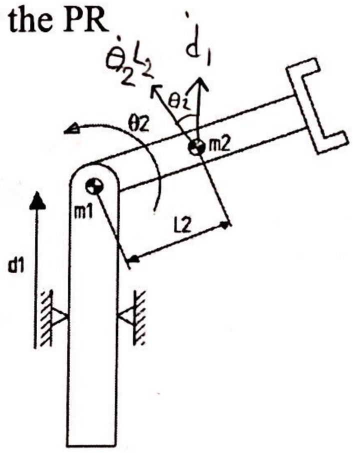
Sheet 5

1- Derive the equations of motion and the torque moment for the PR manipulator shown in the below Fig.



$$K.E_1 = \frac{1}{2} m_1 \dot{d}_1^2 + \cancel{\tfrac{1}{2} m_2 \dot{d}_1}$$

$$P.E_1 = \cancel{\tfrac{1}{2}} m_1 g d_1 + m_1 g d_{1max}$$

$$K.E_2 = \frac{1}{2} m_2 V^2 + \frac{1}{2} I_{zz_2} \dot{\theta}_2^2$$

where $V^2 = \dot{\theta}_2^2 L_2^2 + \dot{d}_1^2 - 2\dot{d}_1 \dot{\theta}_2 L_2 C_2$

$$\therefore K.E_2 = \frac{1}{2} m_2 \dot{\theta}_2^2 L_2^2 + \frac{1}{2} m_2 \dot{d}_1^2 - m_2 \dot{d}_1 \dot{\theta}_2 L_2 C_2 + \frac{1}{2} I_{zz_2} \dot{\theta}_2^2$$

$$P.E_2 = m_2 g (d_1 + L_2 S_2) + m_2 g (d_{max} + L_2)$$

$$\therefore KE = \frac{1}{2} m_1 \dot{d}_1^2 + \frac{1}{2} m_2 \dot{\theta}_2^2 L_2^2 + \frac{1}{2} m_2 \dot{d}_1^2 - m_2 \dot{d}_1 \dot{\theta}_2 L_2 C_2 + \frac{1}{2} I_{zz_2} \dot{\theta}_2^2$$

$$P.E = m_1 g d_1 + m g d_{1max} + m_2 g d_1 + m_2 g L_1 S_2 + m_2 g (d_{max} + L_2)$$

$$\frac{\partial KE}{\partial \dot{\theta}} = \begin{bmatrix} m_1 \dot{d}_1 + m_2 \dot{d}_1 - m_2 \dot{\theta}_2 L_2 C_2 \\ m_2 L_2^2 \dot{\theta}_2 - m_2 \dot{d}_1 L_2 C_2 + I_{zz_2} \dot{\theta}_2 \end{bmatrix}$$

$$\frac{\partial}{\partial t} \frac{\partial KE}{\partial \dot{\theta}} = \begin{bmatrix} m_1 \ddot{d}_1 + m_2 \ddot{d}_1 - m_2 L_2 (C_2 \ddot{\theta}_2 - S_2 \dot{\theta}_2^2) \\ m_2 L_2^2 \ddot{\theta}_2 - m_2 L_2 (\ddot{d}_1 C_2 - S_2 \dot{d}_1 \dot{\theta}_2) + I_{zz_2} \ddot{\theta}_2 \end{bmatrix}$$

$$\frac{\partial KE}{\partial \theta} = \begin{bmatrix} 0 \\ m_2 \dot{d}_1 \dot{\theta}_2 L_2 S_2 \end{bmatrix}$$

$$\frac{\partial PE}{\partial \theta} = \begin{bmatrix} m_1 g + m_2 g \\ m_2 g L_1 C_2 \end{bmatrix}$$

$$\text{equation of motion} = \frac{\partial}{\partial t} \frac{\partial k.E}{\partial \dot{\theta}} - \frac{\partial kE}{\partial \theta} + \frac{\partial PE}{\partial \theta}$$

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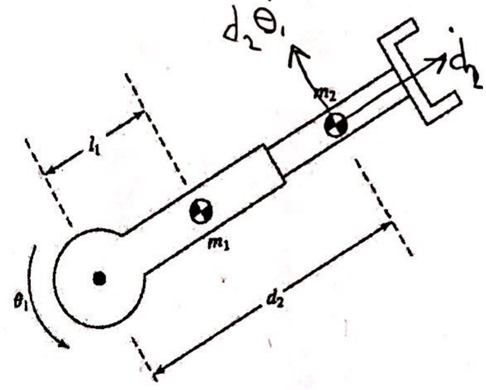
2- **Derive the equations of motion and the torque moment for the RP manipulator shown in the below Fig.**



$$K.E_1 = \frac{1}{2} m_1 (L_1 \dot{\theta}_1)^2 + \frac{1}{2} I_{zz_1} \dot{\theta}_1^2$$

$$P.E_1 = m_1 g L_1 S_1 + m_1 g L_1$$

$$K.E_2 = \frac{1}{2} m_2 \left((d_2 \dot{\theta}_1)^2 + \dot{d}_2^2\right) + \frac{1}{2} I_{zz_2} \dot{\theta}_1^2$$

$$P.E_2 = m_1 g d_2 S_1 + m_2 g d_{max}$$

$$\therefore K.E = \frac{1}{2} m_1 (L_1 \dot{\theta}_1)^2 + \frac{1}{2} I_{zz_1} \dot{\theta}_1^2 + \frac{1}{2} m_2 d_2^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \dot{d}_2^2 + \frac{1}{2} I_{zz_2} \dot{\theta}_1^2$$

$$P.E = m_1 g L_1 S_1 + m_1 g L_1 + m_2 g d_2 S_1 + m_2 g d_{max}$$

$$\frac{\partial KE}{\partial \dot{\theta}} = \begin{bmatrix} m_1 L_1^2 \dot{\theta}_1 + I_{zz_1} \dot{\theta}_1 + m_2 d_2^2 \dot{\theta}_1 + I_{zz_2} \dot{\theta}_1 \\ m_2 \dot{d}_2 \end{bmatrix}$$

$$\frac{\partial}{\partial t} \frac{\partial KE}{\partial \dot{\theta}} = \begin{bmatrix} m_1 L_1^2 \ddot{\theta}_1 + I_{zz_1} \ddot{\theta}_1 + m_2 (d_2^2 \ddot{\theta}_1 + 2 d_1 \dot{d}_1 \dot{\theta}_1) + I_{zz_2} \ddot{\theta}_1 \\ m_2 \ddot{d}_2 \end{bmatrix}$$

$$\frac{\partial KE}{\partial \theta} = \begin{bmatrix} 0 \\ m_2 \dot{\theta}_1^2 d_2 \end{bmatrix}$$

$$\frac{\partial PE}{\partial \theta} = \begin{bmatrix} m_1 g L_1 C_1 + m_2 g d_2 C_1 \\ m_2 g S_1 \end{bmatrix}$$

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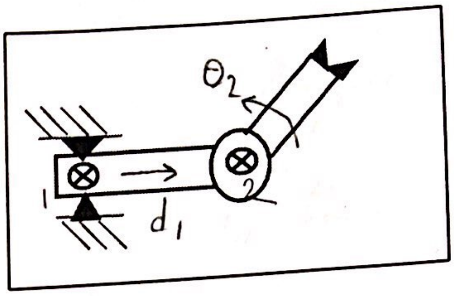
3- Derive the equations of motion and the torque moment for the PR manipulator shown in the below Fig.



$$K.E_1 = \frac{1}{2} m_1 \dot{d}_1^2 \qquad P.E_1 = 0$$

$$K.E_2 = \frac{1}{2} m_2 \dot{d}_1^2 + \frac{1}{2} I_{zz_2} \dot{\theta}_2^2 \qquad P.E_2 = 0$$

$$\therefore K.E_t = \frac{1}{2} m_2 \dot{d}_1^2 + \frac{1}{2} I_{zz_2} \dot{\theta}_2^2 + \frac{1}{2} m_1 \dot{d}_1^2$$

$$P.E = 0$$

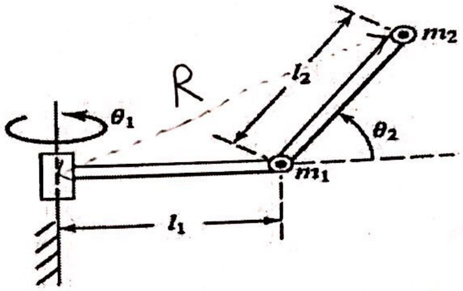
$$\therefore \frac{\partial KE}{\partial \dot{\theta}} = \begin{bmatrix} m_2 \dot{d}_1 + m_1 \dot{d}_1 \\ I_{zz_2} \dot{\theta}_2 \end{bmatrix}$$

$$\frac{\partial}{\partial t} \frac{\partial K.E}{\partial \dot{\theta}} = \begin{bmatrix} m_2 \ddot{d}_1 + m_1 \ddot{d}_1 \\ I_{zz_2} \ddot{\theta}_2 \end{bmatrix}$$

$$\frac{\partial K.E}{\partial \theta} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial P.E}{\partial \theta} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

4- Derive the equations of motion and the torque moment for the RR manipulator shown in the below Fig.



$$K.E_1 = \frac{1}{2} m_1 (l_1 \dot\theta_1)^2 + \frac{1}{2} I_{zz_1} \dot\theta_1^2$$

$$P.E_1 = 0$$

$$K.E_2 = \frac{1}{2} m_2 (\overset{V}{\cancel{R\dot\theta_1}})^2 \cancel{+\frac{1}{2} m_2} + \frac{1}{2} I_{zz_2} \dot\theta_1^2 + \frac{1}{2} I_{zz_2} \dot\theta_2^2$$

where $V = \sqrt{(R\dot\theta_1)^2 + (l_2 \dot\theta_2)^2}$

$$R^2 = l_1^2 + l_2^2 - 2L_1L_2 \cos(180-\theta_2) = L_1^2 + L_2^2 + 2L_1L_2C_2$$

$$\therefore KE_2 = \frac{1}{2} m_2 \left((L_1^2 + L_2^2 + 2L_1L_2C_2)\dot\theta_1^2 + l_2^2 \dot\theta_2^2 \right) + \frac{1}{2} I_{zz_2} \dot\theta_1^2 + \frac{1}{2} I_{zz_2} \dot\theta_2^2$$

$$P.E_2 = m_2 g L_2 S_2 + m_2 g L_2$$

$$\therefore K.E = \frac{1}{2} m_1 (l_1 \dot\theta_1)^2 + \frac{1}{2} I_{zz_1} \dot\theta_1^2 + \frac{1}{2} m_2 L_1^2 \dot\theta_1^2 + \frac{1}{2} m_2 L_2^2 \dot\theta_1^2 + m_2 L_1 L_2 C_2 \dot\theta_1^2 + \frac{1}{2} m_2 L_2^2 \dot\theta_2^2 + \frac{1}{2} I_{zz_2} \dot\theta_1^2 + \frac{1}{2} I_{zz_2} \dot\theta_2^2$$

$$P.E = m_2 g L_2 S_2 + m_2 g L_2$$

$$\frac{\partial K.E}{\partial \dot\theta} = \begin{bmatrix} m_1 L_1^2 \dot\theta_1 + I_{zz_1} \dot\theta_1 + m_2 L_1^2 \dot\theta_1 + m_2 L_2^2 \dot\theta_1 + 2 m_2 L_1 L_2 C_2 \dot\theta_1 + \cancel{m_2 L_2 \dot\theta_2} + I_{zz_2} \dot\theta_1 \\ m_2 L_2^2 \dot\theta_2 + I_{zz_2} \dot\theta_2 \end{bmatrix}$$

$$\frac{\partial}{\partial t} \frac{\partial K.E}{\partial \theta} = \begin{bmatrix} m_1 L_1^2 \ddot\theta_1 + I_{zz_1} \ddot\theta_1 + m_2 L_1^2 \ddot\theta_1 + m_2 L_2^2 \ddot\theta_1 + 2 m_2 L_1 L_2 (C_2 \ddot\theta_1 - S_2 \dot\theta_1 \dot\theta_2) + I_{zz_2} \ddot\theta_1 \\ m_2 L_2^2 \ddot\theta_2 + I_{zz_2} \ddot\theta_2 \end{bmatrix}$$

$$\frac{\partial K.E}{\partial \theta} = \begin{bmatrix} 0 \\ -2 m_2 L_1 L_2 S_2 \dot\theta_1 \end{bmatrix}$$

$$\frac{\partial P.E}{\partial \theta} = \begin{bmatrix} 0 \\ m_2 g L_2 C_2 \end{bmatrix}$$