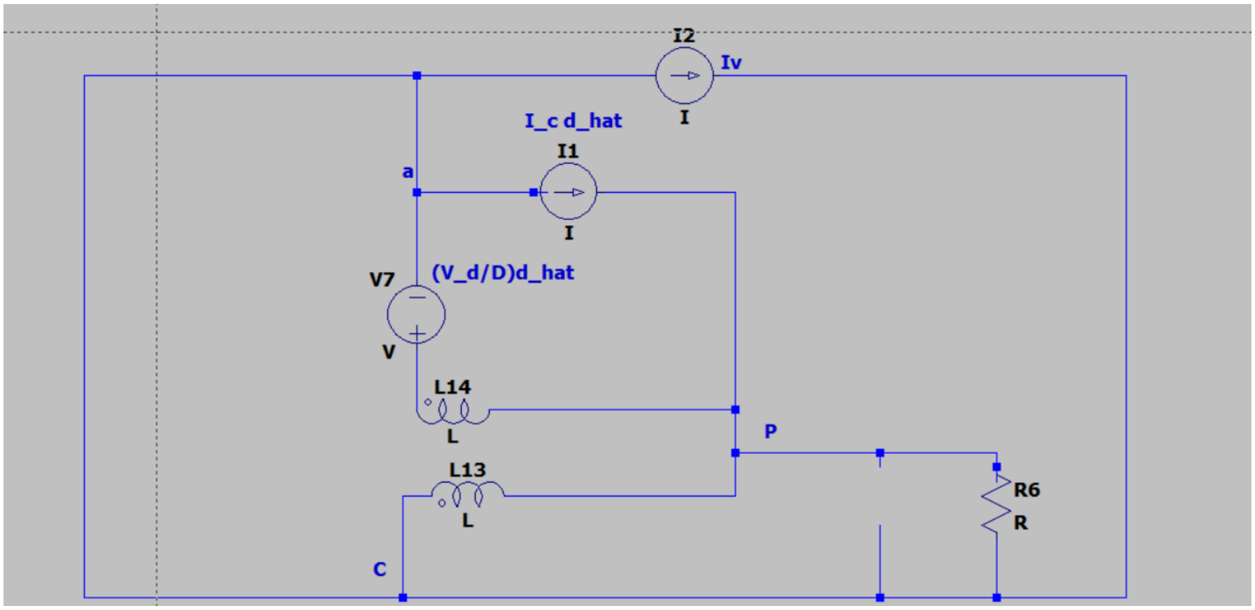
$$V_{ap} = \frac{V_{cp}}{D}$$

$$V_{ac} = V_{ap} - V_{cp}$$

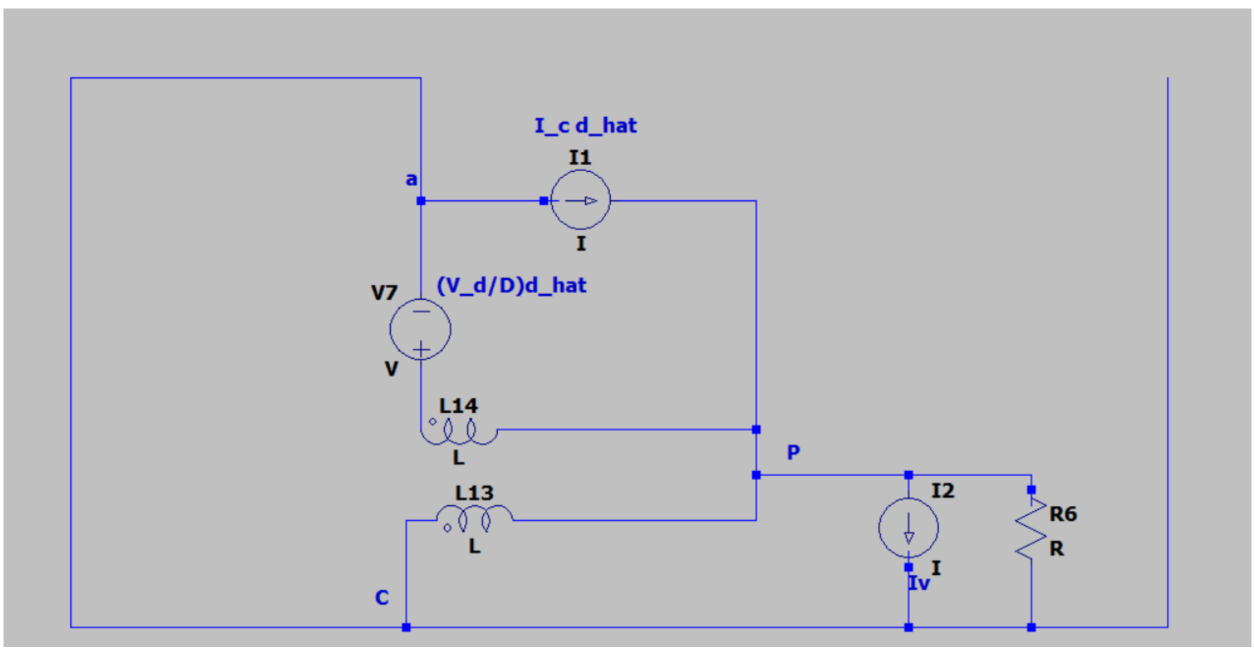
$$V_{cp} = 0, V_{vv} = I_v R$$

$$R(L_2) = R$$

$$For\ R(C_1)$$

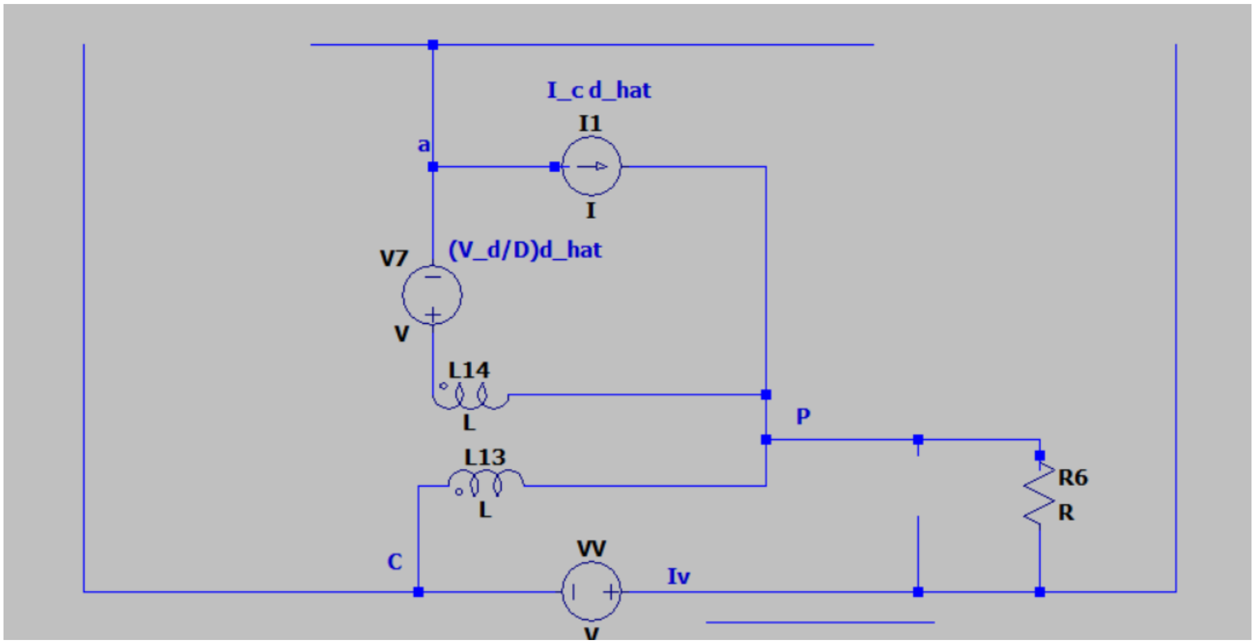


There is a short circuit which leads to $R(C_1) = 0$



There is a short circuit which leads to $R(C_2) = 0$

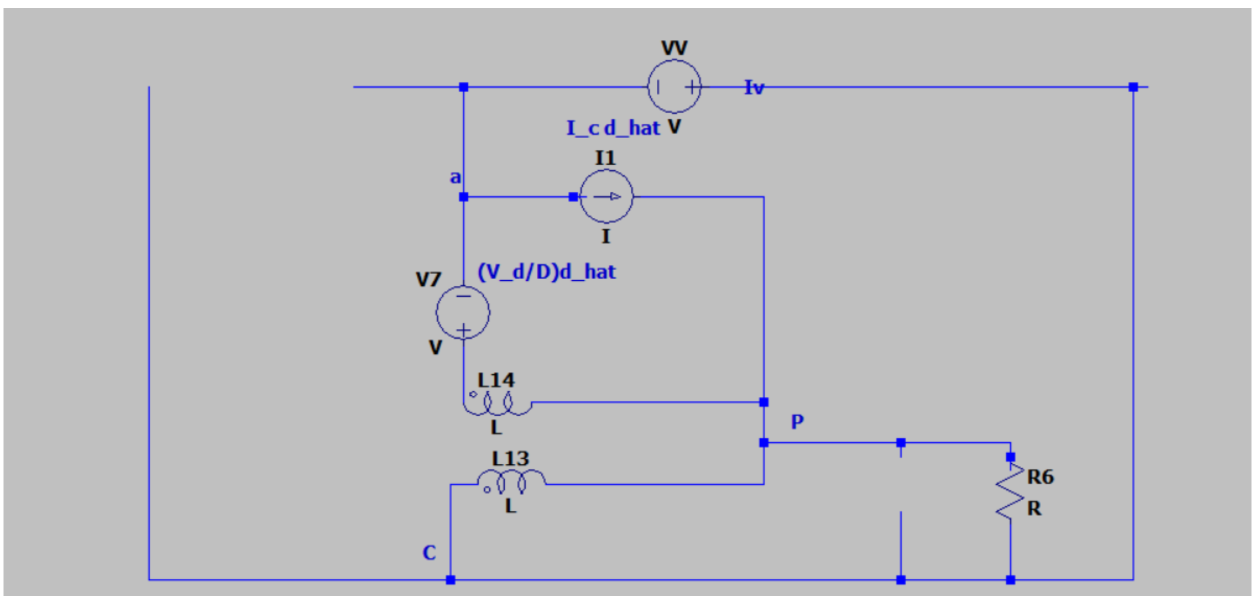
For $R(L_2, L_1)$



$$I_v = 0$$

Then $R(L_2, L_1) = \infty$

For $R(C_1, L_1)$



$$I_v = I_a$$

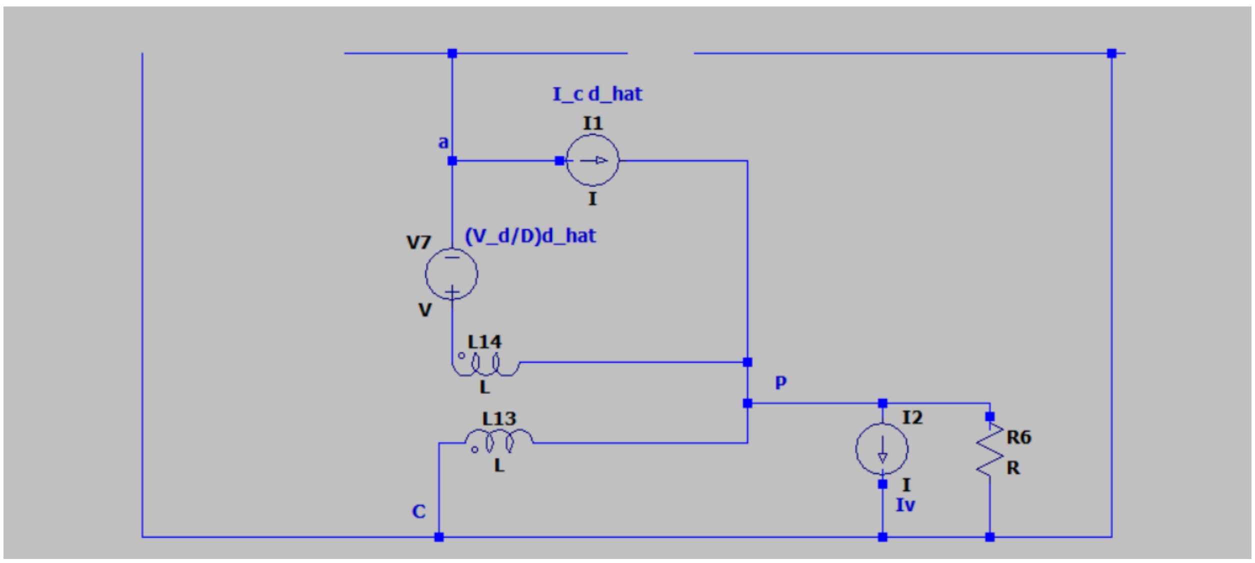
$$I_c = \frac{I_v}{D}\ , I_p = I_a - I_c = I_v - \frac{I_v}{D} = -\frac{D^{`} I_v}{D}$$

$$V_{ap} = \frac{V_{cp}}{D}, V_{cp} = -\frac{D^{`} I_v}{D} R, V_{ac} = -V_{vv}$$

$$V_{ac} = \frac{V_{cp}}{D} - V_{cp} = \frac{D^{`2} I_v}{D^2} R$$

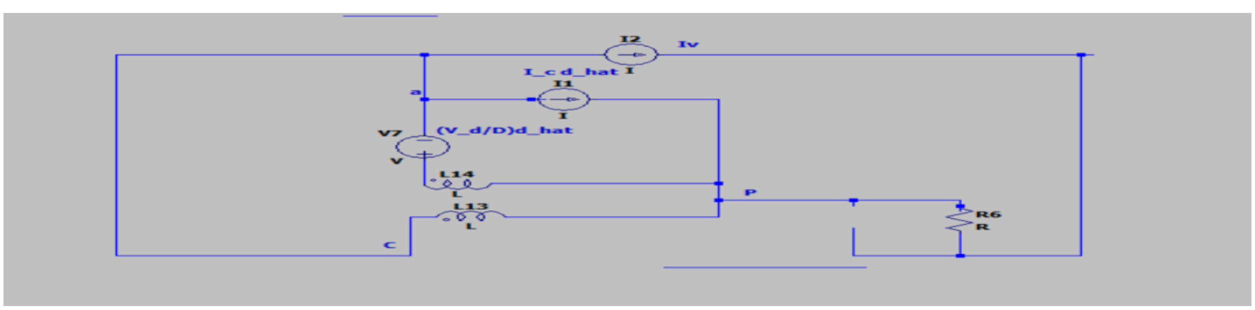
$$R(C_1, L_1) = \frac{D^{`2}}{D^2} R$$

For $R(C_2, L_1)$



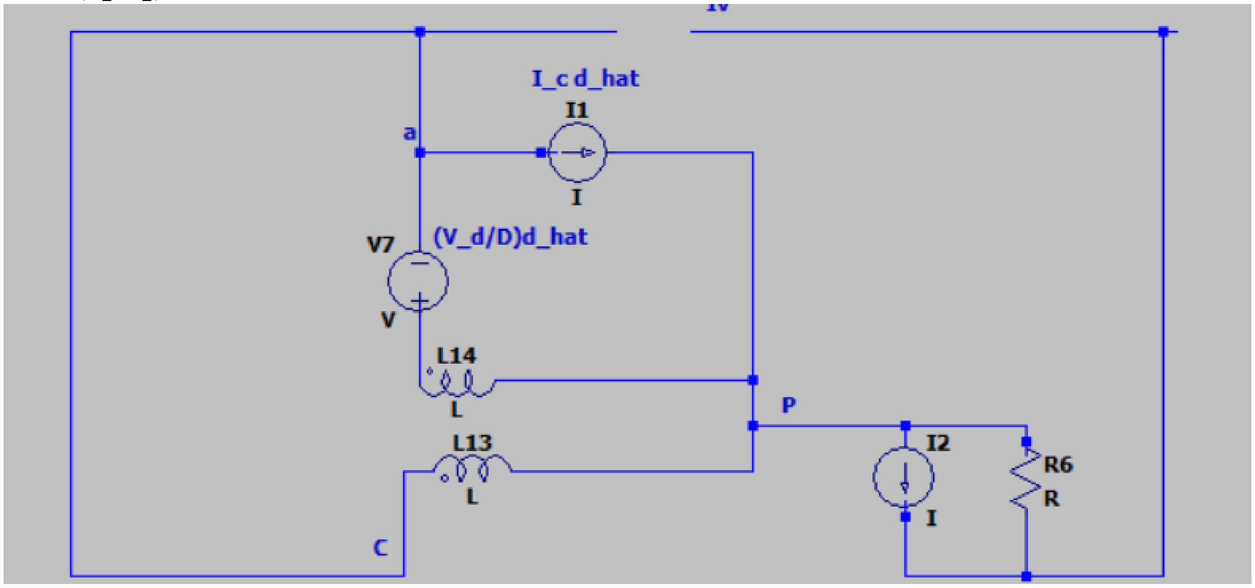
$$R(C_2, L_1) = R$$

For $R(C_1, L_2)$



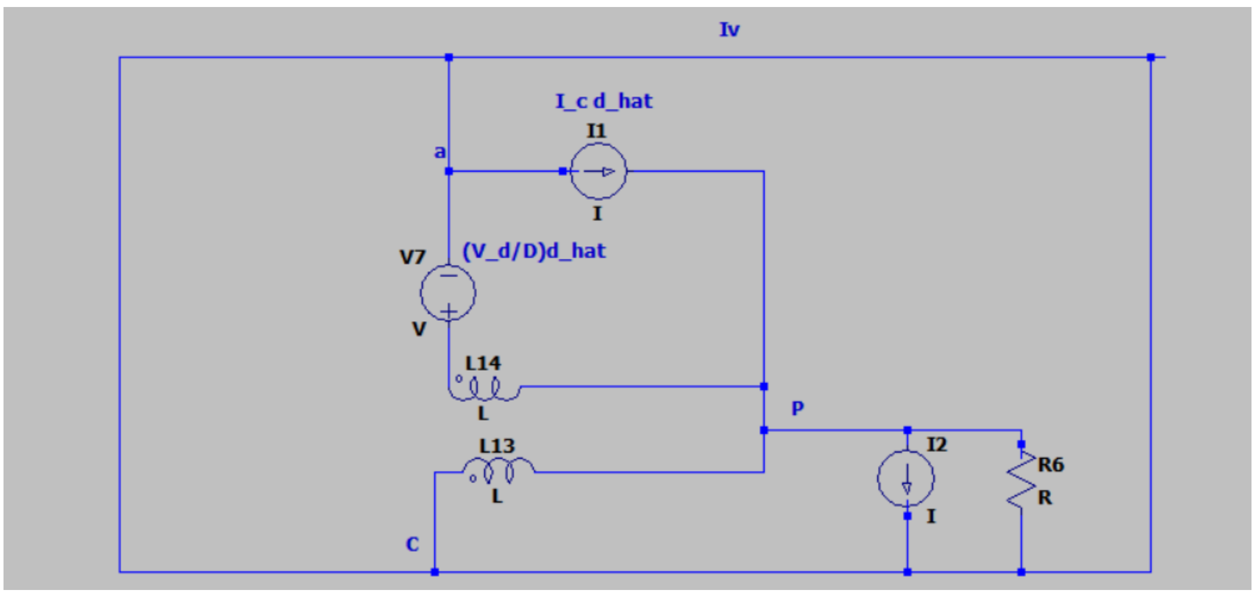
$$R(C_1, L_2) = R$$

For $R(C_2, L_2)$



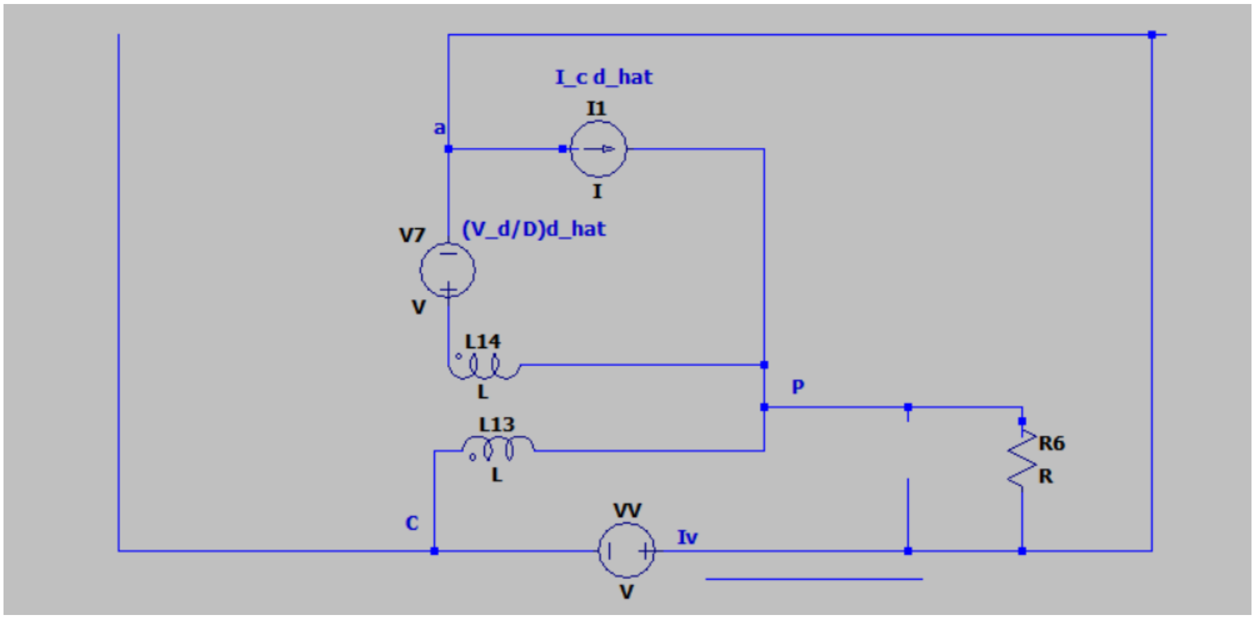
$$R(C_2, L_2) = R$$

For $R(C_2, C_1)$



$$R(C_2, C_1) = 0$$

For $R(L_2, C_{1,}, L_1)$



$$I_c = i_v, i_a = Di_v$$

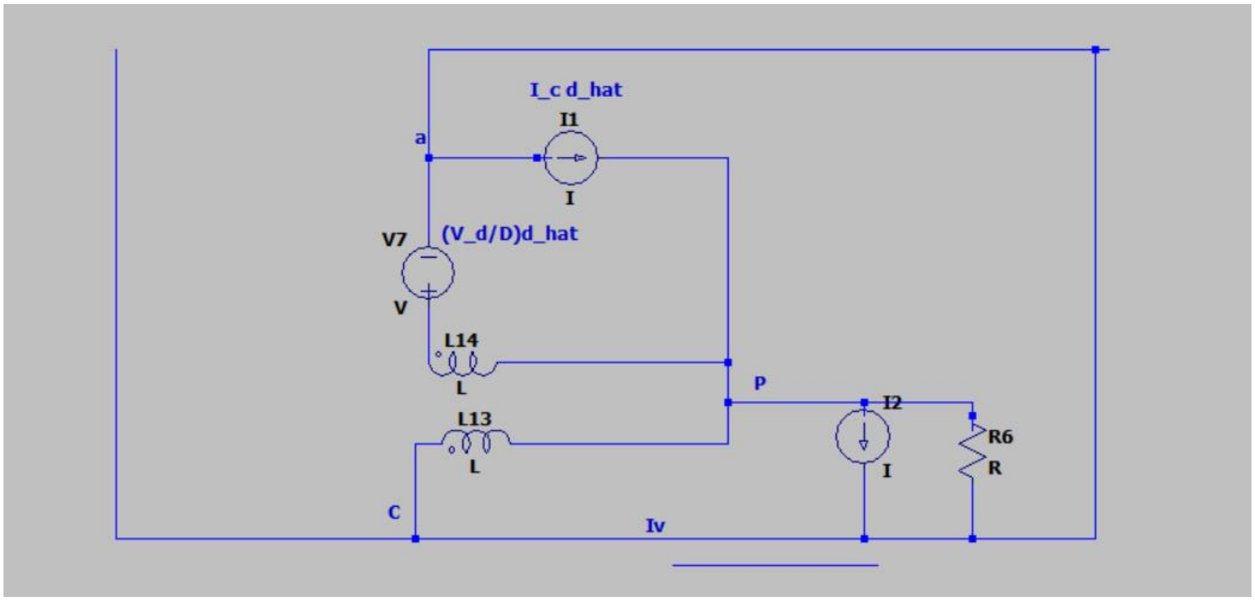
$$V_{ac} = V_{vv}, V_{cp} = DV_{ap}$$

$$V_{ap} = D^{`} i_v R \,, V_{ac} = V_{ap} - V_{cp}$$

$$V_{ac} = V_{ap} - DV_{ap} = D^{`2} i_v R$$

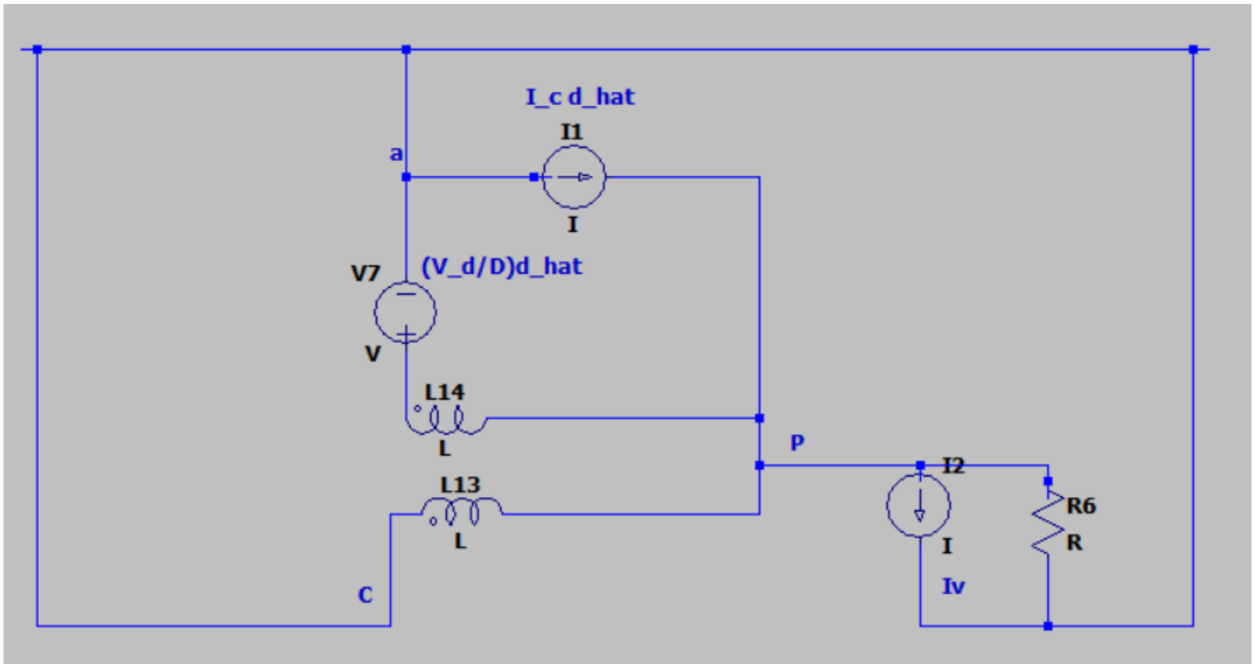
$$R(L_2, C_{1,}, L_1) = \; D^{`2} R$$

For $R(C_2, C_{1,}, L_1)$



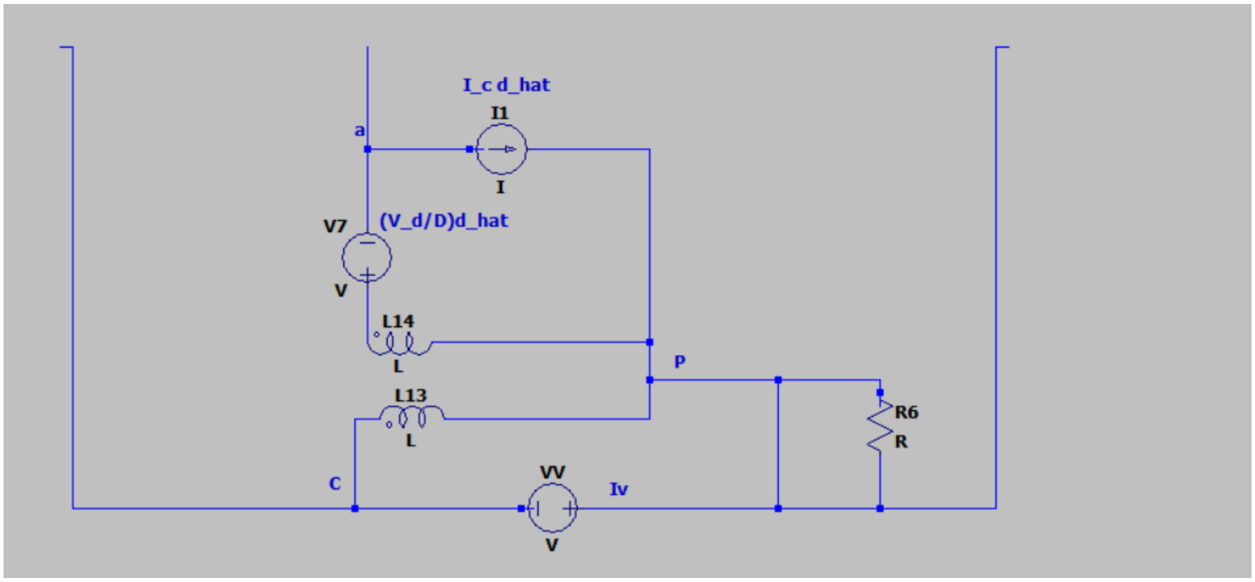
$$R(C_2, C_{1,}, L_1) = 0$$

For $R(C_2, C_{1,}, L_2)$



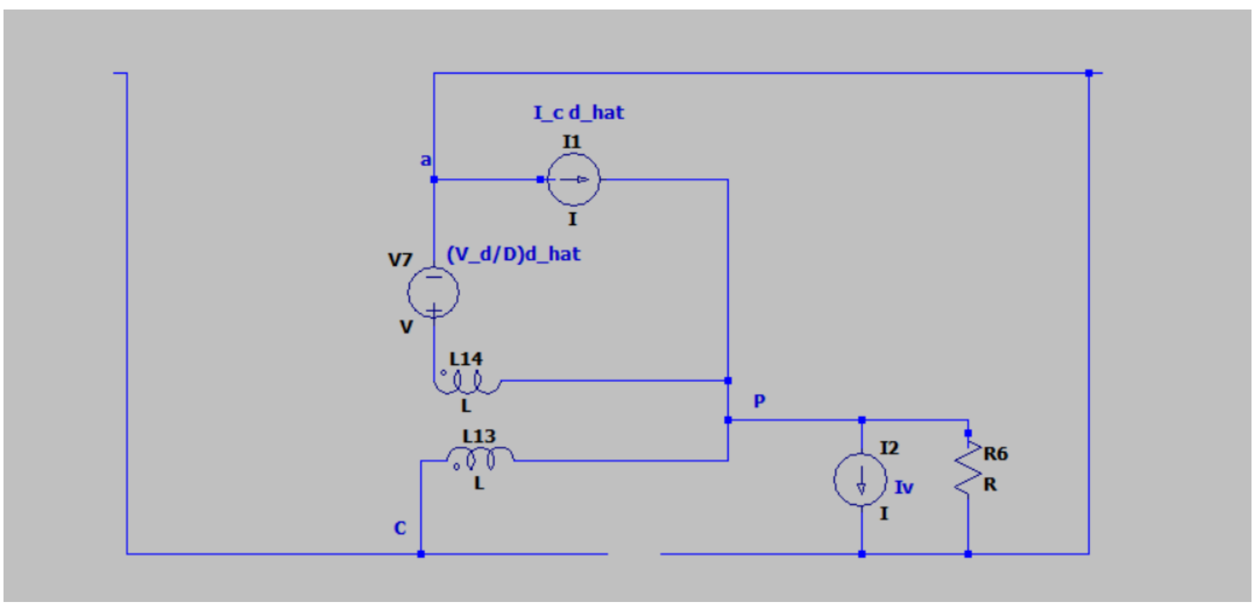
$$R(C_2, C_{1,}, L_2) = 0$$

For $R(L_2, C_2, L_1)$



$$R(L_2, C_2, L_1) = \infty$$

For $R(C_2, L_1, C_1, L_2)$



$$R(C_2, L_1, C_1, L_2) = R$$

Now we got all DEN coefficients

$$a_1 = \frac{L_1}{R\left(\frac{D`}{D}\right)^2} + \frac{L_2}{R}$$

$$a_2 = L_1C_1 + \frac{L_1C_2D^2}{D`^2} + L_2C_1 + L_2C_2$$

$$a_3 = \frac{L_1C_1L_2}{D`^2R}$$

$$a_4 = \frac{L_1C_1L_2C_2}{D`^2}$$

For NEM

$$N(s) = 1 + b_1s + b_2s^2 + b_3s^3 + b_4s^4$$

$$b_1 = \frac{L_1}{R(L_1)} + \frac{L_2}{R(L_2)} + C_1R(C_1) + C_2R(C_2)$$

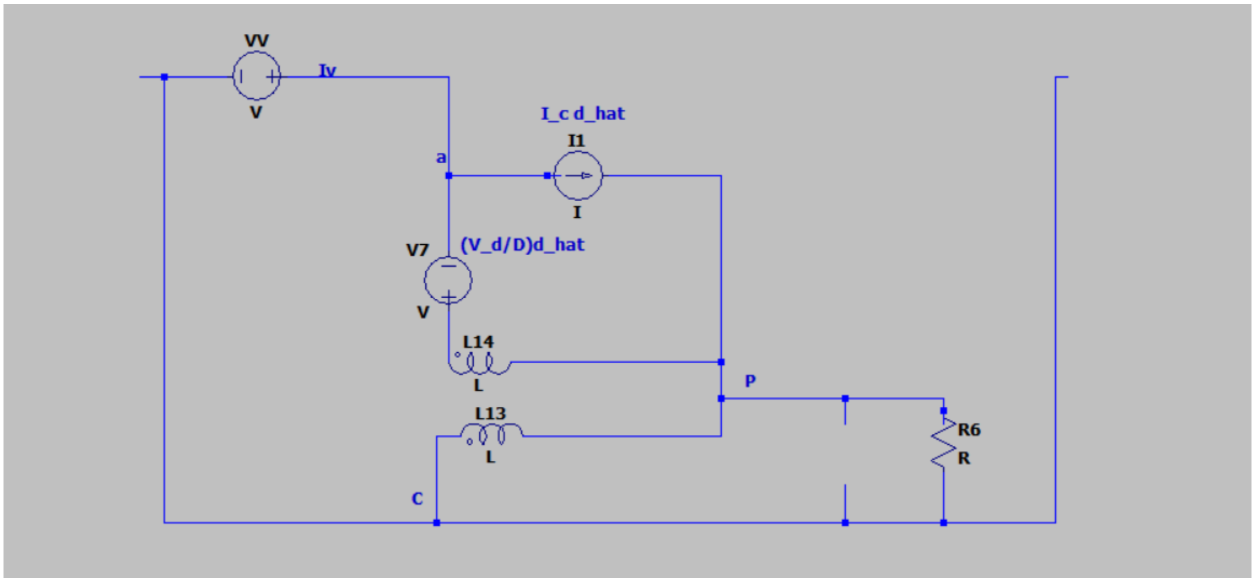
$$b_2 = \frac{L_1}{R(L_1)}\frac{L_2}{R(L_2, L_1)} + \frac{L_1}{R(L_1)}\mathrm{C_1R(C_1, L_1\ )} + \frac{L_1}{R(L_1)}\mathrm{C_2R(C_2, L_1\ )} + + \frac{L_2}{R(L_2)}\mathrm{C_1R(C_1, L_2\ )} + \frac{L_2}{R(L_2)}\mathrm{C_2R(C_2, L_2\ )} + C_1R(C_1)\,C_2R(C_2, C_1)$$

$$b_3 = \frac{L_1}{R(L_1)}\mathrm{C_1R(C_1, L_1\ )}\frac{L_2}{R(L_2, C_1, L_1)} + \frac{L_1}{R(L_1)}\mathrm{C_1R(C_1, L_1\ )}\,C_2R(C_2, C_1, L_1) + \frac{L_2}{R(L_2)}\mathrm{C_1R(C_1, L_2\ )}C_2R(C_2, C_1, L_2) + \frac{L_1}{R(L_1)}\mathrm{C_2R(C_2, L_1\ )}\frac{L_2}{R(L_2, C_2, L_1)}$$

$$b_4 = \frac{L_1}{R(L_1)}\mathrm{C_1R(C_1, L_1\ )}\ \frac{L_2}{R(L_2, C_1, L_1)}\mathrm{C_2R(C_2, L_1, C_1\ , L_2)}$$

To derive the Nem, the output will be nulled.

For $R(L_1)$



$$i_c = i_v, i_a = Di_v$$

$$i_p = i_c - i_a - i_c d^{`} = 0$$

$$i_v = \frac{i_c}{D^{`}} d^{`} = \frac{i_o}{D^{`2}} d^{`} = \frac{V_o}{RD^{`2}} d^{`}$$

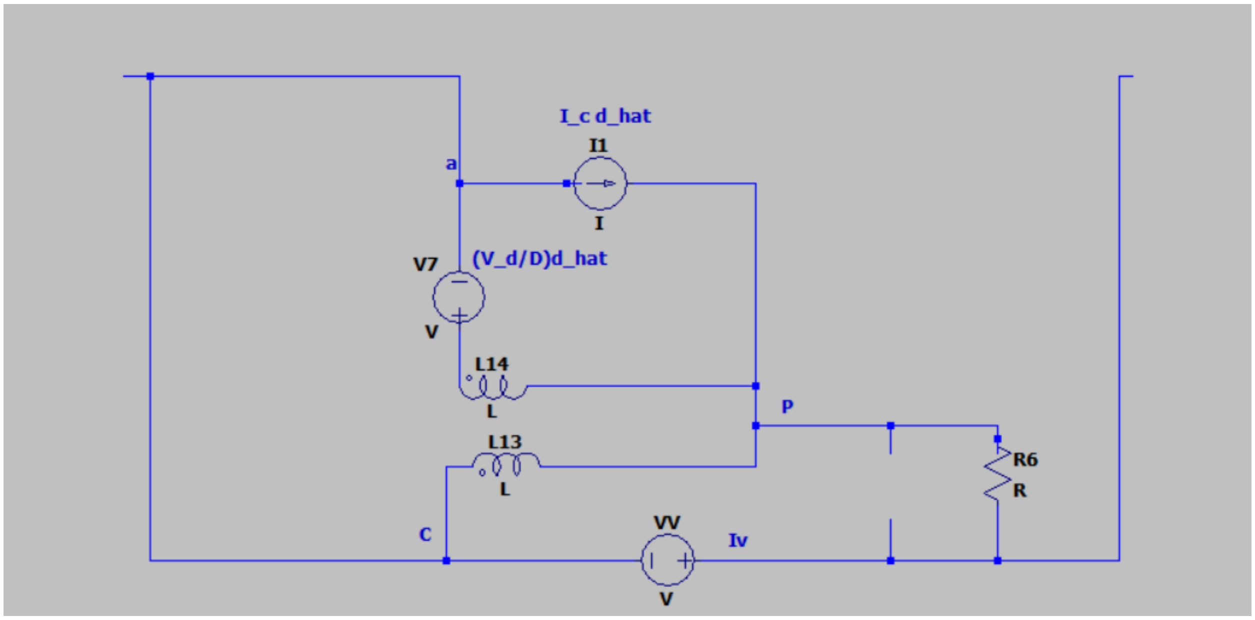
$$V_{ap} = -\frac{V_d}{D} d^{`}$$

$$v_{ac} = v_{vv}, = V_{ap} - V_{cp}$$

$$v_{vv} = -\frac{V_o}{D^2} d^{`}$$

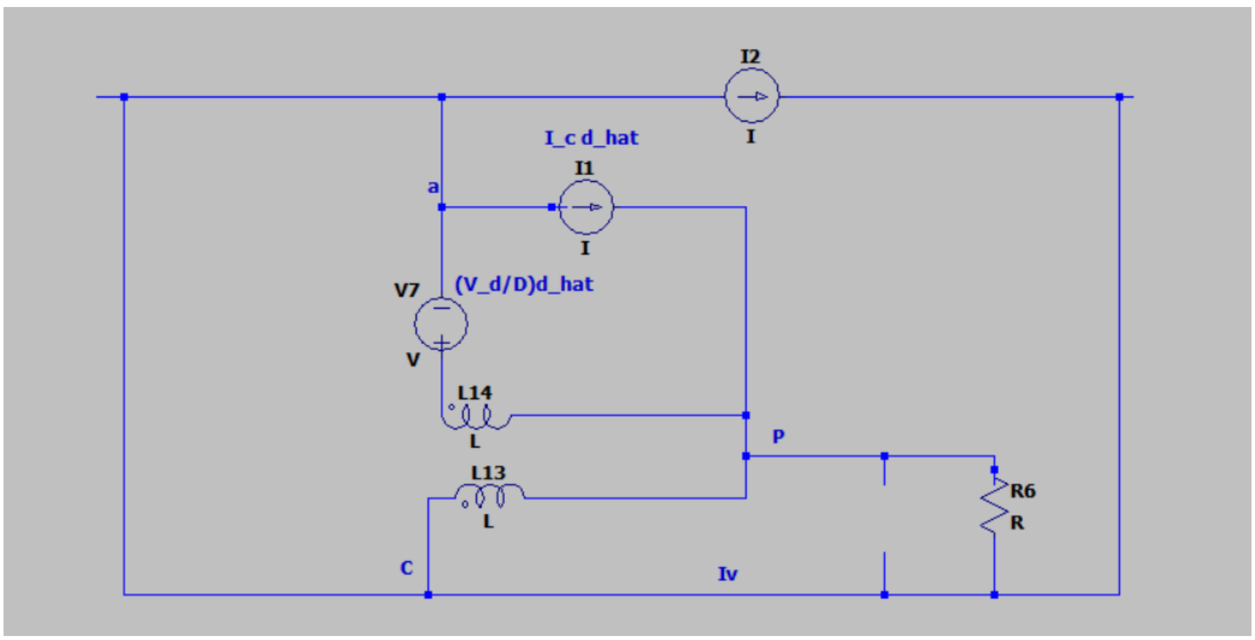
$$R(L_1) = \frac{v_{vv}}{i_v} = -R\left(\frac{D^{`}}{D}\right)^2$$

For $R(L_2)$



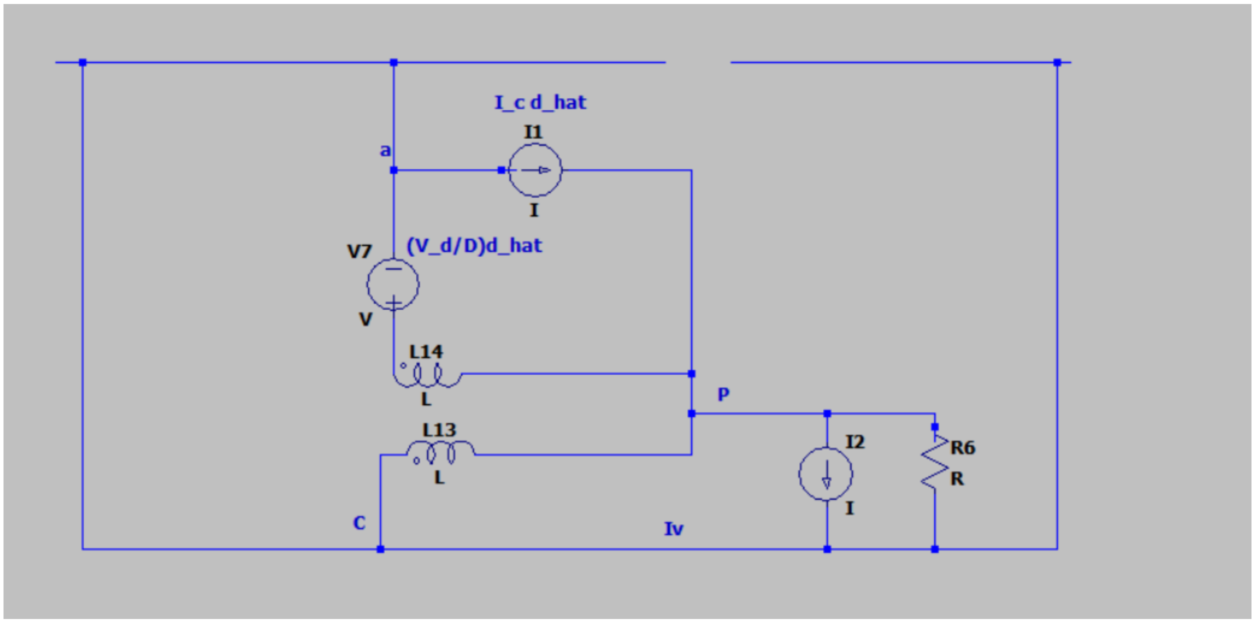
$$R(L_2) = \infty$$

For R(C1)



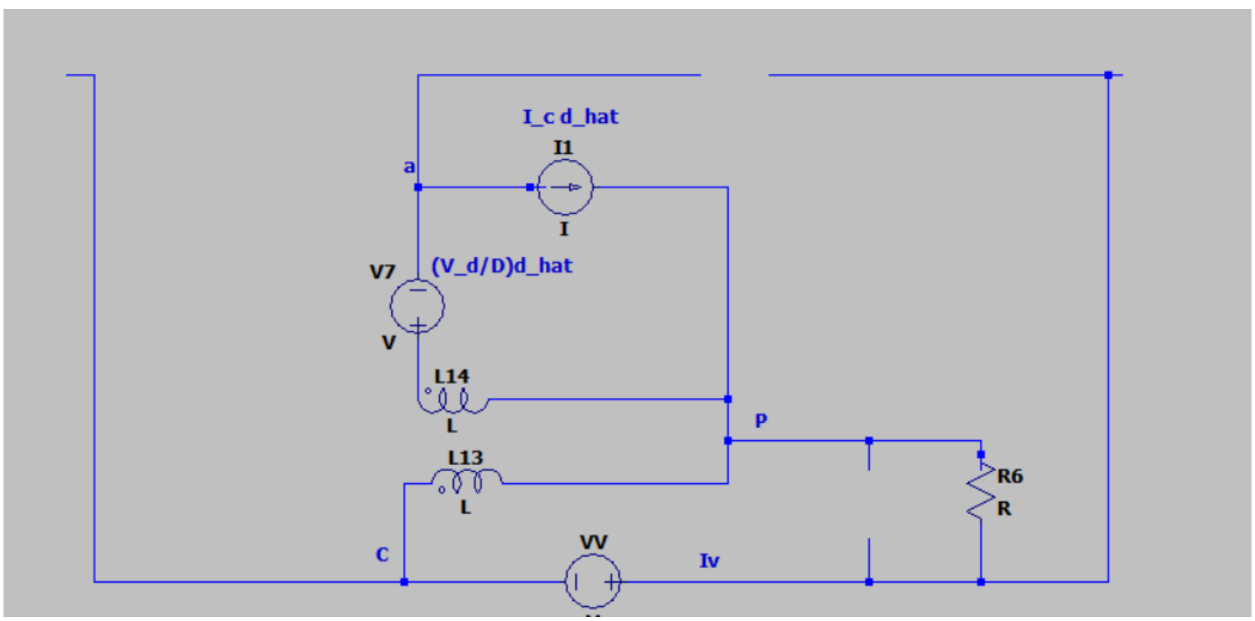
$$R(C_1) = 0$$

For $R(C_2)$



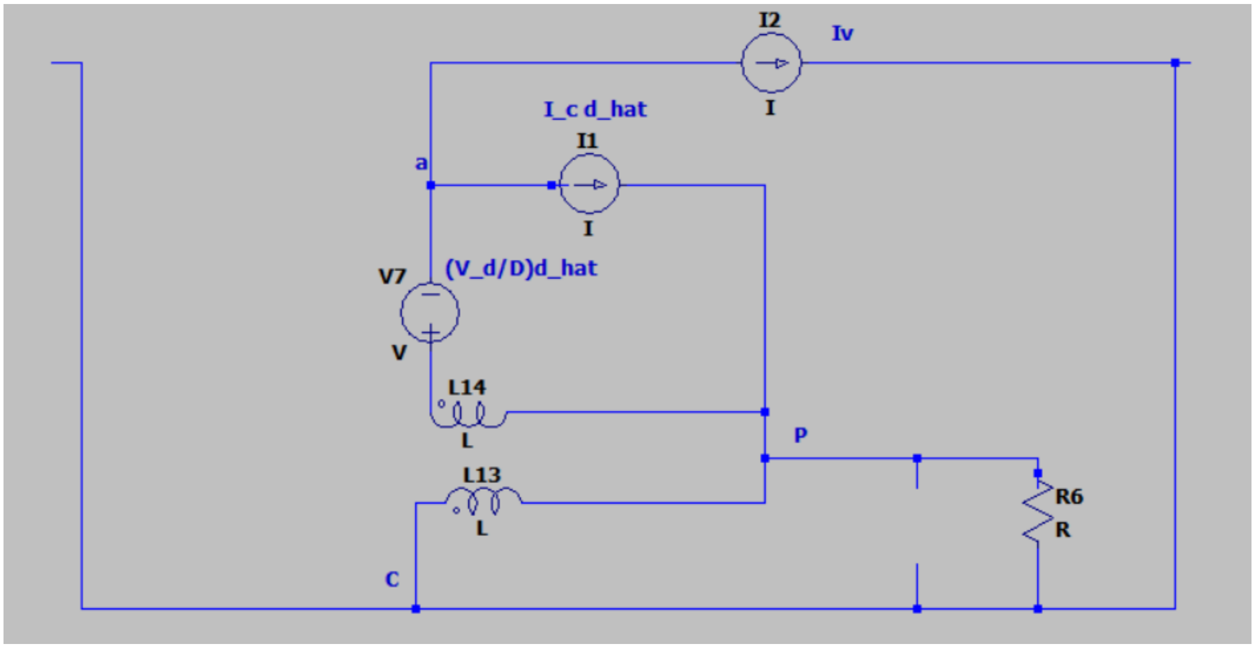
$$R(C_2) = 0$$

For $R(L_2, L_1)$



$$R(L_2, L_1) = \infty$$

For $R(C_1, L_1)$



$$i_c = i_v \, , i_a = D i_v$$

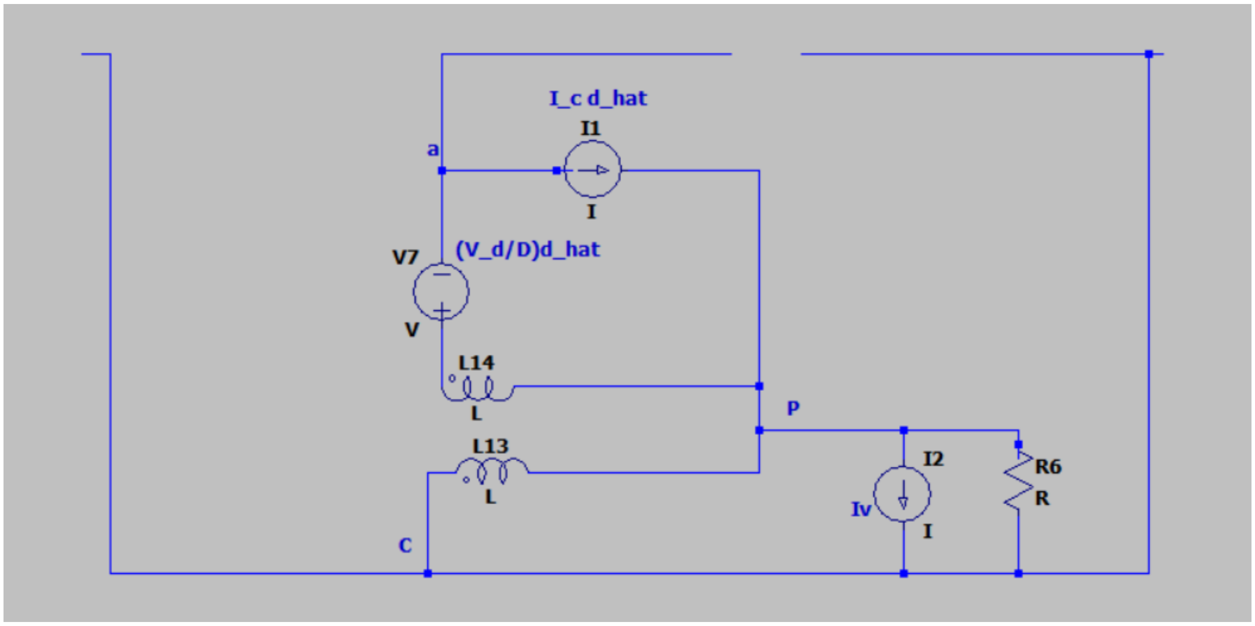
$$V_{ac} = \; -v_{vv}, V_{cp} =$$

$$V_{ap} = -\frac{V_D}{D} d^{`}$$

$$v_{vv} = \frac{V_D}{D} d^{`} = \frac{V_o}{D^2} d^{`}$$

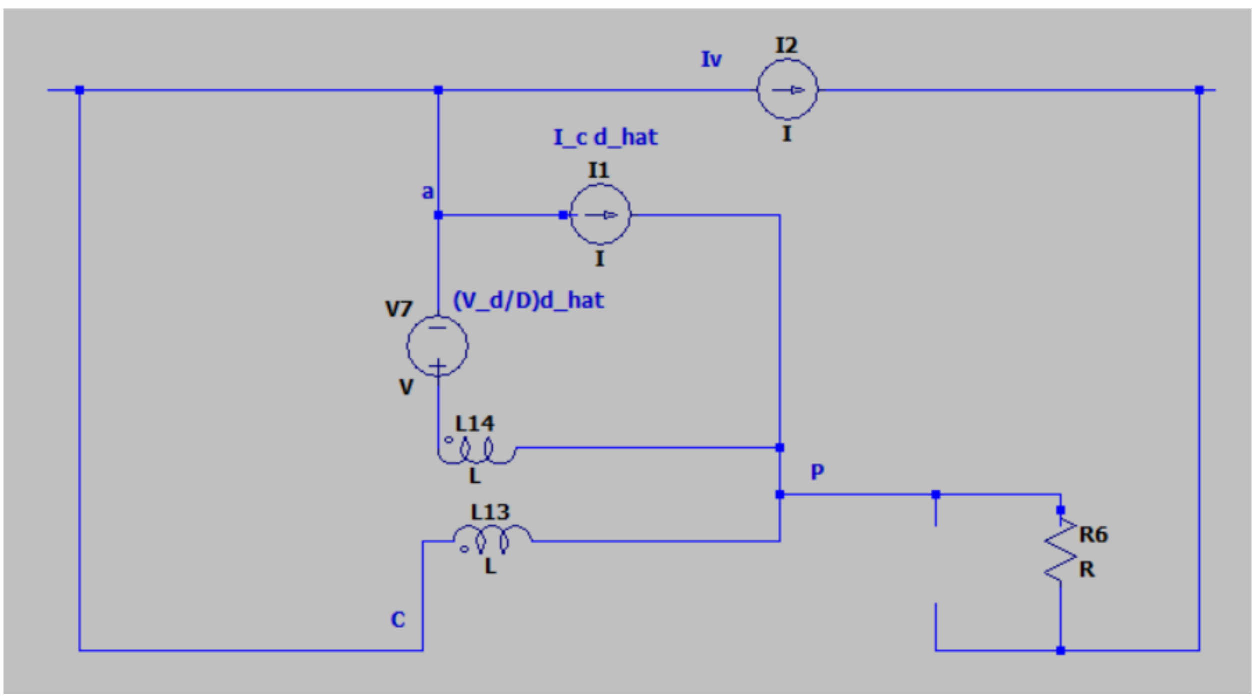
$$R(C_1, L_1) = \; -R \left(\frac{D^{`}}{D}\right)^2$$

For $R(C_2, L_1)$



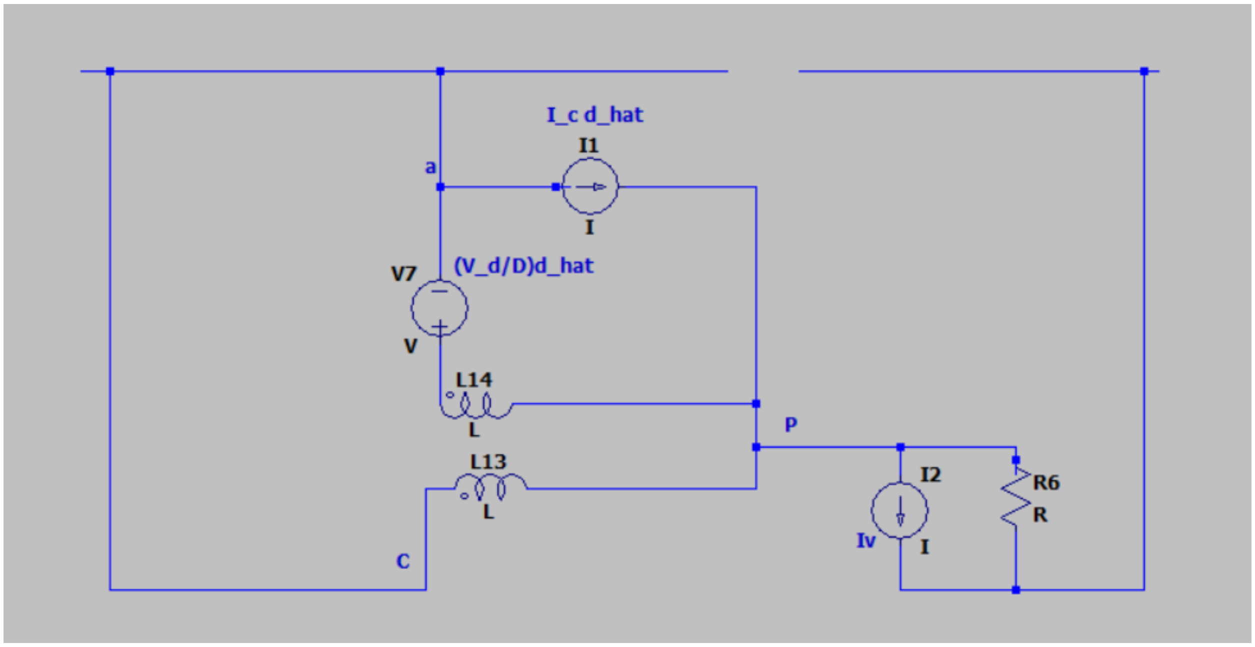
$$R(C_2, L_1) = 0$$

For $R(C_1, L_2)$



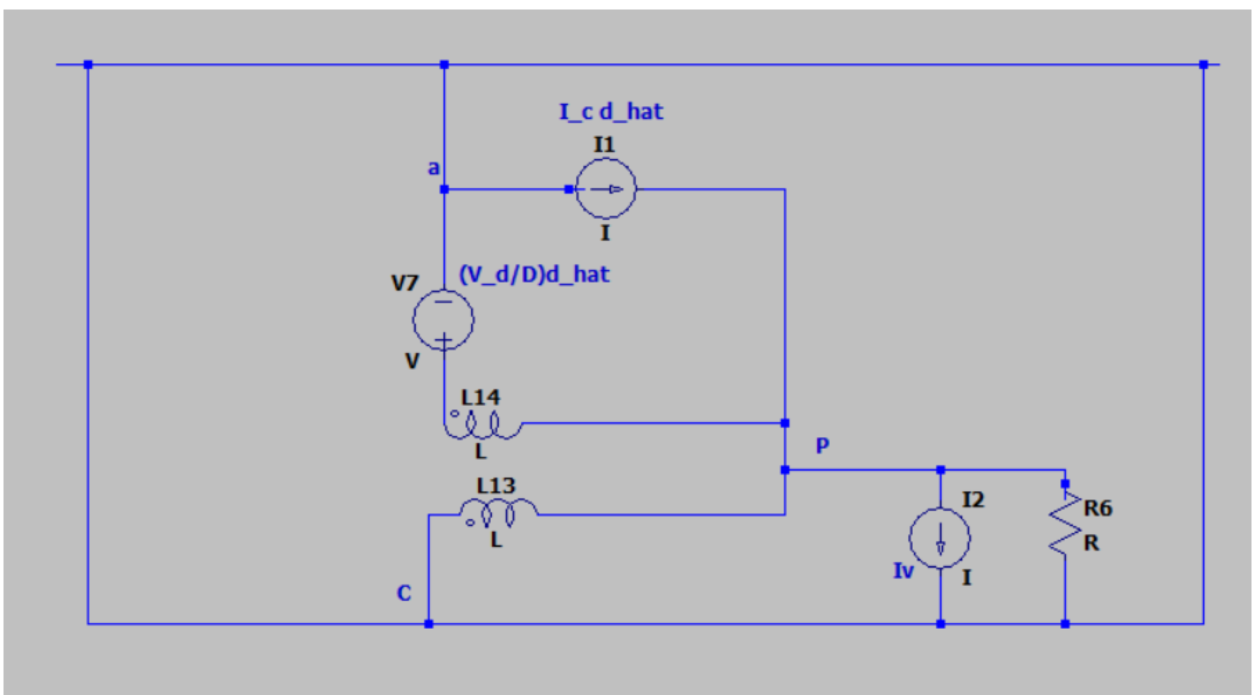
$$R(C_1, L_2) = \infty$$

For $R(C_2, L_2)$



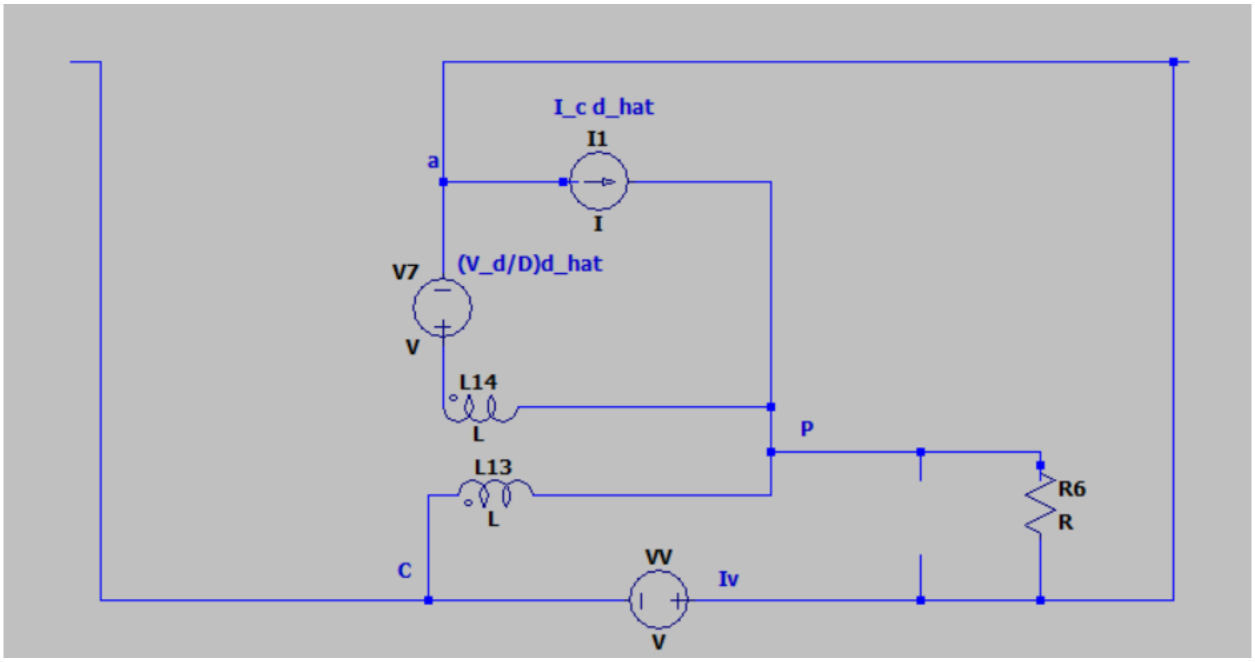
$$R(C_2, L_2) = 0$$

For $R(C_2, C_1)$



$$R(C_2, C_1) = 0$$

For $R(L_2, C_1, L_1)$



$$V_{ac} = v_{vv}\,, V_{cp} = DV_{ap}$$

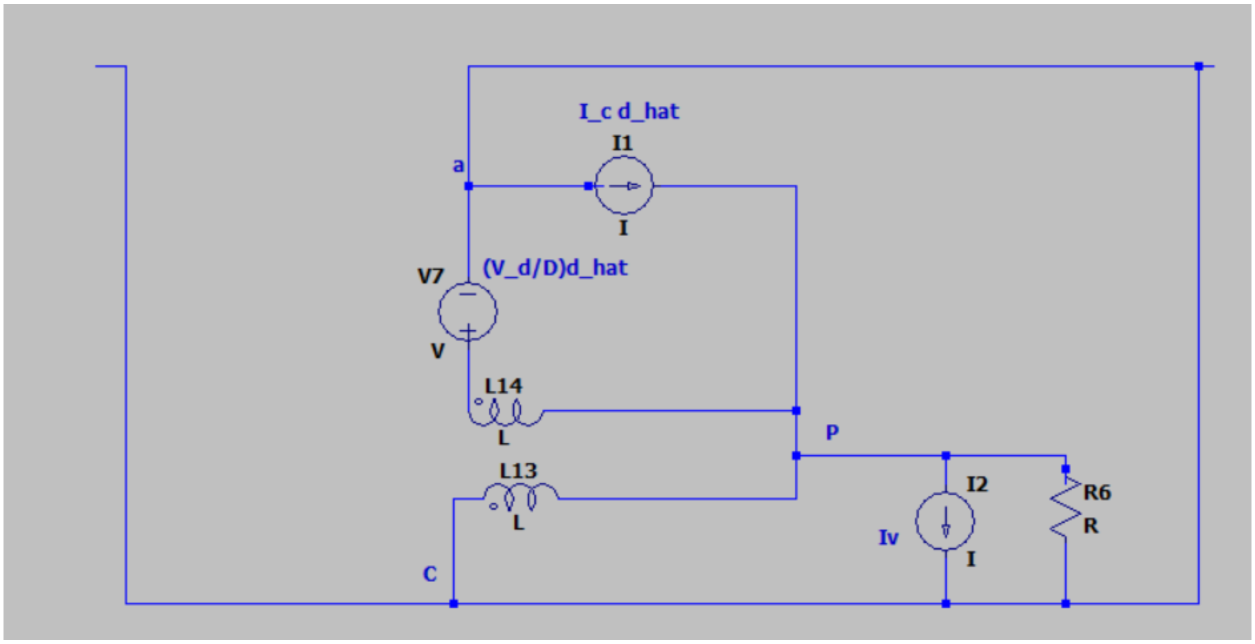
$$V_{ap} = -\frac{V_D}{D} d'$$

$$i_v = \frac{i_c}{D'} d' = \frac{V_o}{RD'^2} d'$$

$$v_{vv} = -V_D d' = -\frac{V_O}{D} d'$$

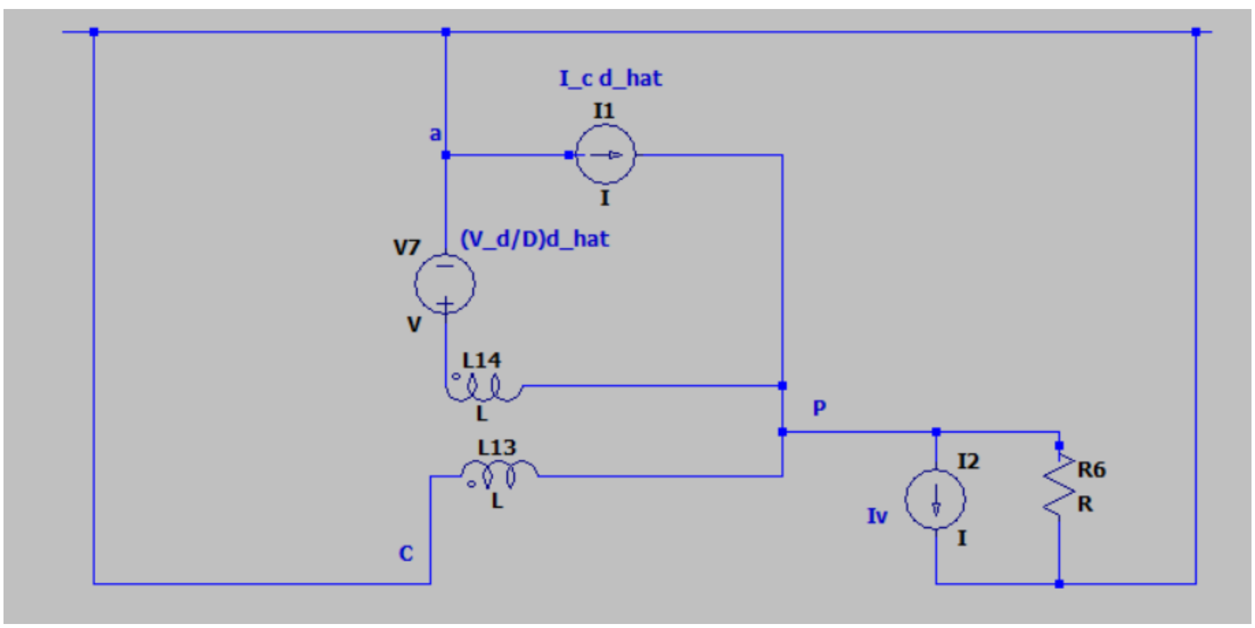
$$R(L_2, C_1, L_1) = -\frac{RD'^2}{D}$$

For $R(C_2, C_1, L_1)$



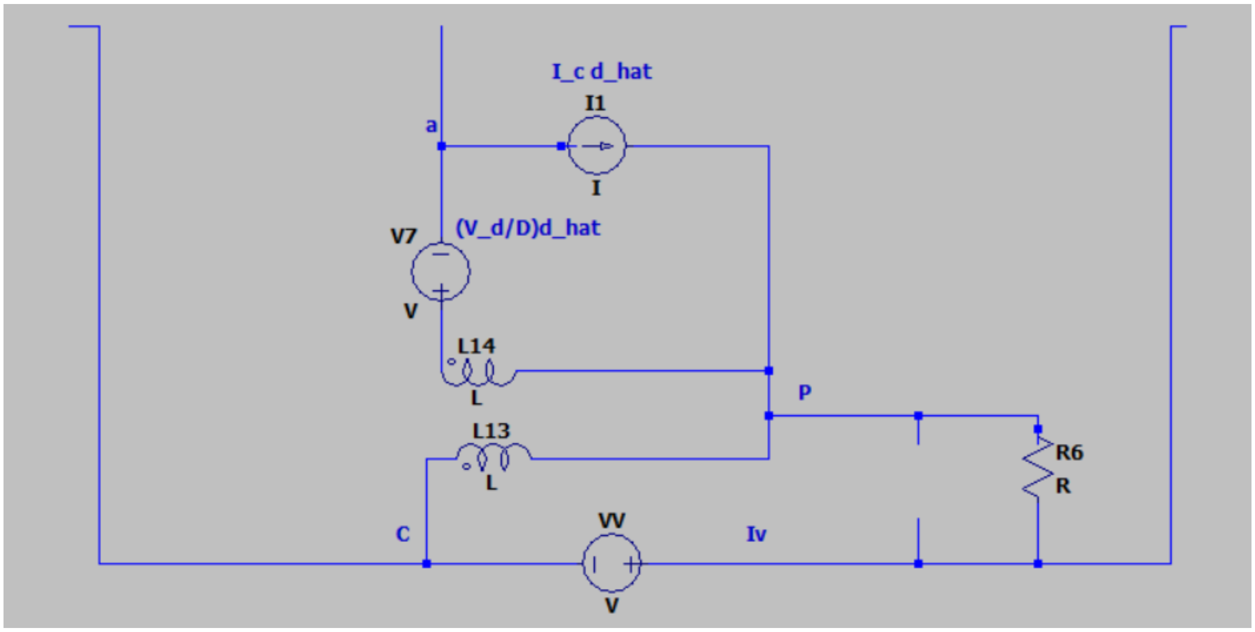
$$R(C_2, C_1, L_1) = 0$$

For $R(C_2, C_1, L_2)$



$$R(C_2, C_1, L_2) = 0$$

For $R(L_2, C_2, L_1)$



$$R(L_2, C_2, L_1) = \infty$$

Then,

$$N(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4$$

$$b_1 = -\frac{L_1}{R\left(\frac{D`}{D}\right)^2}$$

$$b_2 = L_1 C_1 + L_2 C_1$$

$$b_3 = -\frac{L_1 C_1 L_2 D}{R D`^2}$$

$$b_4 = 0$$

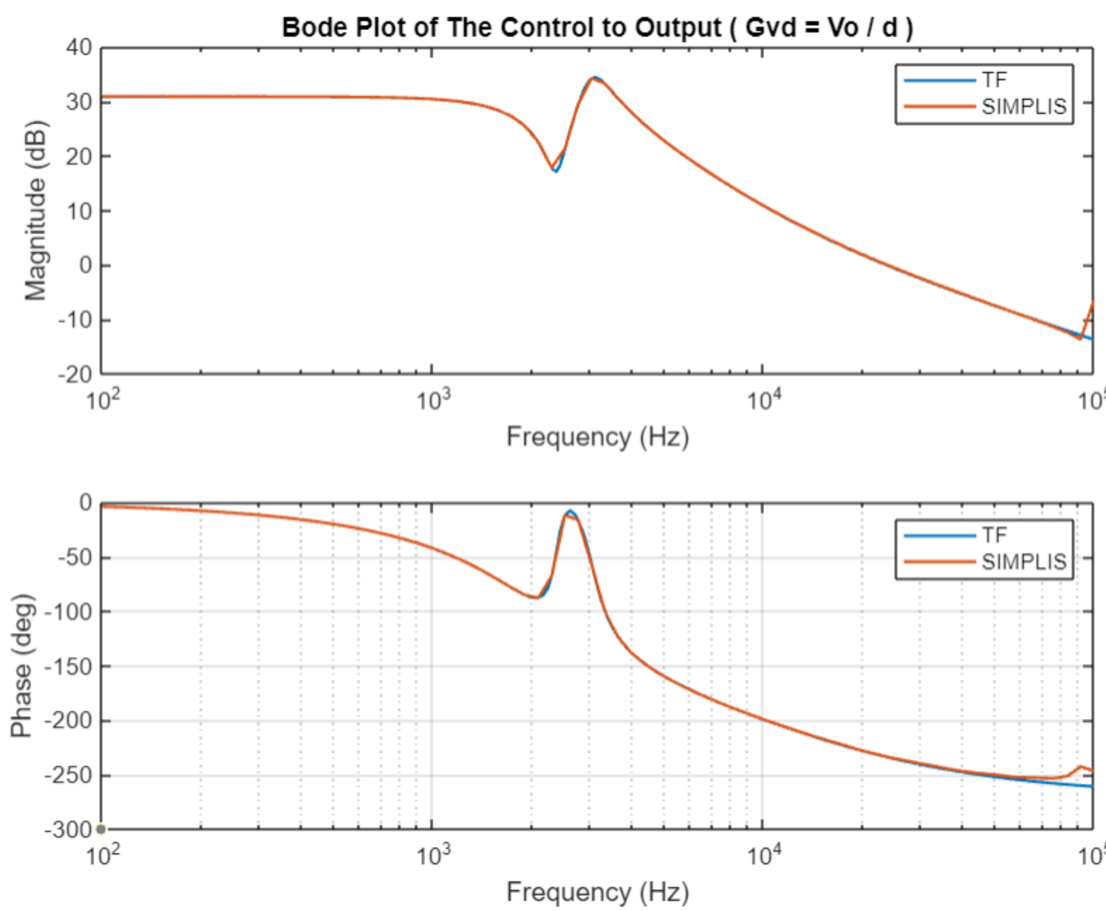
$$G_{vd} = \frac{V_g}{D`^2} \frac{1 - \frac{L_1}{R\left(\frac{D`}{D}\right)^2} s + (L_1 C_1 + L_2 C_1) s^2 - \frac{L_1 C_1 L_2 D}{R D`^2} s^3}{1 + \left(\frac{L_1}{R\left(\frac{D`}{D}\right)^2} + \frac{L_2}{R}\right) s + \left(L_1 C_1 + \frac{L_1 C_2 D^2}{D`^2} + L_2 C_1 + L_2 C_2\right) s^2 + \left(\frac{L_1 C_1 L_2}{D`^2 R}\right) s^3 + \left(\frac{L_1 C_1 L_2 C_2}{D`^2}\right) s^4}$$

$$W_{o1} = \frac{1}{\sqrt{L_1\left(C_2\left(\frac{D}{D`}\right)^2 + C_1\right) + L_2(C_1 + C_2)}}$$

$$Q_{o1} = \frac{R}{W_{o1}\left(L_1\left(\frac{D}{D`}\right)^2 + L_2\right)}$$

$$W_{o2} = \sqrt{\frac{1}{\frac{L_2 C_1}{D^2} || \frac{C_2}{D`^2}} + \frac{1}{L_1 C_1 || C_2}}$$

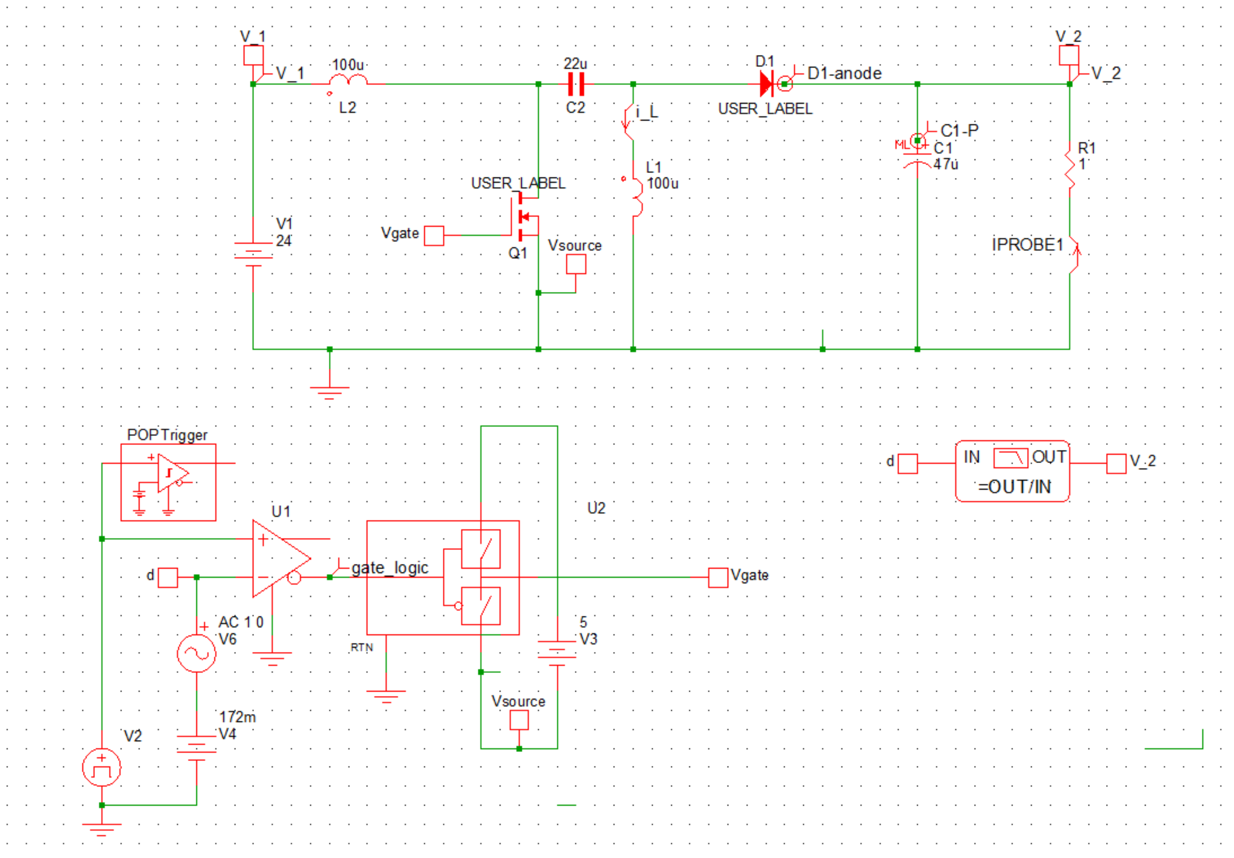
$$Q_{o2} = \frac{R}{\frac{W_{o2}(L_1 + L_2) C_1 W_{o1}^2}{C_2 W_{o2}^2}}$$



/MATLAB Drive/converter/sepic_4.m

```matlab
s =tf('s');
Vg = 24; % input voltage
V = 5; % output voltage
C_1 = 22e-6; % capacitance C1
C_2 = 47e-6; % capacitance C2
L_1 = 100e-6; % inductance L1
L_2 = 100e-6; % inductance L2
R =1; % load resistance
D = V/(V+Vg); % duty cycle


x_1 =  1/((1-D)/D)^2;

x_2 = L_1* C_1+L_2 *C_1;
x_3 = (L_1* C_1* L_2* D)/(R*(1-D)^2 )


a_1 = L_1/(R*((1-D)/D)^2 )+L_2/R;
a_2 = L_2 * C_1 + L_2 *C_2 + L_1 + C_1 + ((L_1 *C_2 *D^2)/(1-D)^2);
a_3 = (L_1* C_1 *L_2)/((1-D)^2 *R);
a_4 = (L_1* C_1 * L_2* C_2)/(1-D)^2 ;


w ={100*2*pi,1000^5*2*pi};

N = 1 - x_1 *s + x_2 * s^2 - x_3 * s^3;

Den = 1+  a_1 *s + a_2 * s^2 + a_3 * s^3 +a_4 * s^4;
G_vdo = Vg/(1-D)^2;
G_vd = G_vdo * (N/Den);
%bode(G_vd,w)
```

```matlab
%bode(G_vd,w)
[mag_1,phase_1,wout]= bode(G_vd,w);
mag_vd=squeeze (mag_1(1,1,:));
ph_vd=squeeze(phase_1(1,1,:));
f_vd=wout/(2*pi);
figure(1);
subplot(2,1,1)
semilogx(f_vd,20*log10(mag_vd),freq_Gvd,Gain_Gvd,'LineWidth',1.2);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Bode Plot of The Control to Output ( Gvd = Vo / d )');
legend('TF','SIMPLIS');
hold on
subplot(2,1,2)
semilogx(f_vd,ph_vd,freq_Gvd,phase_Gvd,'LineWidth',1.2);
xlabel('Frequency (Hz)');
ylabel('Phase (deg)');
legend('TF','SIMPLIS');
grid on;
 hold on
```



Effect of RL:

1- DC gain: it has no effect on DC gain.
2- Poles:

$Q_{o1} and\ Q_{o2}$ are directly proportional with R_L, this is obvious with simulation.

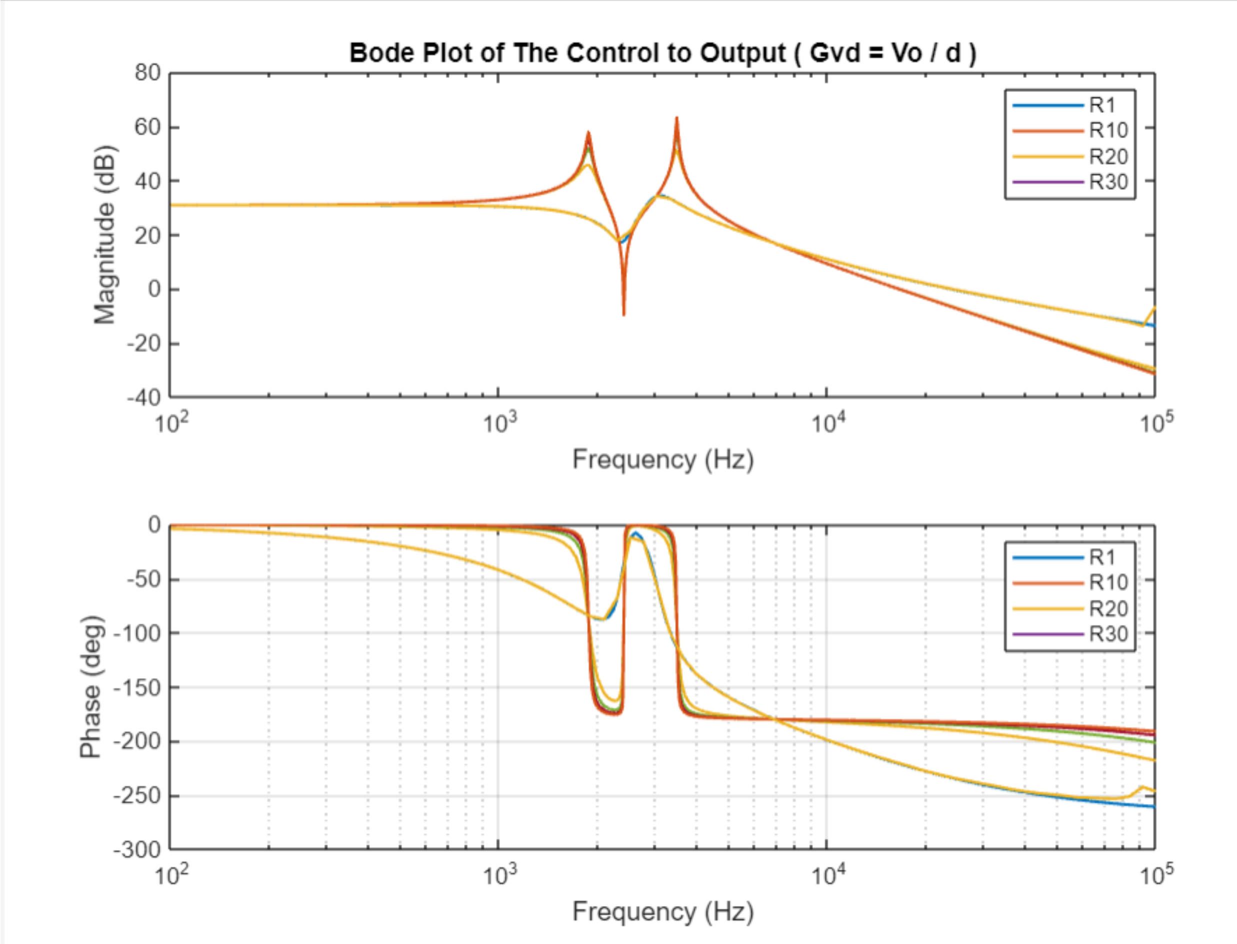
3- Zeros:

FOR RHP zero: $1 - s\frac{L_1 D^2}{RD'^2}$

Increasing the R will lead to increase in W_{rhp} which means the effect of RHP will disappear.

FOR W_z

This means W_z is not depending on R, and Q_z till infinity will increase which leads to decrease the damping.



Effect of L_2:

1- DC gain: there is not affect and independent.

Poles:

W_{o1}: decreases with increase of L_2

$$W_{o1} = \frac{1}{\sqrt{L_1\left(C_2\left(\frac{D}{D'}\right)^2 + C_1\right) + L_2(C_1 + C_2)}}$$

Which means lower bandwidth and reduced stability margin and reduces the overall gain. Consequently, the output voltage (V) will decrease relative to the input voltage (V_g) for a given duty cycle (D).

W_{02} : decreases with increase of L_2

$$W_{o2} = \sqrt{\frac{1}{\frac{L_2C_1}{D^2}||\frac{C_2}{D`^2}} + \frac{1}{L_1C_1||C_2}}$$

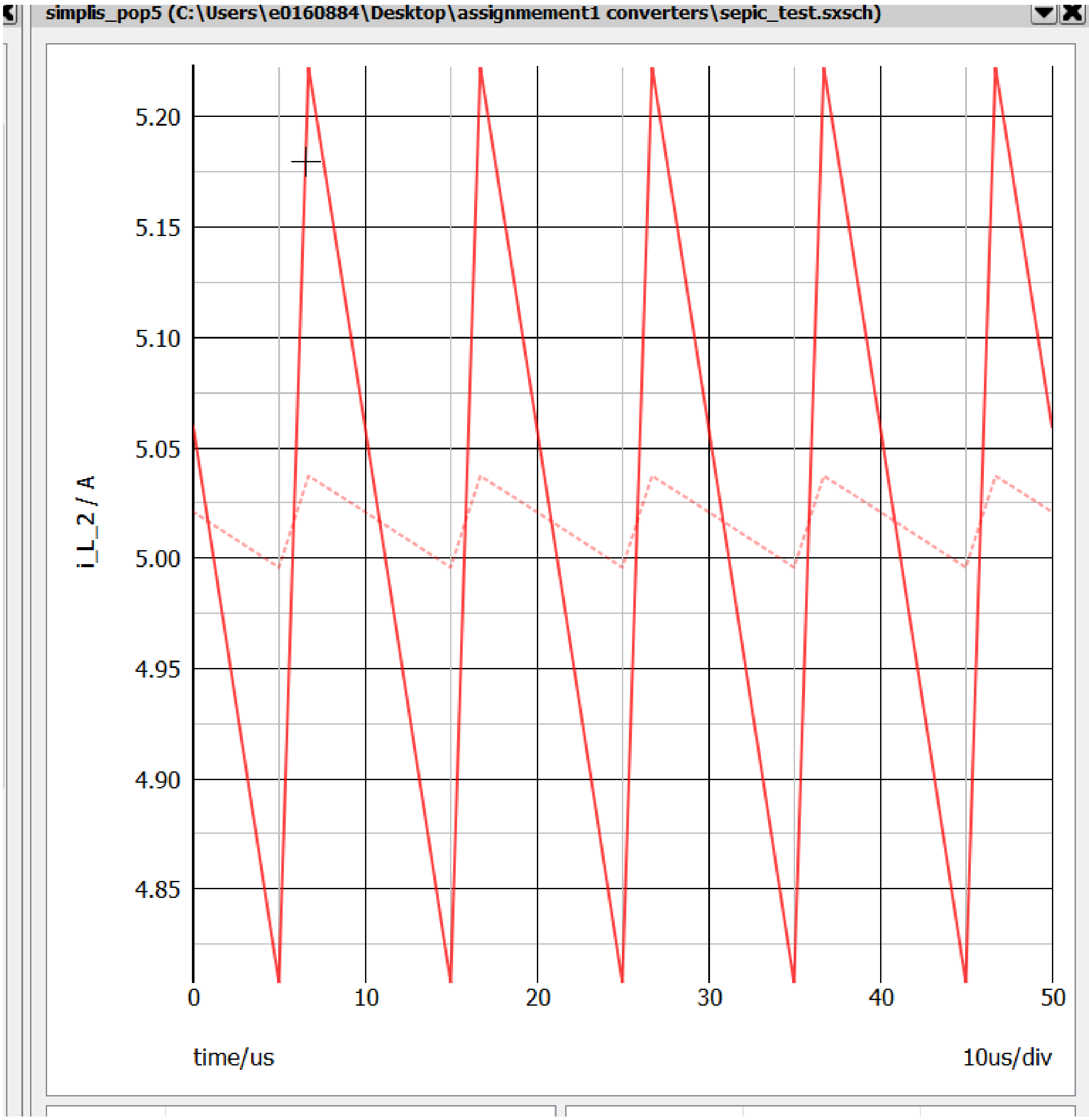
Which means lower bandwidth and reduced stability margin and reduces the overall gain. Consequently, the output voltage (V) will decrease relative to the input voltage (V_g) for a given duty cycle (D).

Q_{o1}: decreases with increase of L_2

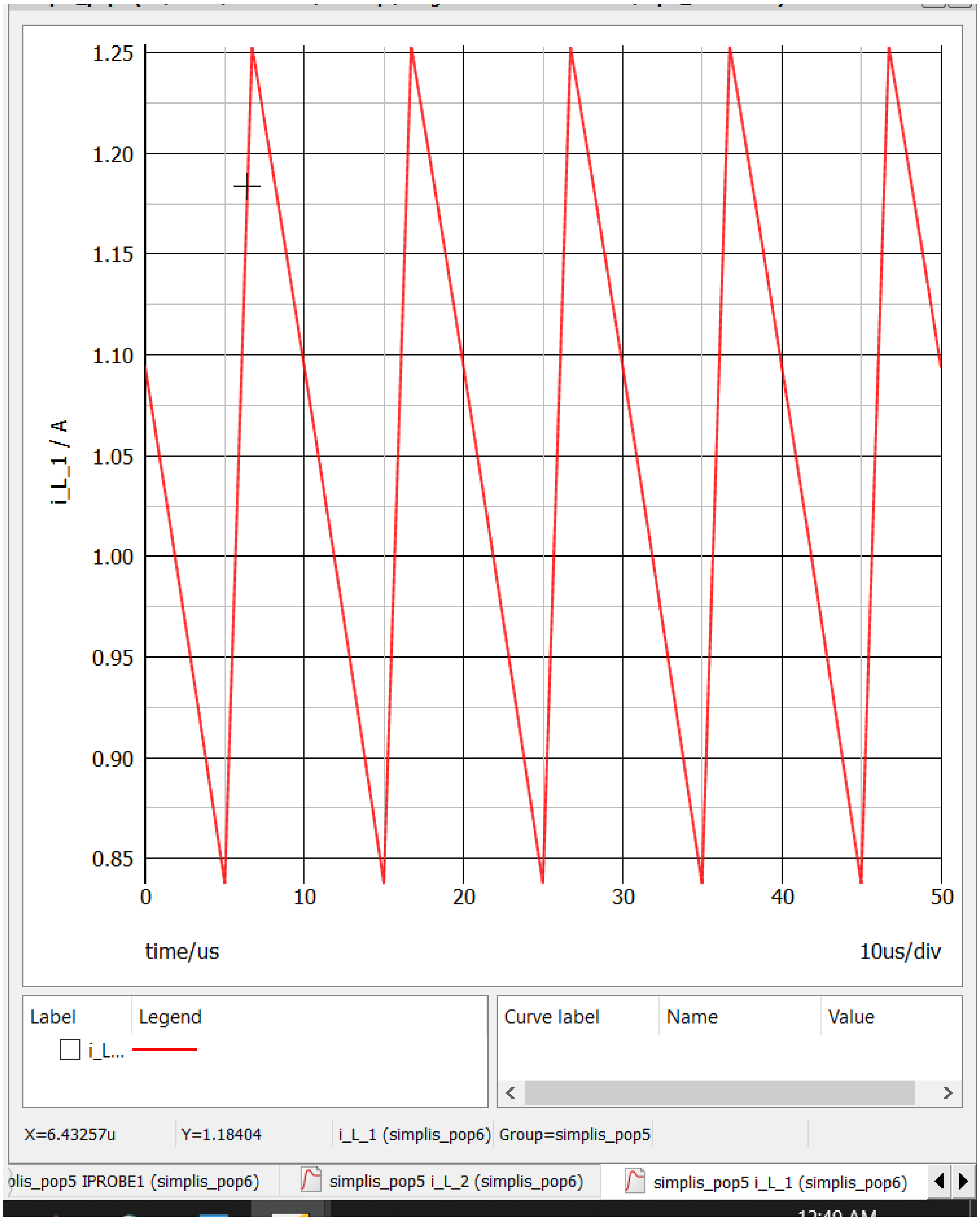
$$Q_{o1} = \frac{R}{W_{o1}\left(L_1\left(\frac{D}{D`}\right)^2 + L_2\right)}$$

Q_{o2}: decreases with increase of L_2

$$Q_{o2} = \frac{R}{\frac{W_{o2}(L_1 + L_2)C_1W_{o1}^2}{C_2W_{o2}^2}}$$



The dashed output current for L_2= 1mH and solid for $L_2 = 100uH$

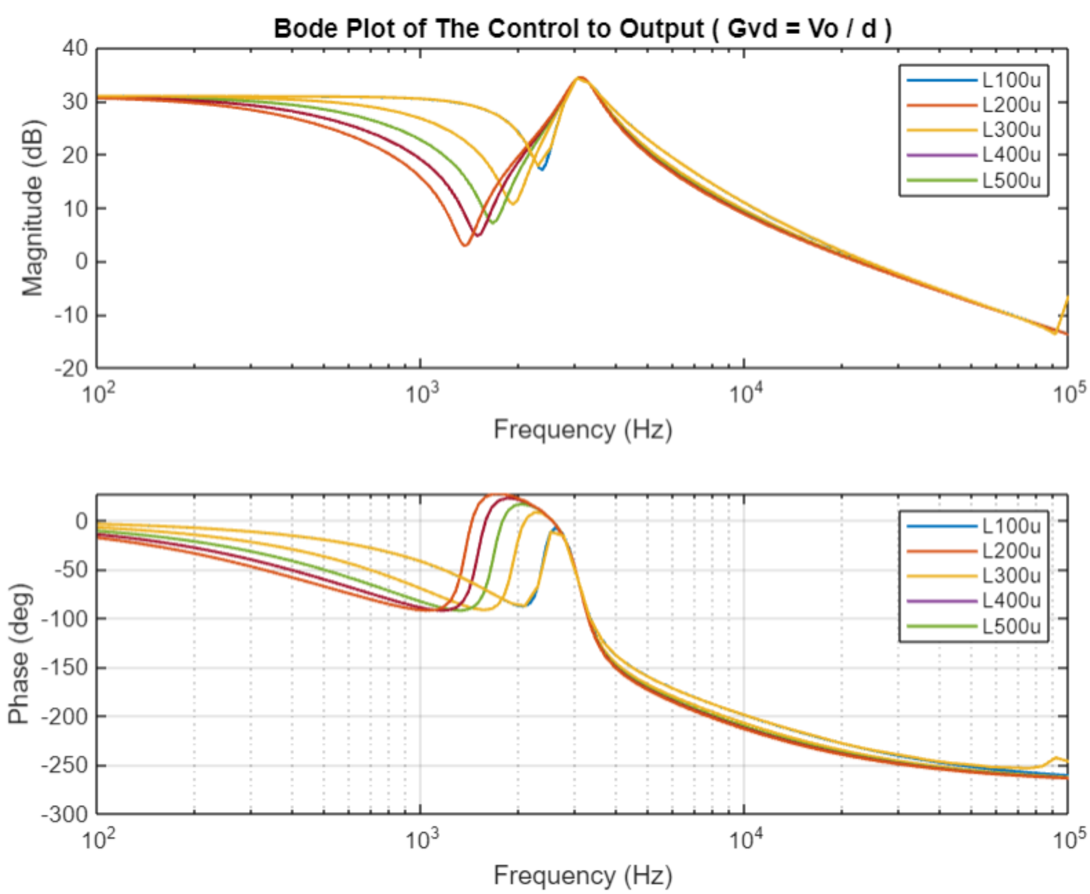


1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
i_L_1 / A
0
10
20
30
40
50
time/us
10us/div
Label
Legend
i_L...
Curve label
Name
Value
X=6.43257u
Y=1.18404
i_L_1 (simplis_pop6)
Group=simplis_pop5
olis_pop5 IPROBE1 (simplis_pop6)
simplis_pop5 i_L_2 (simplis_pop6)
simplis_pop5 i_L_1 (simplis_pop6)

Effect on Zeros:

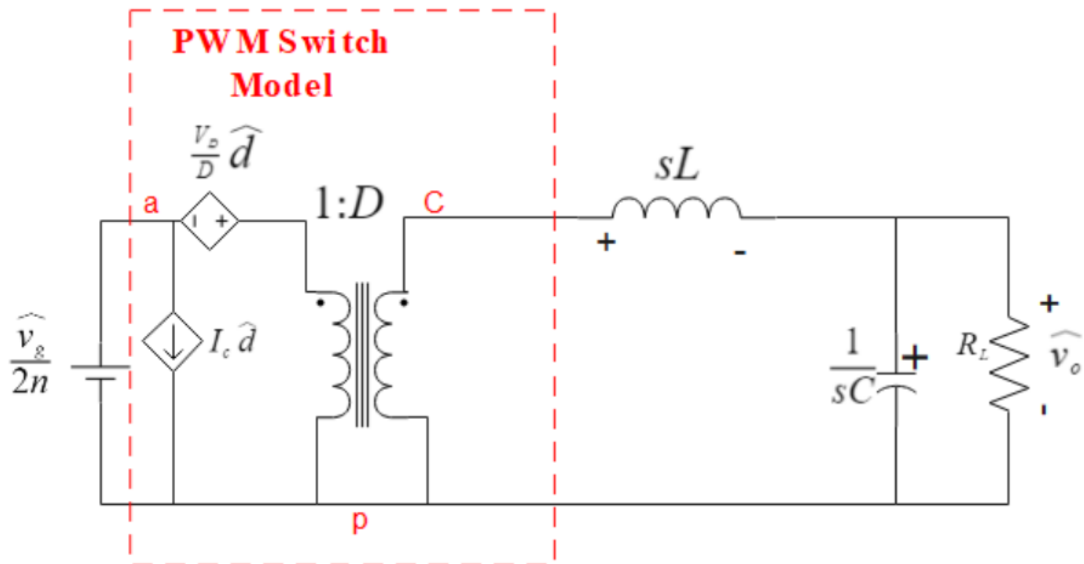
Increasing L_2 will lead to

1- W_z decreases with increase of L_2
2- W_{rhp} decreases with increase of L_2
3- It's also visible the effect of double zero from the following simulation and its effect on phase margin.



Q2

1- For Half bridge converter
   After solving the Half bridge converter, We got the equivalent Buck converter so:
   - Mosfets $Q1$ and $Q2$ is equivalent to the Buck converter switches. So, the duty for the buck switch is the total of the two switches $D = \frac{2T_1}{Ts}$
   - The input voltage of the buck converter is equal to $\frac{Vg}{2n}$ this because the primary voltage of the transformer equal to $\frac{V_g}{2}$ and then the transformer ratio $\frac{1}{n}$
   - D_1 and D_2 are equivalent to the Buck converter diode.



Operation steps:

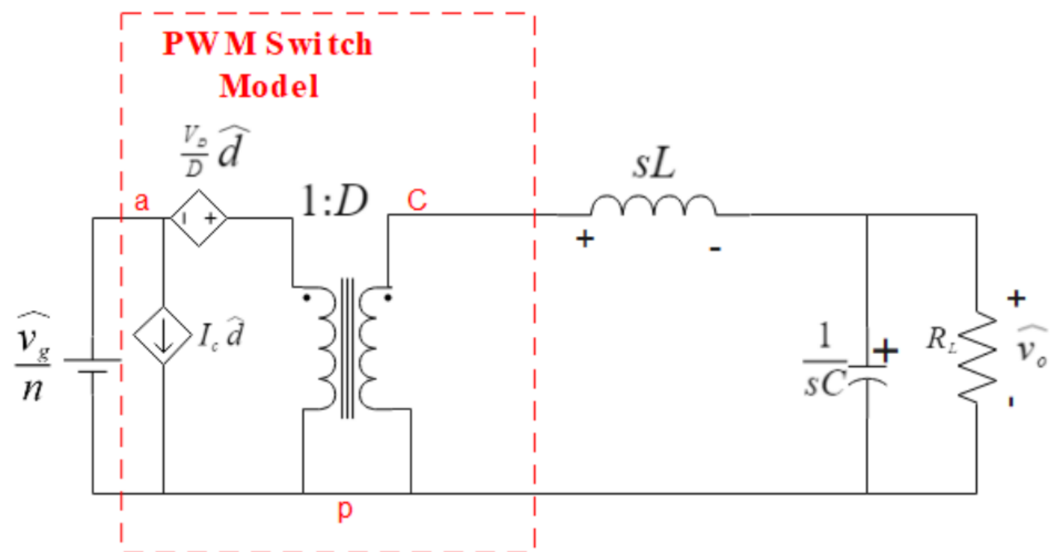
1- C_a and C_b are voltage source and each one equal $\frac{V_g}{2}$
2- Mosfet Q1, and Diode D3 are on, and Mosfet Q2 and Diode D4 are off.
3- Mosfets Q1 & Q2 are off and diodes D3 & D4 are on. The current is freewheeling through both diodes so the supply is disconnected from the load.
4- Mosfet Q2, and Diode D4 are on, and Mosfet Q1, and Diode D3 are off.
5- Mosfets Q1 & Q2 are off and diodes D3 & D4 are on. The current is freewheeling through both diodes so the supply is disconnected from the load.

2- Phase Shift Full Bridge Converter:

For the Phase Shift Full Bridge Converter, we got the equivalent circuit for buck converter and found:

- The input voltage of the buck converter is equal to $\frac{Vg}{n}$ this because the primary voltage of the transformer equal to V_g and then the transformer ratio $\frac{1}{n}$
- D_5 and D_6 are equivalent to the Buck converter diode.
- Mosfets network is equivalent to the Buck converter switch. But the duty for the buck switch is the total of the on periods $D = \frac{2T_1}{T_s}$.

-



Q3

1- For comparison:

Similarities: the two methods give a small signal model that is only valid for $frequncies \leq \frac{F_{sw}}{2}$

Differences:

For the three terminal-PWM Switch we replace the Switch diode network with the equivalent model point by point so the averaging is done only on the switch diode network.

For state space model we make averaging on the circuit during on time and off time.

2- The impact of switching ripple is not considered in small signal model in DCM. As the moving average of the inductor voltage waveform is always zero or approximately zero, and practically, the high-frequency inductor dynamics can usually be neglected in DCM. Therefore, the peak inductor current in the DCM is considered constant regardless the switching ripples and modulated switching input. The model deals with the peak inductor current I_{pk}as a DC operating point that is not affected by switching ripples. All calculations are based on the average inductor current. This assumes that the modulation signal (perturbations) is slower than the switching frequency.

$i_a = \frac{i_{pk}}{2} d \; and \; i_p = \frac{i_{pk}}{2} d$ only the peak current is changing as a representation of the load.

3- Switching frequency influence the small signal model, as the small signal model is based on frequencies $\frac{F_{sw}}{2}$ and assuming the system dynamics is below F_{sw}.