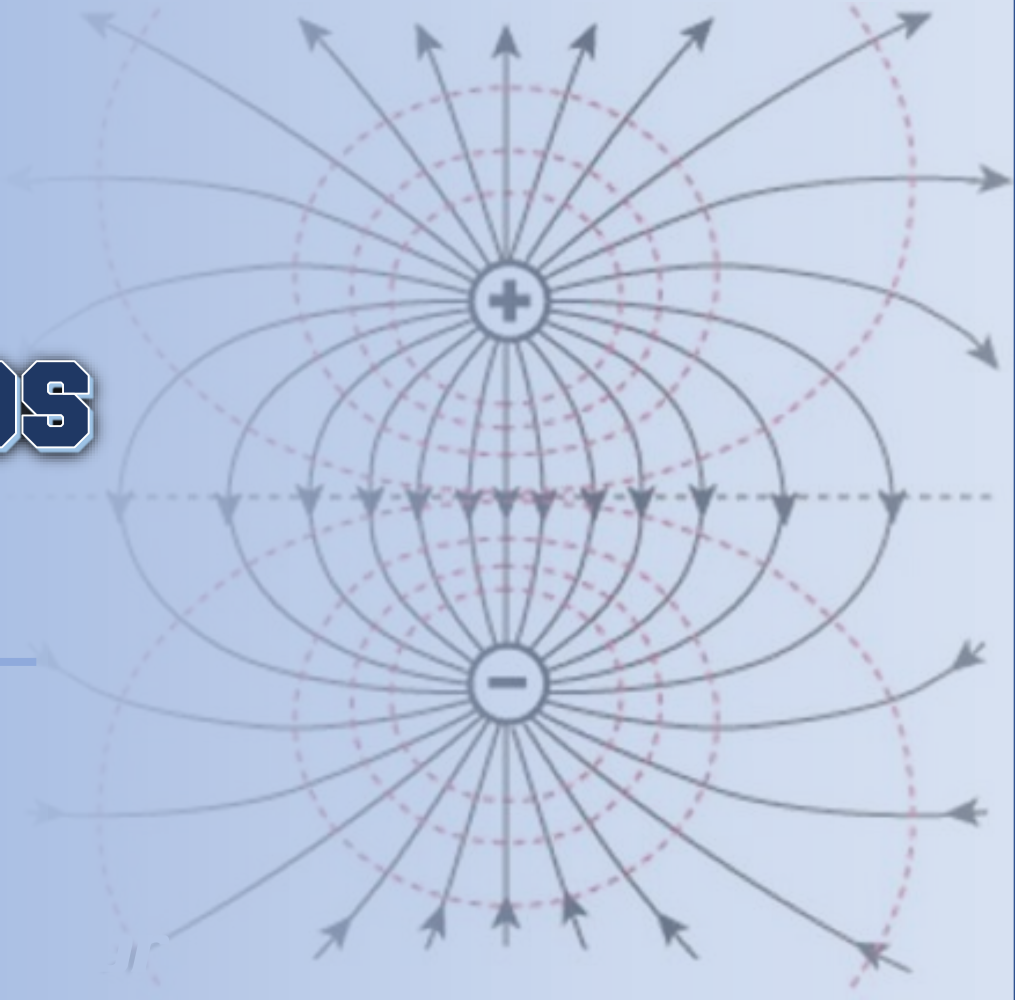
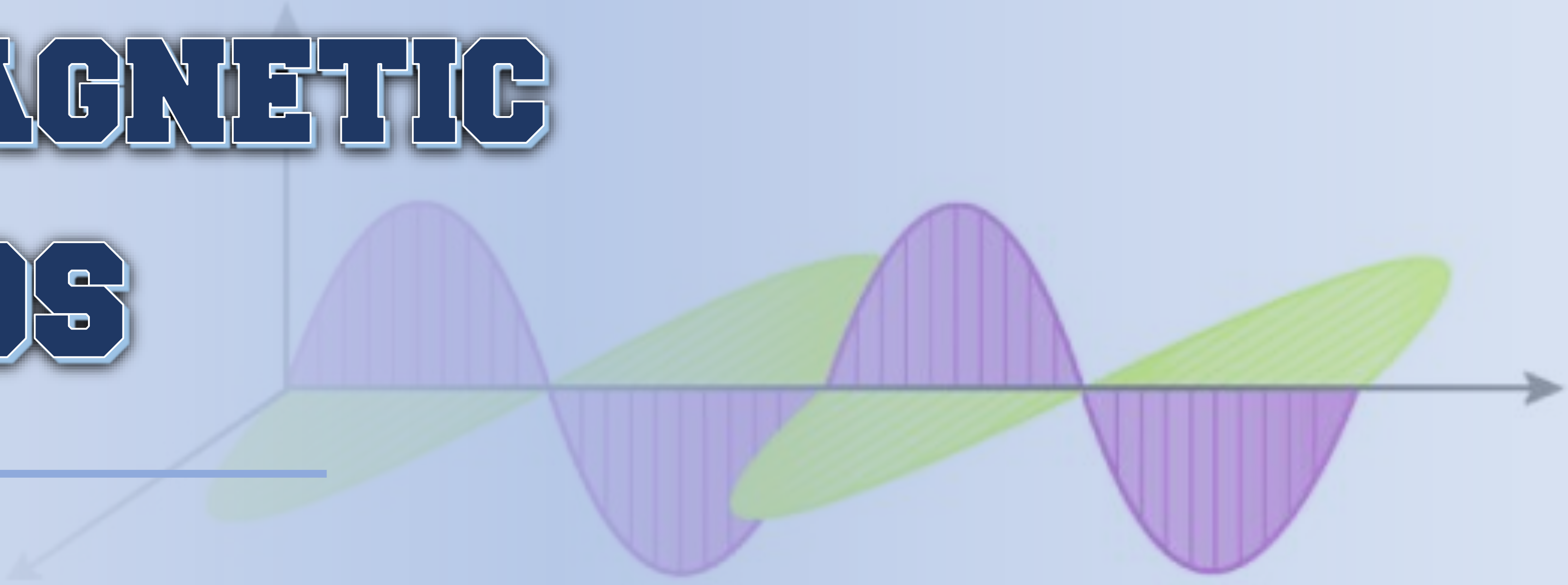
Electromagnetic Fields



Prof. Ragab El-sehiemy

Dr. Mohamed Gabr

Electromagnetic Fields



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Couse Table in Bylaw

Electrical Engineering Department

Second Year

Course Code	Course Name	First Semester						Second Semester						Total Marks
		Hrs. Week		Max Marks			Exam Period	Hrs Week		Max Marks			Exam Period	
		Lec	Tut	Final Exam	Year Work	Oral		Lec	Tut	Final Exam	Year Work	Oral		
EPM2103	Electromagnetic Fields	3	2	90	35	--	3	--	--	--	--	--	--	125

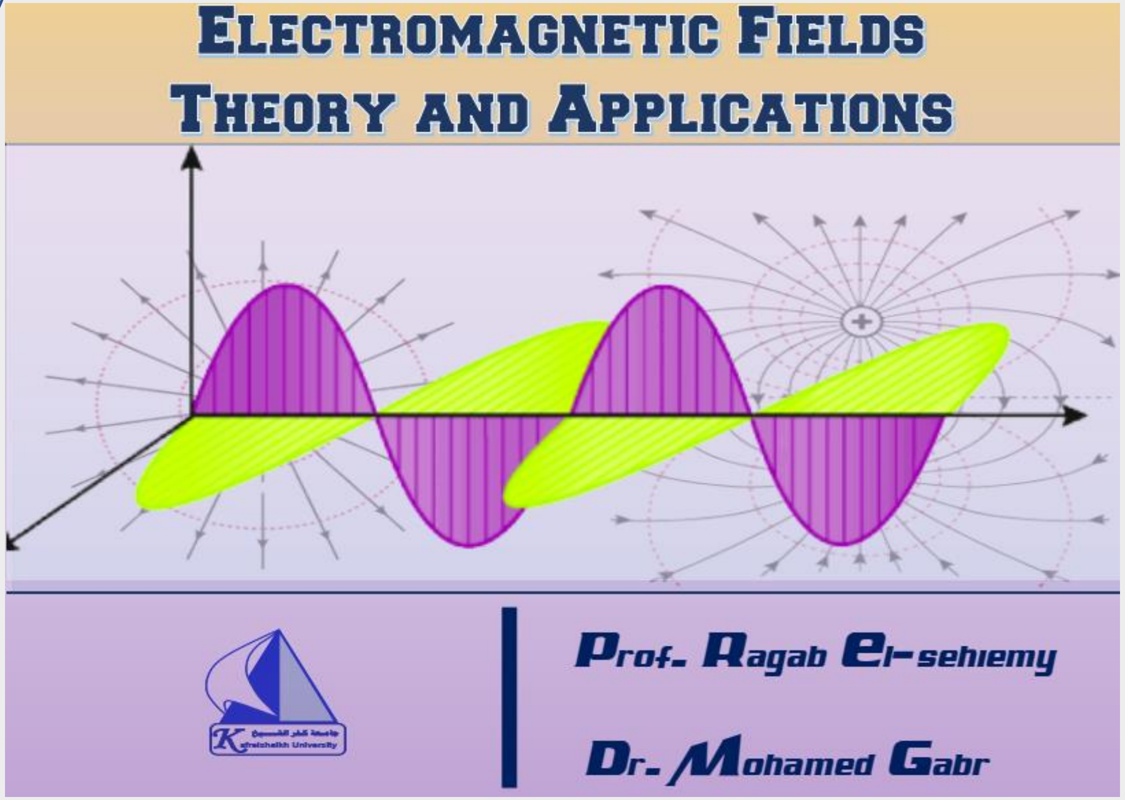
Assessments

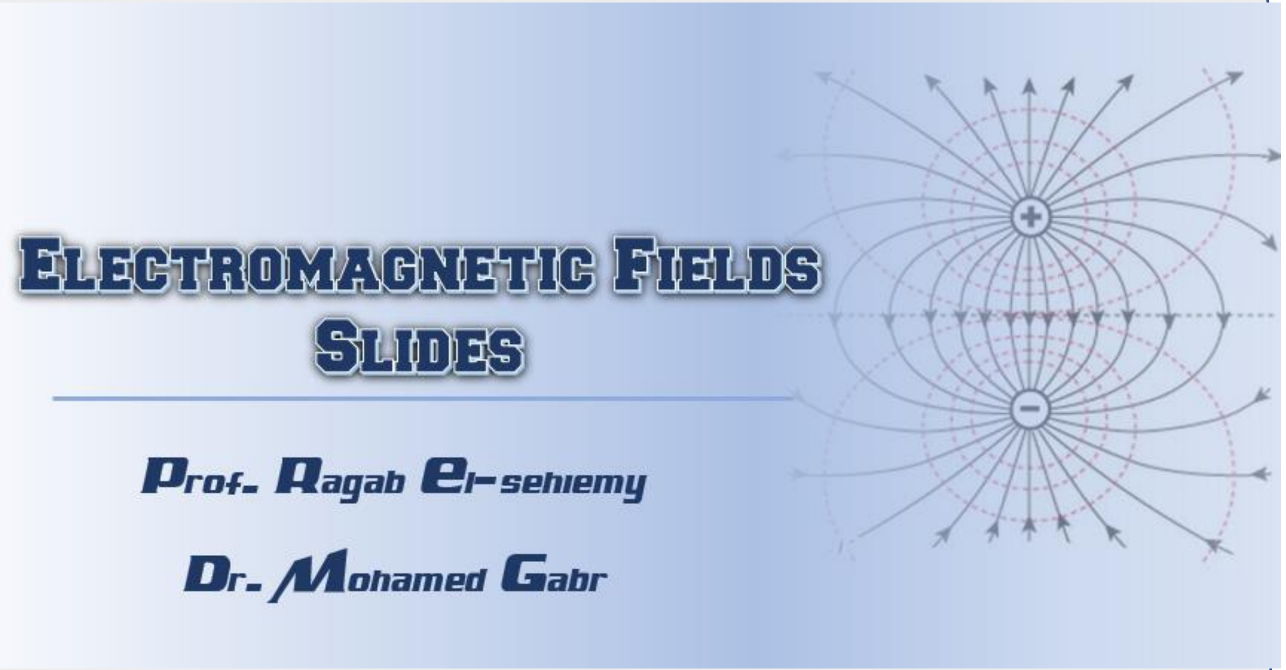
Quizzes	35 Marks
Assignments	
Mid-Term Examination	
In-Class activities and attendance	
Final Exam	90 Marks
Total	125 Marks

Course Outlines

- **Vectors**
- **Electrical Fields**
  - Electric field strength
  - Electrical flux density
  - Potential and energy
  - Dielectric and capacitance
- **Magnetic Fields**
  - Magnetic fields
  - Magnetic forces and torque
  - Inductance
  - Boundary Conditions
- **Time varying magnetic fields**

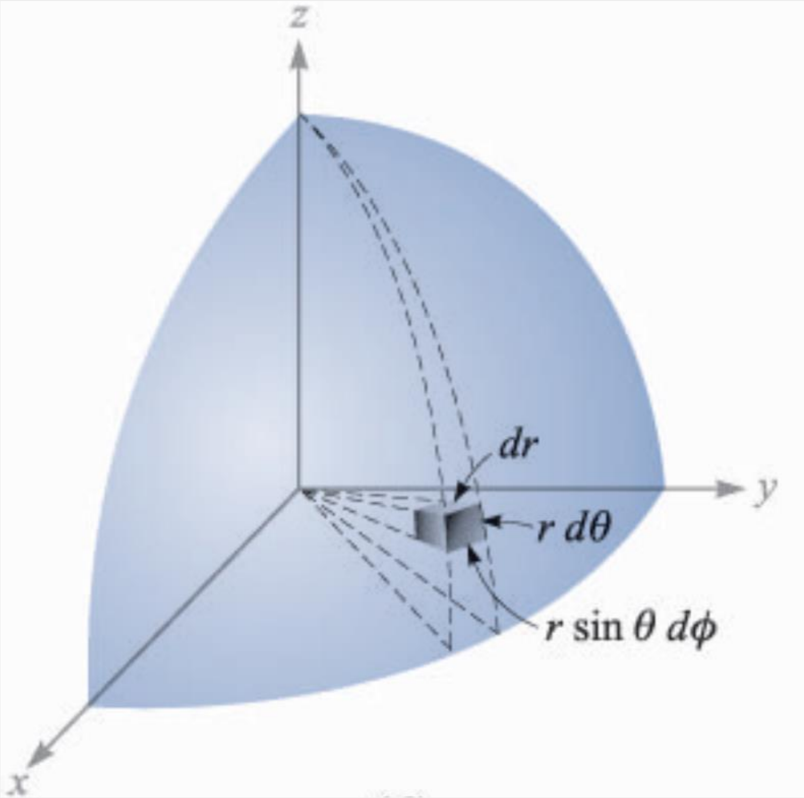
Course Outlines

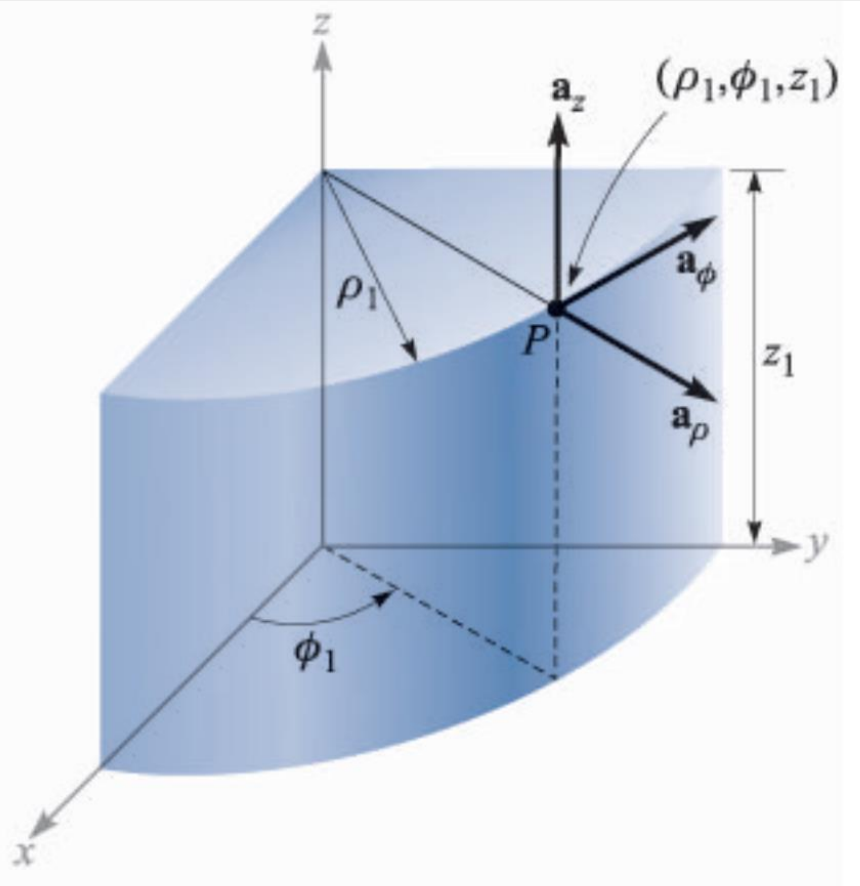




- Lecture Notes Book
- Slides

Vector Analysis



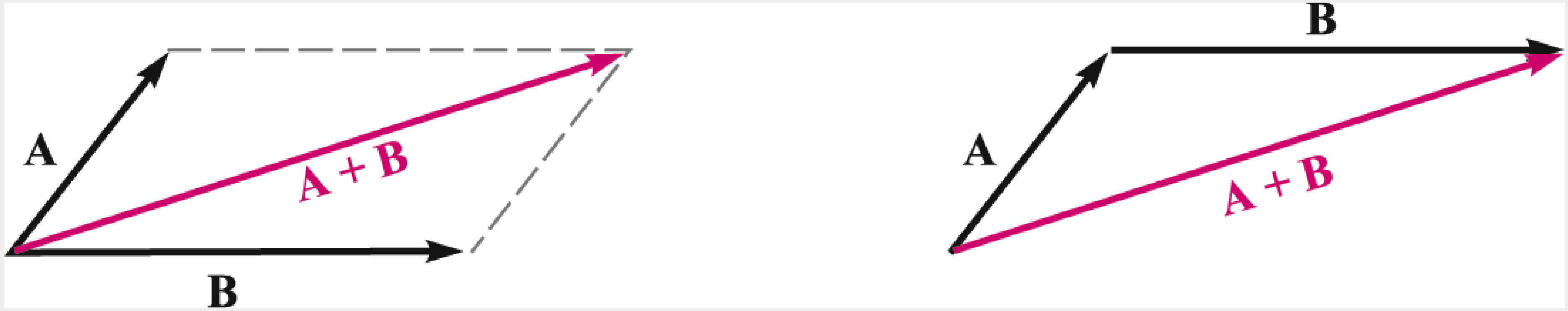


Type of Quantities

Scalar : has a value, but no direction [mass- Length- Flux- Distance- Potential]

Vector : has a magnitude and direction [Force- Electric field]

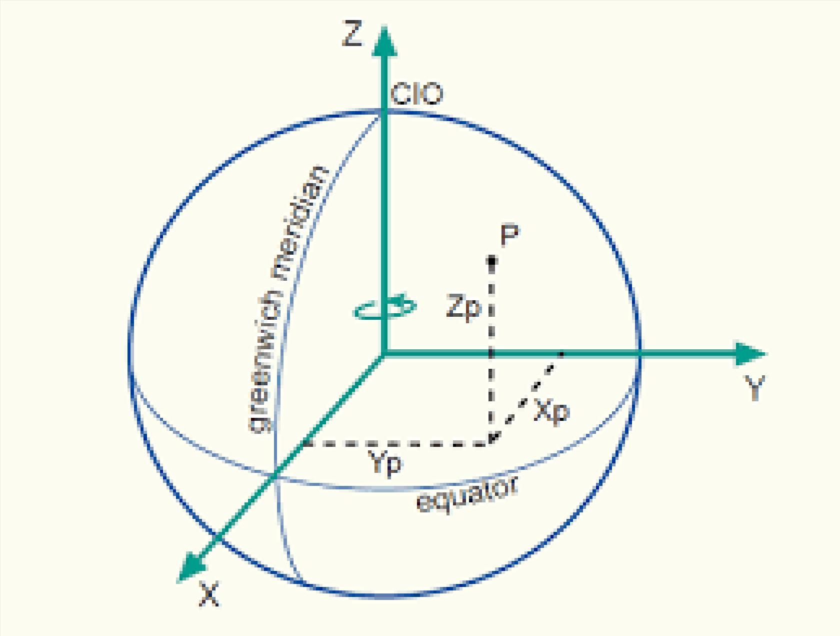
Vector Addition



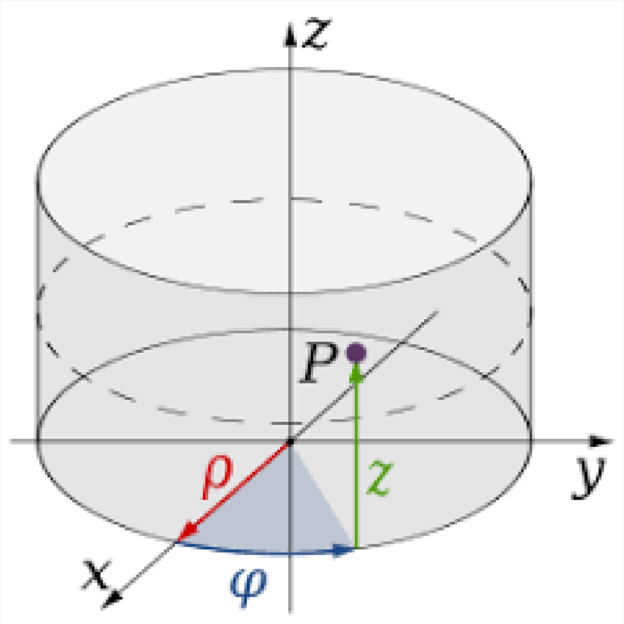
Associative Law: $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$

Distributive Law: $(r + s)(\mathbf{A} + \mathbf{B}) = r(\mathbf{A} + \mathbf{B}) + s(\mathbf{A} + \mathbf{B})$

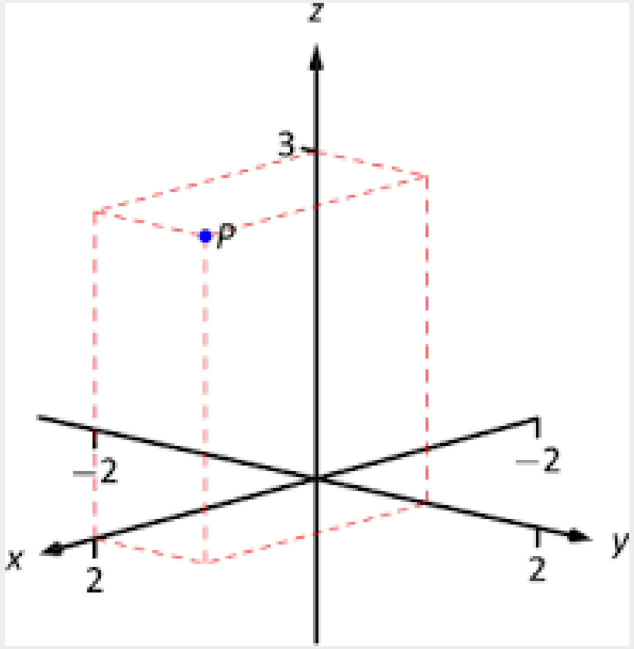
Coordinate Systems



Spherical



Cylindrical



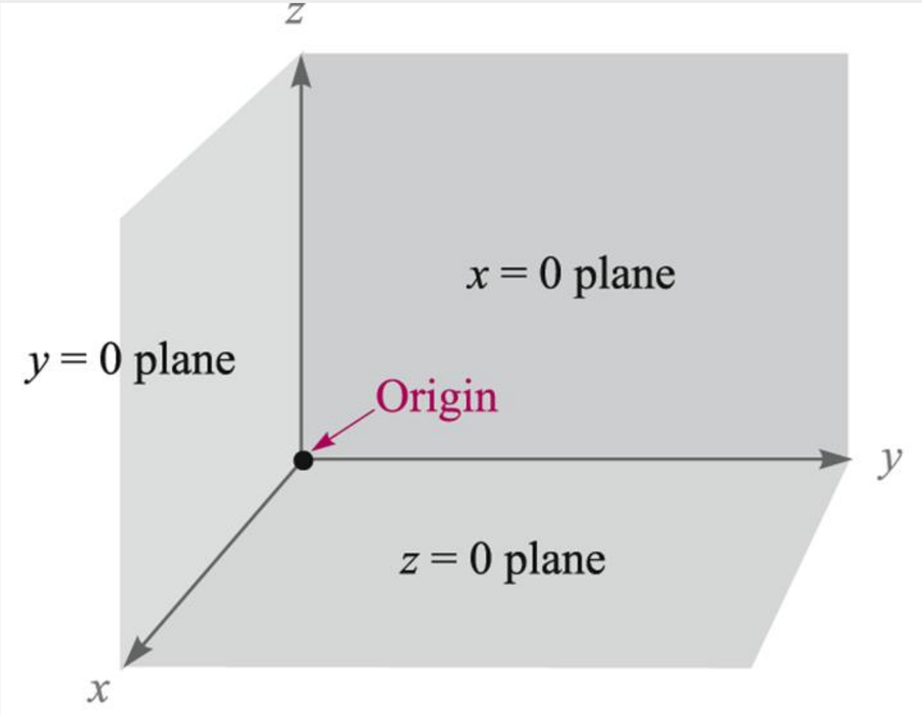
Cartesian

Rectangular/Cartesian Coordinate System

$$-\infty < x < \infty$$

$$-\infty < y < \infty$$

$$-\infty < z < \infty$$



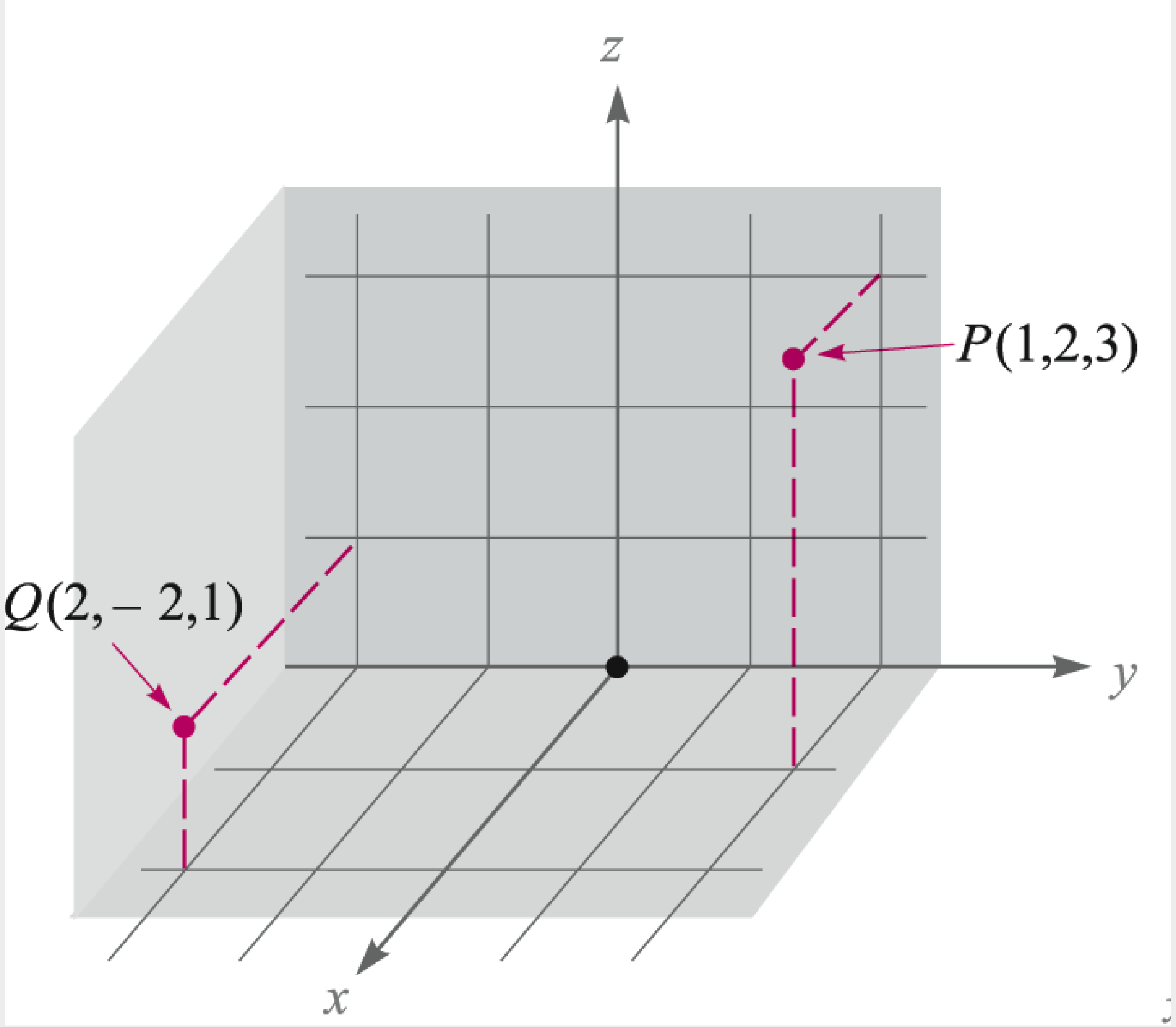
- ❑ Square
- ❑ Rectangle
- ❑ Cube

a constant value of x results a plan surface parallel to yz.

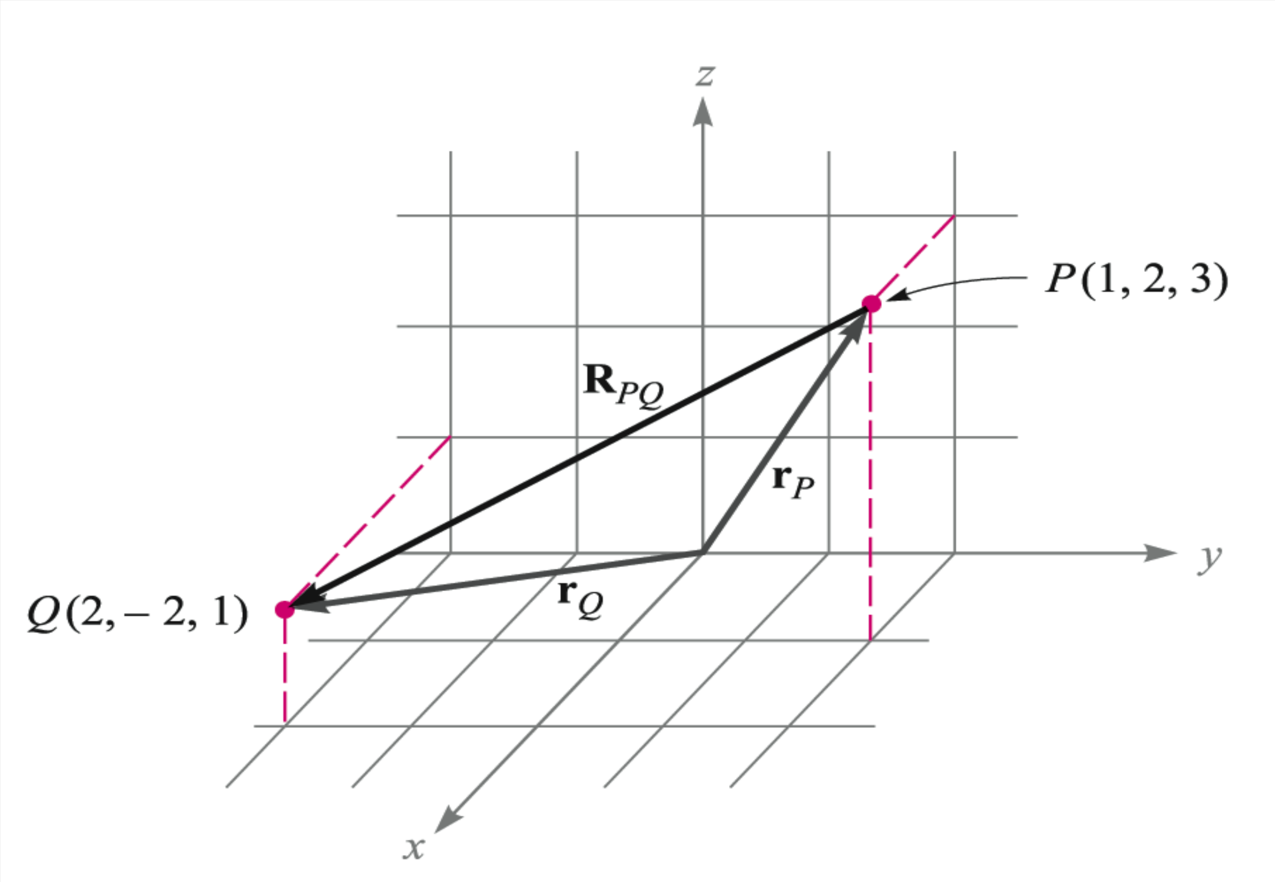
a constant value of y results a plan surface parallel to zx.

a constant value of z results a plan surface parallel to xy

Point Locations in Rectangular Coordinates



Vector Representation in Terms of Orthogonal Rectangular Components



$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (2-1)\mathbf{a}_x + (-2-2)\mathbf{a}_y + (1-3)\mathbf{a}_z$$
$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$

Vector Expressions in Rectangular Coordinates

General Vector, B:

$$\boxed{\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z}$$

Magnitude of B:

$$\boxed{|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}}$$

Unit Vector in the Direction of B:

$$\boxed{\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}}$$

Example

Specify the unit vector extending from the origin toward the point $G(2, -2, -1)$

$$\mathbf{G} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$$

$$|\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

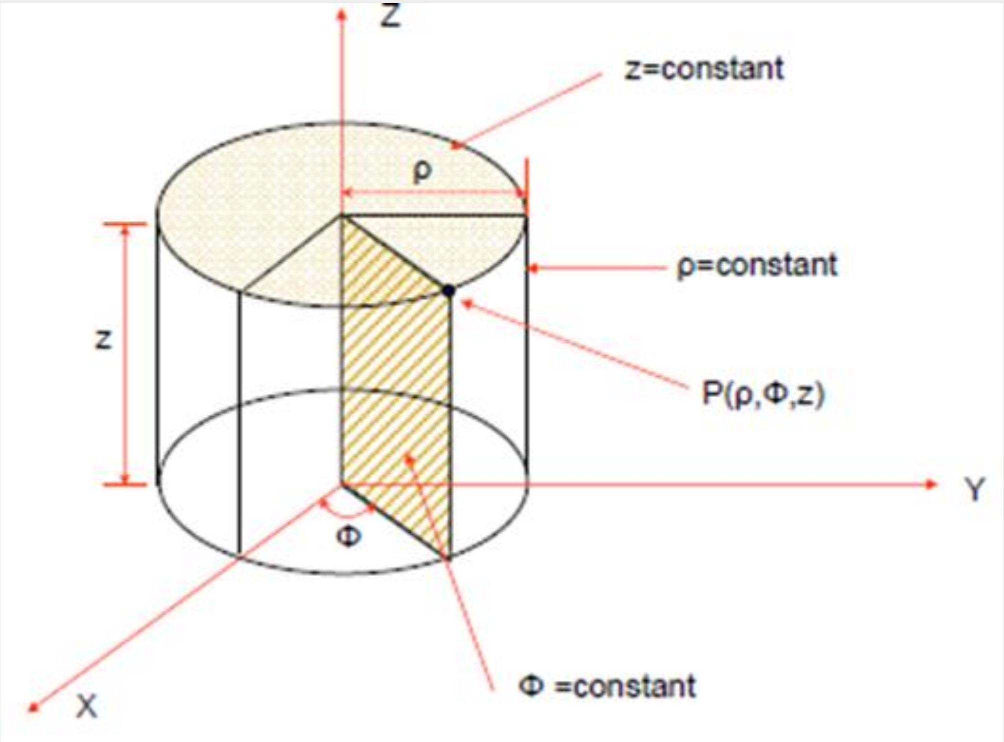
$$\mathbf{a}_G = \frac{\mathbf{G}}{|\mathbf{G}|} = \frac{2}{3}\mathbf{a}_x - \frac{2}{3}\mathbf{a}_y - \frac{1}{3}\mathbf{a}_z = \underline{0.667\mathbf{a}_x - 0.667\mathbf{a}_y - 0.333\mathbf{a}_z}$$

Cylindrical Coordinates System

- Any point in space is considered to be at the intersection of three mutually perpendicular surfaces:
  - A circular cylinder (ρ=constant)
  - A vertical plane (Φ=constant)
  - A horizontal plane (z=constant)
- Any point in space is represented by three coordinates $P(\rho,\Phi,z)$
  - ρ denotes the radius of an imaginary cylinder passing through P, or the radial distance from z axis to the point P.
  - Φ denotes azimuthal angle, measured from x axis to a vertical intersecting plane passing through P.
  - z denotes distance from xy-plane to a horizontal intersecting plane passing through P. It is the same as in rectangular coordinate system.

Cylindrical Coordinates System

$$0 < \rho < \infty$$
$$0 < \phi < 2\pi$$
$$-\infty < z < \infty$$



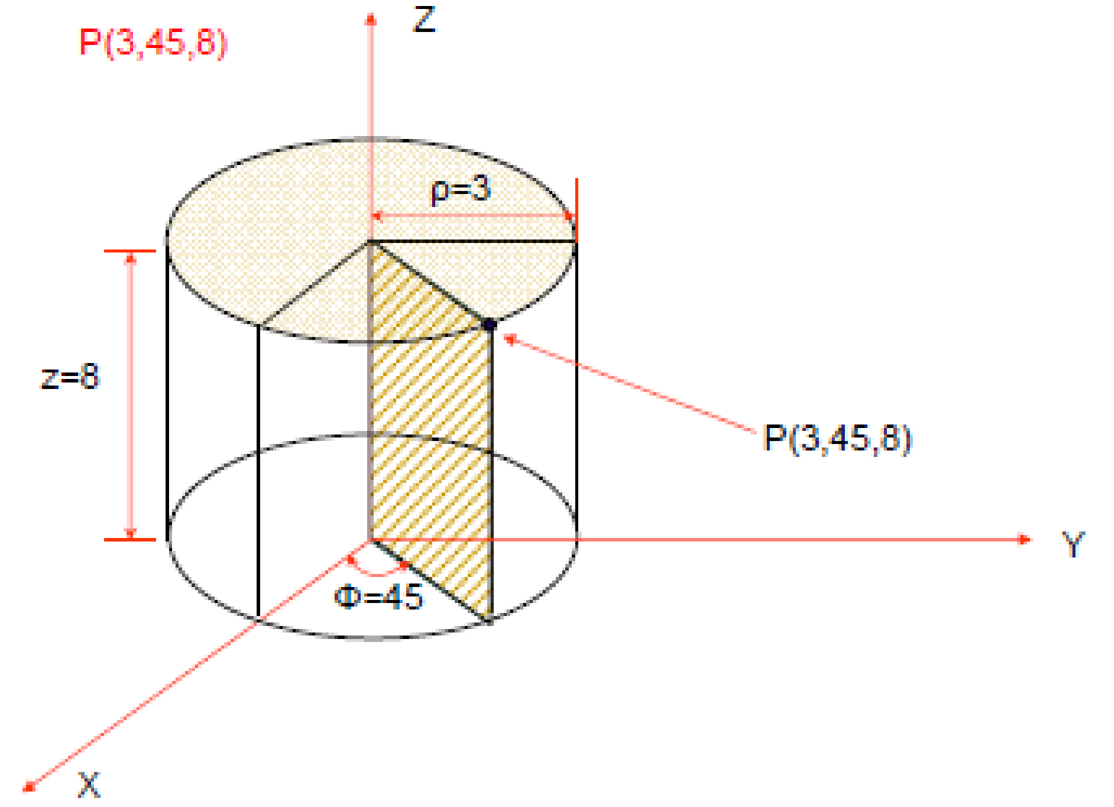
- ❑ Ring
- ❑ Disc
- ❑ Cylinder

a constant z value results a plan surface parallel to xy.

a constant ρ value results a cylindrical surface around z.

a constant φ value results a plan surface normal to both surfaces.

Cylindrical Coordinates System



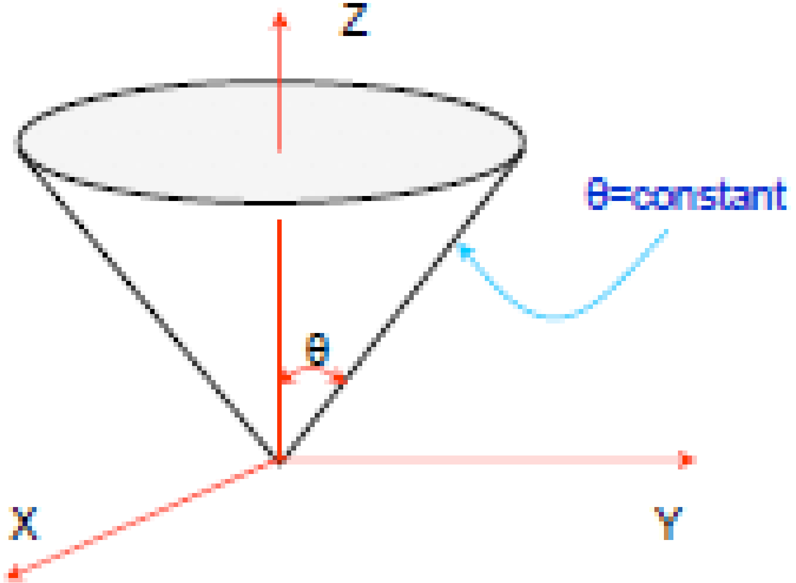
$$\vec{A} = A_\rho \hat{a}_\rho + A_\Phi \hat{a}_\Phi + A_z \hat{a}_z$$

- The magnitude of the vector is given by

$$\left|\vec{A}\right| = \sqrt{A_\rho^{\ 2} + A_\Phi^{\ 2} + A_z^{\ 2}}$$

Spherical Coordinates

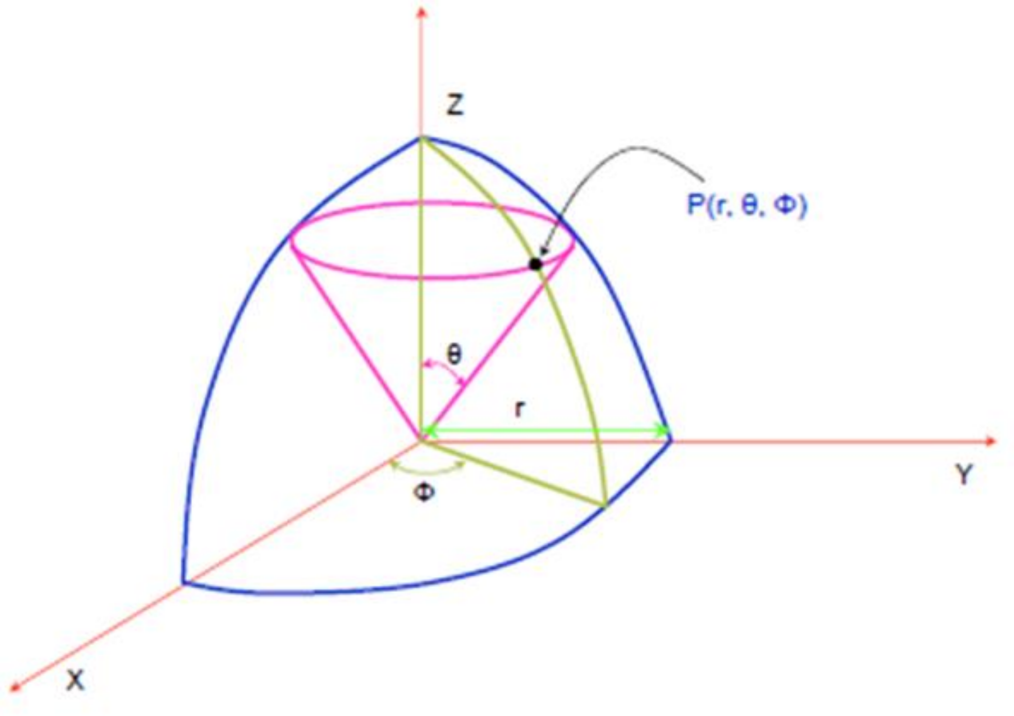
- Any point in space is represented as the intersection of three surfaces:
  - A sphere of radius r from the origin (r=constant)
  - A cone centered around the z axis (θ=constant)
  - A vertical plane (Φ=constant)
- Any point in spherical coordinate system is considered to be at the intersection of the above three planes.



- ❑ Cone
- ❑ Sphere
- ❑ Hemisphere

Spherical Coordinates

$$0 < r < \infty$$
$$0 < \theta < \pi$$
$$0 < \phi < 2\pi$$



a constant r value results a spherical surface.

a constant θ value results a cone surface around z with cone head at the origin.

a constant φ value results a plan surface normal to both surfaces.

Coordinate Transformation

Cylindrical to Cartesian

$$x = \rho \cos(\varphi)$$

$$y = \rho \sin(\varphi)$$

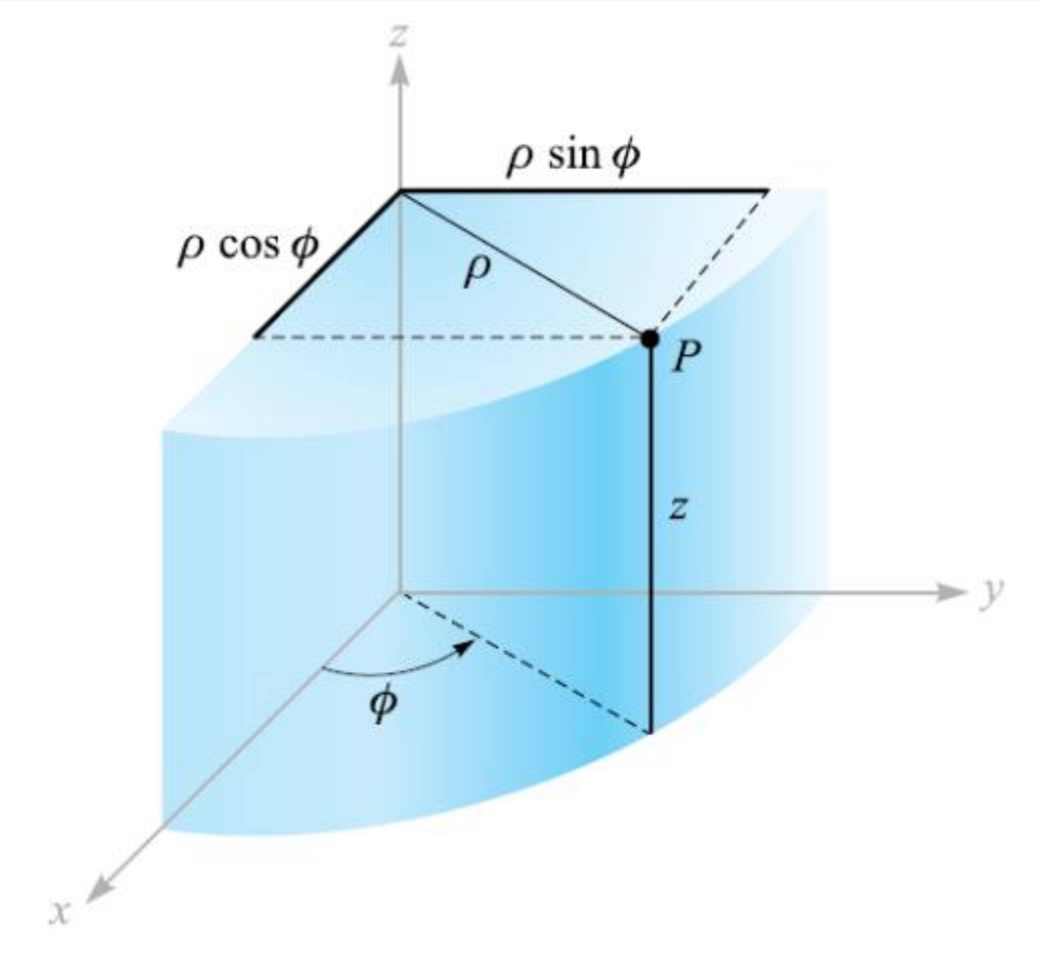
$$z = z$$

Cartesian to Cylindrical

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$



Coordinate Transformation

Spherical to Cartesian

x = r sin (θ) cos (φ)

y = r sin (θ) sin (φ)

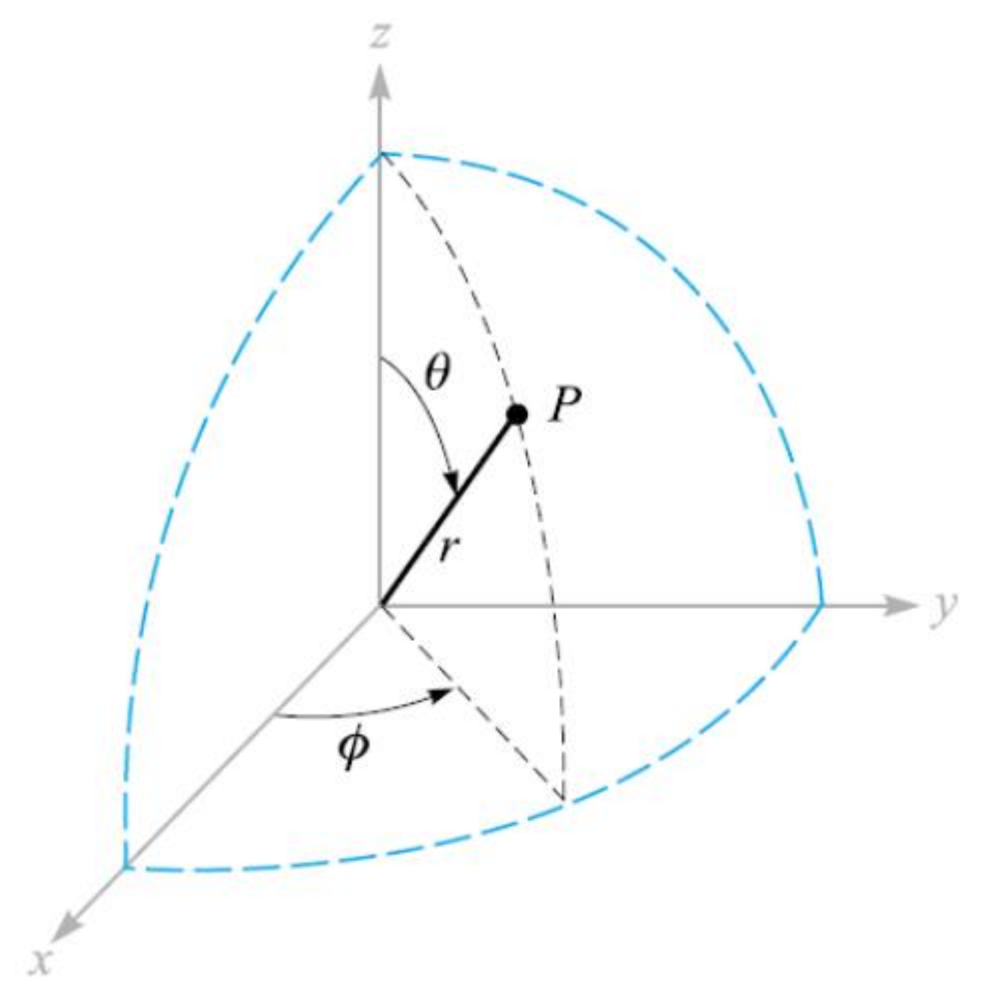
z = r cos (θ)

Cartesian to Spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

φ = tan−1(y/x)



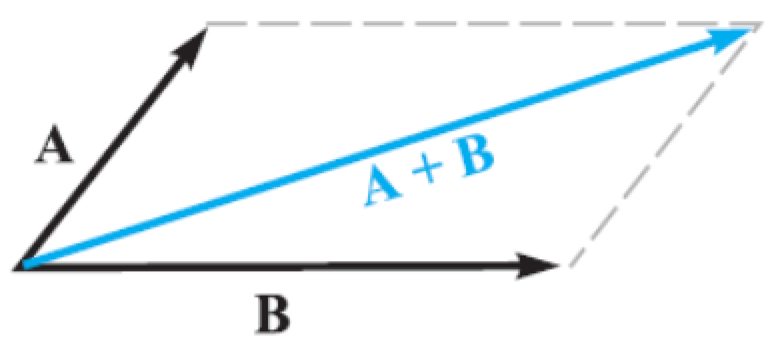
Vector Algebra

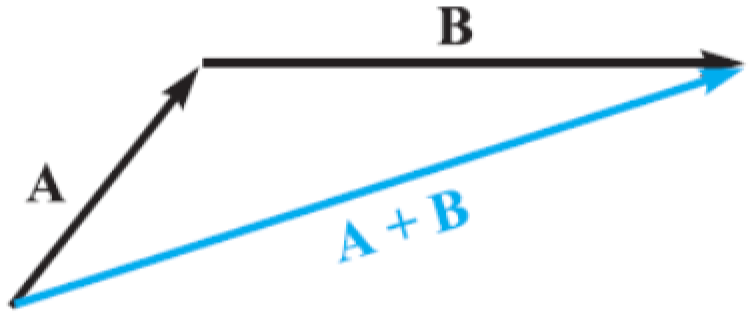
$$\overline{\mathbf{A}} = x_A\overline{\mathbf{a}}_\mathbf{x} + y_A\overline{\mathbf{a}}_\mathbf{y} + z_A\overline{\mathbf{a}}_\mathbf{z}$$

$$\overline{\mathbf{B}} = x_B\overline{\mathbf{a}}_\mathbf{x} + y_B\overline{\mathbf{a}}_\mathbf{y} + z_B\overline{\mathbf{a}}_\mathbf{z}$$

Addition:

$$\overline{\mathbf{A}} + \overline{\mathbf{B}} = \overline{\mathbf{B}} + \overline{\mathbf{A}} = (x_A + x_B)\overline{\mathbf{a}}_\mathbf{x} + (y_A + y_B)\overline{\mathbf{a}}_\mathbf{y} + (z_A + z_B)\overline{\mathbf{a}}_\mathbf{z}$$





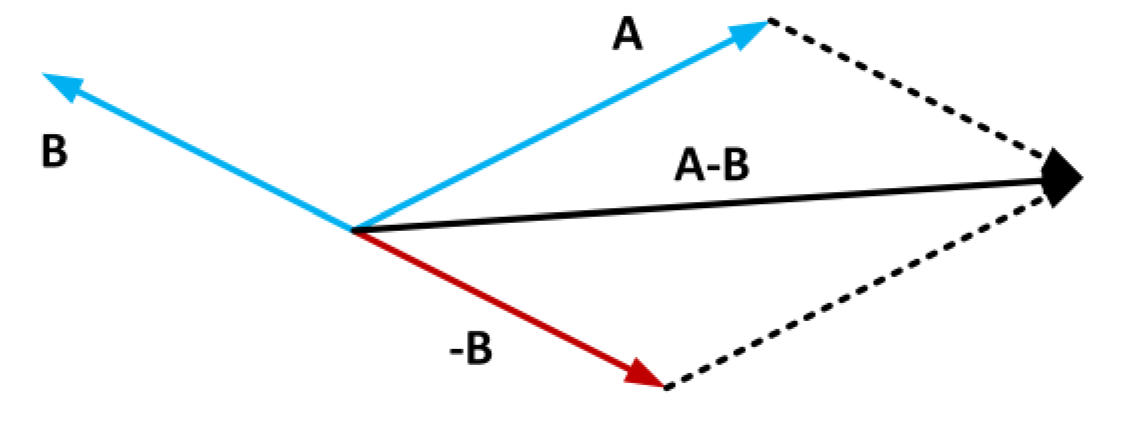
Vector Algebra

$$\bar{\mathbf{A}} = x_A\bar{\mathbf{a}}_\mathbf{x} + y_A\bar{\mathbf{a}}_\mathbf{y} + z_A\bar{\mathbf{a}}_\mathbf{z}$$

$$\bar{\mathbf{B}} = x_B\bar{\mathbf{a}}_\mathbf{x} + y_B\bar{\mathbf{a}}_\mathbf{y} + z_B\bar{\mathbf{a}}_\mathbf{z}$$

Substraction:

$$\bar{\mathbf{A}} - \bar{\mathbf{B}} = -(\bar{\mathbf{B}} - \bar{\mathbf{A}}) = (x_A - x_B)\bar{\mathbf{a}}_\mathbf{x} + (y_A - y_B)\bar{\mathbf{a}}_\mathbf{y} + (z_A - z_B)\bar{\mathbf{a}}_\mathbf{z}$$



Dot Product

Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them,

$$\boxed{\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos \theta_{AB}}$$

Commutative Law: $\boxed{\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}}$

Operational Use of the Dot Product

Given
$$\begin{cases} \mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z \\ \mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z \end{cases}$$

Find
$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$$

Scalar

where we have used:
$$\begin{cases} \mathbf{a}_x\cdot\mathbf{a}_y = \mathbf{a}_y\cdot\mathbf{a}_z = \mathbf{a}_x\cdot\mathbf{a}_z = 0 \\ \mathbf{a}_x\cdot\mathbf{a}_x = \mathbf{a}_y\cdot\mathbf{a}_y = \mathbf{a}_z\cdot\mathbf{a}_z = 1 \end{cases}$$

Note also:
$$\boxed{\mathbf{A}\cdot\mathbf{A} = A^2 = |\mathbf{A}|^2}$$

Operational Use of the Dot Product

Cylindrical Coordinate Systems

$$\hat{a}_\rho \cdot \hat{a}_\rho = \hat{a}_\Phi \cdot \hat{a}_\Phi = \hat{a}_z \cdot \hat{a}_z = 1$$

$$\hat{a}_\rho \cdot \hat{a}_\Phi = \hat{a}_\Phi \cdot \hat{a}_z = \hat{a}_z \cdot \hat{a}_\rho = 0$$

Spherical Coordinate Systems

$$\hat{a}_r \cdot \hat{a}_r = \hat{a}_\theta \cdot \hat{a}_\theta = \hat{a}_\phi \cdot \hat{a}_\phi = 1$$

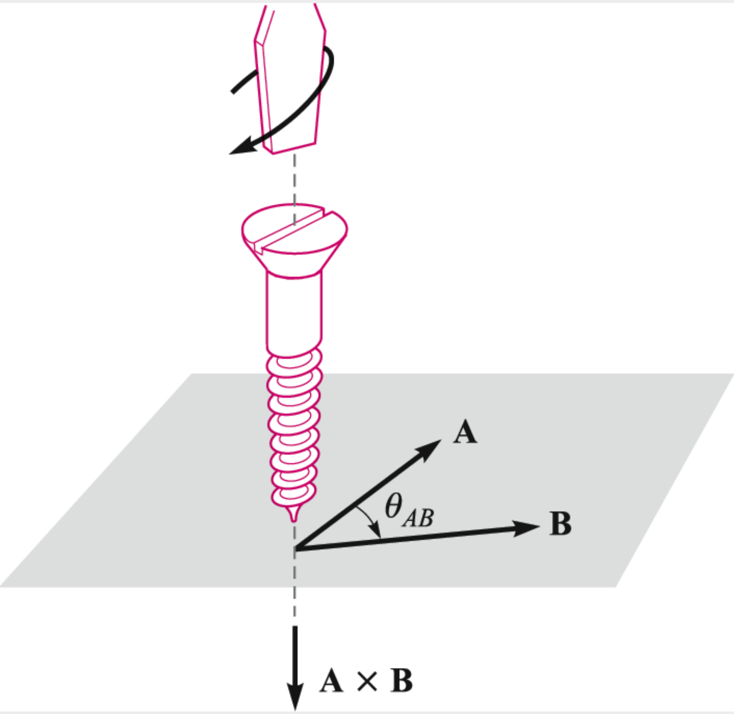
$$\hat{a}_r \cdot \hat{a}_\theta = \hat{a}_\theta \cdot \hat{a}_\phi = \hat{a}_\phi \cdot \hat{a}_r = 0$$

Cross Product

The cross product $\mathbf{A} \times \mathbf{B}$ is a vector; the magnitude of $\mathbf{A} \times \mathbf{B}$ is equal to the product of the magnitudes of $\mathbf{A}$, $\mathbf{B}$, and the sine of the smaller angle between $\mathbf{A}$ and $\mathbf{B}$; the direction of $\mathbf{A} \times \mathbf{B}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ and is along that one of the two possible perpendiculars which is in the direction of advance of a right-handed screw as $\mathbf{A}$ is turned into $\mathbf{B}$.

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\, |\mathbf{B}| \sin\theta_{AB}}$$

Vector



Operational Definition of the Cross Product in Rectangular Coordinates

Begin with:

$$
\begin{aligned}
\mathbf{A} \times \mathbf{B} = {} & A_x B_x \mathbf{a}_x \times \mathbf{a}_x + A_x B_y \mathbf{a}_x \times \mathbf{a}_y + A_x B_z \mathbf{a}_x \times \mathbf{a}_z \\
& + A_y B_x \mathbf{a}_y \times \mathbf{a}_x + A_y B_y \mathbf{a}_y \times \mathbf{a}_y + A_y B_z \mathbf{a}_y \times \mathbf{a}_z \\
& + A_z B_x \mathbf{a}_z \times \mathbf{a}_x + A_z B_y \mathbf{a}_z \times \mathbf{a}_y + A_z B_z \mathbf{a}_z \times \mathbf{a}_z
\end{aligned}
$$

$$
\text{where} \quad \begin{cases} \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z \\ \mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x \\ \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y \end{cases}
$$

Therefore:

$$
\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\mathbf{a}_x + (A_z B_x - A_x B_z)\mathbf{a}_y + (A_x B_y - A_y B_x)\mathbf{a}_z
$$

Or…

$$
\boxed{\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}
$$

Operational Definition of the Cross Product

Cylindrical Coordinate Systems

$$\hat{a}_\rho \times \hat{a}_\Phi = \hat{a}_z$$
$$\hat{a}_\Phi \times \hat{a}_z = \hat{a}_\rho$$
$$\hat{a}_z \times \hat{a}_\rho = \hat{a}_\Phi$$

Spherical Coordinate Systems

$$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\phi$$
$$\hat{a}_\theta \times \hat{a}_\phi = \hat{a}_r$$
$$\hat{a}_\phi \times \hat{a}_r = \hat{a}_\theta$$

Thanks