

The Witnessing Problem

UserID	Name	Job	Salary	maxima
123	Jack	TA	50000	60000
345	Allison	TA	60000	60000
567	Magda	Prof	90000	100000
789	Dan	Prof	100000	100000

Return the person with the highest salary for each job type

The Witnessing Problem

UserID	Name	Job	Salary	maxima
123	Jack	TA	50000	60000
345	Allison	TA	60000	60000
567	Magda	Prof	90000	100000
789	Dan	Prof	100000	100000

Return the person with the highest salary for each job type

```
SELECT P1.Name, MAX(P2.Salary)
FROM Payroll AS P1, Payroll AS P2
WHERE P1.Job = P2.Job
GROUP BY P2.Job, P1.Salary, P1.Name
HAVING P1.Salary = MAX(P2.Salary)
```

The Witnessing Problem

UserID	Name	Job	Salary	maxima
123	Jack	TA	50000	60000
345	Allison	TA	60000	60000
567	Magda	Prof	90000	100000
789	Dan	Prof	100000	100000

Return the person with the highest salary for each job type

the maxima

```
SELECT P1.Name, MAX(P2.Salary)
FROM Payroll AS P1, Payroll AS P2
WHERE P1.Job = P2.Job
GROUP BY P2.Job, P1.Salary, P1.Name
HAVING P1.Salary = MAX(P2.Salary)
```

The Witnessing Problem

```
SELECT P1.Name, MAX(P2.Salary)
  FROM Payroll AS P1, Payroll AS P2
 WHERE P1.Job = P2.Job
 GROUP BY P2.Job, P1.Salary, P1.Name
 HAVING P1.Salary = MAX(P2.Salary)
```

Join on “original”
grouping attributes

P1

UserID	Name	Job	Salary
123	Jack	TA	50000
123	Jack	TA	50000
345	Allison	TA	60000
345	Allison	TA	60000
567	Magda	Prof	90000
567	Magda	Prof	90000
789	Dan	Prof	100000
789	Dan	Prof	100000

P2

UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	TA	60000
345	Allison	TA	60000
123	Jack	TA	50000
567	Magda	Prof	90000
789	Dan	Prof	100000
789	Dan	Prof	100000
567	Magda	Prof	90000

The Witnessing Problem

```
SELECT P1.Name, MAX(P2.Salary)
  FROM Payroll AS P1, Payroll AS P2
 WHERE P1.Job = P2.Job
  GROUP BY P2.Job, P1.Salary, P1.Name
HAVING P1.Salary = MAX(P2.Salary)
```

Group on additional attributes that you are argmax-ing for

P1

P2

UserID	Name	Job	Salary	UserID	Name	Job	Salary
123	Jack	TA	50000	123	Jack	TA	50000
123	Jack	TA	50000	345	Allison	TA	60000
345	Allison	TA	60000	345	Allison	TA	60000
345	Allison	TA	60000	123	Jack	TA	50000
567	Magda	Prof	90000	567	Magda	Prof	90000
567	Magda	Prof	90000	789	Dan	Prof	100000
789	Dan	Prof	100000	789	Dan	Prof	100000
789	Dan	Prof	100000	567	Magda	Prof	90000

The Witnessing Problem

```
SELECT P1.Name, MAX(P2.Salary)
  FROM Payroll AS P1, Payroll AS P2
 WHERE P1.Job = P2.Job
  GROUP BY P2.Job, P1.Salary, P1.Name
  HAVING P1.Salary = MAX(P2.Salary)
```

Group on additional attributes that you are argmax-ing for

P1				P2			
UserID	Name	Job	Salary	UserID	Name	Job	Salary
123	Jack	TA	50000	123	Jack	TA	50000
123	Jack	TA	50000	345	Allison	TA	60000
345	Allison	TA	60000	345	Allison	TA	60000
345	Allison	TA	60000	123	Jack	TA	50000
567	Magda	Prof	90000	567	Magda	Prof	90000
567	Magda	Prof	90000	789	Dan	Prof	100000
789	Dan	Prof	100000	789	Dan	Prof	100000
789	Dan	Prof	100000	567	Magda	Prof	90000

The Witnessing Problem

```
SELECT P1.Name, MAX(P2.Salary)
  FROM Payroll AS P1, Payroll AS P2
 WHERE P1.Job = P2.Job
  GROUP BY P2.Job, P1.Salary, P1.Name
  HAVING P1.Salary = MAX(P2.Salary)
```

Group on additional attributes that you are argmax-ing for

P1				P2			
UserID	Name	Job	Salary	UserID	Name	Job	Salary
123	Jack	TA	50000	123	Jack	TA	50000
123	Jack	TA	50000	345	Allison	TA	60000
345	Allison	TA	60000	345	Allison	TA	60000
345	Allison	TA	60000	123	Jack	TA	50000
567	Magda	Prof	90000	567	Magda	Prof	90000
567	Magda	Prof	90000	789	Dan	Prof	100000
789	Dan	Prof	100000	789	Dan	Prof	100000
789	Dan	Prof	100000	567	Magda	Prof	90000

The Witnessing Problem

```
SELECT P1.Name, MAX(P2.Salary)
  FROM Payroll AS P1, Payroll AS P2
 WHERE P1.Job = P2.Job
 GROUP BY P2.Job, P1.Salary, P1.Name
HAVING P1.Salary = MAX(P2.Salary)
```

P1

P2

UserID	Name	Job	Salary	UserID	Name	Job	Salary
123	Jack	TA	50000	123	Jack	TA	50000
123	Jack	TA	50000	345	Allison	TA	60000
345	Allison	TA	60000	345	Allison	TA	60000
345	Allison	TA	60000	123	Jack	TA	50000
567	Magda	Prof	90000	567	Magda	Prof	90000
567	Magda	Prof	90000	789	Dan	Prof	100000
789	Dan	Prof	100000	789	Dan	Prof	100000
789	Dan	Prof	100000	567	Magda	Prof	90000

The Witnessing Problem

```
SELECT P1.Name, MAX(P2.Salary)
FROM Payroll AS P1, Payroll AS P2
WHERE P1.Job = P2.Job
GROUP BY P2.Job, P1.Salary, P1.Name
HAVING P1.Salary = MAX(P2.Salary)
```

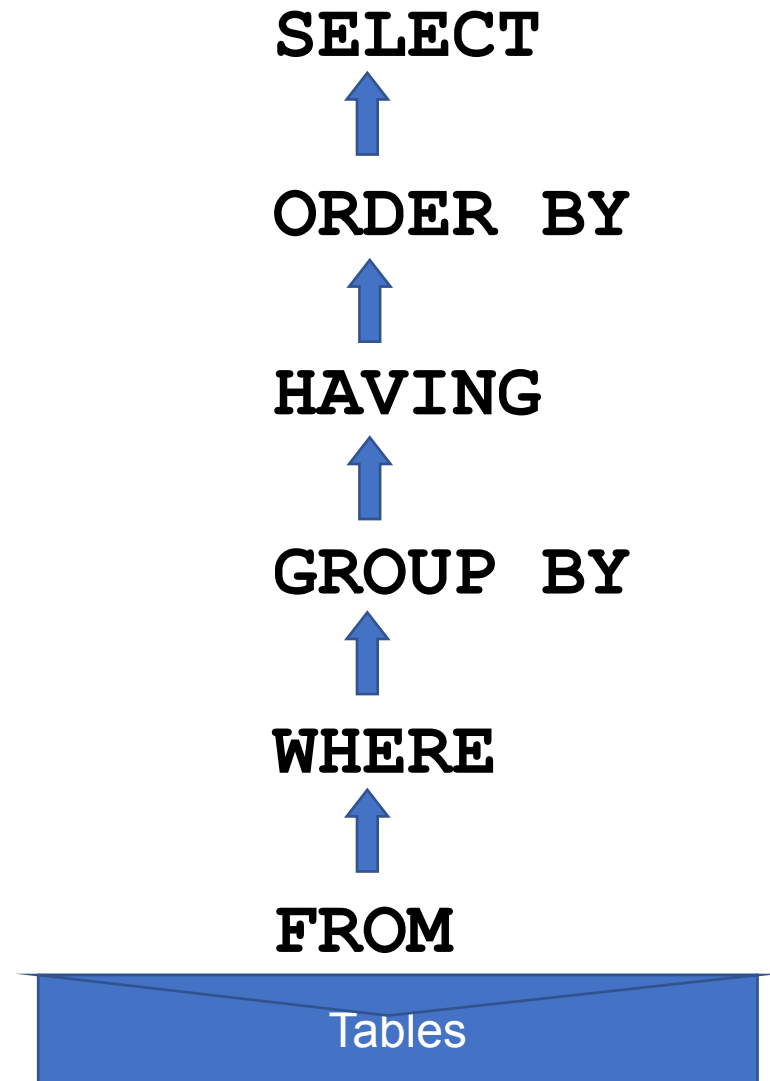
Name	MAX(Salary)
Allison	60000
Dan	100000

Outline

- Introduce relational algebra
- Look at some example RA from previous lectures
- Translating SQL \longleftrightarrow RA

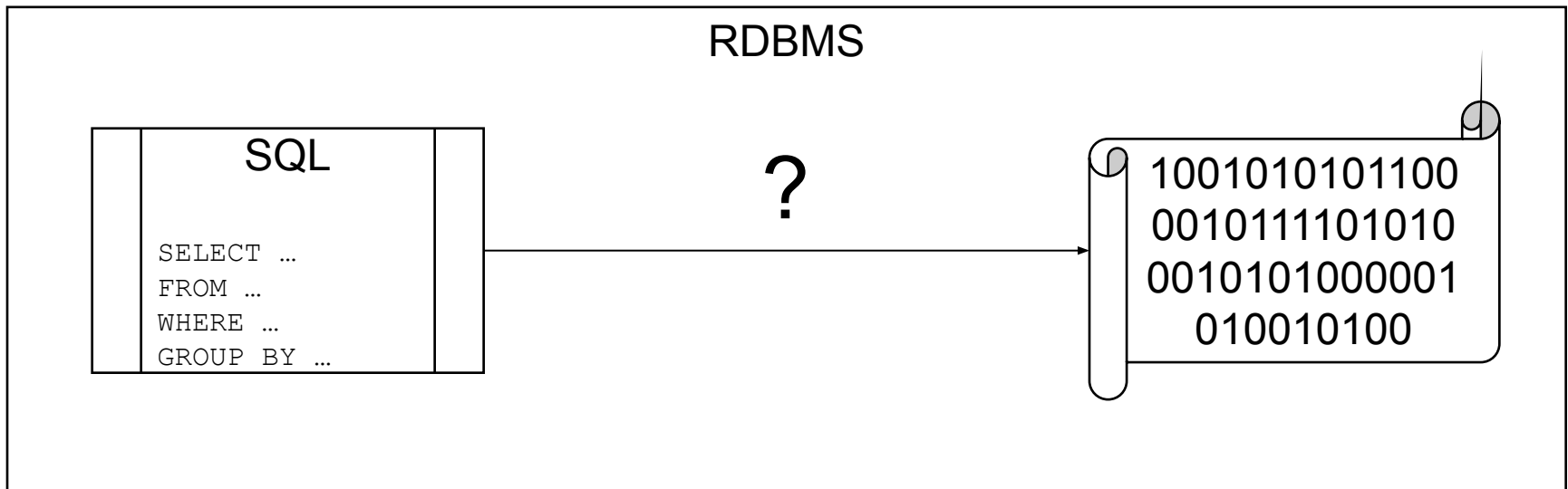
FWGHOS

SELECT ...
FROM ...
WHERE ...
GROUP BY ...
HAVING ...
ORDER BY ...



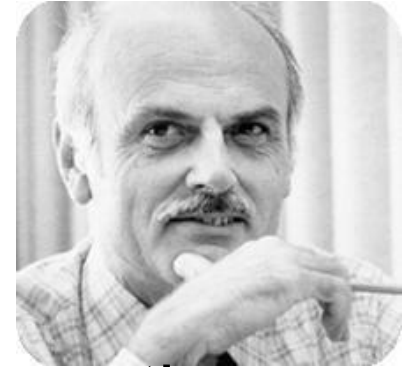
What's the Point of RA?

- SQL is a **Declarative Language**
 - “What to get” rather than “how to get it”
 - Easier to write a SQL query than write a whole Java program that will probably perform worse
- But computers are imperative/procedural
 - Computers only understand the “how”



History of RA

- Invented/Formalized by Ted Codd while working for IBM



- He realized we need a way to describe imperative programming on tables **without knowing physical details**
- IBM initially ignored his techniques

Information Retrieval

A Relational Model of Data for Large Shared Data Banks

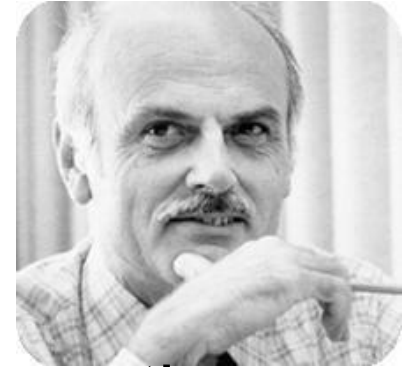
E. F. Codd

IBM Research Laboratory, San Jose, California

Future users of large data banks must be protected from having to know how the data is organized in the machine (the internal representation). A prompting service which supplies such information is not a satisfactory solution. Activities of users at terminals and most application programs should remain unaffected when the internal representation of data is changed and even when some aspects of the external representation are changed. Changes in data representation will often be needed as a result of changes in query, update, and report traffic and natural growth in the types of stored information.

History of RA

- Invented/Formalized by Ted Codd while working for IBM



- He realized we need a way to describe imperative programming on tables **without knowing physical details**

- IBM initially ignored his techniques

- 10 years later he won the Turing Award



Information Retrieval

A Relational Model of Data for Large Shared Data Banks

E. F. Codd

IBM Research Laboratory, San Jose, California

Future users of large data banks must be protected from having to know how the data is organized in the machine (the internal representation). A prompting service which supplies such information is not a satisfactory solution. Activities of users at terminals and most application programs should remain unaffected when the internal representation of data is changed and even when some aspects of the external representation are changed. Changes in data representation will often be needed as a result of changes in query, update, and report traffic and natural growth in the types of stored information.

History of RA

- Invented/Formalized by Ted Codd while working for IBM



- He realized we need a way to describe imperative programming on tables **without knowing physical details**
- IBM initially ignored his techniques
- 10 years later he won the Turing Award



Information Retrieval

A Relational Model of Data for Large Shared Data Banks

E. F. Codd

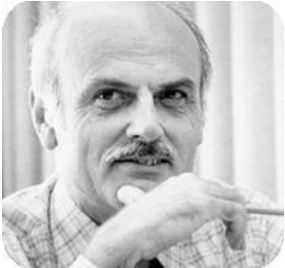
IBM Research Laboratory, San Jose, California

Future users of large data banks must be protected from having to know how the data is organized in the machine (the internal representation). A prompting service which supplies such information is not a satisfactory solution. Activities of users at terminals and most application programs should remain unaffected when the internal representation of data is changed and even when some aspects of the external representation are changed. Changes in data representation will often be needed as a result of changes in query, update, and report traffic and natural growth in the types of stored information.

Turing Awards in Data Management



Charles Bachman, 1973
IDS and CODASYL



Ted Codd, 1981
Relational model



Jim Gray, 1998
Transaction processing

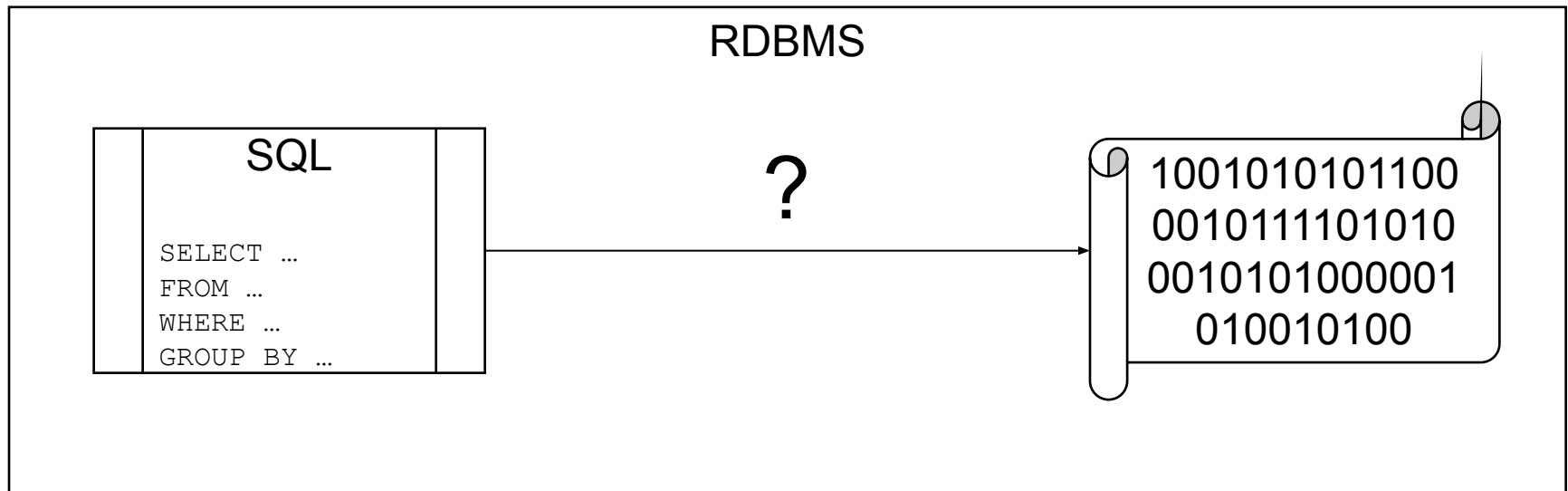


Michael Stonebraker, 2014
INGRES and Postgres



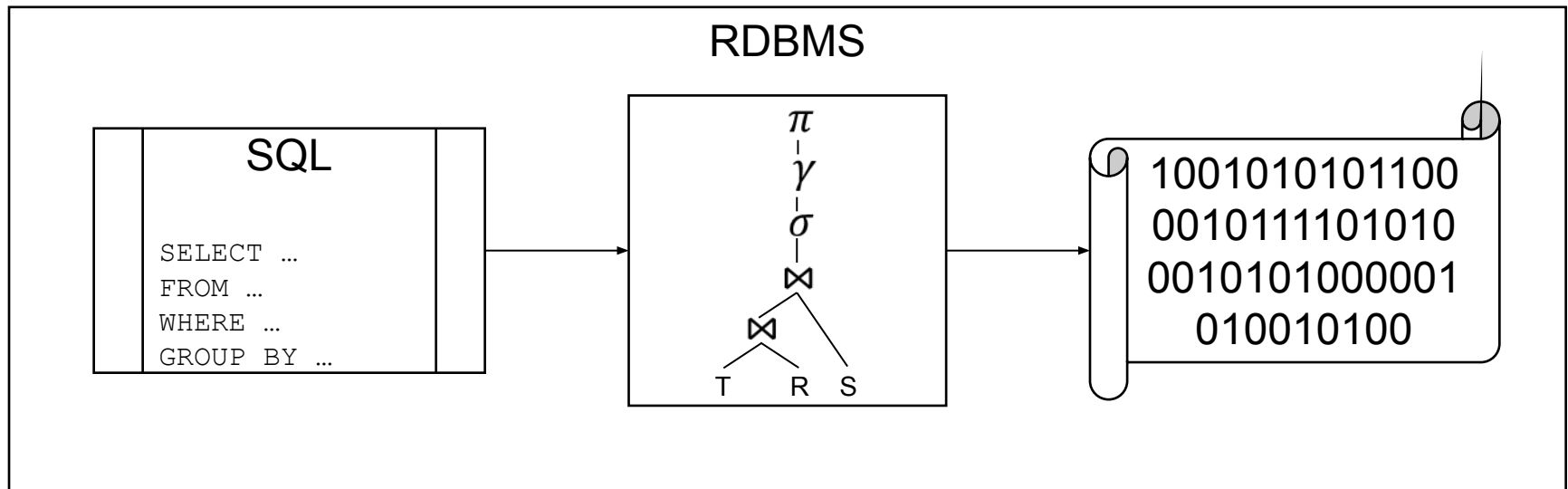
What's the Point of RA?

- We need a language that reads more like **instructions** but still captures the fundamental operations of a query



What's the Point of RA?

- Relational Algebra (RA) does the job
 - When processing your query, the RDBMS will actually store an **RA tree** (like a bunch of labeled nodes and pointers)
 - After some optimizations, the RA tree is converted into instructions (like a bunch of functions linked together)



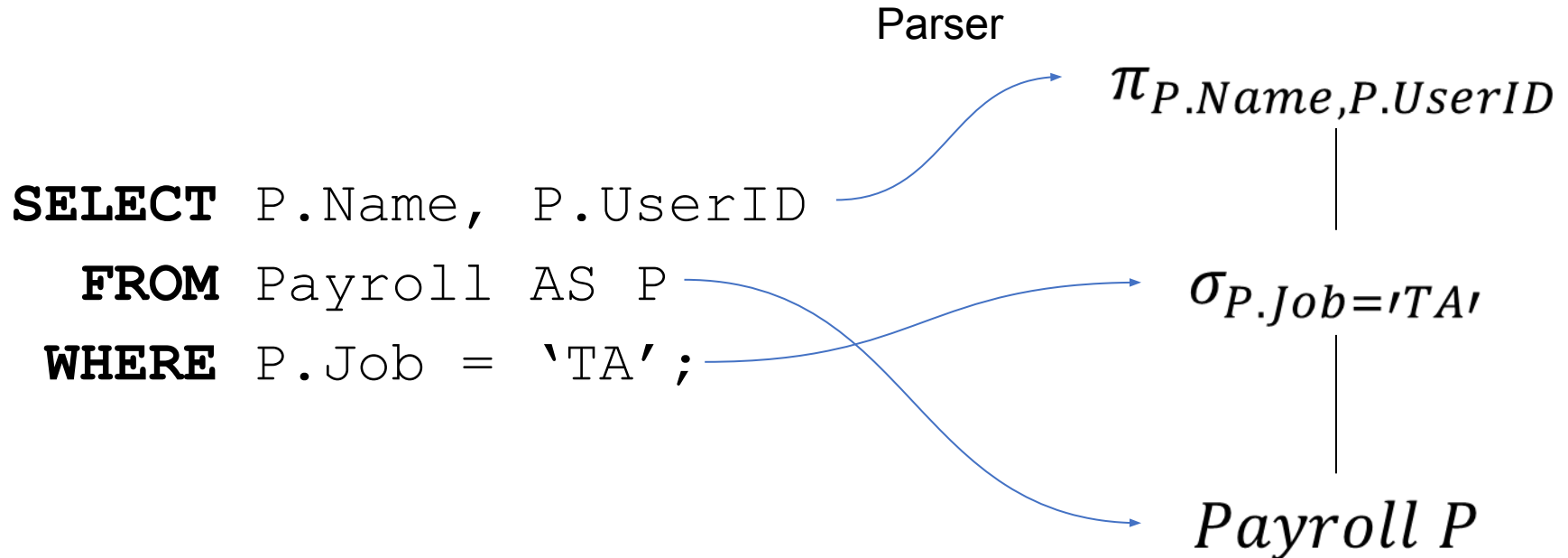
Flashback to our first query

- Code has to boil down to instructions at some point
- Relational Database Management Systems (RDBMSs) use **Relational Algebra** (RA)

```
SELECT P.Name, P.UserID  
  FROM Payroll AS P  
WHERE P.Job = 'TA';
```


Flashback to our first query

- Code has to boil down to instructions at some point
- Relational Database Management Systems (RDBMSs) use **Relational Algebra** (RA)



Flashback to our first query

- Code has to boil down to instructions at some point
- Relational Database Management Systems (RDBMSs) use **Relational Algebra** (RA).

$\pi_{P.Name, P.UserID}$

|

$\sigma_{P.Job='TA'}$

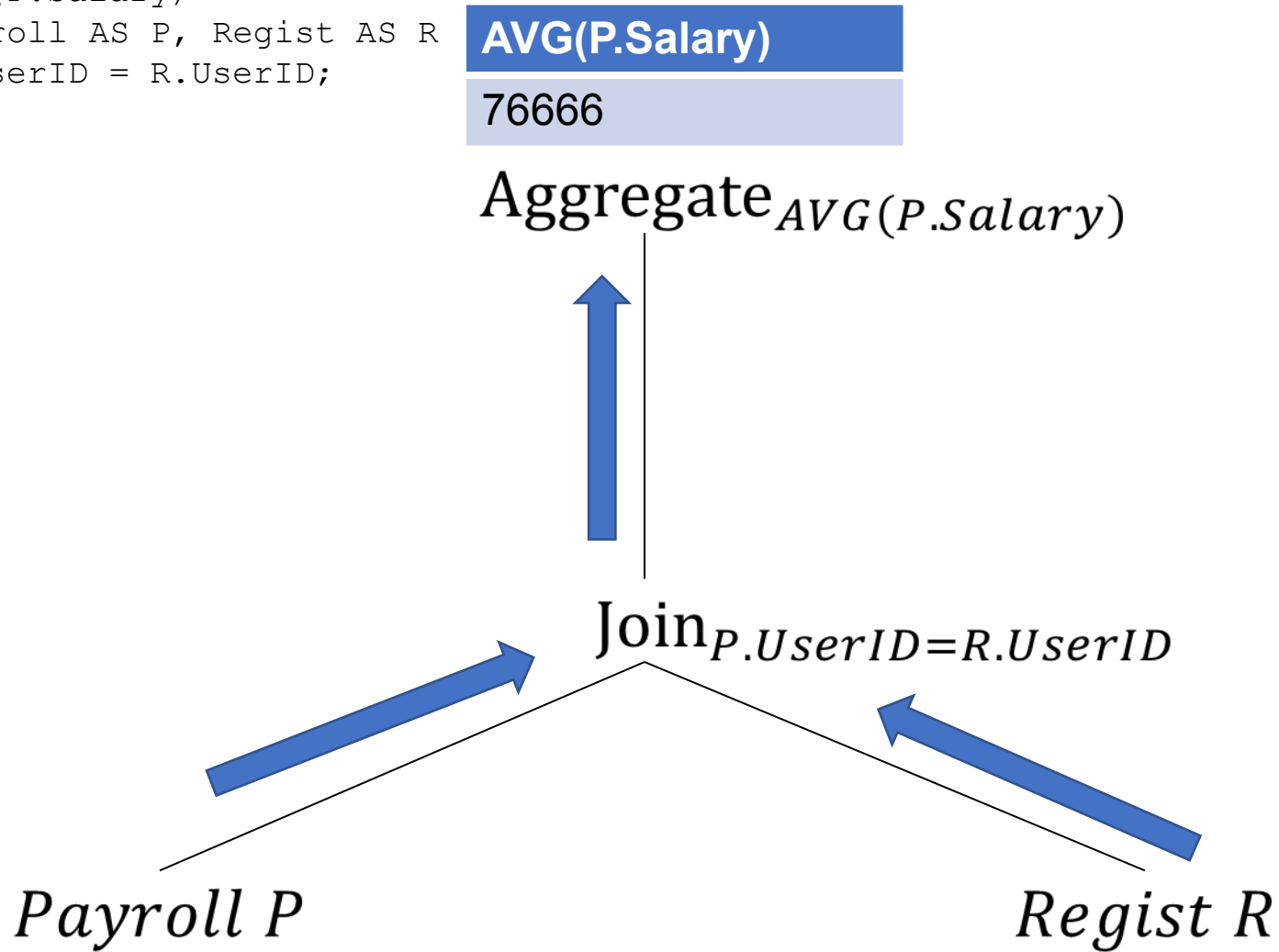
|

Payroll P

Tuples “flow up” the tree, getting modified along the way.

Another example from before...

```
SELECT AVG(P.Salary)
  FROM Payroll AS P, Regist AS R
 WHERE P.UserID = R.UserID;
```

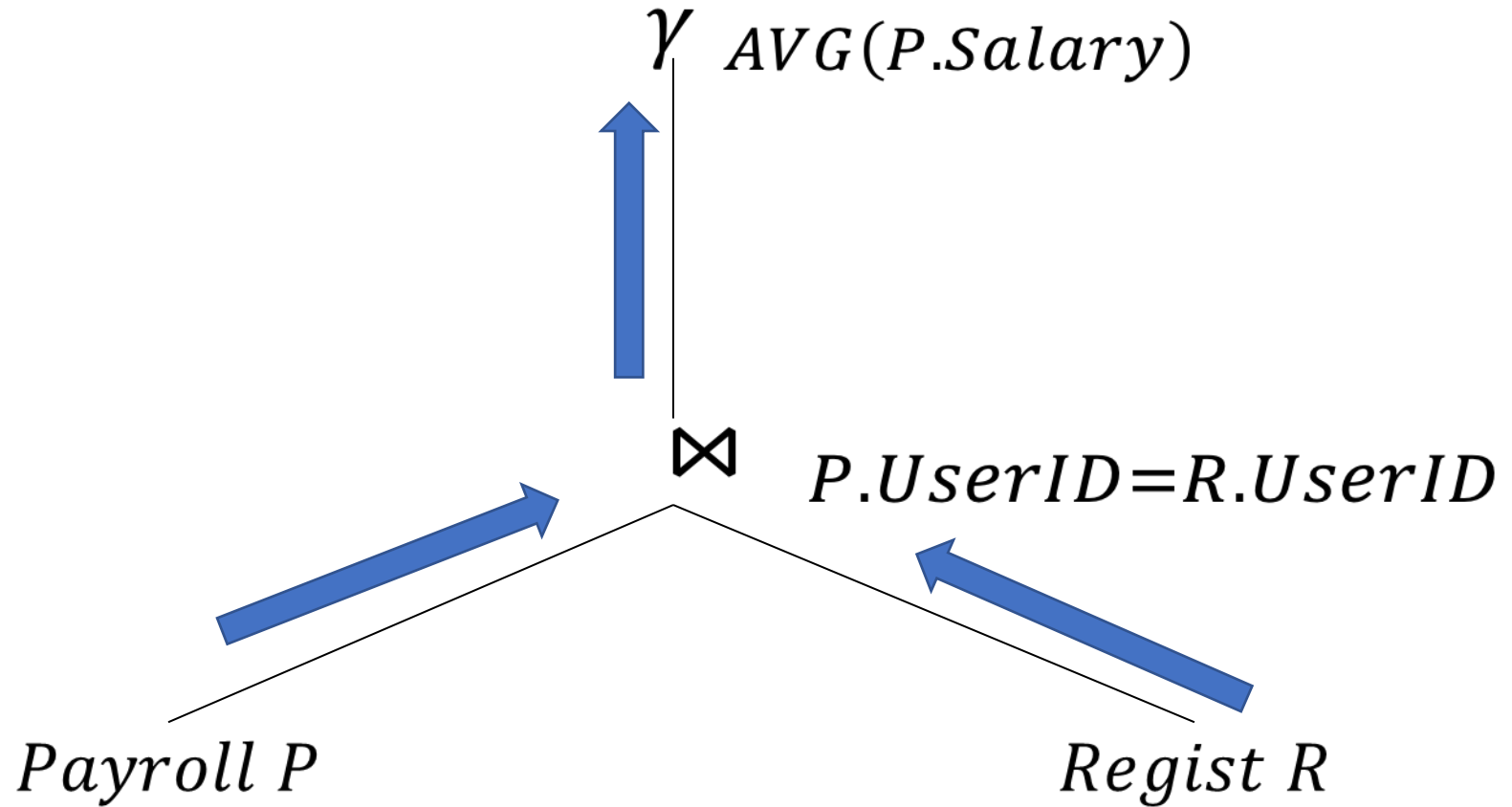


Another example from before...

```
SELECT AVG(P.Salary)
  FROM Payroll AS P, Regist AS R
 WHERE P.UserID = R.UserID;
```

AVG(P.Salary)

76666



RA Operators

- Symbols are mostly Greek letters like π

- σ (sigma)
- γ (gamma)

You don't have to know their Greek names, but this reference may be helpful:

https://www.rapidtables.com/math/symbols/greek_alphabet.html

- Read RA tree from bottom to top

- Bottom \rightarrow Data sources
- Top \rightarrow Query output

- Semantics

- Every operator takes 1 (unary) or 2 (binary) relations as inputs
- Every operator outputs a relation

RA Operators

- These are all the operators you will see in this class
 - We'll profile these one at a time



Join



Grouping &
Aggregation



Sort



Cartesian Product



Union



Duplicate
Elimination



Selection



Intersection



Projection



Difference

RA Operators

- For the curious...



Right Outer Join



Left Outer Join



Full Outer Join



Rename

RA Operators

- Get ready for some definitions...

RA Operators

π Projection \longleftrightarrow **SELECT** ...

- Unary operator
- Projection removes unspecified columns

$$\pi_{A,B}(T(A, B, C)) \rightarrow S(A, B)$$

A	B	C
1	2	3
4	5	6
7	8	9

A	B
1	2
4	5
7	8

RA Operators

σ Selection \longleftrightarrow **WHERE ...**
HAVING ...

- Unary operator
- Selection filters tuples from the input

$$\sigma_{T.A < 6}(T(A, B, C)) \rightarrow S(A, B, C)$$

A	B	C
1	2	3
4	5	6
7	8	9

A	B	C
1	2	3
4	5	6

RA Operators



- Binary operator
- Joins inputs relations on the specified condition

$$T(A, B) \bowtie_{T.B=S.C} S(C, D) \rightarrow R(A, B, C, D)$$

A	B
1	2
3	4
5	6

C	D
2	3
5	6
6	7

A	B	C	D
1	2	2	3
5	6	6	7

RA Operators

\times Cartesian Product

- Binary operator
- Same semantics as in set theory
- Indiscriminate join of input relations

$$T(A, B) \times S(C, D) \rightarrow R(A, B, C, D)$$

RA Operators

γ Grouping & Aggregation \longleftrightarrow **GROUP BY**
(+Aggregates)

- Unary operator
- Specifies grouped attributes and then aggregates
- ONLY operation that can compute aggregates

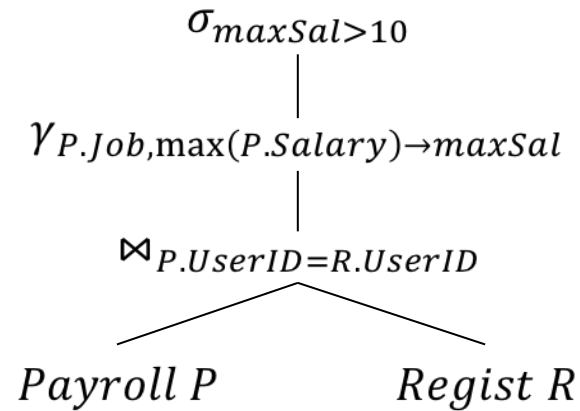
$$\gamma_{T.A, \max(T.B) \rightarrow mB} (T(A, B, C)) \rightarrow R(A, mB)$$

A	B	C
1	2	3
1	5	6
7	8	9

A	mB
1	5
7	8

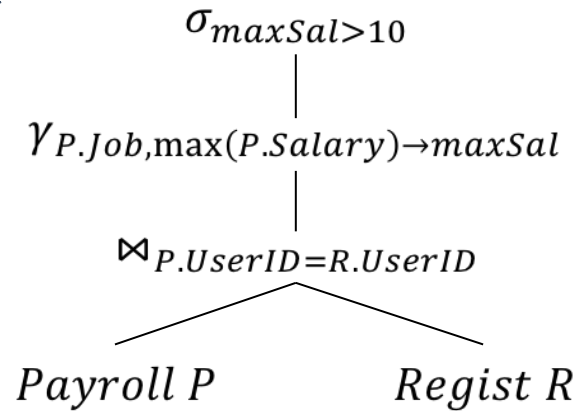
Sometimes RA can be written in-line

$$\sigma_{\text{maxSal} > 10} \\ (\gamma_{P.\text{Job}, \text{max}(P.\text{Salary}) \rightarrow \text{maxSal}} \\ ((\text{Payroll } P) \bowtie_{P.\text{UserID} = R.\text{UserID}} \\ (\\ (\text{Regist } R))))$$



Sometimes RA can be written in-line

The tree style is easier
and preferred!

$$\sigma_{\max Sal > 10}$$
$$(\gamma_{P.Job, \max(P.Salary) \rightarrow \max Sal}$$
$$((Payroll P) \bowtie_{P.UserID=R.UserID}$$
$$(\hspace{10em}(Regist R))))$$


RA Operators

τ Sort

- Unary operator
- Orders the input by any of the columns
- Assume default ascending order like in SQL

$$\tau_{T.A, T.B}(T(A, B, C)) \rightarrow R(A, B, C)$$

A	B	C
7	8	9
1	5	6
1	2	3

A	B	C
1	2	3
1	5	6
7	8	9

RA Operators

δ Duplicate Elimination

- Unary operator
- Deduplicates tuples
- Could get the same effect by grouping on all attributes (if you haven't used a group by yet)

$$\delta(T(A, B, C)) \rightarrow R(A, B, C)$$

A	B	C
1	2	3
1	2	3
4	5	6

A	B	C
1	2	3
4	5	6

RA Operators



Union



Intersection

- Binary operators
- Same semantics as in set theory (but over bags)
- Input tables must have # columns and type

$$T(A, B) \cup S(A, B) \rightarrow R(A, B)$$

A	B
1	2
3	4

A	B
1	2
5	6

A	B
1	2
3	4
1	2
5	6

RA Operators

— Difference

- Binary operator (but direction matters)
- Reads as (left input) – (right input)

$$T(A, B) - S(A, B) \rightarrow R(A, B)$$

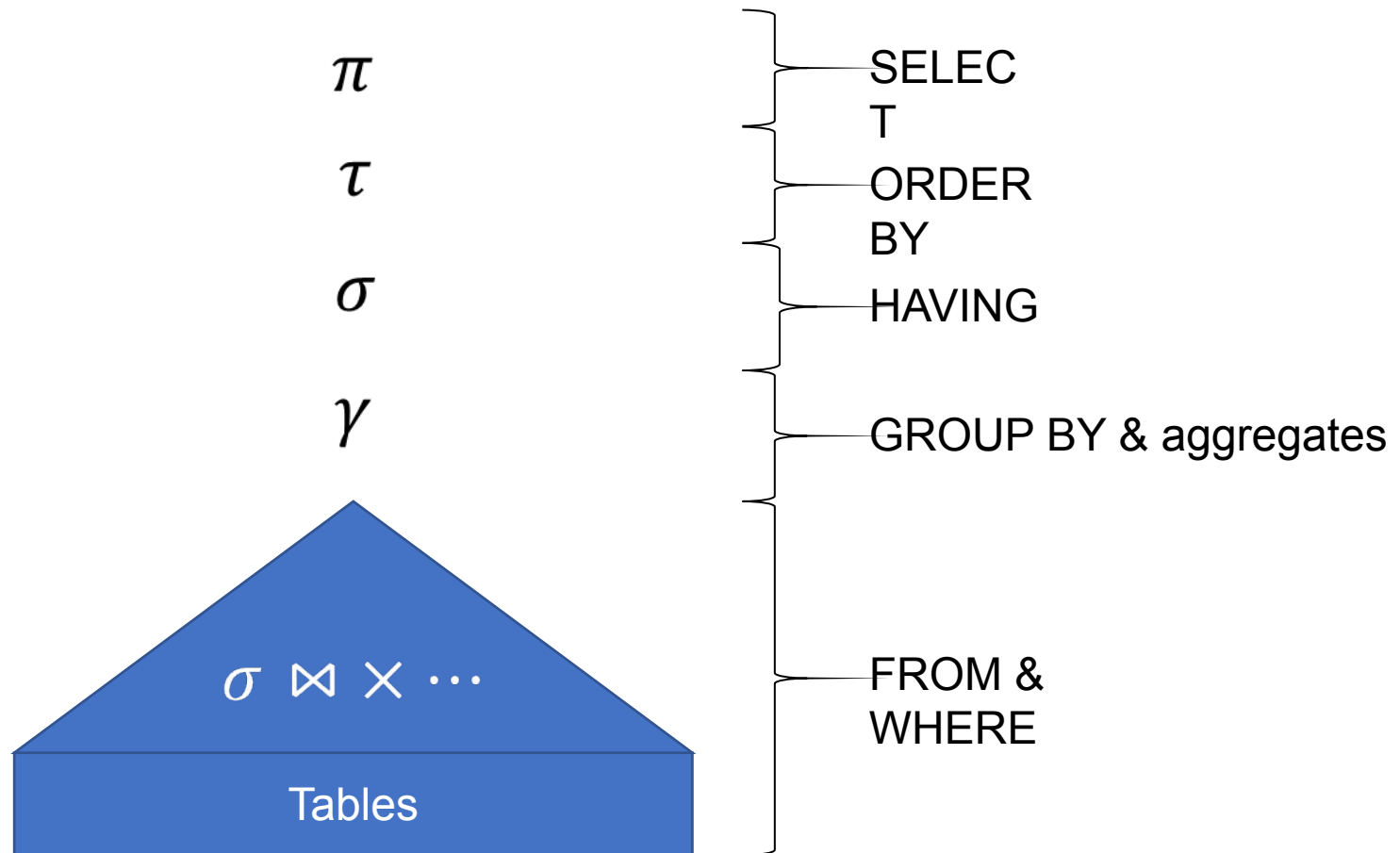
A	B
1	2
3	4

A	B
1	2
5	6

A	B
3	4

Basic SQL to RA Conversion

- The general plan structure for a “flat” SQL query



English to SQL to RA Example

```
CREATE TABLE Payroll (  
  UserID INT PRIMARY KEY,  
  Name   VARCHAR(100),  
  Job    VARCHAR(100),  
  Salary INT);
```

```
CREATE TABLE Regist (  
  UserID INT REFERENCES Payroll,  
  Car     VARCHAR(100));
```

Name all the TAs that drive multiple cars
ordered by the number of cars they drive



```
SELECT DISTINCT P.Name  
  FROM Payroll AS P, Regist AS R  
 WHERE P.UserID = R.UserID AND  
       P.Job = 'TA'  
 GROUP BY P.UserID, P.Name  
 HAVING COUNT(*) > 1  
 ORDER BY COUNT(*)
```

English to SQL to RA Example

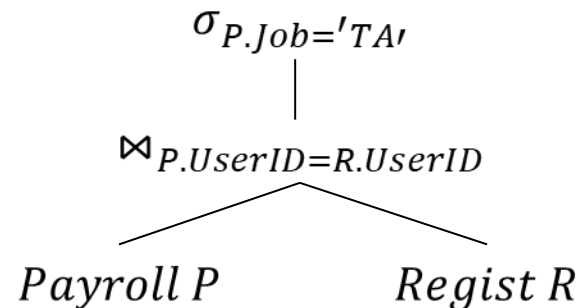
```
CREATE TABLE Payroll (  
  UserID INT PRIMARY KEY,  
  Name    VARCHAR(100),  
  Job     VARCHAR(100),  
  Salary  INT);
```

```
CREATE TABLE Regist (  
  UserID INT REFERENCES Payroll,  
  Car     VARCHAR(100));
```

Name all the TAs that drive multiple cars
ordered by the number of cars they drive



```
SELECT DISTINCT P.Name  
  FROM Payroll AS P, Regist AS R  
 WHERE P.UserID = R.UserID AND  
       P.Job = 'TA'  
  
 GROUP BY P.UserID, P.Name  
HAVING COUNT(*) > 1  
ORDER BY COUNT(*)
```



English to SQL to RA Example

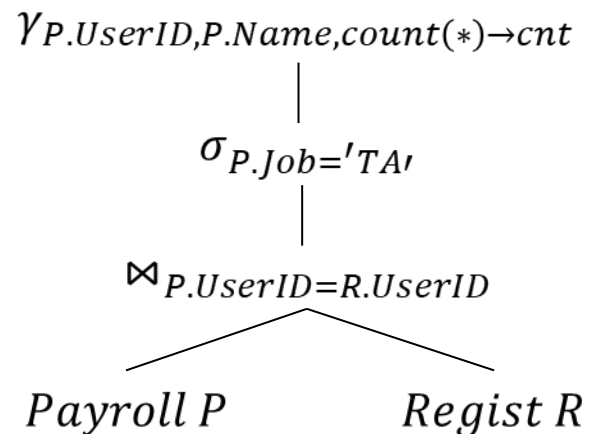
```
CREATE TABLE Payroll (  
  UserID INT PRIMARY KEY,  
  Name    VARCHAR(100),  
  Job     VARCHAR(100),  
  Salary  INT);
```

```
CREATE TABLE Regist (  
  UserID INT REFERENCES Payroll,  
  Car     VARCHAR(100));
```

Name all the TAs that drive multiple cars
ordered by the number of cars they drive



```
SELECT DISTINCT P.Name  
  FROM Payroll AS P, Regist AS R  
 WHERE P.UserID = R.UserID AND  
       P.Job = 'TA'  
GROUP BY P.UserID, P.Name  
HAVING COUNT(*) > 1  
ORDER BY COUNT(*)
```



English to SQL to RA Example

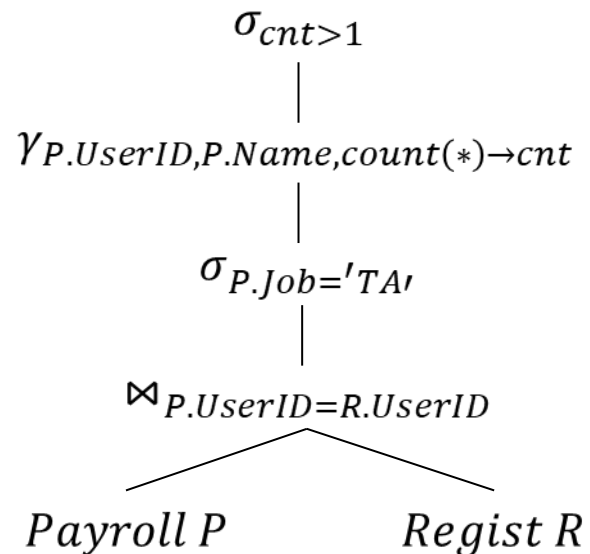
```
CREATE TABLE Payroll (  
  UserID INT PRIMARY KEY,  
  Name    VARCHAR(100),  
  Job     VARCHAR(100),  
  Salary  INT);
```

```
CREATE TABLE Regist (  
  UserID INT REFERENCES Payroll,  
  Car     VARCHAR(100));
```

Name all the TAs that drive multiple cars
ordered by the number of cars they drive



```
SELECT DISTINCT P.Name  
  FROM Payroll AS P, Regist AS R  
 WHERE P.UserID = R.UserID AND  
       P.Job = 'TA'  
 GROUP BY P.UserID, P.Name  
HAVING COUNT(*) > 1  
ORDER BY COUNT(*)
```



English to SQL to RA Example

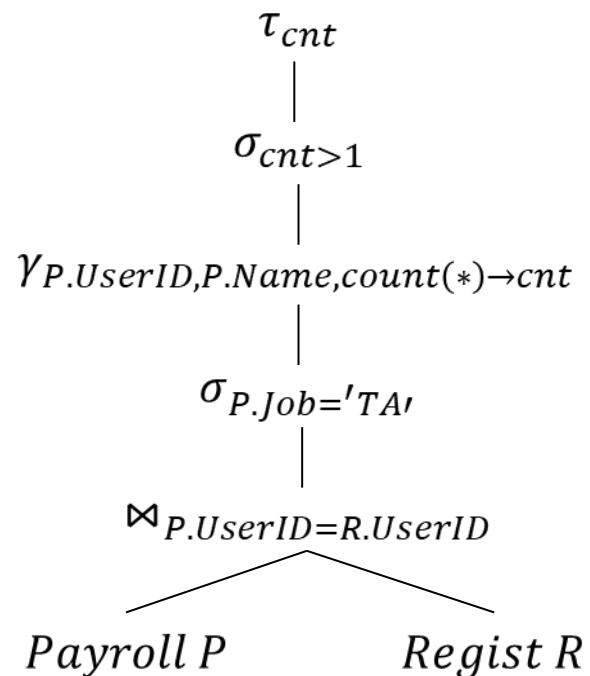
```
CREATE TABLE Payroll (  
  UserID INT PRIMARY KEY,  
  Name    VARCHAR(100),  
  Job     VARCHAR(100),  
  Salary INT);
```

```
CREATE TABLE Regist (  
  UserID INT REFERENCES Payroll,  
  Car     VARCHAR(100));
```

Name all the TAs that drive multiple cars
ordered by the number of cars they drive



```
SELECT DISTINCT P.Name  
  FROM Payroll AS P, Regist AS R  
 WHERE P.UserID = R.UserID AND  
       P.Job = 'TA'  
 GROUP BY P.UserID, P.Name  
HAVING COUNT(*) > 1  
ORDER BY COUNT(*)
```



English to SQL to RA Example

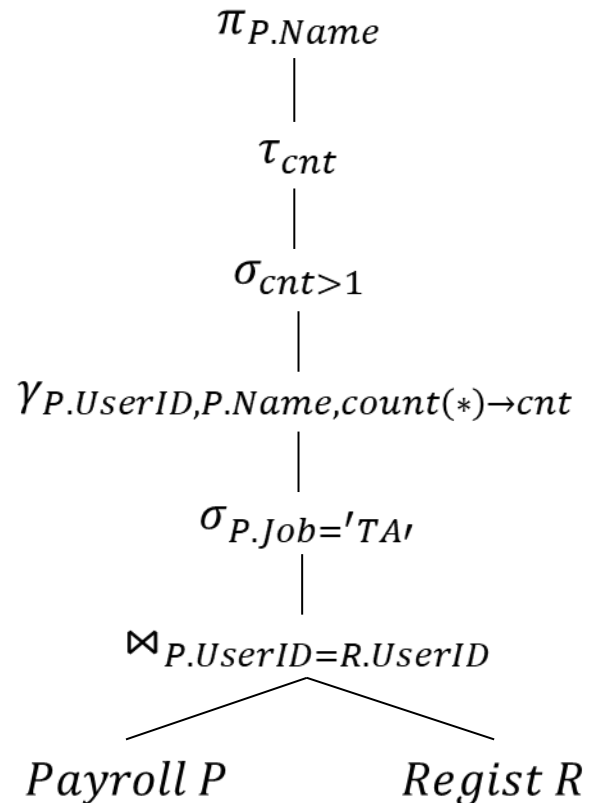
```
CREATE TABLE Payroll (  
  UserID INT PRIMARY KEY,  
  Name    VARCHAR(100),  
  Job     VARCHAR(100),  
  Salary  INT);
```

```
CREATE TABLE Regist (  
  UserID INT REFERENCES Payroll,  
  Car     VARCHAR(100));
```

Name all the TAs that drive multiple cars
ordered by the number of cars they drive



```
SELECT DISTINCT P.Name  
  FROM Payroll AS P, Regist AS R  
 WHERE P.UserID = R.UserID AND  
       P.Job = 'TA'  
 GROUP BY P.UserID, P.Name  
HAVING COUNT(*) > 1  
 ORDER BY COUNT(*)
```



English to SQL to RA Example

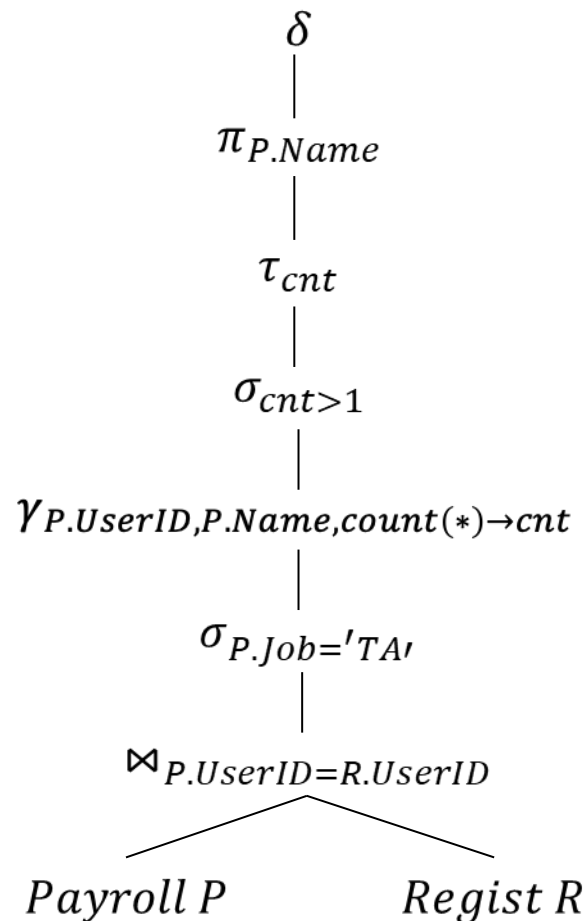
```
CREATE TABLE Payroll (  
  UserID INT PRIMARY KEY,  
  Name    VARCHAR(100),  
  Job     VARCHAR(100),  
  Salary INT);
```

Name all the TAs that drive multiple cars
ordered by the number of cars they drive



```
SELECT DISTINCT P.Name  
  FROM Payroll AS P, Regist AS R  
 WHERE P.UserID = R.UserID AND  
       P.Job = 'TA'  
 GROUP BY P.UserID, P.Name  
 HAVING COUNT(*) > 1  
 ORDER BY COUNT(*)
```

```
CREATE TABLE Regist (  
  UserID INT REFERENCES Payroll,  
  Car     VARCHAR(100));
```



How about Subqueries?

Supplier(sno, sname, scity, sstate)
Part(pno, pname, psize, pcolor)
Supply(sno, pno, price)

```
SELECT  Q.sno
FROM    Supplier Q
WHERE   Q.sstate = 'WA'
       and Q.sno not in
       (SELECT P.sno
        FROM Supply P
        WHERE P.price > 100)
```



How about Subqueries?

Supplier(sno, sname, scity, sstate)
Part(pno, pname, psize, pcolor)
Supply(sno, pno, price)

```
SELECT  Q.sno  
FROM    Supplier Q  
WHERE   Q.sstate = 'WA'  
        and Q.sno not in  
        (SELECT P.sno  
         FROM Supply P  
         WHERE P.price > 100)
```

Remove subquery

```
(SELECT  Q.sno  
 FROM    Supplier Q  
 WHERE   Q.sstate = 'WA')  
EXCEPT  
(SELECT P.sno  
 FROM Supply P  
 WHERE P.price > 100)
```

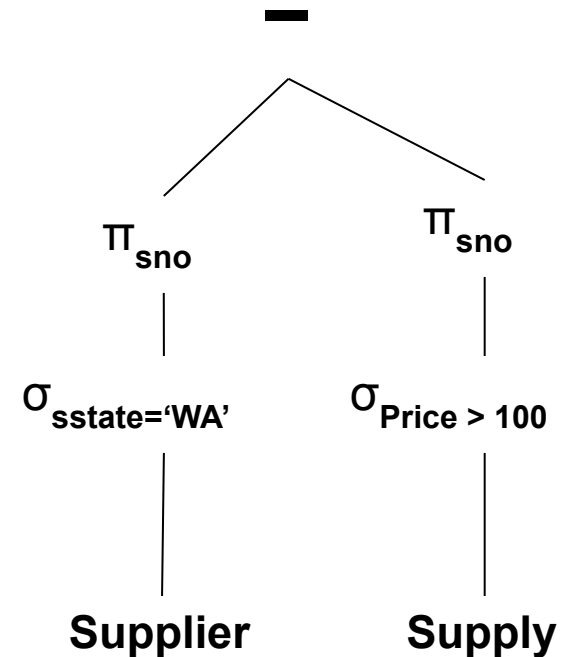
EXCEPT = set difference

How about Subqueries?

Supplier(sno, sname, scity, sstate)
Part(pno, pname, psize, pcolor)
Supply(sno, pno, price)

```
(SELECT Q.sno  
FROM Supplier Q  
WHERE Q.sstate = 'WA')  
EXCEPT  
(SELECT P.sno  
FROM Supply P  
WHERE P.price > 100)
```

Now we have operator



Summary of RA

- SQL = a declarative language where we say ***what*** data we want to retrieve
- RA = an algebra where we say ***how*** we want to retrieve the data
- RDMS translates SQL to RA then optimizes for performance